ELEMENTARY SET THEORY

1) When the number of elements in a given set is finite, the set is called a FINITE SET

2) State whether or not the ff sets are equal

i) $A = \{a, b, d, c\}$ & $B = \{e, d, a, c\}$

ii) $F = \{1, 2, 5\}$ & $H = \{5, 1, 2\}$

Solution

i) Since $A \not\subset B$ and $B \not\subset A$

Therefore $A = B$

ii) Looking at the sets above

$F \subset H$ and $H \subset F$.

Therefore $F = H$

Note :- '$\subset$' means 'is a subset of'.

3) What is the cardinality of the set $A = \{5, 4, 7, 3, 1, 0\}$

Ans

The cardinality of a given set is the number of elements present in the set. The cardinality of A is 6.

4) Given that A is the subset of the universal set $U = \{a, b, c, d, e, f\}$. $A = \{c, e, f\}$. Find the complement of set A.

Ans:

The complement of A are the elements in the universal set that are not elements of A. It is denoted A'

$A' = \{a, b, d\}$.

5) Given that,

$G = \{h, e, a, p\}$

$H = \{1, a, k, e\}$

Find $G \cup H$

Ans

The union of sets G and H, is the set which consists of elements that are either in G or H or both.

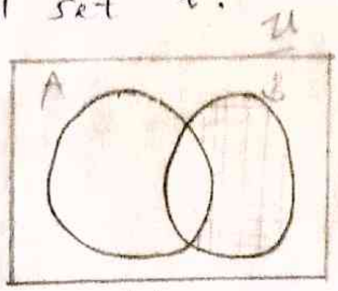
$G \cup H = \{h, e, a, p, 1, k, e\}$

6) Who invented Venn diagrams?

Ans

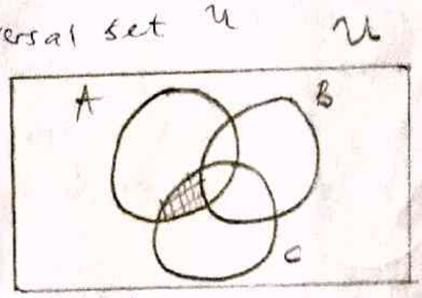
John Venn

7) Represent $A' \cap B$ using venn diagram if they are both subsets of the universal set U.



$A' \cap B$

8) Represent $A \cap B' \cap C$ using venn diagram if they are all subsets of the universal set U



$A \cap B' \cap C$

9) If the universal set

$U = \{x : x \text{ is a natural number and } 1 \le x \le 9\}$

$P = \{x : 1 \le x \le 4\}$

$Q = \{2, 4, 6, 8\}$.

Find $(P \cup Q)'$

Ans

Natural numbers are $1, 2, 3, 4, \ldots \infty$

restructuring U, we have

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$P = \{1, 2, 3, 4\}$

$Q = \{2, 4, 6, 8\}$

$P \cup Q = \{1, 2, 3, 4, 6, 8\}$

$(P \cup Q)' = \{5, 7, 9\}$

10) If P and Q are non empty sets, simplify $(P\cap Q)\cap(P'\cup Q')$

Ans

Using De morgan's law

$P'\cup Q' = (P\cap Q)'$

$P'\cap Q' = (P\cup Q)'$

$(P\cap Q)\cap(P'\cup Q')$

$(P\cap Q)\cap(P\cap Q)'$

Let $(P\cap Q) = A$

$A\cap A'$

using law of complementations

$A\cap A' = \phi$

$A\cup A' = U$

$U' = \phi$

$\phi' = U$

Therefore, $(P\cap Q)\cap(P\cap Q)'$

$A\cap A'$

$= \phi$.

11) How many subsets does the set $A = \{a, b, c, d, e\}$ have?

Ans

Number of subsets a given set have $= 2^n$, where n is the cardinality of the set.

cardinality of $A = 5$

number of subset of $A = 2^n = 2^5$

$= 32$

12) In a class of 80 students, every student had to study Economics or Geography, or both economics and Geography. If 65 students studied Economics and 50 studied Geography, How many studied both subjects.

Ans

From the question given above we have that

$U = 80$, $n(E) = 65$, $n(G) = 50$

Since every student had to study either Economics or Geography or both, This implies that $U = E\cup G$

$n(E\cup G) = n(U) = 80$

using the relation,

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$n(A\cap B) = n(A) + n(B) - n(A\cup B)$

$n(E\cap G) = n(E) + n(G) - n(E\cup G)$

$n(E\cap G) = 65 + 50 - 80$

$= 115 - 80$

$n(E\cap G) = 35$ students

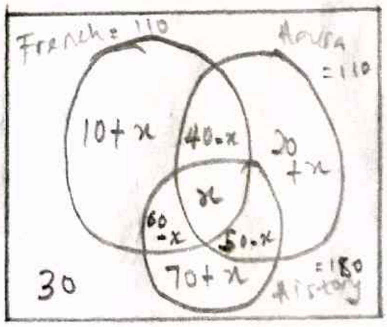
Therefore, 35 students studied both.

13) In a school of 300 students, 110 offered French, 110 Hausa language, 180 History, 40 French and Hausa, 50 Hausa and History, 60 French and History while 30 did not offer any of the three subjects. Find the number of students who offered all the three subjects.

Ans

Representing the information above on a Venn diagram, we have;-

$U = 300$



Let the number of students who offer all three subjects be x

Number of students who studied French alone $= 110 - (40-x) - (x) - (60-x)$

$= 110 - 40 + \cancel{x} - \cancel{x} - 60 + x$

$= 10 + x$

Number of students who studied Hausa alone $= 110 - (40-x) - (x) - (50-x)$

$= 110 - 40 + \cancel{x} - \cancel{x} + 50 + x$

$= 20 + x$

Number of students who study History alone $= 180 - (60-n) - (n) - (50-n)$
$= 180 - 60 + n - n - 50 + n$
$= 70 + n$

Adding all together equals U.
$(10+n) + (40-n) + n + (60-n) + (20+n) + (50-n) + (70+n) + 30 = 300$
$[10+40+60+20+50+70] + (n - n + n - n + n - n + n) + 30 = 300$
$250 + 30 + n = 300$
$n + 280 = 300$
$n = 20$ students

The number of students that offered all three subjects $= 20$.

14) According to question 13, find the number of students who offered History alone.

Ans

History alone $= 70 + n$
$= 70 + 20$
$= 90$ students

15) Find the number of students who studied French and Hausa but not History

Ans

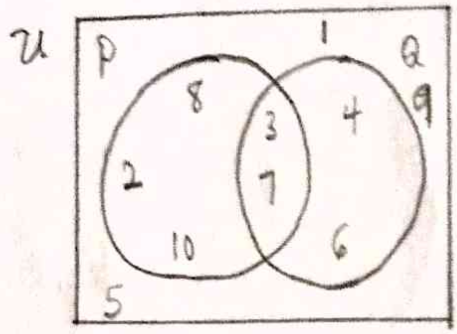
French and Hausa but not History
$= 40 - n = 40 - 20$
$= 20$ students.

16) How many students studied exactly 2 subjects.

Ans

Exactly 2 subjects $= (40-n) + (60-n) + (50-n) = 150 - 3n$
$= 150 - 3(20)$
$= 150 - 60$
$= 90$ students.

Use the Information below to answer questions 17, 18, 19 and 20.
The Venn diagram below represents a universal set U of integers and its subset P and Q. List the elements of the following sets.



17) Find $P \cup Q$

Ans

$P \cup Q = \{2, 3, 4, 6, 7, 8, 10\}$

18) Find $P' \cup Q$

$P' \cup Q = \{1, 4, 5, 6, 9\}$

19) Find $P' \cap Q'$

$P' \cap Q' = \{1, 5, 9\}$

It could be clearly seen that
$(P \cup Q)' = P' \cap Q'$

20) Find $P \cap Q'$

$P \cap Q' = \{2, 8, 10\}$

Key: $P = \{2, 3, 7, 8, 10\}$
$P' = \{1, 4, 5, 6, 9\}$
$Q = \{3, 4, 6, 7\}$
$Q' = \{1, 2, 5, 8, 9, 10\}$.

21) Let the universal set U be the set of integers,
$U = n : 0 < n \leq 10$
Find the complement of the set

$P = x : x \in U$, x is not divisible by 4.

Ans

$U = x : 0 < x \leq 10$

rewriting this in set notation, we have $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$P = \{1, 2, 3, 5, 6, 7, 9, 10\}$

Complement of $P = P'$

$P' = \{4, 8\}$

OR

$P' = x : x \in U$, x is divisible by 4

$= \{4, 8\}$

22) If Q=(all perfect squares, less than 30) and P= (all odd numbers from 1 to 10), find $Q \cap P$

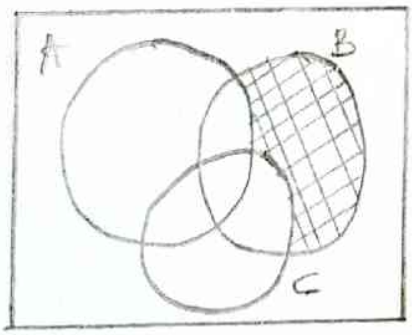
Ans

$Q = \{1, 4, 9, 16, 25\}$

$P = \{1, 3, 5, 7, 9\}$

$Q \cap P = \{1, 9\}$

23) What does the Venn diagram below represent in set notation



Ans

$B \cap A' \cap C'$

$= A' \cap B \cap C'$

24) If $A = \{x : x \text{ is a factor of } 72\}$

$B = \{x : \frac{1}{2}x + 3 < 2x - 3\}$

$C = \{x : x < 20\}$ and $A, B, C \subset U$

where $U = \{\text{integers}\}$, list the elements of $A \cap B \cap C$.

Ans

Listing their various elements, we have;

$U = \{-\infty, \ldots -4, -3, -2, 0, 1, 2, 3, 4, \ldots +\infty\}$

$A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$

$\frac{1}{2}x + 3 < 2x - 3$

solving the inequality, it gives;

$\frac{1}{2}x - 2x < -3 - 3$

$-\frac{3}{2}x < -6$

$x > \frac{-6}{-3/2} \Rightarrow x > 6 \times \frac{2}{3}$

$x > 4$

$B = \{x : x > 4\}$

$B = \{5, 6, 7, 8, 9, \ldots +\infty\}$

$C = \{-\infty, \ldots, -3, -2, -1, 0, 1, 2, \ldots 19\}$

$U = \{-\infty, \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots, +\infty\}$

$A \cap B \cap C = \{6, 8, 9, 12, 18\}$

25) The set of all subsets of a set X is called?

Ans

The power set of the set X.

26) Two sets A and B that do not have any common elements is said to be?

Ans

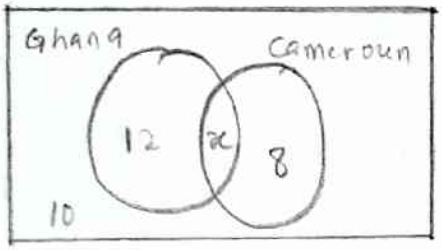
They are said to be DISJOINT.

Use the information below to answer questions 27 & 28.

A survey was carried out on 40 travellers. 12 of the travellers said they had travelled to Ghana, but not to cameroun before. 8 of the travellers claimed they had travelled to cameroun, but not to Ghana before. 10 of the travellers said they had not travelled to any of the two countries

27) Draw a venn diagram to find the number of travellers that had travelled to both countries before.

Ans



Let the number of travellers that had travelled to both countries before be x.

According to the venn diagram above,

$10+12+x+8=40$

$30+x=40$

$x=10$ travellers

28) Find the number of travellers that had travelled to Ghana before

Ans

$n=(12+x)$

$=12+10$

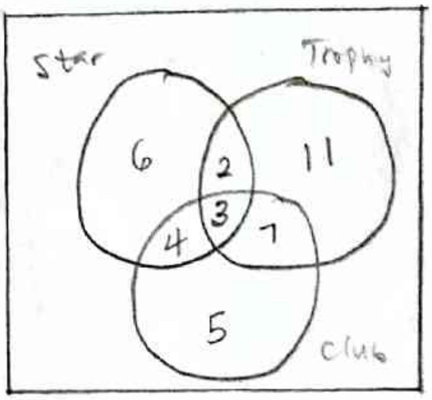
$=22$ travellers

29) Use the information below to answer questions 29 & 30

The members of a college staff club were asked to indicate the brand of beer they drank. 5 members drank star and Trophy beer, 7 members drank star and club beer, while 10 members drank Trophy and club beer. 6 members drank star beer only, 11 drank Trophy beer only, while 5 drank club beer only; 3 members drank all the three bands.

29) Find the number of members that drank star beer.

Ans using venn diagram



Number of members that drank star beer $=6+2+3+4$

$=15$ members

30) Find the total number of members in the club.

Ans

Total number of members in the club

$=6+2+3+4+5+7+11$

$=38$ members

OPERATIONS WITH REAL NUMBERS

1) Evaluate without using Log tables
$\log_{10} 1.44 - \log_{10} 90 + \log_{10} 0.0625$

Ans

From laws of Logarithm,

$\log A + \log B = \log AB$

$\log A - \log B = \log \frac{A}{B}$

$\log_{10} 1.44 - \log_{10} 90 + \log_{10}(0.0625)$

$= \log_{10}\left(\frac{1.44 \times 0.0625}{90}\right)$

$= \log_{10}\left(\frac{1.44 \times 0.0625}{90}\right)$

$= \log_{10} 0.001$

$= \log_{10} 10^{-3}$

$= -3\log_{10} 10$

$= -3$

2) Simplify: $\log 64 + 2\log 5 - 2\log 40$

Ans

$\log 64 + 2\log 5 - 2\log 40$

$\log 8^2 + 2\log 5 - 2\log 40$

From laws of logarithm,

$\log A^B = B\log A$

$2\log 8 + 2\log 5 - 2\log 40$

$2(\log 8 + \log 5 - \log 40)$

$2\left[\log\left(\frac{8 \times 5}{40}\right)\right]$

$2\log 1$

$= 2\log_{10} 10^0$

$= 2 \times 0 = 0$

3) Simplify without using mathematical tables: $\log_{10}\left[\frac{30}{16}\right] - 2\log_{10}\left[\frac{5}{9}\right] + \log_{10}\left[\frac{400}{243}\right]$

Ans

$\log_{10}\left(\frac{30}{16}\right) - 2\log_{10}\left(\frac{5}{9}\right) + \log_{10}\left(\frac{400}{243}\right)$

$\log_{10}\left(\frac{30}{16}\right) - \log_{10}\left(\frac{5}{9}\right)^2 + \log_{10}\left(\frac{400}{243}\right)$

$\log_{10}\left(\frac{30}{16}\right) - \log_{10}\left(\frac{25}{81}\right) + \log_{10}\left(\frac{400}{243}\right)$

$= \log_{10}\left(\frac{30}{16} \div \frac{25}{81} \times \frac{400}{243}\right)$

$= \log_{10}\left[\frac{30}{16} \times \frac{81}{25} \times \frac{400}{243}\right)$

$= \log_{10} 10$

$= 1$

4) Find the value of k, given that $\log k - \log(k-2) = \log 5$

Ans

$\log k - \log(k-2) = \log 5$

$\log\left[\frac{k}{k-2}\right] = \log 5$

Finding the antilog of both sides

$\frac{k}{k-2} = 5$

$k = 5(k-2)$

$k = 5k - 10$

$-4k = -10$

$k = \frac{-10}{-4}$

$k = 5/2$ or $2\frac{1}{2}$

5) Find the value of x if $\log_2 x^2 = -8$

from laws of Logarithm

If $\log_b a = c$, then $\Rightarrow a = b^c$

Ans

$\log_2 x^2 = -8$

$x^2 = 2^{-8}$

Finding the square root of both sides

$\sqrt{x^2} = \sqrt{2^{-8}}$

$x = 2^{-8 \times \frac{1}{2}}$

$x = 2^{-4}$

6) Given $\log_{10} 2.5 = 0.3979$, evaluate $\log_{10} 25 + 2\log_{10} 250$

Ans

$\log_{10} 25 + 2\log_{10} 250$

Lets express this expression in terms of $\log_{10} 2.5$.

$\log_{10} 25 + 2\log_{10} 250$

$\log_{10}(2.5 \times 10) + 2\log_{10}(2.5 \times 100)$

$\log_{10} 2\cdot5 + \log_{10} 10 + 2[\log_{10} 2\cdot5 + \log_{10} 100]$

$\log_{10} 2\cdot5 + 1 + 2(\log_{10} 2\cdot5 + \log_{10} 10^2)$

$\log_{10} 2\cdot5 + 1 + 2(\log_{10} 2\cdot5 + 2\log_{10} 10)$

$\log_{10} 2\cdot5 + 1 + 2(\log_{10} 2\cdot5 + 2)$

$\log_{10} 2\cdot5 + 1 + 2\log_{10} 2\cdot5 + 4$

$3\log_{10} 2\cdot5 + 5$

Substitute $\log_{10} 2\cdot5 = 0\cdot3979$ into the expression, we have

$3(0\cdot3979) + 5$

$= 1\cdot1937 + 5$

$= 6\cdot1937$

7) Given that $\log 8 = 0\cdot9031$, obtain the value of $\log_{10} 4 + \log_{10} 16$

Ans

$\log_{10} 4 + \log_{10} 16$

$\log_{10}(4\times16)$

$\log_{10} 64$

$\log_{10} 8^2$

$2\log_{10} 8$

$2(0\cdot9031)$

$= 1\cdot8062.$

8) Given that $\log_2 a = \log_8 4$, Find a

Ans

$\log_2 a = \log_8 4$

from laws of change of base in logarithm

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$\log_2 a = \log_8 4$

$$\log_2 a = \frac{\log_2 4}{\log_2 8}$$

$$\log_2 a = \frac{\log_2 2^2}{\log_2 2^3}$$

$$\log_2 a = \frac{2\cancel{\log_2 2}}{3\cancel{\log_2 2}} = \frac{2}{3}$$

$\log_2 a = 2/3$

$a = 2^{2/3}$

9) If $9^{2x-1} = \frac{81^{x-2}}{3^x}$, find x

Ans

$$9^{2x-1} = \frac{81^{x-2}}{3^x}$$

expressing every term as a power of 3

$$3^{2(2x-1)} = \frac{3^{4(x-2)}}{3^x}$$

$$3^{4x-2} = \frac{3^{4x-8}}{3^x}$$

$3^{4x-2} = 3^{4x-8-x}$

$\cancel{3}^{4x-2} = \cancel{3}^{3x-8}$

$4x-2 = 3x-8$

$4x-3x = -8+2$

$x = -6$

10) If $\log_{10}(2x+1) - \log_{10}(3x-1) = 1$ find x.

Ans

$\log_{10}(2x+1) - \log_{10}(3x-1) = 1$

$$\log_{10}\left(\frac{2x+1}{3x-1}\right) = 1$$

$$10^1 = \frac{2x+1}{3x-1}$$

$10(3x-1) = 2x+1$

$30x - 10 = 2x+1$

$30x - 2x = 1+10$

$28x = 11$

$x = 11/28$

11) $\log_{10} 5 + \log_{10}(x+2) - \log_{10}(x-1) = 2$ Find the value of x

Ans

$\log_{10} 5 + \log_{10}(x+2) - \log_{10}(x-1) = 2$

$$\log_{10}\left(\frac{5(x+2)}{x-1}\right) = 2$$

$$10^2 = \frac{5(x+2)}{x-1}$$

$$100 = \frac{5(x+2)}{}$$

$\frac{100}{5} = \frac{x+2}{x-1}$

$20 \times \frac{x+2}{x-1}$

$20(x-1) = x+2$

$20x - 20 = x+2$

$20x - x = 2+20$

$19x = 22$

$x = \frac{22}{19}$

12) If $\log_8 2x - \log_8 y = 1$, express y in terms of x

Ans

$\log_8 2x - \log_8 y = 1$

$\log_8 \left(\frac{2x}{y}\right) = 1$

$8^1 = \frac{2x}{y}$

$\frac{2x}{y} = 8$

$2x = 8y$

$y = \frac{2x}{8}$

$y = \frac{x}{4}$ in terms of x

13) Solve the equation

$3(2^{2x+3}) - 5(2^{x+2}) - 156 = 0$

leaving your answer in logarithm form.

Ans

$3(2^{2x+3}) - 5(2^{x+2}) - 156 = 0$

$3(2^{2x})(2^3) - 5(2^x)(2^2) - 156 = 0$

$(3\times8)(2^{2x}) - (5\times4)(2^x) - 156 = 0$

$24(2^{2x}) - 20(2^x) - 156 = 0$

$24(2^x)^2 - 20(2^x) - 156 = 0$

After expressing the equation in terms of 2^x we equate it to any variable p. i.e $2^x = p$

$\Rightarrow 24(p)^2 - 20(p) - 156 = 0$

$24p^2 - 20p - 156 = 0$

dividing through by 4

$\frac{24p^2}{4} - \frac{20p}{4} - \frac{156}{4} = \frac{0}{4}$

$6p^2 - 5p - 39 = 0$

Solving the quadratic equation by factorisation,

$6p^2 - 18p + 13p - 39 = 0$

$6p(p-3) + 13(p-3) = 0$

$(6p+13)(p-3) = 0$

$6p + 13 = 0$ or $p - 3 = 0$

$6p = -13$ or $p = 3$

$p = -13/6$ or 3

Now equating back p to 2^x

$2^x = p$

$2^x = -13/6$

Log both sides

$\log 2^x = \log(-13/6)$

$x \log 2 = \log(-13/6)$

$x = \frac{\log(-13/6)}{\log 2}$

$x = \log_2(-13/6)$

OR

$2^x = 3$

Log both sides

$\log 2^x = \log 3$

$x \log 2 = \log 3$

$x = \frac{\log 3}{\log 2}$

$x = \log_2 3$

14) Solve for x, giving your answer correct to 3 significant figures

$2\log_{10} x + 3\log_{10} 5 = 2$

Ans

$2\log_{10} x + 3\log_{10} 5 = 2$

$\log_{10} x^2 + \log_{10} 5^3 = 2$

$\log_{10} x^2 + \log_{10} 125 = 2$

$\log_{10}(125x^2) = 2$

$10^2 = 125x^2$

$100 = 125x^2$

$x^2 = \frac{100}{125}$

$x^2 = \frac{4}{5}$

$x = \sqrt{4/5} = 0.8944$

≈ 0.894 to 3 ~~sig~~ sig.

15) Solve $9^{x-1} = 27^{x+1}$

Ans

Expressing all terms as a power of 3, we have

$9^{x-1} = 27^{x+1}$

$\cancel{3}^{2(x-1)} = \cancel{3}^{3(x+1)}$

$2(x-1) = 3(x+1)$

$2x - 2 = 3x + 3$

$2x - 3x = 3 + 2$

$-x = 5$

$x = -5$

16) Find the real values of x which satisfy the equation.

$\log_3 x + \log_x 3 = 10/3$

Ans

$\log_3 x + \log_x 3 = 10/3$

applying the change of base principle.

$\frac{\log x}{\log 3} + \frac{\log 3}{\log x} = \frac{10}{3}$

$\frac{(\log x)^2 + (\log 3)^2}{(\log 3)(\log x)} \ast \frac{10}{3}$

$3((\log x)^2 + (\log 3)^2) = 10(\log 3)(\log x)$

$3(\log x)^2 + 3(\log 3)^2 \pm 10(\log 3)(\log x)$

$3(\log x)^2 - 10(\log 3)(\log x) + 3(\log 3)^2 = 0$

Solving the quadratic equation by factorisation method.

$3(\log x)^2 - (\log 3)(\log x) - 9(\log 3)(\log x) + 3(\log 3)^2 = 0$

$\log x[3\log x - \log 3] - 3\log 3[3\log x - \log 3] =$

$[\log x - 3\log 3][3\log x - \log 3] = 0$

$\log x - 3\log 3 = 0$

$\log x = 3\log 3$

$\log x = \log 3^3$

$\cancel{\log} x = \cancel{\log} 27$

$x = 27$

OR

$3\log x - \log 3 = 0$

$3\log x = \log 3$

$\log x = \frac{1}{3}\log 3$

$\cancel{\log} x = \cancel{\log} 3^{1/3}$

$x = 3^{1/3}$.

$\therefore x = 27$ or $3^{1/3}$

17) Express $\frac{2+4\sqrt{6}}{3\sqrt{2}-3}$ in the form $p\sqrt{2} + q\sqrt{3}$ ~~$+ r\sqrt{6} + s$~~ where p, q ~~r & s~~ are rational numbers.

Ans

$\frac{2+4\sqrt{6}}{3\sqrt{2}-3}$

By rationalizing,

$\frac{2+4\sqrt{6}}{3\sqrt{2}-3} \times \frac{3\sqrt{2}+3}{3\sqrt{2}+3}$

$\frac{(2+4\sqrt{6})(3\sqrt{2}+3)}{(3\sqrt{2}-3)(3\sqrt{2}+3)}$

$\frac{6\sqrt{2} + 6 + 12\sqrt{12} + 12\sqrt{6}}{9(2) + 9\sqrt{2} - 9\sqrt{2} - 9}$

$\frac{6\sqrt{2} + 6 + 12(2)\sqrt{3} + 12\sqrt{6}}{18 - 9}$

$$\frac{6\sqrt{2}+6+ +24\sqrt{3}+12\sqrt{6}}{9}$$

$$\frac{6}{9}\sqrt{2}+\frac{24}{9}\sqrt{3}+\frac{12}{9}\sqrt{6}+\frac{6}{9}$$

$$\frac{2}{3}\sqrt{2}+\frac{8}{3}\sqrt{3}+\frac{4}{3}\sqrt{6}+\frac{2}{3}.$$

18) Express $\frac{3\sqrt{2}+2}{1-\sqrt{2}}$ in the form $m+n\sqrt{2}$ where m and n are rational numbers.

Ans

$$\frac{3\sqrt{2}+2}{1-\sqrt{2}}$$

rationalizing the expression above, we have $\frac{3\sqrt{2}+2}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$

$$\frac{(3\sqrt{2}+2)(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$$

$$\frac{3\sqrt{2}+3(2)+2+2\sqrt{2}}{1+\sqrt{2}-\sqrt{2}+2}$$

$$\frac{3\sqrt{2}+6+2+2\sqrt{2}}{1+2}$$

$$\frac{8+5\sqrt{2}}{-1}$$

$$\frac{8}{-1}+\frac{5}{-1}\sqrt{2} = -8-5\sqrt{2}$$

19) simplify

$$\frac{\sqrt{2}}{2\sqrt{2}-\sqrt{3}} - \frac{\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$$

Ans

$$\frac{\sqrt{2}}{2\sqrt{2}-\sqrt{3}} - \frac{\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$$

$$\frac{\sqrt{2}(2\sqrt{2}+\sqrt{3})-\sqrt{3}(2\sqrt{2}-\sqrt{3})}{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})}$$

$$= \frac{2(2)+\sqrt{6}-2\sqrt{6}+3}{4(2)+2\sqrt{6}-2\sqrt{6}-3}$$

$$= \frac{4+\sqrt{6}-2\sqrt{6}+3}{8-3}$$

$$= \frac{7-\sqrt{6}}{5}$$

$$= 7/5 - \frac{1}{5}\sqrt{6}$$

20) If $\frac{1}{10}\left(\frac{1}{5+\sqrt{3}}+\frac{1}{5-\sqrt{3}}\right) = P$, find P

Ans

$$P = \frac{1}{10}\left(\frac{1}{5+\sqrt{3}}+\frac{1}{5-\sqrt{3}}\right)$$

$$= \frac{1}{10}\left(\frac{(5-\sqrt{3})+(5+\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})}\right)$$

$$= \frac{1}{10}\left[\frac{5\cancel{-\sqrt{3}}+5\cancel{+\sqrt{3}}}{25-5\sqrt{3}+5\sqrt{3}-3}\right]$$

$$= \frac{1}{10}\left(\frac{10}{25-3}\right)$$

$$= \frac{1}{10}\left(\frac{10}{22}\right)$$

$$P = \frac{1}{22}$$

21) find P if $\sqrt{2} - \frac{1}{\sqrt{2}} = P\sqrt{2}$

Ans

$$\sqrt{2}-\frac{1}{\sqrt{2}} = P\sqrt{2}$$

$$\frac{2-1}{\sqrt{2}} \times P\sqrt{2}$$

$$1 = (P\sqrt{2})\sqrt{2}$$

$$1 = P(2)$$

$$P = 1/2.$$

22) solve the equation

$$2\sqrt{x+1}+x=7$$

Ans

$$2\sqrt{x+1}+x=7$$

$$2\sqrt{x+1} = 7-x$$

squaring both sides

$$(2\sqrt{x+1})^2 = (7-x)^2$$

$$4(x+1) = 49-14x+x^2$$

$$4x+4 = 49-14x+x^2$$

$0 = 49 - 4 - 14x - 4x + x^2$

$0 = 45 - 18x + x^2$

$x^2 - 18x + 45 = 0$

factorising we have,

$x^2 - 15x - 3x + 45 = 0$

$x(x-15) - 3(x-15) = 0$

$(x-3)(x-15) = 0$

$x = 3$ or 15

23) Express $\frac{1}{(2-\sqrt{2})^4}$ in the form $m + n\sqrt{2}$

Ans

$\frac{1}{(2-\sqrt{2})^4}$

~~first~~ ~~of~~ all, lets evaluate $(2-\sqrt{2})^2$

$(2-\sqrt{2})^2 = 4 - 4\sqrt{2} + 2$

$= 6 - 4\sqrt{2}$

then, $(2-\sqrt{2})^4 = ((2-\sqrt{2})^2)^2$

$(2-\sqrt{2})^4 = (6-4\sqrt{2})^2$

$= 36 - 48\sqrt{2} + 32$

$= 68 - 48\sqrt{2}$

Therefore,

$\frac{1}{(2-\sqrt{2})^4} = \frac{1}{68-48\sqrt{2}}$

rationalizing we have.

$\frac{1}{68-48\sqrt{2}} \times \frac{68+48\sqrt{2}}{68+48\sqrt{2}}$

$\frac{68+48\sqrt{2}}{68^2 + 3264\sqrt{2} - 3264\sqrt{2} - 48^2(2)}$

$= \frac{68+48\sqrt{2}}{4624 - 4608}$

$= \frac{68+48\sqrt{2}}{16}$

$= \frac{68}{16} + \frac{48}{16}\sqrt{2}$

$= \frac{17}{4} + 3\sqrt{2}$

REMAINDER AND FACTOR THEOREM

24) If $x^4 - kx^3 + 10x^2 + px - 3$ is divisible by $x-1$ and if when it is divided by $(x+2)$ the remainder is 27, find the constants k and p.

Ans

put $f(x) = x^4 - kx^3 + 10x^2 + px - 3$

since, $f(x)$ is divisible by $(x-1)$, i.e $(x-1)$ is a factor of $f(x)$, then $f(1) = 0$.

$f(1) = 1^4 - k(1)^3 + 10(1)^2 + p(1) - 3 = 0$

i.e $1 - k + 10 + p - 3 =$

$-k + p + 8 = 0$

$-k + p = -8$ ——— (1)

Since $f(x)$ is divided by $(x+2)$, the remainder is 27, then $f(-2) = 27$.

$f(-2) = (-2)^4 - k(-2)^3 + 10(-2)^2 + p(-2) - 3 = 27.$

i.e $16 + 8k + 40 - 2p - 3 = 27$

$8k - 2p = -26$

$4k - p = -13$ ——— (2)

adding equations (1) and (2)

$3k = -21$

$k = -7,$

from (1), $p = k - 8$

$p = -7 - 8$

$= -15.$

$k = -7$ & $p = -15$.

25) Find the remainder when the polynomial $f(x) = x^3 + 2x^2 - 5x - 6$ is divided by $(x-3)$

Ans

Let $f(x) = x^3 + 2x^2 - 5x - 6$

If $f(x)$ is divided by $(x-3)$

and using the remainder theorem.
Then $f(3)$ is the remainder

$f(3) = (3)^3 + 2(3)^2 - 5(3) - 6$

$= 27 + 18 - 15 - 6$

$= 24$

26) If one of the zeroes of the polynomial $p(x) = x^2 + qx + 6$ is 3, Find the value of constant q.

Ans

$p(x) = x^2 + q(x) + 6$

Since 3 is a zero, then $p(3) = 0$

$p(3) = 3^2 + 3q + 6 = 0$

$\Rightarrow 9 + 3q + 6 = 0$

$3q + 15 = 0$

$3q = -15$

$q = -5.$

27) Use the information below to answer questions 27, 28 & 29

Given that polynomial $f(x) = 6 - x - x^2$ is a factor of the polynomial $g(x) = p(x)^3 + 5x^2 + qx - 18$, where p and q are constants. Find the

27) Values of p and q.

Ans

$g(x) = px^3 + 5x^2 + qx - 18$

and $f(x) = 6 - x - x^2$ is a factor

lets now solve for the zeros of $f(x)$

$6 - x - x^2 = 0$

$6 - 3x + 2x - x^2 = 0$

$3(2-x) + x(2-x) = 0$

$(3+x)(2-x) = 0$

$x = 2$ or -3

The zeros of $f(x)$ are also the zeros of $g(x)$ since $f(x)$ is a factor of $g(x)$.

$\therefore g(2) = 0$ and $g(-3) = 0$

$g(2) = p(2)^3 + 5(2)^2 + q(2) - 18 = 0$

$8p + 20 + 2q - 18 = 0$

$8p + 2q + 2 = 0$

$8p + 2q = -2$

$4p + q = -1$ ——— (1)

$g(-3) = p(-3)^3 + 5(-3)^2 + q(-3) - 18 = 0$

$-27p + 45 - 3q - 18 = 0$

$-27p - 3q = -27$

$9p + q = 9$ ——— (2)

substract equ(1) from (2)

$5p = 10$

$p = 2$

$4p + q = -1$

$4(2) + q = -1$

$8 + q = -1$

$q = -1 - 8 = -9$

$p = 2,\ q = -9$

28) Remainder of $g(x)$ when ~~it~~ is divided by $(x+2)$

Ans

$g(x) = 2x^3 + 5x^2 - 9x - 18$

Using the remainder theorem,

remainder $= g(-2)$

$g(-2) = 2(-2)^3 + 5(-2)^2 - 9(-2) - 18$

$= 2(-8) + 5(4) + 18 - 18$

$= -16 + 20$

$= 4.$

29) Values of x for which $g(x) = f(x)$

$g(x) = f(x)$

$2x^3 + 5x^2 - 9x - 18 = 6 - x - x^2$

$2x^3 + 6x^2 - 8x - 24 = 0$

$x^3 + 3x^2 - 4x - 12 = 0$

Let the expression above be $h(x)$

Using the factor theorem to solve the cubic equation.

factors are $1, -1, 2, -2, 3, -3, -4, +4, -6, +6, -12, +12$

$h(x) = x^3 + 3x^2 - 4x - 12 = 0$

$h(1) = 1^3 + 3(1)^2 - 4(1) - 12$

$= 1 + 3 - 4 - 12$

$= -12 \neq 0$

$h(-1) = (-1)^3 + 3(-1)^2 - 4(-1) - 12$

$= -1 + 3 + 4 - 12$

$= -6 \neq 0$

$h(2) = 2^3 + 3(2)^2 - 4(2) - 12$

$= 8 + 12 - 8 - 12$

$= 0$

$\therefore (x-2)$ is a factor of $h(x)$

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
x-2 & x^3 + 3x^2 - 4x - 12 \\
 & -(x^3 - 2x^2) \\
\hline
 & 5x^2 - 4x - 12 \\
 & -(5x^2 - 10x) \\
\hline
 & +6x - 12 \\
 & -(6x - 12) \\
\hline
 & - \quad -
\end{array}
$$

$\therefore h(x) = (x-2)(x^2+5x+6)$

$= (x-2)(x+2)(x+3)$

The solution of $h(x)$ is

$(x-2)(x+2)(x+3) = 0$

$x = 2, -2$ and -3.

30) If $(x+1)$ ~~and~~ is a factor of $x^3 + 3x^2 + ax + 4$, find the value of a.

Ans

Let $f(x) = x^3 + 3x^2 + ax + 4$

~~If f(x)~~ If $(x+1)$ is a factor of $f(x)$, as per the factor theorem,

$f(-1) = 0$

$f(-1) = (-1)^3 + 3(-1)^2 + a(-1) + 4 = 0$

$= -1 + 3 - a + 4 = 0$

$6 - a = 0$

$a = 6.$

PARTIAL FRACTIONS

31) Resolve into partial fractions

$$\frac{3x}{1 - x - 2x^2}$$

Ans

$$\frac{3x}{1-x-2x^2} = \frac{3x}{1-2x+x-2x^2}.$$

$$= \frac{3x}{(1-2x)+x(1-2x)} = \frac{3x}{(1+x)(1-2x)}$$

$$\frac{3x}{(1+x)(1-2x)} = \frac{A}{1+x} + \frac{B}{1-2x}.$$

Using the cover up Rule to evaluate A.

The denominator of A is $(1+x)$

$(1+x) = 0$

$x = -1$

Substitute $x = -1$ into the initial fraction ignoring value 0 at the denominator

$$\frac{3(-1)}{\cancel{(1+x)}(1-2(-1))} = A = \frac{-3}{(1+2)} = \frac{-3}{3}.$$

$A = -1$

Doing the same for B,

$1 - 2x = 0$

$x = 1/2$

$$B = \frac{3(1/2)}{(1+1/2)(\cancel{1-2(1/2)})} = \frac{3/2}{3/2} = 1$$

$B = 1$

$$\therefore \frac{3x}{(1+x)(1-2x)} = \frac{-1}{1+x} + \frac{1}{1-2x}$$

$$= \frac{1}{1-2x} - \frac{1}{1+x}$$

32) Split into partial fractions

$$\frac{2}{x^2(x+2)}$$

Ans

$$\frac{2}{x^2(x+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2}$$

Using the cover up rule to find A

$(x+2) = 0$

$x = -2$

$$A = \frac{2}{(-2)^2\cancel{(x+2)}} = \frac{2}{4}$$

$A = \frac{1}{2}$

$$\frac{2}{x^2(x+2)} = \frac{1}{2(x+2)} + \frac{Bx+C}{x^2}$$

$$2 \equiv \frac{1}{2}(x^2) + (Bx+C)(x+2)$$

$$2 = \frac{1}{2}x^2 + Bx^2 + 2Bx + Cx + 2C$$

$$2 = (\tfrac{1}{2} + B)x^2 + (2B+C)x + 2C$$

comparing co-efficients, we have

$0 = \frac{1}{2} + B$ —— (1)

$0 = 2B + C$ —— (2)

$2 = 2C$ —— (3)

from equ(1)

$B = -\frac{1}{2}$

from equ(2)

$2B = -C$

$2(-\frac{1}{2}) = -C$

$-1 = -C$

$C = 1$

From equ(3)

$2 = 2C$

$C = 1$

$\therefore\ B = -\frac{1}{2},\ C = 1,\ A = \frac{1}{2}$

$$\frac{2}{x^2(x+2)} = \frac{1}{2(x+2)} + \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2}$$

$$= \frac{1}{2(x+2)} - \frac{x-1}{2x^2} \quad \Bigg|\ = \frac{1}{2(x+2)} - \frac{1}{2x} + \frac{1}{2x^2}$$

$$= \frac{1}{2(x+2)} - \frac{x}{2x^2} + \frac{1}{2x^2}$$

33) Express in partial fractions

$$\frac{9}{(x-1)^2(7x+2)}$$

Ans

$$\frac{9}{(x-1)^2(7x+2)} = \frac{A}{7x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

using the cover up rule to find A

$7x + 2 = 0$

$x = -\frac{2}{7}$

$$A = \frac{9}{(\frac{-2}{7}-1)^2\cancel{(7(-\frac{2}{7})+2)}} = \frac{9}{(-\frac{9}{7})^2}$$

$$A = \frac{9}{\frac{81}{49}} = \cancel{9} \times \frac{49}{\cancel{81}_9}$$

$$A = \frac{49}{9}$$

$$\frac{9}{(x-1)^2(7x+2)} = \frac{49/9}{7x+2} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$9 \equiv \frac{49}{9}(x-1)^2 + B(7x+2)(x-1) + C(7x+2)$$

$$9 = \frac{49}{9}(x^2-2x+1) + B(7x^2-5x-2) + C(7x+2)$$

$$9 = \left(\frac{49}{9} + 7B\right)x^2 + \left(-\frac{98}{9} - 5B + 7C\right)x + \left(\frac{49}{9} - 2B + 2C\right)$$

comparing coefficients

$\frac{49}{9} + 7B = 0$ —— (1)

$-\frac{98}{9} - 5B + 7C = 0$ —— (2)

$\frac{49}{9} - 2B + 2C = 9$ —— (3)

from equ(1)

$$\frac{49}{9} + 7B = 0$$

$$\frac{49}{9} = -7B$$

$$B = -\frac{7}{9}$$

from equ(2)

$$-\frac{98}{9} - 5B + 7C = 0$$

$$-\frac{98}{9} - 5\left(-\frac{7}{9}\right) + 7C = 0$$

$$-\frac{98}{9} + \frac{35}{9} + 7C = 0$$

$$-\frac{63}{9} + 7C = 0$$

$$7C = 7$$

$$C = 1$$

from equ(3)

$$\frac{49}{9} - 2\left(-\frac{7}{9}\right) + 2C = 9$$

$$\frac{49}{9} + \frac{14}{9} + 2C = 9$$

$$7 + 2C = 9$$

$$2C = 2$$

$$C = 1$$

which confirms the value of B & C

$$\therefore \frac{9}{(x-1)^2(7x+2)} = \frac{49/9}{(7x+2)} + \frac{-7/9}{x-1} + \frac{1}{(x-1)^2}$$

$$= \frac{49}{9(7x+2)} + \frac{1}{(x-1)^2} - \frac{7}{9(x-1)}$$

34) If $\frac{10x+1}{(x-2)(x+1)} \equiv \frac{k}{x-2} + \frac{3}{x+1}$

Find the value of k

Ans

$$\frac{10x+1}{(x-2)(x+1)} \equiv \frac{k}{x-2} + \frac{3}{x+1}$$

$$10x+1 \equiv k(x+1) + 3(x-2)$$

$$10x+1 \equiv kx + k + 3x - 6$$

$$10x+1 \equiv (k+3)x + k - 6$$

comparing co-efficients

$$10 = k+3$$

$$k = 7$$

or

$$1 = k-6$$

$$k = 7$$

$$\therefore k = 7.$$

35) If $\frac{3x^2+4x}{(x-2)(x^2+1)} = \frac{P}{x-2} + \frac{Qx+R}{x^2+1}$

Find $P+Q+R$

Ans

Using the cover-up rule, to find the value of P.

$$(x-2) = 0$$

$$x = 2$$

$$P = \frac{3(2)^2+4(2)}{\cancel{(2-2)}(2^2+1)} = \frac{3(4)+8}{4+1}$$

$$P = \frac{20}{5} = 4$$

$$\frac{3x^2+4x}{(x-2)(x^2+1)} = \frac{4}{x-2} + \frac{Qx+R}{x^2+1}$$

$$3x^2+4x \equiv 4(x^2+1) + (Qx+R)(x-2)$$

$$3x^2+4x = 4x^2+4 + Qx^2 - 2Qx + Rx - 2R$$

$$3x^2+4x = (4+Q)x^2 + (-2Q+R)x + (4-2R)$$

Comparing efficients, we have

$$3 = 4+Q \quad \text{——} (1)$$

$$-2Q+R = 4 \quad \text{——} (2)$$

$$4-2R = 0 \quad \text{——} (3)$$

from equ(1)

$$3 = 4+Q$$

$$Q = 3-4$$

$$Q = -1$$

from equ(2)

$$-2Q + R = 4$$

From equ (1)

$\frac{49}{9} + 7B = 0$

$7B = -\frac{49}{9}$

$B = -\frac{7}{9}$

from equ(2)

$-\frac{98}{9} + 5(-\frac{7}{9}) + 7C = 0$

$-\frac{98}{9} + \frac{35}{9} + 7C = 0$

$\cancel{-\frac{63}{9} = 7C}$

$\cancel{C = -\frac{1}{9}}$

From equ(3)

$\frac{49}{9} - 2(-\frac{7}{9}) + 2(C) = 9$

$\frac{49}{9} + \frac{14}{9} + 2C = 9$

$7 + 2C = 9$

$2C = 2$

$C = 1$

---

from equ(2)

$-2Q + R = 4$

$-2(-1) + R = 4$

$2 + R = 4$

$R = 2$

From equ(3)

$4 - 2R = 0$

$2R = 4$

$R = 2$

which confirms the values of Q & R

$P = 4, \quad Q = -1, \quad R = 2$

$\therefore P + Q + R = 4 - 1 + 2$

$= 5$

EQUATIONS AND INEQUALITIES

1) Find the solutions of

$0.5(2x+1) \leq 0.3x + 1.9$

Illustrate your answer on the number line

Ans

$0.5(2x+1) \leq 0.3x + 1.9$

$x + 0.5 \leq 0.3x + 1.9$ ...(open bracket)

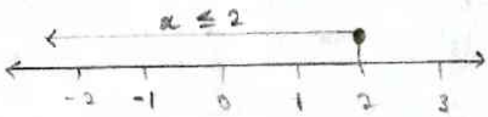
$x - 0.3x \leq 1.9 - 0.5$ ...(subtract 0.5 & 0.3x from both sides)

$0.7x \leq 1.4$

divide both sides by 0.7

$\frac{\cancel{0.7x}}{\cancel{0.7}} \leq \frac{1.4}{0.7}$

$x \leq 2$



2) Find the value of x for which

$\frac{1}{3}(2x+7) - \frac{1}{5}(1-4x) \leq 4 + x$

Ans

$\frac{1}{3}(2x+7) - \frac{1}{5}(1-4x) \leq 4 + x$

opening the brackets.

$\frac{2}{3}x + \frac{7}{3} - \frac{1}{5} + \frac{4}{5}x \leq 4 + x$

collect like terms

$\frac{2}{3}x + \frac{4}{5}x - x \leq 4 - \frac{7}{3} + \frac{1}{5}$

$\frac{7}{15}x \leq \frac{28}{15}$

divide both sides by $\frac{7}{15}$

$\frac{\cancel{7/15}\, x}{\cancel{7/15}} \leq \frac{28/15}{7/15}$

$x \leq \frac{\cancel{28}^{4}}{\cancel{15}} \times \frac{\cancel{15}}{\cancel{7}}$

$x \leq 4$

3) Solve the inequality and show on a number line

$2x - 3 < 5$

Ans

$2x - 3 < 5$

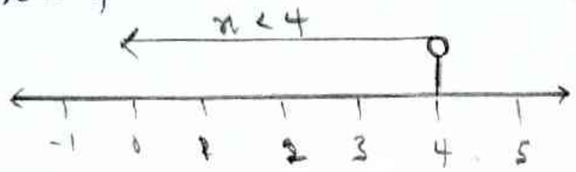
$2x - 3 < 5$

$2x < 5 + 3$

$2x < 8$

$x < \frac{8}{2}$

$x < 4$



Note: (1) When we have just < or > without the equal sign, the circle on the number line won't be shaded and points in the direction of the sign.

(2) But when the sign includes the equals to sign i.e ≤ or ≥, the circle on the number line will be shaded and also points in the direction of the sign.

4) Solve the inequality: $x - 1 \geq 2(x-4)$

Ans

$x - 1 \geq 2(x-4)$

$x - 1 \geq 2x - 8$

$x - 2x \geq -8 + 1$

$-x \geq -7$

divide both sides by -1

$\frac{-x}{-1} \geq \frac{-7}{-1}$

$x \leq 7$

In inequalities, when dividing by a negative sign, the direction of the sign is reversed (i.e in the opposite direction).

5) Solve the inequality: $3 - 4x \leq 7x - 1$

Ans

$3 - 4x \leq 7x - 1$

$3 + 1 \leq 7x + 4x$

$4 \leq 11x$

divide through by 11

$\frac{4}{11} \leq \frac{11x}{11}$

$4/11 \leq x$

$\Rightarrow x \geq 4/11$

6) Solve the inequality and show your answer on a number line

$\frac{1}{3}x + \frac{5}{8} \geq \frac{1}{2}x - \frac{5}{24}$

Ans

$\frac{1}{3}x + \frac{5}{8} \geq \frac{1}{2}x - 5/24$

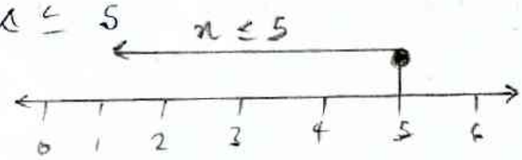
$\frac{1}{3}x - \frac{1}{2}x \geq -5/24 - 5/8$

$-1/6\, x \geq -5/6$

$\frac{-1/6\, x}{-1/6} \geq \frac{-5/6}{-1/6}$

$x \leq \frac{-5}{6} \times \frac{-6}{1}$

$x \leq 5$



7) Solve the inequality $\frac{5-3x}{2} \geq \frac{2x+3}{5}$ and indicate the result on the number line

Ans

$\frac{5-3x}{2} \geq \frac{2x+3}{5}$

Find the Lcm of the denominators of both sides (i.e 2×5) which is 10 multiply all through by their Lcm which is 10.

$10 \times \frac{5-3x}{2} \geq 10 \times \frac{2x+3}{5}$

$5(5-3x) \geq 2(2x+3)$

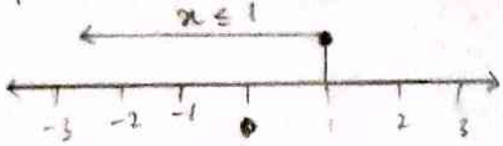
$25 - 15x \geq 4x + 6$

$-15x - 4x \geq 6 - 25$

$-19x \geq -19$

$\frac{-19x}{-19} \geq \frac{-19}{-19}$

$x \leq 1$



8) Solve the inequality $3m + 3 > 9$. Showing your result on a number line

Ans

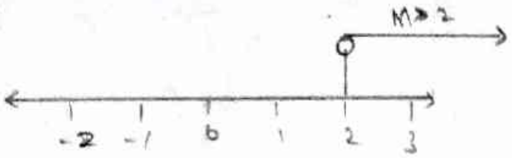
$3m + 3 > 9$

$3m > 9 - 3$

$3m > 6$

$\frac{3m}{3} > \frac{6}{3}$

$m > 2$



9) Find the range of values of n for which $3n^2 + 5n - 28$ is negative.

Ans

For $3n^2 + 5n - 28$ to be negative, then it must be less than 0

$3n^2 + 5n - 28 < 0$

Then, factorising the LHS

$3n^2 + 12n - 7n - 28 < 0$

$3n(n+4) - 7(n+4) < 0$

$(3n - 7)(n+4) < 0$

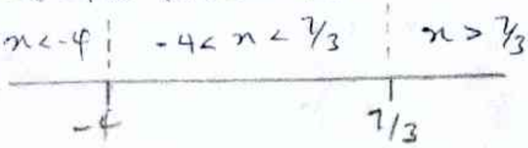
Let $f(n) = (3n-7)(n+4)$

The zeroes of $f(n)$ are

$3n - 7 = 0$ and $n + 4 = 0$

$n = 7/3$ and -4.

Now we consider the signs of $f(n)$ at the intervals



$n < -4$, $-4 < n < 7/3$ and $n > 7/3$

	$n<-4$	$-4<n<7/3$	$n>7/3$
$3x - 7$	-	-	+
$x + 4$	-	+	+
$f(n) = (3n-7)(n+4)$	+	-	+

Explanation of the table

At $n < -4$, we value $-5, -6, -7 \ldots$ making use of -5

$(3n - 7) = 3(-5) - 7 = -22$ ___ (-ve)

$(n+4) = -5 + 4 = -1$ ___ (-ve)

$f(x) = (3n-7)(n+4) = (-ve) \times (-ve) = +ve$

At $-4 < n < 7/3$, we have values $-3, -2, \ldots, 7/3$ making use of -3

$(3n - 7) = 3(-3) - 7 = -16$ ___ (-ve)

$(n+4) = -3 + 4 = 1$ ___ (+ve)

$f(x) = (3n-7)(n+4) = (-ve) \times (+ve) = -ve$

At $n > 7/3$, we have values, $3, 4 \ldots$ making use of 3

$(3n - 7) = 3(3) - 7 = 2$ ___ (+ve)

$(n+4) = 3 + 4 = 7$ ___ (+ve)

$f(n) = (3n-7)(n+4) = (+ve) \times (+ve) = +ve$

From the table above, the range of values of n for which $3n^2 + 5n - 28 < 0$ occur at

$-4 < n < 7/3$

10) Solve the inequality and illustrate your answer on a number line. $2 - n - n^2 \geq 0$

Ans

$2 - n - n^2 \geq 0$

Factorising the LHS

$2 - 2n + n - n^2 \geq 0$

$2(1 - n) + n(1 - n) \geq 0$

$(2 + n)(1 - n) \geq 0$

Let $f(n) = (2+n)(1-n)$

Then the zeroes of $f(n)$

$x<-2$	$-2<x<1$	$x>1$
-2		1

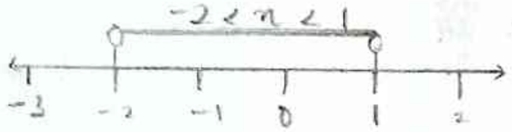
Now we consider the signs of $f(x)$ at the intervals $x<-2$, $-2<x<1$ and $x>1$.

	$x<-2$	$-2<x<1$	$x>1$
$2+x$	$-$	$+$	$+$
$1-x$	$+$	$+$	$-$
$f(x)=(2+x)(1-x)$	$-$	$+$	$-$

From the table above,

$2-x-x^2 \geq 0$ at $-2<x<1$

$\therefore$ The solution of the inequality is $-2<x<1$



11) Find the range of values of x for which $\frac{x-3}{x+3} > 0$ illustrating your answer on a number line.

Ans

Note: its '>' and not '≥'

$\frac{x-3}{x+3} > 0$

which implies that both sides can't be multiplied by the denominator $(x+3)$

$\frac{x-3}{x+3} > 0$

for $\frac{x-3}{x+3}$ to be greater than 0, then

$(x-3)>0$ & $(x+3)>0$

$x>+3$ and $x>-3$

which can be simplified to $x>+3$

or

$(x-3)<0$ & $(x+3)<0$

$x<+3$ and $x<-3$

which can be simplified to $x<-3$

$\therefore$ $x>+3$ or $x<-3$

12) Find the solution set of $|x-2| \leq 1$.

Ans

if $|x-2| \leq 1$

then $x-2 \leq 1$ if $x-2 \geq 0$

$x-2 \leq 1$

$\therefore$ $x \leq 3$

or $-(x-2) \leq 1$ if $x-2 \leq 0$

$\therefore$ $x-2 \geq -1$

$x \geq -1+2$

$x \geq 1$

The solution set is $\{x : 1 \leq x \leq 3\}$

13) Find the solution set of $|2x-1| > 3$

Ans

If $|2x-1| > 3$

then $2x-1 > 3$ if $2x-1 > 0$

$\therefore$ $2x > 4$

$x > 2$

or

$-(2x-1) > 3$ if $(2x-1) < 0$

$\therefore$ $2x-1 < -3$

$2x < -2$

$x < -1$

solution is $x>2$ or $x<-1$

The solution set is

$\{x : x<-1\} \cup \{x : x>2\}$

THEORY OF QUADRATIC EQUATIONS

1) Given that α and β are the roots of the quadratic equation x^2+5x+2. Find $\alpha+\beta$

Ans

x^2+5x+2

we have that

$a=1$, $b=5$ and $c=2$

Sum of roots $\alpha+\beta = -b/a$

$\alpha+\beta = -5/1$

$\alpha+\beta = -5$

2) Find the product of the roots of the quadratic equation $2x^2+3x+7$

Ans

Let the roots be α and β

$2x^2+3x+7$

$a=2$, $b=3$, $c=7$

product of roots $= \alpha\beta = c/a$

$\alpha\beta = 7/2$

$\alpha\beta = 3\frac{1}{2}$

3) Find the value of k for which the equation x^2+6x+k will be a perfect square.

Ans

condition for perfect square is

$b^2 = 4ac$

x^2+6x+k

$a=1$, $b=6$, $c=k$

$b^2=4ac$

$6^2=4(1)(k)$

$36=4k$

$k=\frac{36}{4}=9$ $\therefore k=9$

Use the information below to answer questions 4 - 10

Given the quadratic equation x^2+2x+3 with roots α & β

4) Find $\alpha+\beta$

Ans

$a=1$, $b=2$, $c=3$

$\alpha+\beta = \frac{-b}{a}$

$= -\frac{2}{1}$

$\alpha+\beta = -2$

5) Find $\alpha\beta$

Ans

$\alpha\beta = c/a$

$= 3/1$

$\alpha\beta = 3$

6) Find $\alpha^2+\beta^2$

Ans

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2(3)$

$= 4-6$

$\alpha^2+\beta^2 = -2$

7) Find $\alpha^3+\beta^3$

Ans

$\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$= (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

$= (-2)(-2-3)$

$= (-2)(-5)$

$\alpha^3+\beta^3 = 10$

~~8) Find $\alpha^3-\beta^3$~~

~~Ans~~

~~$\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta$~~

8) Find $\alpha-\beta$

Ans

$\alpha-\beta = \sqrt{(\alpha-\beta)^2} = \sqrt{\alpha^2-2\alpha\beta+\beta^2}$

$\alpha - \beta = \sqrt{\alpha^2 - 2\alpha\beta + \beta^2} = \sqrt{\alpha^2 + \beta^2 - 2\alpha\beta}$

$= \sqrt{(-2)^2 - 2(3)}$

$= \sqrt{-2-6}$

$= \sqrt{-8}$

9) Find $\alpha^3 - \beta^3$

Ans

$\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2)$

$= (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)$

$= \sqrt{-8}\,[-2+3]$

$= \sqrt{-8}\,(1)$

$= \sqrt{-8}$

10) Find $\alpha^2 - \beta^2$

Ans

$\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$

$= (\sqrt{-8})(-2)$

$= -2\sqrt{-8}$

11) Find the value of k for which the quadratic equation has real roots. $kx^2 - 10x - 22$.

Ans

$3x^2 - 10x - 22$

$a = k, \quad b = -10, \quad c = -22$

condition for real roots is that $b^2 > 4ac$ (i.e $D > 0$)

$\Rightarrow (-10)^2 > 4(k)(-22)$

$100 > -88k$

divide both sides by -88

$\frac{100}{-88} > \frac{-88}{-88}k$

$-\frac{25}{22} < k$ (note the direction change)

$k > -\frac{25}{22}$

12) Find the solution of the equation

$x^2 + 3x + 1 = 0$

Ans

$x^2 + 3x + 1 = 0$

clearly, this equation can't be factorised using the quadratic formula.

$a = 1, \quad b = 3, \quad c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$

$= \frac{-3 \pm \sqrt{9-4}}{2}$

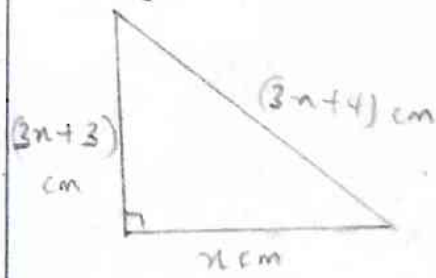
$= \frac{-3 \pm \sqrt{5}}{2}$

$x = \frac{-3}{2} + \frac{\sqrt{5}}{2}$ or $\frac{-3}{2} - \frac{\sqrt{5}}{2}$

13) The length of the sides of a right-angled triangle are $(3x+4)$cm, $(3x+3)$cm and $x$cm. Find x.

Ans

Clearly the longest side is $(3x+4)$cm



Using pythagoras theorem,

$(3x+3)^2 + x^2 = (3x+4)^2$

$9x^2 + 18x + 9 + x^2 = 9x^2 + 24x + 16$

$10x^2 + 18x + 9 = 9x^2 + 24x + 16$

$10x^2 + 9x^2 + 18x - 24x + 9 - 16 = 0$

$x^2 - 6x - 7 = 0$

factorizing

$x^2 - 7x + x - 7 = 0$

$x(x-7) + \cancel{x}(x-7) = 0$

$(x+1)(x-7) = 0$

$x = -1$ or 7.

14) Determine the value of m for which $mx^2 + 2x - 7$ has complex roots.

Ans

$mx^2 + 2x - 7$

$a = m$, $b = 2$, $c = -7$

condition for complex root is

$D = b^2 - 4ac < 0$

$\Rightarrow 2^2 - 4(m)(-7) < 0$

$4 + 28m < 0$

$28m < -4$

$\frac{28m}{28} < \frac{-4}{28}$

$m < -1/7$

15) Find the value of x in the equation: $\frac{8x-1}{2x+3} = x - 2$

Ans

$\frac{8x-1}{2x+3} \times x - 2$

$8x-1 = (2x+3)(x-2)$

$8x - 1 = 2x^2 - 4x + 3x - 6$

$8x - 1 = 2x^2 - x - 6$

$2x^2 - x - 8x - 6 + 1 = 0$

$2x^2 - 9x - 5 = 0$

Factorising, we have

$2x^2 - 10x + x - 5 = 0$

$2x(x-5) + (x-5) = 0$

$(2x+1)(x-5) = 0$

$x = 5$ or $-1/2$.

SEQUENCE & SERIES

1) The nth term of a sequence is $\log_2(n+3)$. What is the difference between the 13th and the first term?

Ans

Let the nth term be denoted by T_n

$T_n = \log_2(n+3)$

$T_1 = \log_2(1+3)$

$= \log_2(4)$

$= \log_2 2^2$

$= 2\cancel{\log_2 2}$

$= 2 \times 1$

$= 2$

$T_{13} = \log_2(13+3)$

$= \log_2 16$

$= \log_2 2^4$

$= 4\cancel{\log_2 2}$

$= 4 \times 1$

$= 4$

Difference $= T_{13} - T_1$

$= 4 - 2 = 2$

2) The third term of a linear sequence (AP) is 16 and its 6th term is 34. Find the second term

Ans

Let the ~~the~~ nth term be T_n

General term for an A.P

$T_n = a + (n-1)d$

a - first term

d - common difference

$T_3 = a + (3-1)d = 16$

$a + 2d = 16$ —— (1)

$T_6 = a + (6-1)d = 34$

$T_6 = a+5d = 34$ ——— ②

subtract equation ① from ②

$(a+5d) - (a+2d) = 34 - 16$

$3d = 18$

$d = \frac{18}{3}$

$d = 6$

substitute for d in equ ①

$a + 2(6) = 16$

$a + 12 = 16$

$a = 16 - 12$

$a = 4$

$\therefore T_2 = a + d$

$= 4 + 6$

$= 10$

3) Eight wooden poles are used for pillars and the lengths of the poles form an Arithmetic Progression (AP). If the second pole is 2m and the sixth is 5m, give the lengths of the poles, in order.

Ans

Number of wooden poles = 8

∴ we have 8 terms of the AP

second pole (T_2) = 2m

sixth pole (T_6) = 5m

$T_2 = a + d = 2$

$a + d = 2$ ——— ①

$T_6 = a + 5d = 5$

$a + 5d = 5$ ——— ②

subtract equ(1) from ②

$4d = 3$

$d = 3/4$

substitute for d in equation ①

$a + 3/4 = 2$

$a = 2 - 3/4 = 5/4 = 1\frac{1}{4}$

Third pole (T_3) = $a + 2d$

$= 5/4 + 2(3/4)$

$= 5/4 + 6/4$

$= 11/4 = 2\frac{3}{4}$

fourth pole (T_4) = $a + 3d$

$= 5/4 + 3(3/4)$

$= 5/4 + 9/4$

$= 14/4$

$= 7/2 = 3\frac{1}{2}$

Fifth pole (T_5) = $a + 4d$

$= 5/4 + 4(3/4)$

$= 5/4 + 3$

$= 17/4 = 4\frac{1}{4}$

seventh pole (T_7) = $a + 6d$

$= 5/4 + 6(3/4)$

$= 5/4 + 9/2$

$= 23/4 = 5\frac{3}{4}$

eighth pole (T_8) = $a + 7d$

$= 5/4 + 7(3/4)$

$= 5/4 + 21/4$

$= 26/4 = 6\frac{1}{2}$

Lengths of pole in order are

$1\frac{1}{4}, 2, 2\frac{3}{4}, 3\frac{1}{2}, 4\frac{1}{4}, 5, 5\frac{3}{4}, 6\frac{1}{2}$.

4) Find the 4th term of an A.P whose first term is 2 and the common difference is 0.5.

Ans

First term, $(a) = 2$

common difference $(d) = 0.5$

4th term = $a + 3d$

$= 2 + 3(0.5)$

$= 2 + 1.5$

$= 3.5$

5) The first term of an AP is equal to twice the common difference d. Find in terms of d, the 5th term of the AP.

Ans

first term = a

common difference = d

$a = 2d$

$T_5 = a + 4d$

$= 2d + 4d$

$= 6d$

6) The arithmetic mean of five numbers is 10. If four of the numbers are 6, 8, 10 and 13, what is the fifth number?.

Ans

Let the fifth number by x

Arithmetic Mean $= \frac{6+8+10+13+x}{5} = 10$

$\frac{37+x}{5} \times 10$

$37 + x = 50$

$x = 50 - 37$

$= 13$

Use the information below to answer no 7 & 8

In an AP, the difference between the 8th and the 4th terms is 20 and the 8th term is 1½ times the fourth term.

7) What is the common difference

Ans

Let the first time and the common difference be 'a' and 'd' respectively

$T_8 = a + 7d$, $T_4 = a + 3d$

$T_8 - T_4 = 20$

$(a+7d) - (a+3d) = 20$

$4d = 20$

$d = \frac{20}{4}$

$d = 5$

8) What is the first term of the sequence

Ans

$T_8 = 1\frac{1}{2} \times T_4$

$a + 7d = 3/2 \times (a + 3d)$

$a + 7(5) = 3/2 (a + 3(5))$

$a + 35 \times 3/2 (a + 15)$

$2(a + 35) = 3(a + 15)$

$2a + 70 = 3a + 45$

$70 - 45 = 3a - 2a$

$25 = a$

First term $a = 25$

9) The common ratio of a GP is 2. If the 5th term is greater than the 1st term by 45, find the 5th term.

Ans

Let the first term and common ratio be 'a' and 'r' respectively

$r = 2$

$T_5 = ar^4$ $[T_n = ar^{n-1}]$

$T_5 - T_1 = 45$

$ar^4 - a = 45$

$a(r^4 - 1) = 45$

$a(2^4 - 1) = 45$

$a(16 - 1) = 45$

$a(15) = 45$

$a = \frac{45}{15}$

$a = 3$

$T_5 = ar^4 = 3(2)^4$

$= 3(16) = 48$

10) The n^{th} term of a sequence is given by $(-1)^{n-2}2^{n-1}$. Find the sum of the second and third terms.

Ans

$T_n = (-1)^{n-2}2^{n-1}$

$T_2 = (-1)^{2-2}2^{2-1}$

$= (-1)^0 2^1$

$= 1 \times 2$

$= 2$

$T_3 = (-1)^{3-2}2^{3-1}$

$= (-1)^1 2^2$

$= (-1)(4)$

$= -4$

Sum of third term and second term

$= T_3 + T_2 = -4 + 2$

$= -2.$

11) ~~State~~ the fifth and seventh terms of the sequence $-2, -3, -4\frac{1}{2} \dots$

Ans

First term, $a = -2$

This is clearly a G.P and not an A.P

$r = \frac{T_{n+1}}{T_n} = \frac{T_n}{T_{n-1}}$

$r = \frac{-4\frac{1}{2}}{-3} = \frac{-3}{-2}$

$r = 3/2$

Fifth term $= ar^4$

$= (-2)(3/2)^4$

$= (-2)(81/16)$

$= -\frac{81}{8}$

$= -10\,1/8$

Seventh term $T_7 = ar^6$

$= (-2)(3/2)^6$

$= (-2)(729/64)$

$= \frac{-729}{32} = -22\frac{25}{32}$

13) If $\frac{16}{9}, x, 1, y$ are in geometric progression, find the product of x and y

Ans

If the sequence is a G.P, then it possesses a common ratio.

$r = \frac{x}{16/9} \cancel{\times} \frac{1}{x}$

$x^2 = \frac{16}{9}$

$x = \pm\sqrt{16/9}$

$= \pm 4/3$

Also,

$r = \frac{1}{x} = \frac{y}{1}$

$\frac{1}{4/3} = y/1$

$1 = 4/3\, y$

$y = 3/4$

or

$r = \frac{1}{-4/3} = y/1$

$1 = -4/3\, y$

$y = -3/4$

The solution of $(x, y) = (4/3, 3/4)$ or $(-4/3, -3/4)$

14) What is the common ratio of the exponential series (G.P)

$(\sqrt{2}-1) + (3-2\sqrt{2}) + \dots$?

Ans

$T_1 = \sqrt{2}-1$, $T_2 = 3 - 2\sqrt{2}$

Common ratio $(r) = \frac{T_2}{T_1}$

$$r = \frac{(3-2\sqrt{2})}{(\sqrt{2}-1)}$$

by rationalizing,

$$r = \frac{3-2\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \frac{(3-2\sqrt{2})(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{3\sqrt{2}+3-2(2)-2\sqrt{2}}{(2)+\cancel{\sqrt{2}}-\cancel{\sqrt{2}}-1}$$

$$= \frac{\sqrt{2}+3-4}{2-1}$$

$$= \frac{\sqrt{2}-4}{2-1}$$

$$r = \sqrt{2}-1$$

15) The sum of the first n terms of a series is given by $S_n = n^2+2n$. Find the nth term. and the first term

Ans

~~$S_n = n^2+2n$~~

~~$T_n = S_{n+1} - S_n$ or $S_n - S_{(n-1)}$~~

~~$T_n = [(n+1)^2+2(n+1)] - [n^2+2n]$~~

~~$= [n^2+2n+1+2n+2 - n^2 - 2n$~~

~~$T_n = 2n+3$~~

~~First term $T_1 = 2(1)+3$~~

~~$= 2+3$~~

~~$= 5$~~

16) The twelveth term of a linear sequence is 47 and the sum of the first three terms is 21. Find the common difference and First term.

Ans

The sum of a sequence is called a series.

Sum of an AP is given by;

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$T_{12} = a+11d = 47$ — (1)

$S_3 = 21$

$\frac{3}{2}[2a+(3-1)d] = 21$

$\frac{3}{2}(2a+2d) = 21$

$3(a+d) = 21$

$a+d = 7$ — (2)

subtract equ(2) from (1)

$10d = 40$

$d = \frac{40}{10}$

$d = 4$

substitute the value of d in equ(2)

$a+4 = 7$

$a = 3$

First term = 3

common difference = 4

15 Ans

$S_n = n^2+2n$

$T_n = S_n - S_{n-1}$

$= n^2+2n - [(n-1)^2+2(n-1)]$

$= n^2+2n - [n^2-\cancel{2n}+1+\cancel{2n}-2]$

$= n^2+2n - (n^2-1]$

$= \cancel{n^2}+2n-\cancel{n^2}+1$

$= 2n+1$

First term $= T_1 = 2(1)+1$

$= 3$.

17) The fourth term of an exponential sequence is 108 and the common ratio is 3. Calculate the value of the eighth term.

Ans

$T_4 = ar^3 = 108$

common ratio $r = 3$

$T_4 = a(3)^3 = 108$

$a(27) = 108$

$a = \frac{108}{27}$

$a = \cancel{36}\ 4$

$T_8 = ar^7$

$= 4(3)^7$

$= 4(2187)$

$= 8748$

18) Calculate the sum of the first five terms of the sequence above

Ans

$n = 5$

$a = 4$

$r = 3$

Sum of a Geometric Progression

$S_n = \frac{a(r^n - 1)}{r-1}, \quad r > 1$

$= \frac{a(1-r^n)}{1-r}, \quad r < 1$

In this case $r = 3 > 1$

$S_5 = \frac{4(3^5 - 1)}{3-1}$

$= \frac{4(243-1)}{2}$

$= 2(242) = 484$

19) Find the sum of all the integers from 1000 to 3500 inclusive.

Ans

The series is of this form

$1000 + 1001 + 1002 + \dots + 3499 + 3500$

From the series above, we discovered that $a = 1000$ $d = 1001$.

Since 1000 and 3500 are inclusive

$n = (3500 - 1000) + 1$

$= 2500 + 1 = 2501$

This is also an Arithmetic progression

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{2501}{2}(2(1000) + (2501-1)1]$

$= \frac{2501}{2}(2000 + 2500)$

$= \frac{2501}{2}(4500)$

$= (2501)(2250)$

$= 5627250$

20) Find the sum of the first eight terms of a linear sequence whose first term is 6 and whose last term is 46.

Ans

$a = 6 \qquad n = 8$

$l = 46$

$S_n = \frac{n}{2}(a + l)$

$= \frac{8}{2}(6+46)$

$= \frac{8}{2}(52)$

$= 4(52)$

$= 208$

21) If 3, p, q, 24 are consecutive terms of an exponential sequence, find the values of p and q.

Ans

3, P, q, 24

Let 3 be the first term i.e $a = 3$

24 will be the 4th term $T_4 = 24$

$T_4 = ar^3 = 24$

$3r^3 = 24$

$r^3 = 8$

$r = 2$

$\therefore T_2 = P = ar$

$= 3(2)$

$= 6$

$T_3 = q = ar^2$

$= 3(2)^2$

$= 12$

Use the information below to answer questions 22 & 23

The sum of the first two terms of an exponential sequence is 135 and the sum of the third and the fourth terms is 60. Given that the common ratio is positive; calculate:

22) The common ratio.

Ans

Let the first term and the common ratio be 'a' and 'r' respectively

$T_1 + T_2 = 135$

$a + ar = 135$ ——— (1)

$T_3 + T_4 = 60$

$ar^2 + ar^3 = 60$ ——— (2)

dividing eq (2) by (1)

$$\frac{ar^2 + ar^3}{a + ar} = \frac{60}{135}$$

$$\frac{\cancel{a}(r^2 + r^3)}{\cancel{a}(1 + r)} = \frac{4}{9}$$

$$\frac{r^2 + r^3}{1 + r} = \frac{4}{9}$$

$$\frac{r^2\cancel{(1+r)}}{\cancel{1+r}} = \frac{4}{9}$$

$r^2 = 4/9$

$r = \pm\sqrt{4/9}$

$= \pm 2/3$

but since r is positive

$r = 2/3.$

23) The first term:

Ans

substitute for r in eqn (1)

$a + ar = 135$

$a + \frac{2}{3}a = 135$

$\frac{5}{3}a = 135$

$a = \overset{27}{\cancel{135}} \times \frac{3}{\cancel{5}}$

$a = 81$

24) If the sum of the first n terms of the series $4 + 7 + 10 + \ldots$ is 209. Find n

Ans

first term $a = 4$

common difference $d = 10 - 7 = 7 - 4$

$= 3$

$S_n = 209$

$\frac{n}{2}[2a + (n - 1)d] = 209$

$\frac{n}{2}[2(4) + (n-1)3] = 209$

$\frac{n}{2}(8 + 3n - 3) = 209$

$n(5 + 3n) = 418$

$5n + 3n^2 = 418$

$3n^2 + 5n - 418 = 0$

$3n^2 + 38n - 33n - 418 = 0$

$n(3n+38) - 11(3n+38) = 0$

$(n-11)(3n+38) = 0$

$n = 11$ or $-\frac{38}{3}$

Since its number of terms, it can't be negative but only positive

$\therefore n = 11.$

25) The fifth term of an exponential sequence is greater than the fourth by $13\frac{1}{2}$ and the fourth term is greater than the third by 9. Find the common ratios.

Ans

$T_5 - T_4 = 13\frac{1}{2}$

$T_4 - T_3 = 9$

$T_5 - T_4 = ar^4 - ar^3 = 13\frac{1}{2}$ —— (1)

$T_4 - T_3 = ar^3 - ar^2 = 9$ —— (2)

divide equ (1) by (2)

$$\frac{ar^4 - ar^3}{ar^3 - ar^2} = \frac{13\frac{1}{2}}{9}$$

$$\frac{\cancel{a}(r^4 - r^3)}{\cancel{a}(r^3 - r^2)} = \frac{27/2}{9}$$

$$\frac{r^4 - r^3}{r^3 - r^2} = \frac{27}{18}$$

$$\frac{r\cancel{(r^3 - r^2)}}{\cancel{r^3 - r^2}} = \frac{3}{2}$$

$r = 3/2$

26) The sum of the first four terms of a linear sequence (AP) is 26 and that of the next four terms is 74. Find the value of the first term

Ans

$S_4 = 26$

The sum of the next four terms can be deduced by subtracting S_4 from S_8 $= S_8 - S_4 = 74.$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_4 = \frac{4}{2}[2a + (4-1)d] = 26$

$= 2(2a + 3d) = 26$

$2a + 3d = 13$ —— (1)

$S_8 - S_4 = \frac{8}{2}[2a + (8-1)d] - \frac{4}{2}[2a + (4-1)d]$

$= 74$

$4[2a + 7d] - 2[2a + 3d] = 74$

$8a + 28d - 4a - 6d = 74$

$4a + 22d = 74$

$2(2a + 11d) = 74$

$2a + 11d = 37$ —— (2)

substract equ (1) from (2)

$8d = 24$

$d = \frac{24}{8}$

$d = 3$

substitute for $d = 3$ in equ (1)

$2a + 3(3) = 13$

$2a + 9 = 13$

$2a = 4$

$a = 2.$

27) Find the sum to infinity of the following progression.

$3 + 2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \cdots$

Ans

First term $a = 3$

Common ratio $r = 2/3$

Sum to infinity = S_∞

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{3}{1-2/3}$$

$$= \frac{3}{1/3}$$

$$= 9$$

28) find ~~the~~ sum of ~~these~~ the first 15 terms of the AP

$\log n, \log n^2, \log n^3 \ldots$

Ans

First term $a = \log n$

common difference $d = \log n^2 - \log n$

$= 2\log n - \log n$

$= \log n$

$$S_{15} = \frac{15}{2}\left[2(\log n) + (15-1)\log n\right]$$

$$= \frac{15}{2}\left[2\log n + 14\log n\right]$$

$$= \frac{15}{2}(16\log n)$$

$$= (15)(8)\log n$$

$$= 120\log n$$

$$= \log n^{120}$$

29) In order to save one million naira for ~~the~~ purchase of some goods, a man saved ₦1, ₦2, ₦4, ₦8 on the first, second, third and fourth day respectively. At this rate, assuming ~~that~~ no interest was added, what amount was saved on the 15th day?

Ans

The amount of money saved daily forms a geometric sequences

i.e ₦1, ₦2, ₦4, ₦8

with common ratio of $\frac{2}{1} = \frac{4}{2} = 2$

$r = 2$

$a = 1$

Amount saved on the 15th day = S_{15}

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_{15} = \frac{1(2^{15}-1)}{2-1}$$

$$= 32768 - 1$$

$$= 32767.$$

30) At least how many days were needed to accumulate the one million naira?

Ans

In this case, our S_n = ₦1m

$$S_n = \frac{a(r^n-1)}{r-1} = 1{,}000{,}000$$

$$\frac{1(2^n-1)}{2-1} = 1{,}000{,}000$$

$$2^n - 1 = 1{,}000{,}000$$

$$2^n = 1000001$$

Log both sides

$$\log 2^n = \log 1{,}000{,}001$$

$$n\log 2 = \log 1{,}000{,}001$$

$$n = \frac{\log 1{,}000{,}001}{\log 2}$$

$$n = \log_2 1{,}000{,}001$$

$$= 19.93$$

$\therefore$ The least number of days needed is 20 since at 19 days it wont be up to a million naira yet.