PHY 112 - BASIC PRINCIPLES OF PHYSICS II

ELECTRICITY AND MAGNETISM

BY

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ELECTROSTATICS

- Definition: Electrostatics is the study of electrical effects of charges at rest. However, when both magnetic and electric effects are present, the interaction between charges is referred to as electromagnetic.

Charges itself is a property of matter that causes it to produce and experience electrical and magnetic effects.

PROPERTIES OF ELECTRIC CHARGES

- **There are two kinds of charges in nature, namely positive and negative charges with the following properties:**

1. Unlike charges attract one another while like charges repel one another
2. The force (of attraction or repulsion) between charges varies as the inverse square of their separation ie $F \propto 1/r^2$
3. Charge is neither created or destroyed, this implies that charge is conserved and can be transferred from one body to another.
4. Electrical charge appears only in discrete amounts ie charge is quantized. Thus, charge $q = ne$ ($n = 0, \pm 1, \pm 2 \ldots\ldots$) and $e = 1.602 \times 10^{-19}C$

Electroscope

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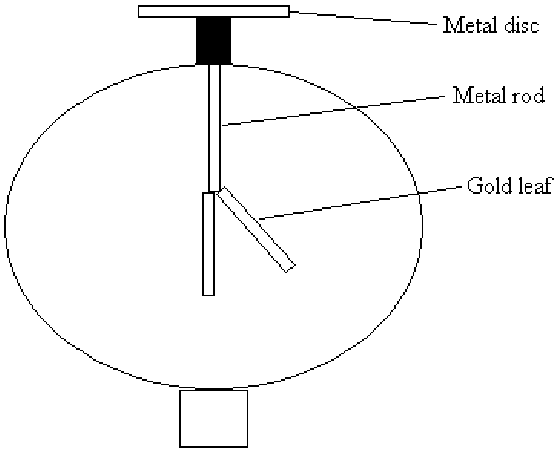


Figure 2: The classical electroscope

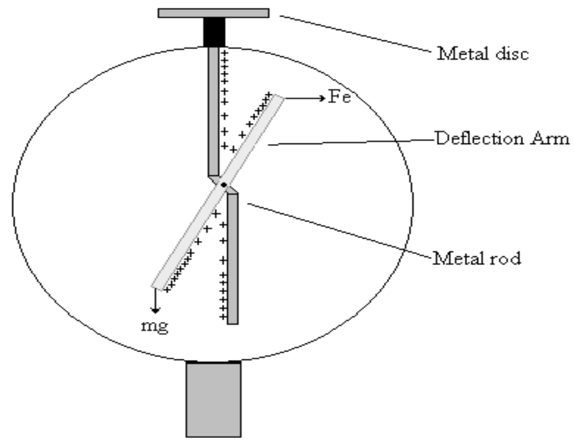


Figure 1: Electroscope

The term electroscope is given to instruments which serve two essential purposes to:
(1) determine if a body is electrified, and (2) determine the nature of the electrification.

In other words an electroscope is a device that can be used to test for the presence of charge, or that can be charged. An electrometer, on the other hand, is a specialized form of electroscope that includes a calibrated scale for reading the strength of the charge. That is to determine the potential associated with the charge.

Illustration

The attraction and repulsion property of charges can be demonstrated as follows;

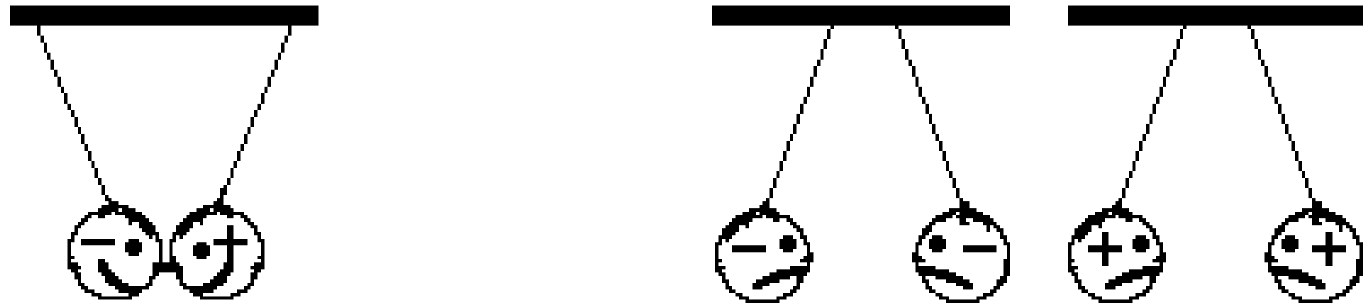
If a glass rod rubbed with silk is brought near a hard rubber rod rubbed with fur and suspended by a non metallic thread, as shown bellow……………(Unlike poles attraction)

It will be observed that the rubber is attracted towards the glass rod.

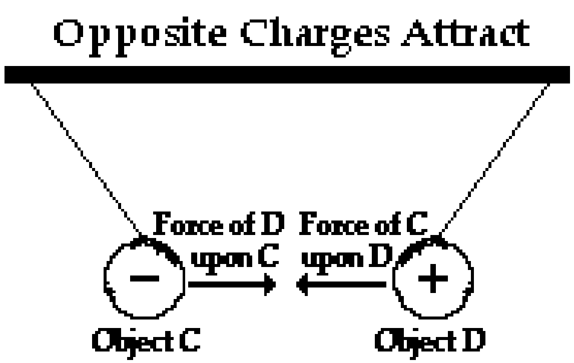
If on the other hand two charged glass rods or two charged rubber rods are brought near each other as shown ……………………(Like poles repulsion) the force between them will be repulsive, that is, the two rods will repel each other.

Using the general convention, the electric charge on the glass is referred to as positive while that on the rubber rod is called negative. However, it is important to note that the signs of acquired charges depend on the electrical properties of the two materials and the condition of their surfaces.

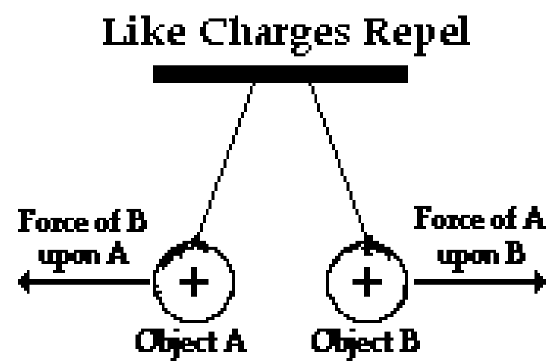
In the world of static electricity ...



oppositely-charged objects attract AND objects with like charges repel



Attractive forces act between opposite charged, pulling them towards each other.



Repulsive forces act between like-charged objects, pushing them away from each other.

Classification of Materials

All materials have electrical properties that allow them to be organized into three broad categories. This categorization is essentially determined or based on the flow of charges in the materials. They are; conductors, insulators and semiconductors materials

- Conductors –they allow charge to flow freely (metals or ionic solutions)
- Insulators – wood, rubber, silk, and glass
- Semiconductors – when pure, they are like insulators but with impurities they can conduct (Ge, Si, C, As)
- Mobility of charges in a substance can be characterised by the relaxation time – how quickly the charge diminishes at a point (Cu = 10^{-12}s; Glass = 2s)
- Charges can be transferred to an object by induction – process of charging without contact

Classification contd..

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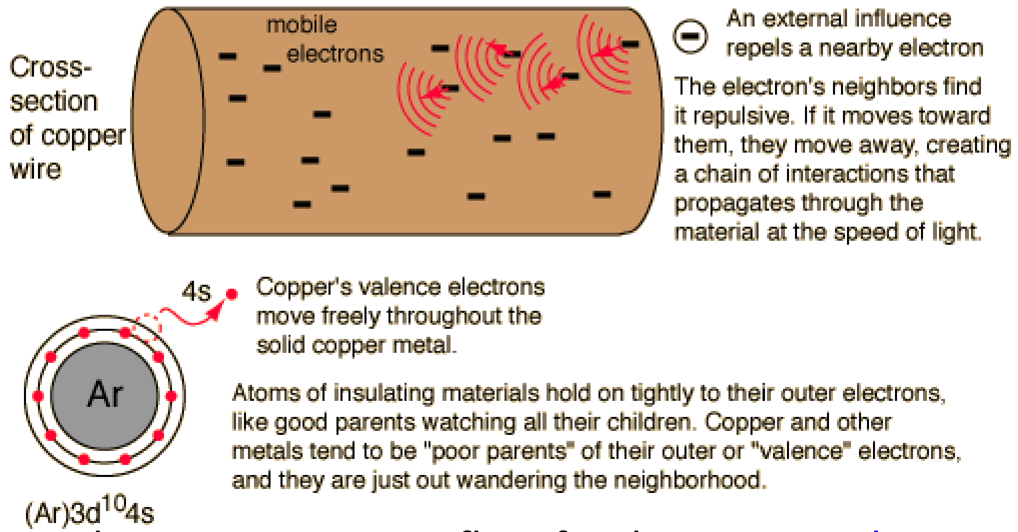


In a conductor, electric current can flow freely, in an insulator it cannot. Metals such as copper typify conductors, while most non-metallic solids are said to be good insulators, having extremely high resistance to the flow of charge through them. "Conductor" implies that the outer electrons of the atoms are loosely bound and free to move through the material. Most atoms hold on to their electrons tightly and are insulators. In copper, the valence electrons are essentially free and strongly repel each other. Any external influence which moves one of them will cause a repulsion of other electrons which propagates, "domino fashion" through the conductor.

Simply stated, most metals are good electrical conductors, most nonmetals are not. Metals are also generally good heat conductors while nonmetals are not

CONDUCTORS AND INSULATORS

- Materials that allow easy flow of electric charges are called conductors. These could metals such as Cu, Al, Ag, etc. or ionic solutions. Insulators on the other hand are materials which do not allow the flow of charges for example, wood, rubber, silk, glass, fur, etc.

In between these two extreme class of materials (i.e. conductors and insulators) is a third class of material known as semiconductors with electrical properties between those of conductors and insulators e.g. Silicon, Germanium and carbon. These materials behave like insulators in their pure state. However, their ability to conduct can be enhanced by increasing the temperature or by careful addition of certain impurity (other elements) such as arsenic and boron. Semiconductors have many practical Applications in solid-state electronic circuits.

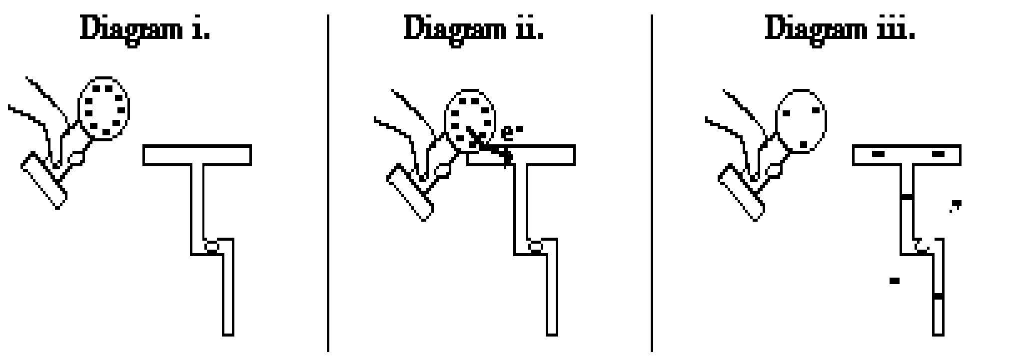
METHODS OF CHARGING

- The presence of different atoms in objects provides different objects with different electrical properties. One of such property is known as electron affinity.
- Simply put, the property of electron affinity refers to the relative amount of *love* which a material has for electrons. If atoms of a material have a high electron affinity, then that material will have a relatively high love for electrons.
- This property of electron affinity will be of utmost importance as we consider the different methods through which an object can be charged.
- There are three ways that objects can be given a net charge. These are:
- (1). charging by conduction: This is useful for charging metals and other conductors.
- (2). charging by friction: This is useful for charging insulators.
- (3). charging by induction: This also useful for charging metals and other conductors

Charging by Conduction

- Charging by conduction involves the contact of a charged object to a neutral object. Suppose that a positively charged aluminum plate is touched to a neutral metal sphere. The neutral metal sphere becomes charged as the result of being contacted by the charged aluminum plate. On the other hand suppose that a negatively charged metal sphere is touched to the top plate of a neutral needle electroscope. The neutral electroscope becomes charged as the result of being contacted by the metal sphere.

Charging a Neutral Object by Conduction



A metal sphere with an excess of - charge is brought near to a neutral electroscope.

Upon contact, e⁻ move from the sphere to the electroscope and spread about uniformly.

The metal sphere now has less excess - charge and the electroscope now has a - charge.

Charging by Friction

The property of electron affinity as previously discussed is of utmost importance as we now consider charging by friction or rubbing, one of the most common methods of charging. Suppose that a rubber balloon is rubbed with a sample of animal fur. During the rubbing process, the atoms of the rubber are forced into close proximity with the atoms of the animal fur. The electron clouds of the two types of atoms are pressed together and are brought closer to the nuclei of the other atoms.

The frictional charging process results in a transfer of electrons between the two objects which are rubbed together. The two objects have become charged with opposite types of charges as a result of the transfer of electrons from the least electron-loving material to the most electron-loving material.

Charging by Induction

- Induction charging is a method used to charge an object without actually touching the object to any other charged object. An understanding of charging by induction requires an understanding of the nature of a conductor and an understanding of the polarization process.

Charging by Induction contd..

The principles of charging by induction are:

- The charged object is never touched to the object being charged by induction.
- The charged object does not transfer electrons to or receive electrons from the object being charged.
- The charged object serves to polarize the object being charged.
- The object being charged is touched by a ground; electrons are transferred between the ground and the object being charged (either into the object or out of it).
- The object being charged ultimately receives a charge that is opposite that of the charged object which is used to polarize it.

Charging by Induction

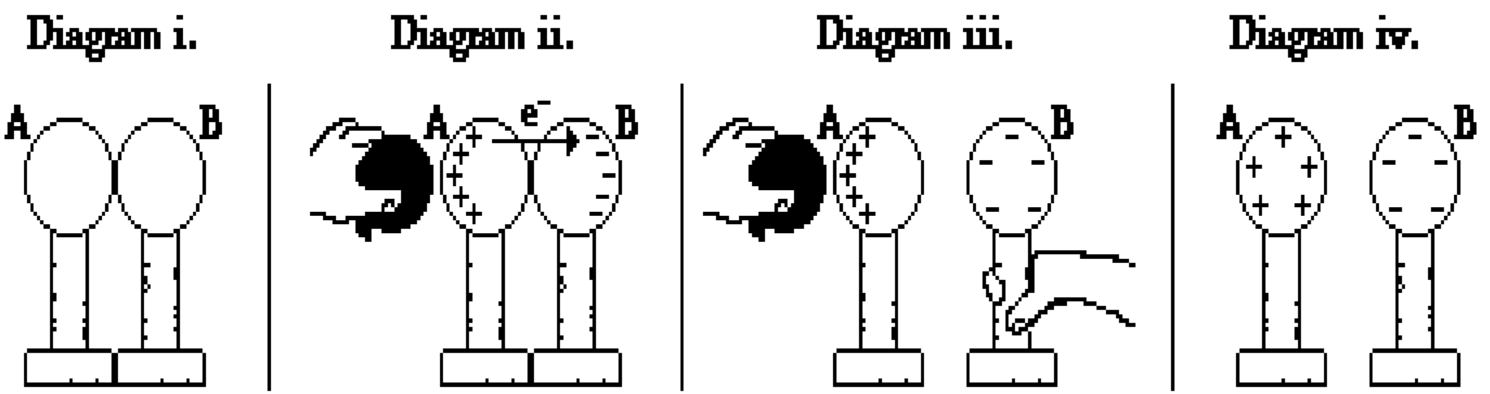


Diagram i. Two metal spheres are mounted on insulating stands.

Diagram ii. The presence of a – charge induces e⁻ to move from sphere A to B. The two-sphere system is polarized.

Diagram iii. Sphere B is separated from sphere A using the insulating stand. The two spheres have opposite charges.

Diagram iv. The excess charge distributes itself uniformly over the surface of the spheres.

Grounding- Removal of charge

Objects with an excess of charge either positive or negative can have this charge removed by a process known as grounding.

- Grounding is the process of removing the excess charge on an object by means of the transfer of electrons between it and another object of substantial size.
- When a charged object is grounded, the excess charge is balanced by the transfer of electrons between the charged object and a ground.
- A ground simply put is an object which serves as a seemingly infinite reservoir of electrons or a large object which serves as an almost infinite source of electrons or sinks for electrons.
- It is capable of transferring electrons to or receiving electrons from a charged object in order to neutralize that object.

Coulomb’s Law

The electric forces between charges can be calculated using Coulomb's law. Coulomb’s law states that the electrical force exerted by one charged body on another is directly proportional to the product of the magnitude of the two charges and inversely proportional to the square of the separation distance between them. For two charges of magnitude Q_1 and Q_2 separated by a distance r, the force of interaction **F** between them according to Coulomb’s law is mathematically expressed as:

Coulomb's Law Contd

$$F \propto \frac{Q_1 Q_2}{r^2} \quad (1)$$

$$F = k \frac{Q_1 Q_2}{r^2} \quad (2)$$

Where $k = \frac{1}{4\pi\varepsilon_o} \approx 9 \times 10^9$ N.m^2 C^{-2} is a constant of proportionality (Coulomb's constant) where ε_o is called the permittivity of free space = 8.854×10^{-12} C^2 N^{-1}m^{-1}. Thus, Coulomb's law is expressed as:

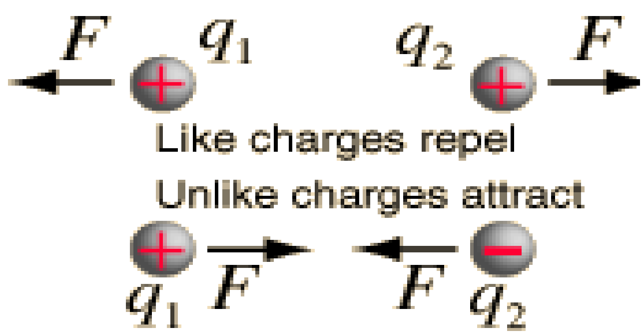
$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2} \quad (3)$$

Coulomb’s Law Contd

The electric force between static electric charges is sometimes referred to as the Coulomb’s force. Coulomb’s force :(i) Obeys the superposition principle. That is $F = \sum F_n$, where n = 1, 2, 3…….n-1, (ii) it is a conservative force and (iii) it is a central force that is, it acts along the line joining the two point charges. The significance of Coulomb’s Law is that it correctly describes the electrical forces that bind the electrons of an atom to the nucleus, the forces that bind the atoms together to form molecules and that bind atoms and molecules to from solids and liquids.

Electric Force

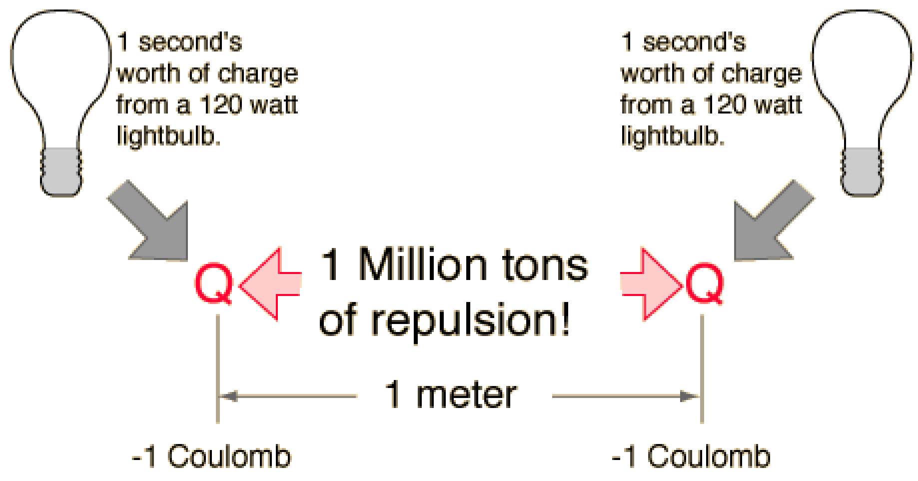
The electric force acting on a point charge q_1 as a result of the presence of a second point charge q_2 is given by Coulomb's Law as illustrated in the figure below.



Electric Force Contd

It could be seen that this satisfies Newton's third law because it implies that exactly the same magnitude of force acts on q_2 . Coulomb's law is a vector equation and includes the fact that the force acts along the line joining the charges. Like charges repel and unlike charges attract. Coulomb's law describes a force of infinite range which obeys the inverse square law, and is of the same form as the gravity force. A negative force implies an attractive force.

Let us use a simple example to illustrate the effect of electric force.



The electric power relationship P = IV tells us that to use power at the rate of P = 120 W on a 120 V circuit would require an electric current of I = 1 A. One ampere of current transports one Coulomb of charge per second through the conductor. So one Coulomb of charge represents the charge transported through a 120 W light bulb in one second. If in one-second collections of 1 Coulomb each were concentrated at points one meter apart, the force between them could be calculated from Coulomb's Law.

- If such enormous forces would result from our hypothetical charge arrangement, then why don't we see more dramatic displays of electrical forces in our everyday life?

- There are three types of force than can act between two charged particles:
- An electrical force
- A magnetic force- if the charged particles are in motion
- A gravitational force

All the three forces are types that varies with distance according to an inverse square law. But they are of different orders of magnitude. Electrical force is incomparably greater than the gravitational force. The magnetic force depends on the speeds of particles but is always less than the electrical forces- much less provided that the speeds do not approach c.

Example 1

- The electron and proton in a hydrogen atom are 0.53 x 10-10m. Compare the electrostatic and gravitational forces between them. (Take m_e= 9.1 x10^{-31}kg, m_p = 1.67 x 10^{-27}kg, e = 1.6 x10^{-19} C and G = 6.67 x 10^{-11} N m^2kg^{-2})

- Solution

Let F_e be the electrical force and F_g be the gravitational force:

$$F_e = \frac{ke^2}{r^2}, \quad F_g = \frac{Gm_e m_p}{R^2}$$

$$\frac{F_g}{F_e} = \frac{ke^2}{Gm_e m_p} = 2.28 \times 10^{-39}$$

Thus the electrical force between charged atomic particles is negligible compared to electrostatic force.

Example 2

- Find the magnitude and the nature of the electrostatic force between two charges q_1 = 5µC and q_2 = -3µC separated by a distance of 0.4m.

Solution

$$F_{12} = k\frac{q_1 q_2}{r^2} = \frac{9.0\times10^{9}\times5.0\times10^{-6}\times-3.0\times10^{-6}}{0.4^{2}} = -0.844N$$

The force F = $-F_{1.2.}$. It is therefore an attractive

Example 3

- A point charge q_1 = -9μc is at x = 0, while q_2 = 4μc at x = 1. At what point beside infinity would the net force on a position charge q_3 be zero.
- $$\overrightarrow{F}_3 = \vec{F}_{31} + \vec{F}_{32} = 0,\ but\, obervethat\ \vec{F}_3 = \vec{F}_{31} - \vec{F}_{32}$$

$$Therefore\ \ \overrightarrow{F}_{31} = \overrightarrow{F}_{32}$$

$$\frac{K|q_3 q_1|}{(1+d)^2} = \frac{K|q_3 q_2|}{d^2}$$

$$\frac{9x10^{-6}}{(1+d)^2} = \frac{4x10^{-6}}{d^2}$$

$$9d^2 = 4(1+d)^2$$

$$5d^2 - 8d - 4 = 0$$

$$\therefore d = 2m$$

Take Home

Three fixed charges $q_1 = 1.0\mu C$, $q_2 = 3.0\mu C$ and $q_3 = -2.0\mu C$ are located at points (0.3m, 0.2m), (0.3m, -0.2m) and (-0.3m, 0.2m) respectively. Sketch the position of the charges and calculate the net force on charge q_1.

Electric Field

Any region of space where an electric charge experiences a force is called electric field. This force is due to the presence of other charges in that region. The direction of an electric field at a point is defined by the direction of the force upon a positive charge placed at that point. If a charge q is in a region of the charges say q_1, q_2, q_3……, it will experience a force F given by:

$$F = f_1 + f_2 + f_3 + \ldots\ldots\ldots\ldots\ldots \quad F = \sum_i^n f_i \qquad (4)$$

The electric field intensity E at a point is equal to the force per unit charge at that point. It is a quantitative measure of the strength of an electric field.

Electric Field Contd

It is a vector quantity defined mathematically as:

$$\vec{E} = \frac{\vec{F}}{q} \qquad \text{(unit :}NC^{-1}\text{)} \qquad (5)$$

If a positive test charge q_0 is placed at a distance r from a point charge q, then the Coulomb's force is:

$$F = \frac{q_o q}{4\pi\varepsilon_o r^2} \qquad (6)$$

Electric Field Contd

- The electric field intensity is therefore:

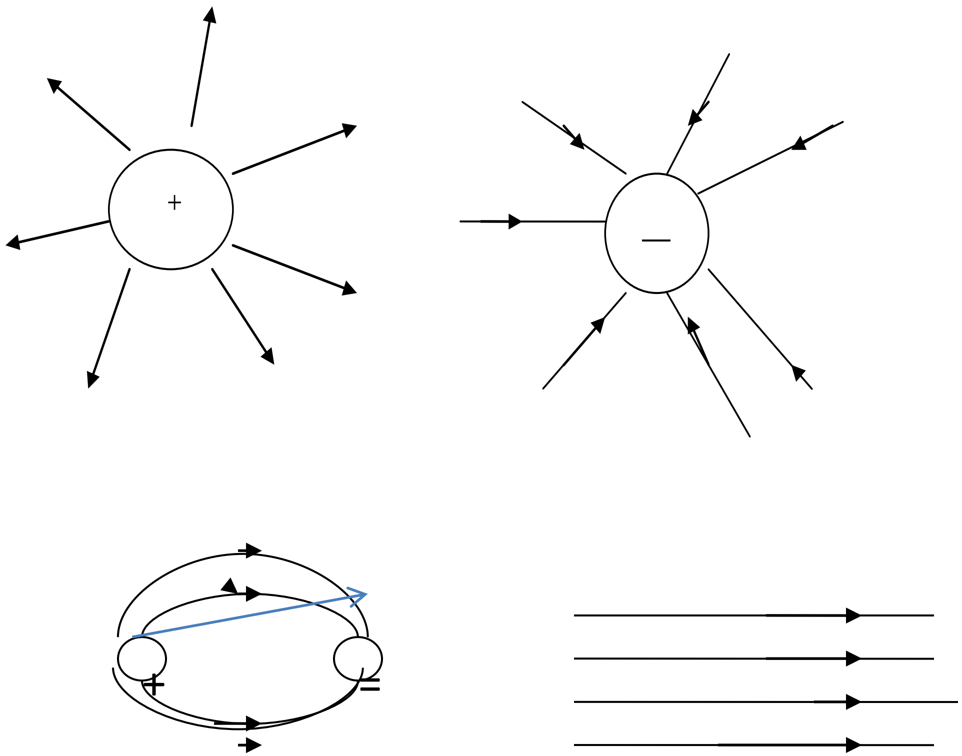
$$E = \frac{F}{q_o} = \frac{q}{4\pi\varepsilon_o r^2} \qquad (7)$$

$$F = qE$$

The direction of electric field is the same as the direction of force along a radial line from q. It points outwards if q is positive and inwards if q is negative.

Electric Field Contd

This is shown in the figure below.



Electric Field Lines

A line of force in an electric field is a line so drawn that a tangent to it at any point shows the direction of the electric field at that point. The total number of lines of electric force is known as the electric flux, symbol ϕ. The electric flux density D is the flux ϕ per unit area A normal to the field at the point of consideration. That is:

$$D = \frac{\phi}{A} \qquad (8)$$

Electric Field Lines Contd

We can therefore represent the electric field intensity by the number of lines of flux per unit area normal to the field at that point of consideration divided by the permittivity of that region. Mathematically, we can write E as:

$$E = \frac{\phi}{\varepsilon_0 A} \qquad (9)$$

Electric Field Lines Contd

The properties of lines of force or electric field lines are:

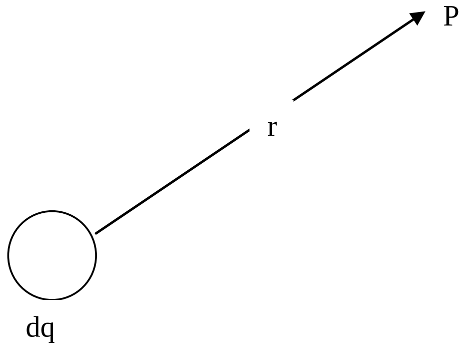
- They begin from positive charge and ends on equal negative charge.
- They do not cross each other.
- They give the direction of electric field at any point.
- Where the field lines are close together, the field is strong, and where they are far apart, the field is weak. Also, where the lines are parallel and equally spaced, the field is uniform.
- When moving along the direction of the arrow of a field line, the electric field potential decreases.
- They cut equipotential surfaces such as conductors at right angles.

Electric Field of Continuous Charge Distribution.

A collection of large number of elementary charges can be regarded as a continuous charge distribution. The field set up by a continuous charge distribution can be obtained by dividing the charge into infinitesimal elements dq. Each element of the charge dq will establish a field dE at a point say, P. The resultant field at the point P is then determined using the principle of superposition or simply by integrating over the total space occupied by the charges.

Electric Field of Continuous Charge Distribution Contd

The infinitesimal elements, dq and the point, P are shown in the figure below.



$$dE = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{dq}{dr^2} \qquad (10)$$

$$E = \int dE = \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{dr^2} \qquad (11)$$

Electric Field of Continuous Charge Distribution Contd

A continuous charge distribution is described by its charge density. In a linear distribution it is described by its linear charge density λ (charge per unit length), on the surface by surface charge density σ (charge per unit area) and volume by the volume charge density ρ (charge per unit volume). If the charge distribution in the object is uniform, taking a linear distribution as an example, the charge density then is constant and it is equal to the total charge q on the object divided by its total length. This treatment is also applicable to the other types of distribution.

$$Linear\ Charge\ density\ \lambda = \frac{dq}{dl}$$

$$Surface\ Charge\ density\ ,\sigma = \frac{dq}{dA}$$

$$Volume\ Charge\ density, \rho = \frac{dq}{dV}$$

Tutorial Questions

Q1. State the properties of the following:

(i) Electric charges and (ii) Electric field lines.

Q2. Sketch the electric field lines between the following combinations:

(i) +q and –q (ii) +2q and –q (iii) +q and -2q.

Q3. Describe the behaviour of the electric field due to point charges and that of a dipole.

Electric Potential

There is a scalar quantity that can be used to describe electric field. This is called the electric potential, usually denoted by the letter V and is related to the amount of work that is done on a test charge (or by a test charge) to move from one point to another in the field. Both E and V are intimately related and indeed, the choice of one over the other depends on what aspect of the field one is interested in. The electric potential energy varies with distance from the centre of the charge producing the field.

Electric Potential Contd

This implies that some work W_{AB} is done whenever a test charge moves from A to B in the electric field in much the same way that work is done when a mass moves between two points, uphill or downhill, in the gravitational field of the earth. The electric potential difference between the two points is the work done per unit charge, and its unit is Joules/Coulomb (J/C) or simply the volt (V).

Electric Potential Contd

- That is:

$$V_B - V_A = \frac{W_{AB}}{q} \qquad (12)$$

Where q is the test charge being moved and V_A and V_B are the absolute potentials at the respective points. Work must be done on the charge by an external agent if $V_B > V_A$ in a similar way to pushing a mass uphill. We observe from equation (12) that the work done W_{AB} is positive in this case, which means increase in the potential energy of the charge.

Electric Potential Contd

When $V_B < V_A$, the work is done by the charge in a similar way to a mass rolling downhill. The work done W_{AB} is negative in this case, which means a decrease in the potential energy of the charge. When point B is very far away from the charge creating the field, we consider this point to be outside the field and as such, the potential energy at this point is zero. Such a point is called infinity and the concept allows us to define the absolute potential at a point. That is, absolute potential at a point is the amount of work per unit charge to move the test charge from infinity to the point, that is, if we set $V_A = 0$ in equation (12) to get the absolute potential at point B.

Electric Potential Contd

This leads us to the general expression for absolute potential at any point.

$$V = \frac{W}{q} \qquad (13)$$

To show how electric potential varies with distance in an electric field, let us apply Coulomb's law.

Electric Potential Contd

This implies that the potential difference between points *A* and *B* is:

$$V_{AB} = \frac{W_{AB}}{q} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \qquad (14)$$

If we take point *A* to be at infinity (i.e. $= \infty$), the work done to bring the test charge to point *B*,

Electric Potential Contd

which is the absolute potential at point *B* becomes:

$$V_B = \frac{Q}{4\pi\varepsilon_0 r_B} \qquad (15)$$

This is true of any point in the electric field. We are however, always more interested in potential difference than in absolute potential in an electric field.

Worked Examples

(1) What positive charge will have an absolute potential +100V at 1 cm from it?

Electric Potential Contd

Solution

From equation (15), the charge is

$Q = V 4\pi\varepsilon_0 r = 1.1 \times 10^{-9}$ Coulombs.

(Remember that all quantities must be in SI units)

(2) How much energy is acquired by an electron in moving through a potential difference of
1 V?

Electric Potential Contd

Solution

From equation (13), $W = QV = 1.6 \times 10^{-19} \times 1 = 1.6 \times 10^{-19} J$. This amount of energy is called an electron volt (eV), the unit of energy in modern physics.

Potential Gradient

- By gradient we mean the space rate of change of a quantity, in the direction of increase. Potential gradient is the rate of change of potential with distance along a line of force. That is, $\Delta V/\Delta x$ in the unit of V/m. For two parallel plates separated by a distance 0.5m, when one is maintained at a potential of 100V above the other, the potential gradient is 200 V/m.

We can get an important relationship between potential gradient and the electric field.

$$E = -\frac{\Delta V}{\Delta x}\left(\frac{V}{m}\right) \qquad (16)$$

Potential Gradient Contd

- Equation (16) is a very useful relationship as it defines the electric field intensity: the electric field intensity at a point is equal to the negative potential gradient of the field at the point.

Electric Potential Energy

The electric potential at point B, which is at distance r_B from the charge Q creating the Coulomb's field, is given by equation (15). Assuming Q is positive and another positive charge q is to be brought from infinity to this point, the work done is $W = qV_B$. This is stored as the electric potential energy U of the system of two charges. That is:

$$U_B = \frac{Qq}{4\pi\varepsilon_o r_B} \qquad (17)$$

Electric Potential Energy

This is true of any pair of charges separated by a distance *r*.

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Worked Examples

(1) Two protons in a nucleus are separated by 6×10^{-15} m. What is their mutual electric potential energy?

Solution

$$U = \frac{Qq}{4\pi\varepsilon_0 r}\frac{9\times10^{9}\times\left(1.6\times10^{-19}\right)^{2}}{6\times10^{-15}} = 3.8\times10^{-14}\ \text{J}$$

Electric Potential Energy Contd

For a system containing more than two charges, the procedure is to compute the potential energy for every pair and then add the results algebraically. That is:

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{pairs} \frac{Q_i Q_j}{r_{ij}} \qquad (18)$$

- This means that if there are three charges in the system, their mutual electric potential energy is:

$$U = \frac{1}{4\pi\varepsilon_o} \left(\frac{Q_1 Q_2}{r_{1,2}} + \frac{Q_1 Q_3}{r_{1,3}} + \frac{Q_2 Q_3}{r_{2,3}} \right)$$

Example

- Three identical charges of 15μC each are located in the vertices of an equilateral triangle that is 9 cm on each side. Calculate the electric potential

$$U = \frac{1}{4\pi\varepsilon_o}\left(\frac{Q_1Q_2}{r_{1,2}} + \frac{Q_1Q_3}{r_{1,3}} + \frac{Q_2Q_3}{r_{2,3}}\right)$$

$$= \frac{1}{4\pi\varepsilon_o}\left[3\times\frac{15\times10^{-6}\times15\times10^{-6}}{9\times10^{-2}}\right] = 67.5J$$

- Observe that we add the result of pair in the above example and U turns to be positive because the charges are identical. If there not, we must consider the signs in the summation. The result of the sum may be positive or negative. A negative means U is attractive and positive implies U is repulsion

Motion of an electric charge in a uniform field

- The acceleration of a body in an electric field depends on the charge per unit mass ratio (q/m). This is true since it is generally different for different charged particles or ions. This gives a clear distinction between the acceleration of charged body in an electric field and that in a gravitational field in which acceleration is the same for all bodies. For a uniform electric field, the acceleration is constant and the path of the charge through is parabolic:

$$x = v_o \times t \ldots\ldots\ldots.1$$

$$y = \frac{1}{2}\left(\frac{q}{m}\right)E\,t^2 \ldots\ldots\ldots.2$$

- By eliminating t from equation (2) we have

$$y = \frac{1}{2}\left(\frac{q}{m}\right)\left(\frac{E}{v_o^{\,2}}\right)x^2 ....3$$

- The deflection experienced by the charge particle is given by

$$\tan\alpha = \left(\frac{dy}{dx}\right)_{X=a} = \frac{qEa}{mv_o^{\,2}} = \frac{d}{L} ....4$$

When stream of particles all having the same ratio q/m passes through an electric field, they are deflected according to their velocities or energies. The application of this is used in deflecting experiments to determine e/m (Thompson experiment) and in cathode ray oscilloscope (CRO).