Solution to the test Questions

MATT 111

(1) $\frac{6x^2+17x+6}{x(x+2)(x+3)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+3}$

Find $A+B-C$

Solu

$$\frac{6x^2+17x+6}{x(x+1)(x+3)} = \frac{A(x+2)(x+3) + Bx(x+1) + Cx(x+2)}{x(x+2)(x+3)}$$

$$6x^2+17x+6 = A(x+2)(x+3) + Bx(x+3) + Cx(x+2)$$

* Put $x=-2$.

$$6(-2)^2+17(-2)+6 = A(0)(0+1) + B(-2)(-2+3) + C(0)$$

$$24-34+6 = B(-2)(-1)$$

$$-4 = 2B \qquad B = -2$$

* Put $x=0$.

$$6(0)^2+17(0)+6 = A(2)(3) + B(0)(3) + C(0)(2)$$

$$6 = 6A$$

$$A = 1$$

* Put $x=-3$.

$$6(-3)^2+17(-3)+6 = A(-3+2)(-3+3) + B(-3)(-3+3) + C(-3)(-3+2)$$

$$54-51+6 = 3C$$

$$9 = 3C$$

$$C = 3$$

$$A+B-C = 1+(-2)-3 \Rightarrow 1-2-3$$

$$\text{Ans} = -4$$

Question 2

$\frac{1}{n(n^2-1)}$ Can also be expressed as

$$\frac{1}{n(n^2-1)} = \frac{1}{n(n^2-1^2)} = \frac{1}{n(n+1)(n-1)}$$

$$\frac{1}{n(n+1)(n-1)} = \frac{A}{n} + \frac{B}{(n+1)} + \frac{C}{(n-1)}$$

$$\frac{1}{n(n+1)(n-1)} = \frac{A(n+1)(n-1) + B(n)(n-1) + C(n)(n+1)}{n(n+1)(n-1)}$$

$$1 = A(n+1)(n-1) + B(n)(n-1) + C(n)(n+1)$$

Put $n = 0$

$$1 = A(1)(-1) + 0 + 0$$

$$-A = 1 \Rightarrow A = -1$$

Put $n = 1$ — (1)

$$1 = A(1+1)(1-1) + B(1)(1-1) + C(1)(1+1)$$

$$1 = 0 + 0 + 2C$$

$$2C = 1 \Rightarrow C = \frac{1}{2} \quad — (2)$$

Put $n = -1$

$$1 = A(-1+1)(-1-1) + B(-1)(-1-1) + C(-1)(-1+1)$$

$$1 = 0 + 2B + 0$$

$$2B = 1 \Rightarrow B = \frac{1}{2}$$

$$\frac{1}{n(n^2-1)} = \frac{-1}{n} + \frac{1}{2(n+1)} + \frac{1}{2(n-1)}$$

Question 3

Find the 6th term in the expansion $(3x - 2y)^7$

From general formula

$${}^nC_r\, a^{n-r} b^r$$

at 6th term $r = 5$

$${}^7C_5 (3x)^{7-5} (-2y)^5$$

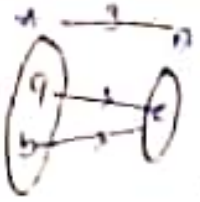
$${}^7C_5 (3x)^2 (-2y)^5$$

$$126 \times 81 x^4 \times (-32) y^5$$

$$-326592\, x^4 y^5$$

Question 4

A mapping $g: A \to B$ in which all element of domain of g are mapped into a single element in the co-domain is called



Onto mapping

Question 5

If Set $F = \{x : 2 < x < 12\}$ list the Members of F

answer

$F = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$

Question 6

What is the Coefficient of the 4th term in the binomial Expansion $(x+2)^4$

Solu

Recall that ${}^nC_r\, a^{n-r}\, b^r$

For forth term $r = 3$

${}^4C_3 (x)^{4-3} (2)^3$

$4 \cdot x \cdot 8 \Rightarrow 32x$.

So Coefficient is 32 only note - 32 only not 32x pls.

Question 7

Evaluate the factorial $\frac{(n+2)!}{n!}$

Solu

$\frac{(n+2)(n+1)(n)!}{n!} = (n+2)(n+1)$

$n^2 + n + 2n + 2 \Rightarrow n^2 + 3n + 2$

Question 8

Evaluate the 5th term of the Binomial $(2x - y)^5$

Solu

4th term has $r = 4$

${}^5C_4 (2x)^{5-4} (-y)^4$

${}^5C_4 (2x)^1 (-y)^4$.

$5 \times 2x \times y^4 \Rightarrow 10xy^4$

Question 9

Find the 4th Term in the Binomial $(x + \frac{1}{2})^8$

Solu

4th term $r = 3$.

${}^8C_3 (x)^{5-3} (\frac{1}{2})^3$

$56 x^5 \times \frac{1}{8} \Rightarrow 7x^5$

Question 10

find the 7th term of $(a+b)^{10}$

Solu

${}^{10}C_6 (a)^4 (b)^6 \Rightarrow a^4 \times 8 \times b^6$

$= 8a^4b^6$

Question 11

The fraction $\frac{2x^3+3x^2-54x+50}{x^2+2x-24}$ is equals ____

Solution

To solve the above by resance.

$$x^2+2x-24 \;\overline{)\; 2x^3+3x^2-54x+50 \;}\; 2x-1$$

$$2x^3+4x^2-48x$$

$$-x^2-6x+50$$

$$-x^2-2x+24$$

$$-4x+26$$

$$2x-1+\frac{-4x+26}{x^2+2x-24}$$

$$(2x-1)+\frac{-4x+26}{x^2+2x-24}$$

Because we are not asked to solve the partial fraction we are just asked to expressed it in another form

Question 12

Express $\frac{10-2x}{(x-3)(x+1)}$ in partial fraction.

$$\frac{10-2x}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

$$\frac{10-2x}{(x-3)(x+1)} = \frac{A(x+1)+B(x-3)}{(x-3)(x+1)}$$

$$10-2x = A(x+1)+B(x-3)$$

Put $x=1$

$$10-2(1) = A(2)+B(1-3)$$

$$8 = -2B \rightarrow B = -4$$

Put $x=3$.

$$10-2(3) = A(3+1)+B(3-3)$$

$$4 = 2A \quad A = 2$$

$$\frac{10-2x}{(x-3)(x+1)} = \frac{2}{(x-3)} + \frac{(-4)}{(x+1)}$$

$$\frac{10-2x}{(x-3)(x+1)} = \frac{2}{(x-3)} - \frac{4}{(x+1)}$$

Question 13

Resolve into partial fraction

$$\frac{4x-13}{(x+5)(x-7)} = \frac{A}{(x+5)} + \frac{B}{(x-7)}$$

$$\frac{4x-13}{(x+5)(x-7)} = \frac{A(x-7)+B(x+5)}{(x+5)(x-7)}$$

$$4x-13 = A(x-7)+B(x+5)$$

Put $x = 7$

$$4(7)-13 = A(0)+B(7+5)$$

$$15 = 15B \qquad B = \frac{15}{15} = 1$$

Put $x = -5$

$$4(-5)-13 = A(-5-7)+B(0)$$

$$-45 = -15A$$

$$A = \frac{-45}{-15} = 3$$

$$A = 3$$

$$\frac{4x-13}{(x+5)(x-7)} = \frac{3}{(x+5)} + \frac{1}{(x-7)}$$

Question 14

If $X \cap Y = \phi$ then X and Y are said to be Disjoint

Disjoint

Question 15

If set P = {even numbers} and $Q = \{x : 7 < x \leq 16\}$ list the element of $P \cap Q$

$P = \{2, 4, 6, 8, 10, 12, \ldots\}$

$Q = \{8, 9, 10, 11, 12, 13, 14, 15, 16\}$

$P \cap Q = \{8, 10, 12, 14, 16\}$

Question

Given Set $x = \{1, 2, 3\}$ and Set $Y = \{3, 1, 2\}$ which of the following statement is true for x and Y (a) $x \neq y$ (b) $x \equiv y$ (c) $x < y$ (d) $x = y$ (e) $x \geq y$

Option d is the Correct Option.

Question 18

Evaluate the 5th term of $(1+x)^n$

Solution

From de Moivre

$T_{r+1} = {}^nC_r\, a^{n-r} b^r$

at 5th term $r = 4$

${}^nC_4\,(1)^{n-4}(x)^4$

${}^nC_4\, x^4$

$\frac{n!}{(n-4)!4!}\, x^4$

$\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!\,4!}\, x^4$

$\frac{n(n-1)(n-2)(n-3)}{4!}\, x^4$

recall that $n = \frac{1}{n}$ from the question

$\frac{\frac{1}{n}\left(\frac{1}{n}-1\right)\left(\frac{1}{n}-2\right)\left(\frac{1}{n}-3\right)}{4!}\, x^4$

Guys check this Question if it is really $\frac{1}{n}$ as Exponent.

Question 18

Determine the domain D of the mapping $f: x \to 2x-3$ if $C = [-1, -1.5]$ is the range and f is defined on D

Solution

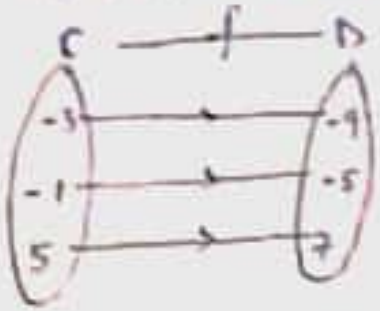
Put -3 into $2x-3$.

$f(x) = 2x - 3$

$f(-3) = 2(-1) - 3 = -9$

$f(-1) = 2(-1) - 3 = -5$

$f(5) = 2(5) - 3 = 7$



$f: C \to D$

Question 19

In the expansion of $(1+5x)^2$ by Binomial theorem the coefficient of x^2

Solution

$T_{r+1} = {}^nC_r\, a^{n-r} b^r \Rightarrow$ at x^2 $r = 2$

${}^2C_2\,(1)^0(5x)^2 \Rightarrow 1 \times 1 \times 25x^2$

$25x^2$

the coefficient of $x^2 = 25$

thus 25

Question 20

Let $f : X \longrightarrow Y$ be a mapping if every element of the Co-domain is an Image of at least one element in the domain. the mapping is Called

Soln

is Called a One to One mapping.