NAME **MATRIC NO** **DEPT** **COLLEGE**

Section I

1. (a) Let A, B, C be nonempty subsets of the universal set X. Show that:
   (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (iii) $(A \cup B)' = A' \cap B'$ (iv) $(A \cap B)' = A' \cup B'$.

   (b) Using your results in (a) or otherwise, show that:
   (i) $(A' \cap B \cap C') \cup (A' \cap B \cap C) = B - A$ (ii) $(A \cup B) \cap (B \cup A') = B$
   (iii) $(A \cap B) \cup (A \cap C) \cup (A \cap B' \cap C') = A$ (iv) $A - (B \cap C) = (A - B) \cup (A - C)$.

   (c) In a survey of 100 families, the number that read recent issues of a certain monthly magazine were found to be: April 26, June 48, April only 18, April but not May 23, April and June 8, June and May 8, none of the three months 24.

   (i) Draw a Venn diagram to represent the data and hence find how many families read(ii)April, May and June issues (iii)May issue only (iv)May issue (v) June issue only (vi)June but not May issue. If a family is chosen at random, what is the probability that the family read issues in one month only ?

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Section II

2. (a) i. Show that $\frac{2+\sqrt{3}}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{2(2+\sqrt{3})} = 5$.

   ii. Solve for x given that $\log_4^x \times \log_8^{x^4} = 32$.

   iii. Prove by induction that $2^{2n+1} + 1$ is divisible by 3.

   (b) i. Find the term in x^2 and the term independent of x in the expansion of $\left(2x - \frac{1}{x}\right)^8$.

   ii. Write down the first five terms of the series $(1 - 2x)^{10}$. Use your expansion to find the value of $(0.998)^{10}$ correct to 7 decimal places.

   (c) i. If $\log x, \log y, \log z$ are three consecutive terms of an AP, show that x, y, z are in GP.

   ii. Let $f(x) = x^4 - 6x^2 - 7x - 6$. Factorize $f(x)$ completely and hence obtain all its zeros.

   iii. Show that the complex conjugate of $1 + i$ is a zero of the polynomial $f(z) = z^4 - z^3 + z^2 + 2$.

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Section III

3. (a) Let $ax^2 - bx + c = 0$ and $bx^2 - cx + a = 0$ be two given quadratic equations.
   - i. If the difference between the roots of the equations are the same, show that $b^4 - a^2c^2 = 4ab(bc - a^2)$.
   - ii. If the two equations are having a common root, show that $a^4 + ab^3 + ac^3 - 3a^2bc = 0$.

   (b)
   - i. Show that the expression $x^2 - (a+b+c)x + a^2 + b^2 + c^2 + 2bc - ca - ab$ can never be negative for all real values of x, a, b, c.
   - ii. Sketch completely the graph of the quadratic function $y = 30 - 11x - 30x^2$.

   (c)
   - i. Find all the roots of the equation $x(x+1) + \frac{12}{x(x+1)} = 8$.
   - ii. Given that $\frac{1-2x}{x^2+2} = k$, form a quadratic equation in x. If x is always real, show that k must lie between two values and find these values. Hence, state the minimum and maximum values of the expression $\frac{1-2x}{x^2+2}$.

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Section IV

4. (a) Find the values of k given that $\begin{vmatrix} k & 3+k & -10 \\ 1-k & 2-k & 5 \\ 2 & 4+k & -k \end{vmatrix} = 48.$

(b) Let $A = \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$ be a given matrix.

i. Show that $A^3 - 5A^2 - A + I = O$, where I is the 3×3 unit matrix.

ii. By pre/post multiply the equation $A^3 - 5A^2 - A + I = O$ by A^{-1}, the inverse of A, show that $A^{-1} = I + 5A - A^2$ and hence compute the inverse of A.

iii. Using the inverse of A obtained in 4(b)(ii), solve the system of linear equations given by

$$\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}.$$