NNAMDI AZIKIWE UNIVERSITY, AWKA
Department Of Mathematics 2005/2006 Session
MAT 102: ELEMENT ACES PROVIDED.
Warning: Mutilation Of Answers Will Be Penalized.

DEPT:..................................REG NO.:..........................
NAME:..

This refers to Q1- 6

Let A = {-3,2,1} Z = Sets of integers, let f :A $\longrightarrow$ Z be defined by f(x) = 2x- 1:

1. Domain of f = ...
2. Codmain of f= ....
3. Range of f =.........
4. Image of f =......................5. Is f one - one?................
6. Is f onto?..
7. If f(x) = sin2x + cos x. (a) f (π/4) = ----------------------

   (b) f(π) = --

8. $f(x) = \begin{cases} -5, & -4 \le x < 2 \\ 0, & 2 < x \le 10 \\ 4, & x > 10 \end{cases}$ f(0) =..........f(12) = ...

9. $\lim_{x \to 1} \dfrac{x - 1}{2x^2 - x - 1}$ Ans: --------------------------------

10. $\lim_{x \to \infty} \dfrac{2x^3 + x^2 + 1}{x^2 - 2x + 3}$ Ans:.................................

11. $\lim_{x \to 0} \dfrac{4x^3 - x^2 + 1}{6x^2 - 2x}$ Ans:.................................

Let f , g:R $\rightarrow$ R be defined by f(x) = x^2-1 , g(x) = x + 2

12. fog(x) = ...

13. $gof(x) =$ ....

14. If $y = \sin^3 x - \cos 3x$, $dy/dx =$ ....

15. Differentiates with respect to x, $y = \dfrac{1}{x^{2/3}} + 3e^{x^2}$

16. The derivative of $2e^{\sqrt{x}} + \log_e x^{1/2} =$ ....

17. If $x^3y^2 - 4y^6 = 3x$, $dy/dx =$ ....

18. The gradient of the curve $y = 4x^3 - 2x + 1$ at the point (2, 3) is --

This refers to Q19 - 22. The distance S of a p article from a fixed point O is given by $S = t^2 - 2t + 3$ meters: ....

19 Its velocity after 2 seconds = ...

20. Its distance from 0 after 3 seconds = ...

21. When is the particle stationary? ....

22. The acceleration of the particle after 2 seconds = ...

23. Evaluate the integral $\int 2x^3 dx$. Ans.................................

24. Evaluate the integral $\int \sin x dx + \int \cos x\, dx$ Ans ....

25. Evaluate the integral $\int e^{3x} dx + \int 3/(3x+2)\, dx$ Ans...........

Suggested Solutions to the Mat 102 (2006) Session For Nau Students - Mr. Ohms:

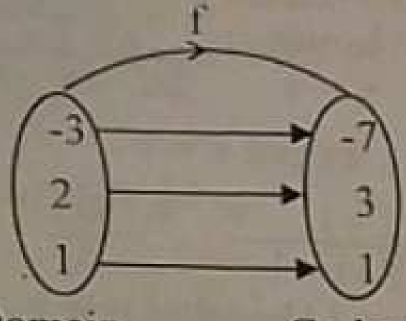
1. ***Working***: $A = \{-3,2,1\}$; $f(x) = 2x - 1$,

:. Domain of f $= \{-3,2,1\}$ (2). Codomain of $f = \{-7,3,1\}$

i.e $f(x) = 2x - 1$, if $x = -3$; => $f(-3) = 2(-3) - 1 = -7$

When $x = 2$, $f(2) = 2(2) - 1 = 2$; $x = 1$:. $f(1) = 2(1) - 1 = 1$

thus we have :



3. Range of f = Sets of all integers
4. Image of -3 is : $f(-3) = 2(-3) - 1 = -7$
5. From the diagram above ; f is one - one . This is so because every pair of different elements in the *domain* have different images in the *codmain*
6. "f" is onto , since every member of the *codmain* is the image of at least one member of the *domain*.
7. (a) Given: $f(x) = \text{Sin}2x + \text{Cos}\, x$

   :. $f(\pi/4) = \text{Sin}2(\pi/4) + \text{Cos}(\pi/4)$; $= \text{Sin}\pi/2 + \cos\pi/4$

   $\text{Sin}\,\pi = 180$, => $\text{Sin}\,\pi/2 + \cos\pi/4 = \text{Sin}90 + \text{Sin}45$

   $= 1 + \sqrt{2}/2 = (1 + \sqrt{2})/2$

   (b) $f(\pi) = \text{Sin}2\pi + \text{Cos}\pi$; Where $\text{Sin}2\pi = 360 = 0$

   :. $f(\pi) = 0 + (-1) = -1$
8. Given:

$$\begin{cases} -5, & -4 \leq x < 2 \\ 0, & 2 < x \leq 10 \\ 4, & x > 10 \end{cases}$$

(a). $f(0) = -5$, since $x = 0$ falls within the interval of $x > 10$

9. Given: $\lim_{x=1} \frac{x - 1}{2x^2 - x - 1}$

First, we put $x = 1$

i.e $\frac{1 - 1}{2(1)^2 - 1 - 1} = \frac{0}{0}$ = Indeterminate

So, we L' hospital rule by differentiating both the numerator and denominator.

i.e $\frac{1}{4x - 1}$, So we let $x = 1$

$\therefore \frac{1}{4(1) - 1} = \frac{1}{3}$

10. $\lim_{x \to \infty} \frac{2x^3 + x^2 + 1}{x^2 - 2x + 3}$

NB: Whenever, the limit of $x \to \infty$, we divide first, each of the term by the highest power of x.

i.e $$\frac{\frac{2x^3}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}}{\frac{x^2}{x^3} - \frac{2x}{x^3} + \frac{3}{x^3}}$$

$$= \frac{2 + 1/x + 1/x^3}{1/x - 2/x^2 + 3/x^3}$$

So, we let $x = \infty$

i.e $\frac{2 + 1/\infty + 1/\infty^3}{1/\infty - 2/\infty^2 + 3/\infty^3} = \frac{2 + 0 + 0}{0 - 0 + 0} = \frac{0}{0}$ = indeterminate

So we differentiate. i.e.

$\frac{6x^2 + 2x}{2x - 2}$, Put $x = \infty$

i.e $\frac{6(\infty)^2 + 2(\infty)}{2(\infty) - 2} = \frac{\infty}{\infty}$ = Indeterminate , so we differentiate again .i.e

$\frac{12x + 2}{2}$, we let $x = \infty$

$$= \frac{12(\infty) + 2}{2} = \frac{\infty}{2} = \infty$$

11. $\lim_{x \to 0} \frac{4x^3 - x^2 + 1}{6x^2 - 2x}$

Put $x = 0$

$= \frac{4(0)^3 - 0^2 + 1}{6(0)^2 - 2(0)} = \frac{1}{0}$ = Indeterminate .

Next, apply L' hospital rule by differentiating continuously until it is determinate.

$= \frac{12x^2 - 2x}{12x - 2}$, Put $x = 0$

$= \frac{12(0)^2 - 2(0)}{12(0) - 2} = 0$

12. $f(x) = x^2 - 1$, $g(x) = x + 2$; => $fog(x) = f(g(x)) = f(x + 2)$

$= (x + 2)^2 - 1 = x^2 + 4x + 3$

13. $gof(x) = g(f(x)) = g(x^2 - 1) = x^2 - 1 + 2 = x^2 + 1$

14. $y = \sin^2 x - \cos 3x$, $dy/dx = 2\sin x \cos x + 3\sin 3x$

15. $y = 1/x^{2/3} + 3e^{x^2}$; $y = x^{-2/3} + 3e^{x^2}$

$\therefore dy/dx = -2/3x^{-2/3-1} + 6xe^{x^2} = \frac{-2}{3x^{5/3}} + 6xe^{x^2}$

16. $y = 2e^{\sqrt{x}} + \ln x^{1/2} = 2e^{x^{1/2}} + \ln x^{1/2}$

$$\therefore \frac{dy}{dx} = \frac{e^{x^{1/2}}}{x^{3/2}} + \frac{1}{2x^2}$$

17. Given ; $x^3y^2 - 4y = 3x$

Differentiating implicitely, we have;

$3x^2y^2 + 2x^3y\,dy/dx - 4dy/dx = 3$

$dy/dx(2x^3y - 4) = 3 - 3x^2y^2$

$$dy/dx = \frac{3 - 3x^2y^2}{2x^3y - 4} = \frac{3(1 - x^2y^2)}{2(x^3y - 2)}$$

18. $y = 4x^3 - 2x + 1$, at point (2,3)

$\therefore dy/dx = 12x^2 - 2$, at $x = 2$

$dy/dx = 12(2)^2 - 2 = 46$

19. $S = t^2 - 2t + 3$; velocity (v) = ds/dt = 2t - 2

at t = 2secs $\therefore$ v = 2(2) - 2 = 2m/secs

20. Dist., $S = t^2 - 2t + 3$: at t = 3 , $S = 3^2 - 2(3) + 3 = 6$m

21. The particle will be at stationary point when ds/dt = 0.

i.e. (v = 0) $\therefore 2t - 2 = 0$; $\therefore t = 1$sec.

22. acceleration, a = dv/dt = 2, at t = 0 ; acc. , a = 2m/sec^2.

23. Given; $\int 2x^3dx = \frac{2x^{3+1}}{3+1} + C = \frac{x^4}{2} + C$

24. Given; $\int \sin x\,dx + \int \cos x\,dx = -\cos x + \sin x + C$

25. Given; $\int e^{3x}dx + \int 3/(3x+2)dx$

First, we integrate $\int e^{3x}dx$; Here , let $u = 3x$, $du/dx = 3$

$dx = du/3$; i.e. $\int e^u du/3 = 1/3\int e^u du = (1/3)e^u = (1/3)e^{3x}$,

Next, we integrate $\int 3/(3x+2)dx = 3\int 1/(3x+2)dx$

$dx = du/3$

:. 3∫1/(3x+2)dx = 3∫1/u.du/3 = ∫1/udu = Inu

In3x + 2. .: ∫e³ˣdx + ∫3/(3x+2)dx is:

$\frac{e^{3x}}{3}$ + In(3x+2) + C

NNAMDI AZIKIWE UNIVERSITY, AWKA.
DEPARTMENT OF MATHEMATICS
MAT 102, ELEMENTARY MATHEMATICS
SECOND SEMESTER EXAMINATIONS 2006/2007
ALTERNATIVE A :TIME: 2 HRS ANSWER ALL

1. Only one of the following sets of ordered pairs is a function. a. {(x, y)}/y > x + 1 b. {x, y}/y + 2 = 0 c. {(x, y)}/x = 6 d. (x,y)/x^2+y^2 < 25
2. Which of the following relations is not a function? a. (1,1) (2,2) (3,3) (4,4) b. (1,2) (2,2) (3,2) (4,2) c. (1,3), (2,3) (1,4) (2,4) d. (4,1), (3,2) (2,3)
3. If f(x) = 1/(1-x) (x ≠ 1) and g(x) = (x-1)/x then gof(x) is : a. x^2 b. 1/x c. x d. 1/(x -1)
4. f(-3)/g(3) is: a. -6 b. 2 2/3 c. 1/6 d. 3.8
5. If the distance between the points (16, C) and (1,1) is 17, find the two values of C. a. (-8, 8) b. (-7, 8) c. (9, 8) d. (-7, 9)
6. The length of the line joining the points (2, 5) and (5, 9) is : a. 7 b. 6 c. 5 d.4
7. The equation of a straight line which passes through the pint (-4, 3) and is parallel to the line y = 2x + 5 is : a. y = 2x + 7 b. y = 2x - 7 c. y = 2x + 1 d. y = 2x+ 11

8. The equation of a straight line which passes through the point (2, -1) and perpendicular to the line $y = -3x + 4$ is a. $3y - x + 7 = 0$ b. $3y + x - 7 = 0$ c. $3y + x + 7 = 0$ d. $3y - x - 7 = 0$

9. Find the point which divides the line joining the points (2, -1) and (3,4) in the ratio 2:1. a. (3/8, 7/8) b. (8/3, 3/7) c. (8/3, 7/3) d. (-8/3, -7/3)

10. Find the centre and radius of the circle $x^2 + y^2 - 6x + 14y + 49 = 0$: a. (3, -7) R3. b. (-3, 7)R3 c. (3,7)R7 d. (-7, 3) R3

11. The equation of a circle which has its centre at (3,5) and radius 6 : a. $x^2+y^2 - 6x - 10y - 2 = 0$ b. $x^2+y^2 + 6x + 10y + 34 = 0$ c. $x^2+y^2 - 6x - 10y + 2 = 0$ d. $x^2 + y^2 - 6x - 10y + 70 = 0$

12. Only one of the following equations represents a circle a. $x^2 - y^2 + 2x + 3y + 7 = 0$ b. $2x+4y - 10 - x^2 + y^2 = 0$ c. $x^2+y^2 - 16 = 0$ d. $3x^2 + 3y^2 + 2xy + 4 = 0$

13. Find the equation of a circle whose diameter is AB where A is the point (-1,3) and B (3,2).
a. $x^2+y^2 - 2x - 5y + 3 = 0$ b. $x^2 + y^2 + 4x - 5y+9 = 0$
c. $x^2 + y^2 + 4x - 5y + 3 = 0$ d. $x^2 + y^2 + 2x - 5y + 9 = 0$

14. If $y = 1/(2x^2+5)$, find dy/dx a. $(4x)(2x^2 + 5)^{-1}$
b. $(-4x)(2x^2+5)^2$ c. $(4)(2x^2 + 5)^2$ d. $(2x)(2x^2 + 5)^{-2}$

15. If y = 1 + tanx/Secx then dy/dx is :
$\frac{\tan x - 1}{\sec x}$ b. $\frac{\sec x}{1 - \tan x}$ c. $\frac{1 - \tan x}{\sec x}$ d. NOTA

16. If $y = \tan^4 x$ find dy/dx

a. $4\tan^3 x\sec^2 x$ b. $3\sec^2 x\tan^3 x$ c. $4\tan^3 x\sec^3 x$ d. NOTA

17. Find the gradient of the curve $y = \sin x + \cos x$ at the points $x = \pi/4$: a. 1 b. 2 c. -1 d. 0

18. Air is being pumped into a spherical balloon at the rate of $4cm^3s^{-1}$. Find the rate of change of the radius of the balloon when the radius of the balloon is 5cm.

a. $\frac{1}{50\pi} cms^{-1}$ b. $\frac{1cms^{-1}}{5\pi}$ c. $\frac{\pi cms^{-1}}{5}$ d. $\frac{1cms^{-1}}{25\pi}$

19. Find the equation of the tangent to the curve $y = 2x^2 - 3x + 1$ at (2 ,3). : a. $y - 5x + 7 = 0$

b. $y + 5x + 7 = 0$ c. $y - 9x + 25 = 0$ d. $y - 5x + 9 = 0$

20. When will it come to rest again?

a. 6s b. 3s c. -3s d. NOTA

21. Where will it come to rest again?

a. 6m b. -12m c. -36m d. 12m

22. Find the area bounded by the curve $y = 2x^2 + 3$ the x-axis and the ordinates $x = 1$ and $x = 3$. a. 23 1/3 b. 24 2/3 c. 23 2/3 d. 24

23. $\int_0^1 (2x - 1)^5 dx$. a. 1/6 b. 0 c. 1/5 d. 1

24. $\int_1^1 dx/\sqrt{x}$. a.0 b. -1 c. 1 d. NOTA

25. $\int (e^x + e^x)dx$. a. $2e^x + k$ b. $e^{x^2} + k$ c. $e^{x^2} + k$ d. $e^{x+2} + k$

26. $\int_0^1 xe^x dx$. a. 2 b. 0 c. 1 d. -1

27. $\int (x + 5)/(x-1)dx$.

a. $x - 4\ln(x-1) + k$ b. $-x + 6\ln(x-1)+k$

c. $-x + 6\ln(x-1)+k$ d. $x + 6\ln(x-1)+k$

28. $\int^{2}_{\pi/4} x\sin 2x$ a. ¼(π -1)   b. ¼ (π + 1)   c. ¼(1 - π)

d. 1/8 (π -1)

29. $\int (x+5)/(x+3)(x+4)$:

a. $\dfrac{\ln(x+3)^2}{x+4} + K$ b. $\ln(x+3)/(x+3) + K$

c. $\ln(x+4)/(x+4)^2 + K$ d. $\ln(x+3)/(x+4)^2$

30. Given $f(x) = x^3 - 6x^2 + 9x + 2$, find the values of x for which f(x) has stationary values and determine the nature of these stationary values a. (1 ,3) Max1Min3 b. (-1,3) Min3 Max -1 c. (-1, -3) Min –1Max -3 d. (1,3) Min1 Max 3

31. Given $y = x^{\sin x}$, dy/dx is :

a. y(sinx/x – cos xinx)   b. y(sinx/x + sinxlnx)

c. y(sinx/x + cosxlnx)   d. y(x/sinx + cosxsinx)

32. Given the function $y = x^2 - 2\ln x$ $(x > 0)$ find the turning point and its nature. : a. (1,max)   b. (1,min)

c. (0,1) min1   d. (0,1) max 1

33. Evaluate $\lim_{x=1} \dfrac{x^2 - 1}{x - 1}$: a. 2   b. 0   c. 1

d. NOTA

Answers To the Mat102 (2007) Session For Nau Students – Mr. Ohms

1.B   2.C

3.C   ***Working:*** $f(x) = 1/(1-x)$, $g(x) = (x-1)/x$

$\Rightarrow gof = g(f(x)) = g(1/(1-x))$

=> $g(1/(1-x)) = \frac{1/(1-x) - 1/1}{1/(1-x)} = x$ => C

4.D *Working:*

$f(x) = 1/(1-x)$, $g(x) = (x-1)/x$

$f(-3) = 1/(1--3) = 1/4$, $g(3) = (3-1)/3 = 2/3$. *thus,*

$f(-3)/g(3) = \frac{1/4}{2/3} = 3/8$

5.D *Working:* Dist. $|AB| = \sqrt{(1-16)^2 + (1-C)^2}$

thus, C = -7 or 9 => D

6.C *Working:* length = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, Where $x_1 = 2$, $x_2 = 5$, $y_1 = 5$, $y_2 = 9$

=> length = $\sqrt{(5-1)^2 + (9-5)^2} = 5$ => C

7.D Given $y = 2x + 5$, => $m = 2$. Using $y - y_1 = m(x - x_1)$, Where $x_1 = -4$, $y_1 = 3$; => $y - 3 = 2(x + 4)$, $y = 2x + 11$ => D.

8.NOTA *Working:* Given $y = -3x + 4$, => $m_2 = -3$ for perpendicularity, $m_1m_2 = -1$; => $m_1m_2 = -1$; => $m_1 = -1/-3$, i.e. $m_1 = 1/3$:Since $y - y_1 = m(x - x_1)$,

=> $y + 1 = 1/3(x - 2)$: $3y - x + 5 = 0$: => NOTA

9.C *Working:* $R_2 = \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}$

Where $m = 2$, $n = 1$, $x_1 = 2$, $y_1 = -1$, $x_2 = 3$, $y_2 = 4$

=> $R_2 = \frac{2(3) + 1(2)}{2+1}, \frac{2(4) + 1(-1)}{2+1} = (8/3, 7/3)$ => C.

10.A *Working:* Given $x^2 + y^2 - 6x + 14y + 49 = 0$, centre = $(-g, -f)$:from the eqn above, $-g = -6$, => $g = 6$

$-f = 14$, $\Rightarrow f = -14$. *Thus*, centre $= (6, -14)$

Or Radius, $R = \sqrt{g^2 + f^2 - c}$, where $c = 49$.

$\Rightarrow R = 3$ ∴ Answer is $(3, -7)$ R3

11. ***Working:*** $(x - a)^2 + (y - b)^2 = r^2$, $a = 3, b = 5, r = 6$

$\Rightarrow (x - 3)^2 + (y - 5)^2 = 6^2$ ∴ $x^2 + y^2 - 6x - 10y - 2 = 0$.

12.C ***Working:*** **Conditions For Equation Of A Circle:**

i. The coeff. Of x^2 and y^2 must be one

ii. The power of x and y must be 2.

iii. The coeff. Of x^2 and y^2 must have the same sign.

13. *Go to Q13 (2008) SESSION for Solution.*

14.B ***Working:*** $y = 1/(2x^2 + 5)$, $y = (2x^2 + 5)^{-1}$,

$dy/dx = -1(2x^2 + 5)^{-2} \times 4x = -4x/(2x^2 + 5)^2 \Rightarrow B$

15.C ***Working:*** $y = \dfrac{1 + \tan x}{\sec x}$. Let $u = 1 + \tan x$, $v = \sec x$

$du/dx = \sec^2 x$, $dv/dx = \sec x \tan x$.

But, $dy/dx = \dfrac{V du/dx - U dv/dx}{V^2}$

$= \dfrac{\sec x(\sec^2 x) - (\sec x \tan x)(1 + \tan x)}{\sec^2 x}$

$= \dfrac{\sec x \sec^2 x - \sec x \tan x - \sec x \tan^2 x}{\sec^2 x}$

Since $\sin^2 x + \cos^2 x = 1$ ________ (1)

Divide through by $\cos^2 x$ in eqn (1). $\dfrac{\sin^2 x}{\cos^2 x} + 1 = \dfrac{1}{\cos^2 x}$

$= \tan^2 x + 1 = \sec^2 x$; thus,

$dy/dx = \dfrac{\sec x(1 + \tan^2 x) - \sec x \tan x - \sec x \tan^2 x}{\sec^2 x}$

$$= \frac{\sec x + \sec x\tan^2 x - \sec x\tan x - \sec x\tan^2 x}{\operatorname{Sec}^2 x}$$

$$= \frac{\operatorname{Sec}x - \operatorname{Sec}\tan x}{\operatorname{Sec}^2}$$

$$= \frac{\operatorname{Sec}x\,(1 - \tan x)}{\operatorname{Sec}^2 x} = \frac{1 - \tan x}{\operatorname{Sec}x} \Rightarrow C$$

16.A ***Working:*** Given that $y = \tan^4 x$, using the short cut method, $dy/dx = 4\sec^2 x\tan^3 x$ OR $= 4\tan^3 x\sec^2 x \Rightarrow A$

17.D ***Working:*** $y = \sin x + \cos x$; $dy/dx = \operatorname{Cos}x - \sin x$; at the point $x = \pi/4 = 45^\circ$;=> $dy/dx = \operatorname{Cos}45 - \sin 45 = 0 \Rightarrow D$

18.D ***Working:*** Volume of a sphere $(V) = 4/3\pi r^3$.
Differentiating both sides with respect to t, we have ;
$dv/dt = 4/3\pi 3r^2\, dr/dt$ i.e. $3dv/dt = 4\pi 3r^2\, dr/dt$,
$dr/dt = 3dv/dt \,.\, 1/4\pi 3r^2$. But $dv/dt = 4$, $r = 5$
$\Rightarrow dr/dt = 3(4).\, 1/4\pi \times 3(5)^2 = 1/25\pi \Rightarrow$ D.

19.A Working: $y = 2x^2 - 3x + 1$, Slope = Gradient = dy/dx = $4x - 3$; at the point (2 ,3) $dy/dx = 4(2) - 3 = 5$. But the eqn of a tangent is given by slope $= \frac{y - y_1}{x - x_1}$

$5 = \frac{y - 3}{x - 2}$, $\therefore\ y - 5x + 7 = 0 \Rightarrow$ (A)

20.A ***Working:*** Given $a = 6 - 2t$, if $v = \int a\,dt = \int (6 - 2t)dt$,
$v = 6t - t^2 + c$, when $v = 0$, $t = 0$ i.e. $0 = 6(0) - 0^2 + c$,
$c = 0$ at final rest, $v = 0$ i.e. $0 = 6t - t^2 \Rightarrow t = 0$ or $6 \Rightarrow$ A

21.D **Working:** $S = \int v\,dt = \int (6t - t^2)dt = 6t^2/2 - (t^3)/2 + C$.
when $S = 0$, $t = 0$, $\Rightarrow C = 0$ i.e. $S = 3t^2 - (t^3)/3$; at $t = 6$

$S = 3(6)^2 - (6^3)/3 = 36m \Rightarrow D$

22.A *Working:* $y = 2x^2 + 3$; Area $= \int_{x_1}^{x_2} y\,dx =$ $\int_1^3 (2x^2 + 3)dx = (2x^3/3 + 3x]_1^3$ = upper Limit - lower limit $= ((2(3)^3)/3 + 3(3)) - ((2(1)^3)/1 + 3(1)), = 27 - 11/3$ $= 23\ 1/3 \Rightarrow A$

23B *Working:* $\int_0^1 (2x - 1)^5 dx$: let $u = 2x - 1 \Rightarrow du/dx = 2$ i.e.

$dx = du/2$.: Substituting , we have . $\int_0^1 u^5 \frac{du}{2}$

$$\frac{1}{2}\left[\frac{u^6}{2}\right]_0^1 = \left[\frac{12x - 1}{2}\right]_0^1$$

= Upper limit – Lower limit

$$= \frac{(2(1) - 1)^6}{12} - \frac{(2(0) - 1)^6}{12} = 0 \Rightarrow B$$

24.A *Working:*
$$\int_{-1}^{1} \frac{1}{\sqrt[3]{x}}\,dx = \int_{-1}^{1} \frac{1}{x^{1/3}}\,dx$$

$$\left[\frac{x^{-1/3+1}}{-1/3+1}\right]_{-1}^{1} = \frac{3x^{2/3}}{2}$$

Upper Limit - Lower Limit
$\frac{3(1)^{2/3}}{2} - \frac{3(-1)^{2/3}}{2} = 0$. *thus*, the correct option is A

25.A *Working:* Given $\int(e^x + e^x)dx$, on integration, we have;
$e^x + e^x + k = 2e^x + k$. thus , the correct option is A

NB; k = constant.

26.C ***Working:*** Given $\int_0^1 xe^x dx$, let u = x , v = e^x :du/dx = 1, => du = dx. Applying the method of integration by part, we have; $\int udv = uv - \int vdu = xe^x - \int e^x dx = (xe^x - e^x)|_0^1 =$ $(1e^1 - e^1) - (0e^0 - e^0)$ => C

27.D ***Working:*** $\int (x+5)dx/(x-1)$ i.e

$$\begin{array}{r|l} & 1 \\ \hline x-1 & x+5 \\ & -(x-1) \\ \hline & 6 \end{array}$$

i.e. $\int (x+5)dx/(x-1) = \int (1 + 6/(x-1))dx$

$= x + 6\ln(x-1) + k$ => D

28. A ***Working:***

Given $\int_{\pi/4}^{\pi/2} x\sin 2x dx$; u = x , => du = dx, v = sin2x

Using $\int udv = uv - \int vdu = x - (-1/2\cos 2x) - \int - \cos 2x dx$
$= -1/2\cos 2x + ½\int \cos 2x dx$. Next, we integrate $\int \cos 2x dx$:
on integration, we have; ½sin2x, *thus*;

$$\int udv = \left[-1/2\cos 2x + 1/2(1/2\sin 2x)\right]_{\pi/4}^{\pi/2}$$

$$= \left[(-\ \pi/2\cos(2\pi)/2 + 1/4\sin(2\pi)/2\right] - \left[(-\ \pi/4\cos 2\pi/4 + 1/4\sin 2\pi/4\right]$$

$= (\pi - 1)$ => A

29.A ***Working:***

$$\int \frac{x+5}{(x+3)(x+4)}dx \quad \text{i.e.} \quad \frac{x+5}{(x+3)(x+4)} \equiv$$

$$\int \left(\frac{A}{x+3} + \frac{B}{x+4} \right) dx$$

$x + 5 \equiv A(x + 4) + B(x+3)$. Solving for A and B. We have ; $A = 2, B = -1$

$$\Rightarrow \int \left(\frac{A}{x+3} + \frac{B}{x+4} \right) dx = \int \left(\frac{2}{x+3} - \frac{1}{x+4} \right) dx$$

$$= \text{In}(x+3)^2 - \text{In}(x+4) + k$$

$$= \text{In}\frac{(x+3)^2}{(x+4)} + k \quad \Rightarrow \quad A$$

30.A ***Working:*** $y = x^3 - 6x^2 + 9x + 2$;
$dy/dx = 3x^2 - 12x + 9$. At the stationary point, $dy/dx = 0$
$\Rightarrow 3x^2 - 12x + 9 = 0$; $(x - 1)(3x - 9) = 0$. Solving, we have, $x = 1$ or 3. To determine the nature of the stationary values, we differentiate dy/dx again; $\Rightarrow d^2y/dx^2 = 6x-12$. Next, apply the following steps.

$d^2y/dx^2 = 0$, (we have point of inflexion)

$d^2y/dx^2 < 0$, (we have maximum point)

$d^2y/dx^2 > 0$, (we have minimum point.)

Recall that; $d^2y/dx^2 = 6x - 12$; at $x = 1$, $\Rightarrow d^2y/dx^2 = 6(1) - 12 = -6$ (max. pt). Again, at $x = 3$ $d^2y/dx^2 = 6(3) - 12 = 6$ (min. pt). Answer is (1, 3) max/min 3 => A.

31.C **Working:** $y = X^{sinx}$. Taking log of both sides we have; log $y = \log X^{sinx}$: $\ln y = \sin x \ln x$. So, we differentiate both sides w.r.t. x : $1/y\,dy/dx = (\sin x(1/x) + \ln x \cos x)$
.: $dy/dx = y(\sin x/x + \cos x \ln x)$ => C

32.B **Working:** $y = x^2 - 2\ln x$; $dy/dx = 2x - 2(1/x) = -2x - 2/x$ $= 2x - 2x^{-1}$; at the turning pt, $dy/dx = 0$: => $2x - 2/x = 0$, Solving for x, we have; $x = 1$. Again; $d^2y/dx^2 = 2 - (-2x^{-2})$ $= 2 + 2/x^2$; Where $x = 1$: => $d^2y/dx^2 = 2 + 2/12 = 4$: => the answer is (1, min) => B.

33.A **Working:** $\lim_{x=1} \frac{x^2 - 1}{x - 1}$

First, put x = 1 into the expression

=> $\frac{x^2 - 1}{x - 1} = \frac{1^2 - 1}{1 - 1} = \frac{0}{0}$ = indeterminate

Next, we apply L' hospital rule by differentiating both the numerator and denominator. i.e. 2x/1 : So , we let x = 1 => 2x/1 = 2(1) = 2 => A.

NNAMDI AZIKIWE UNIVERSITY, AWKA
DEPARTMENT OF MATHEMATICS
2ND SEMESTER EXAMINATIONS 2007/2008
SHADE IN THE CORRECT ALTERNATIVE IN YOUR OMRANSWER ALL :

TIME: 2 hours

Alternative B

1. If f(x) = (2x-3)/(x-3) , determine f(-1/2)

   a. 2/7 b. -7/4 c. 4/7 d. 0

2. If $f(x) = \frac{1}{1 - x}$ $(x \neq 1)$ and $g(x) = \frac{x - 1}{x}$ $(x \neq 0)$

   . Determine $\frac{g(½) - f(½)}{½}$

a. -6 b. 3 c. 6 d. -4

3. Evaluate $\lim_{x \to 1} \frac{x - 1}{(x^3 - 1)}$ a. ¼ b. 1/12 c. 1/3 d. 0

4. Differentiate $f(x) = e^{-2x}\cos(4x - 1)$.
a. $-2e^{2x}[\cos(4x - 1) + 2\sin(4x - 1)]$
b. $2e^{2x}[2\cos(4x - 1) + 2\sin(4x - 1)]$
c. $-2e^{2x}[\cos(4x - 1) + 2\sin(4x - 1)]$
d. $2e^{-2x}[\cos(4x - 1) + 2\sin(4x - 1)]$

5. Determine the dy/dx of y = Incos3x.
a. $\frac{3\cos3x}{\sin3x}$ b. $\frac{3\sin3x}{\cos3x}$
c. $\frac{-3\sin3x}{\cos3x}$ d. $\frac{\cos3x}{3\sin3x}$

6. Find dy/dx of $y = \sin(x^5)$. a. $5x^5\cos(x^4)$ b. $5x^4\cos(x^4)$
c. $5x^4\cos(x^5)$ d. $-5x^4\sin(x^5)$

7. X(2,5) and Y(-4. -7) represent 2 points on the Cartesian plane. Find the equation of the straight line XY.
a. $y + 2x - 1 = 0$ b. $y - 2x - 9 = 0$
c. $y - 2x - 1 = 0$ d. $y - 2x + 9 = 0$

8. Find the domain of the function $y = \sqrt{x - 1}$
a. $(-\infty, 1]$ b. $[1, \infty)$ c. $[2, \infty)$ d. $(-\infty, \infty)$

9. The equation of a circle is $x^2 + y^2 - 6x + 8y - 24 = 0$. Find the equation of the diameter of the circle which passes through the point (-1,1): a. $4y - 5x + 1 = 0$
b. $4y + 5x + 1 = 0$ c. $4y - 3x - 7 = 0$ d. $4y - 3x - 1 = 0$

10. The gradient of the curve $y = \sin x - \cos x$ at the point $x = \pi/4$ is: a. 1 b. [illegible] c. -1 d. 0

11. The equation of a circle which has its centre at (-3,5) and radius 6 is : a. $x^2+y^2-6x-10y-2=0$ b. $x^2+y^2+6x-10y-2=0$ c. $x^2+y^2-6x-10y+2=0$ d. $x^2+y^2-6x-10y+70=0$

12. Only one of the following is a circle.
a. $x^2-y^2+2x+3y+4=0$ b. $x^2+y^2+2x-16=0$
c. $2x^2+y^2+2x+4y+10=16$ d. $3x^2+3y^2+2xy=4$

13. Find the equation of a circle whose diameter is PQ where P is the Point(-1, -3) and Q is (3,2). a. $x^2+y^2-2x-5y+3=0$
b. $x^2+y^2-2x+y-9=0$ c. $x^2+y^2-2x-5y+9=0$
d. $x^2+y^2-2x+y+9=0$

14. $y = \dfrac{1}{2x^2-5}$ find $\dfrac{dy}{dx}$
a. $4x(2x^2+5)^{-2}$
b. $-4x(2x^2-5)^{-3}$ c. $-4x(2x^2-5)^{-2}$ d. $-4x(2x+5)^{-2}$

15. The centre and radius of the circle $6x^2+6y^2-24x+12y+11=0$ are : a.(12,-6)13
b.(6,-12)13 c. (-12, 6)13 d. (-12, -6)13

16. $\displaystyle\int \frac{x+5}{(x+3)(x+4)}dx$ a. $\ln\frac{(x+4)^2}{(x+3)}+k$
b. $\ln\frac{(x+3)}{(x+4)}+k$ c. $\ln\frac{(x+4)^2}{(x+3)}+k$ d. $\ln\frac{(x+3)^2}{(x+4)^2}+k$

17. $\int_0^1 xe^{-x}dx$.
a. 2e+1 b. 1 c. -1 d. $-2e^{-1}+1$

18. $\int(2x+5)^4dx$. a. $\dfrac{(2x+5)^5}{10}+k$
b. $\left(\dfrac{2x+5^5}{10}\right)+k$ c. $\dfrac{(2x+5)^5}{10}$ d. $\dfrac{(2x-5)^5+k}{5}$

19. The equation of a straight line which passes through the point (3,-4) and is parallel to the line $y = 2x + 5$ is.
a. $y = 2x+14$ b. $y = 2x-14$ c. $y = 2x+11$ d. $y = 2x +1$

20. Determine the slope of the tangent line to the curve $y = 2/(2x-1)$ at $x = -1$ a. -4 b. 9/4 c. -4/9 d. 4/9

21. Given $f(x) = x^3-6x^2+9x+2$, find the values of x for which f(x) is minimum and maximum.
(-1, -3) b. (3,1) c. (-3, -1) d. (1, 3)

22. Find the equation of the normal to the curve $y = 3 \tan x$ at $x = \pi/4$. a. $6y - x = 18-\pi/4$ b. $6y+x = 18+\pi/4$
c. $6y - x = 18 + \pi/4$ d. $6y - x = \pi/4 -18$

23. The position of an object moving on the x-axis at time, t is given by $x(t) = 1/3t^3 - 5/2t^2+ 6t + 1$, find the location of the object when velocity is $2ms^{-1}$. a. (6 1/3, 4 1/6)
b. (6, 4 5/6) c. (6 1/3, 4 5/6) d. (6 1/3, 4)

24. The location of the object when acceleration is $1ms^{-2}$
a. 6 1/3 b. 5½ c. 6½ d. 5

25. If $y = \ln(\sin^2 x)$, find dy/dx
a. $\frac{3\text{Cos}2x}{\text{Sin}x}$ b. $\frac{\text{Cos}x}{\text{Sin }x}$ c. $\frac{2\text{Cos}x}{\text{Sin}x}$ d. $\frac{\text{Cos}x+1}{\sin^3 x}$

Suggested Solutions To The Mat 102 (2008) Session For Unizik Students – Mr. Ohms

1.*Nota:* ***Working:*** Given that $f(x) = \frac{2x-3}{x-3}$

thus, $f(-\frac{1}{2}) = \dfrac{2(-\frac{1}{2}) - 3}{-\frac{1}{2} - 3/1} = 8/7 \quad 1\ 1/7 \Rightarrow$ *(none)*

2.A **Working:** Given that $f(x) = \dfrac{1}{1-x}$ and $g(x) = \dfrac{x-1}{x}$

$\therefore g(\frac{1}{2}) = \dfrac{\frac{1}{2} - 1}{\frac{1}{2}} = -1$

and $f(\frac{1}{2}) = \dfrac{1}{1-\frac{1}{2}} = 2$

i.e $\dfrac{g(\frac{1}{2}) - f(\frac{1}{2})}{\frac{1}{2}} = \dfrac{-1-2}{\frac{1}{2}} = -6$

Hence, the correct option is A.

3.C **Working:** $\lim_{x \to 1} \dfrac{x-1}{x^3 - 1}$

First, we put x = 1, i.e. $\dfrac{1-1}{1^3-1} = \dfrac{0}{0}$ = Indeterminate.

Next, we apply L' hospital rule by differentiating both the numerator and denominator respectively. i.e.

$1/3x^2$, then put x = 1

$\Rightarrow \dfrac{1}{3(1)^2} = \dfrac{1}{3} \Rightarrow C$

4.A **Working:** $f(x) = e^{-2x}Cos(4x - 1)$.

Let $u = e^{-2x}$, $v = cos(4x - 1)$; $du/dx = -2e^{-2x}$,

$dv/dx = -4sin(4x-1)$. But $dy/dx = udv/dx + vdu/dx$

$dy/dx = e^{-2x}(-4Sin(4x - 1)) + Cos(4x-1)(-2e^{-2x})$

$= -4e^{-2x} Sin(4x - 1) - 2e^{-2x}Cos(4x-1)$

$= 2e^{-2x}Cos(4x - 1) - 4e^{-2x}Sin(4x-1)$

$= -2e^{-2x}(Cos(4x - 1) + 2Sin(4x - 1))$.

thus, the correct option is A.

5.C

Working: y = lncos3x. Using the short cut method, we have

$$dy/dx = \frac{-3\sin 3x}{\cos 3x}$$:., Correct option is C.

6.C ***Working: Given;*** $y = \sin x^5$, $du/dx = 5x^4\cos x^5$ => C

7.C ***Working:*** Given that x = (2, 5) and y = (-4, - 7);

Gradient, $m = \frac{y_2 - y_1}{x_2 - x_1}$

Where $x_1 = 2$, $x_2 = -4$, $y_1 = 5$, $y_2 = -7$, => $m = \frac{-7-5}{-4-2} = 2$

But ***the*** equation of the straight line xy is;

$y - y_1 = m(x - x_1)$. ***therefore***, $y - 2x - 1 = 0$ => C.

8.A ***Working:***

Given that; $y = \sqrt{1-x}$ .When y = 0 , => $\sqrt{1-x} = 0$

.:1 – x = 0, x = 1 : i.e. x = {∞-, ... -2,-1,0,1}. *HENCE*, the domain is (-∞, 1], thus, the correct option is A

9.B ***Working:*** $x^2+y^2-6x+8y - 24 = 0$, => $x^2-6x+y^2+8y = 24$.

By completing the squares,

$(x-3)^3 + (y+4)^2 = 24 + 9 + 16$. $(x-3)^2 + (y+4)^2 = 49$.

i.e. (3, -4). ***thus***, its centre is (3, -4)

line (Diameter) = (x, y) (3, 4) (-1, 1)

Using gradient from line 1 to line 2 = gradient of line 2 to line 3.

i.e. $\frac{y+4}{x-3} = \frac{-4-1}{3+1}$ $\frac{y+4}{x-3} = \frac{-5}{4}$

therefore, $4y + 5x + 1 = 0$ => B

5.C

Working: $y = \ln\cos 3x$. Using the short cut method, we have

$$dy/dx = \frac{-3\sin 3x}{\cos 3x}$$:., Correct option is C.

6.C ***Working: Given;*** $y = \sin x^5$, $du/dx = 5x^4\cos x^5$ => C

7.C ***Working:*** Given that x = (2, 5) and y = (-4, - 7);

Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Where $x_1 = 2, x_2 = -4, y_1 = 5, y_2 = -7$, => $m = \dfrac{-7-5}{-4-2} = 2$

But *the* equation of the straight line xy is;

$y - y_1 = m(x - x_1)$. ***therefore***, $y - 2x - 1 = 0$ => C.

8.A ***Working:***

Given that; $y = \sqrt{1-x}$ .When y = 0 , => $\sqrt{1-x} = 0$

.:$1 - x = 0$, $x = 1$: i.e. x = {∞-, ... -2,-1,0,1}. *HENCE*,

the domain is (-∞, 1], thus, the correct option is A

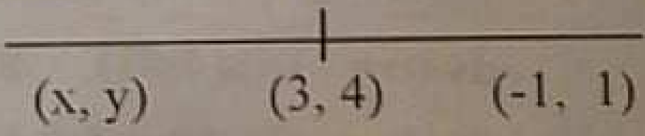
9.B ***Working:*** $x^2+y^2-6x+8y - 24 = 0$, => $x^2-6x+y^2+8y = 24$.

By completing the squares,

$(x-3)^3 + (y+4)^2 = 24 + 9 + 16$. $(x-3)^2 + (y+4)^2 = 49$.

i.e. (3, -4). ***th us***, its centre is (3, -4)

line (Diameter) =



Using gradient from line 1 to line 2 = gradient of line 2 to line 3.

i.e. $\dfrac{y+4}{x-3} = \dfrac{-4-1}{3+1}$ $\dfrac{y+4}{x-3} = \dfrac{-5}{4}$

therefore, $4y + 5x + 1 = 0$ => B

10.B **Working:** Giving that, $y = \sin x - \cos x$
Gradient = $dy/dx = \cos x + \sin x$,
where $x = \pi/4 = 180/4 = 45$. Gradient = $\cos 45^\circ + \sin 45^\circ$
$= \sqrt{2}$. ***thus***, the correct option is B.

11.B **Working:** centre = (-3, 5), r = 6 .Using $(x - a)^2 + (y - b)^2 = r^2$
Where a = -3, b = 5, r = 6, substituting, we have
$(x+3)^2 + (y-5)^2 = 6^2$. OR $x^2 + y^2 + 6x - 10y - 2 = 0$ => B

12.B $x^2 + y^2 + 2x - 16 = 0$

13.B **Working:** PQ = (-1, -3), Q = (3, 2)
Midpoint = (-1+3/2, -3+2/2) = (1, -1/2). i.e.

P(-1,-3) S(1, -1/2) Q(3,2)

Radius, = distance between PS or SQ i.e.
$r = \sqrt{(1-1)^2 + (-1/2+3)^2} = \sqrt{41/4}$
Hence, the equation is; $(x-a)^2 + (y-b)^2 = r^2$
Where a = 1 , b = ½ (i.e. Point on centre)
$r^2 = \left(\sqrt{41/4}\right)^2$. => r = 41/4. => $(x-1)^2 + (y+1/2)^2 = 41/4$
Expanding, we have; $x^2 + y^2 - 2x + y - 9 = 0$ => B

14.C **Working:** $y = 1/2x^2 - 5 = (2x^2 - 5)^1$, $y = (2x^2 - 5)^1$,
$y = (2x^2 - 5)^{-1}$. $dy/dx = -1(2x^2-5)^{-2} \times 4x = -4x(2x^2 - 5)^{-2}$

15.*Nota* **Working:** $6x^2 + 6y^2 - 24x + 12y + 11 = 0$.
Divide through by 6 $x^2 + y^2 - 4x + 2y + 11/6 = 0$
$x^2 - 4x + y^2 + 2y = -11/6$; $(x-2)^2 + (y+1)^2 = -11/6+4+1$,
$(x-2)^2 + (y+1)^2 = 19/6$
:. Centre = (2, -1) radius = 19/6 => NOTA

16.C *Working:* Given that $\int \frac{x+5}{(x+3)(x+4)}dx$

$$\frac{x+5}{(x+3)(x+4)} \equiv \int\left(\frac{A}{x+3} + \frac{B}{x+4}\right)dx$$

$x + 5 \equiv A(x+4) + B(x+3)$. Solving for A and B we have;
$A = 2, B = -1$, =>

$$\int\left(\frac{A}{x+3} + \frac{B}{x+4}\right)dx = \int\left(\frac{2}{x+3} - \frac{1}{x+4}\right)dx$$

On integration we have, $\ln(x+3)^2 - \ln(x+4) + k$

$$= \ln\frac{(x+3)^2}{(x+4)} + k \Rightarrow C$$

17.*Nota*

Working: $\int_0^1 xe^{-x}dx$. Let $u = x$, $v = e^{-x}$. $du = dx$. Applying the method of integration by part, we have;

$\int udv = uv - \int vdu = xe^{-x} - \int e^{-x}dx = (xe^{-x} - (-e^{-x}))$

$= xe^{-x} + e^{-x} - e^{-x}(x+1)$ => (NOTA).

18.A *Working:* $\int(2x+5)^4dx$. Let $u = 2x+5$, => $du/dx = 2$.

$dx = du/2$. i.e. $\int u^4.du/2 = ½\int u^4du, = ½(u^5/5) + k$

$= u^5/10 + k = (2x+5)^5/10 + k$ => A

19.*Nota* *Working:* $y = 2x + 5$, $m = 2$. But the equation of a straight line is given by $y - y_1 = m(x - x_1)$.

Where $x_1 = 3$, $y_1 = -4$, $m = 2$

=> $y + 4 = 2(x - 3)$. :. $y = 2x - 10$ => NOTA.

20.C *Working:* $y = 2/(2x-1) = 2(2x-1)^{-1}$

Slope = dy/dx = $-1(2(2x-1))^{-1-1} \times 2 = -4/(2x-1)^2$.

At x = -1. Slope = $-4/(2(-1)-1)^2 = -4/9$ => C.

21.D ***Working:*** $f(x) = x^3 - 6x^2 + 9x + 2$

$f'(x) = 3x^2 - 12x + 9$. At the turning point, $f'(x) = dy/dx = 0$

=> $3x^2 - 12x + 9 = 0$ OR $x^2 - 4x + 3 = 0$.

x = 1 or 3 . *thus*, x = (1, 3) => D.

22.B ***Working:*** y = 3tanx ; dy/dx = $3\sec^2 x = 3(1+\tan^2 x)$

at x = π/4. dy/dx = $3(1+\tan^2 \pi/4) = 3(1+1) = 6$.

Let dy/dx = $m_1 = 6$. For normal, $m_1 m_2 = -1$ => $m_2 = -1/6$.

:., the required equation is $y - y_1 = m_2(x - x_1)$.

Where $x_1 = \pi/4$, $y_1 = 3\tan\pi/4 = 3$; $y - 3 = -1/6(x - \pi/4)$

.i.e $6y + x = 18 + \pi/4$ => B.

23.C ***Working:*** $x(t) = 1/3t^3 - 5/2t^2 + 6t + 1$ ________(1)

$dx/dt = t^2 - 5t + 6$. At velocity = dx/dt = 2

=> $t^2 - 5t + 6 = 2$; $t^2 - 5t + 4 = 0$, :. t = 1 or 4

Substitute t = 1 or 4 into (1), for t =1, we have;

$1/3(1)^3 - 5/2(1)^2 + 6(1) + 1 = 29/6 = 4\ 5/6$.

For t = 4, we have; $1/3(4)^3 - 5/2(4)^2 + 6(4) + 1 = 19/3$

= 6 1/3 , ***therefore, Answer*** = (6 1/3, 4 5/6)

24.B ***Working:*** remember, $dx/dt = t^2 - 5t + 6$

=> $a = d^2x/dt^2 = 2t - 5$ .At a = 1, we have 2t – 5 = 1,

=> t = 3.Substitute t = 3 into equation (1) ,

$1/3(3)^3 - 5/2(3)^2 + 6(3) + 1 = 11/2 = 5\frac{1}{2}$ => B.

25.C ***Working:*** $y = \ln \sin^2 x$ .Using the short cut method, we have $dy/dx = \frac{2\cos x \sin x}{\sin^2 x} = \frac{\cos x}{\sin x}$ => C

NNAMDI AZIKIWE UNIVERSITY, AWKA
DEPARTMENT OF MATHEMATICS
2ND semester 2008/2009 Examination

Name______________________ Dept______________________

Mat 102 Elementary Mathematic

Reg. No: ______________________ Sign ______________________

Circle the correct option on this question paper. Time: 1½hrs

1. If f(x) = (3x - 1)/(x -2), determine f(-1/3)

   a. 9/7 b. 6/4. c. 6/7 d. 4/9 e. None

2. If f(x) = 1/(1-x) (x not equal to 1) and g (x) = (x -1)/x (x not equal to zero), evaluate (f(0)-g(i))/2

   a. 3 b. -½ c.1/3 d. ½ e. none

3. Let f(x) = x +1 and g(x) = x^2, determine fog(x).

   a. x^2+x+1 b. x^2+1 c. x^2+2x+1
   d. x^2+2 e. none

4. Evaluate lim $(x^2-36)/(x - 6)$ as x tend to 6

   a.12 b.10 c.3 d. 1/6 e. none

5. P(2 ,5) and Q(-3, -5) represent 2 points on the Cartesian plane. Find the equation of the straight line PQ.

   a. y+2x -1= 0 b. y-4x-5 = 0 c. y-3x-2 = 0
   d. y-2x-1 = 0 e. none

6. The equation of a straight line which passes through the point (-3, 5) and is parallel to the line y = x + 5 is

a. x+7 b. y = x + 6 c. y = x + 8 d. y = x+9
e. None

7. The equation of a circle which has it's centre at (-3, 5) radius 7 is: a. $x^2+y^2+6x-10y+9=0$
b. $x^2+y^2+6x+10y-2=0$ c. $x^2+y^2+6x-10y-15=0$
d. $x^2+y^2=25$ e. NONE

8. Find the equation of the circle through the origin with centre at (2, -1): a. $x^2+4x+y^2+2y=0$ b. $x^2-4x+y^2+2y=0$
c. $x^2-4x-y^2+2y=0$ d. $x^2-4x-y^2-2y=0$ e. None

9. The equation of a circle whose diameter is the line joining the points (1, -4) = and (4, 1) is. a. $x^2+y^2-5x+3y=0$
b. $x^2+y^2+5x+3y=0$ c. $x^2+y^2-5x+3y+4=0$
d. $x^2+y^2-5x+3y-4=0$ e. None

10. A point P(x, y) divided the line segment whose end points are (2, 1) and (3, 5) internally in the ratiȯ 2:3. Find this point. a. (-1, 7/2) b. (5/2, 3) c.(12/5, 13/5)
d. (5/5, 6/5) e. None

11. Given that $g(x)=x^3+3$ and $f(x)=x^2+1$. Evaluate lim g(x)/f(x) as x tend to 1. a. 4/5 b. 3/5
c. ½ d. 4/3 e. None

12. Which of the following relations is not a function?
a. (a,a), (a,b) b. (b,d), (-b,d) c. (a,a), (b,b)
d. (a,b), (c,d) e. None

13. If the distance between the points (2, k) and (4,2) is

$2\sqrt{17}$, find the value of k

a. -6, 10 b. -1,7 c. 5, -3 d. 5, 3 e. None

14. If $y = x^4(x^2+1)$, find dy/dx a. $5x^4+3x^2$ b. $6x^5+4x^3$
c. $12x^3+15x^2$ d. $8x^3+10x$ e. None

15 If $y = (x-1)/(x+1)$, find dy/dx : a. $3/(x+1)^2$ b. $3/(x+1)$
c. $4/(x+1)^2$ d. $2/(x+1)$ e. None

16. Differentiate Cos(3x+5) w.r.t x: a. -3sin(3x+5)
b. -4sin(4x+5) c. sin(-4x+5) d. 4 e. none

17. If $y = \tan x^3$, find dy/dx : a. $2\sec^2x$ b. $3x^2\sec^2x^3$
c. $5x^4\sec^2x^5$ d. $\sec x^2$ e. None

18. $\int 1/x^5 dx$ a. $-1/x+c$ b. $-1/3x^3+c$ c. $-1/4x^4+c$
d. infinity e. None

19. Evaluate $\int 10x(x^2-5)^3dx$: a. $(x-5)^4+c$ b. $3/2(x^2-5)^4+c$
c. $5/4(x^2-5)^4+c$ d. infinity e. none

20. Evaluate $\int 10e^{2x}dx$
a. $5/4e^{-x}+c$ b. $5e^{2x}+c$ c. $3e^{3x}+c$ d. infinity

21. Evaluate $\int e^x\ dx$
a. 0 b. infinity c. 1 d. $e^x(x-1)+c$ e. none

22. Find the rate of change of area of a circle with respect to the radius(r).
a. πr^2 b. $2\pi r$ c. $\pi/7$ d. 22/7 e. None

23. Evaluate lim $(2t+5)/(t^2+2)$ as t tends to -2
a. 1/6 b. 7/5 c. -1/6 d. -5/7 e. None

24. Given that k(x) = x. Determine k(-1/3)
a. 2 b. -1/3 c. -1/2 d. k e. None.

Suggested Solutions to 2009 Mat 102 For Unizik Students - Mr. Ohms

1.C **Working:** $f(x) = \frac{3x-1}{x-2}$, => $f(-1/3) = \frac{3(-1/3-1)}{-1/3-2}$

:. f(-1/3) = 6/7 => C

2.E **Working;** $f(x) = \frac{1}{1-x}$, $g(x) = \frac{x-1}{x}$, :. $f(0) = \frac{1}{1-0} = 1$

$g(½) = \frac{½ - 1}{½} = \frac{-½}{½} = -1$, *thus*, f(0) - g(½) = 1 -(-1) = 2

3.B **Working:** f(x) = x + 1, $g(x) = x^3$,

:. $fog(x) = f(g(x)) = f(x^2) = (x^2+1)$ => B

4.A **Working:** $\text{Lim}\ \frac{x^2-36}{x-6}$, x tends to 6, first, we put x = 6

:. $\frac{6^2-36}{6-6} = \frac{0}{0}$ = Indeterminate

Next, we apply L' hospital rule by differentiating both the numerator and denominator respectively. i.e. 2x/1 ,
So put x = 6 . :. (2x6)/1 = 12 => A

5.D **Working:** P = (2, 5), Q = (-3, -5),

Gradient, $m = \frac{y_2-y_1}{x_2-x_1}$

But $x_1 = 2$, $x_2 = -3$, $y_1 = 5$, $y_2 = -5$, so, $m = \frac{-5-5}{-3-2} = 2$

Using $y-y_1 = m(x-x_1)$, :. y-5 = 2(x -2),

:. y - 2x -1 = 0 => D.

6.C **Working:** y = x+5, :. m = 1. But $y - y_1 = m(x - x_1)$.

Where $x_1 = -3$, $y_1 = 5$. :. $y - 5 = 1(x - (-3))$

$y - 5 = x + 3$. :. $y = x + 8$ => C.

7.C *Working:* Center = (-3,5) , r = 7.

Using $(x-a)^2 + (y-b)^2 = r^2$ ---------- (1)

Where a = -3, b = 5, r = 7, substituting into equation (1),

thus, the required equation is; $(x - a)^2 + (y - b)^2 = r^2$,

OR $x^2 + y^2 + 6x - 10y - 15 = 0$ => C

8.B *Working:* from $(x-a)^2 + (y-b)^2 = r^2$, Where a = 2, b = -1

.: $(x-2)^2 + (y+1)^2 = r^2$, since this equation passes through the origin (0,0) , we have $(0-2)^2+(0+1)^2 = r^2$

.: $r^2 = 5$. *thus*, the required equation is:

$(x-2)^2 + (y+1)^2 = 5$ OR $x^2 - 4x + y^2 + 2y = 0$ => B

9.A Working: Let P = (1,4) and Q = (4,1) i.e.

P(1,-4) S(5/2 , -3/2) Q(4,1)

Mid point, S = x_2+x_1 , y_2+y_1

Where $x_1 = 1$, $x_2 = 4$, $y_1 = -4$, $y_2 = 1$

.: Mid point, S = (4+1)/2 , (1-4)/2 = (5/2, -3/2)

Radius = Distance between PS OR SQ

Here, P = (1,4) , S = (5/2, -3/2) ; $x_1 = 1$, $y_1 = -4$, $x_2 = 5/2$, $y_2 = -3/2$. *thus*, radius, $r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

.: $r = \sqrt{(5/2-1)^2 + (-3/2+4)^2} =$

$r = \sqrt{34/4}$

thus, the required equation is

$(x-a)^2 + (y-b)^2 = r^2$

here, a = 5/2, b = -3/2 (i.e. point on centre)

:. $(x-5/2)^2 + (y+3/2)^2 = \left(\sqrt{34/4}\right)^2$

$(x-5/2)(x-5/2) + (y+3/2)(y+3/2) = 34/4$,

Expanding , we have; $x^2+y^2-5x+3y = 0$ => A.

10.C ***Working:*** Let the point = R

But m = 2 , n = 3 , $x_1 = 2$, $y_1 = 1$, $x_2 = 3$, $y_2 = 5$

Remember , R = $\frac{mx_2+nx_1}{m+n}$, $\frac{my_2+ny_1}{m+n}$

:. R = (12/5, 13/4) => C

11.E ***Working: Given;*** $g(x) = x^3+3$, $f(x) = x^2+1$

:. $\lim_{x=1} \frac{x^3+3}{x^2+1} = \frac{1^3+3}{1^2+1} = 2$ => E

12.A.

13.E ***Working:*** Distance , D = $\sqrt{(x_2 - x_1)^2+ (y_2-y_1)^2}$,

Where $x_1 = 2$, $y_1 = k$, $x_2 = 4$, $y_2 = 2$, D = 2/√17,

:. $2/\sqrt{17} = \sqrt{(4-2)^2+ (2-k)^2}$.

$k = 2 - \sqrt{64/17i}$ => E

14.B ***Working: Given;*** $y = x^4(x^2+1) = x^6+x^4$,

:. $dy/dx = 6x^5 + 4x^3$ => B

15.E ***Working:*** $y = \frac{x-1}{x+1}$, Let u = x -1 , v = x+1 ,

du/dx = 1 ; dv/dx = 1.

But $dy/dx = \frac{vdu/dx - udv/dx}{v^2}$

$$:.\ dy/dx = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$= \frac{x+1-x+1}{(x+1)^2} = \frac{2}{(x+1)^2} \quad => E$$

16.A ***Working:*** $y = \cos(3x+5)$. Solving directly, we have;

$dy/dx = -3\sin(3x+5)$ => A

17.B ***Working:*** $y = \tan x^3$, solving directly, we have ;

$dy/dx = 3x^2\sec^2x^3$ => B

18.C ***Working:*** $\int 1/x^5dx = \int x^{-5}dx$, On integration, we have;

$$\frac{x^{-5+1}}{-5+1} + C = \frac{x^{-4}}{-4} + C \quad , = \frac{1}{4x^4} + C \quad => C$$

19.C ***Working:*** $\int 10x(x^2-5)^3dx$, let $u = x^2 - 5$

$=> du/dx = 2x$, :. $dx = du/2x$

$=> \int dy = \int 10x.u^3.du/2x = 5\int u^3du = 5(u^4/4) + C$

$= \frac{5(x^2-5)}{4} + C$ => C

20.B ***Working:***

$\int 10xe^{2x}dx = 10\int e^{2x}dx$. Let $u = 2x$, => $dx = du/2$

:. $10\int e^{2x}dx = 10\int e^u du/2 = 5\int e^u du$, on integration, we have; $5e^u + C = 5e^{2x} + C$ => B.

21.D ***Working:*** $\int xe^xdx$, Let $u = x$, $dv = e^x$, $du/dx = 1$, $v = e^x$ $du = dx$, $v = e^x$. But $\int udu = uv - \int vdu =$ $(x)(e^x) - \int e^xdx = xe^x - \int e^xdx = xe^x - \int e^xdx = xe^x - e^x + c$ $= e^x(x-1) + c$ => D.

22.B ***Working:*** Area of a circle $= \pi r2$, :. Rate of change $= 2\pi r$ => B.

23.A *Working:* $\lim_{t \to -2} \frac{(2t+5)}{(t^2+2)}$

First, we put t = -2, :. $\frac{2(-2)+5}{-2^2+2} = \frac{1}{4+2} = \frac{1}{6}$ => A

24.B *Working:* k(x) = x :. k(-1/3) = -1/3 => B.

Nnamdi Azikiwe University, Awka.

Department of Mathematics MAT 102: Elementary Mathematics II : Date: 29/06/2010 :2009/2010 2nd Semester Examination. Answer all Questions. Time Allowed: 2 Hours.

NAME: ....REG. NO:...................DEPT: ....

1. Find the domain of the function 'f' such that f(x) = (x+2)/(x+1) : A. {x:x ≠ 1} B. {x:x≠ -1} C. {x:x ≠ ±1} D. {x : x ≠ ± 2 }

2. Evaluate (gof)(x) where f (x) = $2x^2$-x and g(x)=3x+2 A. $9x^2$-3x + 6 B. $6x^2$ - 3x + 2 C. $3x^2$ - 3x + 2 D. $18x^2$ + 20x + 4

3. Which of the following relations is a function? A. (a, a,), (a, b) B. (a, b), (c,b) C. (b,d), (b, -d) D. None

4. If f (x) = (x+1)/x (x ≠ 0) and h(x) = x/(1+x) (x ≠ - 1) Determine $\frac{f(2) - h(\frac{1}{2})}{2}$ A. -1 B. - 1/4 C. 7/12 D. 1/4

5. Evaluate $\lim_{x=-3} \frac{x^2 - x - 12}{x+3}$

A. -7

B. does not exists

C. 7

D. -17

6. Evaluate $\lim_{x=\infty} \frac{3x+1}{3x^2+7x+5}$

A. ∞ B. 4/3 C. 0 D. Indeterm: nate

7. Evaluate lim x/cosx as x tends to 0

A. 0 B. 1 C. Undefined D. -1

8. Find the equation of the circle whose center is the same as that of the circle $x^2 + y^2 - 6x + 2y + 4 = 0$ and which passes through the point (7,4) A. $x^2+y^2+6x+2y - 31 = 0$ B. $x^2 + y^2 - 6x - 2y - 4 = 0$ C. $x^2 + y^2 - 6x + 2y - 31 = 0$ D. $x^2 + y^2 + 2y = 0$.

9. Which of the following curves is an ellipse?

A. $\frac{x^2}{36} - \frac{y^2}{49} = 1$ B. $x^2 + 4y^2 - 100 = 0$ C. $2x^2 - 3x + 4y + 2y^2 = 0$ D. $xy = 5$

10. Find the equation of the line tangent to the curve $y = 2x^2+3x - 1$ which passes through p (2,3). A. y=5x - 7 B. y = 15x - 43 C. y = 7x -5 D. y = 11x -19.

11. P(1,5) and Q (2,8) are two points on the cartesian plane, find he equation of the straight line P Q.

A. $y = 3x-3$ B. $y = 3x+2$ C. $y = 3x - 8$ D. $y-3x + 2 = 0$.

12. Which of the following represents a circle?
A. $x^2 + y^2 + 2x + 5xy = 7$ B. $x^2 + 2x + 3y^2 + 4y - 9 = 0$
C. $3x^2 + 9y^2 + 5x + 2y = 6$ D. $x^2 + y^2 + 3x = 16$

13. The distance between the points (1,3) and (5,c) is $\sqrt{41}$. find the two possible values of c. A. {-2,8} B. {2,-8}
C. {2,8} D. {-2,-8}.

14. Find the gradient of the curve $y = \sin x + \cos x$ at the point $x = \pi/2$: a. 1 b. 2 c. -1 d. 0

(15). Which of the following is thesame as d^2y/dx^2
A. $\left(\frac{dy}{dx}\right)^2$ B. $\frac{d}{dy}\left(\frac{dy}{dx}\right)$ C. $\frac{(dy)^2}{dx^2}$ D. None

16. $y = \sin(3x^2 + 5)$. Find y'. A. $y' = -6x\sin(3x^2 + 5)$
B. $y' = 6x\sin(3x^2+ 5)$ C. $y' = 6x\sec^2(3x^2 + 5)$
D. $y' = 6x\cos(3x^2 + 5)$

17. Differentiate $y = (x^2 + 1)(1/(2x) - 3)$ w. r. t x.
A. $6x - \frac{1}{2x^2} + \frac{1}{2}$ B. $2x^3 + 2x - \frac{1}{4x^2} + 1$
C. $-6x - \frac{1}{2x^2} + \frac{1}{2}$ D. $6x - \frac{1}{2x^2} - \frac{1}{2}$

18. If $= \tan^3 x$. Find $\frac{dy}{dx}$.
A. $3\sec^2 x\tan^3 x$ B. $2\sec^2 x\tan x$ C. $3\sec^2 x\tan^2 x$ D. Impossible

19. If $y = x^4-18x^2 +1$. Find d^2y/dx^2

A. $9(4x^2 + 1)$ B. $12x^3 + 4x$ C. $4(9x^2 + 1)$ D. $12x^2 - 36$

20. Evaluate $\int 10x^2(3x^3 + 2)^2 dx$.

A. $\frac{10}{27}(3x^3 + 2)^3 + c$ B. $\frac{10}{9}(3x^3 + 2)^3 + c$

C. $\frac{(2x^3 + 2)^3}{3} + c$ D. None

21. Evaluate $\int 6dx/(6x+5)$

A. $\ln(6x + 5) + c$ B. $\ln\left[\frac{x + 3}{x + 4}\right]^2 + C$

C. $x + 6\ln(6x + 5) + C$ D. $\ln(5x + 1)^3 + C$.

22. Evaluate $\int \frac{x^3+x}{x-1} dx$.

A. $\frac{x^3}{3} + x^2 + 3x + 2\ln(x - 1) + C$

B. $\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\ln(x - 1) + C$

C. $\frac{x^3}{3} + \frac{x^2}{2} + 2x + \ln(x - 1) + C$ D. impossible to integrate

23. Evaluate $\int xe^x dx$.

A. $e^x(x - 1) + C$ B. $e^x(x + 1) + C$ C. $x(e^x - 1) + C$

D. $x(1 - e^x) + C$

24. Evaluate $\int 10x(x^2-5)^3 dx$

A. $(x^2 - 5)^4 + C$ B. $3/2\,(x^2 - 5)^4 + C$ C. $5/4\,(x^2 - 5)^4 + C$

D. $½(x^2 - 5)^4 + C$

Suggested solutions to the Mat 102 (2010) Session for Nau Students -Mr. Ohms.

(1)B

Reason:

The answer is { x : ≠ - 1} , this is so because if we substitute -1 into f(x) = x + 2/x + 1, it will make the function to become undefine. *THUS*, the correct option is (B).

(2)B

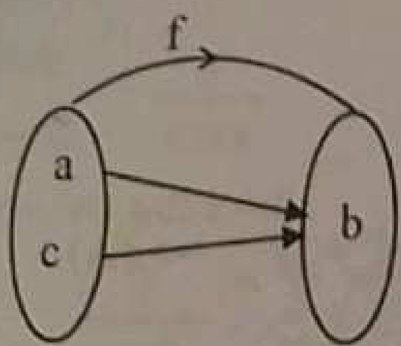
Working: $f(x) = 2x^2 - x$, $g(x) = 3x+2$

$\therefore gof(x) = g(f(x) = g(2x^2-x) = 3(2x^2-x) + 2$

$\therefore gof(x) = 6x^2-3x+2$ => (B)

(3) B

Working:



THUS , the correct optn is (B)

(4)C

Working: f(x) = (x +1)/x , h(x) = x/(1+x) ;

f(2) = (2+1)/2 = 3/2

$h(½) = \frac{½}{1+½} = 1/3$

$\therefore \frac{f(2) - h(½)}{2} = \frac{3/2 - 1/3}{2}$

$= 7/12 \quad => (C)$

(5) A

Working:

$$\lim_{x \to -3} \frac{x^2 - x - 12}{x + 3}$$

First , we put x = -3

$\therefore \frac{-3^2 - (-3) - 12}{-3+3} = \frac{0}{0}$ = Indeterminate.

Next, we apply L' hospital rule by differentiating both the numerator and denominator respectively. i.e.

$$\frac{2x - 1}{1}$$

then , put x = - 3

$\therefore \frac{2(-3) - 1}{1} = - 6 - 1 = -7 \quad => (A)$

(6) C

Working:

$$\lim_{x \to \infty} \frac{3x + 1}{3x^2 + 7x + 5}$$

First, we divide each term by the highest power of x

i.e
$$\frac{\frac{3x}{x^2} + \frac{1}{x^2}}{\frac{3x^2}{x^2} + \frac{7x}{x^2} + \frac{5}{x^2}}$$

$$= \frac{3/x + 1/x^2}{3/x + 7/x^2 + 5}$$

So, we let $x = \infty$

$$= \frac{3/\infty + 1/\infty^2}{3 + 7/\infty + 5/\infty^2} = \frac{0+0}{3+0+0}$$

$$= 0/3 = 0$$

$$\therefore \ \underset{x \to \infty}{\text{Lt.}} \ \frac{3x+1}{3x^2+7x+5} = 0$$

THUS, the correct option is (A)

(7)A

Working:

$$\underset{x \to 0}{\text{Limit}} \ \frac{x}{\cos x}$$

First, we put $x = 0$

i.e. $\frac{0}{\cos 0} = \frac{0}{1} = 0 \quad => (A)$

(8) C

Working:

$x^2 + y^2 - 6x + 2y + 4 = 0$ ________________ (1)

From $x^2 + y^2 + 2gx + 2fy + C = 0$ ____________(2)

Comparing equation (1) and (2), we have;

$dy/dx = 4(2) + 3 = 11$

i.e dy/dx at $x = 2$ $= 11$

But equation of a tangent is given as $y - y_1 = m(x - x_1)$

Where $x_1 = 2$, $y_1 = 3$, $m = 11$

substituting , we have;

$y - 3 = 11(x-2)$ $\therefore$ $y = 11x - 19$ => (D)

(11)B

Working:

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$; Where $x_1 = 1, x_2 = 2$

$y_1 = 5$, $y_2 = 8$

$\Rightarrow m = \dfrac{8 - 5}{2 - 1} = 3$

But the required equation is ; $y - y_1 = m(x - x_1)$

Here , we have P (1,5) as (x_1, y_1)

Substituting , we have; $y - 5 = 3(x-1)$

$\therefore$ $y = 3x + 2$ => (B)

(12)*(None)*

(13)A

Working:

Distance , D $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Where D = √41 , $x_1 = 1$, $x_2 = 5$ $y_1 = 3$, x_2= C dy/dx = 4[illegible] · 3 = 11

i.e dy/dx [illegible] = 11

But equation of a tangent is given as $y - y_1 = m(x - x_1)$

Where $x_1 = 2$, $y_1 = 3$, $m = 11$

substituting , we have;

$y - 3 = 11(x - 2)$:. $y = 11x - 19$ => (D)

(11)B

Working:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad ; \quad \text{Where } x_1 = 1, x_2 = 2, \; y_1 = 5, y_2 = 8$$

$$\Rightarrow m = \frac{8 - 5}{2 - 1} = 3$$

But the required equation is ; $y - y_1 = m(x - x_1)$

Here , we have P (1,5) as (x_1, y_1)

Substituting , we have; $y - 5 = 3(x - 1)$

:. $y = 3x + 2$ => (B)

(12)*(None)*

(13)A

Working:

Distance , D $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Where $D = \sqrt{41}$, $x_1 = 1$, $x_2 = 5$ $y_1 = 3$, $x_2 = C$

$\therefore \sqrt{41} = \sqrt{(5-1)^2 + (C-3)^2}$

$(\sqrt{41})^2 = 4^2 + (C-3)^2$, $\therefore C = 8$

thus , the two values of C is (12,8) => (A).

(14) C

Working:

y = sinx + cosx

Gradient = dy/dx = cosx - sinx

But $x = \pi/_2 = 180/_2 = 90$

$\therefore$ Gradient = cos90 - Sin90

= 0 -1 = -1 => (C)

(15) ***(None)***

(16)D

Working: $y = \sin(3x^2 + 5)$

Using short cut ,

$y^1 = 6x\cos(3x^2 + 5)$ => (D)

(17) (-)

Working:

$y = (x^2+1)(2x^{-1} - 3)$

Let $u = x^2+1$, $V = 2x^{-1} - 3$

$du/dx = 2x$, $dv/dx = -2x^{-2}$

But $\frac{dy}{dx} = \frac{Udv}{dx} + \frac{Vdu}{dx}$

$= (x^2+1)(-2x^{-2}) + (2x^{-1} - 3)(2x)$

:. $dy/dx = 2 - 2/x^2 - 6x$ => (-)

(18) B

Working:

$y = \tan^2 x$,Using short cut,

$dy/dx = 2\sec^2 x \tan x$ => (B)

(19)D

Working:

$y = x^4 - 18x^2 + 1$

$dy/dx = 4x^3 - 36x$

$\frac{d^2y}{dx^2} = 12x^2 - 36$ => (D)

(20) A

Working:

$\int 10x^2(3x^3 + 2)^2 dx$ ——— (1)

Let $u = 3x^3 + 2$, => $du/dx = 9x^2$

i.e. $dx = \frac{du}{9x^2}$

Substituting the values of u and dx into eqn' (1) , we have;

$$\int 10x^2 . u^2 . \frac{du}{9x^2} = \frac{10}{9}\int u^2 du$$

Integrating we have;

$$\frac{10}{9}\left(\frac{u^3}{3}\right) + C = \frac{10}{9}\ \frac{(3x^3+2)^3}{3} + C$$

$$= \frac{10}{27}(3x^3+2)^3 + C \quad => (A)$$

(21) A

Working:

$$\int \frac{6}{6x+5} dx$$

Let u $= 6x + 5$, => $du/dx = 6$

i.e. $dx = du/6$

Substituting the values of U and dx , we have

$$\int \frac{6}{u} . \frac{du}{6} = \int \frac{1}{u} du$$, On integration, we have; Inu -

:. the answer is In $(6x + 5) + C$ => (A)

(22) B

Working: $\int \frac{x^3+x}{x-1} dx$

First , we divide $x^3 + x$ by x-1.

But remember , $x^3 + x \equiv x^3 + 0x^2 + x$

i.e

$$\begin{array}{r|l} & x^2 + x \\ \hline x-1 & x^3 + 0x^2 + x \\ & -(x^3 - x^2) \\ \hline & x^2 + x \\ & -(x^2 - x) \\ \hline & 2x \end{array}$$

$$\text{i.e} \int \frac{x^3+x}{x-1}\,dx = \int\left(x^2 + x + \frac{2x}{x-1}\right)dx$$

$$= \int x^2dx + \int xdx + \int 2x/(x-1)\,dx$$

Where:

$\int x^2dx = \frac{x^3}{3} + C$ ________ (1)

$\int xdx = \frac{x^2}{2} + C$ ________ (2)

$\int 2x/(-1)\,dx = 2x + 2\text{In}(x-1) + C$ ________ (3)

See working of $\int 2x/(x-1)$ below:

$$\begin{array}{r|l} & 2 \\ \hline x-1 & 2x + 0 \\ & -(2x + 2) \\ \hline & 2 \end{array}$$

i.e . $\int \frac{2x}{x-1}\, dx = \int \left(2 + \frac{2}{x-1}\right) dx$

thus , integrating $\int \left(2 + \frac{2}{x-1}\right) dx$

Gives $2x + 2\text{In}(x-1) + C$.

Combining equation (1) , (2) and (3) gives the final answer as;

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\text{In}(x-1) + C$$

thus , the correct option is (B)

(23) A

Working:

$\int xe^x\, dx$

Let $u = x$, $dv = e^x$

$du/dx = 1$, $v = e^x$

But $\int udv = uv - \int vdu$

Substituting , we have;

$\int udv = (x)(e^x) - \int e^x dx$

$= xe^x - \int e^x dx$

$= xe^x - e^x + C$

$= e^x(x - 1) + C$ => (A)

(24) C

Working:

$\int 10x\,(x^2 - 5)^3 dx$

Let $u = x^2 - 5$

$du/dx = 2x$, $\therefore dx = du/2x$

$$\Rightarrow \int dy = \int 10x.\, u^3.\, \frac{du}{2x}$$

$= 5 \int u^3 du$; On integration , we have:

$$\frac{5u^4}{4} + C$$

$$= \frac{5\,(x^2-5)^4}{4} + C \quad \Rightarrow \quad C$$

Nnamdi Azikiwe University, Awka. Department Of Mathematics Mat 102-Elementary. Mathematics 11 Second Semester Examination 2010/2011. Answer All: Time 2Hours Shade The Correct Option Cancellation Not Allowed.

Name:......................Regno:......................Dept:.........................

1. State which of the following sets of ordered pairs is a function. A. $\{(x, y)/y = x + 4\}$ B. $(x, y)\,/x = 6$ C. $\{\,(x, y)/x+y > 4\}$ D. $\{(x,y)/y > x + 1\}$ E. $\{(x, y)/x + 2 = 0\}$

2. If $f(x) = 1/(1-x)$; $(x \neq 1)$ and $g(x) = (x-1)/x$ $(x \neq 0)$ for all real values of x, fog(x) is A. 1/x B. -x C. x D. x/1-x E. 1/1-x

3. $\lim_{x \to -2} \frac{x+2}{\sqrt{x^2-4}}$ is:

A. 0 B. 1/4 C. Does Not Exist D. 2 E. -2

4. $\lim_{x \to 2} \frac{x-2}{x^2-4}$ is

A. 0 B. 1/4 C. -2 D. 2 E. None.

5. Find the slope of the line through the points (4,3) and (2,-5) A. - 1/4 B. -1 C. 1/4 D. 4 E. -4

6. Find the equation of the line perpendicular to y = 1 at the point (0, -1) A. x = 0 B. x = 1 C. y = - 1 D. x + 1= 0 E. NONE.

7. Find an equation of the tangent line to y = 2/x at x = 2.

A. $y = \frac{1}{2}(x-2) + 1$ B. $y = -\frac{1}{2}(x-2) - 1$ C. $y = -\frac{1}{2}(x+2) + 1$ D. $y = -\frac{1}{2}(x-2)+1$ E. NONE.

8. MTN charges N25 for the 1st 10 seconds of call and 20k for each additional seconds of usage. Thus MTN cost function for t > 0 is A. c(t) = 25t B. c(t) = 0.2 (t-10)+25 C. c(t) = 0.2(t-10) D. c(t) = 25(t-10) E. c(t) = 0.2 (t+10) + 25

9. The radius of the circle $x^2 + y^2 - 6x + 14y + 49 = 0$ is

A. 7 B. 3 C. 9 D. 49 E. 10.

10. The equation of the circle whose diameter AB is at points

NB; k = constant.

26.C ***Working:*** Given $\int_0^1 xe^x dx$, let u = x , v = e^x :du/dx = 1.
=> du = dx. Applying the method of integration by part, we have; $\int udv = uv - \int vdu = xe^x - \int e^x dx = (xe^x - e^x)|_0^1 =$
$(1e^1 - e^1) - (0e^0 - e^0)$ => C

27.D ***Working:*** $\int (x+5)dx/(x-1)$ i.e

$$\begin{array}{r|l} & \quad 1 \\ x-1 & \begin{bmatrix} x+5 \\ -(x-1) \end{bmatrix} \\ \hline & \quad 6 \end{array}$$

i.e. $\int (x+5)dx/(x-1) = \int (1 + 6/(x-1))dx$
$= x + 6\ln(x-1) + k$ => D

28. A ***Working:***
Given $\int_{\pi/4}^{\pi/2} x\sin 2x dx$; u = x , => du = dx, v = sin2x

Using $\int udv = uv - \int vdu = x - (-1/2 \cos 2x) - \int -½ \cos 2x dx$
= -1/2 cos2x + ½$\int$cos2xdx. Next, we integrate $\int$cos2xdx :
on integration, we have; ½sin2x, *thus*;

$$\int udv = \Big[- 1/2\cos 2x + 1/2(1/2\sin 2x)\Big]_{\pi/4}^{\pi/2}$$

$$= \Big[(-½.\pi/2\cos(2\pi)/2 + 1/4 \sin(2\pi)/2\Big] - \Big[(-½ \pi/4\cos 2\pi/4 + 1/4\sin 2\pi/4\Big]$$

= ¼ (π 1) => A

A. $9/2\sqrt{3+x^2}+k$ B. $-9\sqrt{3+x^2}+k$ C. $3\sqrt{3+x^2}+k$
D. $9\sqrt{3+x^2}+k$ E. $1/9\sqrt{3+x^2}+k$.

17. $\int_0^1 x(x^2+3)^5\,dx$ A. 337/12 B. 3367/12 C. 2355/8
D. 3366/12 E. 1337/12

18. $$\int_{-3}^{-1}\left(\frac{1}{x^2}-\frac{1}{x^3}\right)dx.$$
A. 8/9 B. 10/9 C. 17/9 D. -5/18 E. 16/9

19. $\int_0^{\pi/4}(\sqrt{2}\cos x+\sec^2 x)dx$ A. $\sqrt{2}$ B. 0 C. -2 D. 2 E. 1

20. Only one of the following equations represents a circle
A. $x^2+2y^2+2x+3y+7=0$ B. $3x^2+3y^2+xy+2=0$
C. $2x+4y-10+x^2+y^2=0$ D. x^2+y^2+xy
E. $x^2-y^2+x+y=3$.

21. Name the conics whose equation is $x^2-4y^2+2x-16y=20$ and its center. A. Circle centre (0,0) B. Hyperbola, centre (-1,2) C. Parabola, centre (-1,-2) D. Ellipse, centre (2,-1) E. Hyperbola, centre (-1, -2)

22. Given $f(x)=x^3-6x^2+9x+2$, the values of x for which f(x) has turning points and nature of these turning points are A. 1max, 3max B. 1max, 3min C. 1min, 3max D. 1min, 3min E. -1max, 3min.

23. Assume that infected area of an injury is circular. If the radius of the infected area is 3mm and growing at rate of 1mm/hr, at what rate is the infected area increasing?
A. 6πmm/hr B. 3πmm/hr C. πmm/hr D. None
E. 2πmm/hr.

The position of an object moving on the X-axis at time t is given by $x = t^3/3 - 5t^2/2 + 6t + 1$.

24. The location of the object when the velocity is 2 ms^{-1} is
A.(25/3 or 59/8) B. (4 or 1) C. (11/3) D. (29/6 or 19/3)
E. (2 or 3/2)

25. The location of the object when acceleration is 1ms^{-1}
A.3 B. -3 C. 25/3 D. 29/6 E. 11/2

ANSWERS to 2011 MAT 102 FOR UNIZIK YR 1 STUDENTS
By MR. OHMS.

1.A ***Working:***

The correct option is A. this is so because:

(i) The function (y = x+4) contains x and y in the equation.

(ii) The function contains only the "equality sign" (=) and does not contains '<' or '>' sign.

2.C ***Working:***

$$f(x) = \frac{1}{1-x} \quad \text{and} \quad g(x) = \frac{x-1}{x}$$

$$\Rightarrow fog(x) = f(g(x)) = f\left(\frac{x-1}{x}\right) = \frac{1}{\frac{1}{1} - \left(\frac{x-1}{x}\right)}$$

$$= \frac{1}{1/x} = x \Rightarrow C$$

3.A ***Working:*** $\lim_{x \to -2} \frac{x+2}{\sqrt{x^2-4}}$

First, we substitute for x = 2, since the limiting value (x = - 2) is a number and not infinity (∞).

i.e $\frac{-2+2}{\sqrt{-2^2-4}} = \frac{0}{0}$ = Indeterminate

So, we apply L' hospital rule by differentiating both the numerator and the denominator respectively. i.e.

$\frac{x+2}{\sqrt{x^2-4}} = \frac{1}{x/(x^2-4)^{½}} = \frac{(x^2-4)^{½}}{x}$

So , we let x = -2 ; $= \frac{((-2)^2-4)^{½}}{-2} = 0$ => A

4.B *Working:*

$\lim_{x=2} \frac{x-2}{x^2-4}$. Put x = 2 i.e $\frac{2-2}{2^2-4} = \frac{0}{0}$ = Indeterminate.

So, we differentiate i.e $\frac{1}{2x}$

Now, put x = 2; $= \frac{1}{2(2)} = \frac{1}{4}$ => B

5.D *Working:*

Given (4,3) and (2,-5); here, $x_1 = 4, x_2 = 2, y_1 = 3, y_2 = -5$

By formula, slope , $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-3}{2-4} = 4$ => D

6.E *Working:*

y = 1 at the point (0,-1) => $\frac{dy}{dx} = m_1 = 0$

For perpendicularity, $m_1m_2 = -1$. => $m_2 = -1/0 = \infty$
By formular, the equation of the line perpendicular to
$y = 1$ at the point (0,-1) is $y - y_1 = m_2(x-x_1)$
Where $x_1 = 0$, $y_1 = -1$, $m_2 = \infty$. Substituting , we have;
$y - (-1) = \infty(x - 0)$; $y + 1 = \infty$ . .: $y = \infty$ => (E).

7.D ***Working:***

$y = 2/x = 2x^{-1}$ i.e $y = 2x^{-1}$ .Where $m = dy/dx = -2x^{-2}$
at the point $x = 2$, $m = dy/dx = -2(2)^{-2} = -½$
Recall that $y = 2/x$,where $x = 2$; => $y = 2/2 = 1$
By formula, the equation of a tangent is $y - y_1 = m(x-x_1)$
But $x_1 = 2$, $y_1 = 1$, $m = -½$. :. $y - 1 = -½(x-2)$
:. $y = -½(x-2) + 1$ => D

8.B ***Working:***

$c(t) = a + bt$ (Partial variation)
Where a = constant value = N25. Since first 10 seconds
is constant, => $c(t) = a + b(t - 10)$, $a = 25$.
For time > 10, cost 20k = N0.2
=> $c(t) = 25 + 0.2(t-10)$. :. $c(t) = 0.2(t-10) + 25$. => B

9.B ***Working:***

Given $x^2 + y^2 - 6x + 14y + 49 = 0$
Re-arranging , $x^2 - 6x + y^2 + 14y = -49$. By completing
the squares in x and y, we have;
$(x-3)^2 + (y+7)^2 = -49 + (-3)^2 + 7^2$
thus, $(x-3)^2 + (y+7)^2 = 3^2$ ____ (1)
Comparing equation (1) above with the general equation

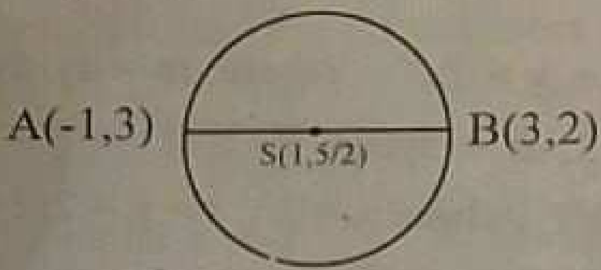
of a circle, with centre (a, b) and radius (r) .i.e.

$(x-a)^2 + (y-b)^2 = r^2$

:. Centre, (a,b) = (3, -7) and radius (r) = 3 => B

10.D ***Working:***

A = (-1,3) and B = (3,2) i.e



Midpoint, $S = \frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}$ .Where $x_1 = -1$, $x_2 = 3$
$y_1 = 3$, $y_2 = 2$

therefore, mid point, S = (3-1)/2 , (2 +3)/2 = (1, 5/2)

Radius = Distance between As or SB. But in this case, we consider 'AS'. Note that A = (-1,3) and S = (1,5/2).

thus, $x_1 = -1$, $x_2 = 1$, $y_1 = 3$, $y_2 = 5/2$

But radius, $r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

:. $r = \sqrt{(1+1)^2 + (5/2 - 3)^2} = \sqrt{17/4}$

Hence, the required equation is $(x-a)^2 + (y-b)^2 = r^2$

Here, a = 1, b = 5/2 (i.e point on centre)

therefore, $(x-1)^2 + (y-5/2)^2 = \left(\sqrt{17/4}\right)^2$

$(x-1)(x-1) + (y-5/2)(y-5/2) = 17/4$

Expanding, we have; $x^2 + y^2 - 2x - 5y + 3 = 0$ => D.

11.B ***Working:*** Let $y = 12x - x^2 - 3x^{-½}$

On differentiation, $dy/dx = 12 - 2x + 3/2x^{-3/2}$

12. ***Answer*** $= \dfrac{-6x^2 + 5 + 3x^{1/2} + 2x^{-3/2}}{x^4 + 2x^{3/2} + x}$

Working: $y = \dfrac{6x - 2x^{-1}}{x^2 + x^{1/2}}$

Let $u = 6x - 2x^{-1}$, => $du/dx = 6 + 2x^{-2}$ and

$V = x^2 + x^{1/2}$, => $dv/dx = 2x + ½x^{-1/2}$

By definition, $\dfrac{dy}{dx} = \dfrac{Vdu/dx - Udv/dx}{V^2}$

$= \dfrac{(x^2+x^{1/2})(6+x^{-2}) - ((6x-2x^{-1})(2x + ½x^{-1/2}))}{(x^2 + x^{1/2})^2}$

Expending, we have: $\dfrac{dy}{dx} = \dfrac{-6x^2+5+3x^{1/2} + 2x^{-3/2}}{x^4 + 2x^{3/2} + x}$

13. ***Answer:*** dy/dx = y(-In4)

Working: $y = 4^{-x+1}$

Taking log of both sides, $\log y = \log 4^{-x+1}$

Iny = -x+1In4 , So we differentiate both sides w. r. t. x.

i.e. $\dfrac{1}{y}\dfrac{dy}{dx} = (-x+1)(0) + \text{In}4(-1) = y(-\text{In}4)$

14.C ***Working:***

$y = \cos x^3$ $u = x^3$, =>dy/dx $= 3x^2$ i.e y = cos u

dy/dx = -sinu. But dy/dx = dy/du x du/dx

$= -\sin u \times 3x^2 = -3x^2 \sin u$

Recall that $u = x^3$

:. dy/dx $= -3x^2 \text{Sin}x^3$ => C

15.A (See Mr. Ohms lecture note for clarification)
(Get Serious).

16.D *Working:*

$$\int \frac{9x}{\sqrt{3+x^2}} dx \qquad \text{Let } u = 3+x^2$$
$$\Rightarrow du/dx = 2x \text{ , } dx = du/2x$$

$$\Rightarrow \int \frac{9x}{\sqrt{3+x^2}} dx = \int \frac{9x}{u^{1/2}} \cdot \frac{du}{2x} = \frac{9}{2}\int u^{-1/2} du$$

On integration, we have; $\frac{9}{2}\left(\frac{u^{1/2}}{½}\right) + k = 9u^{1/2} + k$

$$= 9\sqrt{3+x^2} + k \Rightarrow D.$$

17.B *Working:*

$\int_0^1 x(x^2+3)^5 dx$

Let $u = x^2+3$, $\Rightarrow du/dx = 2x$; $dx = du/2x$

i.e $\int_0^1 x(x^2+3)^5 dx = \int_0^1 x.\ u^5 . \frac{du}{2x}$

$= ½ \int_0^1 u^5 du$, On integration, we have;

$$½\left(\frac{u^6}{6}\right)_0^1 = ½\left(\frac{(x^2+3)^6}{6}\right)_0^1$$

= Upper limit - Lower limit

$$= ½\left(\frac{(1^2+3)^6}{6} - \frac{(0^2+3)^6}{6}\right) = \frac{3367}{12} \Rightarrow B$$

18.B *Working:*

$$\int_{-3}^{-1}\left(\frac{1}{x^2} - \frac{1}{x^3}\right)dx = \int_{-3}^{-1}(x^{-2} - x^{-3})dx$$

On integration, we have;

$$\left(\frac{x^{-2+1}}{-2+1} - \frac{x^{-3+1}}{-3+1}\right)_{-3}^{-1} = \left(\frac{x^{-1}}{-1} + \frac{x^{-2}}{2}\right)_{-3}^{-1} = \left(\frac{-1}{x} + \frac{1}{2x^2}\right)_{-3}^{-1}$$

= Upper limit - lower limit

$$= \left(\frac{-1}{-1} + \frac{1}{2(-1)^2}\right) - \left(\frac{-1}{-3} + \frac{1}{2(-3)^2}\right)$$

= 10/9 => B

19.D ***Working:***

$\int_0^{\pi/4} (\sqrt{2}\cos x + \sec^2 x)dx$

Integrating directly, we have:

$[\sqrt{2}\sin x + \tan x]_0^{\pi/4}$

= Upper limit - Lower limit

$= (\sqrt{2}\ \mathrm{Sin}\ \pi/4 + \tan \pi/4) - (\sqrt{2}\sin 0 + \tan 0)$

$= (\sqrt{2}\sin 45 + \tan 45) - (0 + 0)$

$$= \left(\sqrt{2} \times \frac{\sqrt{2}}{2} + 1\right) - 0$$

$= (1+1) - 0 = 2$ => D

20. C - $2x + 4y - 10 + x_1^2 + x_2^2 = 0$ 21.(-)

22.B ***Working:***

$f(x) = x^3 - 6x^2 + 9x + 2$

Let $y = f(x)$

=> $dy/dx = 3x^2 - 12x + 9$. At the turning point, $dy/dx = 0$

=> $3x^2 - 12x + 9 = 0$ i.e. $(x-1)(3x-9) = 0$, solving, we have $x = 1$ or 3. To determine the nature of the turning point, we differentiate dy/dx again. i.e. $d^2y/dx^2 = 6x-12$.

At $x = 1$, $d^2y/dx^2 = 6(1) - 12 = -6$ (max point) .

Again at $x = 3$, $d^2y/dx^2 = 6(3) - 12 = 6$ (min point)

Answer = 1max , 3min; (1,3) => B.

23. ***Working:***

Area of a circle, $A = \pi r^2$ => $dA/dr = 2\pi r$

Where $dr/dt = 1mm/hr$

Using chain rule, $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 1mm/hr$

Since $r = 3mm$, $dA/dt = 2\pi(3) \times 1mm/hr = 6\pi\ mm^2/hr$

24.D ***Working:***

$x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 1$ ------------- (1)

$dx/dt = t^2 - 5t + 6$. When $V = dx/dt = 2$,

=> $t^2 - 5t + 6 = 2$. ∴ $t = 1$ or 4

Substitute $t = 1$ or 4 into equation (1) for $t = 1$, we have;

$\frac{1^3}{5} - \frac{5}{2}(1)^2 + 6(1) + 1 = \frac{29}{6}$

For $t = 4$, we have;

$\frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 6(4) + 1 = \frac{19}{3}$

∴ Answer = (29/6 , 19/3) => D

25.E ***Working:***

Remember, $dx/dt = t^2 - 5t + 6$

$\Rightarrow a = d^2x/dt^2 = 2t - 5$, at $a = 1$

$\Rightarrow 2t - 5 = 1$, $\therefore t = 3$. Substitute $t = 3$ into equation (1),

$1/3(3)^3 - 5/2(3)^2 + 6(3) + 1 = 11/2 \Rightarrow$ E.

Nnamdi Azikiwe University, Awka. Department Of Maths
(second Semester, 2011 / 2012, Examination)

NAME .... REG NO ....

DEPT .... SEX ....SIGN ....

1. Suppose $f(x) = x^2 - 1$, and $g(x) = 2x - 1$, then find $(fog)(2)$.
A. 6 B. 5 C. 4 D. 8 E. none.

2. A child is flying a kite, if the position of this kite at any time t is given by $f(t) = 2t^3 - 3t^2 - 12t$, where t is in secs, and f is in meters, find the velocity after 3secs. A. 21m/s B. 48m/s C. -9m/s D. 24m/s E. none.

3. Suppose $P(x) = xh(x)$, and $h(1) = 4$, $h'(1) = 6$, find $P'(1)$.
A. 14 B. 10 C. 6 D. -2 E. none.

4. Find the centre of the circle whose equation is given by : $x^2 + y^2 + 2x + 6y + 6 = 0$. A. C(1, -3) B. C (-1, -3) C. C (1, 3) D. C (-1,3) E. none.

5. Find the focal diameter of the parabola that is given by : $x^2 + 6y = 0$.
A. 6 B. -6 C. (0, -3/2) D. y = 3/2 E. x = 3/2.

6. The eccentricity of an ellipse whose equation is given by: $\frac{x^2}{25} + \frac{y^2}{36} = 1$ is ------ ?

A $\frac{\sqrt{11}}{6}$ B. $\frac{2\sqrt{6}}{7}$ C. $\frac{.61}{6}$ D. $\frac{3}{5}$ E. none.

7. Evaluate $\lim_{x \to 0} \frac{\sqrt{x^2+4}-2}{x^2}$ A. 1/4 B. ∞ C. 1/8 D. ½ E. None

8. Suppose $f(x) = kx + 2$, for $x \leq 3$, and $f(x) = kx^2 - 1$, for $x > 3$, then for $f(x)$ to be continuous $\forall x$, $k = -$? A. 1/6 B. ½ C. 0 D. -1/6 E. none.

9. The horizontal asymptote for the graph of the equation given by $f(x) = \frac{2x-3}{x+3}$ is ---- ?

A. $y = 2$ B. $y = 3$ C. $x = 12$ D. $y = 12$ E. $2x = 3$.

10. Find the equation of the tangent line to the curve $xy + 3x^2y = 8x$ at the point (1,2).

A. $y + 3x - 5 = 0$ B. $3x - y - 1 = 0$ C. $2x + y - 4 = 0$ D. $3x + y + 1 = 0$ E. none.

11. If $f(x) = \sin^{-1}5x$, then find $f'(x)$

A. $\frac{5}{\sqrt{1-(5x)^2}}$ B. $\frac{2}{\sqrt{1-(2x)^2}}$ C. $\frac{3}{\sqrt{1-(3x)^2}}$ D. $\frac{-5}{\sqrt{1-(5x)^2}}$

E. none.

12. Suppose $y = \ln|\cosh 2x|$, then find y'

A. $\frac{2\sinh 2x}{\cosh 2x}$ B. $\frac{12\sinh 12x}{\cosh 12x}$ C. $\frac{3\sinh 3x}{\cosh 3x}$ D. $\frac{\sinh 2x}{\cosh 2x}$

E. none.

13. A manufacturer estimates that when x units of lamps are produced, the total profit will be $P(x) = -0.002x^2 + 50x - 1000$ naira. Calculate the marginal profit when 10lamps are produced. A. ₦49.98 B. ₦49.95 C. ₦49.99

D. ₦49. 96 E. none.

14. Evaluate $\int \sin 4x\cos 2x dx$ A. ½ $\left(\frac{\cos 2x}{2} - \frac{\cos 6x}{6}\right) + C$

B. ½ $\left(\frac{\cos 2x}{2} + \frac{\cos 6x}{6}\right) + C$ C. - ½ $\left(\frac{\cos 2x}{2} - \frac{\sin 6x}{6}\right) + C$

D. ½ $\left(\frac{\cos 2x}{2} - \frac{\sin 6x}{6}\right) + C$ E. none.

15. Use the Mean -Value -Theorem to find a value $z \in [1,4]$, where the tangent to the curve $f(x) = x^2 + 2x - 1$ will be parallel to the *secant* line through the points $(1, f(1))$ and $(4, f(4))$.

A. z = 3 B. z = 3/2 C. z = 5/2 D. z = 2 E. z = 4.

16. What will the maximum value of the function given by $f(x) = x^3 - 3x^2$ be on the interval [0,4]. A. 0 B. 16 C. -4 D. 50 E. none.

17. Evaluate $\int_0^{\pi/3} \tan^2 x \sec^2 x dx$.

A. 1/3 B. $\frac{(\sqrt{3})^3}{3}$ C. $\frac{1}{3\sqrt{3}}$ D. $\frac{1}{3(\sqrt{3})^3}$ E. none

18. On which of these interval is $f(x) = x^3 - 3x + 2$ decreasing?

A. [-1,1] B. (-∞, -1] C. [1, ∞) D. [1,1] E. none.

19. Evaluate $\int x^2 \ln x dx$ A. $\frac{1}{x} \ln x + C$ B. $\frac{x^2}{2}(\ln x - ½) + C$

C. $\frac{x^3}{3}(\ln x - ½) + C$ D. $x(\ln x - 1) + C$. E. none.

20. Find the area bounded by the curve $y = 3x^2 + 6x + 8$, the

ordinates $x = 1$, and $x = 3$. A. 14sq.units B. 66sq. units C. 78sq. units D. 84sq. units E. none.

21. Find the foci of the conic $\frac{(y-1)^2}{9} - \frac{(x-4)^2}{4} = 1$.

A. $(-4, -1\pm\sqrt{13})$ B. $(4,3\pm\sqrt{13})$ C. $(4,1\pm\sqrt{13})$ D. $(-4,\pm\sqrt{13})$ E. none.

22. Suppose $f(x) = 3x^2+1$, for $x < 2$, and $f(x) = 2x-1$, for $x \geq 2$, evaluate $\lim_{x\to 2^-} f(x)$

A. 3 B. 13 C. 5 D. ∞ E. limit does not exist.

23. The relationship between the Fahrenheit (F) and Celsius (C) scales is given by $F(C) = \frac{9}{5}C + 32$. Find $F^1(0)$

A. -50/3 B. -160/9 C. 32 D. -155/9 E. none.

24. Evaluate $\int_0^3 8x(x^2-5)^3dx$. A. 256 B. 881 C. 660.75 D. 440.5 E. none.

25. Evaluate $\lim_{x\to\infty} \frac{2x^2}{1 + 5x + 3x^2}$

A. 2/3 B. 2 C. 2/5 D. 5 E. none.

Answers to 2012 Mat 102 for Unizik Students - Mr. Ohms.

1.D *Working:*

Given $f(x) = x^2-1$ and $g(x) = 2x-1$, where $fog(x) = f(g(x))$

$= f(2x-1) = (2x-1)^2-1$

thus, $fog(2) = [2(2)-1]^2-1 = 8 \Rightarrow D.$

2.D ***Working:***

Given $f(t) = 2t^3-3t^2-12t$

By definition, velocity $f'(t)$, $\Rightarrow f'(t) = 6t^2 - 6t - 12$,

at t = 3secs i.e $f'(3) = 6(3)^2 -6(3) -12 = 24m/s \Rightarrow D.$

3.B ***Working:***

Given $P(x) = xh(x)$

thus, $P'(x) = x.h'(x) + h(x) .1$

But $h'(x) = 6$, $h(1) = 4$

Hence, $P'(1) = 1.h'(1) + h(1).1 = 1.(6) + (4).1$

$= 6 + 4 = 10 \Rightarrow$ B.

4.B ***Working:***

Given $x^2+y^2+2x+6y+6 = 0$

Re-arranging, $x^2+2x+y^2+6y = -6$

By completing the squares in x and y, we have;

$(x+1)^2+ (y+3)^2 = -6+1^2+3^2 = 4$

thus, $(x+1)^2 + (y+3)^2 = 2^2$ ________ (1)

Comparing equation(1) above with the general equation of a circle, with centre (a,b) and radius (r)

. i.e. $(x-a)^2 + (y-b)^2 = r^2$

:. centre, (a, b) = (-1, -3) and radius (r) = 2 => B.

5.B ***Working:***

Given $x^2+ 6y = 0$

$\Rightarrow x^2 = - 6y$ ________ (1)

But $x^2 = 4ay$ ______ (2)

Comparing equation(1) and (2), we have $4ay = -6y$

thus, a = -3/2

But by formula, focal diameter = 4a or 4p

Where a = -3/2

=> 4a = 4(-3/2) = -6 => B

6.A **Working:**

Given $\frac{x^2}{25} + \frac{y^2}{36} = 1$ ________ (1)

By formula, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ______ (2)

Since the foci is on the y- axis , $a^2 = 36$, => a = 6

$b^2 = 25$, => b = 5

By definition , e = c/a, Where $c = \sqrt{a^2 - b^2} = \sqrt{36 - 25}$

i.e. $c = \sqrt{11}$, ***therefore***, $e = \sqrt{11}/6$ => A

7.A **Working:**

Observe $\lim_{x \to 0} \frac{\sqrt{x^2+4} - 2}{x^2}$

Rationalizing, we have $\frac{(\sqrt{x^2+4} - 2)(\sqrt{x^2+4}-2)}{x^2(\sqrt{x^2+4}+2)}$

$$= \frac{x^2+4-4}{x^2\left(\sqrt{x^2+4}+2\right)} = \frac{x^2}{x^2\left(\sqrt{x^2+4}+2\right)} = \frac{1}{\sqrt{x^2+4}+2}$$

Now, put $x = 0$

therefore, we have $\frac{1}{\sqrt{0^2+4}+2} = \frac{1}{2+2} = \frac{1}{4}$ => A

8.B ***Working:***

Observe $f(x) = kx+2, x \leq 3$ and $f(x) = kx^2-1, x>3$,

For $f(x)$ to be continuous, $kx+2$ must be equal to kx^2-1

i.e $kx+2 = kx^2-1$

$2+1 = kx^2 - kx$

$3 = kx(x-1)$

Since $x \leq 3$, we substitute for $x = 3$.

i.e $3 = 3k(3-1)$, *thus*, $k = ½$ => B

9.A ***Working:***

Observe $f(x) = \frac{2x-3}{x+3}$

Horizontal asymptote simply is the value of y for which x tends to be ∞

i.e $\lim \frac{2x-3}{x+3}$

We divide by the highest value of x

i.e, we have $\frac{2x/x - 3/x}{x/x + 3/x} = \frac{2 - 3/x}{1 + 3/x}$

Now, we put $x = \infty$

thus, $\frac{2 - 3/\infty}{1 + 3/\infty} = \frac{2-0}{1-0} = 2$ => A

10.E ***Working:***

Observe $xy + 3x2y = 8x$

Differentiating implicitly, we have; $1y + x\frac{dy}{dx} + 6xy + 3x^2\frac{dy}{dx} = 8$

Now, we collect like terms of $\frac{dy}{dx}$. i.e $x\frac{dy}{dx} + 3x^2\frac{dy}{dx} = 8\text{-}y\text{-}6xy$

$(x+3x^2)\frac{dy}{dx} = 8+y+6xy$, *thus*, $\frac{dy}{dx} = \frac{8-y-6xy}{1+3x^2}$

At the point (1, 2) , we have: $\frac{dy}{dx} = \frac{8-2-6(1x2)}{1+3(1)^2} = \frac{-6}{4} = \frac{-3}{2}$

By formula, the equation of a tangent is given by $y\text{-}y_1 = m(x\text{-}x_1)$

Where $y_1 = 2$, $x_1 = 1$, $m = -3/2$, :. $y-2 = \frac{-3}{2}(x-1)$

Expanding, we have; $2y+3x-7 = 0$ => E

11.A ***Working:***

Observe $y = \text{Sin}^{-1}5x$

The above equation can be written as $y = \frac{5x}{\text{Sin}}$

thus, $5x = \text{Siny}$; $x = \frac{\text{Siny}}{5}$ On integration, we have $\frac{dx}{dy} = \frac{\text{Cosy}}{5}$

i.e $\frac{dy}{dx} = \frac{5}{\text{Cosy}}$. But $\text{Cos}^2y + \text{Sin}^2y = 1$

this implies that, $\text{Cos}^2y = 1 - \text{Sin}^2y$

Hence, $\text{Cosy} = \sqrt{1 - \text{Sin}^2y}$. Recall that $\text{Siny} = 5x$

therefore, $\sin^2 y = (5x)^2$, i.e $\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-(5x)^2}$

thus, $\frac{dy}{dx} = \frac{5}{\sqrt{1-(5x)^2}}$ => A

12.E ***Working:***

$y = \ln|\cosh 2x|$

Using short cut to differentiate we have: $y' = \frac{-2\sinh 2x}{\cosh 2x}$ => E

13.D ***Working:***

$P(x) = -0.002x^2 + 50x - 1000$

thus, marginal profit = $P'(x) = -0.004x + 50$

When 10 lamps are produce, we have

$P'(10) = -0.004(10) + 50 = -0.04 + 50 =$ ₦49.96 => D

14.C ***Working:***

From $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

Now, $\sin 4x.\cos 2x = ½(2\sin 4x\cos 2x)$

$= ½[\sin(4x+2x) + \sin(4x-2x)] = ½(\sin 6x + \sin 2x)$

thus, $\int \sin 4x\cos 2x\,dx = ½\int(\sin 6x + \sin 2x)dx$

$$= \left(\frac{-\cos 6x}{12} - \frac{\cos 2x}{4}\right) + C = -½\left(\frac{\cos 2x}{2} + \frac{\cos 6x}{6}\right) + C \Rightarrow C$$

15.D ***Working:***

Observe $f(x) = x^2+2x-1$. Now, for $f(x)$ to be parallel to the secant !ine, it will pass at x axis.

thus, $\lim_{x \to 1} f(x) = 1^2 + 2(1) - 1 = 2$ => D

16. A ***Working:***.

Observe $f(x) = x^3 - 3x^2$, *thus*, $dy/dx = 3x^2 - 6x$

$d^2y/dx^2 = 6x - 6$. At the interval [0, 4], it means that $x = 0$ and $y = 4$, we have $d^2y/dx^2 = 6(0) - 6 = -6$.

Note that for maximum value, d^2y/dx^2 = Negative *hence*, when $x = 0$, the maximum value of

$f(x) = 0^3 - 3(0)^2 = 0$ => A

17.A ***Working:***

Given $\int_0^{\pi/4} \tan^2 x \sec^2 x dx$ .Now, let $u = \tan x$, => $du/dx = \sec^2 x$;

thus, $dx = \frac{du}{\sec^2 x}$; i.e $\int \tan^2 x \sec^2 x dx = \int_0^{\pi/4} \frac{u^2 \sec^2 x \cdot du}{\sec^2 x}$

$= \int u^2 du$. On integration, we have $\frac{u^3}{3}$

Recall that $u = \tan x$. *thus*, we have $\frac{\tan^3 x}{3}$

$= \frac{(\tan x)^3}{3}$. Where $\pi = 180°$, ***thus***, $\pi/4 = 180°/4 = 45°$

Substituting, we have Upper limit - Lower limit

$= \frac{(\tan 45)^3}{3} - \frac{(\tan 0)^3}{3} = \frac{1}{3} - \frac{0}{3} = \frac{1}{3}$ => A

18.A ***Working:***

For $f(x) = x^3 - 3x + 2$ decreasing, it means that

$\frac{dy}{dx} < 0$

i.e $3x^2 - 3 < 0$

$3x^2 < 3$

$x^2 < 3/3$

$x^2 < 1$, *thus*, $x < \pm\sqrt{1}$. *therefore*, $x < 1$ or $x < -1$, *hence*,

answer = [-1, 1]

19.C ***Working:***

Observe, $\int x^2 \ln x dx$,

By integration by part, u = $\ln x$ and dv = x^2

du/dx = 1 , $\int dv = \int x^2$

du = dx/x , v = $x^3/3$

By formula, $\int udv = UV - \int vdu = \ln x . \frac{x^3}{3} - \int \frac{x^3}{3} . \frac{dx}{x}$

$$= \frac{x^3 \ln x}{3} - \frac{1}{3}\int x^2 dx = \frac{x^3 \ln x}{3} - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$$

$$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C = \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C \Rightarrow C$$

20.B ***Working:***

$y = 3x^2 + 6x + 8$. At $x = 1, x = 3$,

We have Area, A = $\int^{x2}_{x1} ydx = \int^{3}_{1}(3x^2+6x+8)dx$

On integration,

$$\left[\frac{3x^3}{3} + \frac{6x^2}{2} + 8x\right]_1^3 = [x^3 + 3x^2 + 8x]_1^3$$

= Upper limit - Lower limit

$= 3^3+3(3)^2+8(3) - 1^3+3(1)^2+8(1) = 66$ sq. units => B

21.C ***Working:***

Observe $\frac{(y-1)^2}{9} - \frac{(x-4)^2}{4} = 1$ ______ (1)

But $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ ______ (2)

Comparing equatn (1) and (2) , we have $a^2 = 9$, => $a = 3$ and $b^2 = 4$, => $b = 2$. $c = \sqrt{a^2+b^2} = \sqrt{9+4} = \sqrt{13}$

thus, foci = $(1+\sqrt{13},4)$ and $(1-\sqrt{13},4)$

OR $(1\pm\sqrt{13},4)$ => C

22.A ***Working:***

$f(x) = 3x^2 + 1$ and $f(x) = 2x - 1$

Now, we have $\lim_{x\to 2^+} f(x)$, meaning $x \geq 2$

Since $x \geq 2$, we use $f(x) = 2x-1$. *thus*, $\lim_{x\to 2^+} 2x - 1$

Put $x = 2$, *therefore*, $f(x) = 2(2) -1 = 3$ => A

23.B ***Working:***

Observe $f(C) = \frac{9C}{5} + 32$

First, we solve for $F^1(C)$ i.e. $F^1(C)$ = inverse of $f(C)$

Let $y = f(C)$; $y = \frac{9C}{5} + 32$

Multiply each term by 5

i.e $5y = 9C + (32x5)$

Making C the subject formula, we have; $C = \frac{5y - 160}{9}$

$\therefore F^1(C) = \frac{5C - 160}{9}$, *thus*, $F^1(0) = \frac{5(0) - 160}{9}$

$= \frac{-160}{9}$ => B

24.E ***Working:***

Observe $\int_0^1 8x(x^2-5)^3dx = 8\int_0^1 x(x^2-5)^3dx$

Let $u = x^2-5$; $du = \frac{2x}{dx}$; $dx = \frac{du}{2x}$

thus, we have $8\int_0^1 x.u^3 \frac{du}{2x} = 4\int_0^1 u^3du$

On integration, $4\left(\frac{u^4}{4}\right)_0^1$

= Upper limit - Lower limit $= 4\left(\frac{(x^2-5)^4}{4}\right)_0^1$

$= 4\left(\frac{(1^2 - 5)^4}{4} - \frac{(0^2 - 5)^4}{4}\right) = -369$ => E

25. A ***Working:***

Observe $\lim\limits_{x \to \infty} \dfrac{2x^3}{1+5x+3x^3}$

Dividing through by the highest power of x, i.e.

$$\frac{2x^3/x^3}{\frac{1}{x^3} + \frac{5x}{x^3} + \frac{3x^3}{x^3}} = \frac{2}{\frac{1}{x^3} + \frac{5}{x} + 3}$$

Now, we put $x = \infty$ i.e. $\dfrac{2}{\frac{1}{\infty^3} + \frac{5}{\infty} + 3} = \dfrac{2}{3} \Rightarrow A$

Nnamdi Azikiwe University, Awka.

Department Of Mathematics, Faculty of Physical Sciences, Uizik, Awka. Second Semester Examination, 2012/2013 Session:

MAT 102 : Elementary Mathematics II: Time: 2hrs Date: 22/01/2014:

INSTRUCTIONS: (1). Write your name(s) in capital letters (No abbreviations), registration number, department and sign your signature (without mutilation or cancellation) on the spaces provided (failure to adhere to this attracts no result). (2). shade the right option in the boxes provided below with BIRO (double shading and mutilations attract zero mark).

NAME(S) .... REG NUMBER...............................

DEPARTMENT ....SIGNATURE ....

1	A	B	C	D	E	11	A	B	C	D	E	21	A	B	C	D	E
2	A	B	C	D	E	12	A	B	C	D	E	22	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E	23	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E	24	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E	25	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E	26	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E	27	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E	28	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E	29	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E	30	A	B	C	D	E

1. Evaluate $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$ A. 2 B. $\frac{1}{8}$ C. $\frac{1}{6}$ D. 6 E. $\frac{3}{10}$

2. Suppose the $\frac{dy}{dx}$ of a given function y is $\frac{x^2-1}{x^3-3x+2}$ then find y

Nnamdi Azikiwe University, Awka.

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2	A	B	C	D	E	12	A	B	C	D	E	22	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E	23	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E	24	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E	25	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E	26	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E	27	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E	28	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E	29	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E	30	A	B	C	D	E

1. Evaluate $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$ A. 2 B. $\frac{1}{8}$ C. $\frac{1}{6}$ D. 6 E. $\frac{3}{10}$

2. Suppose the $\frac{dy}{dx}$ of a given function y is $\frac{x^2-1}{x^3-3x+2}$ then find y.

A. $\frac{1}{3}\ln/x^3-3x+2/+C$ B. $\frac{1}{8}$ C. $\frac{1}{3}\ln/x^3+3x+2/+C$

D. 1/6 E. $\ln/x^3+3x+2/+C$

3. Determine the domain and range of the function {(1,2),(3,5), (-1,17), (5,-3)} A. {1,3 -1,5} and {2,3,-1,-3} B. {1,3,-1,5} and {2,5, 17, -3} C. {1,-1,5} and {2,5,17,-3} D. {1,3,-1,5} and {2,5,17,3} E. {1,3,-1,5} and {2, 17, -3}

4. Determine the domain of $y=\sqrt{x-7}$ A. $\{x\in R/x\leq 7\}$ B. $\{x\in R/x\leq -7\}$ C. $\{x\in R/x\geq 7\}$ D. $\{x\in R/x\geq -7\}$ E. R - {7}

5. Determine the implied range of xy = 1. A. $\{y\in R/x\leq 0\}$ B. $\{y\in R/x\neq 0\}$ C. $\{y\in R/y>0\}$ D. $\{x\in R/x\geq -7\}$ E. R

6. If $f(x)=3x^2+5x+2$, find f(x+1). A. $3x^2+11x-10$ B. $3x^2-x$ C. $3x^2+11x+10$ D. $3x^2+12x-10$ E. $x^2+11x+10$

7. Given $f(x)=x^2$ and b = 3, find the value of $\frac{f(x)-f(b)}{x+b}$ A. x+3 B. x-2 C. x-3 D. $\frac{x+2}{x-b}$ E. x+2

8. Let $f(x)=\sqrt{x}$ and $g(x)=\sqrt[3]{x}$, determine f.g and f ÷ g.

A. $x^{\frac{5}{6}}$ and $x^{\frac{1}{6}}$ B. $2x^{\frac{5}{6}}$ and $x^{\frac{1}{6}}$ C. $x^{\frac{5}{6}}$ and $x^{\frac{7}{6}}$ D. $x^{\frac{5}{6}}$ and $3x^{\frac{1}{6}}$

E. $x^{\frac{-5}{6}}$ and $x^{\frac{1}{6}}$

9. If f(x) = x+10 and $g(x)=\sqrt{x}$, determine (gof)(-1). A. -2 B. 2 C. 3 D. 8 E. -3

10. If $f(x) = \frac{x+4}{2x}$, find $f'(1)$ A. -4 B. 4 C. 2 D. 3 E. 0

11. Determine the center and radius of the circle $x^2+y^2+8x-10y-8=0$
A.(-4, 5) and 9 B. (5, -4) and 7 C. (-4, 5) and 8 D. (4, 5) & 7

12. Determine the equation of the circle that has the center (0, 0) and passes through the point (4, -3). A. $x^2-y^2=25$ B. $x^2+y^2=9$
C. $x^2+y^2=25$ D. $x^2+y^2+4x-3y=25$ E. $x^2+y^2-3x+4y=25$

13. Write the equation of the parabola with focus (5,0) and directrix $x = -5$. A. $y^2=-5x$ B. $x^2=20y$ C. $y^2=-20x$ D. $y^2=20x$ E. $y^2=10x$

14. Obtain the vertices and foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$

A. $(\pm3, 0)$ and $(\pm\sqrt{-5}, 0)$ B. $(\pm5, 0)$ and $(\pm\sqrt{5}, 0)$ C. $(\pm5, 0)$ and $(\pm\sqrt{21}, 0)$ D. $(\pm3, 0)$ and $(\pm\sqrt{13}, 0)$ E. NOTA.

15. Write the given equation of a hyperbola in standard form
$6x^2-9y^2=54$. A. $\frac{x^2}{9}-\frac{y^2}{3}=1$ B. $\frac{x^2}{3}-\frac{y^2}{6}=1$

C. $\frac{x^2}{9}-\frac{y^2}{6}=1$ D. $\frac{x^2}{9}+\frac{y^2}{6}=1$ E. $\frac{x^2}{9}-\frac{y^2}{6}=-1$

16. Evaluate $\lim_{x\to1} \frac{x^2-4x+3}{x^2-6x+5}$ A. 2 B. -2 C. 0 D. ½ E. -5/3

17. Say whether or not the given function is continuous, not continuos, right continuous, left continuous or otherwise at the point $x = 3$

$$f(x) = \begin{cases} \frac{x^2-4x+3}{x^2-9}, & x \neq 3 \\ \frac{1}{2} & x = 3 \end{cases}$$

A. Right continuous
B. Not continuous C. continuous
D. left continuous E. NOTA

18. Find the derivative of $f(x) = \sqrt{2x}$. A. $-\sqrt{x}$ B. $\frac{1}{2\sqrt{-x}}$ C. $-\frac{1}{2\sqrt{x}}$

D. $\frac{1}{\sqrt{x}}$ E. $\frac{1}{2\sqrt{x}}$

19. Find the derivative of $x^2\sin 2x$. A. $-x^2\cos x + 2x\sin x$ B. $2x^2\cos 2x + 2x\sin 2x$ C. $x^2\cos x + 2x\sin x$ D. $x^2\cos x + 2\sin x$ E. $x^2\cos x - 2x\sin x$

20. Find the derivative of $(1 + x^3)^4$ A. $-12x^2(1-x^3)^3$ B. $12x^2(1+x^3)^{-3}$ C. $12x^2(1 + x^{-3})^3$ D. $12x^2(1+x^3)^3$ E. $12x^2(4 + x^3)^3$

21. Find $\frac{dy}{dx}$ if $x^3y^3 + x = y$. A. $-\frac{3x^2y^3 + 1}{3x^3y^2 + 1}$ B. $\frac{3x^2y^3 - 1}{3x^3y^2 - 1}$

C. $-\frac{3x^2y^3 + 1}{3x^3y^2 - 1}$ D. $\frac{3x^2y^3 + 1}{3x^3y^2 - 1}$ E. NOTA

22. Find the area of the region bounded above by $y = x+4$, bounded below by $y = x^2$, and bounded on the sides by the lines $x=0$ and $x = 2$

A. $\frac{34}{3}$ B. $\frac{34}{5}$ C. 34 D. $\frac{22}{3}$ E. NOTA

23. Evaluate $\int_0^2 (x+1)^3 dx$. A. $\frac{34}{8}$ B. 9 C. $\frac{-34}{8}$ D. $\frac{81}{4}$ E. 100

24. Evaluate $\int_0^6 g(x)dx$ if $f(x) = \begin{cases} x^2 & , x < 2 \\ 3x + 2 & , x \geq 2 \end{cases}$ A. 128 B. $\frac{34}{3}$

C. $\frac{128}{3}$ D. $\frac{34}{9}$ E. $\frac{176}{3}$

25. Write your names in full with capital letters, starting with

Surname: .... Sign(a):......................(b) ....

1.C ***Working:***

Observe $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2}$, Rationalizing, we have

$$\frac{\left(\sqrt{x^2+9}-3\right)\left(\sqrt{x^2+9}+3\right)}{x^2\sqrt{x^2+9}+3} = \frac{x^2+9-9}{x^2\left[\sqrt{x^2+9}+3\right]}$$

$$= \frac{x^2}{x^2\left[\sqrt{x^2+9}+3\right]} = \frac{1}{\sqrt{x^2+9}+3}$$

Now put $x = 0$; *therefore*, we have $\frac{1}{\sqrt{0^2+9}+3} = \frac{1}{\sqrt{9}+3}$

$$= \frac{1}{3+3} = \frac{1}{6}$$

2.A ***Working:***

Observe $\frac{dy}{dx} = \frac{x^2-1}{x^3-3x+2}$ i.e $dy = \left(\frac{x^2-1}{x^3-3x+2}\right)dx$

i.e $\int dy = \int\left(\frac{x^2-1}{x^3-3x+2}\right)dx$ _____________ (1)

Now, let $u = x^3 - 3x + 2$; i.e $du/dx = 3x^2 - 3 = 3(x^2-1)$

thus, $dx = \frac{du}{3(x^2-1)}$, ***hence*** substituting the values of u and dx into equation (1), we have

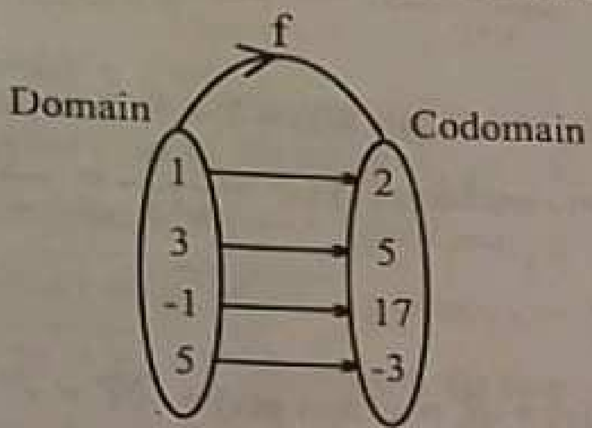
$$\int dy = \int \frac{x^2-1}{u}\cdot\frac{du}{3(x^2-1)} = \frac{1}{3}\int \frac{1}{u}du$$

On integration, $y = \frac{1}{3}\ln u + C$; .: $y = \frac{1}{3}\ln x^3-3x+2 + C$

thus, the correct option is A

3.B ***Working:***

By definition, the ***Domain*** of a function is the departure set while the ***Range*** on the other hand are those elements in the codomain that have a correspondence with those elements in the domain. ***thus,*** from the given function {(1,2) , (3,5) (-1, 17) , (5, -3)} we have



therefore, from the above the; Domain = {1, 3, -1, 5} while the Range = Codomain {2, 5, 17 , -3} . ***thus***, the correct option is B

4.B ***Working:***

Option B is correct, this is so because if we substitute -7 into $f(x) = y = \sqrt{x - 7}$, it will make the function to become undefined. ***thus***, the **correct option** is B

5.B ***Working:***

Observe $xy = 1$

First, we make x the subject ; i.e $x = \frac{1}{y}$

Next, we find the value of y that will make the function ***undefined*** and ***exclude*** it. For $x = \frac{1}{y}$ to be undefined, y must be equal to zero. i.e $x = \frac{1}{0}$ = undefined.

Hence, the ***implied range*** is R\ [0]. i.e all real numbers except 0 ({ y∈R\y ≠ 0}) . ***thus***, the **correct option** is B

6.C *Working:*

Observe $f(x) = 3x^2+5x+2$; ***thus***, $f(x+1) = 3(x+1)^2 + 5(x+1) + 2$

i.e $f(x+1) = 3(x^2+2x+1) + 5x + 7 = 3x^2+11x+10$ => C

7.C *Working:*

Observe $\frac{f(x) - f(b)}{x+b}$ ________ (1) ; Where $f(x) = x^2$, b = 3

this implies that $f(b) = f(3) = 3^2 = 9$. Now, substitute $f(x) = x^2$ and $f(b) = 9$ into equation (1).i.e $\frac{x^2-9}{x+3} = \frac{(x+3)(x-3)}{x+3} = x-3$ => C

8.A *Working:*

Observe $f(x) = \sqrt{x} = x^{½}$ and $g(x) = \sqrt[3]{x} = x^{1/3}$

Where $f.g = (x^{½})(x^{1/3})$, Applying the law of indices, we have $f.g = x^{\frac{5}{6}}$

$f \div g = \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{\frac{1}{2}-\frac{1}{3}} = x^{\frac{1}{6}}$; ***therefore***, f.g and $f \div g = x^{\frac{5}{6}}$ & $x^{\frac{1}{6}}$

thus, the **correct option** is A

9.C *Working:*

Observe $f(x) = x+10$ and $g(x) = \sqrt{x} = x^{½}$; First, we find $gof(x)$

Now, $gof(x) = g(f(x)) = g(x+10) = (x+10)^{½}$

thus, $gof(-1) = (-1+10)^{½} = (9)^{½} = 3$ => C

10.B *Working:*

Observe $f(x) = \frac{x+4}{2x}$; First, we find $f^{-1}(x)$;

Now, $f^{-1}(x)$ = Inverse of $f(x)$; Let $f(x) = y$; => $y = \frac{x+4}{2x}$

Make x the subject formula

i.e $2xy = x+4$; $2xy - x = 4$; $x(y-1) = 4$; $x = \frac{4}{2y-1}$

i.e $f^{-1}(x) = \frac{4}{2x-1}$; ***therefore***, $f^{-1}(1) = \frac{4}{2(1)-1} = 4 \Rightarrow B$

11.E ***Working:***

Given $x^2+y^2+8x-10y-8 = 0$ ______ (1)

From $x^2+y^2+2gx+2fy + c = 0$ ______ (2)

Comparing equation (1) and (2) we have $2gx = 8x$, $\Rightarrow g = 4$

$2fy = -10y$, ***this implies that*** $f = -5$; But Centre is $(-g,-f) = (-4, -5)$

By formula, Radius $r = \sqrt{g^2+f^2-c}$; Where $c = -8$

thus, $r = \sqrt{4^2 + (-5)^2 + 8} = 7$. ***therefore***, the centre $= (-4, 5)$ and radius, $r = 7$. ***thus***, the correct option is E

12.(-) ***Working:***

From $(x-a)^2 + (y-b)^2 = r^2$; Where $a = 4, b = -3$

this implies that $(x-4)^2 + (y+3)^2 = r^2$

Now, since this equation passes through the origin (0,0) we have

$(0-4)^2 + (0+3)^2 = r^2$; $\therefore r^2 = 25$; ***thus***, the required equation is

$(x-4)^2 + (y+3)^2 = 25$, expanding we have $x^2+y^2-8x+6y = 0$

13.D ***Working:***

Let P(x,y) be any point on the parabola, then Q is equidistant from the focus and the directrix. ***thus***, PF = PM i.e $(x-5)^2 + (y-0)^2 = (x+5)^2$.

thus, $y^2 = 20x \Rightarrow D$

14. ***Working:***

Observe $\frac{x^2}{25} + \frac{y^2}{4} = 1$ ______ (1)

Notice that the denominator of x^2 is larger, ***thus***, the ellipse has a horizontal major axis. ***Hence***, From

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ______(2) , We have $a^2 = 25$, => a = 5 and $b^2 = 4$ => b = 2. But $c^2 = a^2 - b^2 = 25 - 4 = 21$

thus, $c = \sqrt{21}$

By formula, vertices for an ellipse = (±a, 0) = (±5, 0)

Foci = (±c, 0) = (±√21, 0) ,***thus***, the ***answer*** = (±5, 0) and (±√21, 0)

therefore, the ***correct option*** is C

15.C ***Working:***

Observe $6x^2 - 9y^2 = 54$ ____________(1)

But the standard equation of an hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ____ (2)

Transform equation (1) to equation (2), we have

$\frac{6x^2}{54} - \frac{9y^2}{54} = \frac{54}{54}$; $\frac{x^2}{9} - \frac{y^2}{6} = 1$ => C

16.D ***Working:***

Observe $\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 - 6x + 5}$

First, we put x = 1 i.e $\frac{1^2 - 4(1) + 3}{1^2 - 6(1) + 5} = \frac{0}{0}$ = Indeterminate

So, we apply L'hospital rule by differentiating both the numerator and denominator respectively .i.e $\frac{x^2-4x+3}{x^2-6x+5} = \frac{2x-4}{2x-6}$

So, we let x = 1 .i.e $\frac{2(1) - 4}{2(1) - 6} = \frac{1}{2}$ => D

17.B Not Continuous

18.E ***Working:***

Observe $f(x) = y = \sqrt{2x} = x^{½} = ½x^{-½} = \frac{1}{2x^{½}} = \frac{1}{2\sqrt{x}}$ => E

19.B ***Working:***

Observe $x^2\sin 2x$; Let $y = x^2\sin 2x$; Let $u = x^2$, => $\frac{du}{dx} = 2x$

And $v = \sin 2x$, $\frac{dv}{dx} = 2\cos 2x$

By formula, $\frac{dy}{dy} = u\frac{dv}{dx} + v\frac{du}{dx} = x^2\cos 2x + \sin 2x.2x$

therefore, $\frac{dy}{dx} = 2x^2\cos 2x + 2x\sin 2x$ => B

20.D . ***Working:***

Given $y = (1+x^3)^4$,

Using short cut, $dy/dx = 4(1+x^3)^{4-1} \times 3x^2 = 12x^2(1+x^3)^3$ => D

21.C ***Working:***

Observe $x^3y^3 + x = y$

By implicit differentiation, $3x^2y^3 + 3x^3y^2\frac{dy}{dx} + 1 = \frac{dy}{dx}$

Collecting like terms of $\frac{dy}{dx}$; $\frac{dy}{dx} - 3x^3y^2\frac{dy}{dx} = 3x^2y^3 + 1$

i.e $\frac{dy}{dx}(1 - 3x^3y^2) = 3x^2y^3 + 1$

.: $\frac{dy}{dx} = \frac{3x^2y^3 + 1}{-3x^3y^2 + 1}$; Multiply each term by '-' sign

thus, $\frac{dy}{dx} = -\left(\frac{3x^2y^3 + 1}{3x^3y^2 - 1}\right)$ => C

22.A ***Suggested Solution:***

Given $y = x+4$ ________________ (1)

$y = x^2$ ________________ (2)

Multiply equation (1) by equation (2)

$y^2 = x^2(x+4)$; Expanding, we have $y^2 = x^3 + 4x^2$; $y = (x^3 + 4x^2)^{1/2}$

By formula, Area A = $\int_{x_1}^{x_2} y\,dx = \int_0^2 (x^{3/2} + 4x)dx = \left[\frac{x^{3/2+1}}{3/2+1} + \frac{4x^2}{2}\right]_0^2$

$$\text{Area} = \frac{2x^{5/2}}{5} + 2x^2 = \frac{2(2)^{5/2}}{5} + 2(2)^2 - \frac{2(0)^{5/2}}{5} + 2(0)^2$$

$$= \frac{2(2)^{5/2}}{5} + 8 \Rightarrow A$$

23(-) ***Working:***

Observe $\int_0^2 (x+1)^3 dx$; Let $u = x+1$; $\frac{du}{dx} = 1$, i.e. $dx = du$

thus, $\int_0^2 (x+1)^3 dx = \int_0^2 u^3 du$; On integration, $\left[\frac{u^4}{4}\right]_0^2 = \left[\frac{(x+1)^4}{4}\right]_0^2$

$= \frac{(2+1)^4}{4} - \frac{(0+1)^4}{4} = 20$sq.units => (-)

24.(-)

25. ***Answer*** = TITIGWAYE MATTHEW OHMS

MAT 102-ELEMENTARY MATHEMATICS II -FOR NAU STUDENTS

Nnamdi Azikiwe University, Awka.
Department of Mathematics, Faculty of Physical Sciences, Unizik , Awka. Second Semester Examination, 2013/2014 Session: MAT 102: Elementary Mathematics II: Time: 2hrs: Date: 26/08/2014

NAME(S) .... REG NUMBER.........................
DEPARTMENT.............................. SIGNATURE ....

INSTRUCTIONS:
(1). Write your name(s) in capital letters (No abbreviations), registration number, department and sign your signature (without mutilation or cancellation) on the spaces provided (failure to adhere to this attracts no result). (2). Shade the right option in the boxes provided below with BIRO (double shading and mutilations attract zero mark).

1. State which of the following sets of ordered pairs is a function
a. $\{(x,y)/x+2 = 0\}$ b. $\{(x,y)/y-6x+2 = 0\}$ c. $\{(x,y)/y = x^4\}$
d. $\{(x,y)/x^2+y^2 > 0$

2. If $f(x) = 1-x$ and $g(x) = \frac{x}{x+1}$ $(x \neq -1)$. Then for real values of x, $fog(x)$ is a. $\frac{1}{1+x}$ b. $x+1$ c. x d. -1

3. The curve $y = \frac{2x+1}{x+1} + 2$, has a horizontal asymptote at
a. $x = -1$ b. $y = 4$ c. $y = 2$ d. $y = 0.5$

4. Evaluate $\lim_{x\to 1} \frac{x-1}{x^3-1}$, a. 0 b. $\frac{-1}{12}$ c. $\frac{1}{3}$ d. undefined

5. The slope of the line through the points (3,4) and (5,3) is
a. -1 b. -0.5 c. 1 d. undefined

6. The curve $y = \frac{2x-1}{x-2} + 2$ has a vertical symptote at a. $x = 2$
b. $x = -1$ c. $y = 1$ d. $x = 1$

7. The equation of the tangent line to the curve $y = \frac{2}{x^2}$ at $x = -1$

is a. $y = 4x-6$ b. $y = 6x+9$ c. $y+6x = 9$ d. $y = 4x+6$

8. Evaluate $\lim_{x \to -3} \frac{x^2+1}{x^2-9}$ a. 0 b. $\frac{1}{4}$ c. does not exist d. 1

9. Name the conics and centre whose equation is $9x^2-4y^2+36x+8y+4 = 0$. a. Hyperbola, centre (-2,1) b. Ellipse, centre (-2,1) c. Circle, centre (-2,1) d. Parabola, centre (-2,1)

10. MTN charges N10 for the first 5 seconds of call and 25kobo for each additional seconds of usage. The cost function for $t \geq 5$ is a. 0.25(t-10) + 5 b. 0.25(t-5) + 10 c. 0.25(t+5) - 10 d. 0.25(t-15)

11. Assume that the infected area of an injury is circular. If the radius of the infected area is 1mm and is growing at a rate of 2mm/hr, at what rate is the infected area increasing?
a. $2\pi mm^2/hr$ b. $4\pi mm^2/hr$ c. 4mm/hr d. 0.5mm/hr

12. Evaluate $\int \frac{2x+5}{x+1}dx$. a. $2x+6In/x+1/ + k$ b. $2x-3In/x+1/ + k$ c. $2x+3In/x+1$ d. $2x+3In(x+1)$

13. Find the area enclosed by the curve $y = 3x^2+x$ and the lines $x = 1, x = 2$ and the x-axis. a. 21½ b. 22½ c. 10½ d. 21

14. Differentiate $y = \frac{x^4-1}{2x^3+3}$ a. $\frac{2(x^6+6x^3+3x^2)}{(2x^3+3)^2}$ b. $\frac{x^6+12x^3+6x^2}{(2x^3+3)^2}$
c. $\frac{12x^3+6x^2}{(2x^3+3)^2}$ d. $\frac{2(x^6+6x^3+3x^2)}{(x^4+1)^2}$

15. Only of the following is not a circle. $x^2+y^2 = 3$
b. $x^2+y^2-3x+4y = 8$ c. $4x^2-4y^2+16x+8y+4 = 0$ d. $x^2+y^2+2x = 3$

16. The radius of the circle $2x^2+2y^2-6x+4y-3 = 0$ is a. 2 b. 4 c. 3 d. 1

17. The equation of a circle with centre at (-3,5) and radius 6 is
a. $x^2+y^2-6x-10y+2 = 0$ b. $x^2+y^2-6x+10y-2 = 0$
c. $x^2+y^2+6x-10y-2 = 0$ d. $x^2+y^2-10x-6y-2 = 0$

18. Evaluate $\int_1^2 (3x-6)^6 dx$ a. 43/7 b. 20 c. -19 d. 19

19. Differentiate with respect to x, $\tan^3(x^6)$. a. $24x^7\tan^2(x^8)\sec^2(x^7)$
b. $18x^6\tan^2(x^6)\sec^2(x^7)$ c. $24x^7\tan^3(x^8)\sec^2(x^8)$
d. $18x^5\tan^2(x^6)\sec^2(x^6)$

20. Find the gradient of the curve $x^3+xy^3 = 3xy$ at the point (1,2)
a. 1 b. -1 c. 4/5 d. 2

21. Find $\int \frac{\ln^3 x}{x}\,dx$ a. $\frac{\ln^4 x}{4}+k$ b. $\frac{x^4}{4}+k$ c. $\frac{\ln x^4}{4}+k$ d. $\frac{\ln^4 x}{4}$

22. Given the function $f(x) = 2x^3-9x^2+12x-1$, find the values of x for which the stationary value is maximum and the corresponding value of y. a. (2,3) b. (2,1) c. (1,4) d. (3,2)

23. Evaluate $\int_0^{\pi} x^2 \sin x\,dx$. a. π b. -2π c. $-\pi^2$ d. 2π

24. Find the area of the region bounded by the curve $y = x^2-1$ and the straight line $y = x+1$. a. 4½ b. -6½ c. 3½ d. 2

25. Differentiate with respect to x, $(2x^3-1)(5x^3+7)$. a. $3x^2(20x^3+9)$
b. $3x^3(20x^3+9)$ c. $3x^2(20x^3+9x)$ d. $3x^2(20x^3-9)$

Answers to 2014 MAT 102 For Unizik Students - By Mr Ohms:

1.B $\{(x,y)/y-6x+2 = 0\}$

2.A ***Working:***

$f(x) = 1-x$ and $g(x) = \frac{x}{x+1}$

this implies that, $fog(x) = f(g(x)) = f\left(\frac{x}{x+1}\right) = 1-\left(\frac{x}{x+1}\right) = \frac{x+1-x}{x+1}$

$= \frac{1}{x+1} = \frac{1}{1+x}$ => A

3.B ***Working:*** $\frac{2x+1}{x+1}+2$

Horizontal asymptote simply is the value of y for which x tends to be ∞. So, we divide each term by the highest power of x

i.e $\dfrac{\frac{2x}{x} + \frac{1}{x}}{\frac{x}{x} + \frac{1}{x}} + 2 = \dfrac{2 + \frac{1}{x}}{1 + \frac{1}{x}} + 2$

Now, we put $x = \infty$, *thus,* $\dfrac{2 + \infty}{1 + \infty} + 2 = 2 + 2 = 4$

therefore, the curve has a horizontal asymptote at $y = 4$ => B

4.C ***Working:***

Given $\lim_{x\to 1} \dfrac{x-1}{x^3-1}$; First, we put $x = 1$ i.e $\dfrac{1-1}{1^3-1} = \dfrac{0}{0}$ = Indeterminate

thus, we apply L' hospital rule by differentiating both the numerator and denominator respectively. i.e $\dfrac{x-1}{x^3-1} = \dfrac{1}{3x^2}$; So, we let $x = 1$

i.e $\dfrac{1}{3(1)^2} = \dfrac{1}{3}$ => C

5.B ***Working:*** Let $(3,4) = (x_1, y_1)$ and $(5,3) = (x_2, y_2)$; Now, by definition,

slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - 4}{5 - 3} = \dfrac{-1}{2} = -0.5$ => B

6.A ***Working:*** $y = \dfrac{2x-1}{x-2} + 2$ ________ (1)

For vertical asymptote, the value of y tends to be infinity. Now from equation (1), $y-2 = \dfrac{2x-1}{x-2}$; $x-2 = \dfrac{2x-1}{y-2}$; Now, we put $y = \infty$

i.e $x-2 = \dfrac{2x-1}{\infty - 2} = \dfrac{2x-1}{\infty} = 0$ $\left[\text{since } \dfrac{2x-1}{\infty} = 0\right]$; i.e $x-2 = 0$, => $x = 2$

7.D ***Working:***

Observe $y = \dfrac{2}{x^2} = 2x^{-2}$; Where $m = \dfrac{dy}{dx} = -4x^{-3} = \dfrac{-4}{x^3}$

At the point $x = -1$, we have $m = \dfrac{dy}{dx} = \dfrac{-4}{(-1)^3} = 4$

Recall that $y = \dfrac{2}{x^2}$, Where $x = -1$, => $y = \dfrac{2}{(-1)^2} = 2$

By formula, the equation of a tangent is $y-y_1 = m(x-x_1)$

But $x_1 = -1, y_1 = 2$; $\therefore y-2 = 4[x-(-1)]$; $\therefore y = 4x + 6$ => D

8.C ***Working:*** Observe $\lim_{x\to -3} \dfrac{x^2+1}{\sqrt{x^2-9}}$; Putting $x = -3$ we have

$\frac{(-3)^2+1}{\sqrt{(-3)^2-9}} = \frac{10}{0}$ = (or Undefined or does not exist) => C

9.A *Working:* Given $9x^2-4y^2+36x+8y+4=0$

Rearrange, $9x^2+36x-4y^2+8y=-4$; By completing the squares in x & y, we have $9(x^2+4x+(2)^2) - 4(y2-2y+(-1)^2) = -4+4+36$

$9(x+2)^2 - 4(y-1)^2 = 36$;Divide through by 36; $\frac{9(x+2)^2}{36} - \frac{4(y-1)^2}{36} = 1$

$\frac{(x+2)^2}{4} - \frac{(y-1)^2}{9} = 1$ ______ (1)

For a hyperbola, (i.e shifted hyperbola), the general equation is

$\frac{(x+h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ ______ (2)

Comparing equation (1) and (2) we have Centre = (-2, 1). *thus*, the *answer* = Hyperbola, centre (-2, 1). **Option A is Correct**

10.B *Working:* By partial variation c(t) = a + bt

But a = constant value = N10; Since the first 5 seconds is constant, we have c(t) = a + b(t-5); For t≥5, cost = 25k = 0.25

therefore, c(t) = 10 + 0.25(t-5) = 0.25(t-5) + 10 => B

11. This is a repeated Question. GOTO Q25 (2011) Session for the solution

12.C *Working:* Observe $\int \frac{2x+5dx}{x+1}$; First, we divide 2x+5 by x+1

i.e long division: $x+1 \overline{)\,2x+5\,}$ gives quotient 2; $-(2x+2)$, remainder 3 ; i.e $\int \frac{2x+5dx}{x+1} = \int\left(2 + \frac{3}{x+1}\right)dx = \int 2dx + \int \frac{3\,dx}{x+1}$

; On Integration, we have 2x + 3ln/x+1/ + K => C

13. Another repeated Question. See previous *workings* for the answer

14.A *Working:* Observe $y = \frac{x^4-1}{2x^3+3}$; Let $u = x^4-1$, => $\frac{du}{dx} = 4x^3$

$v = 2x^3+3$, => $\frac{dv}{dx} = 6x^2$; By formula, $\frac{dy}{dx} = \frac{vdu/dx - udv/dx}{v^2}$

i.e $\frac{dy}{dx} = \frac{(2x^3+3)(4x^3) - (x^4-1)(6x^2)}{(2x^3+3)^2}$; Expanding, we have;

$\frac{dy}{dx} = \frac{2(x^6+6x^3+3x^2)}{(2x^3+3)^2}$ => A

15.C *Answer* is $4x^2-4y^2+16x+8y+4=0$

16.A *Working:* Observe $2x^2+2y^2-6x+4y-3=0$

Divide through by 2 ; $x^2+y^2-3x+2y-\frac{3}{2}=0$ ________ (1)

From the general equation of a circle, $x^2+y^2+2gx+2fy+c=0$ ____ (2)

Comparing equation (1) and (2), we have; $2gx=-3x$, => $g=-3/2$

$2fy=2y$, => $y=1$, $c=-3/2$

By formula, the radius (r) of a circle = $\sqrt{g^2+f^2-c}$ ________ (A)

Substituting the values of g, f and c into equation (A), we have

$r=\sqrt{(-3/2)^2+1^2-(-3/2)}=2$. **Option A is Correct.**

17. (C) Another repeated Question. See previous workings. (***Get Serious***)

18. Another repeated Question. GOTO Q23 (2013) Session for the answer

19.D *Working:* Let $y=\tan^3(x^6)$; Now, Let $u=\tan x^6$, $du/dx=6x^5\sec^2x^6$

$y=u^3$, => $dy/du=3u^2$; But $dy/dx=dy/du \times du/dx=3u^2\times 6x^5\sec^2x^6$

$=3\tan^2x^6\times 6x^5\sec^2x^6=18x^5\tan^2x^6\sec^2(x^6)$ => D

20. Another repeated Question. See previous workings for the answer.

21.A *Working:* Observe $\int \frac{In^3x}{x}dx$ __________ (1)

Let $u=Inx$; $du/dx=1/x$; $dx=xdu$

Substituting in equation (1), $\int \frac{u^3}{x}xdu=\int u^3du=\frac{u^4}{4}+K$

$=\frac{(Inx)^4}{4}+K=\frac{In^4x}{4}+K$; **Option A is Correct**

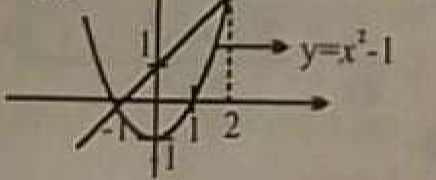
22. Another repeated Question. See Previous workings for the answer

23. Another repeated Question. Go to Q28 (2007) Session for solution

24.A *Working:* Area bounded by the curve

$$A=\int_{y_1}^{y_2} ydx \text{ ;}$$

i.e



Let $y_2=x^2-1$, $y_1=x+1$

$y_2-y_1=x^2-1-(x+1)=0$

$0=x^2-1-x-1$; $0=x^2-x-2$

$0=(x-2)(x+1)$;*thus*, $x=2$ or -1

therefore, $A = \int_{-1}^{2}(y_2 - y_1)dx = \int_{-1}^{2}(x^2 - x - 2)dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{2}$

$= \left[\frac{2^3}{3} - \frac{2^2}{2} - 2(2)\right] - \left[\frac{(-1)^3}{3} - \frac{(-1)^2}{2} - 2(-1)\right] = \frac{-9}{2}$ = -4½ sq units

Area is +ve, .: A = 4½ sq units. **Option A is Correct.**

25.D ***Working:***

Let $y = (2x^3 - 1)(5x^3 + 7)$; Expanding, $y = 10x^6 + 9x^3 - 7$

* *thus*, $dy/dx = 60x^5 - 27x^2 = 3x^2(20x^3 - 9)$ => D

$10x^6 + 9x^3 - 7 \Rightarrow \frac{dy}{dx} = 60x^5 + 27x^2$

$3x^2(20x^3 + 9)$ (A)

Nnamdi Azikiwe University, Awka.
Department of Mathematics, Faculty of Physical Sciences, Unizik , Awka. Second Semester Examination, 2014/2015 Session:
MAT 102: Elementary Mathematics II: Time: 1½ hrs:
NAME(S) .... REG NUMBER........................
DEPARTMENT.......................... SIGNATURE ....

R = Real numbers, Q = Rational numbers, P = Prime numbers, Z = Integers, N = Natural numbers.

1. What is the domain of the function $f(x) = \sqrt{x^2 - 4}$? a. $(-\infty, \infty)$ b. $[-2, \infty)$ c. R d. $(-\infty, -2] \cup [2, \infty)$ e. NOTA
2. Determine the range of values for which $f(x) = \dfrac{x^2 + 7x - 11}{x^2 - 4}$ is defined a. {2, -2} b. $\{x \in R | x \neq \pm 2\}$. c. $\{x \in R | x \neq \pm 1\}$ d. $\{x \in R | x \neq -1 \pm \sqrt{7}\}$ e. NOTA
3. Find the domain of the function f + g if $f(x) = x - 3$ & $g(x) = \sqrt{x - 1}$ a. $\{x \in R | x \geq 1\}$ b. $\{x \in R | -1 \leq x \leq 3\}$ c. $\{x \in R | x > 1\}$ d. $\{x \in R | x \geq 3\}$ e. NOTA
4. Given $f(x) = x^2 + 1$ and $g(x) = \sqrt{x - 2}$, determine the domain of the composite function fog a. $(-\infty, -1] \cup [1, \infty)$ b. R c. N d. $\{x \in R | x \geq 2\}$ e. NOTA
5. Evaluate the $\lim_{x \to -3} \dfrac{3x + 9}{x^2 - 9}$ a. ½ b. -½ c. 0 d. does not exist e. NOTA
6. Compute the $\lim_{x \to 0} \dfrac{\text{Sin}x}{x}$ a. does not exist b. -1 c. 1 d. 2 e. NOTA
7. Given the line equation $x - 3y = 10$, find the gradient and y-intercept (m, c) a. (-2/3, 10/3) b. (2/5, 21/5) c. (1/3, 10/3) d. (-2/3, 2) e. nota
8. Find the equation of the line drawn through the point of interception of the lines $3x - 5y = 1$ and $x + 7y = 9$; perpendicular to $5y + 2x = 0$. a. $5x - 2y - 8 = 0$ b. $2x + 5y - 9 = 0$ c. $5x - 2y + 8 = 0$ d. $2x - y + 9 = 0$ e. NOTA
9. Determine the coordinate of the foot of the perpendicular from the point (2, 3) to the line $x + y - 11 = 0$. a. (-5, -6) b. (5, 6) c. (6, 5)

d. (2, 3) e. NOTA

10. The equation of the straight line which passes through (-3, 5) and perpendicular to $2x-4y+3=0$ is a. $2x+y+1=0$ b. $3x+5y-16=0$ c. $x-2y-13=0$ d. $x-2y+13=0$ e. NOTA

11. What is the equation of the locus of point Q(x, y) that is equidistant from the points A(2,3) and C(4,5)? a. $x-y+7=0$ b. $x+y-7=0$ c. $x+y-5=0$ d. $x-y+5=0$ e. NOTA

12. Compute the equation of the circle whose diameter has the end point A(5,4) and B(7,4) a. $x^2+4y^2-16x-24y+38=0$ b. $x^2+y^2-10x+8y=59$ c. $x^2+y^2-8y-10x=0$ d. $4x^2+4y^2-16x-24y=0$ e. NOTA

13. Determine the centre and radius of a circle whose equation is $x^2+y^2-6x+4y-3=0$ a. (4, -2) r = 1 b. (1/3, ½) r = 1 c. (4, -2) $r=7\frac{\sqrt{3}}{3}$ d. (3, -2) r = 4 e. NOTA

14. Find the equation of the tangent to the circle $x^2+y^2-3x+4y-19=0$ at the point (2,3) a. $4x-7y-22=0$ b. $7y+4x-22=0$ c. $x+10y-32=0$ d. $16y+5x-28=0$ e. NOTA

15. Which of the following curves is a circle? a. $x^2+y^2+2x+3y=0$ b. $2x^2+3y^2-x+2y+1=0$ c. $x^2-6x-4y+13=0$ d. $25x^2-4y^2-50x-16y-91=0$ e. NOTA

16. Write the equation of the parabola $y^2+4y+4x+16=0$ in its canonical form

a. $\frac{(x-3)^2}{5}+\frac{(y-2)^2}{4}=1$ b. $\frac{(x-3)^2}{5}-\frac{(y-2)^2}{4}=1$

c. $\frac{(x+1)^2}{10}-\frac{(y+1)^2}{4}=1$ d. $(y+2)^2=-4(x+3)$ e. NOTA

17. *Determine the vertex and focus of the parabola $y^2+4y+4x+16=0$ a. (3, 2) and (3, -1) b. (-1, -1) & $F_1(2\sqrt{29}-1, -1)$, $F_2(-2\sqrt{29}-1, -1)$ c. (3, 2) and $F_1(3, 5)$, $F_2(3,-1)$ d. (-3, -2) and (-4, -2) e. NOTA

18. What is the equation of the tangent to the hyperbola $9x^2-4y^2=36$ at the point (-2, 2)? a. $9x+4y+18=0$ b. $8x-18y+52=0$ c. $3x+2y+9=0$ d. $2x-9y+13=0$ e. NOTA

19. Compute the differential coefficient of the equation $x^2y+y^2x+4x=1$ with respect to x at the point (1, 1) a. $\frac{2}{-1}$ b. $\frac{-7}{3}$ c. $\frac{-1}{2}$ d. $\frac{1}{-1}$

e. NOTA

20. Obtain the differential coefficient of the function $\cot^2 x^3$

a. $-6x^2\cot x^2\sec^2 x^2$ b. $6\cot x^2\sec^2 x^2$ c. $-8\sin^2 x^2\cos^2 x^2$

d. $\frac{-3}{2}x\cot\frac{3x^2}{4}\sec\frac{3x^2}{4}$ e. NOTA

21. Find the instantaneous rate of change of the function $\log_e\sqrt{1+x}$

a. $\frac{-2}{1-x^2}\log_e 3$ b. $\frac{-2}{1-x^2}\log_3 e(1-x)$ c. $\frac{1}{2+2x}$ d. $\frac{8}{4x-1}$ e. NOTA

22. What is the turning points on the curve $f(x) = \frac{x^4}{2} + \frac{5x^3}{3} - 2x^2 - 3x + 1$

a. 2, -2, -4 b. 2 twice c. -½, 1, 3 d. -½, -3, 1 e. NOTA

23. Evaluate the integral $\int\frac{x}{1-x}$ a. $-(x+\ln|1-x|) + C$ b. $\frac{x^2}{2} - 2x + 4\ln|x+2| + C$

c. $x + \ln|1-x| + C$ d. $\frac{x^2}{2} - x + \ln|1+x| + C$ e. NOTA

24. What is the integral coefficient of the function $\frac{\exp\sqrt{x}}{\sqrt{x}}$ a. $\frac{(\ln x)^4}{4} + C$

b. $2\exp\sqrt{x} + C$ c. $2\sqrt{\cos x} + C$ d. $2\sqrt{\sin x}$ e. NOTA

25. Compute $\int_{-\pi/3}^{\pi}\sec x\tan x\,dx$ a. $\frac{2\sqrt{2}-2}{\sqrt{2}}$ b. -3 c. 3 d. 1 e. NOTA

Answers to 2015 Unizik Mat 102 - By Mr Ohms:

1.D ***Working:*** Observe $\sqrt{x^2-4} = \sqrt{(x-2)(x-2)}$

Step 1: Set the expression inside the square root greater than or equal to zero i.e $(x-2)(x-2) \geq 0$

Step 2: Solve the equation in step 1: $x-2 \geq 0$ or $x+2 \geq 0$; $x \geq 2$ or $x \geq -2$

Step 3: Write the answer using interval form: i.e $(-\infty, -2] \cup [2, \infty)$ => D

2.B ***Working:*** $f(x) = \frac{x^2+7x-11}{x^2-4}$; at $x^2 - 4 \neq 0$, f(x) will be defined

$\therefore x^2 \neq 4$; $x \neq \pm\sqrt{4}$; $x \neq \pm 2$; ***thus***, the range of values of f(x) are

$\{x : x \in R | x \neq \pm 2\}$ => B

3.A *Working:* $f(x) = x - 3$ & $g(x) = \sqrt{x-1}$; *thus*, $f + g = x - 3 + \sqrt{x-1}$
For $\sqrt{x-1}$, for this to exist, $x - 1 \geq 0$; *therefore*, $x \geq 1$
Hence, the domain of f + g is $\{x \in R | x \geq 1\}$ => A

4.B *Working:* $f(x) = x^2 + 1$; $g(x) = \sqrt{x-2}$; $fog = f(g(x)) = \left[\sqrt{x-2}\right]^2 + 1$
thus, $fog = x - 2 + 1 = x - 1$. This function is defined at all values of x
therefore, Domain of the composite function, fog is R => B

5.B *Working:* Observe $\lim_{x \to -3} \frac{3x+9}{x^2-9}$; At $x = -3$, we have $\frac{-3(-3)+9}{(-3)^2-9}$
$= \frac{0}{0}$ = Indeterminate. Applying L'Hopitals rule, we have

$$\lim_{x \to -3} \frac{3}{2x} = \frac{3}{2(-3)} = \frac{3}{-6} = \frac{-1}{2} \Rightarrow (B)$$

6.C *Working:* $\lim_{x \to 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0}$ (Indeterminate)
Applying L'Hopital rule, we have $\lim_{x \to 0} \frac{\cos x}{1} = \frac{1}{1} = 1$ => (C)

7.E *Working:* $x - 3y = 10$, => $3y = x - 10$; $y = \frac{x-10}{3} = \frac{x}{3} - \frac{10}{3}$
Comparing with $y = mx + c$, we have $m = 1/3$, $c = -10/3$
therefore, (m, c) = (1/3, -10/3) => E

8.A *Working:* The point of intersection of the 2 lines is obtained by solving the 2 lines simultaneously.
$3x - 5y = 1$ ---------- (1)
$x + 7y = 9$ ---------- (2)

Solving equation (1) and (2) simultaneously, we have $x = 2$, $y = 1$.
therefore, the point of intersection is defined by (2, 1). For the line,
$5y + 2x = 0$, => $y = \frac{-2x}{5}$; .: Slope of the line, $m = \frac{-2}{5}$
For the perpendicular line to it, $m_1 = \frac{-1}{m}$; .: $m_1 = \frac{-1}{-2/5} = \frac{5}{2}$
Using the equation of a line with a point and a slope, $y - y_1 = m(x - x_1)$
At point $(x_1, y_1) = (2, 1)$ and $m = 5/2$; i.e $y - 1 = 5/2(x - 2)$

Solving, we have $5x - 2y - 8 = 0 \Rightarrow A$

9.B *Working:* For the line $x + y - 11 = 0$; $y = -x + 11$; The slope of the line is -1. For a line perpendicular to a point, $m = -1/m = -1/-1 = 1$
Using the equation; $y - y_1 = m(x - x_1)$; Where $(x_1, y_1) = (2, 3)$
$\therefore y - 3 = 1(x - 2)$; *thus,* $x - y = -1$.
Finding the point of intersection of the 2 lines we have
$x + y - 11 = 0$; $x + y = 11$ ---------- (1)
$x - y = -1$ ------------- (2)

Solving equation (1) and (2) simultaneously, we have $x = 5, y = 6$
therefore, the point of intersection are $(x, y) = (5, 6) \Rightarrow B$

10.A *Working:* For the line $2x - 4y + 3 = 0$; Making y subject formula, we have $4y = 2x + 3$; $y = \frac{x}{2} + \frac{3}{4}$; $\therefore$ Slope, $m_1 = ½$

For a line perpendicular to this, the slope, $m = -1/m_1 = \frac{-1}{½} = -2$

$\therefore$ for the line passing through (-3, 5), the equation is given by
$y - y_1 = m(x - x_1)$; $y - 5 = -2(x - -3)$; $y + 2x + 1 = 0 \Rightarrow A$

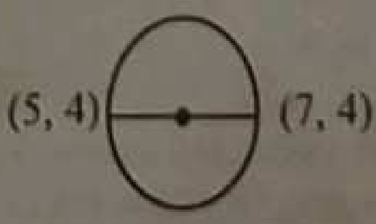
11.B *Working:* For the point equidistant between 2 points A(2, 3) and C(4, 5) ; The midpoint of line AC is $x = \frac{4 + 2}{2} = 3$; $y = \frac{5 + 3}{2} = 4$

Midpoint = (3, 4) ; Equation of line AC is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$

For $(x_1, y_1) = (2, 3)$; $(x_2, y_2) = (4, 5)$; *thus,* $\frac{5 - 3}{4 - 2} = \frac{y - 3}{x - 2}$

i.e $y = x + 1$; ***this implies that*** the slope of the line, $m_1 = 1$
$\therefore$ for the line perpendicular to this line AC passing through the mid point (3, 4) ; Slope, $m = -1/m_1 = -1/1 = -1$; $y - y_1 = m(x - x_1)$
i.e $y - 4 = -1(x - 3)$; ***therefore,*** $x + y - 7 = 0 \Rightarrow B$

12.E *Working:* Observe



Finding the midpoint of the diameter line, we have;
$x = \frac{5+7}{2} = 6$; $y = \frac{4+4}{2} = 4$
$\therefore$ midpoint $(h, k) = (6, 4)$
By equation of a circle,

$(x-h)^2 + (y-k)^2 = r^2$; $d = \sqrt{(7-5)^2 + (4-4)^2} = 2$; *thus*, $r = d/2 = 1$
.: the equation of the circle becomes $(x - 6)^2 + (y - 4)^2 = 1^2$
Expanding, we have $x^2 + y^2 - 12x - 8y + 51 = 0$ => E

13.D ***Working:*** Equation of circle ; $x^2 + y^2 - 6x + 4y - 3 = 0$
By completing the square, $(x^2 - 6x + 9) - 9 + (y^2 + 4y + 4) - 4 - 3 = 0$
$(x - 3)^2 + (y + 2)^2 - 16 = 0$; $(x - 3)^2 + (y + 2)^2 = 16$;
Comparing with $(x-h)^2 + (y-k)^2 = r^2$; $h = 3, k = -2, r^2 = 16$; $r = 4$
therefore, centre $(h, k) = (3, -2)$, $r = 4$ => D

14.C ***Working:*** Observe $x^2 + y^2 - 3x + 4y - 19 = 0$; Obtain dy/dx i.e the tangent; ***thus***, $2x + 2y\,dy/dx - 3 + 4dy/dx = 0$; $\frac{dy}{dx} = \frac{3 - 2x}{2y + 4}$

At the point (2, 3) ; $\frac{dy}{dx} = \frac{3 - 2(2)}{2(3) + 4} = \frac{-1}{10}$, .: Slope, $m = \frac{-1}{10}$

Apply $y - y_1 = m(x - x_1)$; $y - 3 = \frac{-1}{10}(x - 2)$; ***thus***, $10y + x - 32 = 0$ => C

15.A $x^2 + y^2 + 2x + 3y + 1 = 0$
Working: B is not circle because the coefficient of x^2 and y^2 are not equa.: C is not a circle because no y^2 term exists D is not a circle because the coefficient of x^2 and y^2 are not equal and also contains coefficients of 25 and -4 respectively.

16.D ***Working:*** Observe $y^2 + 4y + 4x + 16 = 0$; $y^2 + 4y = -4x - 16$
Completing the square of the L.H.S, we have
$y^2 + 4y + (4/2)^2 - (4/2)^2 = -4x - 16$; $(y^2+4y+4) - 4 = -4x - 16$
$(y+2)^2 - 4 = -4x - 16$; $(y+2)^2 = -4x + 4 - 16 = -4x - 12 = -4(x+3)$
therefore, $(y+2)^2 = -4(x+3)$ => D

17.D ***Working:*** From the equation above, $y^2 + 4y + 4x + 16 = 0$, we have that $(y+2)^2 = -4(x+3)$; Comparing with the standard equation of a parabola $(y-k)^2 = 4a(x-h)$; => $k = -2$, $h = -3$; $4a = -4$; .: $a = -1$
Focus, $f = (h+a, k) = (-1 + -3, -2) = (-4, -2)$; Vertex, $V = (h, k) = (-3, -2)$. .: Vertex and focus are (-3, -2) and (-4, -2) => D

18.E ***Working:*** Equation of the hyperbola is $9x^2 - 4y^2 = 36$; Performing Implicit differentiation $18x - 8y\,dy/dx = 0$; $\frac{dy}{dx} = \frac{18x}{8y} = \frac{9x}{4y}$

At (-2, 2) ; $\frac{dy}{dx} = \frac{9(-2)}{4(2)} = \frac{-9}{4}$; ***therefore***, slope, $m = \frac{-9}{4}$

At point (-2, 2), we use the equation of a line and a point;

$y - y_1 = m(x - x_1)$; $y - 2 = \frac{-9}{4}(x+2)$; $4y - 8 = -9x - 18$

$4y + 9x - 8 + 18 = 0$; ***Hence***, $4y + 9x + 10 = 0$ => E

19.B ***Working:*** $x^2y + y^2x + 4x = 1$; Implicit differentiation gives

$x^2dy/dx + 2xy + y^2 + 2xydy/dx + 4 = 0$;

$x^2dy/dx + 2xydy/dx = -2xy - y^2 - 4$; $\frac{dy}{dx} = \frac{-2xy - y^2 - 4}{x^2 + 2xy}$

Putting (1, 1) ; $\frac{dy}{dx} = \frac{-2(1)(1) - 1^2 - 4}{1^2 + 2(1)(1)} = \frac{-2 - 1 - 4}{1 + 2} = \frac{-7}{3}$ => B

20.A ***Working:*** Observe $y = \cot^2x^3$; Let $u = x^3$; $du/dx = 3x^2$

$y = \cot^2u$; $dy/du = -\text{cosec}^2u \, . \, 2\cot u$

By formula, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3x^2.2. -\text{cosec}^2u\cot u$

therefore, $dy/dx = -6x^2\text{cosec}^2x^3\cot x^3$ => A

21.C ***Working:*** Observe $y = \log_e\sqrt{1+x} = \ln\sqrt{1+x} = \ln(1+x)^{1/2}$

Rate of change, $\frac{dy}{dx} = 1 \, . \, \frac{1}{2}(1+x)^{1/2} \, . \, \frac{1}{(1+x)^{1/2}} = \frac{1}{2\sqrt{1+x}} \cdot \frac{1}{\sqrt{1+x}}$

$= \frac{1}{2(1+x)} = \frac{1}{2 + 2x}$ => C

22.D ***Working:*** $f(x) = \frac{x^4}{2} + \frac{5x^3}{3} - 2x^2 - 3x + 1$

Turning points occur at $dy/dx = 0$; Differentiating f(x), we have

$2x^3 + 5x^2 - 4x - 3 = 0$; Using trial and error method; Let $x = 1$

i.e $2(1)^3 + 5(1)^2 - 4(1) - 3 = 0$; ∴ $(x-1)$ is a factor of the polynomial.

We then use long division method to divide to obtain other roots.

i.e

$$
\begin{array}{r|l}
 & 2x^2 + 7x + 3 \\
\hline
x-1 & 2x^3 + 5x^2 - 4x - 3 \\
 & -\,(2x^3 - 2x^2) \\
\hline
 & 7x^2 - 4x \\
 & -\,(7x^2 - 7x) \\
\hline
\end{array}
$$

$$-\begin{pmatrix} 3x-3 \\ 3x-3 \end{pmatrix}$$
$$0$$

.: the quotient is $2x^2 + 7x + 3$; Obtaining the value of x from this quotient, we have $x = -\frac{1}{2}$, $x = -3$. ***therefore***, the turning points on the curve are $-\frac{1}{2}, -3, 1 \Rightarrow D$

23.A ***Working:*** Observe $\int \frac{x\,dx}{1-x}$; Let $u = 1-x$; $x = 1-u$; $dx = -du$

$$\therefore \int \frac{x}{1-x}\,dx = \int \frac{1-u}{u}\,.-du = -\int \frac{1-u}{u}\,du = -\int \left(\frac{1}{u} - \frac{u}{u}\right)du$$

$$= -\int \left(\frac{1}{u} - 1\right)du = [\ln u - u] + C = -\ln u + u + C = \ln(1-x) + 1-x + C$$

Method 2: $\int \frac{x\,dx}{1-x} = -\int \frac{-x\,dx}{1-x} = -\int \left(\frac{1-x-1}{1-x}\right)dx = -\int \left(\frac{1-x}{1-x} - \frac{1}{1-x}\right)dx = -\int \left(1 - \frac{1}{1-x}\right)dx$

$= -[x + \ln(1-x)] + C \Rightarrow A$

24.B ***Working:*** Integral coefficient of $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$; Let $u = \sqrt{x} = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$; $\Rightarrow dx = 2\sqrt{x}\,du = 2u\,du$; $\therefore \int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int \frac{e^{u}}{u}.2u\,du$

$= 2e^{u} + C = 2e^{\sqrt{x}} + C = 2\exp\sqrt{x} + C \Rightarrow B$

25.B ***Working:*** Observe Secxtanxdx; Integrating, we have Secx

$$= \Big|_{-\pi/3}^{\pi} \text{Sec}\pi - \text{Sec}(-\pi/3) = \left(\frac{1}{\text{Cos}\pi} - \frac{1}{\text{Cos}(-\pi/3)}\right)_{-\pi/3}^{\pi} = \left(\frac{1}{-1} - \frac{1}{\frac{1}{2}}\right)_{-\pi/3}^{\pi} = -3 \Rightarrow B$$

Nnamdi Azikiwe University, Awka.
Department of Mathematics, Faculty of Physical Sciences, Unizik, Awka. Second Semester Examination, 2015/2016 Session:
MAT 102: Elementary Mathematics II: Time: 2hrs:
NAME(S) .... REG NUMBER........................
DEPARTMENT........................ SIGNATURE ....
Instruction: Answer All Questions:

R = Real numbers, Q = Rational numbers, P = Prime numbers, Z = Integers, N = Natural numbers.

1. Compute the values f(-6), f(-5), f(16) of the function:

$$f(x) = \begin{cases} 3; & \text{if } t < -5 \\ t+1; & \text{if } -5 \le t \le 5 \\ \sqrt{t}, & \text{if } t > 5 \end{cases}$$

a. 2, 4, -4 b. 3, 4, 3 c. 0, 1, 4 d. 3, -4, 4 e. NOTA

2. Given the functions $f(x) = 2x - 1$ for $-2 < x < 4$ and $g(x) = \frac{4}{x-2}$ for $3 < x < 5$. What is the domain of the function $h(x) = f(x) - g(x)$?
a. $3 < x < 4$ b. $-2 < h(x) < 5$ c. $1 < h(x) < 5$ d. $-2 < x < 3$ e. NOTA

3. Compute the coordinate of the maximum point of the curve $f(x) = 2x^3 - 9x^2 + 12x - 1$ a. (1, 4) b. (2, 3) c. (2, 1) d. (1, 2) e. NOTA

4. Given the function $\int_0^2 f(x)dx = 3$, $\int_2^7 f(x)dx = 13$, $\int_0^9 f(x)dx = 56$,

obtain $\int_7^9 f(x)dx$; a. 43 b. 18 c. 40 d. NOTA e. 50

5. Evaluate $\int x\log_e x\,dx$ a. $\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + K$ b. $\ln|\sec^2 x| + K$

c. $\frac{x^2}{4}\left(\ln x - \frac{1}{4}\right) + K$ d. $\ln|\sec x\tan x| + K$

6. Compute $\int_0^{\pi/2} \cos^2 x \tan x\,dx$ a. π b. $-\pi$ c. ½ d. $-\pi/2$ e. NOTA

7. What is the integral coefficient of the function $f(x) = \frac{e^{3x}}{e^{3x}+6}$ w.r.t x

a. $\frac{e^{x^2}}{4} + C$ b. $3\text{In}(e^{3x}+6) + C$ c. $\frac{1}{3}\text{In}(e^{3x}+6) + C$

d. $-\frac{e^{-2x^2}}{4} + C$ e. NOTA

8. Compute the differential coefficient of $\cos^2 x + \text{In}\,x^3$

a. $-2\cos x\sin x + \frac{3}{x}$ b. $\frac{1}{2}e^{x/2} - \sin 2x$ c. $-2\cos x\sin^2 x + \frac{3}{x^2}$

d. $-\frac{1}{2}e^{x/2} - 2\sin 2x$ e. NOTA

9. The gradient of the curve $x^2 - y^3 + 3x - 5y = 22$ at $(1, -2)$ is a. 6/17 b. 5/17 c. 1/13 d. 2/17 e. NOTA

10. Find y' given that $14x^3 + y^2 = 3xy$ a. $\frac{42x^2 - 3}{2y}$ b. $\frac{36x^2 - 3y}{5y}$

c. $\frac{36x^2}{y} - 3$ d. $\frac{42x^3}{y} - 3$ e. NOTA

11. Compute the derivative of $h(x) = \tan(x^3 - 7x)$ a. $\frac{3x^2+7}{x^3+7x}$ b. NOTA

c. $\frac{3x^2 - 7}{x^3 + 7x}$ d. $(3x^2 - 7)\sec^2(x^3 + 7x)$ e. $3x^2\tan(x^3 + 7x)$

12. What is the area of the region R bounded by the line $y = 2x$, the x-axis and the vertical line $x = 2$? a. 36 b. 2 c. 24 d. 4 e. NOTA

13. Determine the equation of the line which is parallel to the line $2y + x = 3$ and passes through the midpoint of $(-2, 3)$ and $(4, 5)$.
a. $2y = x + 8$ b. $y - 2x = 2$ c. $2y + x = 9$ d. $y - x = 4$ e. NOTA

14. Which of these points is inside the circle, $x^2 + y^2 - 3x + 4y - 12 = 0$?
a. $(-3, 2)$ b. $(3, -1)$ c. $(1, -1)$ d. $(3, 1)$ e. NOTA

15. The equation of a circle with centre at $(-3, 5)$ and radius 6 is
a. $x^2 + y^2 - 6x + 10y + 25 = 0$ b. $x^2 + y^2 - 6x + 10y - 2 = 0$

c. $x^2 + y^2 + 6x + 10y - 2 = 0$ d. $x^2+y^2 - 6x + 10y - 25 = 0$ e. NOTA

16. Which of these curves is an ellipse a. $\frac{x^2}{36} - \frac{y^2}{49} = 1$ b. $x^2 + 4y^2 = 100$ c. $(x - 1)^2 = -4(y - 2)$ d. $xy = 5$ e. NOTA

17. Compute $\frac{dy}{dx}$ if $y = (x^2 + 1)^3 - 4(x^2 + 1)^2 + 5(x^2 + 1) + 2$ for the value $x = 1$ a. 1 b. -1 c. NOTA d. 2 e. -2

18. What is the vertical asymptote of the function, $f(x) = \frac{x - 2}{2x + 3}$ a. $f(x) = 0.5$ b. $f(x) = 1$ c. $x = -3/2$ d. $x = ½$ e. NOTA

19. Evaluate the $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$ a. 0 b. ½ c. ∞ d. -½ e. NOTA

20. Compute the slope of the line tangent to the graph $f(x) = x^3$ when $x = 1$ and obtain the equation of the normal at same point a. 3, $3y + x = 4$ b. 4, $3y = 3x + 4$ c. $-\frac{11}{5}$, $y = 4x - 30$ d. $-\frac{11}{4}$, $11y - 4x + 30 = 0$ e. NOTA

21. The equation of a line passing through the point (3, -1) and parallel to the straight line $y = 2x - 3$ is a. $y = 2x + 7$ b. $2y + x - 1 = 0$ c. $y = 2x - 7$ d. $y - 2x + 6 = 0$ e. NOTA

22. The points (1, a), (3, 2) and (4a, -1) are collinear. Find the possible values of a a. 0 or 11/4 b. 7 or 3 c. 4/3 or -5/3 d. 3 or 2 e. NOTA

23. Determine fog given $f(x) = x + 1$ and $g(x) = x + \frac{1}{x}$ a. $(1 + x)^2$ b. $\frac{(1+x)^2 + 1}{1 + x}$ c. $x + \frac{1}{x} + 1$ d. $\frac{x + 1}{x}$ e. NOTA

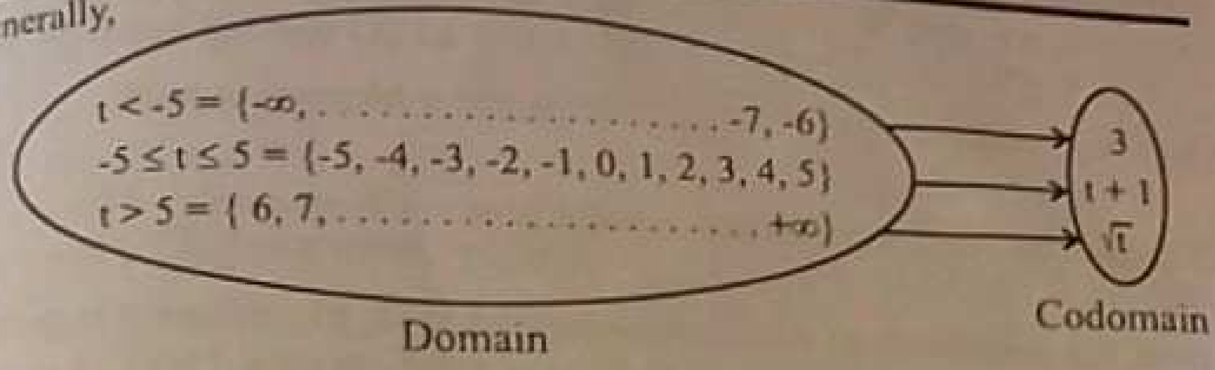
24. Assume that the infected area of an injury is circular. If the radius of the infected area is 0.5mm, and is growing at a rate of 0.5mm/hr; at what rate is the infected area increasing? a. $2.5\pi mm^2/hr$ b. $1.5\pi mm^2/hr$ c. $2\pi mm^2/hr$ d. $0.5\pi mm^2/hr$ e. NOTA

25. Determine the centre and radius of the circle $x^2 + y^2 = 6x$. a. (0, 0), 3 b. (0, 2), 4 c. (3, 0), 3 d. (-2, 0), 4 e. NOTA

Answers to 2016 Unizik MAT 102 - By Mr Ohms

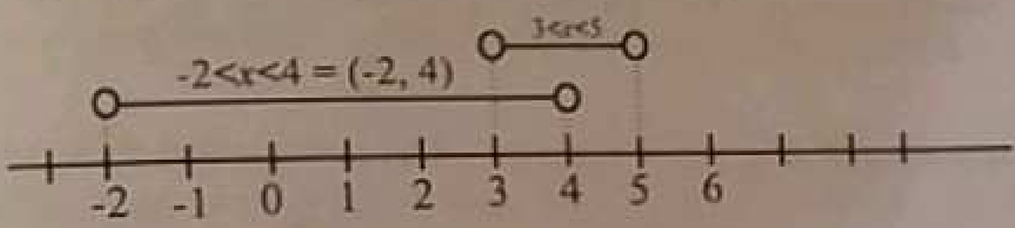
1.D Working:

Generally,



thus, $f(-6) = 3$; $f(-5) = t + 1 = -5 + 1 = 4$; $f(16) = \sqrt{16} = 4$
therefore, answer = (3, -4, 4) => D

2.A *Working:*
For $-2 < x < 4$, we have $x = -1, 0, 1, 2, 3$; i.e $-2 < x < 4 = (-2, 4)$ and for $3 < x < 5$, we have $x = 4$; i.e $3 < x < 5 = (3, 5)$; *thus*, we have



The domain of the function $h(x) = f(x) - g(x)$ is the intersection of (-2, 4) and (3, 5). i.e $(-2, 4) \cap (3, 5) = (3, 4) = 3 < x < 4$ => A

NB: (i). (3, 4) is gotten from the diagram of the number line above
(ii). The symbol, "( )", is called open end interval.

3.A *Working:* $f(x) = 2x^3 - 9x^2 + 12x - 1$
Let $f(x) = y$; => $y = 2x^3 - 9x^2 + 12x - 1$ ________ (1)
i.e $\frac{dy}{dx} = 6x^2 - 18x + 12$; At the turning point, $\frac{dy}{dx} = 0$
=> $6x^2 - 18x + 12 = 0$; $x^2 - 3x + 2 = 0$; Factorizing, we have
$x = (1, 2)$; To test for maximum point, we differentiate again
i.e $\frac{d^2y}{dx^2} = 12x - 18$

At $x = 1$, we have $\frac{d^2y}{dx^2} = 12(1) - 18 = -6 < 0 =$ Maximum point

At $x = 2$, we have $\frac{d^2y}{dx^2} = 12(2) - 18 = 6 > 0 =$ Maximum point

The coordinate of the maximum point is at $x = 1$, since $\frac{d^2y}{dx^2} < 0$.

thus, the coordinate of the maximum point is (1, y) ; Where y is gotten by putting $x = 1$ into equation (1). i.e $y = 2(1)^3 - 9(1)^2 + 12(1) - 1 = 4$
.: the coordinate of the maximum point is (1, 4) => A

4.C **Working:** By formula, $\int_0^9 f(x)dx = \int_0^2 f(x)dx + \int_2^7 f(x)dx + \int_7^9 f(x)dx$

$$=> \int_7^9 f(x)dx = \int_0^9 f(x)dx - \int_0^2 f(x)dx - \int_2^7 f(x)dx = 56 - 3 - 13 = 40 => C$$

5.A **Working:** $\int x\log_e xdx = \int x\ln xdx$; Let $u = \ln x$ and $dv = xdx$

i.e $\frac{du}{dx} = \frac{1}{x}$, => $du = \frac{1}{x}dx$ and $\int dv = \int xdx$; i.e $v = \frac{x^2}{2}$

By Integration by part, $\int udv = uv - \int vdu = \frac{x^2\ln x}{2} - \frac{1}{2}\int xdx$

$$= \frac{x^2\ln x}{2} - \left(\frac{1}{2}\right)\left(\frac{x^2}{2}\right) + K = \frac{x^2}{2}\left(\ln x - \frac{1}{2}\right) + K => A$$

6.C **Working:** Observe $\int_0^{\pi/2} \cos^3 x\tan xdx$ ________ (1)

Let $u = \cos x$, => $\frac{du}{dx} = -\sin x$; i.e $dx = -\frac{du}{\sin x}$ ____________ (2)

Put equation (2) into (1) i.e $\int_0^{\pi/2} u^2 \times \frac{\sin x}{\cos x} \times \frac{-du}{\sin x} = \int_0^{\pi/2} u^2 \times \frac{\sin x}{u} \times \frac{-du}{\sin x}$

$= -\int_0^{\pi/2} u\,du$; On integration, we have $-\left(\frac{u^2}{2}\right)_0^{\pi/2}$; But $\frac{\pi}{2} = \frac{180^\circ}{2} = 90^\circ$

$= \left[\frac{\cos^2 x}{2}\right]_0^{\pi/2} = -\left[\frac{\cos^2 90}{2} - \frac{\cos^2 0}{2}\right]_0^{\pi/2} = -(0 - ½) = ½ \Rightarrow C$

7.B *Working:* Observe $\int \frac{e^{3x}}{e^{3x}+6}\,dx$ ______________ (1)

Let $u = e^{3x} + 6$, $\Rightarrow dx = \frac{3\,du}{e^{3x}}$; Put equation (2) into (1)

i.e $\int \frac{e^{3x}}{u} \times \frac{3}{e^{3x}}du = 3\int \frac{1}{u}du = 3\ln u + C = 3\ln(e^{3x}+6) + C \Rightarrow B$

8.A *Working:* Let $y = \cos^2 x + \ln x^3$;
By short cut, $\frac{dy}{dx} = -2\sin x\cos x + \frac{3x^2}{x^3} = -2\cos x\sin x + \frac{3}{x} \Rightarrow A$

9.B *Working:* Observe $x^2 - y^3 + 3x - 5y = 22$
$2x - 3y^2\frac{dy}{dx} + 3 - 5\frac{dy}{dx} = 0$; $2x + 3 = 3y^2\frac{dy}{dx} + 5\frac{dy}{dx}$
$2x + 3 = (3y^2 + 5)\frac{dy}{dx}$; $\frac{dy}{dx} = \frac{2x+3}{3y^2+5}$
At the point 1, -2; i.e x = 1, & y = -2 ; i.e $\frac{dy}{dx} = \frac{2(1)+3}{3(-2)^2+5} = \frac{5}{17}$

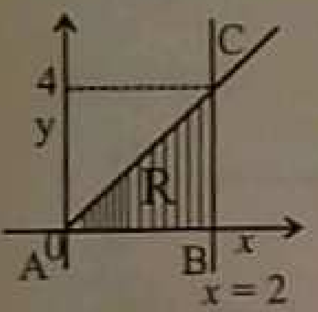
10.E *Working:* Observe $14x^3 + y^2 = 3xy$; Differentiating implicitly, we have

$42x^2 + 2y\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$; $42x^2 - 3y = (3x - 2y)\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{42x^2 - 3y}{3x - 2y}$ => E

11.B ***Working:*** $h(x) = \tan(x^2 - 7x)$;
By short cut, $h'(x) = (3x^2 - 7)\sec^2(x^2 - 7x)$ => B

12.D ***Working:*** For $y = 2x$, we have



Method 1: When $x = 2$, $y = 2x = 2(2)$
=> $y = 4$; Area bounded = ΔABC
i.e Area = ½ x b x h = ½ x 2 x 4 = 4

Method 2: $y = 2x$, => $x = \frac{y}{2}$ ____(1)

$x = 2$ ____(2); At intersection; $x = \frac{y}{2}$

i.e $2 = \frac{y}{2}$; => $y = 2 \times 2 = 4$; ∴ Area of R = ½ x bh = ½ x 2x4

therefore, Area of R = 4 => D

13.C ***Working:*** Given $2y + x = 3$; We first make y the subject of formula; i.e $y = -\frac{x}{2} + \frac{3}{2}$

thus, the slope of the line, $m_1 = -½$; For the midpoint (N) of (-2, 3) and (4, 5), we have $N = \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}$

i.e $N = \frac{4 + (-2)}{2}, \frac{5 + 3}{2} = (1, 4)$

Since the lines are parallel, $m_1 = m_2 = -½$ and the required equation is $y - y_1 = m_1(x - x_1)$; Where $m_1 = m_2 = -½$, $x_1 = 1$, $y_1 = 4$; ∴ $y - 4 = -½ (x - 1)$; *thus*, $2y + x = 9$ => C

14.E NOTA. This is so because, if we put any of the points (coordinates) into $x^2 + y^2 - 3x + 4y - 12 = 0$, the result will not be equal to zero. Generally, for any point inside a circle, the result (outcome) must be equal to zero, when the given points are inserted into the circle equation. **Option E is Correct**

15.E ***Working:*** By formula, the equation of a circle with centre (a, b) and radius, r is given by $(x - a)^2 + (y - b)^2 = r^2$
From the question, a = -3, y = 5, r = 6 ; .: $(x+3)^2 + (y-5)^2 = 6^2$
Expanding, we have $x^2 + 6x + 9 + y^2 - 10y + 25 - 36 = 0$
.: $x^2 + y^2 + 6x - 10y - 2 = 0$ => E

16.B ***Working:*** The standard form (canonical form) of an ellipse when the centre coordinate is at the origin, is written as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; For $x^2 + 4y^2 = 100$, we divide through by 100
to get $\frac{x^2}{100} + \frac{y^2}{25} = 1$ => B

17.D ***Working:*** Observe $y = (x^2 + 1)^3 - 4(x^2 + 1) + 5(x^2 + 1) + 2$
thus, $\frac{dy}{dx} = [3(x^2+1)^{3-1} \times 2x] - [(4\times2)(x^2+1)^{2-1} \times 2x)$
$+ [5(x^2+1)^{1-1} \times 2x] + 0 = 6x[x^2+1]^2 - 16x[x^2+1] + 10x$
At x = 1 ; $\frac{dy}{dx} = [6(1)((1)^2 + 1)^2] - [16(1)((1)^2 + 1)] + 10(1)$
therefore, $\frac{dy}{dx} = 24 - 32 + 10 = 2$ => D

18.C ***Working:*** For vertical asymptote, the value of y tends to be infinity. Now, from the equation, $f(x) = \frac{x - 2}{2x + 3}$

We let f(x) = y ; => $y = \frac{x - 2}{2x + 3}$; i.e $2x + 3 = \frac{x - 2}{y}$

Now, we put $y = \infty$; i.e $2x + 3 = \frac{x - 2}{\infty} = 0$; $x = \frac{-3}{2}$ => C

19.B ***Working:*** Observe $\lim_{0 \to \infty} \frac{e^x - x - 1}{x^2}$

First, we put x = 0 ; i.e $\frac{e^0 - 0 - 1}{0^2} = \frac{0}{0}$ = Indeterminate

So, we apply L' hospital rule by differentiating both the numerator and denominator respectively. i.e $\frac{e^x - x - 1}{x^2} = \frac{e^x - 1}{2x}$

So, we let $x = 0$; i.e $\frac{e^0 - 1}{2(0)} = \frac{0}{0}$ = Indeterminate

Hence, we differentiate again; i.e $\frac{e^x - 1}{2x} = \frac{e^x}{2}$

Put $x = 0$; i.e $\frac{e^0}{2} = \frac{1}{2}$ => B

20.A ***Working:*** $f(x) = x^3$; Let $y = f(x)$; => tangent slope, $\frac{dy}{dx} = 3x^2$

At $x = 1$; tangent slope, $\frac{dy}{dx} = 3(1)^2 = 3$

Recall; Slope of normal, $m_1 = \frac{-1}{m} = \frac{-1}{3}$

But $y = x^3$, and $x = 1$; => $y = (1)^3 = 1$; Point = (1, 1)

The equation of a tangent of a line is $y - y_1 = m_1(x - x_1)$

$y - 1 = \frac{-1}{3}(x - 1)$; $3y + x = 4$

therefore, answer = 3, $3y + x = 4$ => A

21.C ***Working:*** Point (3, -1) & $y = 2x - 3$, compare with $y = mx + c$

slope, m = 2 ; since it is parallel, $m_1 = m = 2$

Using equation of a line given the slope and a point;

$y - y_1 = m_1(x - x_1)$; $y - (1) = 2(x - 3)$; $y = 2x - 7$ => C

22.A ***Working:*** Let A = (1, a), B = (3, 2) and C = (4a, -1). Generally, three or more points are collinear if slope of any two pairs of points is the same. Since the points are collinear, slope AB = slope BC = slope AC. i.e Slope of AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - a}{3 - 1}$

Slope BC = $\frac{-1 - 2}{4a - 3}$

Since the points are collinear, slope AB = slope BC i.e $\frac{2-a}{3-1} = \frac{-1-2}{4a-3}$; $(2-a)(4a-3) = (3-1)(-1-2)$

$= 11a = 4a^2$; $4a^2 - 11a = 0$; $a(4a-11) = 0$; $a = 0$ or $\frac{11}{4}$ => A

23.C *Working:* Given; $f(x) = x + 1$ and $g(x) = x + \frac{1}{x}$

thus, fog = fog(x) = f(g(x)) = $f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right) + 1$

therefore, fog = $\frac{x^2 + 1 + x}{x} = x + \frac{1}{x} + 1$ => C

24.D *Working:* Area of a circle, $A = \pi r^2$; => $\frac{dA}{dr} = 2\pi r$

But $\frac{dr}{dt} = 0.5$mm/hr ; By chain rule, $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

i.e $\frac{dA}{dt} = 2\pi r \times 0.5$mm/hr ; But r = 0.5mm

.: $\frac{dA}{dt} = 2\pi \times 0.5$mm $\times 0.5$mm/hr $= 0.5\pi$mm2/hr => D

50.C *Working:* Given $x^2 + y^2 = 6x$; => $x^2 + y^2 - 6x = 0$ ______ (1)
From $x^2 + y^2 + 2gx + 2fy + c = 0$ ______ (2); Comparing equation (1) and (2), we have $2gx = -6x$, => $g = -3$ & $2fy = 0$, => $f = 0$; But centre is $(-g, -f) = (3, 0)$

Radius, $r = \sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + 0^2 - 0} = 3$

therefore, answer = (3, 0), 3 => C

Nnamdi Azikiwe University, Awka.
Department of Mathematics, Faculty of Physical Sciences, Unizik, Awka. Second Semester Examination, 2016/2017 Session:
MAT 102: Elementary Mathematics II: Time: 2hrs
NAME(S) .... REG NUMBER.......................
DEPARTMENT.......................... SIGNATURE ....
R = Real numbers, Q = Rational numbers, P = Prime numbers, Z = Integers, N = Natural numbers:
Instruction: Answer All Questions:

1. Evaluate $\lim_{x\to 3} \frac{4 - x^2}{x - 2}$ a. 5 b. 4 c. 3 d. 2 e. NOTA

2. Evaluate $\int_1^2 xe^x d\theta$ a. e^2 b. $x\ln x - x + C$ c. $\ln x(1+\cos^2\theta) + C$ d. $-\ln(1 + \cos^2\theta) + C$ e. NOTA

3. Consider the function, $f(x) = \frac{-x^2 + 3x}{x^2 - 2x + 5}$, what is the $\lim_{x\to\infty} f(x)$? a. $-\infty$ b. 1 c. 0 d. $+\infty$ e. NOTA

4. Determine the domain of the function $f(x) = \sqrt{(1 - x)^2}$ a. R b. R - {0} c. $(-\infty, -1] \cup [1, +\infty)$ d. $[-1, 1]$ e. NOTA

5. Find the area of the region bounded by the curves $y = 1 - x^2$, $y = x + 2$ and the lines $x = 1, x = 3$ a. 1 square units b. 44/3 square units c. 1/3 square units d. 1/4 square units e. NOTA

6. Suppose the functions f, g, h : R→R are defined by $f(x) = 5 - 3x$, $g(x) = x^4$ and $h(x) = 2x^3$. Compute *foh* a. $(5 - 3x)^4$ b. $5 - 6x^3$ c. $5 - 3x^4$ d. $(3x - 5)^4$ e. NOTA

7. Find the slope of the equation $-\frac{x}{3} - \frac{y}{2} = 1$ a. $\frac{2}{3}$ c. $\frac{-2}{3}$ c. $\frac{3}{2}$ d. -3/2 e. NOTA

8. Determine the center and radius of the circle $x^2 - 2x - 4y + y^2 - 4 = 0$ respectively. a. (0, 0) and 3 b. (1, 2) and $\sqrt{5}$ c. (2, 3) and 2 d. (1, 2) and 3 e. NOTA

9. If $f : R \rightarrow R$ is defined by $f(x) = \sqrt{3 + 2x}$, then $f' = ?$

a. $\left\{(x, y) : y = \frac{x^2}{2} - \frac{3}{2}\right\}$ b. $\left\{(x, y) : y = \frac{x}{7} + \frac{5}{7}\right\}$

c. $\left\{(x, y) : y = \frac{x}{7} - \frac{5}{7}\right\}$ d. $\left\{(x, y) : y = \frac{-x}{7} + \frac{5}{7}\right\}$ e. NOTA

10. Obtain the vertex and focus of the parabola $x = 8y^2$ respectively. a. (0, 0); (1/8, 0) b. (0, 0); (1/24, 0) c. (0, 0); (1/16, 0) d. (0, 0); (1/32, 0) e. NOTA

11. Obtain the vertices and foci of the ellipse $16x^2 + 9y^2 = 144$ respectively. a. $(0, \pm3); (0, \sqrt{\pm5})$ b. $(0, \pm4); (0, \sqrt{\pm7})$ c. $(0, \pm5); (0, \sqrt{\pm2})$ d. $(0, \pm6); (0, \sqrt{\pm13})$ e. NOTA

12. Determine the constant a and b for which the function $f(x) = x^3 + ax^2 + bx$ have stationary points $x = -1$ and $x = 3$ a. a = -15/8 & b = -27/4 b. a = -3 & b = -10 c. a = 30/8 & b = -9/2 d. a = -3 & b = -12 e. NOTA

13. Given $y = x^x$, find dy/dx a. $-(1 + \ln x)x^x$ b. $(1 + \ln x)x^x$ c. $[1 + \ln(x+1)](x+1)^{(x+1)}$ d. $(1 + \ln x)xx^{-1}$ e. NOTA

14. Compute the differential coefficient of the equation $2(x^3 + 2y^3)^2 = 4$ and obtain y' a. $y' = -x^2/y^2$ b. $y' = x^2/y^2$ c. $y' = x^2/2y^2$ d. $y' = -x^2/2y^2$ e. NOTA

15. Find the turning point(s) of $y = (1 - x)e^x$ a. $x = -2$, maximum b. $x = 0$, maximum c. $x = 0$, minimum d. $x = -2$, minimum e. NOTA

16. Find the differential coefficient of $x^3 + y^3 - xy^2 + 1 = 0$ at (1, 1) a. 5/2 b. -5/2 c. -2 d. -2/5 e. NOTA

17. What is the slope of the line Normal to the graph of $y = 3x^2 + 1$ at the point (½, 7/4) a. -1/3 b. 6 c. -1/6 d. 3 e. NOTA

18. What is the equation of the line that has x intercept at 3 an

parallel to the line $x - y + 1 = 0$? a. $x - y + 1 = 0$
b. $x - y - 2 = 0$ c. $x - y - 3 = 0$ d. $x - y - 1 = 0$ e. NOTA

19. Compute the equation of the circle with radius 2 and center at the point (-2, 0). a. $x^2 + y^2 - 4x = 0$ b. $x^2 + y^2 - 4y = 0$
c. $x^2 + y^2 + 4y = 0$ d. $x - y - 1 = 0$ e. NOTA

20. Find the equation of the line which makes an angle 135° with the positive direction of the x-axis and passes through the point (-2, -1) a. $y + x + 2 = 0$ b. $y - x + 1 = 0$
c. $y + x - 3 = 0$ d. $y - x + 3 = 0$ e. NOTA

21. At what point is the function $f(x) = \frac{1}{1 + (x-1)}$ discontinuous

a. 0 b. -1 c. -2 d. 1 e. NOTA

22. Compute the vertices and foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$

a. $(\pm 5, 0); (\pm\sqrt{29}, 0)$ b. $(\pm 5, 0); (\pm\sqrt{39}, 0)$
c. $(\pm 3, 0); (\pm\sqrt{13}, 0)$ d. $(\pm 4, 0); (\pm 5, 0)$ e. NOTA

23. What is the equation of the tangent plane to the curve $y = 2x^2 - 3x$ at the point (0, 1)? a. $y = 3x + 1$ b. $y = 1$
c. $y = -3x + 1$ d. $y = -1$ e. NOTA

24. Supposed $f(x) = (x + 1)(x + 2)$ is a real valued function, what is the domain of f(x)? a. $(-\infty, -1]\cup[2, +\infty)$
b. $(-\infty, -1]\cup[-2, -\infty)$ c. $(-\infty, -2]\cup[1, +\infty)$ d. $(-\infty, -2]\cup[2, +\infty)$
e. NOTA

25. If the point (-7, -6) divides a line segment whose endpoints are on the axes in the ratio 1:5 internally, then find the coordinates of these points which are on the axes
a. (-35/4, 0); (0, 30) b. (-42/5, 0); (0, 36) c. (-28/3, 0);(0, 24)
d. (-21/2, 0); (0, 18) e. NOTA

MAT 102 2017 Answers for Unizik Students - By Mr Ohms

1..E *Working:* Observe $\lim_{x \to 3} \frac{4 - x^2}{x - 2}$, at $x = 3$, we have

$$\frac{4 - (3)^2}{3 - 2} = -5 \Rightarrow E$$

2.A *Working:* Observe $\int_1^2 xe^x$; Let u = dx and dv = e^x

thus, du = dx and v = e^x ; Apply Integration by part formula

$$\int u dv = uv - \int v du = xe^x - \int_1^2 e^x dx = \left[xe^x - e^x\right]_1^2 = (2e^2 - e^2) - (e^1 - e$$

$$= e^2 - 0 = e^2 \Rightarrow A$$

3.C *Working:* Observe $\lim_{x \to \infty} \frac{-x^2 + 3x}{x^3 - 2x + 5}$

Since $x \to \infty$, we apply the Thumb's Rule by dividing through by the highest power of x.

i.e $$\frac{-x^2/x^3 + 3x/x^3}{x^3/x^3 - 2x/x^3 + 5/x^3} = \frac{-1/x + 3/x^2}{1 - 2/x^2 + 5/x^3}$$

Now, we put $x = \infty$

thus, $$\frac{-1/\infty + 3/\infty^2}{1 - 2/\infty^2 + \infty^3} = \frac{0}{1} = 0 \Rightarrow C$$

4.A *Working:* Observe $f(x) = \sqrt{(1 - x)^2} = (1 - x) = R \Rightarrow A$

$(1 - x)$ is a **linear function**. **Linear functions** are lines that continue forever in each direction. Any real number can be used for x to get a meaningful output. **Domain is all real (R) numbers. Option A is Correct.**

5.B *Working:* Area, $A = \int_{x_1}^{x_2} y dx$;

Where $y = y_2 - y_1 = 1 - x^2 - (x+2) = -x^2 - x - 1$; $x_1 = 1$, $x_2 = 3$

thus, $A = \int_{1}^{3} (-x^2 - x - 1)dx$. On integration, we have $\left[\frac{-x^3}{3} - \frac{x^2}{2} - x\right]_1^3$

$$= \left(\frac{-3^3}{3} - \frac{3^2}{2} - 3\right) - \left(\frac{-1^3}{3} - \frac{1^2}{2} - 1\right) = \frac{-44}{3} = \frac{44}{3} \text{ square units} \Rightarrow B$$

6.B ***Working:*** Given $f(x) = 5 - 3x$; $g(x) = x^4$; $h(x) = 2x^3$
thus, $foh = f(h(x)) = f(2x^3) = 5 - 3(2x^3) = 5 - 6x^3 \Rightarrow B$

7.B ***Working:*** Observe $\frac{-x}{3} - \frac{y}{2} = 1$
Differentiating, we have $-\frac{1}{3} - \frac{1}{2}\frac{dy}{dx} = 0$; $\frac{1}{2}\frac{dy}{dx} = \frac{-1}{3}$
thus, slope $= \frac{dy}{dx} = \frac{-2}{3} \Rightarrow B$

8.D ***Working:***
Given $x^2 - 2x - 4y + y^2 - 4 = 0 \equiv x^2 + y^2 - 2x - 4y - 4 = 0$ ____(1)
From $x^2 + y^2 + 2gx + 2fy + c = 0$ __________ (2)
Comparing equation (1) and (2) : $2gx = -2x$, => $g = -1$ and $2fy = -4y$, => $f = -2$; But center $= (-g, -f) = (1, 2)$

Radius, $r = \sqrt{g^2 + f^2 - c}$; $c = -4$; *thus*, $r = \sqrt{(-1)^2 + (-)^2 + 4}$
i.e $r = 3$; ***therefore***, Answer is (1, 2) and 3 => D

9.A ***Working:*** $f(x) = \sqrt{3 + 2x}$; But $f^{-1}(x)$ = Inverse of $f(x)$

Let $f(x) = y$, => $y = \sqrt{3 + 2x}$; Make x the subject formula
i.e $y^2 = 3 + 2x$; $x = \frac{y^2 - 3}{2}$; *thus*, $f^{-1}(x) = \frac{x^2 - 3}{2} = \frac{x^2}{2} - \frac{3}{2}$

Option A is Correct.

10.D ***Working:*** Given $x = 8y^2$; Make y^2 the subject formula i.e $y^2 = \frac{x}{8}$ ________ (1)

The general equation of a parabola is $y^2 = 4ax$ ________ (2)
Comparing equation (1) and (2), we have $4ax = \frac{x}{8}$; $a = \frac{1}{32}$

By definition, **focus**, F = (a, 0) = (1/32, 0).
Vertex by definition is the central base of a parabola. Since the **vertex** is at the center of the parabola, we have V = (0, 0)
∴ Answer is (0, 0) ; (1/32, 0) => D

11.B *Working:* Given: $16x^2 + 9y^2 = 144$; Divide through by 144 to get $\frac{x^2}{9} + \frac{y^2}{16} = 1$ ________ (1)

But the standard form of equation of an ellipse is:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since the ellipse is on the vertical axis, $a = 16$, => $a^2 = 4$; $b^2 = 9$, => $b = 3$; But $c^2 = a^2 - b^2 = 16 - 9 = 7$; i.e $c = \sqrt{7}$
By formula, **vertices** for this ellipse = (0, ±a) = (0, ±4)
Foci = (0, ±c) = (0, ±$\sqrt{7}$) ; ∴ Answer = (0, ±4) ; (0, ±$\sqrt{7}$) => B

12.E *Working:* Given: $f(x) = x^3 + ax^2 + bx$; $dy/dx = 3x^2 + 2ax + b$
At the stationary point, dy/dx = 0 ; At x = -1, we have:
$3(-1)^2 + 2a(-1) + 6 = 0$; $2a - b = 3$ ________ (1)
At x = 3; $3(3)^2 + 2a(3) + b = 0$; $6a + b = -27$ ________ (2)
Equation (1) + (2): 8a = -24 ; a = -3 ; Put a = -3 into equation (1): 2(-3) - b = 3, => b = -9 ; *therefore*, a = -3, b = -9 => E

13.A *Working:* $y = x^{-x}$; Take ln of both sides: $\ln y = \ln x^{-x} = -x\ln x$
Differentiating, $\frac{1}{y}\frac{dy}{dx} = -x.\frac{1}{x} + (-1)\ln x = -1 - \ln x = -(1 + \ln x)$

$\frac{dy}{dx} = -y(1 + \ln x) = -(1 + \ln x)x^{-x}$ => A

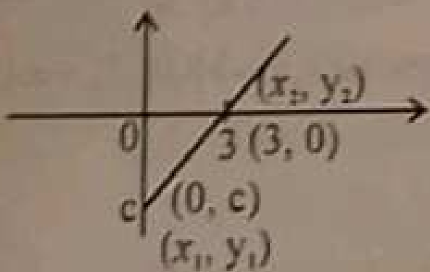
14.D **Working:** Observe: $2(x^3 + 2y^3)^2 = 4$; Dividing through by 2, we have $(x^3 + 2y^3)^2 = 2$; $x^3 + 2y^3 = 2$
Differentiating, we have: $3x^2 + 6y^2 dy/dx = 0$; $6y^2 dy/dx = -3x^2$
$\frac{dy}{dx} = \frac{-3x^2}{6y^2} = \frac{-x^2}{2y^2}$ => D

15.E **Working:** Given: $y = (1 - x)e^x$; Differentiating, we have;
$\frac{dy}{dx} = (1-x)e^x + e^x(-1) = e^x - xe^x - e^x = -xe^x$
At the turning point, $\frac{dy}{dx} = 0$; i.e $-xe^x = 0$; $e^x = 0$
Take ln of both sides; $\ln e^x = \ln 0$; $x = \ln 0 = -\infty$ => E

16.C **Working:** Observe $x^3 + y^3 - xy^2 + 1 = 0$
Differentiating implicitly, $3x^2 + 3y^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} + 0 = 0$
$3x^2 - y^2 + 3y^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = 0$; $\frac{dy}{dx}(3y^2 - 2xy) = y^2 - 3x^2$
$\frac{dy}{dx} = \frac{y^2 - 3x^2}{3y^2 - 2xy}$; At the point (1, 1) ; $\frac{dy}{dx} = \frac{1^2 - 3(1)^2}{3(1)^2 - 2(1)(1)} = -2$

17.A **Working:** Given: $y = 3x^2 + 1$; $dy/dx = 6x$; At the point $(\frac{1}{2}, 7/4) \equiv (x, y)$; *thus*, $dy/dx = 6(\frac{1}{2}) = 3$
Hence, $dy/dx = m_1 = 3$; For normal, $m_1m_2 = -1$; $m_2 = -1/m_1$
i.e $m_2 = -1/3$; *therefore*, the slope of the line Normal to the graph is $-1/3$ => A

18.C **Working:** The line $x - y + 1 = 0$, => $y = x + 1$ ______ (1)
Comparing equation (1) with $y = mx + c$, we have $m = 1$
But intercept on x-axis = 3 i.e



From the diagram, slope, $m = \frac{0 - c}{3 - 0}$
thus, $m = \frac{-c}{3}$; But $m = 1$; i.e $c = -3$

∴, the new equation $y = mx + c$ becomes $y = (1)(x) - 3$
i.e $y = x - 3$; ***therefore,*** $x - y - 3 = 0$ => C

19.D ***Working:*** Apply $(x - a)^2 + (y - b)^2 = r^2$; Where $(-2, 0) \equiv (a, b)$
i.e $a = -2$, $b = 0$; ∴ $(x + 2)^2 + (y - 0)^2 = 2^2$; $x^2 + y^2 + 4x = 0$

20.E ***Working:*** Slope = $\tan 135° = -1$; $(x_1, y_1) = (-2, -1)$;
Apply $y - y_1 = m(x - x_1)$; $y + 1 = -1(x + 2)$; $y + x + 3 = 0$ => E

21.A ***Working:*** For the function to be discontinuous, the denominator must be equal to zero; i.e $1 + (x-1) = 0$, => $x = 0$

22.D ***Working:*** Given: $\frac{x^2}{16} - \frac{y^2}{9} = 1$ ______ (1)

The standard equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ______ (2)

Comparing equation (1) and (2); $a^2 = 16$, => $a = 4$; $b^2 = 9$, => $b = 3$; But $c^2 = a^2 + b^2 = 16 + 9 = 25$, => $c = 5$

By definition, vertices for hyperbola = $(\pm a, 0) = (\pm 4, 0)$ and Foci, $F = (\pm c, 0) = (\pm 5, 0)$; Answer is $(\pm 4, 0)$; $(\pm 5, 0)$ => D

23.C ***Working:*** Given $y = 2x^2 - 3x$; $dy/dx = 4x - 3$; At the point (0, 1) ; Where $x = 0$, $y = 1$; $dy/dx = 4(0) - 3 = 3 = m$;
By formula, the equation of a tangent is $y - y_1 = m(x - x_1)$
But $x_1 = 0$, $y_1 = 1$; $y - 1 = -3(x - 0)$; ∴ $y = -3x + 1$ => C

24.B ***Working:*** Given $f(x) = \sqrt{(x+1)(x+2)}$

Step 1: Set the expression inside the square root greater than or equal to zero. i.e $(x+1)(x+2) \geq 0$

Step 2: Solve the equation found in step 1: i.e $x + 1 \geq 0$ or $x + 2 \geq 0$
thus, $x \geq -1$ or $x \geq -2$

Step 3: Write the answer using interval form. i.e $(,\infty -1] \cup (-2, +\infty]$.
Option B is Correct.

25.(-)

Nnamdi Azikiwe University, Awka.
Department of Mathematics, Faculty of Physical Sciences, Unizik, Awka. Second Semester Examination, 2018/2019 Session: (CBT)
MAT 102: Elementary Mathematics II: (Preparatory Questions):
R = Real numbers, Q = Rational numbers, P = Prime numbers, Z = Integers, N = Natural numbers:
Instruction: Answer All Questions:

1. If $y = -\sin^2 x - \cos 2x$ then, find dy/dx A. $-\sin 2x$ B. $-3\sin 2x$ C. $3\sin 2x$ D. $\sin 2x$ E. NOTA
2. Find the equation of the straight line which passes through the point (-2, 1) and is parallel to the line $y - 3x = 2$. A. $3y + x - 1 = 0$ B. $y - 3x - 7 = 0$ C. $3y + x - 5 = 0$ D. $y - 3x + 5 = 0$ E. NOTA
3. Find the equation of a straight line which passes through the point (-2, 3) and is perpendicular to the line $y - 2x = 4$ A. $y - 2x - 7 = 0$ B. $2y - x - 6 = 0$ C. $2y + x - 4 = 0$ D. $y + 2x + 7 = 0$ E. NOTA
4. The endpoints of the diameter of a circle are (-4, 0) and (4, 0). Find the equation of the circle. A. $x^2 + y2 - 4 = 0$ B. $x^2 + y^2 - 8 = 0$ C. $x^2 + y^2 - 16 = 0$ D. $x^2 + y^2 + 8 = 0$

   E. NOTA

5. Evaluate $\int x^2 \ln x dx$ A. $(x^3/3)[\ln x + 1/3] + C$

   B. $(x^3/3)[\ln x - 1/3] + C$ C. $(x^2/6)\ln x - 1/9 + C$ D. $-(x^3/3)[\ln x - 1/3] + C$ E. NOTA
6. Identify a stationary point of $f(x) = x + \sin x$ A. -1 B. π C. 1 D. -2π E. NOTA
7. Find the centre of the ellipse having foci at (0, 1) and (6, 1) and a major axis of length 8. A. (3, 1) B. (6, 0)

C. (3, 0) D. (0, 0) E. NOTA

8. Define g: R → R by $g(x-3) = 3x^2 - 2x + 1$, compute g(0)
A. 20 B. 22 C. 21 D. 16 E. NOTA

9. Determine the maximum point of $x(12 - 2x)$ A. 0 B. 1
C. 2 D. 3 E. NOTA

10. If $2y^3 - x^2 = 4xy$ then, find dy/dx A. (2y + x)/3y2 - 2x)
B. $(y + x)/(3y^2 - 2x)$ C. $(y - x)/(3y^2 - 2x)$
D. $(2y - x)/(3y^2 + 2x)$ E. NOTA

11. Find the domain of $\sqrt{(16 - x^2)}$ A. [-4, 4] B. $x \leq \pm 4$
C. [0, 4] D. $x \leq 4$ E. NOTA

12. Given $y = x^2 - x$. Evaluate y'(0) A. -1 B. 1 - x C. 2
D. 2 - x E. NOTA

13. Find the derivative of $(\sin x - 3x)^4$
A. $4(\sin x - 3x)^3(\cos x - 3)$ B. $4(\sin x - 3)^3(\cos x - 3)$
C. $4(\cos x - 3)^3(\sin x - 3x)$ D. $4(\cos x - 3)^3(\sin x - 3x)$
E. NOTA

4(sinx-3 ... cos x

14. Find the equation of the tangent line to $y = x^3 - 2x^2 + 4$ at (2, 4). A. y = x - 4 B. y = 2x - 4 C. y = 3x - 4
D. y = 4x - 4 E. NOTA

15. Evaluate y'' at (1, 1) given $y = 3x^2 + 4x - 2$ A. 2 B. 4
C. 6 D. 8 E. NOTA

16. Evaluate $\lim_{x \to 0} \frac{2x + \sin x}{2x - \sin x}$
A. 0 B. 1 C. 2 D. 3 E. NOTA

17. Find the equation of normal line to $x^2 + 3xy = 5$ at (1, 1)
A. y = 3x + 2 B. y = 1/3(3x + 2) C. y = 1/5(3x + 2)
D. y = 1/7(3x + 2) E. NOTA

18. Determine the minimum point of $f(x) = (x - 2)^2$

A. 0 B. 1 C. 2 D. 3 E. NOTA

19. Find the derivative of $\ln\left(\frac{x+2}{x-2}\right)$ A. $4/(x^2+4)$
B. $4(x^2-4)$ C. $(x+2)/(x-2)$ D. $-4/(x^2-4)$ E. NOTA

20. Find the derivative of $x\sqrt{x}+2/x^2$ A. $(2/3)\sqrt{x}-2/x^3$
B. $(3/2)\sqrt{x}-4/x^3$ C. $(3/2)\sqrt{x}+4/x^3$ D. $(2/3)\sqrt{x}-4/x^3$
E. NOTA

21. Find the equation of the tangent line for $x^2y^2-2x=4-4y$ at the point (2, -2). A. $6y+7x+26=0$
B. $7x-6y-26=0$ C. $6y-7x-26=0$
D. $6y-7x+26=0$ E. NOTA

22. Find the domain of the function $(x+5)/(x^2-2)$
A. $R\backslash(\sqrt{2}, \sqrt{-2})$ B. $R\backslash(\pm\sqrt{2})$ or $R\backslash(-\sqrt{2})$ C. $R\backslash(2)$ or $R\backslash(-2)$
D. R E. NOTA

23. Compute $\lim_{x\to 1} \frac{x^2+x-2}{x-1}$ A. 1 B. 2 C. 3 D. 4
E. NOTA

24. Find the derivative of $2x^{x-1}$ A. $4x^{x-1}[\ln x+(x-1)/x]$
B. $[\ln x-(x-1)/x]$ C. $2x^{x-1}[\ln x+(x-1)/x]$ D. $x^{x-1}[\ln x-(x-1)]$

25. Evaluate $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$ A. $\sqrt{2}/4$ B. $\sqrt{1/2}$ C. $2/\sqrt{3}$
D. $4/\sqrt{2}$ E. NOTA

26. Define f, g, h : R → R by $f(x)=2x$, $g(x)=x-5$, $h(x)=\sin x$, Compute $h(f(g(x)))$. A. $\sin(2x-10)$
B. $(2\sin x-10)$ C. $2\sin x-5$ D. $2\sin(x-5)$ E. NOTA

27. If $e^{2x+y}=xy$, then find dy/dx. A. $(y+e^{2x+y})/(e^{2x+y}+x)$
B. $(y-2e^{2x+y})/(e^{2x+y}-x)$ C. $(2y+e^{2x+y})/(e^{2x+y}+2x)$
D. $(e^{2x+y}+x)/(y+e^{2x+y})$ E. NOTA

28. Find the equation of the normal line at the point ($\pi/4$, 0)

on the curve that is given by the equation $x\cos y + y\sin x = \pi/2$. A. $y + x\sqrt{2} - (\pi/4)\sqrt{2} = 0$
B. $2y + x\sqrt{2} - (\pi/4)\sqrt{2} = 0$ C. $y + x\sqrt{2} + (\pi/4)\sqrt{2} = 0$
D. $2y - x\sqrt{2} + (\pi/4)\sqrt{2} = 0$ E. NOTA

29. If f : R→R is given by $f(x) = (x/3) + 1$. Find the inverse of f. A. $3(x - 1)$ B. $3x + 1$ C. $(1/3)(x - 1)$
D. $(-1/3)(x + 1)$ E. NOTA

30. Find the centre and radius of the circle given by the equation $2(x^2 + 1) + 2(y^2 - 2) + 8 = 0$. A. (-1, 0) & 2
B. (1, 2) & 4 C. (-1, 2) & 2 D. (-1, 2) & 4 E. NOTA

* 31. If $y = -2\sin^2 x + \cos 2x$ then, find dy/dx A. $3\sin 2x$
B. $-4\sin 2x$ C. $-\sin 2x$ D. $-3\sin 2x$ E. NOTA

32. Define g: R → R by $g(x-4) = 2x^2 + x - 1$. Compute g(0)
A. -1 B. 27 C. 9 D. 35 E. NOTA

MAT 102 2018/2019 Answers for Unizik Students - Mr Ohms

1.E ***Working:*** Observe $y = -\sin^2 x - \cos 2x$

By short cut, $\frac{dy}{dx} = -2\cos x\sin x - 2(-\sin 2x)$

.: $\frac{dy}{dx} = -2\cos x\sin x + 2\sin 2x$ => E

2.B ***Working:*** $y - 3x = 2$

Make y the subject formula in order to get the slope

i.e $y = 3x + 3$; ***thus***, slope = m = 3

But $y - y_1 = m(x - x_1)$

At the point (-2, 1), meaning $x_1 = -2$; $y_1 = 1$

=> $y - 1 = 3(x - (-2))$: $y - 1 = 3(x + 2)$

.: $y - 3x - 7 = 0$ => B

3.C ***Working:*** $y - 2x = 4$

Again, we find the slope, ***thus***, $y = 2x + 4$

i.e slope = $m_1 = 2$; For perpendicularity, $m_1m_2 = -1$

$\Rightarrow m_2 = \frac{-1}{m_1} = \frac{-1}{2}$

Equation of the required line is $y - y_1 = m_2(x - x_1)$

Where $x_1 = -2$; $y_1 = 3$ at the point (-2, 3)

thus, $y - 3 = -\frac{1}{2}(x - (-2))$; $\therefore 2y + x - 4 = 0 \Rightarrow C$

4. This is a repeated Question. Goto Q12 (2015) session for solution. (***Get Serious***).

5.B ***Working:*** $\int x^2 \ln x dx$; Apply $\int u dv = uv - \int v du$ —— (1)

Now, we let $u = \ln x$ and $dv = x^2 dx$

i.e $\frac{du}{dx} = \frac{1}{x}$; $\Rightarrow du = \frac{1}{x}dx$ and $\int dv = \int x^2 dx$

thus, $v = \frac{x^3}{3}$

Substituting into equation (1) we have

$$\int u dv = \ln x\left(\frac{x^3}{3}\right) - \int \frac{x^3}{3}.\frac{1}{x}dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$$

$$= \frac{x^3}{3}\ln x - \frac{x^3}{9} + C = \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C \Rightarrow B$$

6.B ***Working:*** Observe $f(x) = x + \sin x$; Let $y = f(x)$

$\Rightarrow y = x + \sin x$; i.e $dy/dx = 1 + \cos x$

At the stationary point, $dy/dx = 0$; i.e $1 + \cos x = 0$

thus, $\cos x = -1$; $\therefore x = \cos^{-1}(-1) = 180 = \pi$

Option B is Correct.

7.A ***Working:***

Remember, an ellipse is like an egg i.e

thus, $D = \sqrt{(6-0)^2 + (1-1)^2} = 36$

i.e $f = 6/2 = 3$

the value of y is constant ; $g = 1$

therefore, the centre of the ellipse, $(f, g) = (3, 1)$ => A

8.B ***Working:*** Observe $g(x-3) = 3x^2 - 2x + 1$

We let $g(x-3) = g(0)$; i.e $x - 3 = 0$; => $x = 3$.

This means that at $g(0)$, $x = 3$. Putting $x = 3$ into $3x^2 - 2x + 1$,

we have $g(0) = 3(3)^2 - 2(3) + 1 = 27 - 5 = 22$ => B

9. D ***Working:*** Observe $x(12 - 2x)$; Let $y = x(12 - 2x)$

=> $y = 12x - 2x^2$; $dy/dx = 12 - 4x$; $4x - 12 = 0$; $x = 3$

Generally, if $\frac{d^2y}{dx^2} < 0$, we have maximum point.

So we differentiate again. ***Hence,*** $\frac{d^2y}{dx^2} = -4$

Since $d^2y/dx^2 = -4 < 0$, we have maximum point

.: $x = 3$ is a maximum point => D

10.A ***Working:*** $2y3 - x^2 = 4xy$; By implicit differentiation,

$$6y^2\frac{dy}{dx} - 2x = 4y + 4x\frac{dy}{dx}$$

Collect like terms of $\frac{dy}{dx}$; $6y^2\frac{dy}{dx} - 4x\frac{dy}{dx} = 4y + 2x$

i.e $\frac{dy}{dx}(6y^2 - 4x) = 4y + 2x$; $\frac{dy}{dx} = \frac{4y + 2x}{6y^2 - 4x} = \frac{2y + x}{3y^2 - 2x}$

11.A **Working:** Observe $\sqrt{16 - x^2} = \sqrt{(4 - x)(4 + x)}$
Now, we let $16 - x^2 \geq 0$; $-x^2 \geq -16$
$x^2 \leq 16$ (the sign '$\geq$' changes to '$\leq$' because we divide both sides by '-' sign).
thus, $x \leq \pm\sqrt{16} = \pm 4$; i.e $x \leq -4$ and $x \leq 4$
therefore, we have $-4 \leq x \leq 4 = [-4, 4]$ => A

12.A **Working:** $y = x^2 - x$; $y' = 2x - 1$; At $y'(0)$, we have $x = 0$
.: $y' = 2(0) - 1 = -1$ => A

13.B **Working:** Observe $(\sin x - 3x)^4$; Let $y = (\sin x - 3x)^4$
Differentiating directly, we have:
$\frac{dy}{dx} = 4(\sin x - 3x)^3 \text{ x } (\cos x - 3) = 4(\sin x - 3x)^3(\cos x - 3)$

14.D **Working:** $y = x^3 - 2x^2 + 4$; $\frac{dy}{dx} = 3x^2 - 4x$
At the point (2, 4), we have $x = 2$, $y = 4$
=> $\frac{dy}{dx} = 3(2)^2 - 4(2) = 3(4) - 8 = 4 = m$
By formula, the equation of a tangent is $y - y_1 = m(x - x_1)$
But $x_1 = 2$, $y_1 = 4$; *thus*, $y - 4 = 4(x - 2)$; $y - 4 = 4x - 8$
.: $y = 4x - 4$ => D

15.C **Working:** $y = 3x^2 + 4x - 2$; $y' = 6x + 4$; $y'' = 6$ => C

16.D **Working:** Observe $\lim\limits_{x\to 0} \frac{2x + \sin x}{2x - \sin x}$
At $x = 0$, we have $\frac{2(0) + \sin(0)}{2(0) - \sin(0)} = \frac{0}{0}$ = Indeterminate
Apply L'Hospital rule by differentiating both the numerator and denominator respectively. i.e

$\frac{2x + \cos x}{2x - \sin x}$; At $x = 0$; $\frac{2 + \cos 0}{2 - \cos 0} = \frac{2 + 1}{2 - 1} = 3 \Rightarrow D$

17.E ***Working:*** Observe $x^2 + 3xy = 5$; Obtain dy/dx

i.e $2x + 3y + 3x\frac{dy}{dx} = 0$; $\frac{dy}{dx} = \frac{-2x - 3y}{3x}$

At the point $(1, 1) = (x, y)$; => $\frac{dy}{dx} = \frac{-2(1) - 3(1)}{3(1)} = \frac{-5}{3}$

Hence, $\frac{dy}{dx} = m_1 = \frac{-5}{3}$

For normal, $m_1m_2 = -1$; $m_2 = \frac{-1}{m_1} = \frac{-1}{(-5/3)} = \frac{3}{5}$

For equation of normal, apply $y - y_1 = m_2(x - x_1)$

Where at the point $(1, 1) \equiv (x_1, y_1)$; ***thus,*** $y - 1 = \frac{3}{5}(x - 1)$

.: $y = \frac{3}{5}(x - 1) + 1 \Rightarrow E$

18.B ***Working:*** $f(x) = (x - 2)^2$; Let $y = f(x)$; => $y = (x - 2)^2$

$\frac{dy}{dx} = 2(x - 2) \times 1 = 2x - 2$; i.e $x = 1$

If $\frac{d^2y}{dx^2} > 0$, we have minimum point ; ***thus,*** $\frac{d^2y}{dx^2} = 2$

thus, at $x = 1$, $d^2y/dx^2 = 2 > 0$ (we have minimum point)

therefore, $x = 1$ is a **minimum point** and the **minimum value is 1** => B

19.D ***Working:*** Let $y = \ln\frac{x+2}{x-2}$; i.e $y = \ln(x+2) - \ln(x-2)$

$\frac{dy}{dx} = \frac{1}{x+2} - \frac{1}{x-2} = \frac{(x-2) - (x+2)}{(x+2)(x-2)} = \frac{-4}{x^2 - 4} \Rightarrow D$

20.B ***Working:*** Let $y = x\sqrt{x} + \frac{2}{x^2}$; $y = x.x^{1/2} + 2x^{-2} = x^{3/2} + 2x^{-2}$

$\frac{dy}{dx} = \frac{3x^{1/2}}{2} - 4x^{-3} = \frac{3\sqrt{x}}{2} - \frac{4}{x^3}$ => B

21.B ***Working:*** $x^2y^2 - 2x = 4 - 4y$

i.e $2xy^2 + 2x^2y\frac{dy}{dx} - 2x = -4\frac{dy}{dx}$; $2x^2y\frac{dy}{dx} + 4\frac{dy}{dx} = 2 - 2xy^2$

$\frac{dy}{dx}(2x^2y + 4) = 2 - 2xy^2 = \frac{2 - 2xy^2}{2x^2y + 4} = \frac{1 - xy^2}{x^2y + 2}$

At the point (2, -2) ; $\frac{dy}{dx} = \frac{1 - 2(-2)^2}{(2)^2(-2) + 2} = \frac{7}{6} = m = \text{slope}$

Apply $y - y_1 = m(x - x_1)$; But $x_1 = 2, y_1 = -2$

$y + 2 = \frac{7}{6}(x - 2)$; $6(y + 2) = 7(x - 2)$; .: $7x - 6y - 26 = 0$

22.B ***Working:*** Generally, the domain of the function are all possible values of x that will make the function $\frac{x+5}{x^2-2}$ to be defined.

So when $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

thus, the domain of the function now are all real numbers except $\pm\sqrt{2}$, which can be express mathematically as $R\backslash[\pm\sqrt{2}]$. **Option B is Correct.**

23.C ***Working:*** Observe $\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$

At $x = 1$, we have $\frac{1^2 + 1 - 2}{1 - 1} = \frac{0}{0}$ = Indeterminate

Apply L' Hospital rule, we have $\frac{2x + 1}{1}$

At $x = 1$, we have $\frac{2(1) + 1}{1} = 3 \Rightarrow C$

24.C ***Working:*** Let $y = 2x^{x-1}$; Take ln of both sides

$\ln y = \ln 2 + \ln x^{x-1}$;

$\ln y = \ln 2 + (x-1)\ln x$; Differentiating both sides

$$\frac{1}{y}\frac{dy}{dx} = 0 + \left[(x-1).\frac{1}{x} + \ln x.(1)\right] = \left[\frac{x-1}{x} + \ln x\right]$$

$$\frac{dy}{dx} = y\left[\frac{x-1}{x} + \ln x\right] = 2x^{x-1}\left[\frac{x-1}{x} + \ln x\right] \Rightarrow C$$

25.A ***Working:*** Observe $\lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$

At $x = 2$, we have $\frac{\sqrt{2} - \sqrt{2}}{2 - 2} = \frac{0}{0}$ = Indeterminate

Next, we apply L'Hospital rule ; i.e $\frac{\frac{1}{2}x^{-\frac{1}{2}}}{1} = \frac{1}{2\sqrt{x}}$

So, we put $x = 2$: i.e $\frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \Rightarrow A$

26.A ***Working:*** $f(x) = 2x$; $g(x) = x - 5$; $h(x) = \sin x$

thus, $h(f(g(x))) = h(f(x-5)) = h(2(x-5)) = h(2x-10)$

$= \sin(2x - 10) \Rightarrow A$

27.B ***Working:*** Observe $e^{2x+y} = xy$

Differentiating, $\left(2 + \frac{1dy}{dx}\right)(e^{2x+y}) = x\frac{dy}{dx} + y$

$$2e^{3x+y} + \frac{dy}{dx}e^{3x+y} = x\frac{dy}{dx} + y$$

$$\frac{dy}{dx}e^{3x+y} - x\frac{dy}{dx} = y - 2e^{3x+y}\ ;\ \frac{dy}{dx}(e^{3x+y} - x) = y - 2e^{3x+y}$$

$$\therefore \frac{dy}{dx} = \frac{y - 2e^{3x+y}}{e^{3x+y} - x} = (y - 2e^{3x+y})/(e^{3x+y} - x)$$

28.D ***Working:*** Observe $x\cos y + y\sin x = \pi/2$

By implicit differentiation we have

$$1.\cos y + (-x\sin y)\frac{dy}{dx} + \sin x\frac{dy}{dx} + y\cos x = 0$$

$$\cos y - x\sin y\frac{dy}{dx} + \sin x\frac{dy}{dx} + y\cos x = 0$$

Collect like terms of dy/dx

$$\text{i.e } \cos y + y\cos x = x\sin y\frac{dy}{dx} - \sin x\frac{dy}{dx}$$

$$\cos y + y\cos x = \frac{dy}{dx}(x\sin y - \sin x)$$

$$\frac{dy}{dx}(x\sin y - \sin x) = \cos y + y\cos x$$

$$\frac{dy}{dx} = \frac{\cos y + y\cos x}{x\sin y - \sin x}$$

thus, $\left.\frac{dy}{dx}\right|(\pi/4, 0) = \frac{\cos 0 + \cos\pi/4}{\pi/4\sin 0 - \sin\pi/4} = \frac{1}{-\sin 45}$

$$\frac{dy}{dx} = \frac{-1}{1/\sqrt{2}} = -\sqrt{2}$$

For normal, $m_1m_2 = -1$; $\Rightarrow m_2 = \frac{-1}{m_1} = \frac{-1}{-\sqrt{2}} = \frac{1}{\sqrt{2}}$

Hence, the required equation is $y - y_1 = m_2(x - x_1)$

Where $x_1 = \pi/4$; $y_1 = 0$; $m_2 = 1/\sqrt{2}$

thus, $y - 0 = \frac{1}{\sqrt{2}}(x - \pi/4)$; $y = \frac{1}{\sqrt{2}}(x - \pi/4)$

Rationalizing, we have $y = \frac{\sqrt{2}}{2}(x - \pi/4)$

i.e $2y = \sqrt{2}(x - \pi/4)$; $2y - x\sqrt{2} - \pi/4(\sqrt{2}) = 0$

.: $2y - x\sqrt{2} + \pi/4(\sqrt{2}) = 0 \Rightarrow$ D

29.A ***Working:*** $f(x) = (x/3) + 1$; Let $y = f(x)$; $\Rightarrow y = (x/3) + 1$

thus, $y - 1 = \frac{x}{3}$

To get the inverse of f, make x the subject formula. i.e

$x = 3(y - 1)$

.: the inverse of $f = f^{-1}(x) = 3(x - 1) \Rightarrow$ A

NB: We simply replace y with x to get $3(x-1)$ (the final answer)

30.E ***Working:*** Observe $2(x^2 + 1) + 2(y^2 - 2) + 8 = 0$

Divide through by 2 ; $(x^2 + 1) + (y^2 - 2) + 4 = 0$

thus, we have $x^2 + y^2 + 3 = 0$

i.e $x^2 + y^2 = -3$ ------------ (1)

But the equation of a circle when the centre is at the origin (0, 0) is $x^2 + y^2 = r^2$ ---------- (2)

Compare equation (1) and (2), we have $r^2 = \sqrt{-3} = \sqrt{3}i$ and the centre = (0, 0).

.: ***Answer*** = (0, 0) & $\sqrt{3}i$; **Option E is Correct.**

31. ***Working:*** Observe $y = -2\sin^2 x + \cos 2x$

By shot cut $\frac{dy}{dx} = -4\sin x\cos x - 2\sin 2x$

$\therefore \frac{dy}{dx} = -2(2\sin x\cos x + \sin 2x)$ => E

32.D ***Working:*** Observe $g(x - 4) = 2x^2 + x - 1$

We let $g(x - 4) = g(0)$

i.e $x - 4 = 0$

$x = 4$

thus, at $g(0)$, $x = 4$

Now, put $x = 4$ into $2x^2 + x - 1$

i.e $g(0) = 2(4)^2 + 4 - 1 = 2(16) + 3 = 32 + 3 = 35$ => D