EEE 511: ELECTRICAL POWER SYSTEMS

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Course Outline

<table>
<tr><td rowspan="7">Course Syllabus</td><td colspan="2">Faults and concept of protection in power systems, symmetrical and unsymmetrical fault analysis, fuses, circuit breakers, basic principles of relay design, construction, characteristics and applications, protective relays: distance relay, differential relay, etc, protection of generators, motors, busbars and transformers.
30hrs(T); Status: Compulsory/Elective</td></tr>
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Assessment Methods		
	Method	**Weighting**
	Quizzes	10 %
	Assignments	10 %
	Tests	20 %
	Final Exam	60 %
	Total	**100%**

References

1.	Saadat, H. (1999). Power system analysis (Vol. 2). McGraw-Hill.
2.	Grainger, J. J., Stevenson, W. D., & Stevenson, W. D. (2003). Power system analysis.
3.	Grigsby, L. L. (2006). Electric power engineering handbook. CRC Press LLC, London.
4.	Glover, J. D., Sarma, M. S., & Overbye, T. (2012). Power system analysis & design, SI version. Cengage Learning.

Course Lecture Timetable and Venue

- Friday: 8:00 - 10:00 am; Venue: Online

Chapter 5

POWER SYSTEM FAULT ANALYSIS

Introduction

- Electrical networks, machines and equipment are often subjected to various types of faults while they are in operation.
- When a fault occurs, the characteristic values (such as impedance) of the machines may change from existing values to different values till the fault is cleared.
- A fault is any abnormal condition in a power system that results in the electrical failure of an equipment.
- Under normal or safe operating conditions, the electric equipment in a power network operate at normal voltage and current ratings.
- However, once a fault takes place in the circuit or device, voltage and current values deviates from their nominal ranges.

Introduction contd.

- Faults in power system cause over current, under voltage, unbalance of the phases, reversed power and high voltage surges.
- They result in the interruption of the normal operation of the network, failure of equipment, electrical fires, etc.
- Electrical faults in three-phase power system are mainly classified into two types, namely open and short circuit faults.
- Further, these faults can be symmetrical or unsymmetrical faults.

Causes of Electrical Faults

- **Weather conditions:** Include lighting strikes, heavy rains, heavy winds, salt deposition on overhead lines and conductors, snow and ice accumulation on transmission lines, etc. These environmental conditions interrupt the power supply and also damage electrical installations.

- **Equipment failures:** Various electrical equipment like generators, motors, transformers, reactors, switching devices, etc, cause short circuit faults due to malfunctioning, ageing, insulation failure of cables and windings. These failures result in high current flow through the devices or equipment which further damages it.

- Broken conductor and malfunctioning of circuit breaker in one or more phases could result in an open circuit fault.

Causes of Electrical Faults

- **Human errors: H**uman errors such as selecting improper rating of equipment or devices, forgetting the metallic or electrical conducting parts after servicing or maintenance, switching the circuit while it is under servicing, etc.

- **Smoke of fires:** Due to smoke particles, ionization of air surrounding the overhead lines could result in spark between the lines or between conductors to insulator. This flashover causes insulators to lose their insulating capacity due to high voltages.

Causes of Electrical Faults

- **Animal factors:** Birds could short overhead lines while rodents could enter the switchgear to cause total non-availability or malfunctioning of the equipment.

- **Other factors:** Vehicles colliding with towers or poles, aircraft colliding with lines, vandalisation of power system equipment, line breaks due to excessive loading, etc, could all result in faults on the network.

Fault Studies

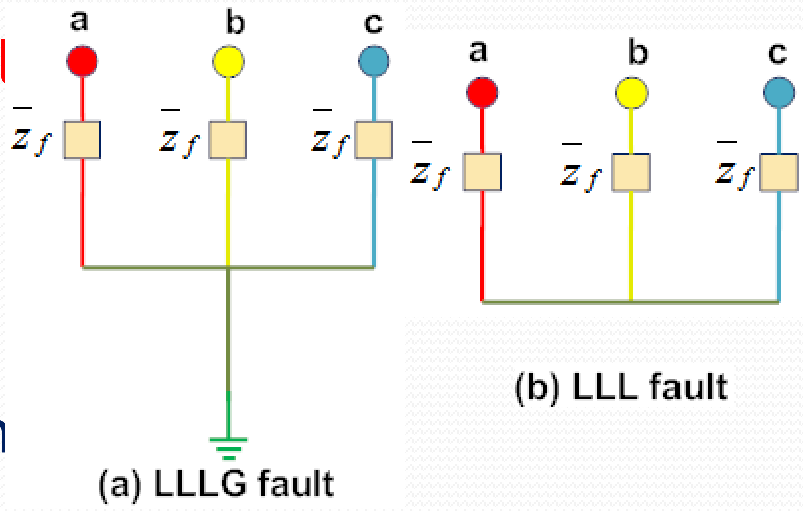
- Fault studies form an important part of power system analysis.
- The studies entail determining bus voltages and line currents during various types of faults.
- Faults on power systems are divided into 3-Φ balanced (symmetrical) faults and unbalanced (unsymmetrical) faults.
- Different types of unbalanced faults are *single line-to-ground fault*, *line-to-line fault*, and *double line-to-ground fault*.
- The information gained from fault studies are used to for proper relay setting and coordination.
- The 3-Φ balanced fault information is used to select and set phase relays, while the line-to-ground fault is used for ground relays.
- Fault studies are also used to obtain the rating of the protective switchgears.

Fault Studies Contd.

- The magnitude of the fault currents depends on the internal impedance of the generators plus the impedance of the intervening circuit.
- It should be noted that the reactance of a generator under short circuit condition is not constant; but a time-varying quantity.
- For the purpose of fault studies, the gen behaviour can be divided into 3 periods: the *subtransient period*, lasting for the first few cycles; the *transient period*, covering a relatively longer time; and finally, the *steady-state period*.
- In fault analysis, transmission lines are represented by their π-models with all impedances referred to a common base.

Three-phase Symmetrical Faults

- This type of fault is defined as the simultaneous short circuit across all three phases.
- It occurs infrequently, as only about 5% of system faults are 3-Φ faults, but it is the most severe type of fault encountered.
- Because the network is balanced, it is solved on per-phase basis.
- The behaviour of LLLG fault and LLL fault is identical due to the balanced nature of the fault.
- The other two phases carry identical currents except for the phase shift.
- Since the duration of the short cct current depends on the time of operation of the protective system, it is not always easy to decide which gen reactance to use.



Three-phase Symmetrical Faults

- Generally, the subtransient reactance, x_d'' is used for determining the interrupting capacity of the cct breakers.
- In fault studies required for relay setting and coordination, transient reactance, x_d' is used.
- Transient reactance is also used in typical transient stability studies.
- A fault represents a structural netwrk change equivalent with that caused by addition of an impedance at the place of fault.
- If the fault impedance is zero, the fault is referred to as *bolted fault* or *solid fault*.
- The 3-Φ symmetrical faulted netwrk can be solved conveniently by the Thevenin's method (impedance diagram method) or using the Bus impedance matrix method.
- The Thevenin's method procedure is demonstrated in the following example.

Thevenin's Method

Example 1.1

The one-line diagram of a simple three-bus power system is shown in Figure 9.1. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common 100 MVA base, and for simplicity, resistances are neglected. The following assumptions are made.

(*i*) Shunt capacitances are neglected and the system is considered on no-load.

(*ii*) All generators are running at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current, the bus voltages, and the line currents during the fault when a balanced three-phase fault with a fault impedance $Z_f = 0.16$ per unit occurs on Bus 3.

Example 1.1 contd.

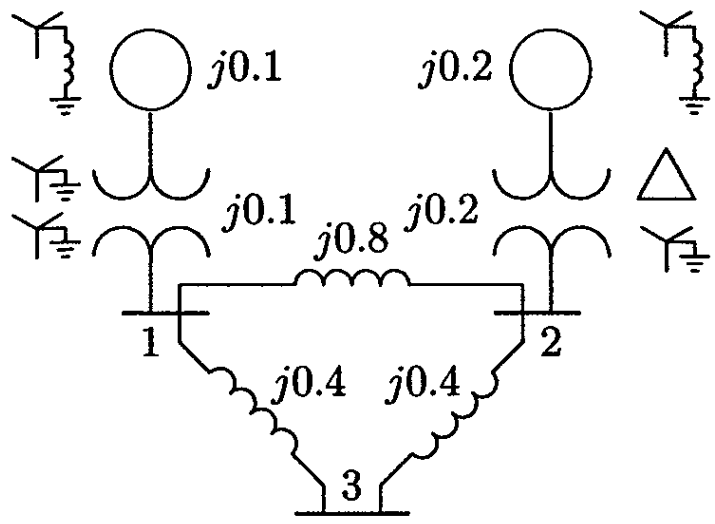


FIGURE 9.1
The impedance diagram of a simple power system.

The fault is simulated by switching on an impedance Z_f at bus 3 as shown in Figure 9.2(a). Thévenin's theorem states that the changes in the network voltage caused by the added branch (the fault impedance) shown in Figure 9.2(a) is equivalent to those caused by the added voltage $V_3(0)$ with all other sources short-circuited as shown in Figure 9.2(b).

Example 1.1 contd.

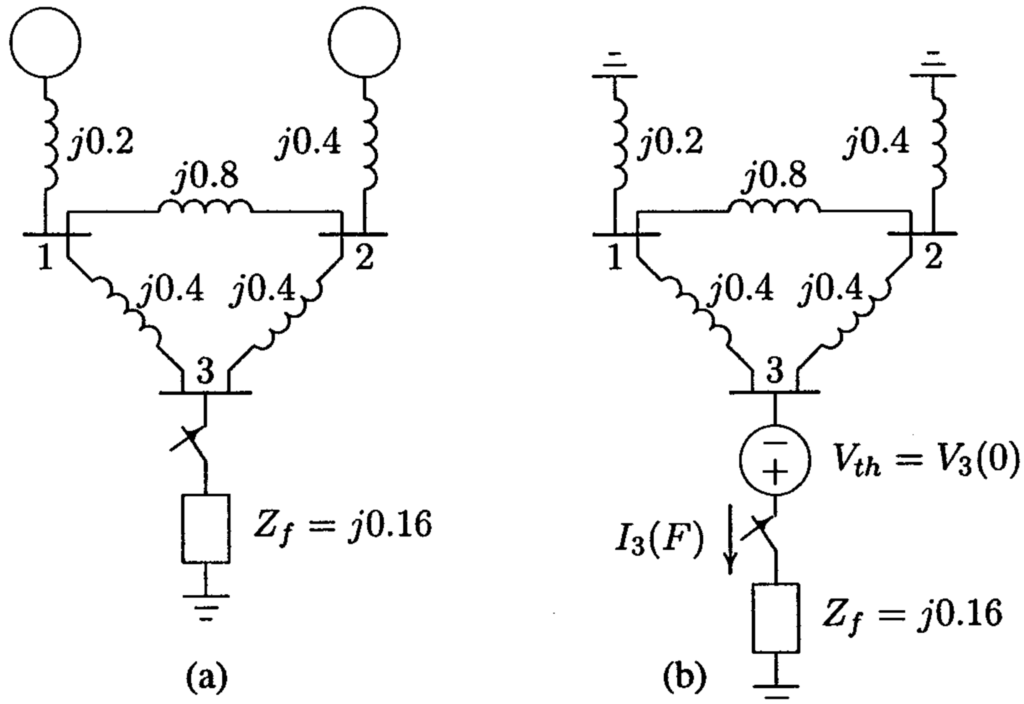


FIGURE 9.2
(a) The impedance network for fault at bus 3. (b) Thévenin's equivalent network.

From 9.2(b), the fault current at bus 3 is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f}$$

Example 1.1 contd.

where $V_3(0)$ is the Thévenin's voltage or the prefault bus voltage. The prefault bus voltage can be obtained from the results of the power flow solution. In this example, since the loads are neglected and generator's emfs are assumed equal to the rated value, all the prefault bus voltages are equal to 1.0 per unit, i.e.,

$$V_1(0) = V_2(0) = V_3(0) = 1.0 \quad \text{pu}$$

Z_{33} is the Thévenin's impedance viewed from the faulted bus.

To find the Thévenin's impedance, we convert the Δ formed by buses 123 to an equivalent Y as shown in Figure 9.3(a).

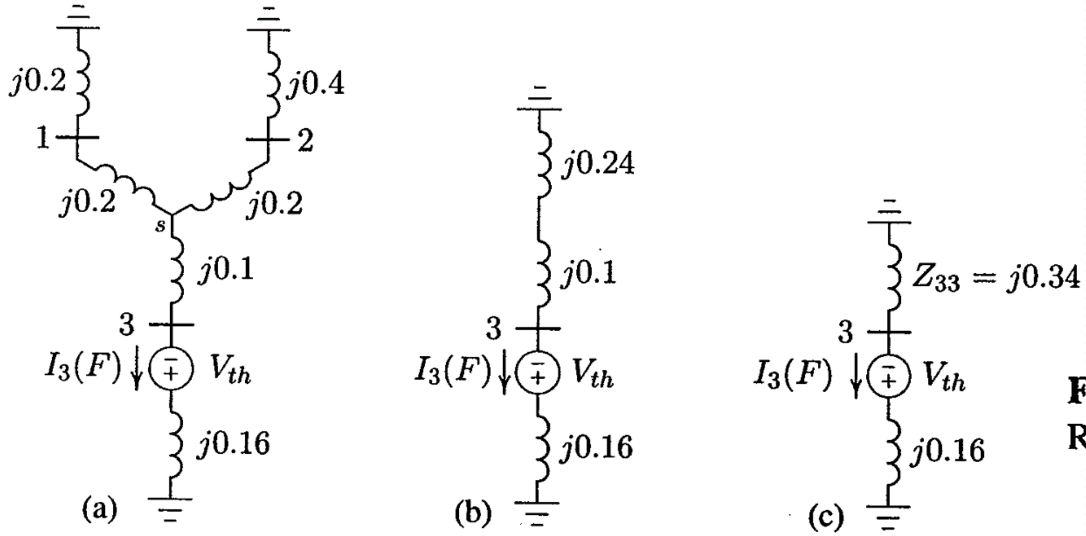


FIGURE 9.3
Reduction of Thévenin's equivalent network.

Example 1.1 contd.

$$Z_{1s} = Z_{2s} = \frac{(j0.4)(j0.8)}{j1.6} = j0.2 \qquad Z_{3s} = \frac{(j0.4)(j0.4)}{j1.6} = j0.1$$

Combining the parallel branches, Thévenin's impedance is

$$\begin{aligned} Z_{33} &= \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1 \\ &= j0.24 + j0.1 = j0.34 \end{aligned}$$

From Figure 9.3(c), the fault current is

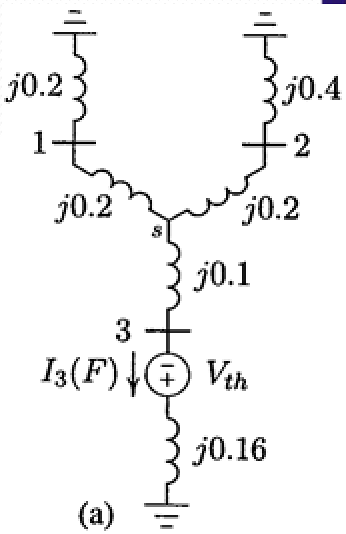
$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

With reference to Figure 9.3(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.6}{j0.4 + j0.6} I_3(F) = -j1.2 \quad \text{pu}$$

$$I_{G2} = \frac{j0.4}{j0.4 + j0.6} I_3(F) = -j0.8 \quad \text{pu}$$

Example 1.1 contd.

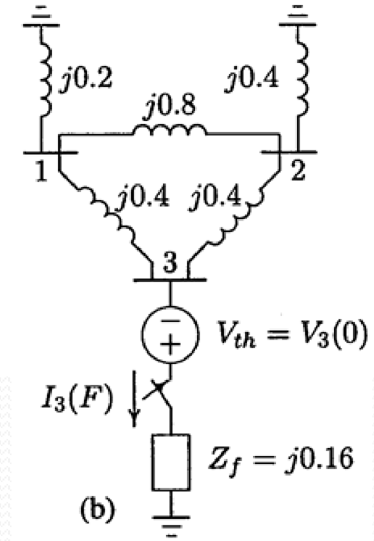


For the bus voltage changes from Figure 9.3(a), we get

$$\Delta V_1 = 0 - (j0.2)(-j1.2) = -0.24 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j0.8) = -0.32 \quad \text{pu}$$
$$\Delta V_3 = (j0.16)(-j2) - 1.0 = -0.68 \quad \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.2(b), i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.24 = 0.76 \quad \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.32 = 0.68 \quad \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.68 = 0.32 \quad \text{pu}$$



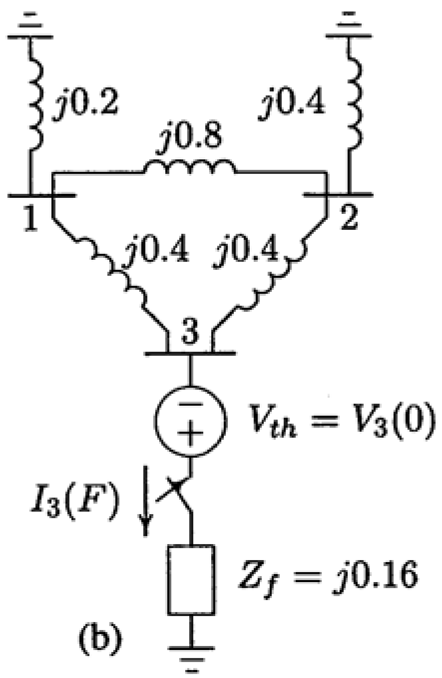
Example 1.1 contd.

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$

$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$



Practice Question 1

Solve the above example now with a symmetrical three-phase fault at bus 2

Practice Question 2

The one-line diagram of a simple power system is shown in Figure 9.20. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.

(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.

(b) Determine the bus voltages and line currents during fault.

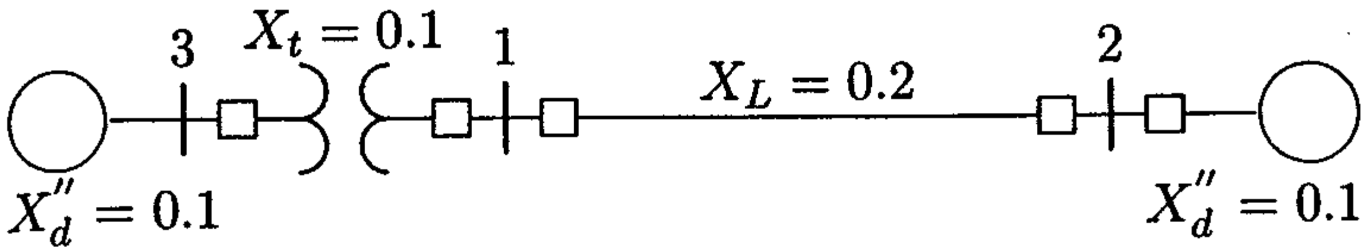
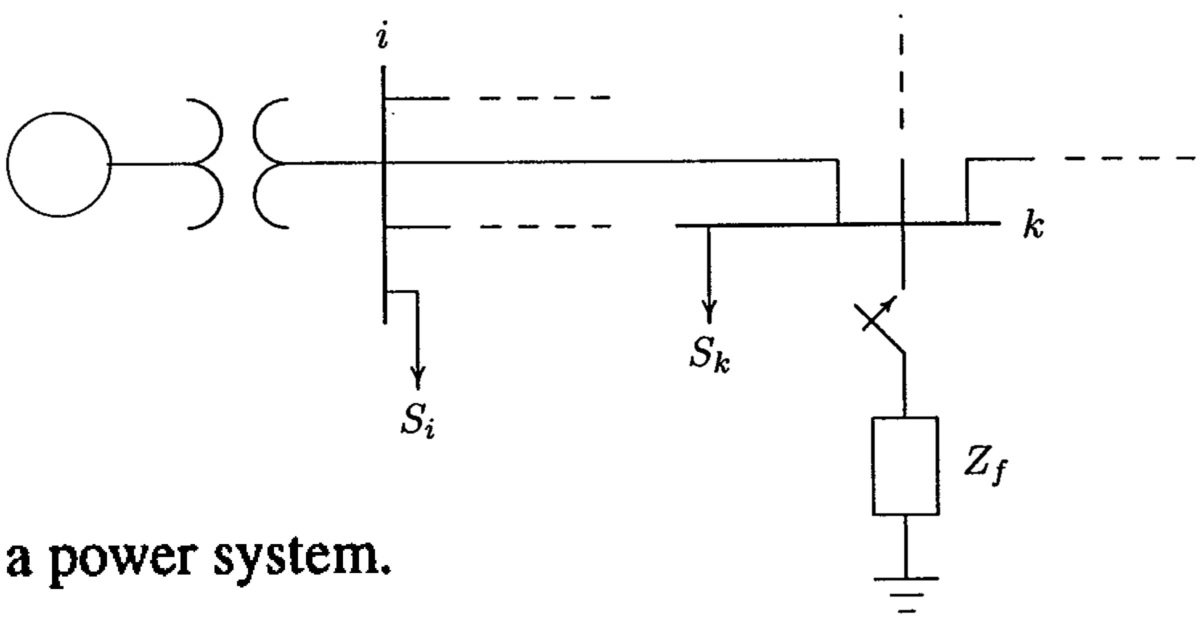


FIGURE 9.20

Bus Impedance Matrix Method

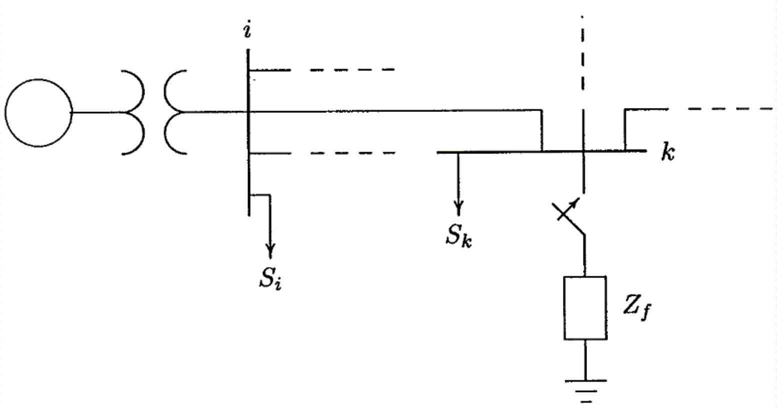
- The bus impedance matrix is convenient to use for fault studies as its diagonal elements are the Thevenin's impedance of the network as seen from different buses.
- Consider the circuit below.



A typical bus of a power system.

Bus Impedance Matrix Method

- A symmetrical fault is applied at bus k through Z_f.
- The prefault bus voltages are obtained from the power flow solution, and are represented by the column vector.



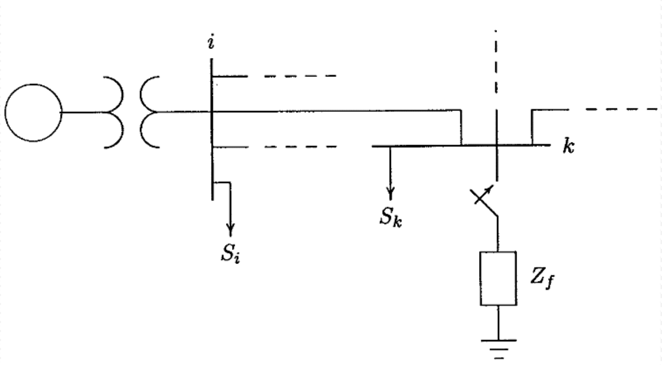
$$\mathbf{V}_{bus}(0) = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix}$$

- We represent the bus load by a constant impedance evaluated at the prefault bus voltage.

$$Z_{i\,L} = \frac{|V_i(0)|^2}{S_L^*}$$

Bus Impedance Matrix Method

- The bus voltage changes caused by the fault is represented by the column vector.



$$\Delta\mathbf{V}_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

- From Thevenin's theorem, bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages, given by

$$\mathbf{V}_{bus}(F) = \mathbf{V}_{bus}(0) + \Delta\mathbf{V}_{bus}$$

- The injected bus currents are expressed in terms of bus voltages (with 0 as ref)

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\mathbf{V}_{bus}$$

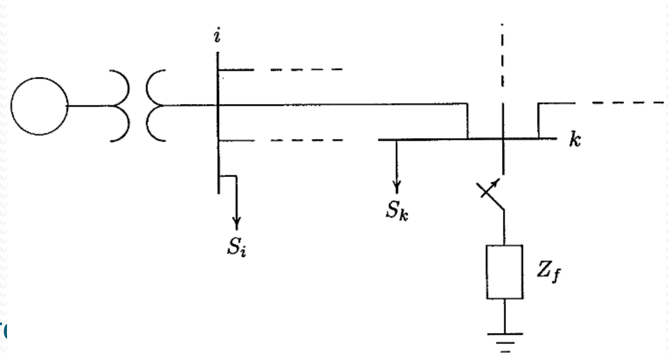
where I_{bus} is the bus current vector entering the bus.

Bus Impedance Matrix Method

- Since the current at faulted bus is leaving the bus, it is taken as a –ve current entering bus k.
- Thus the nodal eqn applied to the Thevenin's circuit becomes

$$\begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & \cdots & y_{1k} & \cdots & y_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{k1} & \cdots & y_{kk} & \cdots & y_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{n1} & \cdots & y_{nk} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

$$\mathbf{I}_{bus}(F) = \mathbf{Y}_{bus} \Delta \mathbf{V}_{bus}$$



Bus Impedance Matrix Method

- Solving for $\Delta \mathbf{V}_{bus}$, we have $$\Delta \mathbf{V}_{bus} = \mathbf{Z}_{bus}\mathbf{I}_{bus}(F)$$
- Thus the bus voltage vector during the fault becomes
- Therefore $$\mathbf{V}_{bus\,(F)} = \mathbf{V}_{bus}(0) + \mathbf{Z}_{bus}\mathbf{I}_{bus}(F)$$

$$\begin{bmatrix} V_1(F) \\ \vdots \\ V_k(F) \\ \vdots \\ V_n(F) \end{bmatrix} = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} + \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix}$$

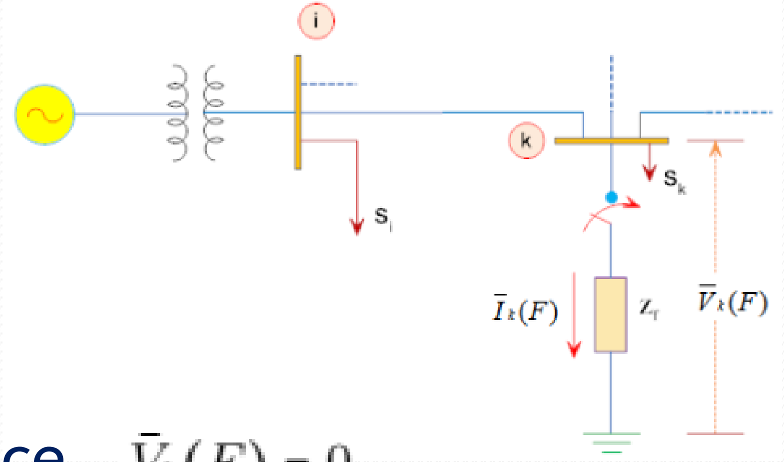
Bus Impedance Matrix Method

- The bus voltage of k^th^ bus can be expressed as:

$$\bar{V}_k(F) = \bar{V}_k(0) - \bar{Z}_{kk}\bar{I}_k(F)$$

- Also from Fig. below

$$\bar{V}_k(F) = \bar{Z}_F\bar{I}_k(F)$$



- For a bolted fault $\bar{Z}_f = 0$ and hence, $\bar{V}_k(F) = 0$
  Thus the fault current for bolted fault can be expressed

$$\bar{I}_k(F) = \frac{\bar{V}_k(0)}{\bar{Z}_{kk}}$$

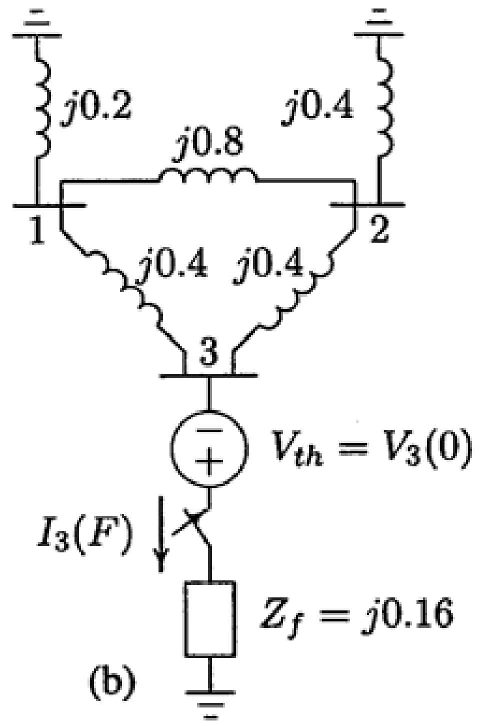
- Then we follow procedure used in Thevenin's method.

BIM Method Example

Solve Example 9.1 using BIM method.

Solution

- To find bus admittance matrix, the Thevenin's circuit in Fig. 9.2(b) is redrawn with impedances converted to admittances as shown below.



- So

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.5 \\ j1.25 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix}$$

BIM Method Example

Inverting the Y_{bus}, we have

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.12)(-j2.0) = 0.76 \quad \text{pu}$$
$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.16)(-j2.0) = 0.68 \quad \text{pu}$$
$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.34)(-j2.0) = 0.32 \quad \text{pu}$$

BIM Method Example

Therefore

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$

$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$

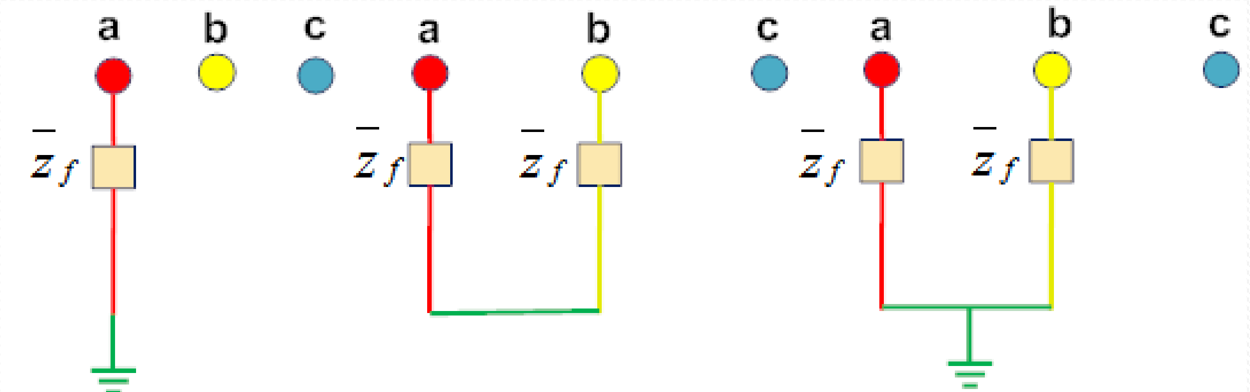
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$

- Note that the values of the diagonal elements in the bus impedance matrix are the same as the Thevenin's impedances found in Example 9.1, thus eliminating the repeated need for ntwrk reduction for each fault location.
- The off-diagonal elements are utilised to obtain bus voltages during the fault.

Unsymmetrical Faults

- Faults in which the balanced state of the network is disturbed are called unsymmetrical or unbalanced faults.
- The most common type of unbalanced fault in a system is a single line to ground fault **(LG fault)**.
- Almost 60 to 75% of faults in a system are LG faults.
- The other types of unbalanced faults are line to line faults (LL faults) and double line to ground faults **(LLG faults)**.
- About 15 to 25% faults are LLG faults and 5 to 15% are **LL faults**.

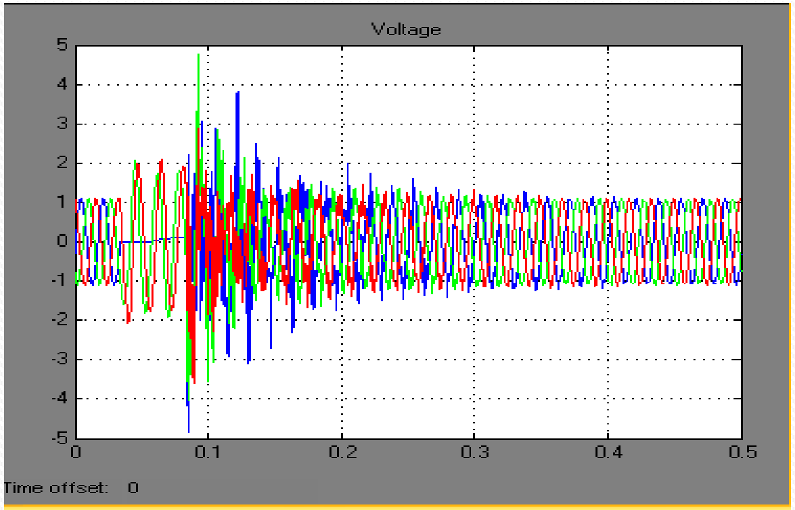
Unsymmetrical Faults



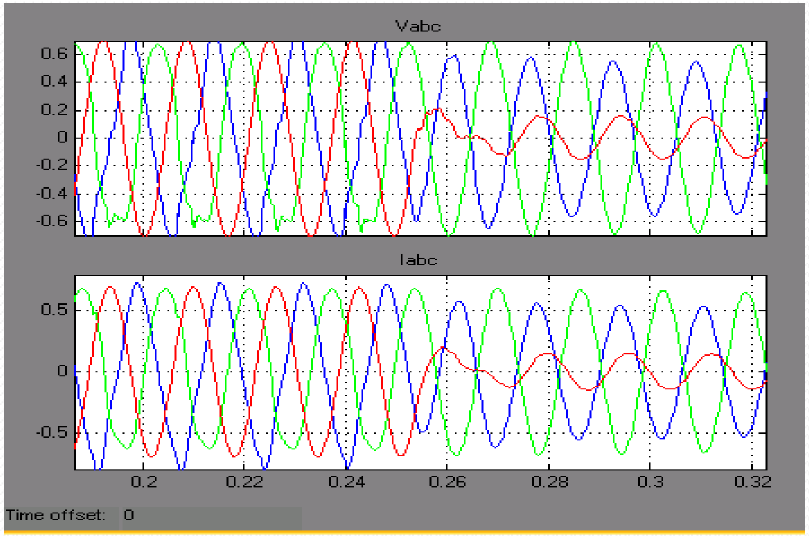
(a) LG fault (b) LL fault (c) LLG fault

Unsymmetrical Faults

- While symmetrical faults are analysed on per phase basis, unsymmetrical faults are analysed using symmetrical components.
- The key idea of symmetrical component analysis is to decompose the system into three sequence networks.
- The networks are then coupled only at the point of the unbalance (i.e., the fault).



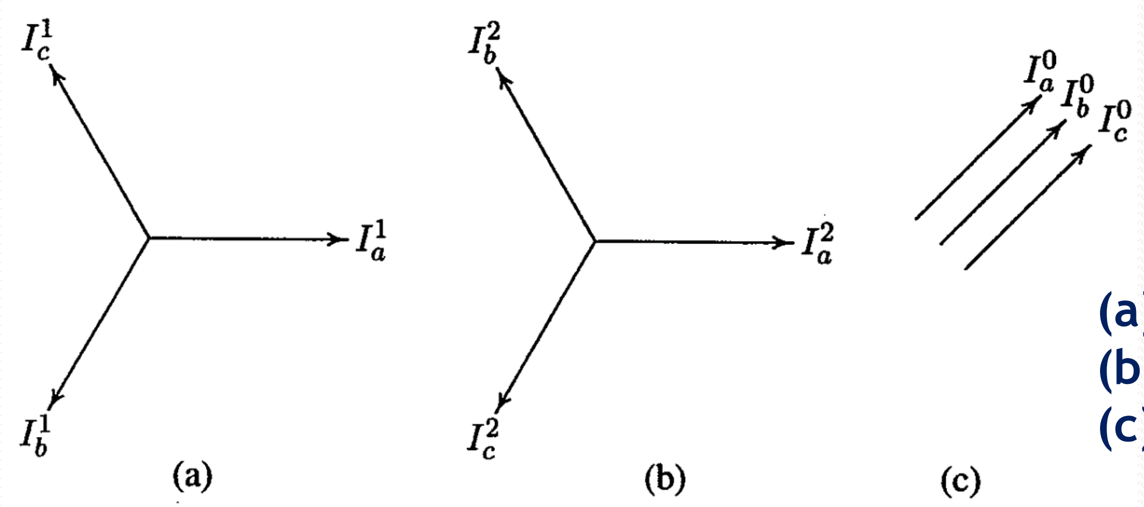
SLG Fault



One phase open circuit Fault

Symmetrical Components

- Any unbalanced set of three phase voltage or current phasors can be replaced by three balanced sets of three phase voltage or current phasors.
- These three balanced set of voltage or current phasors are called symmetrical components of voltages or currents.
- Let I_a, I_b, and I_c be an arbitrary set of three current phasors representing phase currents.



(a) Positive phase sequence
(b) Negative phase sequence
(c) Zero phase sequence

Symmetrical Components

- The positive sequence sets have three phase currents/voltages with equal magnitude, with phase ***b*** lagging phase ***a*** by 120°, and phase ***c*** lagging phase ***b*** by 120°.
- Positive sequence sets have zero neutral current.
- The negative sequence sets have three phase currents/voltages with equal magnitude, with phase ***b*** leading phase ***a*** by 120°, and phase ***c*** leading phase ***b*** by 120°.
- Negative sequence sets are similar to positive sequence, except the phase order is reversed.
- Negative sequence sets also have zero neutral current.
- Zero sequence sets are three vectors that are equal in magnitude and phase.
- Hence they form what is known as uniphase system.

Symmetrical Components

By convention, the direction of rotation of the phasors is taken to be counterclockwise. The three phasors are written as

$$I_a^1 = I_a^1 \angle 0^\circ \quad = I_a^1$$
$$I_b^1 = I_a^1 \angle 240^\circ = a^2 I_a^1$$
$$I_c^1 = I_a^1 \angle 120^\circ = a I_a^1$$

where we have defined an operator a that causes a counterclockwise rotation of 120°, such that

$$a \; = 1\angle 120^\circ = -0.5 + j0.866$$
$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$
$$a^3 = 1\angle 360^\circ = 1 + j0$$

$$1\angle 120^o = 1\left(\cos 120^o + j\sin 120^o\right)$$
$$= -0.5 + j0.866$$

From above, it is clear that

$$1 + a + a^2 = 0$$

Symmetrical Components

Function	Polar	Rectangular
a	$1\angle 120^\circ$	$-0.5 + j0.866$
a^2	$1\angle 240^\circ$	$-0.5 - j0.866$
a^3	$1\angle 0^\circ$	$1.0 + j0$
a^4	$1\angle 120^\circ$	$-0.5 + j0.866$
$1 + a = -a^2$	$1\angle 60^\circ$	$0.5 + j0.866$
$1 + a^2 = -a$	$1\angle -60^\circ$	$0.5 - j0.866$
$1 - a$	$\sqrt{3}\angle -30^\circ$	$1.5 - j0.866$
$1 - a^2$	$\sqrt{3}\angle 30^\circ$	$1.5 + j0.866$
$a - 1$	$\sqrt{3}\angle 150^\circ$	$-1.5 + j0.866$
$a^2 - 1$	$\sqrt{3}\angle -150^\circ$	$-1.5 - j0.866$
$a - a^2$	$\sqrt{3}\angle 90^\circ$	$0.0 + j1.732$
$a^2 - a$	$\sqrt{3}\angle -90^\circ$	$0.0 - j1.732$
$a + a^2$	$1\angle 180^\circ$	$-1.0 + j0$
$1 + a + a^2$	0	0

Symmetrical Components

The negative phase sequence quantities are represented as

$$I_a^2 = I_a^2\angle 0^\circ \quad = I_a^2$$
$$I_b^2 = I_a^2\angle 120^\circ = aI_a^2$$
$$I_c^2 = I_a^2\angle 240^\circ = a^2I_a^2$$

When analyzing certain types of unbalanced faults, it will be found that a third set of balanced phasors must be introduced. These phasors, known as the *zero phase sequence*, are found to be in phase with each other.

Zero phase sequence currents, from the above figure would be designated

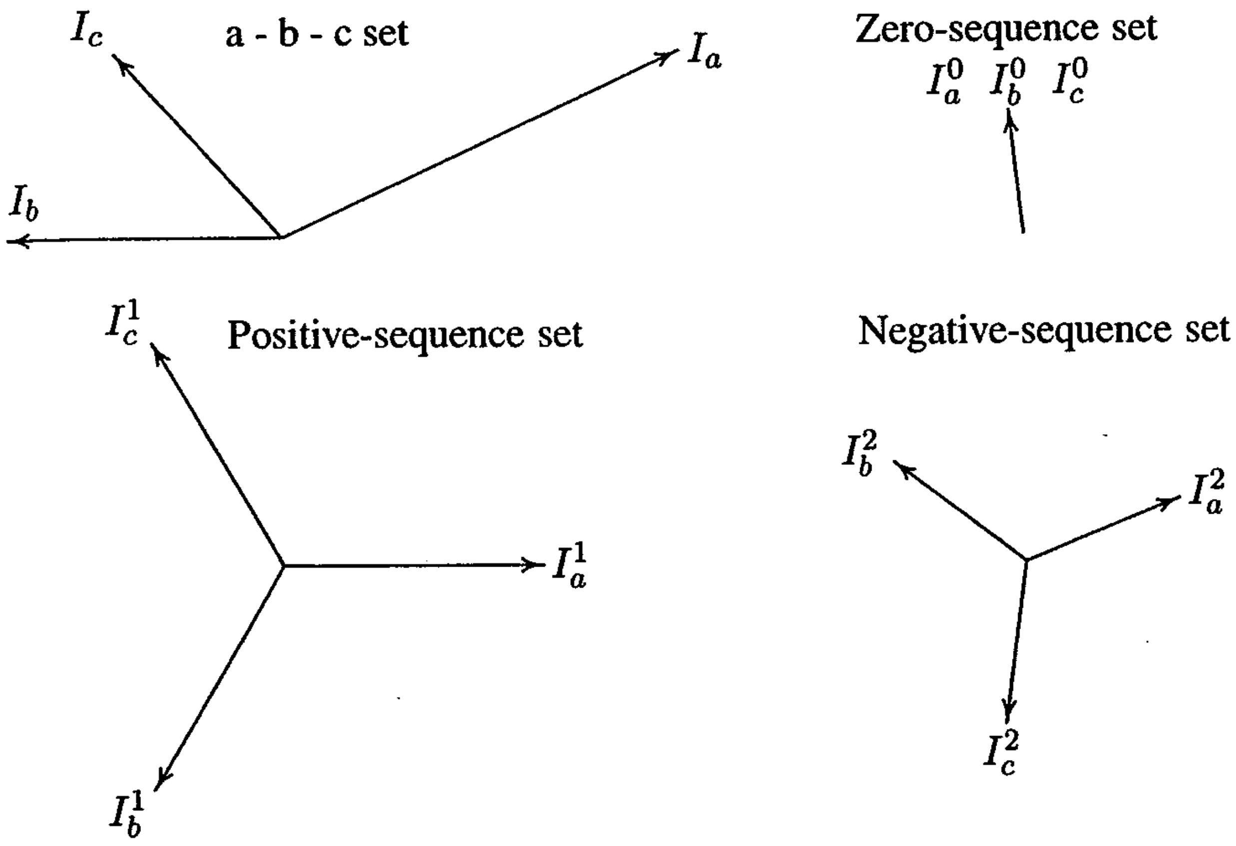
$$I_a^0 = I_b^0 = I_c^0$$

Note that the zero sequence sets have neutral current.

The superscripts 1, 2, and 0 are being used to represent positive, negative, and zero-sequence quantities, respectively.

Symmetrical Components

- Consider the three-phase unbalanced currents I_a, I_b, and I_c shown below.



Resolution of unbalanced phasors into symmetrical components.

Symmetrical Components

- We are seeking to find the three symmetrical components of the currents such that

$$I_a = I_a^0 + I_a^1 + I_a^2$$
$$I_b = I_b^0 + I_b^1 + I_b^2$$
$$I_c = I_c^0 + I_c^1 + I_c^2$$

- We can re-write these in terms of phase ***a*** components according to symmetrical components.

$$I_a = I_a^0 + I_a^1 + I_a^2$$
$$I_b = I_a^0 + a^2 I_a^1 + a I_a^2$$
$$I_c = I_a^0 + a I_a^1 + a^2 I_a^2$$

or

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix}$$

Symmetrical Components

We rewrite the above equation in matrix notation as

$$\mathbf{I}^{abc} = \mathbf{A}\,\mathbf{I}_a^{012} \qquad \textbf{(i)}$$

where $\mathbf{A}$ is known as *symmetrical components transformation matrix* (SCTM) which transforms phasor currents $\mathbf{I}^{abc}$ into component currents $\mathbf{I}_a^{012}$, and is

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad \textbf{(1)}$$

Solving $\mathbf{I}^{abc} = \mathbf{A}\,\mathbf{I}_a^{012}$ for the symmetrical components of currents, we have

$$\mathbf{I}_a^{012} = \mathbf{A}^{-1}\,\mathbf{I}^{abc} \qquad \textbf{(2)}$$

The inverse of $\mathbf{A}$ is given by

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \qquad \textbf{(3)}$$

Symmetrical Components

- From (1) and (3), we conclude that $\mathbf{A}^{-1} = \frac{1}{3}\mathbf{A}^*$ **(4)**
- Substituting for A^{-1} in (2), we have

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

or in component form, the symmetrical components are

$$I_a^0 = \tfrac{1}{3}(I_a + I_b + I_c)$$
$$I_a^1 = \tfrac{1}{3}(I_a + aI_b + a^2 I_c)$$
$$I_a^2 = \tfrac{1}{3}(I_a + a^2 I_b + aI_c)$$

- From this, we note that the zero sequence component of current is equal to one-third of sum of the phase currents.

Symmetrical Components

- Therefore, when the phase currents sum to zero, e.g. in a three-phase system with ungrounded neutral, the zero sequence current does not exist.
- However, if the neutral of the power system is grounded, zero sequence current flows btw the neutral and the ground.
- Similar expressions exist for voltages.
- Thus, the unbalanced phase voltages in terms of the symmetrical components voltages are

$$V_a = V_a^0 + V_a^1 + V_a^2$$
$$V_b = V_a^0 + a^2 V_a^1 + a V_a^2$$
$$V_c = V_a^0 + a V_a^1 + a^2 V_a^2$$

or in matrix notation

$$\mathbf{V}^{abc} = \mathbf{A}\,\mathbf{V}_a^{012} \qquad \textbf{(ii)}$$

Symmetrical Components

- The symmetrical components in terms of the unbalanced voltages are

$$V_a^0 = \frac{1}{3}(V_a + V_b + V_c)$$
$$V_a^1 = \frac{1}{3}(V_a + aV_b + a^2V_c)$$
$$V_a^2 = \frac{1}{3}(V_a + a^2V_b + aV_c)$$

in matrix form $$\mathbf{V}_a^{012} = \mathbf{A}^{-1}\,\mathbf{V}^{abc}$$

- The apparent power may also be expressed in terms of the symmetrical components.
- The three-phase complex power is

$$S_{(3\phi)} = \mathbf{V}^{abc^T}\mathbf{I}^{abc^*} \qquad \textbf{(iii)}$$

- Substituting eqns (i) and (ii) into (iii), we obtain

Symmetrical Components

$$S_{(3\phi)} = \left(\mathbf{A}\mathbf{V}_a^{012}\right)^T \left(\mathbf{A}\mathbf{I}_a^{012}\right)^* = \mathbf{V}_a^{012^T}\mathbf{A}^T\mathbf{A}^*\mathbf{I}_a^{012^*}$$

$$\mathbf{A}^{-1} = \frac{1}{3}\mathbf{A}^* \quad (4)$$

- Since $A^T=A$, then from eqn (4), $A^TA^*=3$, and the complex power becomes

$$S_{(3\phi)} = 3\left(\mathbf{V}^{012^T}\mathbf{I}^{012^*}\right) = 3V_a^0 I_a^{0^*} + 3V_a^1 I_a^{1^*} + 3V_a^2 I_a^{2^*}$$

- This eqn shows that the total unbalanced power can be obtained from the sum of the symmetrical components powers.
- Often the subscript ***a*** of the symmetrical components are omitted, e.g., I^0, I^1, and I^2 are understood to refer to phase ***a***.

Symmetrical Components

Example 1

$$\text{Let } \mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 10\angle -120^\circ \\ 10\angle 120^\circ \end{bmatrix} \qquad \text{Then}$$

$$\mathbf{I}_s = \mathbf{A}^{-1}\mathbf{I} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 10\angle 0^\circ \\ 10\angle -120^\circ \\ 10\angle 120^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 10\angle 0^\circ \\ 0 \end{bmatrix}$$

$$\text{If } \quad \mathbf{I} = \begin{bmatrix} 10\angle 0^\circ \\ 10\angle +120^\circ \\ 10\angle -120^\circ \end{bmatrix} \quad \rightarrow \quad \mathbf{I}_s = \begin{bmatrix} 0 \\ 0 \\ 10\angle 0^\circ \end{bmatrix}$$

Symmetrical Components

Example 2

$$\text{Let } \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 5\angle 90° \\ 8\angle 150° \\ 8\angle -30° \end{bmatrix}$$

Then

$$\mathbf{V}_s = \mathbf{A}^{-1}\mathbf{V} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 5\angle 90° \\ 8\angle 150° \\ 8\angle -30° \end{bmatrix} = \begin{bmatrix} 1.67\angle 90° \\ 3.29\angle -135° \\ 6.12\angle 68° \end{bmatrix}$$

Symmetrical Components

Example 3

$$\text{Let } \mathbf{I}_s = \begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ -10\angle 0° \\ 5\angle 0° \end{bmatrix}$$

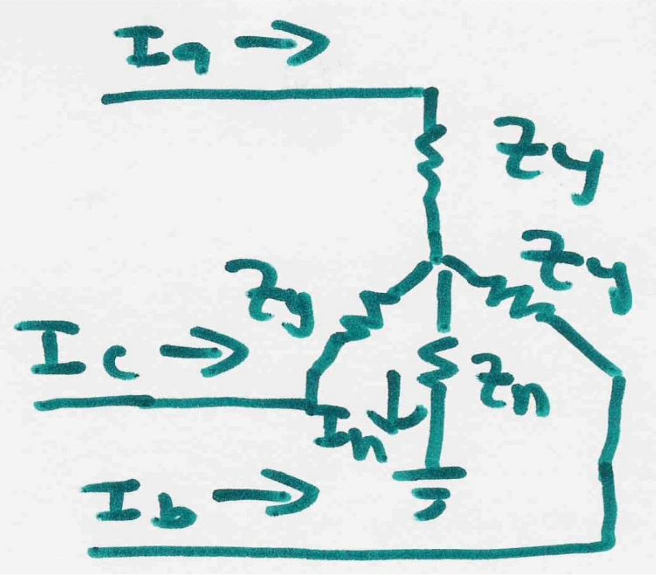
Then

$$\mathbf{I} = \mathbf{A}\mathbf{I}_s = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 10\angle 0° \\ -10\angle 0° \\ 5\angle 0° \end{bmatrix} = \begin{bmatrix} 5.0\angle 0° \\ 18.0\angle 46.1° \\ 18.0\angle -46.1° \end{bmatrix}$$

Questions?

Use of Symmetrical Components

- Consider the following wye-connected load:



$$I_n = I_a + I_b + I_c$$

$$V_{ag} = I_a Z_y + I_n Z_n$$

$$V_{ag} = (Z_Y + Z_n) I_a + Z_n I_b + Z_n I_c$$

$$V_{bg} = Z_n I_a + (Z_Y + Z_n) I_b + Z_n I_c$$

$$V_{cg} = Z_n I_a + Z_n I_b + (Z_Y + Z_n) I_c$$

Z_n is the impedance in the neutral circuit which is grounded and draws current I_n.

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Use of Symmetrical Components

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\mathbf{V} = \mathbf{Z}\,\mathbf{I} \qquad \mathbf{V} = \mathbf{A}\,\mathbf{V}_s \qquad \mathbf{I} = \mathbf{A}\,\mathbf{I}_s$$

$$\mathbf{A}\,\mathbf{V}_s = \mathbf{Z}\,\mathbf{A}\,\mathbf{I}_s \quad \rightarrow \quad \mathbf{V}_s = \mathbf{A}^{-1}\,\mathbf{Z}\,\mathbf{A}\,\mathbf{I}_s$$

$$\mathbf{A}^{-1}\,\mathbf{Z}\,\mathbf{A} = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix}$$

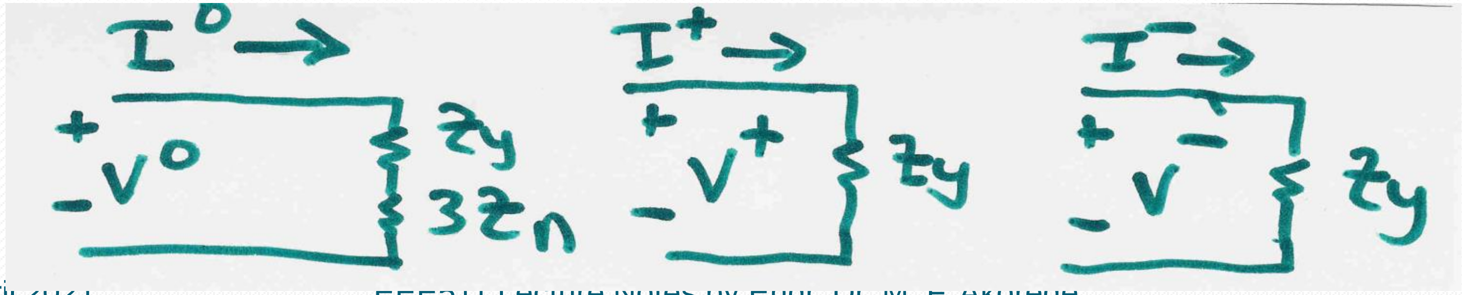
Use of Symmetrical Components

$$\begin{bmatrix} V^0 \\ V^+ \\ V^- \end{bmatrix} = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \begin{bmatrix} I^0 \\ I^+ \\ I^- \end{bmatrix}$$

Systems are decoupled

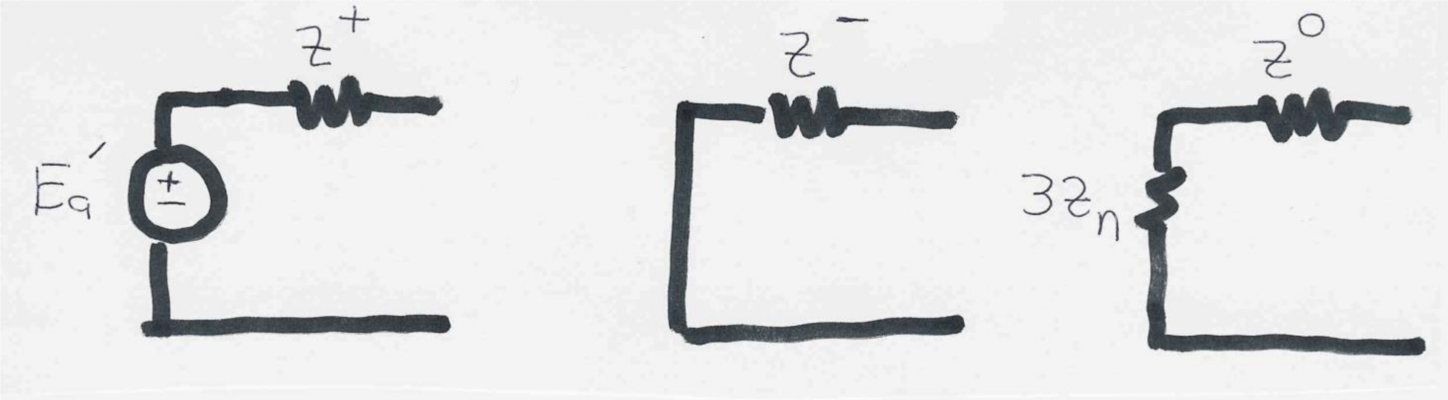
$$V^0 = (Z_y + 3Z_n)\, I^0 \qquad V^+ = Z_y\, I^+$$

$$V^- = Z_y\, I^-$$



Sequence Diagrams for Generators

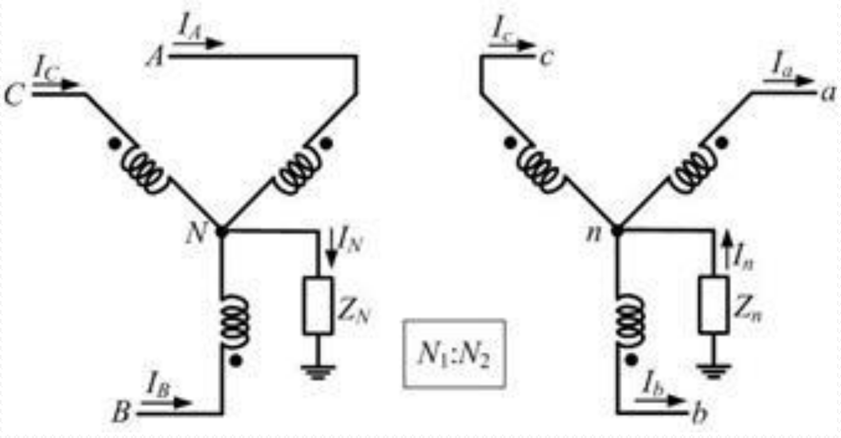
- Key point: generators only produce positive sequence voltages; therefore only the positive sequence has a voltage source.



- During a fault, $Z^+ \approx Z^- \approx X_d''$.
- The zero sequence impedance is usually substantially smaller.
- The value of Z_n depends on whether the generator is grounded or not.

Sequence Diagrams for Transformers

- The positive and negative sequence diagrams for transformers are similar to those for transmission lines.
- The zero sequence network depends upon both how the transformer is grounded and its type of connection.
- The easiest to understand is a double grounded wye-wye.



- The turns ratio of the transformer is given by $\alpha = N_1 : N_2$.

Sequence Diagrams for Transformers

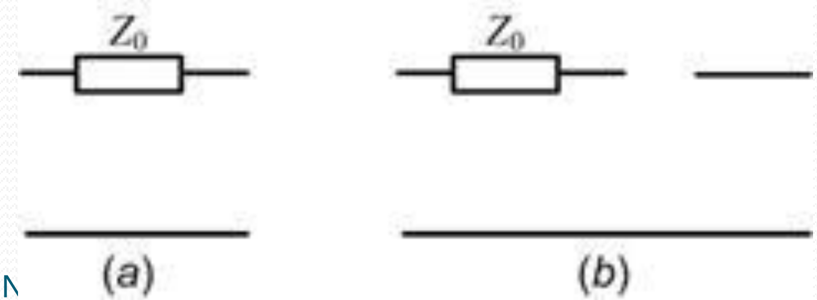
- The voltage of phase-a of the primary side is

$$V_A = V_{AN} + V_N = V_{AN} + 3Z_N I_{A0}$$

- The total zero sequence impedance is given by

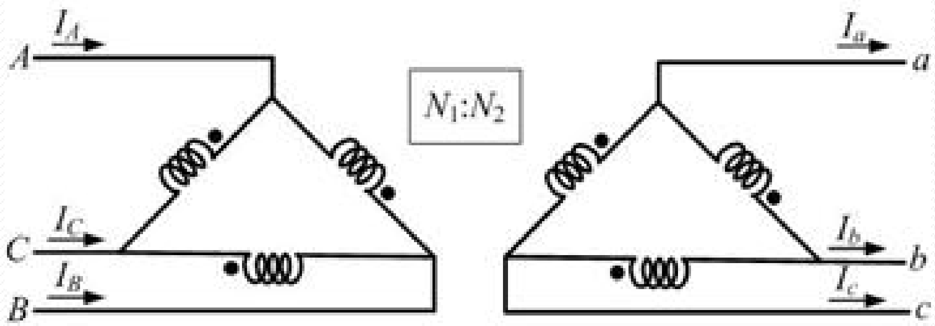
$$Z_0 = Z + 3Z_N + 3(N_1/N_2)^2 Z_n$$

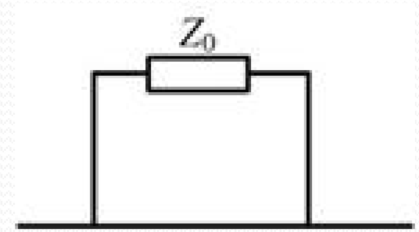
- The zero sequence diagram of the grounded neutral Y-Y connected transformer is shown below.
- If both the neutrals are solidly grounded, i.e., $Z_n = Z_N = 0$, then Z_0 is equal to Z, we have (a).
- If however one of the two neutrals or both neutrals are ungrounded, then we have either $Z_n = \infty$ or $Z_N = \infty$ or both, then we have (b).



Sequence Diagrams for Transformers

- The schematic diagram of a Δ - Δ connected transformer is shown.

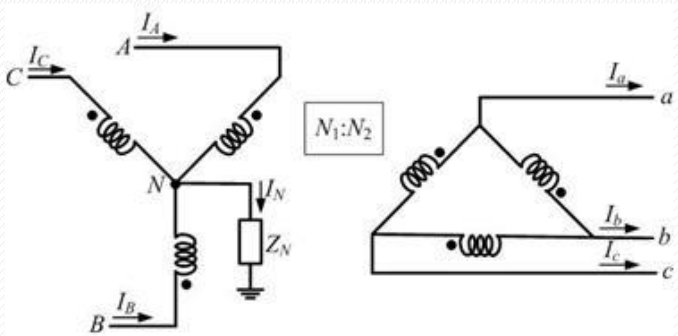




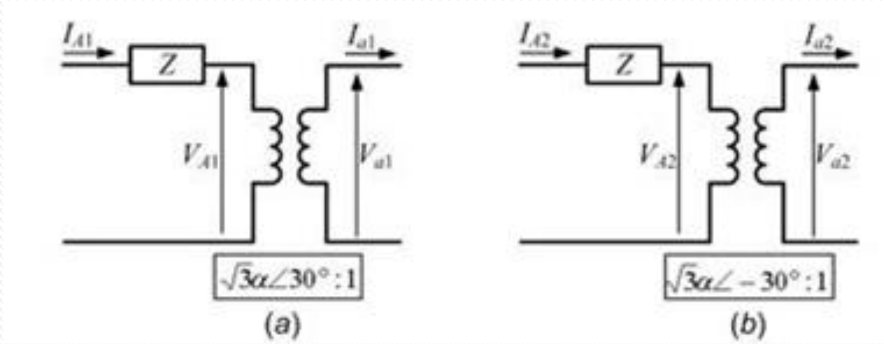
- Thus the positive and negative sequence equivalent circuits are represented by a series impedance that is equal to the leakage impedance of the transformer.
- Since the Δ -connected winding does not provide any path for the zero sequence current to flow, $I_{A0} = I_{a0} = 0$.
- However the zero sequence current can sometimes circulate within the Δ windings.
- We can then draw the zero sequence equivalent circuit as shown on the top right Fig.

Sequence Diagrams for Transformers

- The schematic diagram of a Y- Δ connected transformer is shown.

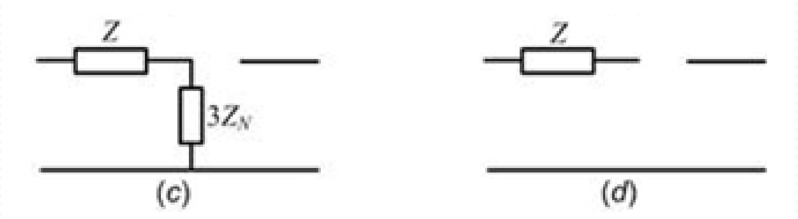


- The positive sequence equivalent cct is shown in Fig. (a).
- The negative sequence circuit is the same as that of the positive sequence circuit except for the phase shift in the induced emf.
- This is shown in Fig. (b).

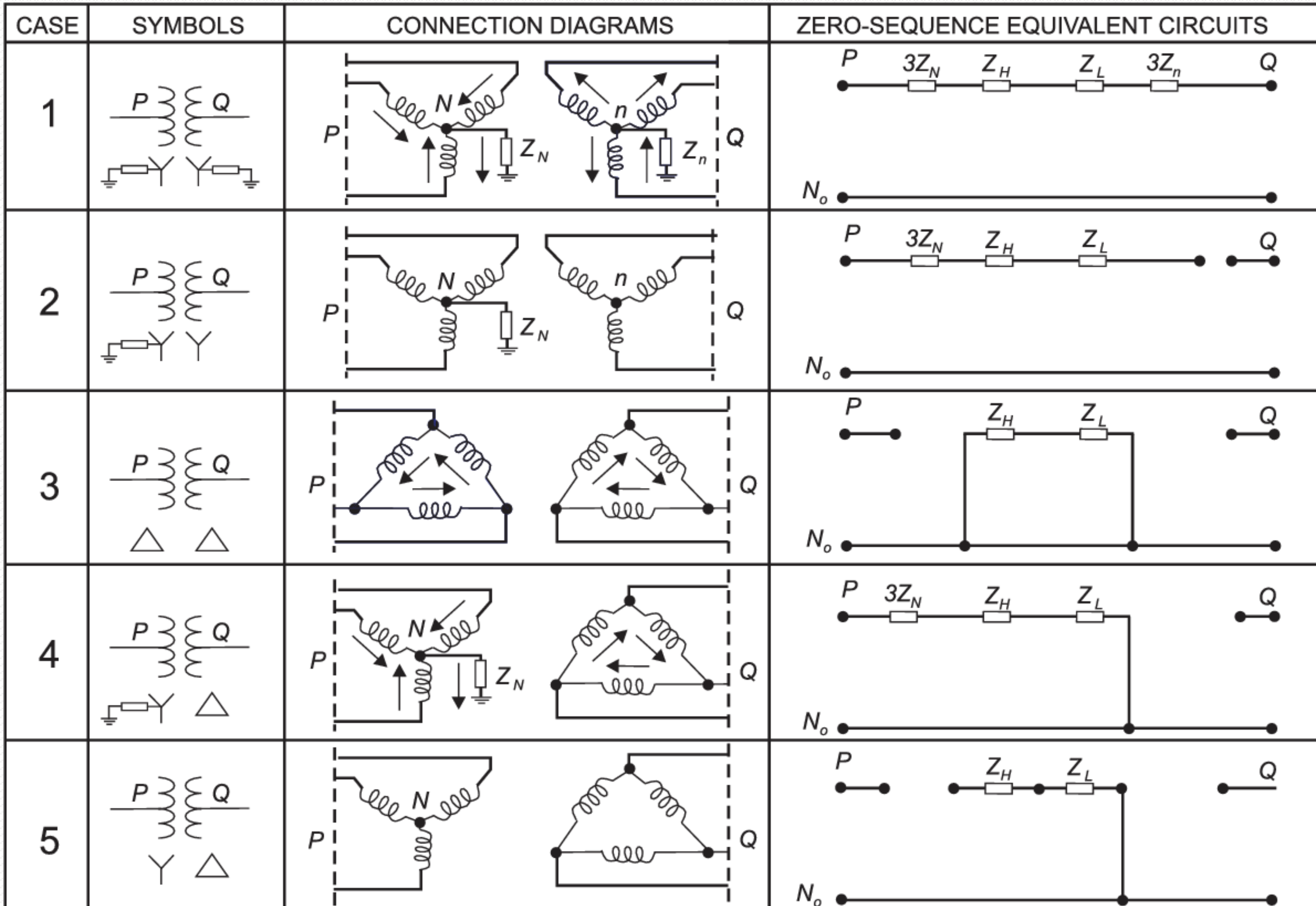


Sequence Diagrams for Transformers

- The zero sequence equivalent circuit is shown in Fig. (c) where $Z_0 = Z + 3Z_N$.
- Note that the pry and sec sides are not connected and hence there is an open circuit between them.
- However since the zero sequence current flows through primary windings, a return path is provided through the ground.
- If however, the neutral in the primary side is not grounded, i.e., ZN = ∞ , then the zero sequence current cannot flow in the primary side as well.
- The sequence diagram is then as shown in Fig. (d) where $Z_0 = Z$.

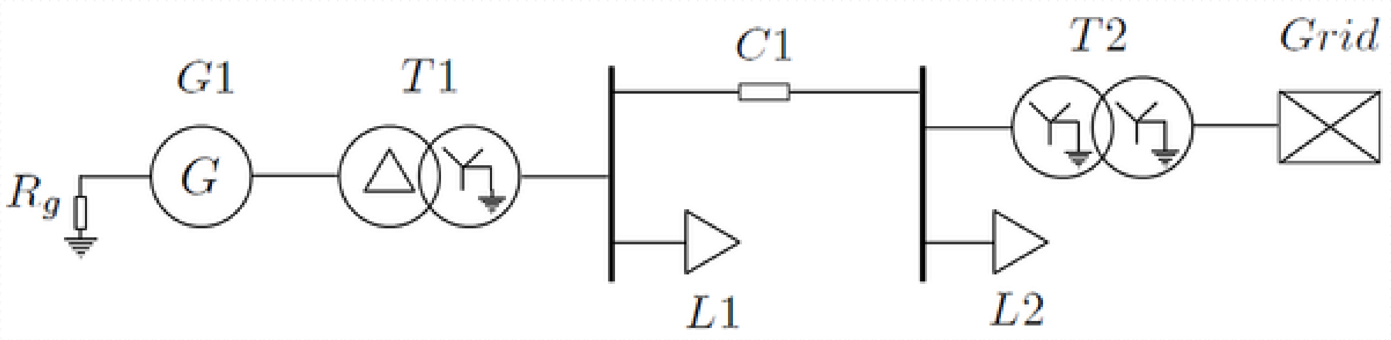


Transformer Sequence Diagrams



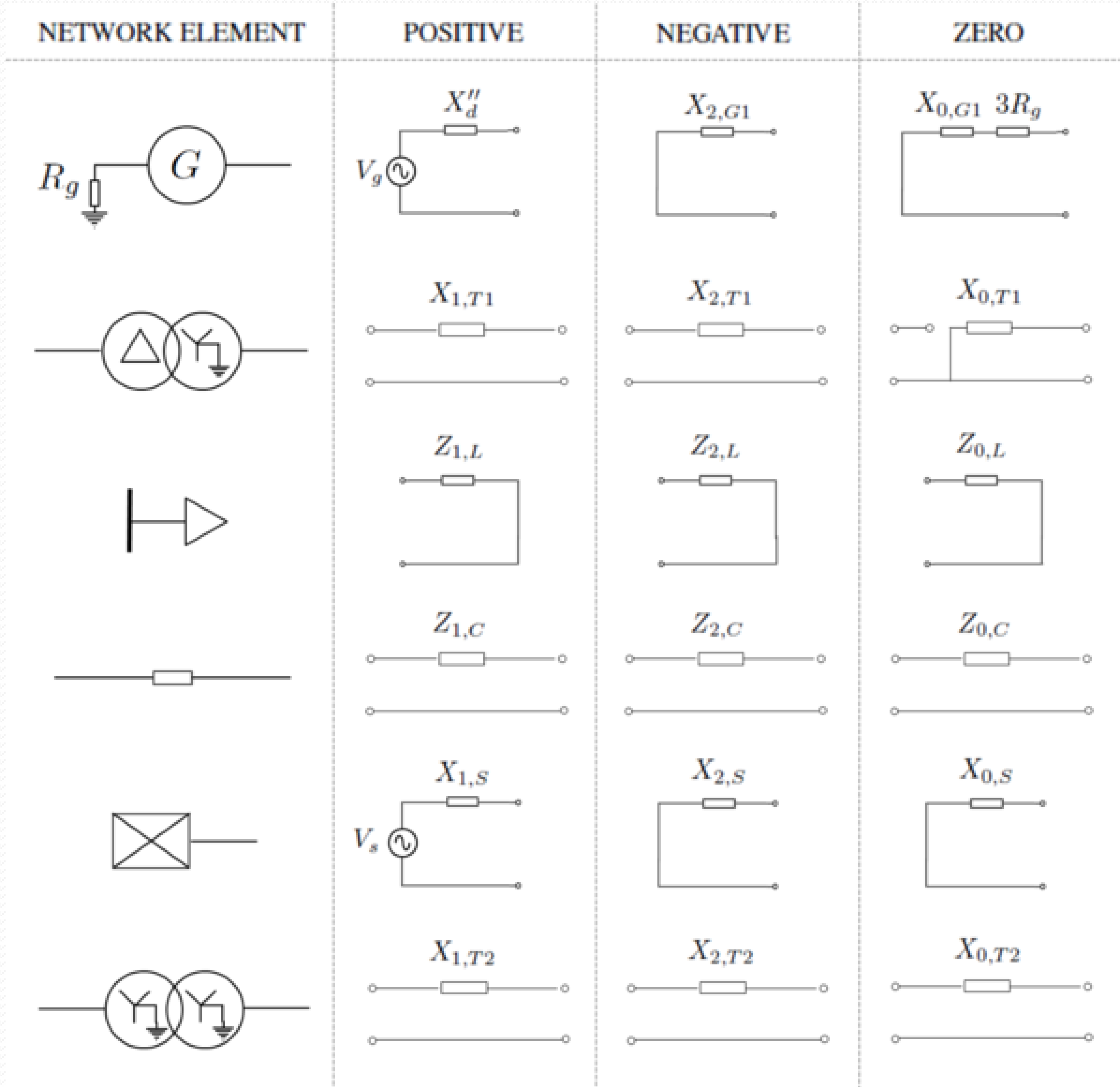
Constructing Sequence Networks

- Sequence networks are constructed from two-port sequence networks of individual elements. This is best illustrated by example. Given the system below:



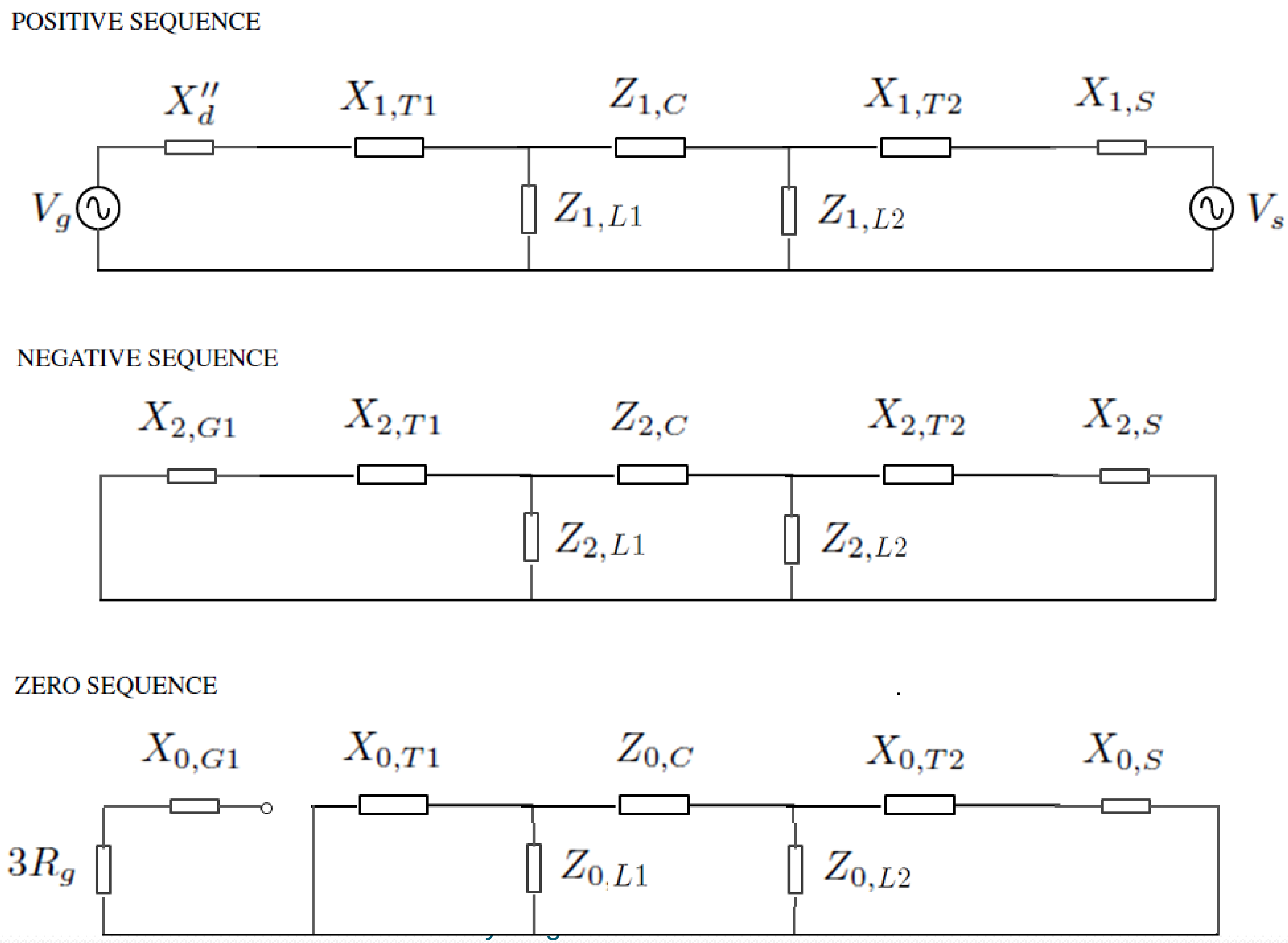
- The individual positive, negative and zero sequence networks for each of the network elements are shown in the figure below:

Constructing Sequence Networks



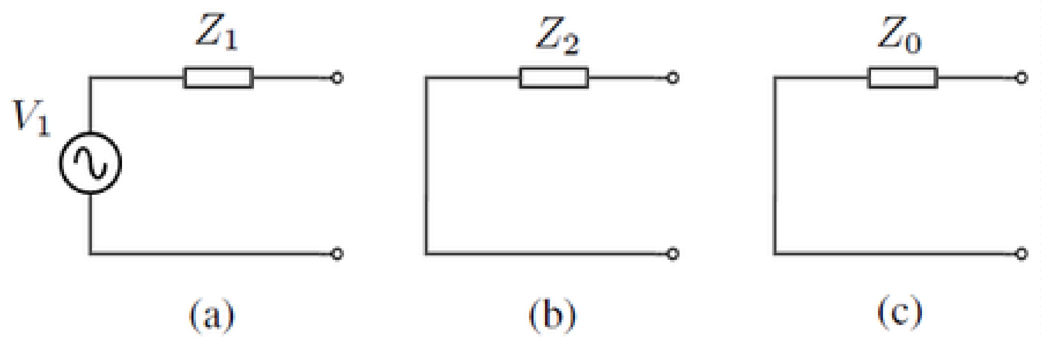
Constructing Sequence Networks

- These individual networks are then connected together to form the positive, negative and zero sequence networks for the overall system:



Constructing Sequence Networks

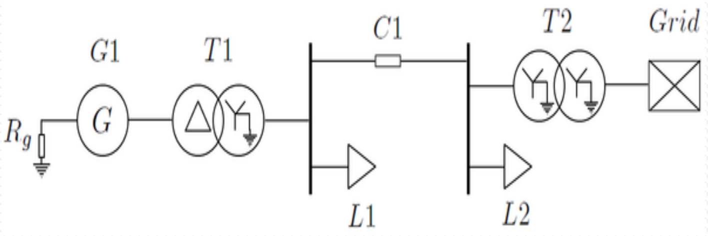
- Consider a fault on the grid side of cable C1 of the original network.
- The sequence networks can be reduced to Thevenin's equivalent networks:



$$Z_1 = \left[\left(X_d'' + X_{1,T1}\right) || Z_{1,L1} + Z_{1,C}\right] || \left(X_{1,T2} + X_{1,S}\right) || Z_{1,L2}$$

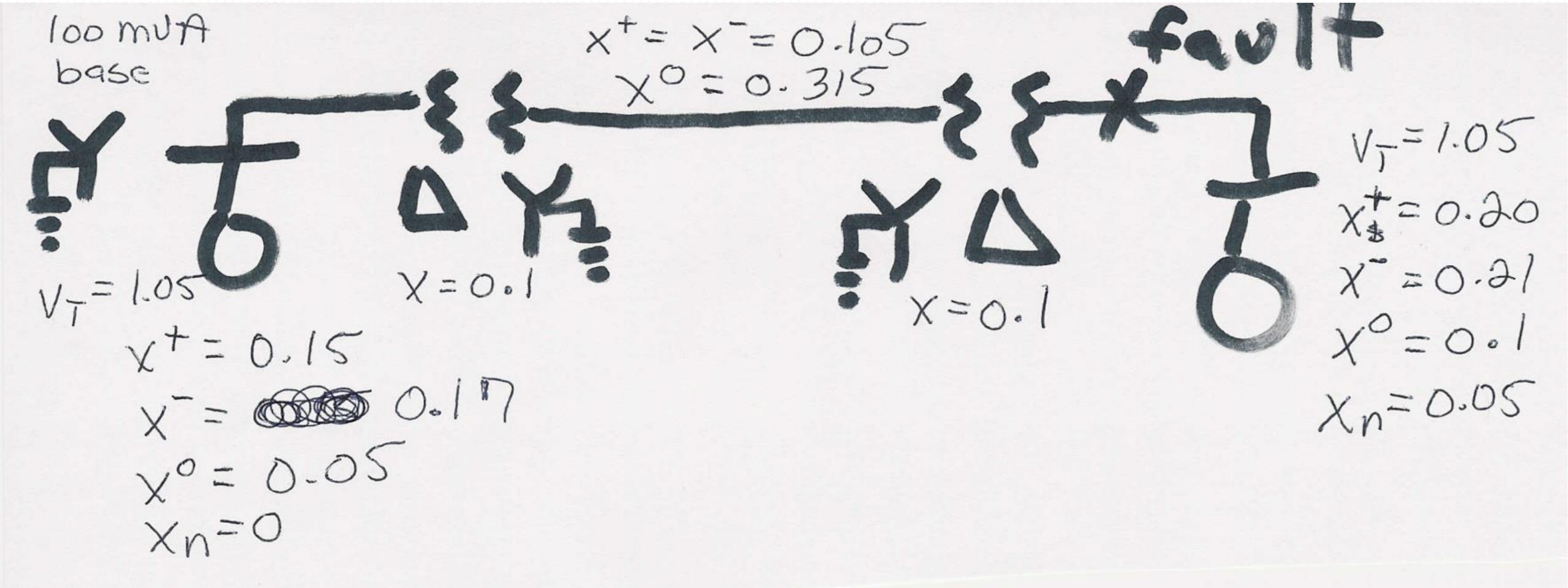
$$Z_2 = \left[\left(X_{2,G1} + X_{2,T1}\right) || Z_{2,L1} + Z_{2,C}\right] || \left(X_{2,T2} + X_{2,S}\right) || Z_{2,L2}$$

$$Z_0 = \left(X_{0,T1} || Z_{0,L1} + Z_{0,C}\right) || \left(X_{0,T2} + X_{0,S}\right) || Z_{0,L2}$$



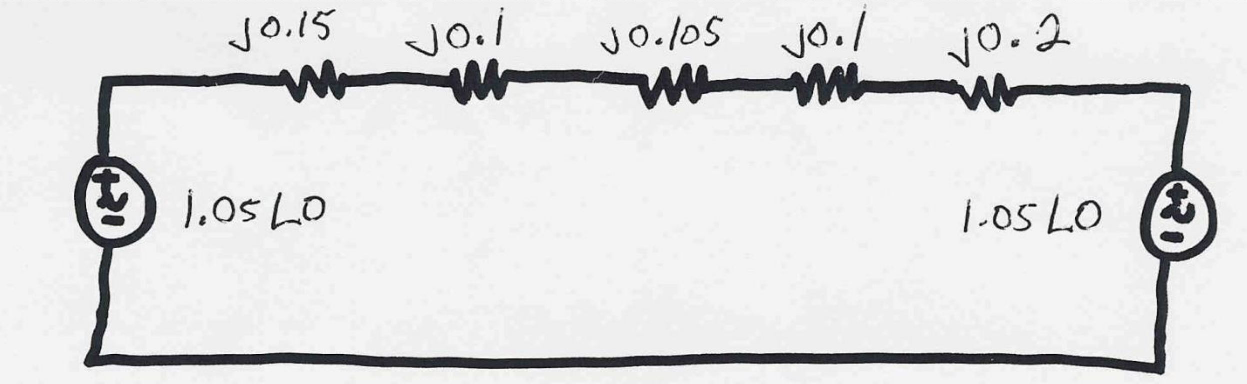
Unbalanced Fault Analysis

- The first step in the analysis of unbalanced faults is to assemble the three sequence networks.
- For example, for a single generator, single motor example let's develop the sequence networks.

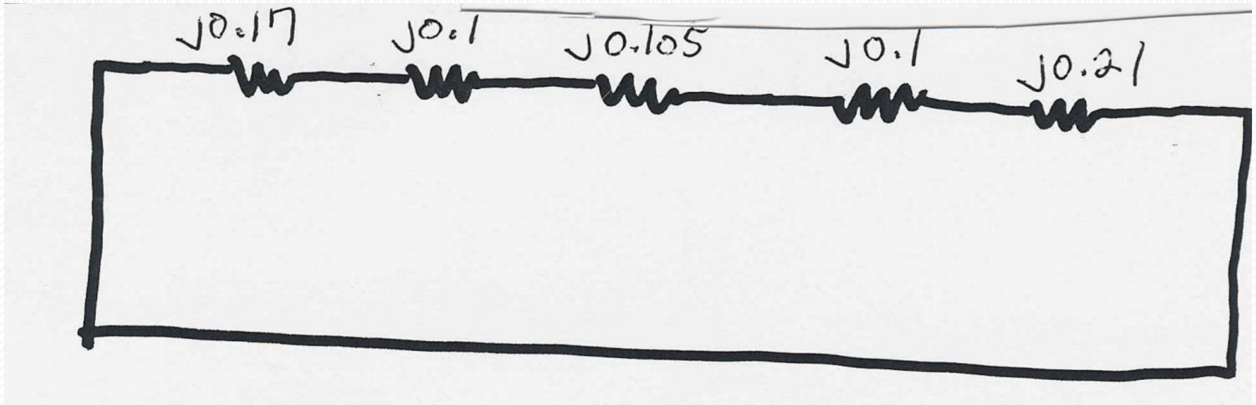


Sequence Diagrams for Example

Positive Sequence Network

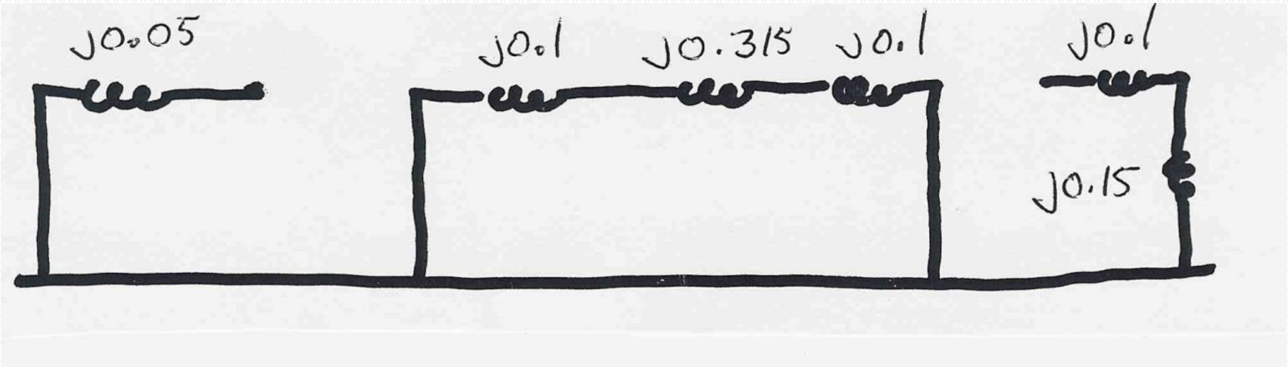


Negative Sequence Network



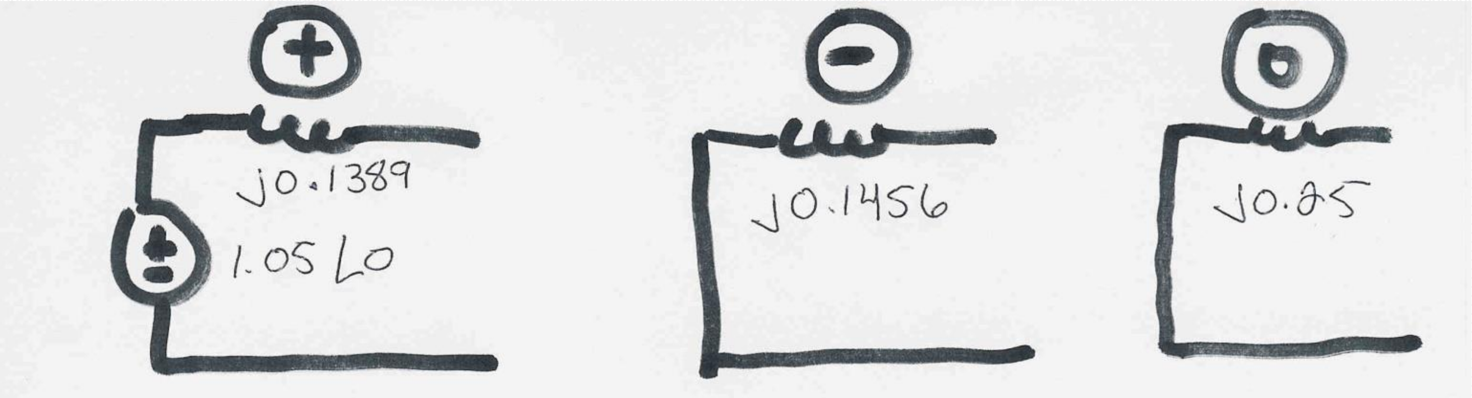
Sequence Diagrams for Example

Zero Sequence Network



Create Thevenin Equivalents

- To do further analysis, we first need to calculate the thevenin equivalents as seen from the fault location.
- In this example the fault is at the terminal of the right machine, so the thevenin equivalents are:



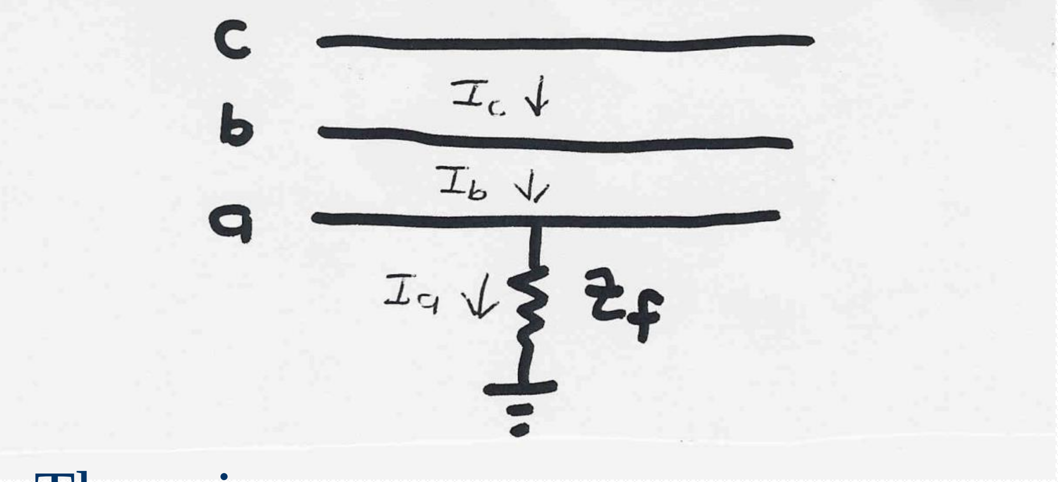
$$Z_{th}^{+} = j0.2 \text{ in parallel with j0.455}$$

$$Z_{th}^{-} = j0.21 \text{ in parallel with j0.475}$$

Single Line-to-Ground (SLG) Faults

- Power system operates under balanced steady-state conditions before the fault occurs.
- Therefore, the positive, negative and zero seq. networks are uncoupled before the occurrence of the fault.
- So when unbalanced faults occur, they unbalance the network, but only at the fault location.
- This causes a coupling of the sequence networks.
- How the sequence networks are coupled depends upon the fault type.
- With a SLG fault, only one phase has non-zero fault current -- we'll assume it is phase A.

SLG Faults contd.



$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \begin{bmatrix} ? \\ 0 \\ 0 \end{bmatrix}$$

Then since

$$\begin{bmatrix} I_f^0 \\ I_f^+ \\ I_f^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} ? \\ 0 \\ 0 \end{bmatrix} \rightarrow I_f^0 = I_f^+ = I_f^- = \frac{1}{3} I_a^f$$

SLG Faults contd.

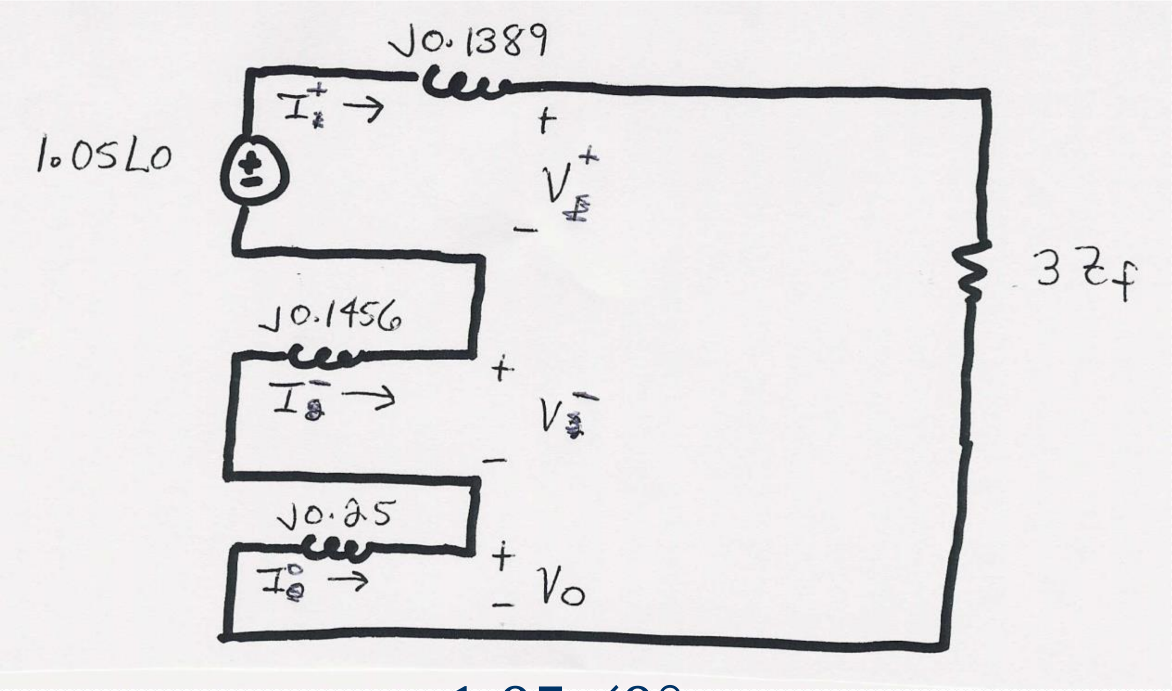
$$V_a^f = Z_f I_a^f$$

$$\begin{bmatrix} V_a^f \\ V_b^f \\ V_c^f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix}$$

This means $V_a^f = V_f^0 + V_f^+ + V_f^-$

The only way these two constraints can be satisified is by coupling the sequence networks in series

SLG Faults contd.



With the sequence networks in series, we can solve for the fault currents (assume Z_f=0)

$$I_f^+ = \frac{1.05\angle 0^\circ}{j(0.1389+0.1456+0.25+3Z_f)} = -j1.964 = I_f^- = I_f^0$$

$$\mathbf{I} = \mathbf{A}\,\mathbf{I}_s \rightarrow I_a^f = -j5.8 \;\; \text{(of course, } I_b^f = I_c^f = 0\text{)}$$

Fault Current Formulae

Three Phase Fault :- For a Three Phase fault only Positive Sequence Network is considered. The fault currents are given by the following equations.

- $I_1 = \frac{V}{Z_1}$ $\quad$ *(solid Fault)*
- $I_1 = \frac{V}{Z_1+Z_f}$ $\quad$ *(Fault Through impendenceZ_f)*

Single Line To Ground Fault(SLG) :- The Positive Sequence, negetive Sequence and Zero Sequence Fault currents are given by

- $I_1 = I_2 = I_0 = \frac{V}{Z_1+Z_2+Z_0}$ $\quad$ *(Solid Fault)*
- $I_1 = I_2 = I_0 = \frac{V}{Z_1+Z_2+Z_0+3Z_f}$ $\quad$ *(Fault Through impendence Z_f)*
- $I_{aF} = I_1 + I_2 + I_0 = 3I_1 = 3I_2 = 3I_0$

Fault Current Formulae

LL fault :- The Zero Sequence Data is not required for this fault.

- $I_1 = -I_2 = \frac{V}{Z_1+Z_2}$ (*solid Fault*)
- $I_1 = -I_2 = \frac{V}{Z_1+Z_2+Z_f}$ (*Fault Through impendence* Z_f)

Line to Line Ground Fault (LLG) :-

1. solid Fault :-
   - $I_1 = \frac{V}{Z_1+\frac{z_2 Z_0}{Z_2+Z_0}}$
   - $I_2 = -I_1 \frac{Z_0}{Z_2+Z_0}$
   - $I_0 = -I_1 \frac{Z_2}{Z_2+Z_0}$

Fault Current Formulae

2. Fault Through impendence Z_F

- $I_1 = \dfrac{V}{Z_1 + \frac{Z_F}{2} + \frac{(Z_2 + \frac{Z_F}{2})(Z_2 + \frac{Z_F}{2} + 3Z_{FG})}{Z_2 + Z_0 + Z_F + 3Z_{FG}}}$
- $I_2 = -I_1 \dfrac{(Z_2 + \frac{Z_F}{2} + 3Z_{FG})}{Z_2 + Z_0 + Z_F + 3Z_{FG}}$
- $I_0 = -I_1 \dfrac{(Z_2 + \frac{Z_F}{2})}{Z_2 + Z_0 + Z_F + 3Z_{FG}}$

Z_f is Fault impedence between the lines,
While Z_{FG} is the Fault impendence to Ground.

Questions?