Physics Majors Tutorial Class

Stepical Solutions to Phy 114 Tutorials

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Miss Odunayo Fowler

[illegible]

(1) If the distance x covered by a body in time t is given by $x = ut + \frac{1}{2}at^2$: find the velocity $v = \frac{dx}{dt}$ if u and a are constants.

Solution:

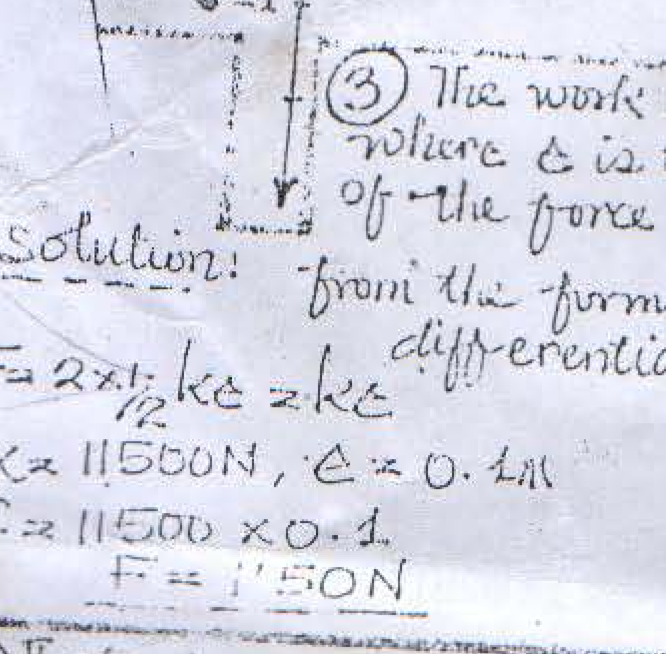
$V = \frac{dx}{dt} = U + at$, $a = \frac{d^2x}{dt^2} = \frac{dV}{dt}$.

(2) Sketch the function $y = 5x^2 + 10x + 4$ on a graph and find the slopes of the graph at points $x = 0$ and $x = 2$, from the graph and by differentiation. Find the point at which the slope is zero.

Solution: To find the gradient of the slope we use the differentiation of the function. $-3 \leq x \leq 3$

$y = 5x^2 + 10x + 4$

x	-3	-2	-1	0	1	2	3
y	19	4	-1	4	19	44	79



$\frac{dy}{dx} = 10x + 10$

$\therefore$ The gradient at $x = 0 \Rightarrow 10(0) + 10 = 10$

" " " $x = 2 \Rightarrow 10(2) + 10 = 30$

(3) The work done by a force is given by $W = \frac{1}{2}k\epsilon^2$ where ϵ is the distance covered. Find the magnitude of the force at point $\epsilon = 10cm$ if $k = 11500N/m$ (dw = Fdx)

Solution: from the formula work = force × distance.

differentiating $dW = f \cdot dx$ where x is the distance.

$\left\{ \therefore f = \frac{dW}{dx} = \frac{dW}{d\epsilon} \right.$ given that ϵ is distance covered.

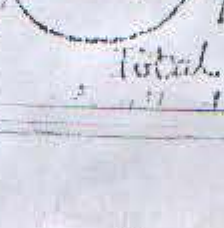
$F = 2 \times \frac{1}{2} k\epsilon = k\epsilon$

$K = 11500N$, $\epsilon = 0.1m$

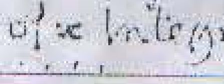
$f = 11500 \times 0.1$

$F = 1150N$

(4) The surface area of a sphere is $4\pi r^2$. By dividing the sphere into an infinite number of spheres each of very small thickness Δx when radius is x, show that the volume of the sphere of $\frac{dV}{dx}$



thickness = δy



Surface area $4\pi r^2$

dx

$V =$ vol of all tiny spheres

Total V of = integration of all vol

[illegible]

1) The velocity V of transverse wave along a stretched string depends on
(i) Its stress P i.e Force per unit area
(ii) the density ρ of the material
(iii) Its length. Find the actual equation
$(V = k\sqrt{P/\rho})$

Soln $V \propto P^x \rho^y l^z$

$\therefore V = k\, P^x \rho^y l^z$

$$P = \frac{\text{Force}}{\text{Area}} = \frac{M \times LT^{-2}}{L^2} = \left[\frac{\text{Mass} \times \text{acceleration}}{\text{Length} \times \text{Breath}}\right]$$
$$= ML^{-1}T^{-2}.$$

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = \frac{\text{Mass}}{\text{Length} \times \text{Breath} \times \text{height}}$$
$$= ML^{-3}.$$

Length $= L$

$$V = k\, P^x \rho^y l^z$$
$$LT^{-1} = [ML^{-1}T^{-2}]^x [ML^{-3}]^y [L]^z$$
$$LT^{-1} = M^x L^{-1x} T^{-2x} . M^y L^{-3y} . L^z$$
$$M^0 LT^{-1} = M^{(x+y)} . L^{-x-3y+z} . T^{-2x}$$

Equating Indices:

$M: \quad x + y = 0 \; \ldots\ldots\; (i)$

$T: \quad -2x = -1 \; \ldots\ldots\; (ii)$

$L: \quad -x - 3y + z = 1 \; \ldots\; (iii)$

From Equ (ii): $x = \frac{1}{2}$.

From (i): $\frac{1}{2} + y = 0 \Rightarrow y = -\frac{1}{2}$

From (iii): $-(\frac{1}{2}) - 3(-\frac{1}{2}) + z = 1$
$$-\tfrac{1}{2} + \tfrac{3}{2} + z = 1$$
$$\tfrac{2}{2} + z = \cancel{0} + 1$$
$$z = 1 - 1 = 0$$

$$\therefore V = k\, P^x . \rho^y l^z$$
$$V = k\, P^{1/2} . \rho^{-1/2} . l^0$$
$$V = k\frac{P^{1/2}}{\rho^{1/2}} = V = k\sqrt{P/\rho}$$

The force stretching a spring is proportional the extension produced. Find the dimension the constant of proportionality $(k = MT^{-2})$

: $F \propto e \Rightarrow F = ke$

$$\frac{F}{e} = \frac{\text{Mass} \times \text{acceleration}}{\text{extension}} = \frac{MT^{-2}L}{L}$$
$$= MT^{-2}$$

3) The volume of liquid flowing per unit time depends on the co-efficient of viscosity η, radius r of the pipe and the pressure gradient P/l. Using their dimensions and noting that $\eta = \frac{Fl}{AV}$ where F is force, A is area and V is the Velocity, Find the expression for volume flowing per second $\left(\frac{kPr^4}{\eta l}\right)$.

Soln

$$\frac{V}{t} = \eta^x r^y (P/l)^z$$
$$\frac{V}{t} = k\, \eta^x r^y (P/l)^z$$

$$\eta = \frac{Fl}{AV} = \frac{\text{Force} \times \text{length}}{\text{Area} \times \text{Velocity}} = \frac{ML^2T^{-2}}{L^2 \times L T^{-2}}$$
$$= ML^{-1}T^{-1}$$

$$\frac{\text{Pressure}}{\text{Length}} = \frac{\text{Force}}{\text{Area}} \times \frac{1}{\text{Length}} = \frac{MLT^{-2}}{L^2} \times \frac{1}{L}$$
$$= \frac{MLT^{-2}}{L^3} = ML^{-2}T^{-2}$$

$$\frac{\text{Volume}}{\text{Time}} = \frac{V}{T} = \frac{L^3}{T} = L^3T^{-1}.$$

$$\frac{V}{t} = \eta^x \; r^y \; (P/l)^z$$
$$L^3T^{-1} = [ML^{-1}T^{-1}]^x [L]^y [ML^{-2}T^{-2}]^z$$
$$L^3T^{-1} = M^x L^{-1x} T^{-1x} . L^y . M^z L^{-2z} T^{-2z}$$
$$M^0 L^3 T^{-1} = M^{(x+z)} . L^{(-x+y+(-2z))} . T^{(-x+(-2z))}.$$

Equating Indices

$M: \quad x + z = 0 \; \ldots\ldots\ldots\ldots\; (i)$

$L: \quad -x + y - 2z = 3 \; \ldots\ldots\; (ii)$

$T: \quad -x - 2z = -1 \; \ldots\ldots\; (iii)$

From (i): $x = -z$

(iii): $-(-z) - 2z = -1$
$$z - 2z = -1$$
$$-z = -1 \qquad z = 1.$$

(i): $x + 1 = 0 \qquad x = -1.$

(ii): $-(-1) + y - 2z = 3$
$$1 + y - 2(1) = 3.$$
$$y - 1 = 3$$
$$y = 4$$

$$\frac{V}{t} = k\, \eta^{-1} r^4 (P/l)^1$$
$$\frac{V}{t} = \frac{kr^4P}{\eta l}$$

4. The Velocity of a physical system is given by
$$V = \sqrt{\frac{P + \frac{1}{n}}{x}}$$
where.

P is the pressure [illegible]

must have the dimensions with Pressure since they are additive. $(P + \frac{1}{\eta})$

$\therefore P \equiv \frac{1}{\eta}$.

$$P = \frac{Force}{Area} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2} = \frac{1}{\eta}$$

$$V^2 = \frac{P + \frac{1}{\eta}}{x} \Rightarrow x = \frac{P + \frac{1}{\eta}}{V^2}$$

$$x = \frac{ML^{-1}T^{-2}}{LT^{-1}.LT^{-1}} = \frac{ML^{-1}T^{-2}}{L^2T^{-2}} = ML^{-3}$$

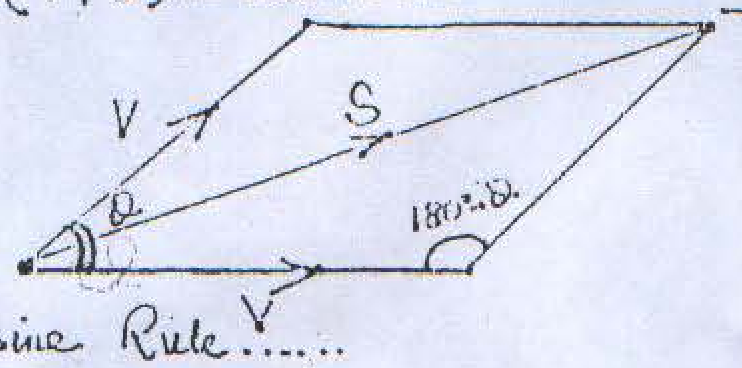
Note: $Velocity = \frac{Distance}{Time} = \frac{L}{T} = LT^{-1}$.

TUTORIAL 5

(1.) Prove that if 2 vectors have the same magnitude V and make an angle θ their sum has magnitude $S = 2V\cos\frac{1}{2}\theta$ and their difference is $D = 2V\sin\frac{1}{2}\theta$.

[Hint: $\cos(A+B) = \cos A\cos B - \sin A \sin B$]

Soln:



Using Cosine Rule.....

$$S^2 = V^2 + V^2 - 2 \times V \times V\cos(180° - \theta)$$
$$S^2 = 2V^2 - 2V^2\cos(180° - \theta)$$
$$S^2 = 2V^2 + 2V^2\cos\theta$$
$$S^2 = 2V^2(1 + \cos\theta)$$

Recall MAT121: $\left\{ \begin{array}{l} \sin^2\theta + \cos^2\theta = 1 \\ \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} \end{array} \right\}$

$$S^2 = 2V^2(\sin^2\theta + \cos^2\theta + \cos^2\tfrac{\theta}{2} - \sin^2\tfrac{\theta}{2})$$

$$S = \sqrt{2V^2 . 2\cos^2\theta/2} = 2V\cos\theta/2$$

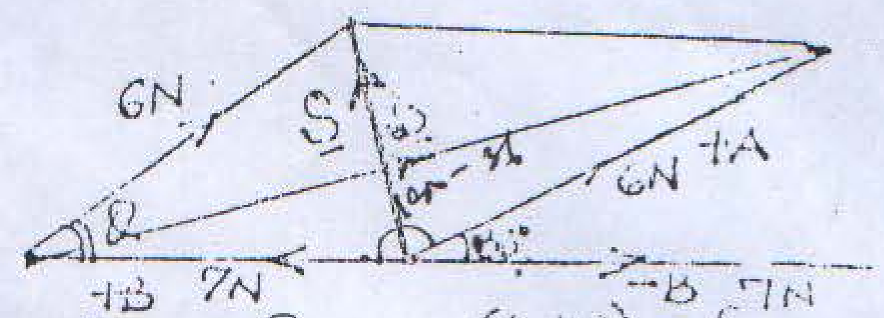
$$S = 2V\cos\theta/2$$

$$D^2 = 2V^2\left[\cos^2\theta/2 + \sin^2\theta/2\right] - \left[\cos^2\theta/2 - \sin^2\theta/2\right]$$
$$= 2V^2(\sin^2\theta/2 + \sin^2\theta/2)$$
$$D^2 = 2V^2 \times 2\sin^2\theta/2$$
$$D = \sqrt{2V^2 \times 2\sin^2\theta/2}$$
$$D = 2V\sin\theta/2$$

(2.) 2. Forces A and B are 6N at 36° to the positive x-axis and 7N along the negative x axis, respectively. Find A+B and A−B by both geometrical and analytical methods.

Soln Geometrical Method:

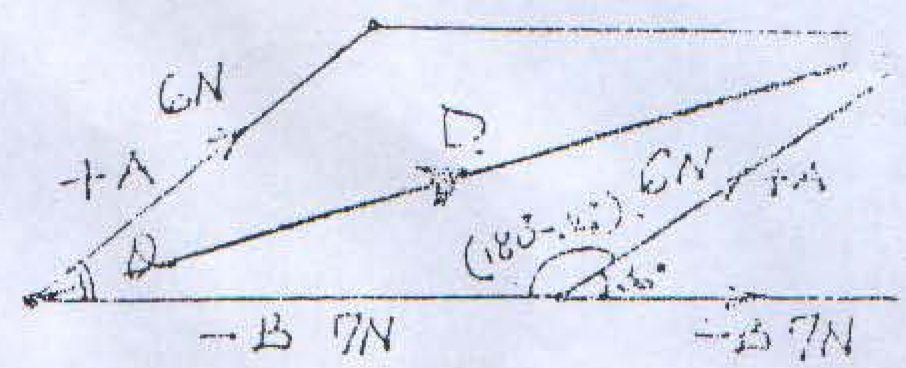
(a)



Using Cosine Rule: $(A+B) = S$

$$S^2 = 7^2 + 6^2 - 2 \times 7 \times 6\cos\theta(36)$$
$$S^2 = 49 + 36 - 84 \times 0.8090$$
$$S^2 = 85 - 84 \times 0.8090$$
$$S^2 = 85 - 67.957$$
$$S^2 = 17.04$$
$$S = \sqrt{17.04} \quad S = 4.127 \approx 4.13N$$

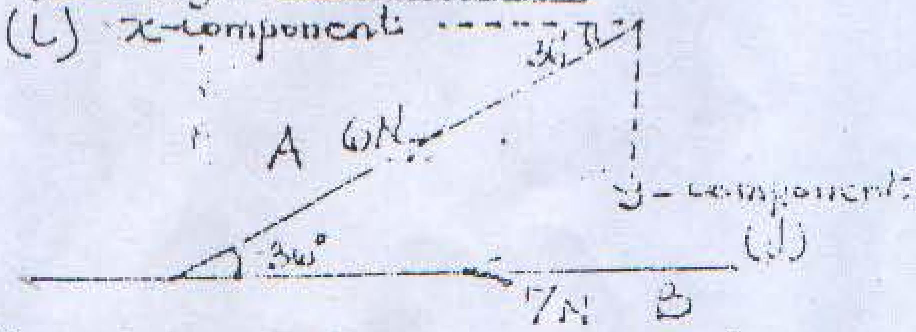
(b)



$$D = (A - B) = A + (-B)$$
$$D^2 = 7^2 + 6^2 - 2.7.6\cos\theta\,[144]$$
$$D^2 = 49 + 36 - 2.42 \times -\cos 0.8090$$
$$D^2 = 85 + 84 \times 0.8090$$
$$D^2 = 152.69. \quad D = \sqrt{152.69}$$
$$D = 12.367 \approx 12.37N$$

Analytical Method

(i) x-component



Resolution of the forces.

$A = (6\cos 36)i + (6\sin 36)j$

$B = (-7i)$ (No y component).

For Sum $(A+B) = S$.

$A + B = (6\cos 36)i + (6\sin 36)j$

$= (6 \times 0.8090)i + (6 \times 0.5878)j$

$A = (4.85)i + (3.52)j$

$B = \{-7\}i$

$= (-2.15)i + (3.52)j$

$$|\vec{A}+\vec{B}| = \sqrt{(2.15)^2+(3.53)^2}$$
$$= \sqrt{4.41 + 12.461}$$
$$= \sqrt{16.871}$$
$$S = 4.107 \simeq 4.12\,N.$$

For Difference $\underset{\sim}{D} = A - B$

$$A = (4.85)i + (3.53)j$$
$$-B = (-7i) + 0j$$
$$(11.85)i + (3.53)j$$

$$|A-B| = \sqrt{(11.85)^2+(3.53)^2}$$
$$= \sqrt{140.42 + 12.461}$$
$$= \sqrt{152.88}$$
$$= 12.365$$
$$\underset{\sim}{D} \simeq 12.37$$

(3.) Find the angle between $A = 2i+j$ and $B = -i+j$

Soln

$$\vec{a}\cdot\vec{b} = |a||b|\cos\theta$$
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|}$$
$$\vec{a}\cdot\vec{b} = (2i+j)(-i+j)$$
$$= -2+1 = -1.$$
$$|a| = \sqrt{4+1} = \sqrt{5}$$
$$|b| = \sqrt{1+1} = \sqrt{2}$$
$$\cos\theta = \frac{-1}{\sqrt{2}.\sqrt{5}} = \frac{-1}{1.4142\times 2.236}$$
$$\cos\theta = \frac{-1}{3.1623} = -0.31623$$
$$\theta = -\cos^{-1} 0.31623$$
$$180°-\theta = \cos^{-1} 0.31623$$
$$-\theta = +71.56 - 180$$
$$-\theta = -108.44$$
$$\theta = 108.44°$$

(4.) A force $F = 5i+3j$ acts on a body and changes its position from $P_1(4,5)$ to $P_2(-3,6)$ in a plane. Find the work done on the body if the force is in newtons and displacement is in metres.

Soln

$$\vec{P_2} - \vec{P_1} = \{-3i, 6j\} - \{4i, 5j\}$$
$$= \{-7i, j\}.$$

$$\text{Work} = F\cdot\{P_2 - P_1\}$$
$$= \{5i+3j\}\{-7i+j\}$$
$$= -35+3$$
$$= -32\,J$$

(5) Prove that for any three vectors A, B and C.

$$A\cdot(B\times C) = (A\times B)\cdot C$$

$C = C_1i + C_2j + C_3k$

$$B\times C = \begin{bmatrix} i & j & k \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} = i(B_2C_3 - B_3C_2) - j(B_1C_3 - B_3C_1) + k(B_1C_2 - B_2C_1)$$

$$\Rightarrow (A_1i + A_2j + A_3k)\cdot\left(i(B_2C_3 - B_3C_2) - j(B_1C_3 - B_3C_1) + k(B_1C_2 - B_2C_1)\right)$$
$$\Rightarrow A_1(B_2C_3 - B_3C_2) + A_2(B_3C_1 - B_1C_3) + A_3(B_1C_2 - B_2C_1)$$
$$\Rightarrow A_1B_2C_3 - A_1B_3C_2 + A_2B_3C_1 - A_2B_1C_3 + A_3B_1C_2 - A_3B_2C_1 \text{ ---- (i)}$$

Let $A = A_1i + A_2j + A_3k$.

$$A\times B \begin{bmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{bmatrix} = i(A_2B_3 - A_3B_2) - j(A_1B_3 - A_3B_1) + k(A_1B_2 - A_2B_1)$$

$$(C_1i + C_2j + C_3k)\cdot\left(i(A_2B_3 - A_3B_2) - j(A_1B_3 - A_3B_1) + k(A_1B_2 - A_2B_1)\right)$$
$$C_1(A_2B_3 - A_3B_2) + C_2(A_3B_1 - A_1B_3) + C_3(A_1B_2 - A_2B_1)$$
$$A_2B_3C_1 - A_3B_2C_1 + A_3B_1C_2 - A_1B_3C_2 + A_1B_2C_3 - A_2B_1C_3 \text{ ---- (ii)}$$

Compare equation (i) and (ii). They are equal in magnitude.

Q.E.D.

(6.) Two insects A and B fly in space with Uniform Velocities $U_A = i + 4j + 3k$ and $U_B = 4i + 2j - 4k$ in m/s with respect to a stationary observer at the origin (0,0,0). Show that the insects fly at right angle to each other and determine their distances apart after 5s.

Soln: $U_a = i + 4j + 3k$

$U_b = 4i + 2j - 4k$.

$\vec{Va} \cdot \vec{Ub} = |Va||Vb| \cos\theta$

$\cos\theta = \dfrac{\vec{Ua} \cdot \vec{Ub}}{|Va||Ub|}$

$\Rightarrow (4 \times 1)\, i \cdot i + (4 \times 2)\, j \cdot j + (-4 \times 3)\, k \cdot k$

$4 + 8 - 12 = 0$

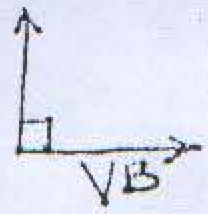
$|V_a||V_b| \therefore |Va| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26}$.

$|Ub| = \sqrt{4^2 + 2^2 + (-4)^2} = \sqrt{36}$

$\cos\theta = \dfrac{0}{\sqrt{26} \cdot \sqrt{36}} = 0$

$\theta = \cos^{-1} 0$

$= 90°$



Distance between them:

$U_b - U_a = [4i + 2j - 4k] - [i + 4j + 3k]$

$= 3i - 2j - 7k$

$|V_b - V_a| = \sqrt{(3)^2 + (-2)^2 + (-7)^2}$

$= \sqrt{9 + 4 + 47} = \sqrt{62} = 7.874\,ms^{-1}$

Distance = Velocity × Time

$= 7.874 \times 5$

$D = 39.37\,m$:

TUTORIAL 4:

(1) A bullet is fired vertically upwards with an initial Velocity of $98\,ms^{-1}$ from the top of a building 100m high find.

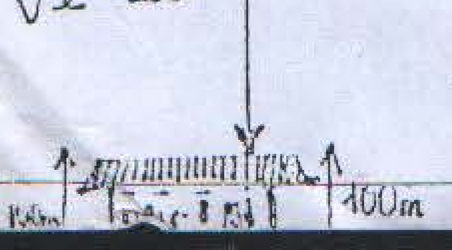
(i) The maximum height reached above the ground?

(ii) The Total time before reaching the ground?

(iii) The velocity on landing.

u=0 v=0

v u



Using $V^2 = U^2 + 2as$

$V = 0,\ a = -g,\ S = h$

$\therefore U^2 = -2gh$

$H = \dfrac{U^2}{2g} = \dfrac{96.04}{19.6}$

$= 490m$

Total height = (490 + 100)m = 590m

t_1 to reach 490m with $U = 98\,m/s$.

$V = U + at,\ V = 0,\ a = -g,\ U = -gt_1$

$t_1 = U/g = \dfrac{98}{9.8} = 10\,s$.

t_2 to reach the ground from a distance 590m with V.

$V = U + at$; $V^2 = U^2 + 2aS$. $a = g$

$U = 0$ at maximum height.

$V^2 = 0^2 + 2 \cdot 9.8 \cdot 590$

$V = \sqrt{1564} = 107.54\,m/s$

$V = U + at,\ V = 0 + gt$

$gt = V \equiv g\ V/t \Rightarrow t_2 = \dfrac{107.54}{9.8}$

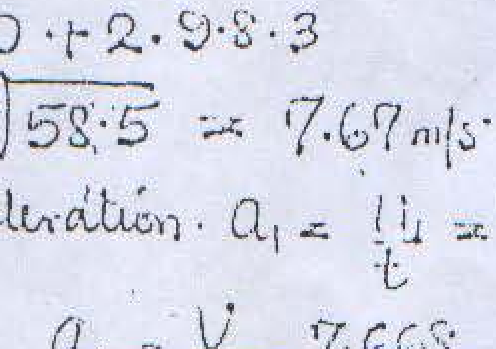
$t_2 = 10.97s$

The total time $= t_1 + t_2 = (10 + 10.97)s$

$= 20.97s$

(2) A Table tennis ball is dropped unto the floor from a height of 4m and rebounds to a height of 3m. If the time of contact with the floor is 0.01s, what is the magnitude and direction of the acceleration during the contact.

Soln



$V^2 = U^2 + 2aS$

$0 = U^2 - 2gS$

$U^2 = 2gS$

$\therefore U = \sqrt{2gS}$

$U = \sqrt{2 \cdot 9.8 \cdot 4}$

$= \sqrt{78.5}$

$= 8.85$

$V^2 = U^2 + 2gS$

$V^2 = 0 + 2 \cdot 9.8 \cdot 3$

$V = \sqrt{58.5} = 7.67\,m/s$

acceleration. $a_1 = \dfrac{U}{t} = \dfrac{8.854}{0.01} = 885.4\,m/s^2$

$a_2 = \dfrac{V}{t} = \dfrac{7.668}{0.01} = 766.8\,m/s^2$

$a = a_1 + a_2 = (885.4 + 7.668)\,m/s^2$

$= 1652.2\,m/s^{-2}$ ↑

(3) The first 3 runners in a 100m race were clocked 9.5s, 10.0s, 10.5s respectively. How far apart were the first and third runners when the first runner reached the finish line?

Soln.

The Speed of third runner $= \dfrac{Dist.}{Time}$

$= \dfrac{100}{10.5} = 9.52$

The distance the 3rd has covered when the 1st runner has reached finish line at 9.5s.

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

Speed = speed to run 100m
Distance = The distance he has covered at time = 9.5s.

$$\text{Distance} = 9.5238 \times 9.5$$
$$= 90.48\,m$$

at finish line time 9.5s their distance apart $= (100 - 90.48)m = \underline{9.52m}$ *

(1b) The equation of motion of a body is given by $x = 3 - 5t + 12t^2$
Find the velocity and acceleration at time $t = 5s$. Is this motion due to a steady force?

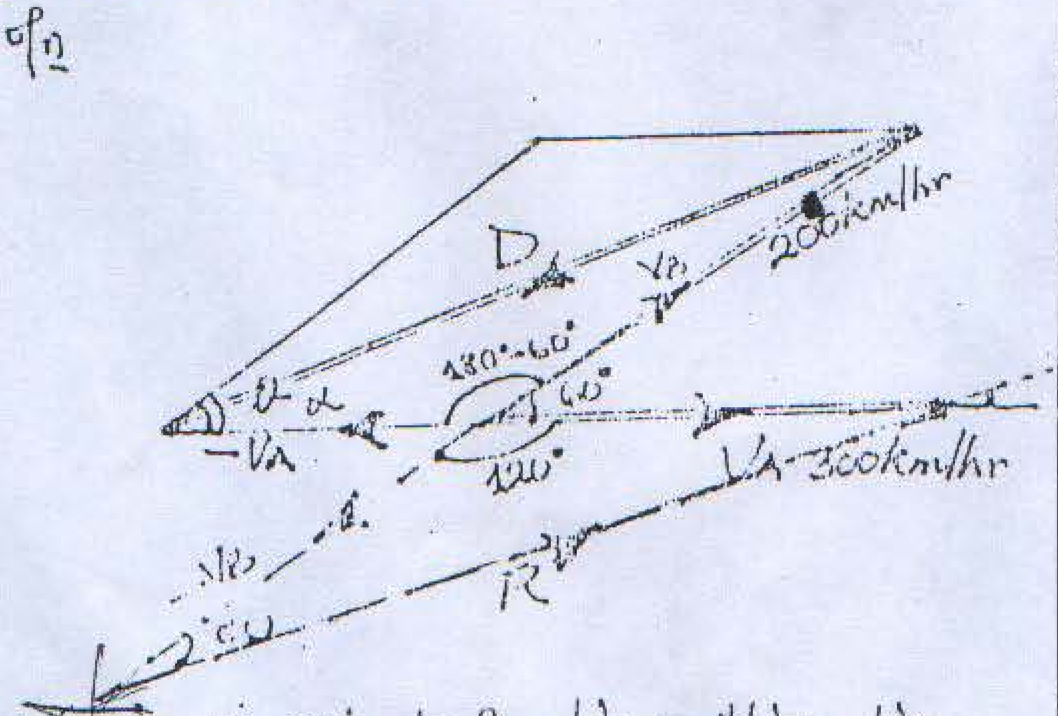
$$\text{Velocity} = \frac{dx}{dt} = -5 + 24t.$$
$$= -5 + 24(5)$$
$$V = -5 + 120 = \underline{115\,m/s}$$

$$\text{acceleration} = \frac{dV}{dt} = \frac{d^2x}{dt^2} = 24$$
$$a = \underline{24\,m/s^2}$$

No!, The motion is not due a steady force because there is a change of acceleration. $F = ma$ but $\underline{F = m\Delta a}$

(4). An airplane A flies due north at 300km/hr relative to the ground at the same time, another plane B flies at 200km/hr at 60°W of N, also relative to the ground find the velocity of (i) A relative to B (ii) B relative to A (ii) B relative to A.

Soln



(i) A relative to B. $V_{AB} = V_A - V_B$
$= V_A + (-V_B)$

$$R^2 = 200^2 + 300^2 - 2 \times 200 \times 300 \cos 60$$
$$= 40000 + 90000 - 120000 \times 0.5$$
$$= 130,000 - 60,000$$
$$= 70,000$$

$$R = \sqrt{70,000} = \underline{264.6\,km/hr}$$ *

~~for Direction Using Sine Rule~~

$$\frac{200}{\sin\infty} = \frac{264.6}{\sin 120} \Rightarrow \frac{200\sin 120}{264.6} = \sin$$

$$\infty = \sin^{-1}\frac{200\sin 120}{264.6} = \sin^{-1}\frac{173.205}{264.6}$$

$$\infty = \sin^{-1} 0.6546 = \underline{40.9°N \text{ of } W}$$

$$D^2 = 200^2 + 300^2 - 2 \times 200 \times 300 \times \cos 60$$
$$= 40000 + 90000 - 120,000 \times 0.5$$
$$= 130,000 - 60,000 = 70,000$$
$$D = \sqrt{70,000} = \underline{264.6\,km/hr}$$ *

For Direction $\frac{200}{\sin\alpha} = \frac{264.6}{\sin 120}$

$$\Rightarrow \frac{200\sin 120}{264.6} = \sin\alpha \Rightarrow \alpha = \sin^{-1}\frac{200\sin 120}{264.6}$$

$$\sin^{-1}\frac{173.205}{264.6}, \alpha = 0.6546$$

$$= \underline{40.9° \text{ S of } W}$$ *

(5) When a balloon is 1960m above the ground and rising at 98 m/s, a stone is thrown vertically out of the balloon which hits the ground below in 20s. What is the initial velocity of the stone
(i) relative to the ground
(ii) relative to the balloon.

Soln:

1. The ground is the frame of reference. The stone fell towards the ground it has an initial vel. of 98 m/s relative to the ground

$V_s - V_g = V_{sg}$
$= (98 - 0) = 98\,m/s.$

2. As at the altitude when the stone was released from the balloon, they where at that instant inline in the altitude. Therefore their velocities at that point is same and their relative velocity = 0
$V_{sB} = V_s - V_B = 0$

(6) The engine rotating a shaft is shut off when the angular speed of the shaft is 1,800 r.p.m. It stops rotating 15s later what is (i) the angular acceleration.
(ii) Angular displacement before coming to rest after the engine has been shut off.

Soln

$2\pi \, rad = 1 \, rev \Rightarrow 30.005^{-1} \times 2\pi \times 30$

$= 188.5 \, rads^{-1}$, $w = w_0 + \alpha t$

$\therefore w_0 = \alpha t$, $\alpha = \frac{-188.5}{15} = -12.57$

$\theta = w_0 t + \frac{1}{2}\alpha t^2$

$\theta = 188.5 \times 15 + \frac{1}{2} - 12.57 \times (15)^2$

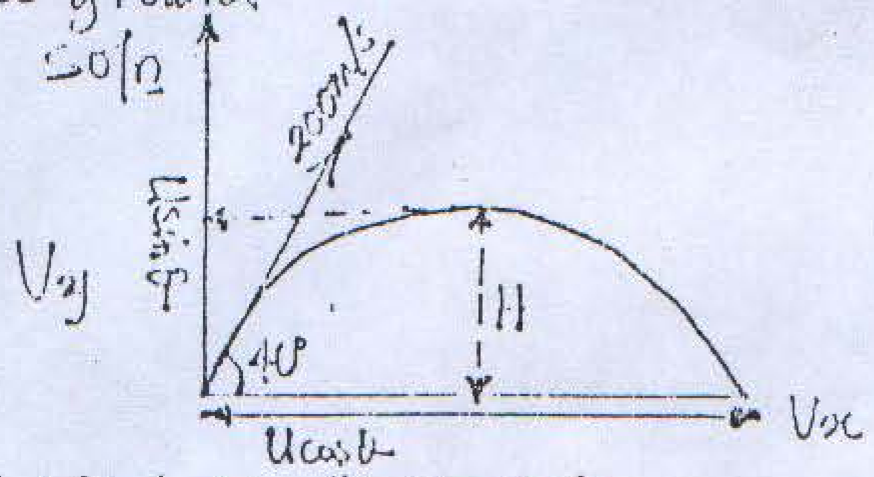
$= 2827.5 - 1414.125$

$= 1413.375 \, rad$

$= \underline{225.1 rev}$ *

(7) A gun fires a bullet with a velocity of 200m/s at an angle of 40° to the ground. find the velocity and position of the bullet after 20s. Find also the range and the time required for the bullet to return to the ground.

Soln



Velocity along the x-axis

$U_x = U\cos\theta + gt$ where $g = 0$

$= 200 \times \cos 40° = 200 \times 0.7660$

$= 153.21 \, m/s.$

$V_y = U\sin\theta - gt$, $-g$ ↑ against gravity

$V_y = 200 . \sin 40° - 9.8 \times 20$

$= 128.56 - 196$

$= -67.44.$

$U^2 = U_x^2 + U_y^2$

$U = \sqrt{(153.21)^2 + (-67.44)^2}$

$= \sqrt{4548.15 + 23472.96}$

$= \sqrt{28021.11}$

$= \underline{167.4 \, ms^{-1}}$

Displacement along x-axis

$x = Ut\cos\theta$ x = velocity × time along x.

$= 200 \times 20 \times \cos 40$

$4000 \times 0.7660 = 3064m.$

along Y-axis

$y = Ut\sin\theta + \frac{1}{2} - gt^2$

$y = 200 \times 20 \times \sin 40° - \frac{1}{2} \cdot 9.8 \times 2^2$

$= 2571.15 - 1960$

$R = \frac{U^2 \sin 2\theta}{g} = \frac{200^2 \sin 2 \times 40}{9.8}$

$= \underline{4020m}$ *

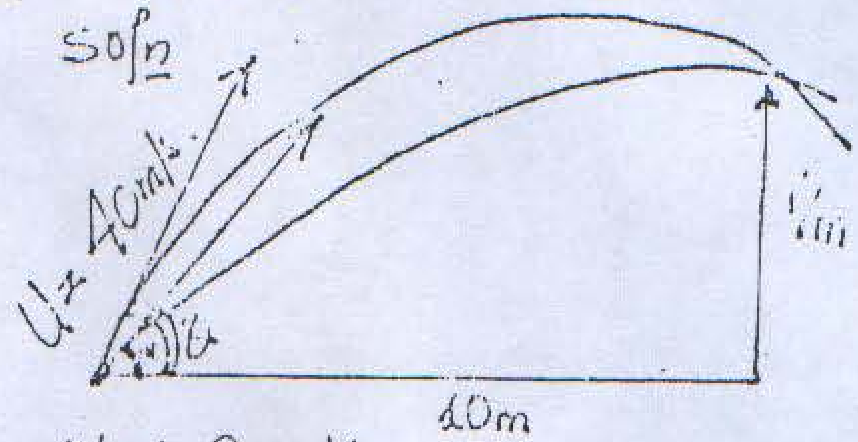
$T = \frac{2U\sin\theta}{g} = \frac{2 \times 200 \times \sin 40°}{9.8}$

$= \frac{257.115}{9.8} = \underline{26.24s}$ *

(8) A ball is thrown towards a building 10m away with a velocity of 40m/s. At what angle must it be thrown if it is to pass through a window 7m from the ground, neglecting wind effects.

[Hint: $\frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$]

Soln



$y = Ut\sin\theta - \frac{1}{2}gt^2$

$7 = 40t\sin\theta - \frac{1}{2} . 9.8 . t^2$

$7 = 40t\sin\theta - 4.9t^2 \; .... (i).$

x
$10 = U\cos\theta \times t$

$10 = 40\cos\theta \times t$

$t = \frac{10}{40\cos\theta} \; .... (ii)$

Substituting (ii) into (i)

$7 = 40 . \frac{10}{40\cos\theta} . \sin\theta - \frac{1}{2} . 9.8 \left\{\frac{10}{40\cos\theta}\right\}^2$

$= \frac{10\sin\theta}{\cos\theta} - 4.9 . \frac{1}{4\cos\theta} . \frac{1}{4\cos\theta}$

$= 10\tan\theta - \frac{4.9}{16\cos^2\theta}.$

$= 10\tan\theta - \frac{4.9}{16}\left[1 + \tan^2\theta\right] = \frac{1}{\cos^2\theta}$

$= 10\tan\theta - \frac{4.9}{16} - \frac{4.9}{16}\tan^2\theta$

$= 10\tan\theta - 0.306 - 0.306\tan^2\theta$

$7 + 0.306 = 10\tan\theta - 0.306\tan^2\theta$

$7.306 = 10\tan\theta - 0.306\tan^2\theta$

let x rep $\tan\theta$.

$7.306 = 10x - 0.306x^2$

$0.306x^2 - 10x + 7.306 = 0$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 0.306,\ b = -10,\ c = 7 + 0.306$
$c = 7.306$

$$x = \frac{10 \pm \sqrt{10^2 - 4 \times 7.306 \times 0.306}}{2 \cdot 0.306}$$

$$= \frac{10 \pm \sqrt{100 - 8.94}}{0.612}$$

$$\frac{10 \pm \sqrt{91.06}}{0.612} = \frac{10 + 9.54}{0.612} \text{ or } \frac{10 - 9.54}{0.612}$$

$$= \frac{19.54}{0.612} \text{ or } \frac{0.46}{0.612}$$

$$= 31.928 \text{ or } 0.751$$

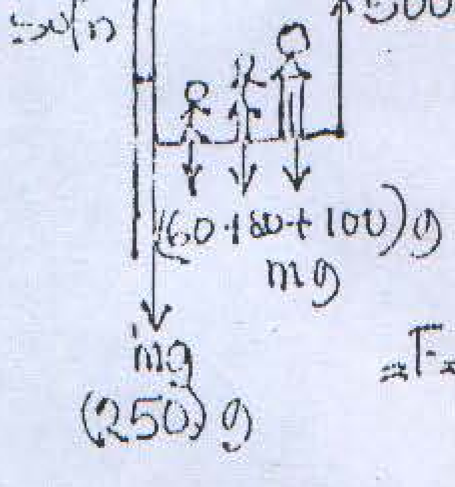
$\theta = \tan^{-1} 31.928 = 88.2° \simeq \underline{88°}$

$\theta = \tan^{-1} 0.751 = 36.9° \simeq \underline{37°}$

TUTORIAL 5

(1) An elevator of mass 250kg is carrying 3 persons whose masses are 60kg 80kg and 100kg and the force exerted by the motor is 5000N (a) with what acceleration will the elevator ascend ? (b) Starting from rest how far will it go in 5s.

Sol/n



Total mass downward
$(60 + 80 + 100 + 250)$
$= 490kg$

Downward Force ↓

$F = mg = (490)g$
$= 490 \times 9.8$
$= 4802 N = W$

Net Force $= ma$

$R - W = ma$

$5000 - 4802 = 490a$

$198 = 490a,\ a = \frac{198}{490}$

$= \underline{0.40 m/s}$

$S = ut + \frac{1}{2}at^2$ where $u = 0$ {from rest}

$S = \frac{1}{2} \times 0.404 \times 25$

$= 0.5 \times 10.10$

$= \underline{5.05m}$

(2) A block of mass 0.2kg starts up an inclined plane of 30° to the horizontal with a velocity of 12m/s. If the co-efficient of sliding friction is 0.16 (i) Determine how far up the plane the block travel before stopping. (ii) If the block returns ... [illegible]

$F = mg\sin\theta + Fr$

$mg\sin\theta = 0.2 \times 9.8 \times \sin 30$

$= 0.98N$

$Fr = \mu R \quad R = mg\cos\theta = 0.2 \times 9.8 \times \cos 30$
$= 1.699$

$\mu = 0.16$.

$Fr = 0.16 \times 1.699$
$= 0.27N$.

$F = 0.98 + 0.27N = 1.25N$.

$F = ma \Rightarrow a = \frac{F}{M} = \frac{1.25}{0.2}$

$a = 6.250 m/s$.

$V^2 = U^2 + 2as,\ -U^2 = -2as,\ \begin{matrix} a = - \\ V = 0 \end{matrix}$

$S = \frac{-U^2}{-2a} = \frac{12^2}{-2 \times 6.250} = \frac{144}{12.51} = \underline{11.5}$

When the body is coming down the friction force Fr is opposite to mg sinθ.

$F = 0.98 - 0.27 = 0.71N$

$\frac{F}{M} = a = \frac{0.71}{0.2} = 3.55 m/s^2$.

Distance travel $= 11.50$

$V^2 = U^2 + 2aS,\ V^2 = 2aS \quad u = 0$

$V = \sqrt{2 \times 3.55 \times 11.50}$

$= \sqrt{81.65} = 9.04 \quad V = 9.04 m/s$

(3) A body of mass 2kg is moving on a smooth horizontal surface under the action of a horizontal force $F = 55 + t^2$ (N). Calculate the velocity of the body at $t = 5s$, assuming the body was at rest at $t = 0$.

Sol/n $F = 55 + t^2$

$F = ma = \frac{m\delta v}{\delta t} = 55 + t^2$

$m\delta v = (55 + t^2)\delta t$

$m\int \delta v = \int_0^5 (55 + t^2)\, dt$

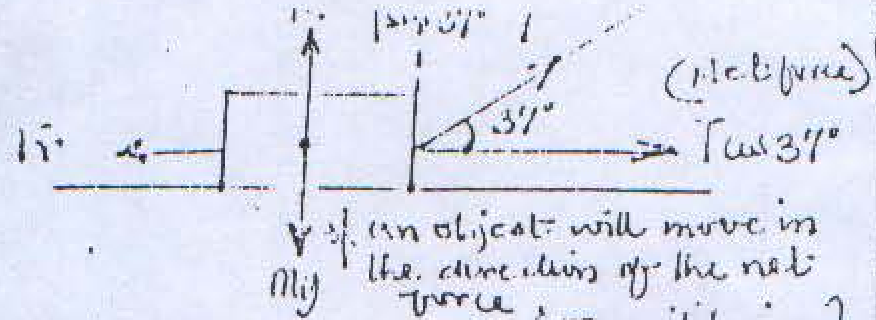
$mv = \left[55t + \frac{t^3}{3}\right]_0^5$

$mv = \left[55(5) + (5)^3/3\right] - \left[55(0)\right]$

$mv = 275 + 41.67 = 316.67$.

$V = \frac{316.67}{M} = \frac{316.67}{2} = \underline{158.3 m/s}$

(4) A rope inclined at angle 37° to the horizontal is used to drag a [illegible] along a level floor with an acceleration of 1m/s². The co-efficient of friction b/w the block and the floor is 0.2. What is the tension in the rope.



$R + T\sin 37 - mg = 0$ {Equilibrium}.

(Net force) $T\cos 37 - Fr = ma$

$R + 0.6018T - 50 \times 9.8 = 0$

$R + 0.6018T = 490$

$R = 490 - 0.6018T$ ......(1)

$T\cos 37 - Fr = ma \Rightarrow Fr = \mu R$

$T\cos 37 - \mu R = ma$

$0.7986T - 0.2R = 50 \times 1$

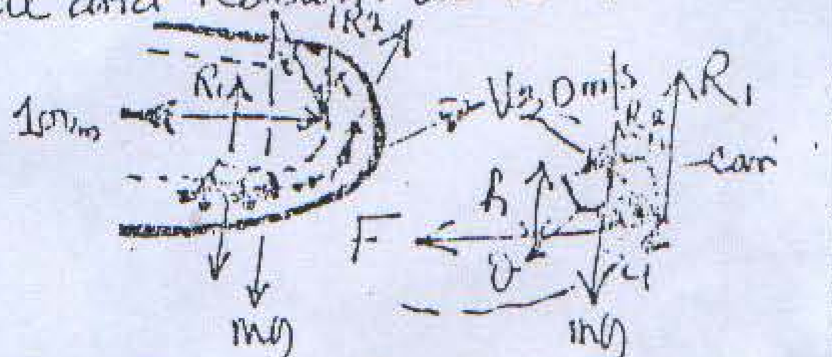
$0.7986T - 0.2(490 - 0.6018T) = 50$

$0.91896T = 148$

$T = \frac{148}{0.9186} = \underline{161.05N}$.

(5) A racing car 1000kg moves round a banked track at a constant speed of 30m/s. Assuming the total reaction at the wheels is normal to the track, and the horizontal radius is 100m, calculate the angle of inclination of the track to the horizontal and reaction at the wheels?

Sol/n



$(R_1 + R_2)\sin\theta = \frac{mV^2}{r}$ = Centripetal force

$(R_1 + R_2)\cos\theta = mg$ = gravitational force.

$\frac{(R_1+R_2)\sin\theta}{(R_1+R_2)\cos\theta} = \frac{mV^2}{r} \times \frac{1}{mg} = \frac{V^2}{gr}$

$\tan\theta = \frac{V^2}{gr}$ $\left\{ Ra = Fh = 0,\ \frac{a}{h} = \frac{F}{R} \right.$

$\frac{a}{h} = \tan\theta$, $R = mg$, $F = \frac{mV^2}{r}$

$\tan\theta = \frac{V^2}{gr} \Big\} = \frac{(30)^2}{100 \times 10} = \frac{9}{10}$

$\tan\theta = 0.9$

$\theta = \tan^{-1} 0.9 = 41.987°$

$= \underline{42°}$

$(R_1 + R_2)\sin\theta = \frac{mV^2}{r}$

$(R_1 + R_2)\sin 41.987 = \frac{1000 \times 30^2}{100}$

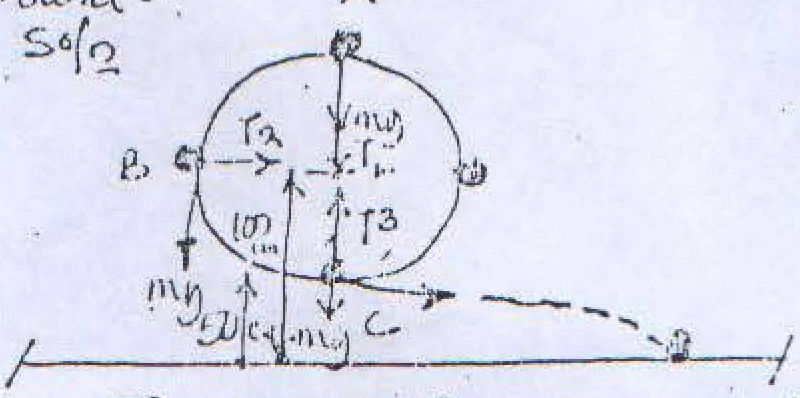
$(R_1 + R_2)\sin 41.987 = 9000$

$(R_1 + R_2) = \frac{9000}{\sin 41.987}$

9000 ...

(6) A stone of mass 0.5kg is attached to a string of length 50cm which will break when the tension in it exceeds 20N. The stone is whirled in vertical circle, the axis of rotation being at a height of 100cm above the ground. At what position and angular speed is the break most likely to occur? Where will the stone hit the ground.

Sol/n



$T_1 + mg = \frac{mV^2}{r}$ at A minimum

$F = T_2 = \frac{mV^2}{r}$ at B

$T_3 - mg = \frac{mV^2}{r}$ at C maximum

The stone will break off at C {overhead whirling}

$T - mg = \frac{mV^2}{r} \Rightarrow T - 0.5 \times 9.8 = \frac{0.5V}{0.5}$

$T - 4.9 = V^2 \Rightarrow 20 - 4.9 = V^2 = 15.1$

$V = \sqrt{15.1} = 3.89$ m/s.

$V = wr$, $W = V/r = \frac{3.89}{0.5} = \underline{7.78}$ rad/s

$S = ut + \frac{1}{2}at^2$

$S = \frac{1}{2}gt^2$

$0.5 = \frac{1}{2} \times 9.8t^2$

$0.5 = 4.9t^2 \Rightarrow t^2 = \frac{0.5}{4.9} \Rightarrow t = \sqrt{0.10}$

$t = 0.3194$.

$V = $ Displacement $\times$ time

$D = 3.89 \times 0.3194$

$= \underline{1.24m}$

TUTORIAL 6.

1.(a) The period of Jupiter is 11.88y. find its distance from the centre of the Sun if the distance b/w the earth and the sun is 1.49×10^8km. Sol/n

Using Keplers Law $T^2 = kR^3$

$k = \frac{4\pi^2}{GM}$ $T^2 = kR^3$ $t = 1s$

$1^2 = k(1.49 \times 10^8)^3$

$k = \frac{1}{R^3} = 3.02 \times 10$

$T^2 = kR^3$

$R^3 = \frac{T^2}{k} = \left(\frac{11.88}{3.02 \times 10^{-25}}\right)^{1/3}$

$= (7.758 \times 10^{11})$ m

$= 7.758 \times 10^8$ km

(1) A Russian satellite moves round the earth in an orbit at an altitude of about 800 km. Determine (i) The value of g at this altitude (ii) the linear velocity of the satellite and (iii) the period of the satellite.

g above earth surface

$mg' = \frac{GMm}{(r+R)^2}$ $g' = \frac{GM}{(r+R)^2}$

$GM = gr_E^2$ $g' = \frac{gr_E^2}{(r+R)} = \frac{9.8 \times (6.4\times10^6)^2}{(7.2\times10^6)^2}$

earth r

$= 7.74 m/s^2$

$V = \sqrt{R'g'} = \sqrt{7.2\times10^6 \times 7.74}$ $R' = (r+R)$

$= \sqrt{55758000} = \underline{7465 m/s}$

$W = \frac{2\pi}{T}$, $Wr = V$

$T = \frac{2\pi R'}{V} = \left\{\frac{2\times3.142\times7.2\times10^6}{7465.12}\right\}$

$= 6060.0455 = \frac{6060.04}{3600}$

$= \underline{1.65 hr}$

(2) Assuming that the radius of the earth is $6.38\times10^6 m$ and $G = 6.67\times10^{-11} Nm^2kg^{-2}$ find the mean density of the earth.

Soln Density $= \frac{mass}{Volume}$

Assume that earth is sperical

$V = \frac{4}{3}\pi r^3$

$mg = \frac{GMm}{r_E^2}$ $\Rightarrow M = \frac{gr_E^2}{G}$

$D = \frac{gr_E^2}{G} \times \frac{3}{4\pi r_E^3} = \frac{3g}{4\pi G r_E}$

$= \frac{3\times9.8}{4\times6.67\times10^{-11}\times3.142\times6.38\times10^6} = \frac{3\times9.8}{5.37\times10^{-3}}$

$= 5.5\times10^3 kg/m^3$

$\frac{3\times9.8\times10^3}{5.347}$

(3) Compare the value of acceleration due to gravity on the surface of mars with its value on the earth's surface assuming that the radii of mars and earth are the ratio 0.53:1 and their mean densities are in the ratio 0.71:1.

Soln $D = M/V$, $V = \frac{4}{3}\pi r^3$

$mg = \frac{GMm}{r^2} \Rightarrow \frac{gr^2}{G} = M$

$D = \frac{gr^2}{G} \times \frac{3}{4\pi r^3} \Rightarrow D = \frac{3g}{4G\pi r}$

on the surface of the mars let the gravity ... and density equal ...

unit $g' = \frac{4G\pi RD}{3}$ Surface let the ... gravity equal g and the density ... g and the density equal d.

$g = \frac{4G\pi rd}{3}$

$\frac{g'}{g} = \frac{4G\pi RD}{3} \times \frac{3}{4G\pi rd} = \frac{R}{r}\cdot\frac{D}{d}$

$= 0.53\times0.71 = 0.3763$

$\frac{g'}{g} = \underline{0.376:1}$

(3) A satellite is to revolve round the earth's surface in the equatorial plane find its period in hrs, if the mass of the earth is $5.95\times10^{24} kg$ and other constant areas in the text. Comment on the result

Soln

$\frac{2\pi R}{T} = V \Rightarrow V^2 = \frac{4\pi^2R^2}{T^2} = \frac{GM}{R}$

$T^2 = \frac{4\pi^2R^3}{GM}$

$T = \sqrt{\frac{4\times3.142^2\times(4.228\times10^7)^3}{6.67\times10^{-11}\times5.95\times10^{24}}}$

$= 86708 s \Rightarrow \underline{24 hr}$

(4) Assuming the Kepler's Laws, show that the acceleration of a planet is inversely proportional to the square of its distance from the Sun. Explain the significance of this and show how it leads to Newton's Law of Universal gravitation.

[Hint: $a = \frac{v^2}{r}$ and also $v = \frac{2\pi r}{T}$]

Soln

Centripetal force = Gravitational force

$\frac{GMm}{R^2} = mR\omega^2 \Rightarrow GM = R\cdot R^2\omega^2$

Since $r\omega^2 = a$ $\therefore GM = aR^2$

$a = \frac{GM}{R^2}$ {GM is constant}

Alternatively: $\frac{mv^2}{r} = mr\omega^2 \Rightarrow v^2 = r^2\omega^2$

$a = \frac{v^2}{r} \Rightarrow ar = v^2 = r^2\omega^2$

$\underline{a = r\omega^2}$

(5) Neglecting the gravitational effect of the sun and assuming the moon rotates the ... in a circular path 13 round the earth in a circular path 13 times a year, calculate the distance of the moon from the earth.

... $GMm = mR\omega^2$

$T = \left\{\frac{365.25}{13}\right\} \times 24 \times 60 ...$

$GM = R^3\omega^2$

$= 365$ days $\times 24 \times 60 \times 60$ ... $= 31557600s$.

$T = \frac{31557600}{18} = 1753200$

$$R = \sqrt[3]{\frac{g r_e^2 T^2}{4\pi^2}} = \sqrt[3]{\frac{9.8(6.4\times10^6)^2 \times (1753200)^2}{4\times3.142\times3.142}}$$

$$= \sqrt[3]{\frac{4.096\times10^3 \times 9.8\times3.07371024\times10^{12}}{39.478}}$$

$$= \sqrt[3]{\frac{4.096\times9.8\times3.07371024\times10^{25}}{39.478}}$$

$$= \sqrt[3]{\frac{123.38\times10^{25}}{39.478}} = \sqrt[3]{3.125\times10^{25}}$$

$= 314989836.5m = 3.15\times10^8 m$

$3.15\times10^5 km$

TUTORIAL 7.

(1) Sand falls at the rate of $0.15kgs^{-1}$ on a conveyor belt moving horizontally at a constant speed of $2m/s$. Calculate.

(i) the extra force necessary to maintain this speed

(ii) the rate at which work is done by this force

(iii) the change in K.E per second of the sand on the belt. Account for the diff. b/w (ii) & (iii)

Soln (i) $\frac{dm}{dt} = 0.15kgs^{-1}$ $V = 2m/s$.

$\frac{dm}{dt}\times V = 0.15\times2 = 0.3N$

(ii) work = force $\times$ distance or displacement $= 0.3\times2 = 0.6Js^{-1}$ or W

(iii) $KE = \frac{1}{2}mv^2 = \frac{1}{2}\times0.15\times2^2 = 0.3Js^{-1}$ or W

Out of the work done, some of the energy is used to overcome friction while the rest is used for the movement and that is why the work done is more than the kinetic energy of the sand.

(5) A 10^3 kg rocket is set vertically on its launching pad. The propellant is expelled at the rate of $2kgs^{-1}$. Find the min. velocity of the exhaust gases so that the rocket just begins to move.

Soln momentum balances

$V\frac{dm}{dt} = Mg = F$ {but $mv = gMt$}

$V = \frac{Mgt}{m} = \frac{10^3\times9.8\times1}{2} = 4900m/s$.

(2) Distinguish b/w conservative and non-conservative fields. A particle is subject to a force associated with the potential energy $E = 3x^2 - x^3$ where x is in cm from the origin. Find the position of min. force.

Soln.
Conservative fields are fields in which work is conserved at the points of work done and ... path taken ... while non-conservative ...

the work done by or upon 2 points ... on the path taken b/w the 2 points.

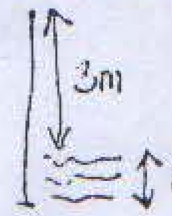
$mgh = 3x^2 - x^3$, $Fh = 3x^2 - x^3$

$F = \frac{3x^2 - x^3}{h} = \frac{3x^2}{h} - \frac{x^3}{h} = 3x - x^2$

when $x = 1$. $F = 3(1) - (1)^2$

$F = 3 - 1 = 2cm$

(3) A body of mass 0.10kg falls through a height of 3m onto a sand pile. If the body penetrates a distance of 3cm before stopping, what is the constant force exerted on the body by sand. It is a conservative field?



$V^2 = U^2 + 2gS$ $U = 0$

$V^2 = 2\times9.8\times3$

$V = \sqrt{58.8} = 7.668m/s$

Since the field is conservative.

$KE = PE = \frac{1}{2}mv^2 = mgh$

$mg = F \Rightarrow$ used by the sand to oppose the movement of the body.

$\frac{1}{2}\times0.7668 = Fh$

$F = \frac{\frac{1}{2}0.1\times7.668}{0.03} = \frac{2.94}{0.03} = 98$

(4) In an α-decay, a radioactive nucleus mass A units emits an α-particle of mass 4 units with an energy E_α. Show that total energy of disintegration $Q = E_\alpha\left(\frac{A}{A-4}\right)$. Calculate the energy of daughter nucleus in an α-decay of nuclide $A = 210$ with $E_\alpha = 5.5Mev$.

{Hint: Both energy and momentum are conserved}

M_α - mass of particle α, V_α = velo of α

M_o = mass of daughter nucleus

V_o = Vel " " "

Momentum $= M_\alpha V_\alpha + M_o V_o = 0$

$M_\alpha V_\alpha = -M_o V_o$ .... (1)

Energy : $\frac{1}{2}M_\alpha V_\alpha^2 + \frac{1}{2}M_o V_o^2$ = Total E ... -2

Squaring both sides

$M^2_\alpha V_\alpha^2 = M^2_o V_o^2 \Rightarrow M_o V_o^2 = \frac{M_\alpha^2 V_\alpha^2}{M_o}$ -(3)

Substitute (3) into (2)

$\frac{1}{2}M_\alpha V_\alpha^2 + \frac{\frac{1}{2}M_\alpha^2 V_\alpha^2}{M_o} = Q$

$\frac{1}{2}M_\alpha V_\alpha^2\left(1 + \frac{M_\alpha}{M_o}\right) = Q$

Let $E_\alpha = \frac{1}{2}M_\alpha V_\alpha^2 \Rightarrow E_\alpha = \left[1 + \frac{M_\alpha}{M_o}\right] = Q$

$\Rightarrow E_\alpha\left[\frac{M_o + M_\alpha}{M_o}\right] = Q$

where $M_\alpha = 4$ and $M_o = A - 4$

$\therefore Q = E_\alpha\left(\frac{A}{A-4}\right)$

$\boxed{Q = E_\alpha\frac{A}{A-4}}$

(5) a. A body of mass 20kg slides down a hill from an altitude of 20m. If its velocity is 16m/s at the bottom of the hill, Calculate the loss of energy due to friction:

$mgh - \frac{1}{2}mv^2 = E$

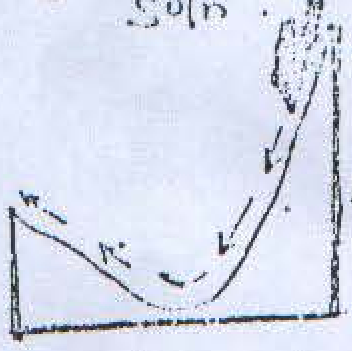
$20 \cdot 9.8 \times 2 \times 20 \times 16^2$

$3920 - 2560 = E$

$E = 1360J$

(b) A car starting from rest travels 300m down a 1% slope. With the momentum thus acquired, it goes 60m up a 2% slope and comes to rest. Calculate the constant force of resistance to the motion of the car.

Soln. lost in PE − gain in KE = work done against friction.

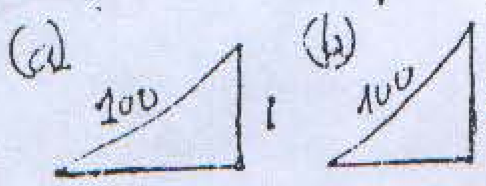


$mg(h_1 - h_2) + \frac{1}{2}m(V_i^2 - V_f^2) = F.D$

where the initial velocity $= 0$ m/s

V_f = final velocity $= 0$ m/s

F = force of resistance



from (a). for every 100m travel there is 1m descend. for 300m there, the height descended

$= \frac{1}{100} \times \frac{300}{1} = 3m - h_1$

from (b) for every 100m travelled, there is 2m ascended for 60m, the height ascended

$= \frac{2}{100} \times 60 = 1.2M - h_2$

Total distance travelled $= 300 + 60 = 360m$

$mg(h_1 - h_2) + \frac{1}{2}m(V_i^2 - 0_f^2) = F.S.$

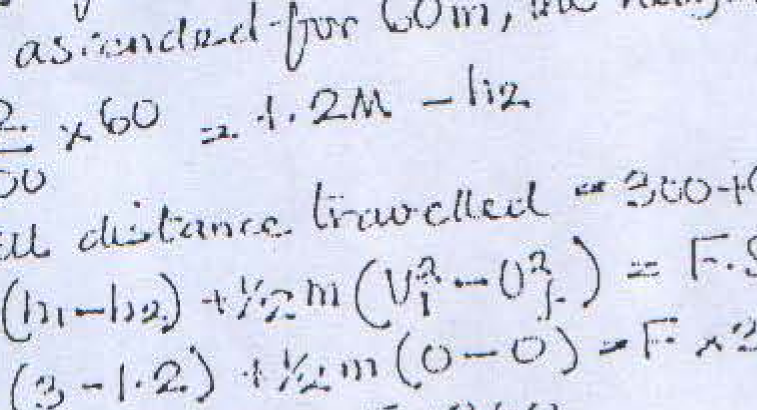
$mg(3 - 1.2) + \frac{1}{2}m(0 - 0) = F \times 360$

$1.8mg = F \times 360$

$F = \frac{1.8}{360} mg = \underline{5 \times 10^{-3} mg}$

(6) A particle, whose mass is 0.2kg is moving at $0.4ms^{-1}$ along the x-axis when it collides with another particle of mass 0.3kg, which is at rest. After the collision, the first particle moves at $0.2ms^{-1}$ in a direction making angle 40° with the x-axis. Determine the magnitude and direction of the velocity of the second particle after collision.

Soln



$m_1 = 0.2kg$

$m_2 = 0.3kg$

$u_1 = 0.4$

$u_2 = 0$

$V_1 = 0.2$

$V_2 = ?$

$m_1u_1 = m_1V_1\cos 40 + m_2V_2\cos\alpha$

$0.2 \times 0.4 = 0.2 \times 0.2 \cos 40 + 0.3V_2 \cos\alpha$

$0.2 \times 0.2 \sin 40 = 0.3V_2 \sin\alpha$

$0.08 = 0.03 + 0.3V_2\cos\alpha$

$0.05 = 0.3V_2\cos\alpha$

$0.026 = 0.3V_2 \sin\alpha$

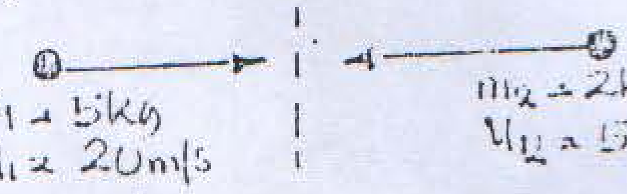
$\frac{0.026}{0.05} = \frac{0.3V_2\sin\alpha}{0.3V_2\cos\alpha} \Rightarrow 0.5142 = \tan$

$\alpha = \tan^{-1} 0.5142, \ \alpha = \underline{27.2°}$

$\frac{0.05}{0.3} = V_2 \cos 27°$

$V_2 = \frac{0.05}{0.3 \times 0.889} \Rightarrow V_2 = \underline{0.187m/s}$

(7). Two spheres, of masses 5kg and 2kg respectively, collide when moving directly towards each other at speeds 20m/s and 50m/s respectively, the co-efficient of restitution being 0.4. Find the velocities after collision



$m_1 = 5kg$, $u_1 = 20m/s$; $m_2 = 2kg$, $u_2 = 50m/s$

$\left\{\frac{V_2 - V_1}{u_1 - u_2}\right\} = 0.4 \quad m_1u_1 - m_2u_2 = m_2V_2 - m_1V_1$

$5 \times 20 - 2 \times 50 = 2V_2 - 5V_1$

$0 = 2V_2 - 5V_1 \ldots$

$(V_2 - V_1) = 0.4(u_1 - u_2)$

$V_2 - V_1 = 0.4u_1 - 0.4u_2 = 0.4(20 - 50)$

$V_2 - V_1 = -12 \Rightarrow V_2 = -12 + V_1$

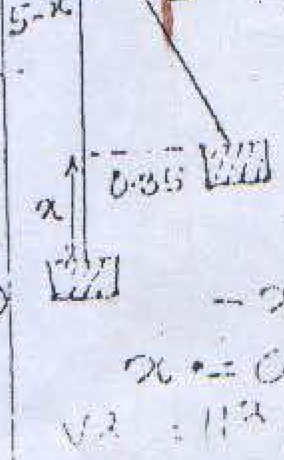
$2(-12 + V_1) - 5V_1 = 0 \Rightarrow -24 + 2V_1 - 5V_1 = 0$

$24 = -3V_1 \Rightarrow \underline{V_1 = -8m/s}$

from $0 = 2V_2 - 5V_1 \Rightarrow 0 = 2V_2 - 5(-8)$

$-2V_2 = 40 \Rightarrow V_2 = 40/-2 = -20$

(8). A block of wood of mass 2kg is suspended by a fine wire 5m long from a fixed point. A bullet of mass 2gm is fired horizontally into the wood and remains embedded in it. As a result of the impact, the block swings outwards through a horizontal distance of 35cm. Calculate the velocity of the bullet at impact and the loss of energy that takes place Soln



$5^2 = (5 - x)^2 + (0.35)^2$

$(5 - x)^2 = 5^2 - 0.35^2$

$= 25 - 0.1225$

$5 - x = \sqrt{24.8775}$

$= 4.9877$

$-x = 4.9877 - 5$

$-x = -0.01227$

$x = 0.01227$

$V^2 = u^2 + 2gs, \ u = 0 \Rightarrow V = \sqrt{2gs}$

$= \sqrt{2 \times 9.8 \times 0.01227} = \sqrt{0.2405}$

$= 0.4904 m/s$

$2.002 \times 0.4904 = 0.98178$

$\therefore u = \frac{0.98178}{2\times10^{-3}} = 490.89\ m/s$

ke   pe

$\frac{1}{2}mu^2 - mgh$ = loss energy due to friction

$\frac{1}{2}\times 2\times10^{-3}\times 490.89^2 - 2.002\times 9.8\times 0.0122$

$240.97 - 0.241$

$= 240.7\ J$

TUTORIAL 4

(1) What is a rigid body?

(b) A body consists of 2 spherical masses of 5.0kg each connected by a light rigid rod of length 1m. Neglecting the weight of the rod, determine the moment of inertia of the body (i) about its mid-point (ii) about an axis normal to it through one sphere. Comment on your result.

Soln

moment of inertia $= ML^2$

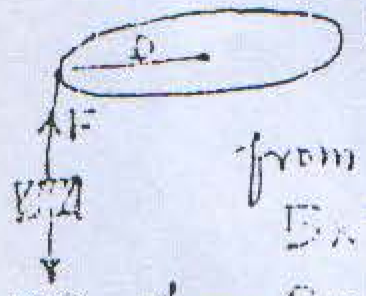
Considering the 2 sides when pivoted at the middle - moment of inertia $= 2ML^2 = 5(\frac{1}{2})^2\times 2$

$= 5\frac{1}{4}\times 2 = 2.5\ kgm^2$.

The moment of inertia about an axis normal to it through one sphere

$ML^2 = 5(\ )^2 = 5kgm^2$.

(2.) A flywheel with horizontal axle, 2cm in diameter, has a cord wound round the axle, to the end of which is attached a weight of 5kg. The weight is allowed to fall, causing the wheel to rotate. The weight falls a distance of 4m in 10s from rest. What is the moment of inertia of the flywheel. Soln



Diameter = 2cm

radius = 1cm = 0.01m.

from the diagram $mg - F = ma$

$5\times 9.8 - F = 5\times a$.

from $S = ut + \frac{1}{2}at^2$

$4 = \frac{1}{2}\times a(10)^2$

$8 \times 100a \Rightarrow a = 0.08m/s^2$

$5\times 9.8 - F = 5\times 0.08 = 4.9 - F = 0.4$

$49 - 0.4 = F \Rightarrow F = 48.6N$.

Torque $= F.r = 48.6\times 0.01$

$= 0.486 Nm$

$a = \alpha r \Rightarrow \alpha = a/r = \frac{0.08}{0.01} = 8\ rads^{-2}$

$\boxed{\alpha = a/r}$ $T = I\alpha$ and $I = \frac{T}{\alpha}$

$I = \frac{0.486}{8} = 6.075\times10^{-2}\ kgm^2$

about a vertical axis makes 100 rpm. A piece of wax of mass 100g falls vertically on to the disc and adheres to it a distance of 9cm from the axis. If the no of revolutions per minute is thereby reduced to 90, Calculate the moment of inertia of the disc

{Hint: Angular momentum is always conserved

$I\omega = \{I + I_m\}\omega_o \Rightarrow I\omega = I\omega_o + I_m\omega_o$

$\omega = 100r.p.m$, $\omega_o = 90r.p.m$

$I\times 100 = I\times 90 + I_m\times 90$

$I100 - I90 = 90I_m$

$10I = 90\times Mr^2 = 90\times\frac{100}{1000}\times\frac{9}{100}\times\frac{9}{100}$

$\frac{729}{10000} \Rightarrow I = \frac{729}{100000} = 7.29\times10^{-3}kgm^2$

(4.) The flywheel of a steam engine has a mass of 200kg and a radius of gyration of 2m. When it is rotating at 120r.p.m, the steam inlet is closed. Supposing that the flywheel stops in 5min, what is the torque due to friction on the axis of flywheel? what is the work done by the torque during this time?

Soln $I = Mk^2 = 200\times 2^2 = 800\ kgm^2$

2π rad = 1rev, $2\pi\ rads^{-1}$ = 1revs^{-1}

120 r.p.m $= \frac{120}{60} = 2revs^{-1}$

$2revs^{-1} = 2\times 2\pi\ rads^{-1} = 4\times 3.142...$

$\omega = 12.56\ rads^{-1}$

$-\alpha = \omega/t = \frac{12.56}{5\times 60} = -0.042\ rads^{-2}$

Torque $= -I\alpha = 800\times 0.042 = -33.5...$

(ii) $ke = \frac{1}{2}\omega^2 I = \frac{1}{2}(12.56)^2\times 800$

$= 400\times(12.56)^2 = 400\times 157.75$

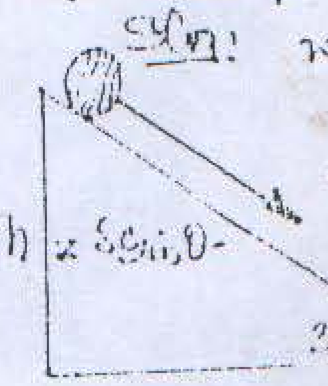
$= 6.32\times10^4 J$

(5) A 1kg solid ball rolling on a horizontal surface at 20m/s comes to the bottom of an inclined plane of angle 30° to the horizontal. Calculate its ke at the bottom of the incline and (ii) how far the ball will roll up the plane, neglecting friction

Soln: when the ball roll down the incline

$= 9.8\times\sin 30 = 9.8\times 0.5$

$= 4.9m/s^2$



Total ke $= \frac{1}{2}mu^2 + \frac{1}{5}mu^2$

$= \frac{7}{10}mu^2$

$ke = (\frac{1}{2}mu^2 + \frac{1}{2}I\omega^2)$

$I = \frac{2}{5}mr^2$, $ke = \frac{1}{2}mu^2 + \frac{1}{2}\times\frac{2}{5}mr^2$

... $\frac{7}{10}mu^2$

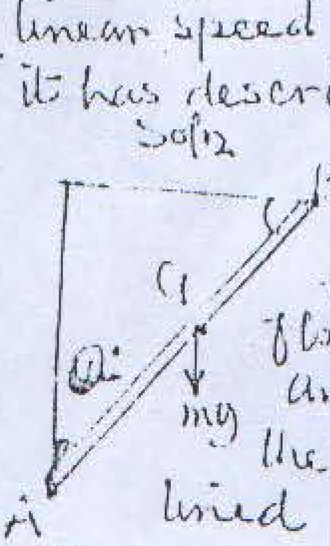
$ke = \frac{1}{2}mu^2 + \frac{1}{5}mu^2 = \frac{7}{10}mu^2$

$= \frac{7}{10}\times 1\times 20\times 20 = ...$

$\frac{7}{10}mu^2 = mgh$ ... [neglecting friction]

(6) A rod, of 3m long is hinged at one end so that it can turn in a vertical plane. It is held horizontally and then released. Cal- the angular speed of the rod and the linear speed of its free end at the instant it has described a angle of 60°

Soln



B Note: when a thin uniform rod AB of mass m and length L is hinged at end A to the level floor as shown below. The angular speed it will rise to strike the floor can be calculated as out lined below.

The moment of inertia about a transverse axis through end A is ...

$I_A = I_G + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}$

when the rod falls down the centre of mass of G falls a distance $\frac{L}{2}$ we can write

PE lost = KE gained

$mg\frac{L}{2} = \frac{1}{2}\left(\frac{mL^2}{3}\right)w^2$

$w = \sqrt{3g/L}$

Initially "g" is not in action because it is held horizontally but "g" is equal to $9.8 \times \sin\theta$ when the body fall through $60°$

$g = 9.8 \times \sin 60° = 9.8 \times 0.8660 = 8.487\,m/s^2$

$w = \sqrt{\frac{3 \times 8.487}{3}} = \sqrt{8.487}$

$w = \underline{2.91\,rads^{-1}}$

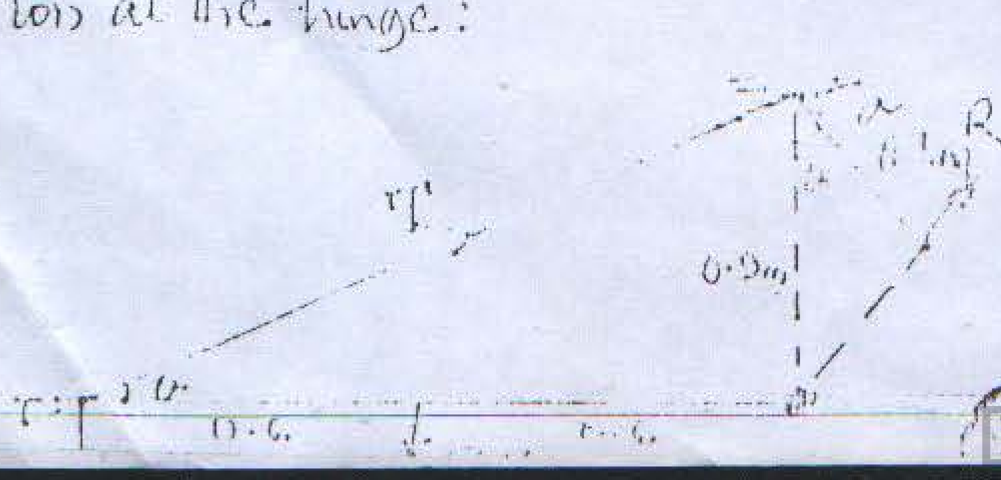
$\{V = wr\}$ r = radius of arc produced by the rod when it falls via 60°

$r = l\cos 60 = 3 \times 0.5 = 1.5$

$V = wr = 2.91 \times 1.5$

$\underline{V = 4.37\,m/s}$

(7) A uniform bar 120 cm long is kept horizontally by a string attached to the end opposite to that of the hinge. The other end of the string is tied to a point 90cm vertically above the hinge. If the bar weighs 50N, Cal- the Tension in the string and the reaction at the hinge:



$\tan\theta = \frac{0.9}{1.2} = 0.75$

$\theta = 37°$

$\cos 37° = \frac{a}{0.9} \quad a = 0.72.$

Clockwise moment = anticlockwise moment.

$T \times 0.72 = 0.6 \times 50$

$T = \frac{0.6 \times 50}{0.72}$

$T = \underline{41.7\,N}$

$T = R$

$\therefore R = \underline{41.7\,N}$

TUTORIAL 9.

(1) A particle is oscillating with a frequency of 100Hz and with an amplitude of 3cm. Cal. its velocity and acceleration at the middle and at the extremes of its motion. Write the equation of motion assuming zero initial phase. What is the displacement at time $t = 4s$.

Soln.

$W = \frac{2\pi}{T} = 2 \times 3.142 \times = 628.32\,rads^{-1}$

$V = Wr = 628.32 \times 3 \times 10^{-2} = 18.85\,m/s$

$(18.85, 0)\,m/s$ — (middle & End)

$a = w^2 r = (628.32)^2 \times 3 \times 10^{-2}$

$= 394786.022 \times 3 \times 10^{-2}$

$= 11843.58\,m/s^2$

$\{11843.58\,m/s^2, 0\}$ middle & End.

$x = A\sin wt = A\sin(2\pi f)t$

$x = 3 \times 10^{-2}\sin 2\pi 100 \times 4$

$= 3 \times 10^{-2}\sin 800\pi$

$\sin 800\pi = 0$

$x = 0.$

(2) What should be the percentage increase of a pendulum in order that they it has the same period at a place where g = 9.8 and another where g = 9.81 m/s²?

$T = 2\pi\sqrt{l/g} \Rightarrow T^2 = \frac{4\pi^2 l}{g}$

$\frac{T^2 g}{4\pi^2} = l, \quad l_1 = \frac{T^2\,9.8}{4 \times 3.142^2}$

$l_1 = \frac{T^2\,9.80}{39.478} = 0.248T^2$

% Change in length.

$\frac{l_2-l_1}{l_1} \times 100 = 0.2485T^2 - 0.2482T \times 100$

$= \frac{3\times10^{-4}T^2}{0.2482T^2} \times 100$

$= 1.208 \times 10^{-3} \times 100\% = \underline{0.12\%}$

(3) A body moving with SHM has a velocity 0.4m/s at a distance of 30cm from the mean position, and 0.3m/s at a distance 40cm from the mean position. Find the velocity at the mean position.

$0.4 = W\sqrt{(A)^2-(0.3)^2}$ --- (i)

$0.3 = W\sqrt{A^2-(0.4)^2}$ --- (ii)

$0.4^2 = W^2(A^2-0.3^2) = 0.3 = W^2(A-0.4^2)$

$\frac{0.4^2}{0.3^2} = \frac{W^2(A^2-0.3^2)}{W^2(A^2-0.4^2)}$

$0.4^2(A^2-0.4) = 0.3^2(A-0.3^2)$

$0.16A^2 - 0.0256 = 0.09A^2 - 8.1\times10^{-3}$

$0.16A^2 - 0.09A^2 = -8.1\times10^{-3} + 0.0256$

$0.07A^2 = 0.0175$

$A^2 = \frac{0.0175}{0.07} = 0.25$

$A = \sqrt{0.25} = \underline{0.5m}$

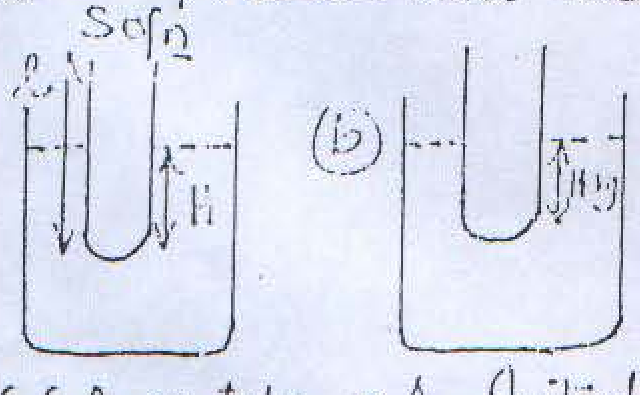
$0.4^2 = w^2(0.25-0.09)$

$w^2 = \frac{0.42}{0.16} = 1 \text{ rad s}^{-1}$

$V = WA = 0.5\times1 = \underline{0.5m/s}$

(4) Show that a test tube floating vertically in water performs S.H.M when slightly depressed. If such a test tube has an external diameter of 2cm and weighs 4gm, find the period of oscillation when slight depressed and released:

Soln

(a)  (b)

The C.S.A. of tube = A. Initial depth = H. Depth after slight depression $(H+y)$ [illegible] of tube l. density of $H_2O = \rho w$ length of tube L.

The test tube was floating the [illegible] of H_2O displaced = Mw g mass of H_2O displaced? [illegible]

Vol. of H_2O displaced = Vol. of Test tube initially immersed = C.S.A of tube × length immersed. $A\times H$. [A = is C.S.A].

$M_W = A\times H\times \rho w$

After the test tube has been depressed the volume of liquid displaced will increase $A(H+y)$ because the length immersed has increased by y.

Mass of H_2O now displaced = $M_W = A(H+y)\rho w$

weight of H_2O displaced before depression $AH\rho wg = w_1$, weight of H_2O displaced during depression = $A(H+y)\rho wg = w_2$

diff. in weight = restoring force.

$F = w_1 - w_2 = A\cdot H\rho wg - [A(H+y)\rho wg]$

$F = A\cdot H\rho wg - AH\rho wg - Ay\rho wg = Ay\rho wg$

$F = ma \Rightarrow ma = -\frac{Ay\rho wg}{m} = -w^2y$

$$\boxed{a = -\frac{Ay\rho wg}{m} = -w^2y}$$

The motion is S.H. ... ope!

$w^2y = \frac{Ay\rho wg}{m} = w^2 = \frac{A\rho wg}{m}$

$W = \sqrt{\frac{A\rho wg}{m}}$; $T = \frac{2\pi}{W} = \frac{2\pi}{\sqrt{\frac{A\rho wg}{m}}}$

$w = \sqrt{\frac{\pi\times1\times10^{-2}\times1\times10^{-2}\times9.8\times10^3}{4\times10^{-3}}}$

$= \sqrt{\frac{\pi\times10^2\times9.8}{4}} = \sqrt{\frac{3078.76}{4}}$

$= \sqrt{769.6902} = 27.74 rad$

$T = \frac{2\pi}{27.74} = 2\pi\times0.036 = \underline{0.226s}$

(5) Find the average values of kinetic and potential energies in SHM relative to (i) time and (ii) position.

* plz check the Tutorial Class for solution

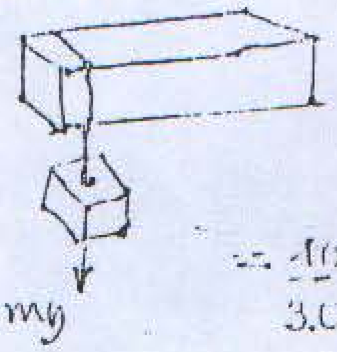
Physics Majors Tutorial Class [illegible] Physics Dept.

Signed

[illegible]

TUTORIAL 10

(1b) A steel bar of length 4m and of rectangular section 1.5cm × 2.0cm Supports a load of 100kg. By how much is the bar stretched?



$E = \frac{F}{A}\cdot\frac{L}{e} = Y$

$A = \frac{1.5}{100} \times \frac{20}{200} = 3\times10^{-4}$

$= \frac{1000\times 4}{3.0\times10^{-4}} = 2\times10^{11} e$

$e = 6.53\times10^{-7}$

$= 6.53\times10^{-5}$ m

(2) What is the Young's modulus of a cylindrical bone specimen of cross-sectional area of 1.5m² if a load of 10kg produces a decrease of 0.0065% in its length

$L = 100,\ e = 0.0065 \quad F = 9.8\times10 = 98N$

Area $= 1.5cm = 1.5\times10^{-4}$

$Y = \frac{FL}{Ae} = \frac{100\times98}{1.5\times10^{-4}\times6.5\times10^{-3}}$

$= \frac{9.8\times10^{3}}{1.5\times6.5\times10^{-7}} = \frac{9.8}{1.5\times65}\times10^{10}$

$= \frac{9.8}{9.75}\times10^{10} = 1.005\times10^{10}\ N/m^2.$

(b) A cylindrical copper wire and cylindrical steel wire each of length 1.5m and diameter 2mm are joined at one end to form a composite wire 3m long. The wire is loaded until its length becomes 3.003m Cal. the strains in the copper steel wire and the applied force to the wire.

Sol^n

$Y_c = 1.2\times10^{11}\ Nm^{-2},\ Y_s = 2\times10^{11}\ Nm^{-2}$

$e_c + e_s = 0.003m \Rightarrow Y = \frac{FL}{Ae}$

$\frac{Y_c A e_c}{l} = F = \frac{1.2\times10^{11}\times(1\times10^{-3})\times\pi e_c}{1.5}$

$= 2.5136\times10^{5} e_c = f.$

$\frac{Y_s A e_s}{l} = f = \frac{2\times10^{11}\times(1\times10^{-3})^2 e_s \times \pi}{1.5}$

$= 4.1888\times10^{5} e_s$

$4.1888\times10^5 e_s - 2.5136\times10^5 e_c = 0$

$e_c = 0.003 - e_s.$

$4.1888\times10^5 e_s - 2.5136\times10^5(0.003 - e_s) = 0$

$6.7024\times10^5 e_s = 754.08$

$e_s = \frac{754.08}{6.7024\times10^5} = 1.125\times10^{-3}m$

$e_s + e_c = 0.003.$

$1.125\times10^{-3} + e_c = 0.003.$

$e_c = 0.003 - 1.125\times10^{-3} = 1.875\times10^{-3}$

$F = 2.5136\times10^5\times1.875\times10^{-3}$

$F = 4.713\times10^{2} = 471.3N$

(3) The rubber cord of a catapult has a C.S.A of 1.00mm² and a total unstretched length of 10.0cm. It is stretched to 12.00cm and then released project a missle of mass 5g. Calculate the velo. of projection if the Young's modulus of rubber is $5.0\times10^{8}\ N/m^2$. State your assumptions.

Sol^n

Energy $= \frac{1}{2}\frac{EAe^2}{l}$

$= \left\{\frac{\frac{1}{2}\times5\times10^{8}\times1\times10^{-6}\times10^{-4}\times4}{0.1}\right\}$

$= \left\{\frac{20\times10^{-2}}{0.2}\right\} = 1J.$

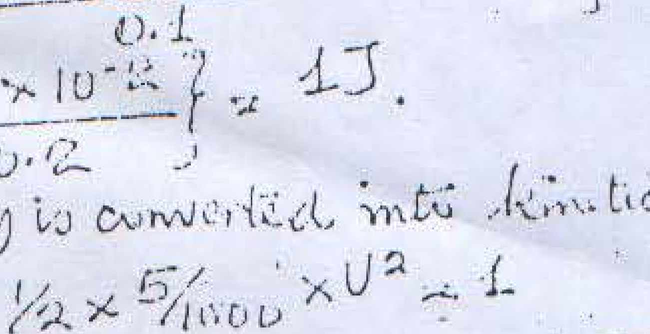
The energy is converted into kinetic energy $= \frac{1}{2}\times\frac{5}{1000}\times U^2 = 1$

$U^2 = \left\{\frac{2\times1\times1000}{5}\right\} = 400,\ U = \sqrt{400}$

$U = 20m$

(4) A wire of length 2m and C.S.A a.c 1mm is tightly stretched in a horizontal line. When a mass of 10gm is hung from its midpoint, it sags by 1cm at this point. Cal. the Young's modulus of material.

Sol^n



$l^2 = 1^2 + (0.01)^2$

$= 1^2 + 0.0001$

$= 1.0001$

$l = \sqrt{1.0001}$

$l/2 = 1.0005$

$e = 1.0005 - 1$

$= 0.00005$

$l = 0.00005\times2$

$= 0.0001.$

$Y = \frac{FL}{Ae}$

$Y = \frac{0.1\times2}{1\times10^{-6}\times10}$

$Y = 2.0\times10^{9}$

Continued in Page (17)