FEDERAL UNIVERSITY OF TECHNOLOGY, OWERRI
SCHOOL OF PHYSICAL SCIENCES DEPARTMENT OF MATHEMATICS
2018/2019 HARMATTAN SEMESTER EXAMINATIONS
MTH 403: TOPOLOGY I TIME: 2.5 HOURS

Instructions: Answer ALL questions in Section A and any five (5) in Section B, write your FULL NAMES with ink on your question paper.

Section A

1. Which of the following sets are not countable; $\mathbb{N}, \mathbb{Q}, \mathbb{Z}$?
2. Describe the set $\mathbb{N} \setminus \mathbb{Z}$.
3. Given $\mathbb{R}$ with the usual metric, describe $[a, b]$ as a closed ball.
4. Given (X, τ) a topological space, is $\emptyset$ closed or open?
5. Given $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x^2 - 1$, is f injective?
6. Is every linear function injective? Justify.
7. Given $X = \{1, 2, 3\}$ and $\tau = \{\emptyset, X, \{1\}, \{2, 3\}\}$, is (X, τ) connected?
8. If $X = \{a, b, c, d\}$, let $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{b\}\}$, obtain the smallest topology containing τ_1 and τ_2.
9. Describe the set $\mathbb{N} \setminus \mathbb{Z}^c$.
10. A set with only one (1) element has how many possible topology(ies)?

Section B

4 1 (a) What is a topological space?
(b)Given $X = \{a, b, c, d, e\}$, describe one(1) topology on X which is not a discrete topology and has a minimum of six (6) elements.
(c) Let X be an infinite set and let $\tau = \{U \subset X : U^c \text{ is countable or } U^c = X\}$. Show that τ is a topology on X.
(d) Why is X assumed to be infinite?

2 (a) What is a metric space?
(b) Let $X = C[-1, 1]$ and $d : X \times X \to [0, +\infty)$ be defined as $d(f, g) = \int_{-1}^{1} |f(t) - g(t)|dt$ for all $f, g \in X$. Prove that d is a metric on X. Hence, obtain $d(f, g)$ if $f(t) = 1 + 2t$ and $g(t) = 3 - 6t$.
(c) Consider the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2x^2 + 4$ for all $x \in \mathbb{R}$, obtain the following sets; (i) $g^{-1}([1, 2])$ (ii) $g^{-1}([6, 12])$.

3 (a) Let (X, d) be a metric space and let $A \subset X$. When is A said to be open?
(b)(i) Let $\mathbb{R}$ be endowed with the usual metric. Prove that $[0, 2)$ is not open.
(b)(ii) Is $[0, 2)$ closed? (c) Describe the sets $B_{\frac{2}{3}}(4)$, $\overline{B}_2(1)$, and $S_{\frac{3}{2}}(1)$ if $X = \mathbb{R}$ with metric d_0 defined by

$$d_0(x, y) = \begin{cases} 1 & if\ x \neq y \\ 0 & if\ x = y \end{cases}$$

4 (a) When is a function $f : X \to Y$ said to be an open function? (b) Let $g : \mathbb{R} \to \mathbb{R}$ (where $\mathbb{R}$ is endowed with the usual metric) be given by

$$g(x) = \begin{cases} -1 & if\ x \geq 0 \\ 1 & if\ x < 0 \end{cases}$$

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Is g continuous?

(c) Is the discrete topology a connected space?

5 (a) Show that (a, b) and $(0, 1)$ are homeomorphic. [Hint: Consider the function $f : (a, b) \to (0, 1)$ defined as $f(x) = \frac{x-a}{b-a}$ for all $x \in (a, b)$]

(b) Let (X, τ) be a topological space, and let $x \in X$. When is $U \subsetneq X$ said to be a neighborhood of x.

(c) Let (X, τ) be a topological space, and let $A, U \subset X$. If U is open in X, A is closed in X, show that $A \setminus U$ is closed in X.

6 (a) What is a subspace topology?

(b) Consider the following subset of the real line with the usual topology, $Y = [0, 1) \cup (2, 3]$:

(i) Is $[0, 1)$ open in $\mathbb{R}$?, (ii) Is $[0, 1)$ open in Y?, (iii) Is $(2, 3]$ open in Y?, and (iv) Is $(2, 3]$ open in $\mathbb{R}$?

(c) Given $X = \{1, -1.2, 0.6\}$, is X compact?

7 (a) When is a topological space (X, τ) said to be Hausdorff?

(b) When is a function $f : X \to Y$ said to be a bijection?

(c) Let (X, τ) be a cofinite topology for an infinite set X, show that X is a Fréchet space.

GOOD LUCK