MTS 104 LECTURE NOTE: MODULE 1

Lecturer: Mr. Samuel D. J.

TOPIC: Displacement, Speed, Velocity and Acceleration of a Particle

Vectors

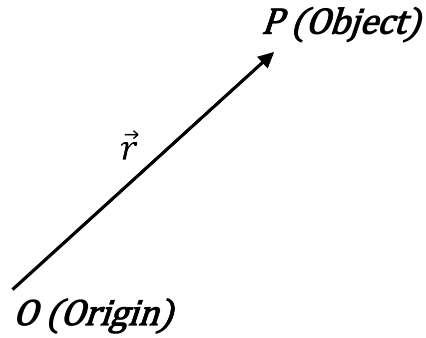
A vector is a quantity that has both magnitude and direction. It is typically represented symbolically by an arrow in the proper direction, whose length is proportional to the magnitude of the vector. Vectors are essential to physics and engineering. A lot of fundamental physical quantities are vectors, including displacement, velocity, force, and electric and magnetic vector fields. In the language of mathematics, physical vector quantities are represented by mathematical objects called **vectors.** We can add or subtract two vectors, and we can multiply a vector by a scalar or by another vector, but we cannot divide by a vector. The operation of division by a vector is not defined.

The equations of mechanics are usually written in terms of Cartesian coordinates. At a certain time t, the position of a particle may be specified by giving its coordinates $x(t)$, $y(t)$, and $z(t)$ in a particular Cartesian frame of reference. However, another observer of the same particle might choose a differently oriented set of mutually perpendicular axes, say, x', y', and z'. The motion of the particle is then described by the first observer in terms of the rate of change of $x(t)$, $y(t)$, and $z(t)$, while the second observer would discuss the rates of change of $x'(t)$, $y'(t)$, and $z'(t)$. We can say from this analogy that both observers see the same particle executing the same motion and obeying the same laws, but they use different equations to describe the situation.

Position and Displacement Vectors

In this section we are going to look at what position vector is, what displacement vector is, and what the difference between the position vector and displacement vector is.

Position Vector: the position vector is used to specify the position of a certain body. Knowing the position of a body is vital when it comes to describing the motion of that body. The position vector of an object is measured from the origin, in general. Suppose an object is placed in the space as shown below



Mathematically, position vector can be written as

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$$

Where,

$\hat{\imath}$ = unit vector along *x*-direction

$\hat{\jmath}$ = unit vector along *y*-direction

$\hat{k}$ = unit vector along *z*-direction

So if an object is at a certain point *P* (say) at a certain time, its position vector is given as described above.

Displacement Vector: the change in the position vector of an object is known as the displacement vector. Suppose an object is at point *A* at time = *0* and at point *B* at time = *t*. The position vectors of the object at point *A* and at point *B* are given as:

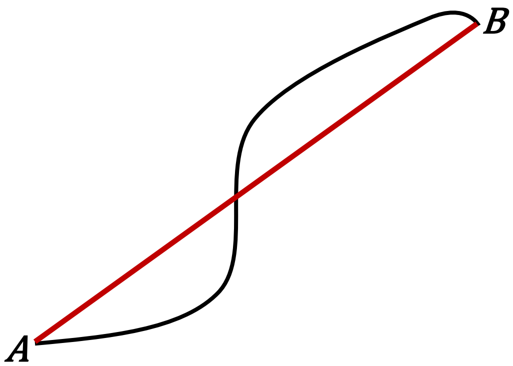
Position vector at point *A* = $\widehat{r_A} = 5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

Position vector at point *B* = $\widehat{r_B} = 2\hat{\imath} + 2\hat{\jmath} + 1\hat{k}$.

Now, the displacement vector of the object from time interval *0* to *t* will be:

$\widehat{r_B} - \widehat{r_A} = -3\hat{\imath} - \hat{\jmath} - 3\hat{k}$.

The displacement of an object can also be defined as the vector distance between the initial point and the final point. Suppose an object travels from point *A* to point *B* in the path shown in the black curve.



The displacement of the particle would be the vector line *AB*, headed in the direction *A* to *B*. The direction of the displacement vector is always headed from the initial point to the final point.

Example 1

Brownian motion is a chaotic random motion of particles suspended in a fluid, resulting from collisions with the molecules of the fluid. This motion is three-dimensional. The displacements in numerical order of a particle undergoing Brownian motion could look like the following, in micrometers as delineated in the figure below.

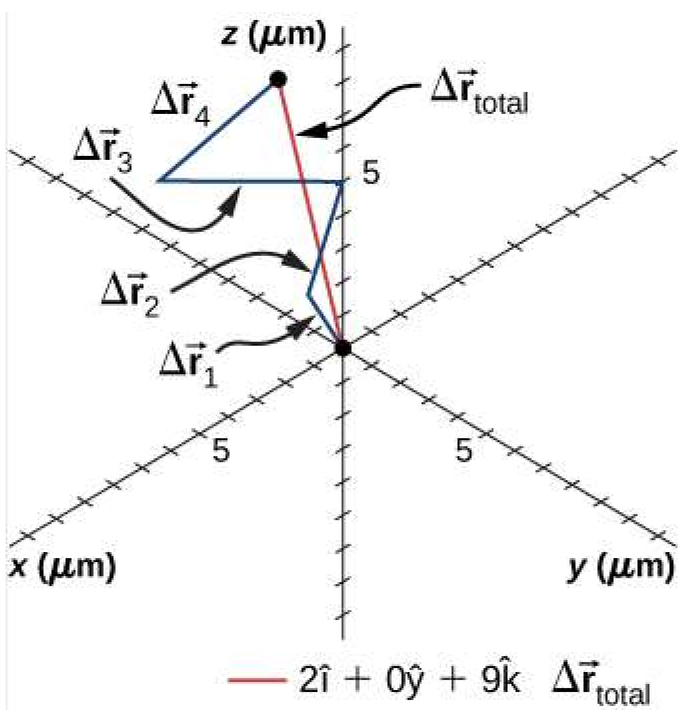
$$\Delta\vec{r}_1 = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$$

$$\Delta\vec{r}_2 = -\hat{\imath} + 3\hat{k}$$

$$\Delta\vec{r}_3 = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$$

$$\Delta\vec{r}_4 = -3\hat{\imath} + \hat{\jmath} + 2\hat{k}$$

What is the total displacement of the particle from the origin?



Solution

We form the sum of the displacements and add them as vectors:

$$\Delta\vec{r}_{Total} = \sum \Delta\vec{r}_i = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 + \Delta\vec{r}_4$$

$$= \left(2\hat{\imath} + \hat{\jmath} + 3\hat{k}\right) + \left(-\hat{\imath} + 3\hat{k}\right) + \left(4\hat{\imath} - 2\hat{\jmath} + \hat{k}\right) + \left(-3\hat{\imath} + \hat{\jmath} + 2\hat{k}\right)$$

$$= (2 - 1 + 4 - 3)\hat{\imath} + (1 + 0 - 2 + 1)\hat{\jmath} + (3 + 3 + 1 + 2)\hat{k}$$

$$= 2\hat{\imath} + 0\hat{\jmath} + 9\hat{k}\boldsymbol{\mu m}.$$

To complete the solution, we express the displacement as a magnitude and direction.

$$|\Delta\vec{r}_{Total}| = \sqrt{(2)^2 + (0)^2 + (9)^2} = 9.2\boldsymbol{\mu m}, \qquad \theta = tan^{-1}\left(\frac{z}{x}\right) = tan^{-1}\left(\frac{9}{2}\right) = 77^o,$$

with respect to the *x* axis in the *xz*-plane.

Velocity Vector

A velocity vector represents the rate of change of the position of an object and it is also known as instantaneous velocity. ***The magnitude of a velocity vector gives the speed of an object*** while the vector direction gives its direction. Velocity vectors can be added or subtracted according to the principles of vector addition. Considering a two and three dimensions, the instantaneous velocity vector is

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} = \frac{d\vec{r}}{dt},$$

Since $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$,

we have,

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}\right)$$

$$= \frac{dx(t)}{dt}\hat{\imath} + \frac{dy(t)}{dt}\hat{\jmath} + \frac{dz(t)}{dt}\hat{k},$$

$$\Longrightarrow \vec{v}(t) = v_x(t)\hat{\imath} + v_y(t)\hat{\jmath} + v_z(t)\hat{k},$$

where,

$$v_x(t) = \frac{dx(t)}{dt}, v_y(t) = \frac{dy(t)}{dt}, v_z(t) = \frac{dz(t)}{dt}.$$

The magnitude of a velocity vector is the speed of an object/a particle and it can be written as

$$|\vec{v}(t)| = \sqrt{(v_x(t))^2 + \left(v_y(t)\right)^2 + (v_z(t))^2}.$$

To find the average velocity, we make use of the formula below

$$\vec{v}_{avg} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}.$$

Example 2

If the position function of a particle is $\vec{r}(t) = 2t^2\hat{\imath} + (2 + 3t)\hat{\jmath} + 5t\hat{k}$.

(a) What is the instantaneous velocity and speed at $t = 2s$?

(b) What is the average velocity between 1 s and 3 s.

Solution

(a) The velocity vector is calculated as follows

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(2t^2\hat{\imath} + (2 + 3t)\hat{\jmath} + 5t\hat{k}\right)$$

$$= 4t\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\boldsymbol{m/s},$$

when $t = 2s$, the instantaneous velocity is

$$\vec{v}(2) = 4(2)\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\boldsymbol{m/s},$$

$$= 8\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\boldsymbol{m/s}.$$

Hence, the instantaneous velocity is $8\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\boldsymbol{m/s}$.

For speed,

Speed is calculated as $|\vec{v}(t)| = \sqrt{(v_x(t))^2 + \left(v_y(t)\right)^2 + (v_z(t))^2}$,

$$\Longrightarrow |\vec{v}(t)| = \sqrt{(8)^2 + (3)^2 + (5)^2}$$

$$= \sqrt{64 + 9 + 25} = 9.9\boldsymbol{m/s}.$$

Therefore speed is $9.9\boldsymbol{m/s}$.

(b) The average velocity is

$$\vec{v}_{avg} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1},$$

where $t_1 = 1s$ and $t_2 = 3s$.

$$\vec{r}(t_1) = \vec{r}(1) = 2(1)^2\hat{\imath} + (2 + 3(1))\hat{\jmath} + 5(1)\hat{k}$$

$$= 2\hat{\imath} + 5\hat{\jmath} + 5\hat{k},$$

$$\vec{r}(t_2) = \vec{r}(3) = 2(3)^2\hat{\imath} + (2 + 3(3))\hat{\jmath} + 5(3)\hat{k}$$

$$= 18\hat{\imath} + 11\hat{\jmath} + 15\hat{k}.$$

$$\vec{v}_{avg} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1} = \frac{\left(18\hat{\imath} + 11\hat{\jmath} + 15\hat{k}\right) - \left(2\hat{\imath} + 5\hat{\jmath} + 5\hat{k}\right)}{3 - 1}$$

$$= \frac{16\hat{\imath} + 6\hat{\jmath} + 10\hat{k}}{2} = 8\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\boldsymbol{m/s}.$$

Acceleration Vector

In addition to obtaining the displacement and velocity vectors of an object in motion, we often want to know its acceleration vector at any point in time along its trajectory. This acceleration vector is the instantaneous acceleration and it can be obtained from the derivative with respect to time of the velocity function. Taking the derivative with respect to time $\vec{v}(t)$, we find

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t} = \frac{d\vec{v}(t)}{dt},$$

Since $\vec{v}(t) = v_x(t)\hat{\imath} + v_y(t)\hat{\jmath} + v_z(t)\hat{k}$,

the acceleration in terms of components is now

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x(t)\hat{\imath} + v_y(t)\hat{\jmath} + v_z(t)\hat{k}\right)$$

$$= \frac{dv_x(t)}{dt}\hat{\imath} + \frac{dv_y(t)}{dt}\hat{\jmath} + \frac{dv_z(t)}{dt}\hat{k},$$

$$\vec{a}(t) = a_x(t)\hat{\imath} + a_y(t)\hat{\jmath} + a_z(t)\hat{k},$$

Also, since the velocity is the derivative of the position function, we can write the acceleration in terms of the second derivative of the position function:

$$\vec{a}(t) = \frac{d^2x(t)}{dt^2} + \frac{d^2y(t)}{dt^2} + \frac{d^2z(t)}{dt^2}.$$

The magnitude of the acceleration vector is $|\vec{a}(t)| = \sqrt{(a_x(t))^2 + \left(a_y(t)\right)^2 + (a_z(t))^2}$.

Example 3

A particle has a velocity of $\vec{v}(t) = 5t\hat{\imath} + t^2\hat{\jmath} - 2t^3\hat{k}\boldsymbol{m/s}$.

(a) What the acceleration function?

(b) What is the acceleration vector at *t* = 2.0 *s*? Find its magnitude

Solution

(a) We take the first derivative with respect to time of the velocity function to find the acceleration. The derivative is taken component by component:

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(5t\hat{\imath} + t^2\hat{\jmath} - 2t^3\hat{k}\right)$$

$$= 5\hat{\imath} + 2t\hat{\jmath} - 6t^2\hat{k}.$$

Therefore, the acceleration function $\vec{a}(t) = 5\hat{\imath} + 2t\hat{\jmath} - 6t^2\hat{k}$.

(b) To find acceleration vector at *t* = 2.0 *s*, we are to evaluate $\vec{a}(2)$

$$\Longrightarrow \vec{a}(2) = 5\hat{\imath} + 2(2)\hat{\jmath} - 6(2)^2\hat{k}$$

$$= 5\hat{\imath} + 4\hat{\jmath} - 24\hat{k}.$$

Therefore, the acceleration vector at t = 2.0 *s* is $\vec{a}(2) = 5\hat{\imath} + 4\hat{\jmath} - 24\hat{k}$ which gives us the direction in unit vector notation.

The magnitude of the acceleration vector is $|\vec{a}(t)| = \sqrt{(a_x(t))^2 + \left(a_y(t)\right)^2 + (a_z(t))^2}$

$$\Rightarrow |\vec{a}(2)| = \sqrt{(5)^2 + (4)^2 + (-24)^2}$$

$$= \sqrt{25 + 16 + 576} = 24.8\boldsymbol{m/s^2}.$$

MTS 104 LECTURE NOTE: MODULE 2

Lecturer: Mr. Samuel D. J.

TOPIC: Newton's Laws of Motion

Newton's Laws of Motion

In this section, we are going to study Newton's Laws of Motion. But before we proceed into this, we need to remind ourselves of what ***force*** is. A force is a push or pull. An object at rest needs a force to get it moving; a moving object needs a force to change its velocity. Furthermore, force is a vector, having both magnitude and direction. The magnitude of a force can be measured using a spring scale. Force may be a contact force or a field force. Contact forces result from physical contact between two objects while field forces act between disconnected objects. Types of force include gravitational force, tension force, normal force, friction force spring force etc.

Newton's First Law of Motion

Newton's first law states that,

> "**if a body is at rest or moving at a constant speed in a straight line, it will remain at rest or keep moving in a straight line at constant speed unless it is acted upon by a force. This postulate is also known as the law of inertia**."

There are many applications of Newton's first law of motion. Consider some of your experiences in an automobile. Have you ever observed the behavior of coffee in a coffee cup filled to the rim while starting a car from rest or while bringing a car to rest from a state of motion? Coffee "keeps on doing what it is doing." When you accelerate a car from rest, the road provides an unbalanced force on the spinning wheels to push the car forward; yet the coffee (that was at rest) wants to stay at rest. While the car accelerates forward, the coffee remains in the same position; subsequently, the car accelerates out from under the coffee and the coffee spills in your lap. On the other hand, when braking from a state of motion the coffee continues forward with the same speed and in the same direction, ultimately hitting the windshield or the dash. Coffee in motion stays in motion.

Example 1

A $200kg$ spaceship travels in the vacuum of space at a constant speed of $200m/s$. Ignoring any gravitational forces, what is the net force on the spaceship?

Solution

In a vacuum, there is no friction due to air resistance. Newton's first law states that an object in motion stays in motion unless acted upon by a net force. Thus the spaceship will travel at the constant speed (zero acceleration) of $200m/s$ indefinitely and the net force on the spaceship must be zero. This can also be shown mathematically:

$F = ma$

$F = (200kg)(0m/s^2) = 0N.$

Therefore the net force on the spaceship is $0N$.

Example 2

You are traveling on an airplane at constant speed of $2000mph$. Your friend is traveling in his car at a constant speed of $20mph$. Who experiences a larger acceleration?

A. *You*
B. *Your friend*
C. *Neither you nor your friend*
D. *Cannot be determined; we must know the force due to friction*

Solution

The answer is **option *C* (***Neither you nor your friend* ***).***

The reason is that both you and your friend are traveling at a constant speed, the acceleration of you and your friend is zero. Thus, neither you nor your friend experiences any acceleration.

Also note that when acceleration is zero, so is the net force.

Example 3

A van is driving around with a bowling ball in the back, free to roll around. The van approaches a red light and must decelerate to come to a complete stop. As the van is slowing down, which direction is the bowling ball rolling?

A. *The bowling ball does not move*
B. *To the right side of the van*
C. *To the front of the van*
D. *To the left side of the van*
E. *To the back of the van*

Solution

The answer is **option *C***

According to Newton's First Law of Motion, an object that is in motion will stay in motion unless acted on by another force. When the van slows down, the ball will want to continue moving forward, and the friction between it and the floor of the van is not strong enough to keep the ball back.

Newton's Second Law of Motion

Newton’s second law is the relation between acceleration and net force. Newton’s second law is quantitative and is used extensively to calculate what happens in situations involving a force. It states that,

> “**The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass**.”

Mathematically,

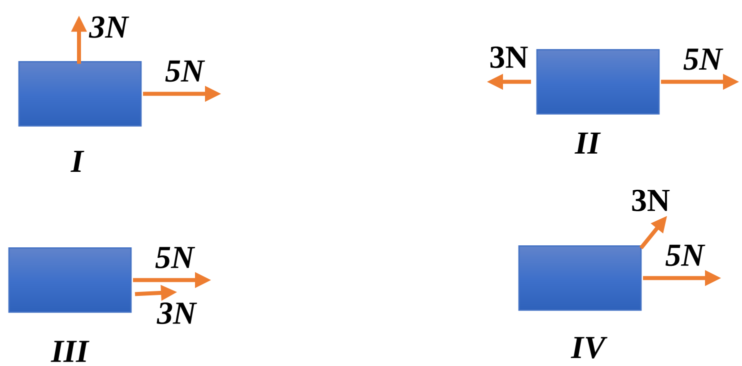
$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{\sum \vec{F}}{m},$$

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}.$$

Example 4

The figures below show overhead views of four situations in which two forces accelerate the same block across a frictionless surface. Rank the situations below according to the magnitude of the horizontal acceleration of the block, greatest first.

A. *I, IV, III, II*
B. *III, IV, I, II*
C. *II, III, I, II*
D. *III, IV, II, I*



Solution

Option B is the answer to the given problem.

Example 5

A man of mass $50kg$ on the top floor of a skyscraper steps into an elevator. What is the man's weight as the elevator accelerates downward at a rate of $1.5m/s^2$. (where $g = 10m/s^2$)

Solution

Using Newton's second law to solve this problem, we have

$F = ma,$

when the elevator is not moving, we obtain,

$F = mg = (50kg)(10m/s^2),$

$\quad\quad = 500N.$

However, when the elevator is accelerating downward, the man appears to be lighter since the elevator is negating some of the force from gravity. Written as an equation, we have:

$F = m(g - a),$

where g is acceleration due to gravity and a is the acceleration of the elevator.

$\Longrightarrow F = (50kg)(10m/s^2 - 1.5m/s^2),$

$\quad\quad = 425N.$

Example 6

A skydiver of mass $50kg$ is mid jump and has an instantaneous acceleration of $4m/s^2$. What is the force exerted on the diver from the air? (where $g = 10m/s^2$)

Solution

In this scenario, there are two forces in play. The first is gravity, and the second is air resistance. Since they are opposing each other, we can write:

$F_{net} = F_g - F_{air}.$

Substituting in Newton's second law, we get

$ma_{inst} = mg - F_{air}.$

Rearranging for the force of air resistance, we obtain,

$F_{air} = mg - ma_{inst},$

$F_{air} = (50kg)(10m/s^2) - (50kg)(4m/s^2)$

$\quad\quad = 300N$

Example 7

If we apply a constant force of $100N$ on a $10kg$ object that is located on a frictionless surface, what is the acceleration of the object?

Solution

In the given problem, $F = 100N$ and $m = 10kg$

By Newton's second law

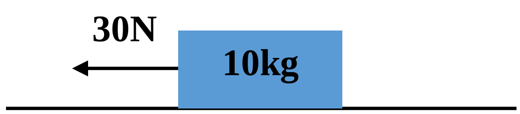
$F = ma,$

$a = \frac{F}{m} = \frac{100N}{10kg},$

$a = 10m/s^2.$

Example 8

A constant force of $30N$ acts on a $10kg$ box as shown in the diagram below. If the box is originally at rest, what will be its velocity after 5s?



Solution

Since the constant force that is acting on the box is pulling it towards the left. Therefore we can write this as:

$F = -30N$, where the negative sign indicates that the force is directed towards the left.

Since the force is constant, this means that it is causing the box to move with a constant acceleration that can be calculated using Newton's second law of motion:

$F = ma,$

$-30N = (10kg)a,$

$a = \frac{-30N}{10kg} = -3m/s^2.$

Now that we know what the acceleration is, we can calculate the final velocity after 5 seconds:

$a = \frac{final\ velocity - initial\ velocity}{time} = \frac{V_f - V_i}{t},$

In this scenario, $t = 5s$ and $V_i = 0$ since the box is originally at rest,

So we have that,

$-3m/s^2 = \frac{V_f - 0m/s}{5s},$

$V_f = (-3m/s^2)(5s) = -15m/s.$

Note that the negative sign indicates that the box is moving to the left.

Example 9

A $5kg$ box is being pushed across a frictionless field by two people. The box is moving with an acceleration of $1m/s^2$. What is the force applied by the weaker person if the stronger person can push twice as hard?

Solution

Since two people are pushing the box, using Newton's second law, we can write that the net force applied by the two people is equal to the product of the mass of the object and the acceleration of the object, i.e.

$$\sum F = ma$$

If the force applied by the weaker person is x, then the force applied by the stronger person will be $2x$.

$$\sum F = 2x + x = ma$$

$$3x = 5 \times 1$$

$$\Rightarrow x = \frac{5}{3}N.$$

Therefore the force applied by the weaker person is $\frac{5}{3}N$.

<u>Newton's Third Law of Motion</u>

Suppose object *A* and object *B* interact, the force exerted by object *A* on object *B* is equal in magnitude but opposite in direction to the force exerted by object *B* on object *A*. Mathematically,

$$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}.$$

$\vec{F}_{A\ on\ B}$ may be called the action force and $\vec{F}_{B\ on\ A}$ the reaction force. Either force can be the action or the reaction force. Newton's third law of motion states that,

"**To every action, there is an equal and opposite reaction**."

Example 10

If a bird collides with the windshield of a fast moving plane, which experiences an impact force with a larger magnitude?

A. The bird.

B. *The plane.*

C. The same force is experienced by both.

D. Not enough information is given

Solution

Option *C* is the right answer.

Example 11

If a bird collides with the windshield of a fast moving plane, which experiences greater acceleration?

A. The bird.

B. *The plane.*

C. The same force is experienced by both.

D. Not enough information is given

Solution

Option *A* is the right answer.

Example 12

A man shoots a rifle and the force of the shot results in recoil. The magnitude of the force on the rifle __________ the magnitude of force on the bullet, and the magnitude of acceleration of the rile __________ that of the bullet.

A. equals . . . is less than
B. equals . . . equals
C. is greater than . . . is less than
D. is less than . . . is less than
E. is less than . . . is greater than

Solution

Option *A* is the right answer.

According to Newton's third law, which states that every force has an equal and opposite reaction, the force on the rifle is equal to the force on the bullet. However, the riffle has a larger mass, so the magnitude of its acceleration is less than that of the bullet.

Example 13

Two buses, one of mass *400kg* and one of mass *250kg*, collide head on. The bus with more mass experiences a(n) __________ force and a(n) __________ acceleration with respect to the smaller bus.

A. *larger . . . smaller*

B. *equal . . . smaller*

C. *larger . . . larger*

D. *equal . . . larger*

Solution

Option *B* is the right answer.

The buses will experience the same force due to Newtown's third law. Because the larger bus experiences the same force and has a larger mass, by Newton's first law, it will have a smaller acceleration.

MTS 104 LECTURE NOTE: MODULE 3

Lecturer: Mr. Samuel D. J.

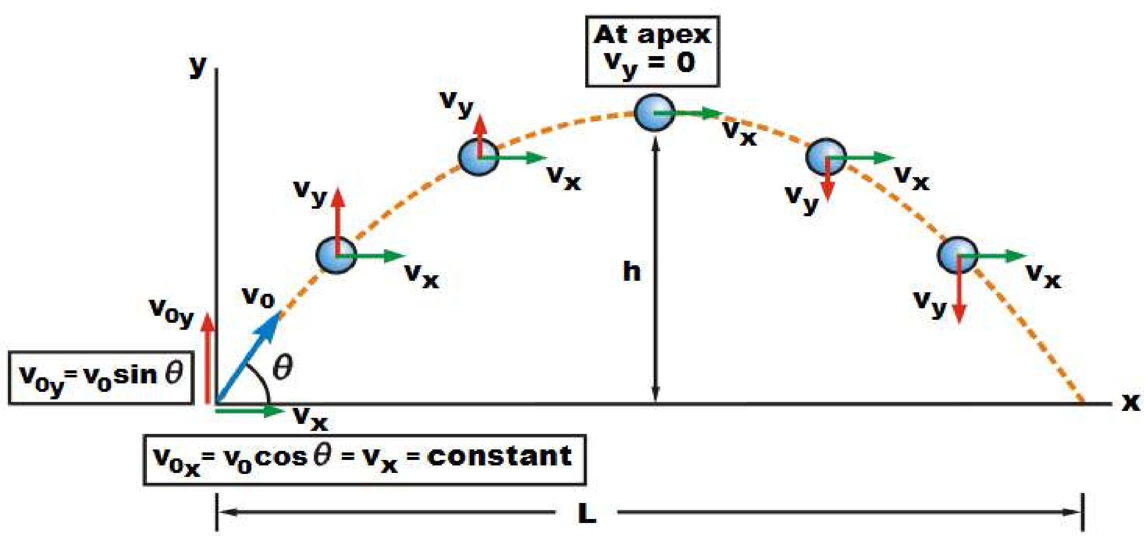
TOPIC: Projectile Motion

PROJECTILE MOTION

Projectile motion is the motion of an object thrown or projected into the air, subject only to acceleration as a result of gravity. The applications of projectile motion in physics and engineering are numerous. Some examples include meteors as they enter Earth's atmosphere, fireworks, and the motion of any ball in sports. Such objects are called projectiles and their path is called a trajectory. In this section, we consider two-dimensional projectile motion, and our treatment neglects the effects of air resistance. Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., gravity).

Any object flying through the vacuum you can breathe in both the x and y directions is in projectile motion. When solving a projectile motion problem you need to separate the x and y direction variables.

Ideal Projectile: If the only force is weight, then the x velocity stays constant. The y velocity changes with time and position.



Ideal Projectile Equations

If the only force is weight, then the x velocity stays constant ($a_x = 0$). The y velocity changes with time and position (y acceleration $a_y = -g$).

	x direction	y direction
Acceleration	$a_x = 0$	$a_y = -g = -9.81 m/s^2$
Velocity	$v_x = v_0 cos\theta$	$v_y = v_{0y} - gt,$ Where $v_{0y} = v_0 sin\theta$
Position	$x = x_0 + v_x t$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ An additional y equation is $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

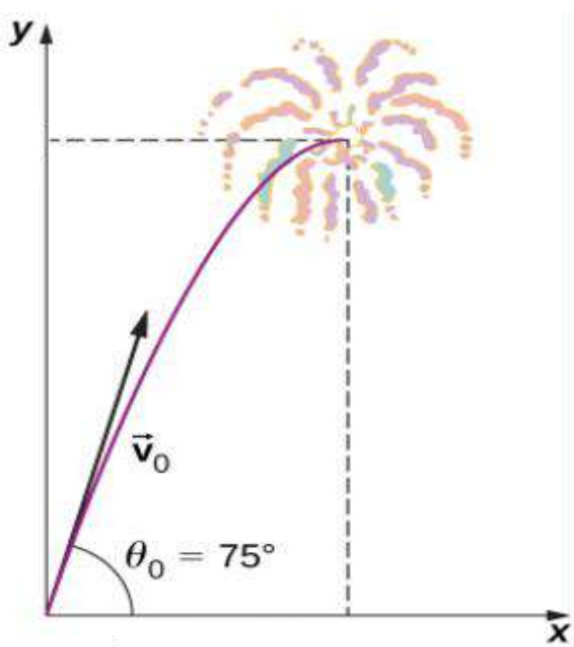
Example 1

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in the Figure below. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. Calculate

(a) the height at which the shell explodes.

(b) How much time passes between the launch of the shell and the explosion?

(c) What is the horizontal displacement of the shell when it explodes?



Solution

(a) The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -$g. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when v_y=0. Since we know the initial and final velocities, as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy$$

$$\Rightarrow y = \frac{v_{0y}^2}{2g}.$$

Now we must find v_{0y}, which is the component of the initial velocity in the y direction. It is given by v_{0y} =$v_0$sinθ, where v_0 is the initial velocity of 70.0 m/s and θ = 75° is the initial angle. Thus,

$$v_{0y} = v_0 \sin\theta = (70.0 \text{ m/s})(sin75)$$

$$= 67.6\text{m}/s.$$

Therefore,

$$y = \frac{(67.6m/s)^2}{2(9.80m/s^2)}.$$

Thus, we have

$$y = 233m.$$

(b) To solve for the time passes between the launch of the shell and the explosion, we can make you of the formula below

$$v_y = v_{0y} - gt,$$

since $v_y = 0$ at the apex, the equation reduces to

$$0 = v_{0y} - gt,$$

$$\Rightarrow t = \frac{v_{0y}}{g} = \frac{67.6m/s}{9.80m/s^2}$$

$$= 6.90s.$$

(c) To find the horizontal displacement of the shell when it explodes. By definition, the horizontal displacement is the horizontal velocity multiplied by time, it can be expressed as

$$x = x_0 + v_x t,$$

here x_0 is equal to zero. Thus,

$$x = v_x t$$

where v_x is the *x*-component of the velocity, which is given by

$$v_x = v_0 cos\theta = (70m/s)(cos75^0)$$
$$= 18.1m/s,$$

So we have,

$$x = (18.1m/s)(6.90s) = 125m.$$

Example 2

While in a car moving at 10.0 miles per hour, Mr. P drops a ball from a height of 0.70m above the top of a bucket. Calculate how far in front of the bucket he should drop the ball such that the ball will land in the bucket.

Solution

This is a projectile motion problem, so we should list our variables in the x and y directions,

x-direction: $x = ?$, $v_x = 10.0m/h$

$$= 10.0\frac{mile}{hour} = 10.0 \times \frac{1609meter}{3600sec} = 4.4694m/s$$

y-direction: $y = -0.70m$, $a_y = 9.8m/s^2$, $v_{0y} = 0$.

Because we know three variables in the *y* direction, we should start there to find the change in time

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$\Rightarrow -0.7 = 0 + (0)t - \frac{1}{2}(9.81)t^2$$

$$\Rightarrow -0.7 = -\frac{1}{2}(9.81)t^2$$

$$\Rightarrow (2)(0.7) = (9.81)t^2$$

$$t = \sqrt{\frac{(2)(0.7)}{9.81}} = 0.377772s$$

To find how far in front of the bucket he should drop the ball such that the ball will land in the bucket which is the horizontal distance *x*, we can use the following formula to solve this

$$x = x_0 + v_x t = 0 + (4.4694m/s)(0.377772s) = 1.7m.$$