Catalog

DEPT OF MATHS FUNAAB MTS 101 CAT 23/03/16 ANSWER ALL 1 HOUR
NAME MATRIC NO DEPT COLLEGE

1. (a) Let $P = \{3, 5, 7\}, Q = \{2, 4, 6\}, R = \{1, 9\}$ be subsets of $X = \{x \in \mathbb{N} : 1 \leq x \leq 10\}$. Compute: (i) 2^P (ii) $(P \triangle Q \triangle R)^c$ (iii) $P \times Q \times R$.

   (b) Prove by induction that $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$ and deduce that $\sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)} = \frac{1}{2}$.

   (c) (i) Show that $\log^x_{(a/b)} = \frac{\log^x_a \log^x_b}{\log^x_b - \log^x_a}$ (ii) Find x in the form $\log_p^{(q+\sqrt{r})}$ given that $10^x + 10^{-x} = 4$. (iii) If $a * b = (a +_3 b) - (a \times_8 b)$ where $+_3$ and $\times_8$ are respectively addition modulo 3 and multiplication modulo 8 and $a, b \in \mathbb{Z}$, compute $-20 * (10 * -30)$ leaving your answer in modulo 5.

2. (a) If α and β are the roots of the equation ax^2+bx+c, show that $(1-\alpha^3)(1-\beta^3) = \frac{1}{a^3}(a^3+b^3+c^3-3abc)$.

(b) Factorize completely the polynomial $p(x) = x^4-6x^2-7x-6$. Hence or otherwise, obtain its real roots.

DEPT OF MATHS FUNAAB MTS 101 CAT 21/06/17 ANSWER ALL 1 HOUR
NAME MATRIC NO DEPT COLLEGE

1. (a) Let A, B and C be nonempty subsets of the reference set X.
   - i. Show that $(A - B) - C = (A - C) - (B - C) = A \cap (B \cup C)'$.
   - ii. If $X = \mathbb{Z}$ and $A = \{2n : n \in \mathbb{Z}\}, B = \{3n : n \in \mathbb{Z}\}$ and $C = \{6n : n \in \mathbb{Z}\}$, find an expression connecting A, B and C.
   - iii. If $\circ$ is a binary operation on X defined by $A \circ B = A \cup B$. Show that $\circ$ is commutative and associative.

   (b)
   - i. Show that $\frac{2+\sqrt{3}}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{2(2+\sqrt{3})} = 5$.
   - ii. Solve for x given that $\log_4^x \times \log_8^{x^4} = 32$.

2. (a) Let $f(x) = 6x^3 - 7x^2 - 7x + 6$ be a given polynomial. Factorize $f(x)$ completely and hence state the zeros of $f(x)$.

(b) i. Find S_n, the sum of the first n terms of the series $\frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \cdots$ and hence find S_∞, the sum to infinity of the series.

ii. Find the ranges of values of x for which $\frac{2x-1}{x+1} \leq 1$.

DEPT OF MATHS FUNAAB

2019 MTS 101 OPEN CAT1 TIME ALLOWED: 2 HOURS

INSTRUCTION: Answer All Questions inside well Stapled Plain Sheets

Submit Through Your Class Rep on Tuesday, July 16, 2019 at Exactly 12pm

INVIGILATORS: Honesty and Sincerity

1. (a) Let A, B and C be nonempty subsets of a universal set X. Show that:

   i. $(X - A) \cap (X - B) = X - (A \cup B)$,

   ii. $(X - A) \cup (X - B) = X - (A \cap B)$,

   iii. $(A - B) \cap (A - C) = A - (B \cup C)$,

   iv. $\mid A \cup B \mid = \mid A \mid + \mid B \mid - \mid A \cap B \mid$,

   v. $\mid A\cup B\cup C \mid = \mid A \mid + \mid B \mid + \mid C \mid - \mid A\cap B \mid - \mid B\cap C \mid - \mid C\cap A \mid + \mid A\cap B\cap C \mid$.

   (b) Given that $X = \{x \in \mathbb{Z}^+ : 1 \leq x \leq 100\}$, $A = \{x \in X : \text{ x is a multiple of 2}\}$ and $B = \{x \in X : \text{ x is a multiple of 3}\}$, show that:

   i. $\mid A \mid = 50$,

   ii. $\mid B \mid = 33$,

   iii. $\mid A \cap B \mid = 16$,

   iv. $\mid A \cup B \mid = 67$.

   (c) In the year 2018, 112 PHS students sat for MTS 101 examination. Question 1 was attempted by 50 students, Question 2 by 66 students and Question 3 by 38 students. 32 students attempted both Questions 1 and 2,22 students attempted Questions 2 and 3 and 20 students Questions 1 and 3. If only 8 students attempted all the Questions 1,2 and 3, how many students attempted none of these three Questions ?

2. (a) i. Show that

$$\sqrt{x^{2/3}y^{1/2}} \times x^{2/3}y^{3/2} \times \sqrt[3]{x^{3/4}y^{-1/2}} = \frac{y^{19/12}}{x^{1/12}}.$$

ii. Given that $x = \sqrt[3]{p+q} + \sqrt[3]{p-q}$ and $p^2 - q^2 = r^3$, show that

$$x^3 - 3rx - 2p = 0.$$

(b) i. Without using tables and calculator, show that

$$\frac{\log\sqrt{27} + \log\sqrt{8} - \log\sqrt{125}}{\log 6 - \log 5} = \frac{3}{2}.$$

ii. Show that

$$\frac{1}{\log_x(xyz)} + \frac{1}{\log_y(xyz)} + \frac{1}{\log_z(xyz)} = 1.$$

iii. Solve the equation

$$5^{2x} - 5^{x+1} + 4 = 0.$$

(c) i. Show that

$$\left[\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right]^2 = 49 - 20\sqrt{6}.$$

ii. Solve the equation

$$\sqrt{3x+4} - \sqrt{x+2} = \sqrt{x-3}.$$

3. (a) i. Expand $(x+y)^5$.

ii. By synthetic long division, divide $x^3 + 2x^2y + 2xy^2 + y^3$ by $(x+y)$.

iii. Show that $f(x,y) = x^3 + 2x^2y + 2xy^2 + y^3$ is a symmetrical function of order 3 and show that

$$f(x,y) = (x+y)(x^2 + xy + y^2).$$

iv. Using all your answers in (i)-(iii) or otherwise, show that

$$(x+y)^5 - x^5 - y^5 = 5xy(x+y)(x^2 + xy + y^2).$$

(b) Resolve into partial fractions the rational function $r(x)$ given by

$$r(x) = \frac{2x^3 + 2x^2 + 2}{x^4 + 2x^3 + 2x^2 + 2x + 1}.$$