UNIVERSITY OF NIGERIA, NSUKKA.
Faculty of Physical Sciences
Department of Mathematics.
2017/2018 First semester Examination
MTH 211: Sets, Logic and Algebra.

INSTUCTIONS: Answer Question 1 and any other Two questions.
TIME: 2 HOURS

1. (a) Given that U={1,2,...,6} X={x: $x^2-5x+6=0$}, Y={x: $x^2-6x+9=0$} and Z={1,2,3,5}. Find (i) X∩Y (ii) X∩Z (iii) X∪Y (iv) Y∪Z (v) N(X∪Y) (vi) N{Y∪Z) (vii) XΔY (viii) X' ∪ Y' (ix)Z' (x) p(Y).
(b) (i)When is a relation R on a non empty set X said to be an equivalence relation?
(ii) Let X = {1,2,3} and a relation R on X be defined as R={(1,1),(1,2),(2,1),(2,2),(3,3)}. Verify if R is an equivalence relation.
2. Let (X,*) and (Y,+) be two groups. What do you understand by saying that a function f:(X,*)→(Y,+) is a homeomorphism?
(b) When do we say that a homeomorphism is (i) monomorphism (ii) isomorphism?
(c) Let f:(R,+)→(R\{0},*) be defined by f(n)=3^n. Show that f is a monomorphism but not isomorphism.
(d). (i) When do we say that (X,*) is a groupoid?
(ii) Let S=R\{1} and define a binary operation * on S as follows:
x*y=x+y+xy for all x,y∈S. Show that (S,*) is a groupoid and find the identity element of (S,*).
3. Consider the set S={a,b,c,d,e,f} and the relation * defined on S by the table below;

*	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d	f	c
b	b	e	a	f	c	d
c	c	f	d	e	b	a
d	d	c	f	a	e	b
f	f	d	c	b	a	e

(a). Find (i) the identity element of the algebraic system A,S. (ii) the inverse of each element of the 3set S. (iii) show that the above A,S is associative.
(b). The above A,S is a group. Justify.
(c). What do you understand by the statement "the operation * is distributive over the operation of + on the set X?
4. Let R={u, v, w, x} and define addition + and multiplication * on R by means of the following tables;

+	u	v	w	x
u	u	v	w	x
v	v	u	x	w
w	w	x	u	v
x	x	w	v	u

*	u	v	w	x
u	u	u	u	u
v	u	v	w	x
w	u	w	w	u
x	u	x	u	x

(a). (R,+,*) is a ring. Justify.
(b). What is the unity of the ring?
(c). What is the zero of the ring?
(d). If a binary operation * on a set of real numbers is defined by x*y= ½(x+y), show that this binary operation (*) is commutative but not associative on R.

5. (a). What do you understand by a word "proposition"?
(b). Consider P,Q,R,S as four propositional variables. Find , in a tabular form, all the possible truth combinations of the above variables.
(c). Use truth table to show that $P \cup (Q \cap R) \Leftrightarrow (P \cup Q) \cap (P \cup R)$ is a tautology.
(d). Let B be the set of mappings from $X \rightarrow X$, and n be a fixed non negative integer. If f and g are in B , define a relation R on B as

$$f \; R \; g \quad \text{iff} \quad \lim_{t \to 0} \frac{f(t) - g(t)}{t^n} = 0$$

Show that R is an equivalence relation on B.