

FEDERAL UNIVERSITY OF TECHNOLOGY OWERRI
 SCHOOL OF ENGINEERING AND ENGINEERING TECHNOLOGY
 2017/2018 HARMATTAN SEMESTER EXAMINATIONS
 ENG 307- ENGINEERING MATHEMATICS 1 ; DATE: MONDAY APRIL 23, 2018
 INSTRUCTION: ATTEMPT ANY 5 QUESTIONS TIME ALLOWED: 3 HOURS

Question 1a) If for $f(x, y)$, $x = r \cos \theta$ and $y = r \sin \theta$. Develop a relationship for: $w = \frac{\partial^2 f}{\partial r^2} + \frac{\partial^2 f}{\partial \theta^2}$. (10 Marks)

b) Given that $s = ut + \frac{1}{2}at^2$, what is the change in s if $\Delta u = 2m/s$, $\Delta t = 5\text{sec}$ and $\Delta a = 1m/s^2$ for initial values of $u = 5m/s$, $t = 10\text{sec}$ and $a = 9m/s^2$. [Recall: $\Delta s = \frac{\partial s}{\partial t} \Delta t + \frac{\partial s}{\partial u} \Delta u + \frac{\partial s}{\partial a} \Delta a$]. (10 Marks)

Question 2a) Enumerate the 4 steps that should be employed in the use of Laplace Transforms for solution of Ordinary Constant Coefficient Linear Differential equations. (6 Marks)

b) Using the Power Series method (i.e. Long Division method), compute the first 4 f_n 's of the following Z-transform. $F(z) = \frac{z+1}{2z^4 - 2z^3 + 2z - 2}$ (14 Marks)

Question 3a) Find a differential equation from the function: $y = Ae^{-4x} + Be^{-6x}$. (10 Marks)

b) Solve the initial value problem: $y'' - 2y' + 10y = 0$, $y(0) = 4$ and $y'(0) = 1$. (10 Marks)

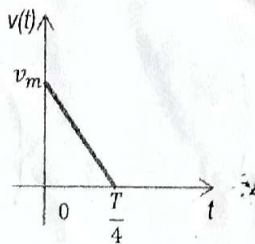
Question 4a) Find the power series solution of: $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 1$, when at $x = 0$, $y = 2$ and $\frac{dy}{dx} = 1$. (10 Marks)

b) Given an ODE: $x \frac{d^2 y}{dx^2} + \frac{C}{y} = 0$, where C is a constant and at $x = 0$, $y = b$ and $\frac{dy}{dx} = 0$. Using your knowledge of gamma function, show that at $x = b \left(\frac{\pi}{2C} \right)^{\frac{1}{2}}$. (10 Marks)

Question 5a) The periodic function $v(t)$ shown below is odd and has both half-wave and quarter-wave symmetry. Sketch one full cycle of the function over the interval $-\frac{T}{4} \leq t \leq \frac{3T}{4}$, and also a full cycle of $f(t)$ over the same interval given that $f(t) = \frac{dv(t)}{dt}$. (4 Marks)

b) Find the Fourier series for $f(t)$ showing the series in the alternative trigonometric form. (10 Marks)

c) Estimate the rms value of $f(t)$ using the first five nonzero terms in its Fourier series representation, and determine the percentage of error in the estimation. [Take $\frac{v_m}{T} = 0.125$]. (6 Marks)



Question 6a) Two quantities, x and y , are related by the law $y = \frac{a}{1 - bx^2}$, where a and b are constants. Using the values given below, draw a suitable graph and hence determine the best values of a and b . (6 Marks)

x	4	6	8	10	11	12
y	4.89	5.49	6.62	9.00	11.4	16.1

b) The force, F in Newtons, required to keep an object moving at a speed, S in meters per second, was recorded.

F	126	178	263	398	525	724
S	1.86	2.34	2.75	3.63	4.17	4.79

If the law connecting F and S is of the form $S = bF^c$, where b and c are constants, apply the method of least squares to obtain the values of b and c that give the best fit to the given set of values. (6 Marks)

Handwritten notes:
 $y'' - 2y' + 10y = 0$, $y(0) = 4$
 (i) transform the given equation into the Laplace form
 (ii) input the initial value into
 (iii) ...