DEPARTMENT OF ELECTRICAL ENGINEERING

FACULTY OF TECHNOLOGY

BAYERO UNIVERSITY, KANO

EGR2208: PRINCIPLES OF ELECTRICAL ENGINEERING II

(LECTURE NOTES)

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CHAPTER ONE: ELECTROMECHANICAL ENERGY CONVERSION

1.1 INTRODUCTION TO MAGNETISM:

The knowledge of magnetism and electromagnetism is very essential to Electrical Engineers. Almost all electromechanical power generations and electrical derives involve the phenomena of magnetism and electromagnetism.

Before analyzing the behavior of magnets, it is important to know the following terminologies with regard to electromagnetism.

1.1.1 ABSOLUTE AND RELATIVE PERMEABILITY OF A MEDIUM

Permeability of a medium is the ability of the medium to become magnetized when it comes closer to a magnetic field. It can also be defined as the readiness of the medium to allow magnetic flux to pass through it. The process of magnetism and electromagnetism depend on this property of the medium.

The relative permeability of a medium on the other hand is the ratio of the permeability of the medium to the permeability of free space (vacuum). The permeability of free space is chosen as a reference and it has a value of $\mu_o = 4\pi \times 10^{-7}$ Henry per meter (H/m). The relative permeability of ferromagnetic materials is not constant, it vary with the magnetic flux density established in the medium. On the other hand the relative permeability of air and other non magnetic materials is almost always unity.

Now if the absolute permeability of a medium is μ then the relative permeability of that medium is giving by

$$\mu_r = \frac{\mu}{\mu_o} \Rightarrow \mu = \mu_r \mu_o \quad \ldots\ldots(1.1)$$

Where μ is the absolute permeability of the medium

μ_o is the permeability of free space and

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μ_r is the relative permeability of the medium.

The relative permeability can also be expressed in terms of the flux density of the medium and flux density of free space when they are subjected to a uniform field strength. Suppose a magnetic material such as iron or nickel is placed in a uniform field of strength H and it was found that a flux density of B Wb/m^2 is developed in it, while B_o Wb/m^2 is the flux density of the surrounding then:

$$\mu_r = \frac{B}{B_o} \quad \text{......(1.2)}$$

1.1.2 MAGNETIC FLUX (ϕ):

The magnetic flux per pole is the amount of lines of magnetic field radiated out by the pole. It is important to note that the lines of magnetic field are pointing out ward in the case of north pole and inward in the case of south pole. This is illustrated in figure 1.1.

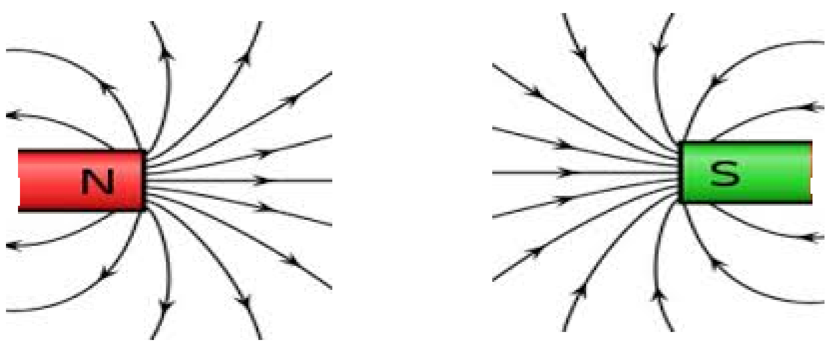

Figure 1.1 Magnetic Flux

The unit of magnetic flux is Weber (Wb), hence a unit N pole is suppose to radiate out a flux of 1 Weber. Sometimes the flux is expressed in Maxwell or just lines in which one Weber equals 10^8 Maxwells. When the magnetic field intersects the area at an angle, then the normal component of the field is used in calculating the magnetic flux.

1.1.3 MAGNETIC FLUX DENSITY (B):

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Suppose we have a magnetic pole radiating out its flux, the amount of flux (that is the lines of magnetic field) passing out per unit area is referred to as magnetic flux density. It is measured in Weber per square meter (Wb/m^2) or Tesla (T).

In some applications maxwells per square inch or lines per square inch are used. And if ϕ Wb is the total magnetic flux passing through an area of A m^2 then:

$$B = \frac{\phi}{A} \text{ Wb/m}^2 \text{ or Tesla} \ldots\ldots (1.3)$$

1.1.4 MAGNETIC FIELD STRENGTH (H):

Magnetic field strength at any point within a magnetic field is the force experienced by a unit pole (1 Weber) placed at that point due to the magnetic field. If the unit pole and the field are the same in nature (i.e. N & N or S &S) the force will be repulsive, whereas it will be attractive if the unit pole and the field are opposite in nature.

The magnetic field strength is numerically given by:

$$H = \frac{m}{4\pi\mu_o r^2} \text{ N/Wb or AT/m} \ldots\ldots (1.4)$$

Where **m** is the magnetic flux of the field

r is the distance from the field source to the unit pole and

μ_o is the permeability of free space.

The magnetic field strength can also be expressed in terms of magneto motive force as follows:

$$H = \frac{\Im}{L}$$

where $\Im$ is the mmf and L is the length of the magnetic path

1.1.5 MAGNETIC POTENTIAL (M):

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The magnetic potential at any point within a magnetic field is the work done in carrying a unit pole from infinity to that point against the force of magnetic field. It is given by the following expression:

$$M = \frac{m}{4\pi\mu_o r}$$ Joules per Weber (J/Wb)…………………………………………………………..(1.5)

1.1.6 INTENSITY OF MAGNETIZATION (I):

Intensity of magnetization is a term related to a substance undergoing magnetization. It can be defined as the induce pole strength developed per unit area of the bar or it can also be defined as the magnetic moment developed per unit volume of the bar. (i.e. the bar undergoing the magnetization).

Let **m** be the pole strength induced in the bar and **A** be the cross sectional area of the bar. Then the intensity of magnetization is given by

$$I = \frac{m}{A}$$ Wb/m^2………………………………………….……………………………..(1.6)

In essence, the intensity of magnetization of a substance is the flux density induced in it due to its own induced magnetism. If the magnetic length of the bar is l then:

$$I = \frac{m}{A} = \frac{m \times l}{A \times l} = \frac{M_t}{V}$$ = Magnetic moment / Volume…………………………………………(1.7)

Where $\mathbf{M_t}$ is the magnetic moment and ***V*** is the volume of the bar.

1.1.7 SUSCEPTIBILITY (K):

Susceptibility is defined as the ratio of intensity of magnetization **I** and the magnetic field strength **H**. That is K = I/H……………………………………………………………(1.8)

When an iron bar is placed in a magnetic field it gets magnetized by induction hence the total flux density in the iron bar consists of two parts;

(i) B_o, flux density of the air (surrounding medium) even if the bar is not present and

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(ii) B_i, induced flux density in the bar

Therefore $B = \mu H = B_o + B_i = \mu_o H + m/A$

Now the permeability of the iron bar is given by

$$\mu = \frac{\mu_o H + I}{H} = \mu_o + \frac{I}{H} \text{ hence } \mu = \mu_o + K \quad \ldots\ldots(1.9)$$

The relative permeability of the bar is therefore;

$$\mu_r = \frac{\mu}{\mu_o} = 1 + \frac{K}{\mu_o} \quad \ldots\ldots(1.10)$$

1.1.8 BOUNDARY CONDITIONS OF MAGNETIC FIELD:

Boundary conditions describe the behavior of magnetic field when it passes through media of different permeabilities. Consider figure 1.2 in which the magnetic field passes from medium 1 (of permeability μ_1) to medium 2 (of permeability μ_2).

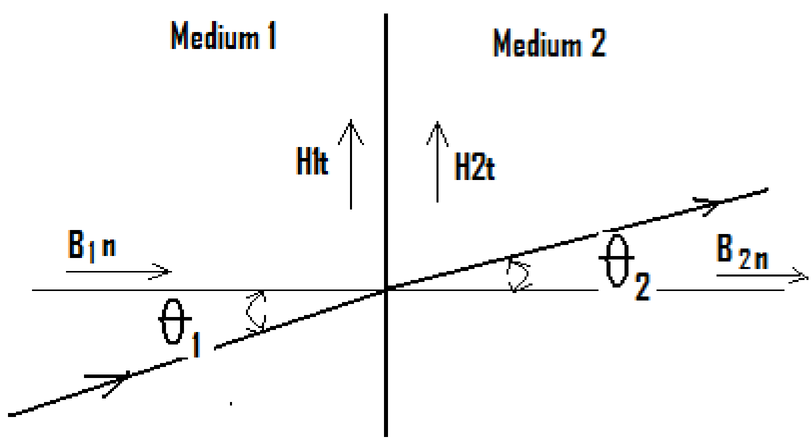

FIG. 1.2 THE BOUNDARY CONDITIONS

These boundary conditions are as follows:

(i) The normal component of the flux density is continuous across the boundary. That is $B_{1n} = B_{2n}$.

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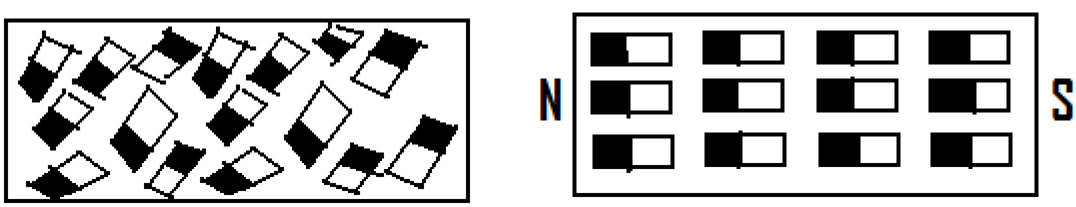
(ii) The tangential component of the magnetic field strength H is continuous across the boundary that is $H_{1t} = H_{2t}$.

From (i) above we have $B_1\cos\theta_1 = B_2\cos\theta_2$ and from (ii) we have $\frac{B_1 \sin\theta_1}{\mu_1} = \frac{B_2 \sin\theta_2}{\mu_2}$ dividing (i) by (ii) we have $\frac{\tan\theta_1}{\mu_1} = \frac{\tan\theta_2}{\mu_2}$ from which

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2} \quad \text{.......(1.11)}$$

1.1.9 WEBER AND EWINGS MOLECULAR THEORY:

According to this theory the molecules of all substances are inherently magnets in themselves, each (molecule) having a north pole and south pole. When the substance is not magnetized the molecules are not arrange in order, in other words they lie in a haphazard manner forming a close loop. These closed magnetic circuits are satisfied internally hence there is no external magnetism produced by the bar. That is the magnetic effect of some of the molecules cancel the effects of other molecules. This is illustrated in figure 1.3(a) below.



(a) Un magnetize state (b) Magnetized state

FIG 1.3 MOLECULES OF A SUBSTANCE

But when the iron bar is placed in a magnetic field the molecular magnets will arrange themselves more or less along a straight line parallel to the direction of the magnetizing force. This linear arrangement of the molecules result in N polarity at one end of the bar and S polarity at the other end as shown in figure 1.3 (b). The formation of this magnet by the material is in order because its molecular elements consist of nucleus of positive (electrical) polarity

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surrounded by moving electrons in the shells. The moving electrons constitute a current, which produces a magnetic field according to Ampere's law (section 1.2.2).

1.1.10 THE CURIE POINT:

When a magnetic material is heated, its molecules vibrate violently as the result of increase in temperature. As the result of this the molecular magnets get out of alignment thereby reducing the magnetic strength of the substance.

As the temperature is increased a point will be reached when the magnetic property of the substance is partially or completely destroyed. The temperature at which such complete demagnetization occur is called the "curie point".

The curie point can also be defined as a critical temperature above which a Ferro magnetic material becomes paramagnetic.

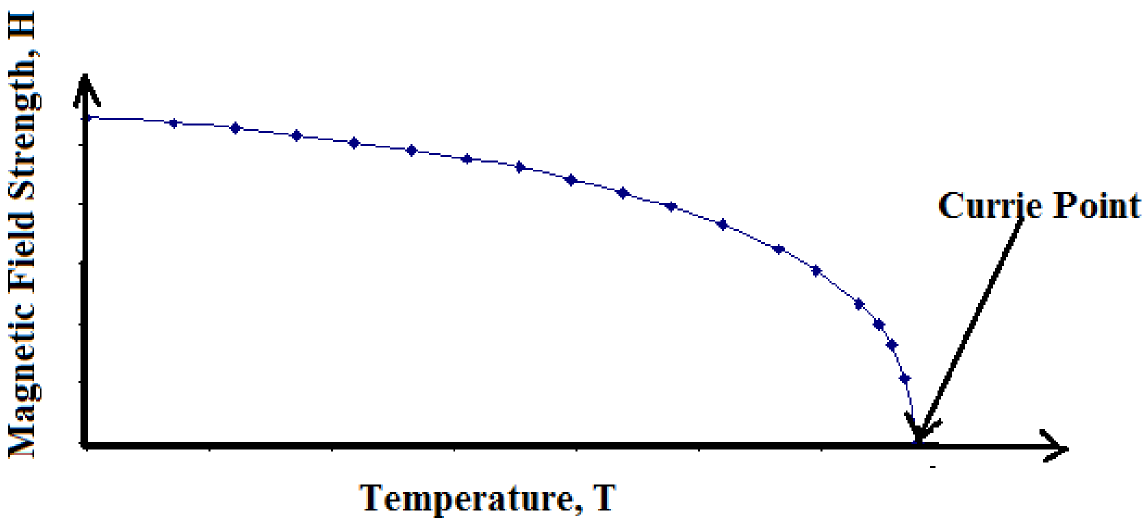

Figure 1.4 Curie Point

1.2 MAGNETIC CIRCUIT :

Magnetic circuit, like electrical circuit is the path which is followed by magnetic flux (current is the quantity that flows in electric circuit). Before we go to the magnetic circuits, it is important to treat the following topics.

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1.2.1 FORCE ON A CURRENT CARRYING CONDUCTOR LYING IN A MAGNETIC FIELD:

It is found that when a current carrying conductor is placed in a magnetic field it will experience a force F which is perpendicular to both the direction of current (positive to negative) and the direction of the magnetic field (from North to South), this can be illustrated in figure 1.5 below:

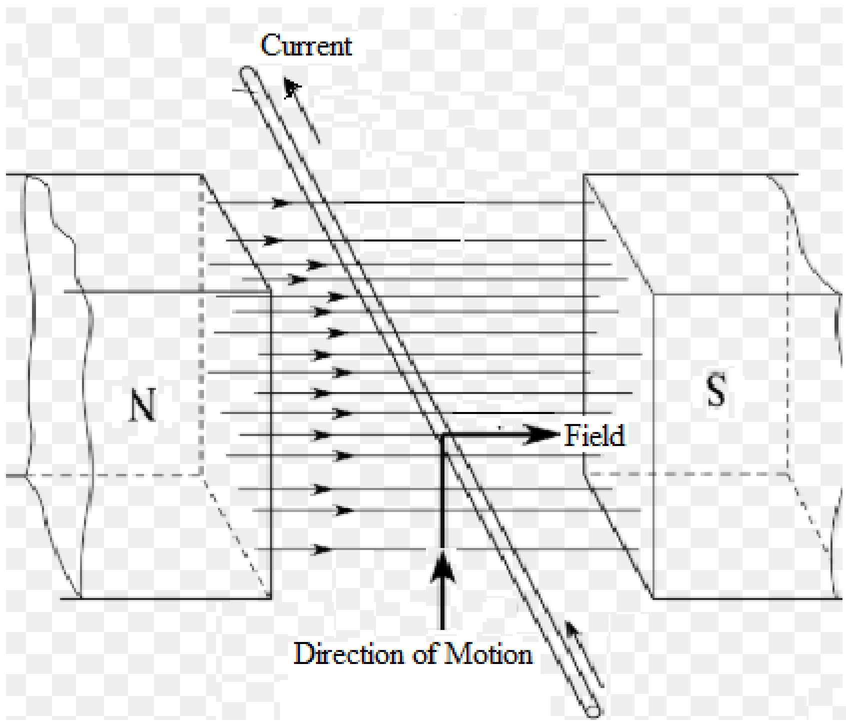

Figure 1.5 Conductor Carrying Current in a Magnetic Field

The direction of the force can be found using Fleming's left hand rule. Which says "**if we stretch out the thumb and first two fingers of our left hand such that they are perpendicular to one another and we placed the fore finger in the direction of field and the second finger the direction of current then the thumb will give the direction of motion of the current carrying conductor in the magnetic field**".

Now let **L** be the length of the conductor in the magnetic field, **B** be the magnetic flux density of the field (Wb/m^2) and **I** be the current flowing through the conductor (A) then the magnitude of this force is given by:

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$F = BIL = \mu HIL = \mu_r\mu_o HIL$………………………………………………..……………(1.12)

If the conductor **L** is at an angle θ to the direction of the field, then this force will be given by:

$F = BIL\sin\theta = \mu_r\mu_o HIL\sin\theta$…………………………………………….……………..(1.13)

This is because we only use the tangential component of the force.

1.2.2 AMPERE'S CIRCUITAL LAW:

This law also known as Ampere's work law states that **the magneto motive force (mmf) around a close path is equal to the current enclosed by the path**. It can be mathematically written as:

 Amperes……………………………………………..………………(1.14)

Where **H** is the vector representing magnetic field strength and **dS** is the vector of the enclosing path around the current.

The above law is used for obtaining the value of the mmf around a long straight current carrying conductor and along a solenoid.

1.2.3 MMF AROUND A LONG STRAIGHT CONDUCTOR

When a conductor carries an electric current a magnetic field is established around the conductor.

Figure 1.6 shows a long conductor carrying a current of **I** Amperes, the magnetic field consists of a circular lines of force centered at the center of the conductor.

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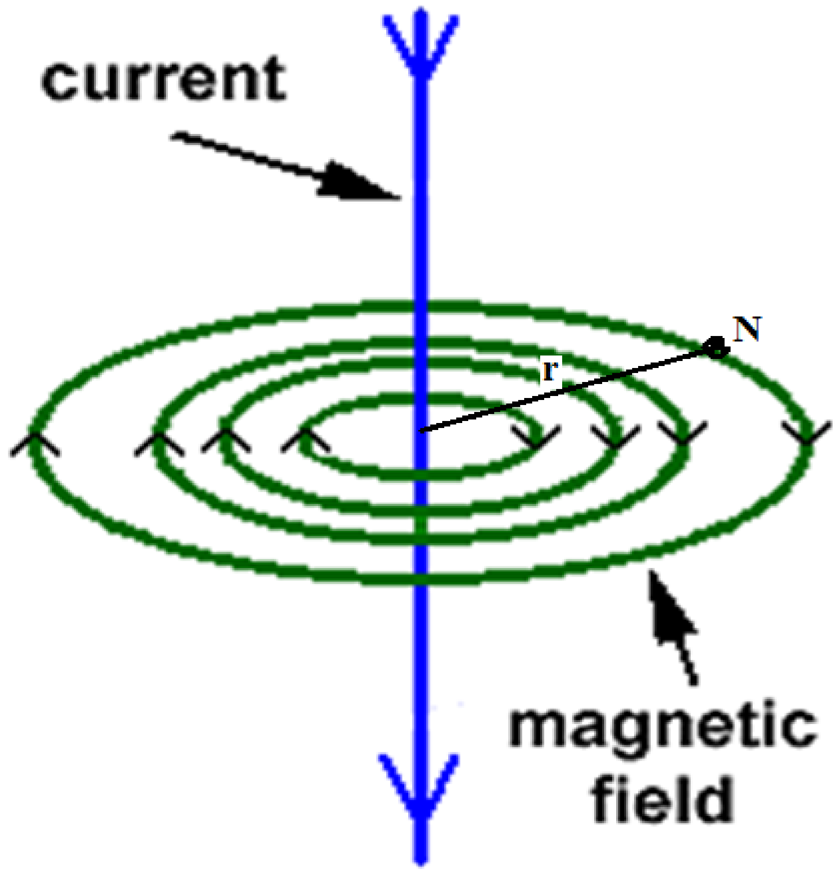

Figure 1.6 Magnetic Field Around a Current Carrying Conductor

Now suppose that the field strength is **H** and that a unit **North** pole (1 Weber) is placed at a distance of **r** meters from the centre of the conductor. The unit pole will experience a force of **H** Newtons (that is going by the definition of field strength **H**). If this unit pole is moved round the conductor against the magnetic force then the work done is given by:

mmf = force x distance = H x 2πr = I (Ampere's law)

Hence $H = \frac{I}{2\pi r}$ ………………………………………………………….………………(1.14)

And if there are N conductors arranged together then the field strength is given by:

$H = \frac{NI}{2\pi r}$ A/m or Oersted……………………………………………….………………(1.15)

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And since the flux density, B =μH then we have the following relationship:

$$B = \frac{\mu_r \mu_o NI}{2\pi r} \text{ Wb/m}^2 \quad \text{...}(1.16)$$

N/B The conductors must be carrying the same current and in the same direction otherwise some magnetic fields (of the individual conductors) may cancel the effect of others.

1.2.4 MMF AND FIELD STRENGTH OF A LONG SOLENOID:

A solenoid is a wire or conductor made into some number of turns (coils) on a magnetic material with **N** as the total number of turns. These turns could be wound on a steel rod or bar of any good magnetic material.

As current pass through the coil the steel rod becomes magnetized and behave like a permanent magnet. This magnetic property will remain in the steel rod as long as the current is passing through the solenoid. Figure 1.7 illustrate the magnetic behavior of the solenoid.

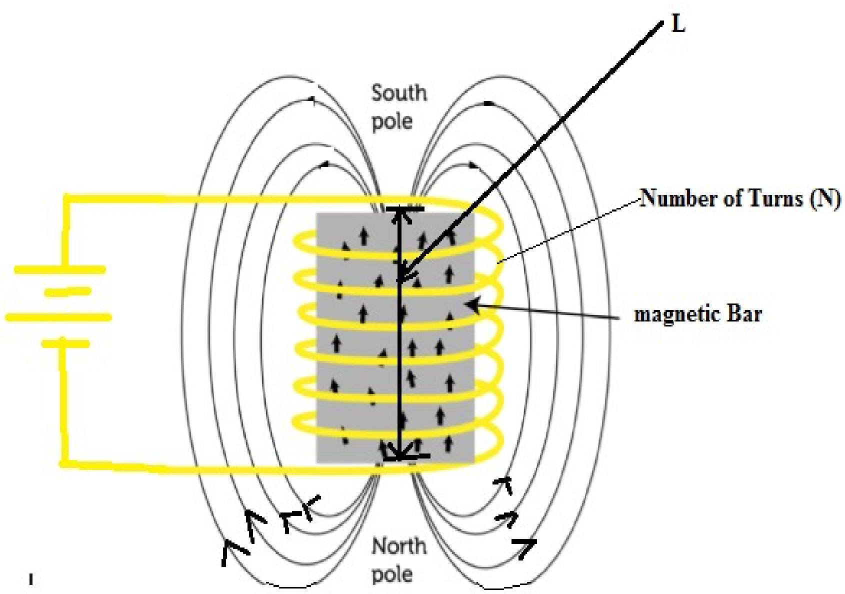

Figure 1.7 Solenoid with Lines of Magnetic Field

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Let the magnetic field strength along the axis of the solenoid be **H**. and let us have the following assumptions:

(i) The value of H remains constant throughout the length **L** of the solenoid and

(ii) The value of H outside the solenoid is negligible because the permeability of the bar is much higher than the permeability of the surrounding ($\mu_{steel} >> \mu_o$).

Then the work done in carrying a unit **North** pole around a complete circle along the bar is given by:

Work done = H x L Amperes

This is because **H** is assumed to exist only along the **L**. And since there are **N** turns in the solenoid then ampere-turns (total current) linked to this path is **NI**

Therefore H x L = NI, or $H = \frac{NI}{L}$ A/m (rather AT/m)……………………………(1.17)

Also the flux density is given by:

$$B = \mu H = \frac{\mu_r \mu_o NI}{L} \quad \text{…………(1.18)}$$

1.2.5 FORCE BETWEEN TWO PARALLEL CONDUCTORS:

In the above sections we mentioned that a magnetic field is established along a conductor carrying current. Now if we have two parallel conductors close to each other they both have magnetic field around them.

If the currents in them are in the same direction the two fields will be in opposite to each other (one being north and the other one being south) therefore the two conductors will attract each other. On the other hand if the current in them are in opposite direction, then the field produced are in the same direction and therefore the two conductors will repel each other. Figure 1.8 illustrate the two phenomenon.

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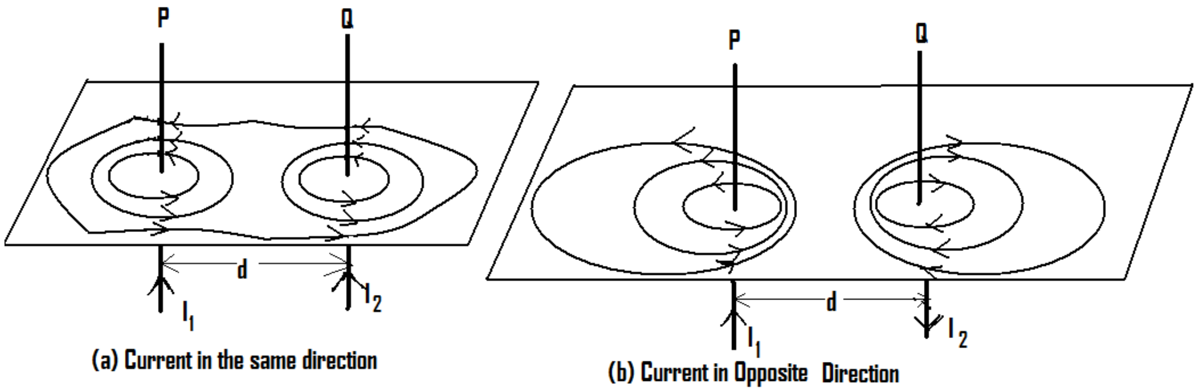

FIG. 1.8 FORCE BETWEEN TWO PARALLEL CONDUCTORS

Now it is clear that each of the conductors lies in the magnetic field of the other. And by taking conductor P as the reference, its flux density at a distance of d meters is given by:

$$B = \mu H = \frac{\mu_o I_1}{2\pi d}$$

And if the length of the conductor Q lying in the flux density B is L then the force is given by:

$$F = BI_2L = \frac{\mu_o I_1 I_2 L}{2\pi d} \quad \text{...(1.19)}$$

N/B The force in figure 1.8(a) and (b) have the same magnitude only that the force is attractive in (a) and repulsive in (b).

Equation (1.19) can be used to define what an Ampere is, it is as follows:

The force per unit length is given by:

$$F_L = \frac{\mu_o I_1 I_2}{2\pi d} \text{ N/m} \quad \text{...(1.20)}$$

Now if we make $I_1 = I_2 = 1$A and the distanced = 1m then

$$F_L = \frac{\mu_o \times 1 \times 1}{2\pi \times 1} = \frac{\mu_o}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{N}$$

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Therefore one Ampere is the current which when flowing in each of the two conductors situated in vacuum (free space) and separated by a distance of 1m, produces on each of the conductor a force of 2 x 10^{-7}N per unit length (1 meter).

EXAMPLE 1.1

The force between two long parallel conductors is 15Kg/m. the conductor spacing is 10 cm. If one conductor carries twice the current of the other, calculate the current in each conductor.

SOLUTION:

Force per unit length F_L = 15Kg/m = 15 x g = 15 x 9.81 = 147N/m, d = 10cm = 0.1m

Let I_1 = I and I_2 = 2I

Using the relation, $F_L = \frac{\mu_o I_1 I_2}{2\pi d}$ we have $147 = \frac{4\pi \times 10^{-7} \times I \times 2I}{2\pi \times 0.1} \Rightarrow I^2 = \frac{147 \times 0.1}{4 \times 10^{-7}} = 36,750,000$

Therefore I = 6,062A and hence I_1 = 6,062A & I_2 = 12,124A

EXAMPLE 1.2

Two long straight parallel wires, standing in air, 2m apart, carry currents I_1 and I_2 in the same direction. The magnetic intensity at a point mid way between the wires is 7.954AT/m. If the force on each wire per unit length is 2.4 x 10^{-4}N. Evaluate I_1 and I_2.

SOLUTION:

F_L = 2.4 x 10^{-4}N, H = 7.95AT/m.

The magnetic field strength of a long straight current carrying conductor at a distance **r** from the conductor is given by:

$$H = \frac{I}{2\pi r} \text{ AT/m}$$

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Now the magnetic intensity at a point between two current carrying conductors is the sum of individual intensities if the currents in the conductors are in opposite direction, whereas the difference is taken if the currents are in the same direction.

The midway between the conductors is a point where r = 2/2 = 1cm

Let the currents be I_1 and I_2

The total intensities is therefore $\frac{I_1}{2\pi r} - \frac{I_2}{2\pi r} = 7.95$

$$\therefore \frac{I_1}{2\pi} - \frac{I_2}{2\pi} = 7.95 \Rightarrow I_1 - I_2 = 50 \text{ -----------------(i)}$$

The force per unit length is given by: $F_L = \frac{\mu_o I_1 I_2}{2\pi d} \Rightarrow 2.4\times10^{-4} = \frac{4\pi\times10^{-7}\times I_1\times I_2}{2\pi\times2}$

From which $I_1I_2 = = \frac{2.4\times10^{-4}}{10^{-7}} = 2400$ ………(ii)

Substituting (i) into (ii) we have

$(50 + I_2)I_2 = 2400$ from which $I_2^2 + 50I_2 - 2400 = 0 \Rightarrow (I_2 + 80)(I_2 - 30) = 0$

Therefore I_2 = <u>30A</u> and $I_1 = 50 + I_2$ = <u>80A</u>

1.2.6 DEFINITIONS CONCERNING MAGNETIC CIRCUIT:

One of the characteristics of lines of magnetic flux is that each line forms a closed loop (Fig. 1.7) The complete close path followed by any group of magnetic flux lines is referred to as magnetic circuits.

Consider a toroidal iron ring (fig 1.9) having a magnetic path of L meter, cross sectional area A and a coil of N turns carrying a current of I amperes.

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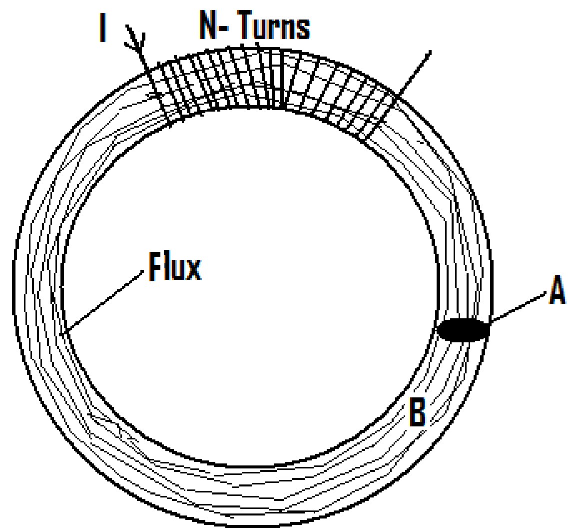

FIG. 1.9 MAGNETIC FLUX OF A TOROID

The field strength inside the solenoid is given by:

$H = \frac{NI}{L}$ and B = μH $= \frac{\mu_o \mu_r NI}{L}$ ……………………….(1.21)

Therefore the total flux produced is given by: $\phi = B \times A = \frac{\mu_o \mu_r NI}{L} = \frac{NI}{L/\mu_o \mu_r A}$ …..(1.22)

The numerator of equation 1.22, NI is known as magneto motive force (mmf) and its unit is Ampere turns (AT), whereas the denominator is called the reluctance which is expressed in AT/Wb. As such mmf $\Im = NI$ …………………………..(1.23)

It is analogous to emf in electrical circuit and the Reluctance is given by

$\Re = \frac{L}{\mu_o \mu_r A}$ ………………………………………………..(1.24)

It is similar to resistance in electrical circuit.

The terms concerning magnetic circuit can be summarized as follows:

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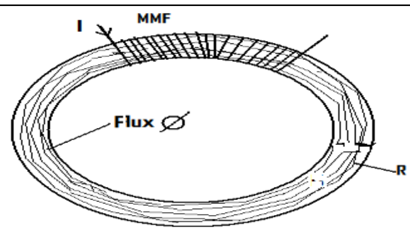
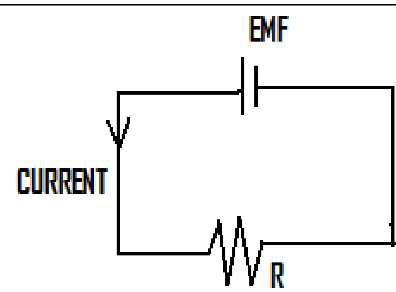
(1) Magneto motive force (mmf): It drives the flux through the magnetic circuit and it is defined as the work done in carrying a unit magnetic pole through the entire magnetic circuit. Its unit is Ampere Turns (AT).

(2) Reluctance: This is the measure of the opposition to the flow of magnetic flux offered by the material (it is analogous to resistance in electric circuit). Its unit is AT/Wb.

(3) Permeance: It is the reciprocal of reluctance and it is the readiness with which magnetic flux is developed in a material. It is analogous to conductance in electrical circuit. It is measured in Wb/AT or Henry.

(4) Reluctivity: This is the reluctance per unit length for each unit cross sectional area of the magnetic path. It is analogous to resistivity in electrical circuit.

The following table gives the comparison between Magnetic and Electric circuits.

	MAGNETIC CIRCUIT	ELECTRIC CIRCUIT
S/N		
1	Flux = mmf/Reluctance	Current = emf/Resistance
2	mmf (Ampere – Turns) AT	emf (volts)
3	Flux Density B (Wb/m^2)	Current density δ (A/m^2)
4	Flux (Weber, Wb)	Current (Ampere, A)
5	Reluctance $\Re = \frac{L}{\mu A}$	Resistance $R = \frac{L}{\rho A}$

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6	Permeance $= \frac{1}{\Re}$	Conductance $= \frac{1}{R}$
8	Reluctivity	Resistivity
9	Permeability = 1/Reluctivity	Conductivity = 1/ Resistivity
10	Total mmf = $\phi S_1 + \phi S_2 + \phi S_3$……. For series connections	Total emf = $IR_1 + IR_2 + IR_3$……. For series connections

1.2.7 LEAKAGE FLUX AND LEAKAGE FACTOR (COEFFICIENT):

The leakage flux is the flux that follow a path not intended for it. This is because, unlike electric current the magnetic flux does not have a perfect insulator (even free space). Therefore some flux will always leak to the surrounding medium. The other flux that remains after the leakage flux is called the useful flux. (Figure 1.10).

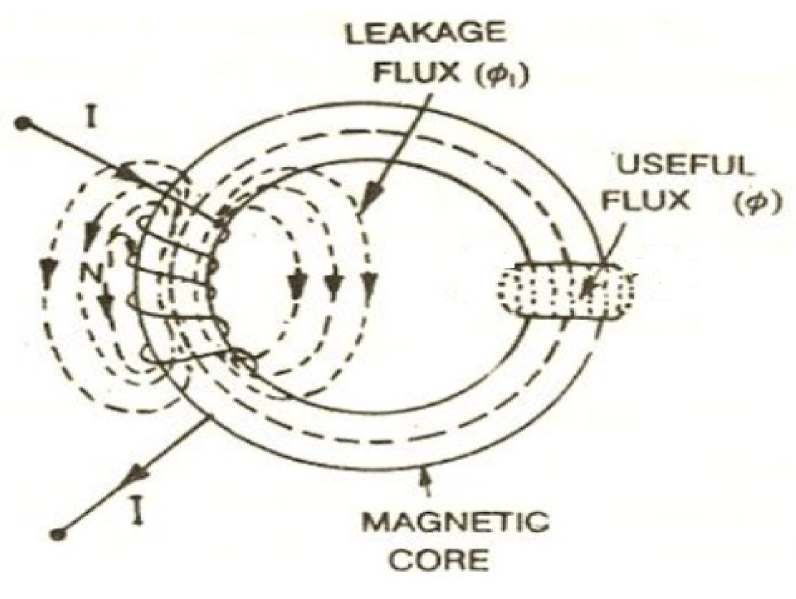

Figure 1.10 Leakage Flux

The leakage factor is given by:

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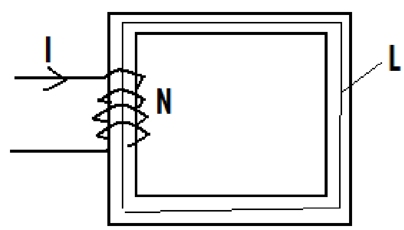
Leakage factor $= \frac{Total\ Flux}{Useful\ Flux} = \lambda = \frac{\phi_t}{\phi}$ ……………………………………….(1.25)

EXAMPLE 1.3

A square iron core has a mean length of magnetic path of 120cm, cross sectional area of 4 cm^2 and relative permeability of 1400. A coil of 600 turns is wound on one of the arms of the core. Calculate the magnetic flux density and field strength in the iron core if a current of 4.5A is supplied to the coil. Take $\mu_o = 4\pi \times 10^{-7}$ Henry/meter.

SOLUTION:

The core is represented in the following figure:

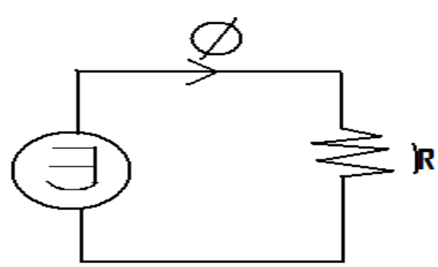


Mean length of the core L = 120 cm = 1.2m

Cross sectional Area A = $4cm^2 = 0.0004m^2$

N = 675 turns, I = 4.5A and $\mu_r = 1400$

The above circuit can be represented as follows:



mmf, Э = NI = 675 x 4.5 = 3,037.5 AT

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Reluctance of the path is given by $\Re = \dfrac{L}{\mu A} = \dfrac{L}{\mu_o \mu_r A}$ = 1.705 x 10^6 AT/Wb

And therefore the flux is given by $\phi = \dfrac{\Im}{\Re} = 1.78 \times 10^{-3}$ Wb

The flux density B = ϕ/A = 4.45 Tesla and the field strength is given by H = B/μ = B/$\mu_o\mu_r$ = 2,531.25 AT/m.

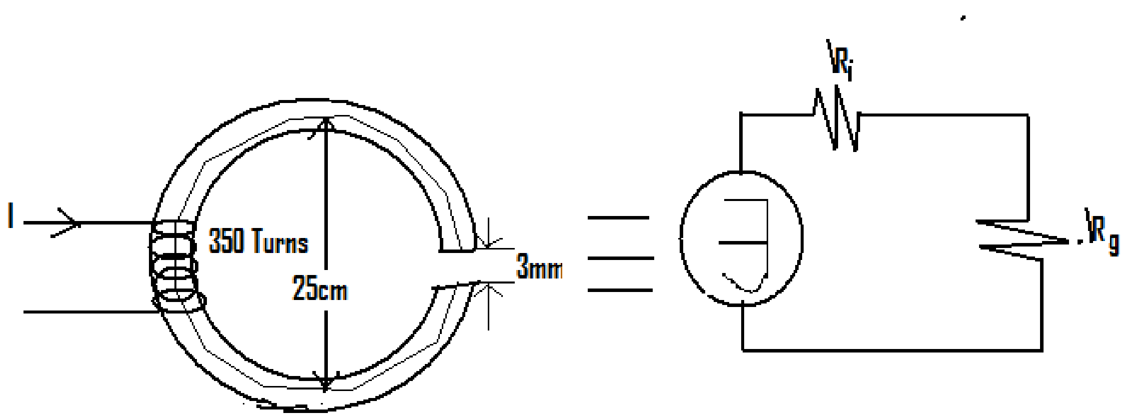
H can also be found using H = Э/L = 3,037.5/1.2 = 2,531.25 AT/m.

EXAMPLE 1.4

A steel ring has a mean diameter of 25cm and a cross sectional area of 8cm^2. The ring has a coil of 350 turns on one side and an air gap of 3mm on the other side. Calculate the current required in the coil to generate a flux of 0.5mWb in the air gap. Given that μ_r =1200 and the leakage factor for the steel is 1.22.

SOLUTION:

The steel ring can be represented by the following circuit.



N = 350 turns, ϕ_g = 0.5mWb = 0.0005Wb , μ_r =1200, λ = 1.22, Area A = 8 x 10^{-4}m^2, Length of the air gap L_g = 0.003m, diameter of the ring d = 0.25m and the length of the iron ring = πd - L_g = π x 0.25 – 0.003 = 0.7824m.

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$$\Re_i = \frac{L_i}{\mu A} = \frac{L_i}{\mu_o \mu_r A} = 6.486 \text{ x } 10^5 \text{AT/Wb}$$

$$\Re_g = \frac{L_g}{\mu A} = \frac{L_g}{\mu_o A} = 2.984 \text{ x } 10^6 \text{AT/Wb}$$

The total mmf required = $\phi_i \Re_i + \phi_g \Re_g$

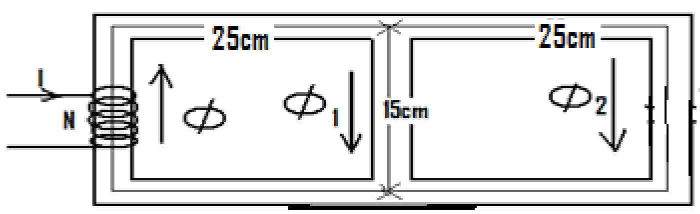
Now $\lambda = \frac{Total\ Flux}{Useful\ Flux} = \frac{\phi_i}{\phi_g}$ therefore $\phi_i = \lambda \times \phi_g = 6 \text{ x } 10^{-4}$Wb

Hence $\Im = \phi_i \Re_i + \phi_g \Re_g = 1{,}881.16$ AT

And therefore the current I = $\frac{mmf}{N} = \frac{\Im}{N} = 5.37$A

EXAMPLE 1.5

A cast steel magnetic structure made of a bar of section 8cm x 2cm (as shown). Determine the current that the 500 turns magnetizing coil on the left limb will take so that a flux of 2mWb is produced at the right limb. Take μ_r =600 and neglect leakage.



SOLUTION:

The circuit can be represented by the following circuit:

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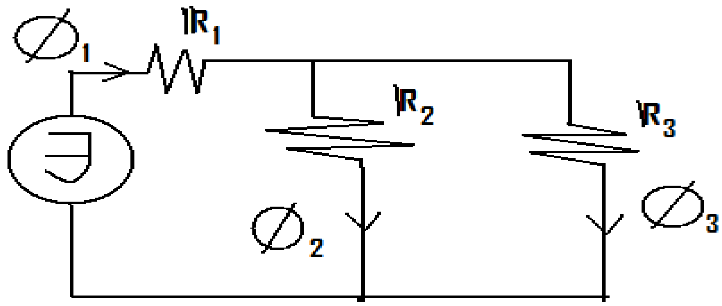

Here L_1 = 25cm = 0.25m, L_2 = 15cm = 0.15m and L_3 = 25cm = 0.25m

The cross sectional area A = 8cm x 2cm = 0.0016m^2, N = 500, ϕ_3 = 2mWb = 0.002Wb and the relative permeability μ_r = 600.

The reluctances are:

$$\Re_1 = \frac{L_1}{\mu A} = \frac{L_1}{\mu_o \mu_r A} = 2.072 \text{ x } 10^5 \text{ AT/Wb}$$

$$\Re_2 = \frac{L_2}{\mu A} = \frac{L_2}{\mu_o \mu_r A} = 1.243 \text{ x } 10^5 \text{ AT/Wb and}$$

$$\Re_3 = \frac{L_3}{\mu A} = \frac{L_3}{\mu_o \mu_r A} = 2.072 \text{ x } 10^5 \text{ AT/Wb}$$

Now using KCL formula we have:

$$\phi_3 = \frac{\phi_1 \Re_2}{\Re_2 + \Re_3} \Rightarrow \phi_1 = \frac{\phi_3 \left(\Re_2 + \Re_3\right)}{\Re_2} = 5.33 \text{ x } 10^{-3}\text{Wb}$$

The total reluctance is given by $\Re_T = \Re_1 + \Re_2 // \Re_3 = \Re_1 + \frac{\Re_2 \bullet \Re_3}{\Re_2 + \Re_3} = 2.85 \text{ x } 10^5\text{Wb}$

And therefore $\Im = \phi_1 \Re_T = 1519.05$ AT from which the current is $I = \frac{\Im}{N} = \frac{1519.05}{500} = \underline{3.04\text{A}}$

1.2.8 HYSTERESIS LOSES:

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Hysteresis is the property of a magnetic substance in which the flux density B lags behind the magnetizing force (magnetic field strength), H as the substance undergoes magnetization. It can also be defined as the quality of a magnetic substance, due which energy is dissipated in the substance on reversal of its magnetism. The loses due this characteristic of the magnetic material is called " Hysteresis losses".

Let us consider a cycle of magnetization of an un magnetized iron bar, that is by placing it in a field of solenoid (Figure 1.11). The field produced by the solenoid can be increased by increasing the supply current I (i.e. H = NI/L).

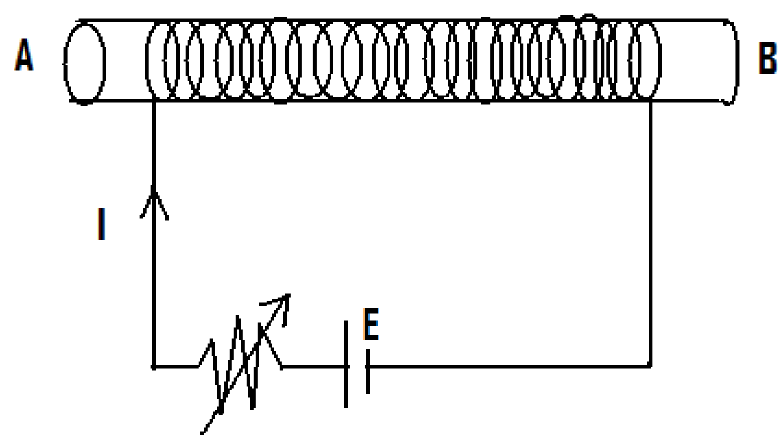

FIG. 1.11 A SOLENOID

Let H be increase in steps from zero to certain maximum value and the corresponding values of flux density B are taken. If we plot the graph of B against H a curve like OA in figure 1.12 is obtained. The material is saturated at H = OM and has corresponding maximum flux density of B_m.

If H is decreased gradually the flux density will not decrease along OA, rather it will decrease along AC due to the property of the material.

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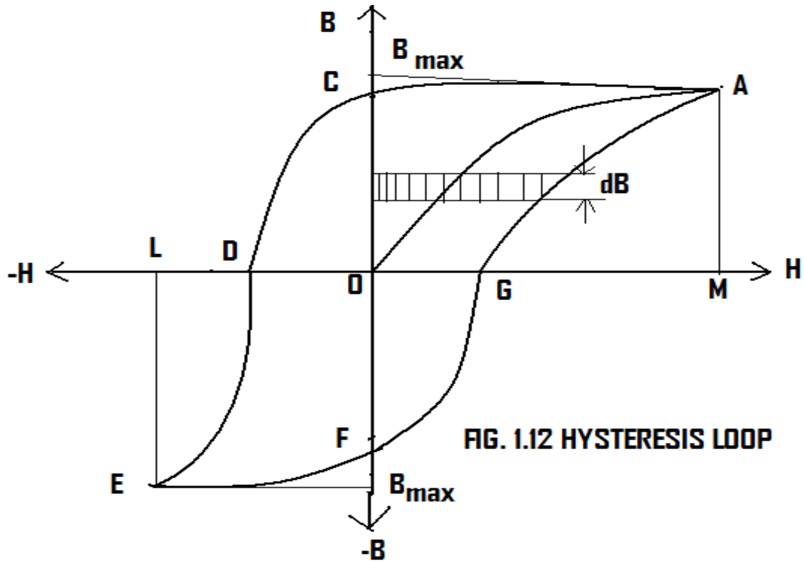

FIG. 1.12 HYSTERESIS LOOP

When H is decrease to zero, B is not zero but has a value B_r = OC. This means by removing the magnetizing force H, the iron bar is not completely demagnetized. This value of B that remains (OC) measure the "**retentivity or remanence**" of the material and B_r is called the "**residual flux".**

To demagnetize the bar completely we have to apply the magnetizing force in the reverse direction (i.e. by reversing the current through the solenoid). In this case B reduce to zero at point D where H = OD. This value of H required to wipe out the residual magnetism is known as "Coercive force".

If after the residual magnetization has been removed, value of H is further increased in the negative direction the iron bar will reach a magnetic saturation at point E (i.e. where H = OL). Similarly if H is taken from its minimum value OL to its maximum value OM a similar curve EFGA is obtained. If we go again from G to L back to G, the same curve GACDEFG is obtained.

The above loop GACDEFG represents what is called "**hysteresis loop**" and its area represent the energy spent in taking the iron bar through one cycle of magnetization. The bigger the area of the loop the more retentive (ability to retain residual magnetism) is the material and the more suitable is the material for permanent magnet.

Let us find this energy per cycle of magnetization.

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The induce emf in the coil as the result of changing the flux (i.e. the current) is given by

$$e = N\frac{d\phi}{dt} = N\frac{d(BA)}{dt} = NA\frac{dB}{dt}$$

The power rate of expenditure of energy is given by:

$$P = eI = NAI\frac{dB}{dt}$$

But $HL = NI \Rightarrow I = \frac{HL}{N}$

$$\therefore P = NA \times \frac{HL}{N} \bullet \frac{dB}{dt} = ALH\frac{dB}{dt}$$

And the Energy spent in time dt = Pdt = ALHdB……………………(1.26)

Total net work done for one cycle of magnetization is given by:

$W = AL$□ ……………………………………………………..(1.27)

Now the □ = Area of loop i.e. the area between B/H curve. Therefore

Work done / cycle = AL x Area of loop or

Work done/cycle/m^3 = Area of loop.

Now the scale factors of B and H should be taken into consideration. Let the scale be

1cm = xAT/m on H axis and

1cm = yWb/m^2 on B axis then:

Work done/cycle/m^3 = $xy\times$ Area of loop(cm^2)……………………………………(1.28)

The hysteresis loss can also be found using Steinmetz Hysteresis law which says:

Hysteresis loss $W_h \alpha B_{\max}^{1.6}$

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$\Rightarrow W_h = \eta B_{max}^{1.6}$ Joule/m^3/cycle……………………………………………………………….(1.29)

The index 1.6 is empirical and it holds for values of B_{max} from 0.1 T to 1.2T and if the B_{max} is out of this range (B_{max} < 0.1 or B_{max} > 1.2) then the index is greater than 1.6 (mostly 1.8). The constant η is called the Steinmetz hysteresis coefficient.

N/B the above loses are calculated in J/m^3/cycle and therefore to obtain each one of them in watt, we need to multiply them by fV where f = frequency and V = volume of the core material. As such:

$W_H = xy \times$ Area of loop(cm^2) x fV Watts…………………………………………(1.30) and

$W_H = \eta B_{max}^{1.6} \bullet fV$ Watts……………………………………………………………(1.31)

1.2.9 EDDY CURRENT LOSES:

When the magnetic flux of the solenoid changes from maximum value to minimum value and back to maximum value again (cycle of magnetization). The core (iron bar) itself will have a small emf induced in it (i.e. the iron bar being a conductor also). This induced emf will create a circulating current in the bar, which will heat up the bar and create a loss.

This loss due to the circulating current in the core of an armature or solenoid is called the "**eddy current loss".** It can be minimized by laminating the armature core.

The eddy current loss is given by the following relation:

$W_e = KB_{max}^2 f^2 t^2 V^2$ Watt……………………………………………………(1.32)

Where:

B_{max} = maximum flux density, f = frequency, t = thickness of lamination, V = volume of the core material and K = constant.

N/B The thickness of lamination should be very thin to minimize the eddy current loss.

EXAMPLE 1.6

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Determine the hysteresis loss in an iron core weighing 50 Kg having a density of 7.8 x 10^3 Kg/m^3 when the area of the hysteresis loop is 150cm^2, frequency is 50Hz and scale on X and Y axis are 1cm = 30AT/cm and 1cm = 0.2wb/m^2 respectively.

SOLUTION:

Area of loop = 150cm^2, f = 50Hz, ρ =7.8 x 10^3 Kg/m^3, x = 30AT/cm = 3000AT/m, y = 0.2T

Volume of 50Kg of iron $V = \frac{M}{\rho} = \frac{50}{7.8 \times 10^3}$ = 6.41 x 10^3 m^3.

The hysteresis loss = = $xy \times$ Area of loop(cm^2) J/m^3/cycle = 90,000 J/m^3/cycle

And the total loss =>

W_H = $xy \times$ Area of loop(cm^2) x fV = 90,000 x 50 x 6.41 x 10^{-3} = 28846.2W = 28.85KW

EXAMPLE 1.7

Calculate the loss of energy caused by hysteresis in one hour in 50Kg of iron when subjected to cyclic magnetic changes. The frequency is 25Hz and the area of the loop represents 240J/m^3/cycle. Take the density of iron to be 7800Kg/m^3.

SOLUTION:

M = 50Kg, ρ = 7800Kg/m^3, f = 50Hz, time t = 1hour = 3600s,

Hysteresis loss = $xy \times$ Area of loop(cm^2) = 240 J/m^3.cycle, the volume is given by V = M/ρ = 50/7800.

Therefore W_H = $xy \times$ Area of loop(cm^2) x fV = 240 x 25 x 50/7800 = 38.4 watts

The total loss = W_H x t = W_H x 3600 = 138,240J = 138.24KJ

EXAMPLE 1,8

A certain transformer has core volume of 6.41 x 10^{-3}m3. The maximum flux density is 0.9T and the frequency is 50Hz. Calculate the hysteresis loss under these conditions. If the maximum flux

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density was increased to 1.1Wb/m^2 and the frequency reduced to 40Hz what will be the new loss. Take the Steinmetz coefficient to be 1120 Kg/m^3 and assume the hysteresis loss over the range to be proportional to $B_{\max}^{1.7}$.

SOLUTION:

V = 6.41 x 10^{-3}m^3, η = 1120

N/B The coefficient 1.7 is specified in the question. If it is not given we can use 1.6 as illustrated in equation (1.31).

$\therefore W_H = \eta B_{\max}^{1.7} \bullet fV$

Now for the first case f = 50Hz and B_{max} = 0.9T we have:

$W_H = \eta B_{\max}^{1.7} \bullet fV = 1120 \times 0.9^{1.7} \times 50 \times 6.41 \times 10^{-3} =$ <u>300W</u>

And for the second case f = 40Hz and B_{max} = 1.1T we have:

$W_H = \eta B_{\max}^{1.7} \bullet fV = 1120 \times 1.1^{1.7} \times 40 \times 6.41 \times 10^{-3} =$ <u>337.8W</u>

EXAMPLE 1.9

The area of the hysteresis loop obtained with a certain specimen of iron was 9.3cm^2. the coordinate scales were 1cm = 1000AT/m and 1cm = 0.2 Wb/m^2. Calculate (a) The hysteresis loss per m^3 if the test was done at a frequency of 50Hz and a maximum flux density of 1.5Wb/m^2 (b) the hysteresis loss per m3 for f =30Hz and max flux density of 1.2Wb/m^2. Assuming the loss to be proportional to $B_{\max}^{1.8}$.

SOLUTION:

(a) The hysteresis loss per m^3 is given by:

W_H/ m^3 = $xy \times$ Area of loop(cm^2) x f W/m^3 = 1000 x 0.2 x 9.3 x 50 = <u>93,000Watts/m^3</u>

(b) The hysteresis loss in the second case can be found using Steinmetz equation

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$W_H = \eta B_{\max}^{1.8} \bullet fV$ Watts $\Rightarrow W_H / m^3 = \eta B_{\max}^{1.8} \bullet f$

The Steinmetz constant can be found using the values in (a) above

i.e. $W_H / m^3 = \eta B_{\max}^{1.8} \bullet f$ from which

$93,000 = \eta \times 1.5^{1.8} \times 50 \Rightarrow \eta = 896.5$

Hence for (b) the loss is given by:

$W_H / m^3 = \eta B_{\max}^{1.8} \bullet f$ = 896.5 x $1.2^{1.8}$ x 30 = $\underline{37{,}342 \text{ Watts/m}^3}$

EXAMPLE 1.10

An iron ring of mean length 30cm is made of three pieces of cast iron, each have the same length but their respective diameters are 4, 3 and 2.5cm. An air gap of length 0.5mm is cut in the 2.5cm piece. If a coil of 1,000 turns is wound on the ring, find the value of current it has to carry to produce a flux density of 0.5T in the air gap. B/H characteristic of cast iron may be drawn from the following readings:

B (Tesla)	0.1	0.2	0.3	0.4	0.5	0.6
H (AT/m)	280	620	990	1,400	2,000	2,800

Permeability of free space = 4π x 10^{-7} Henry/meter. Neglect leakage and fringing.

(AK & BL Theraja)

SOLUTION:

The cast iron can be represented in the following figure: L_1=9.95cm = .0995m, L_2 =L_3= 10cm =0.1m and L_g = 0.5mm = 0.0005m

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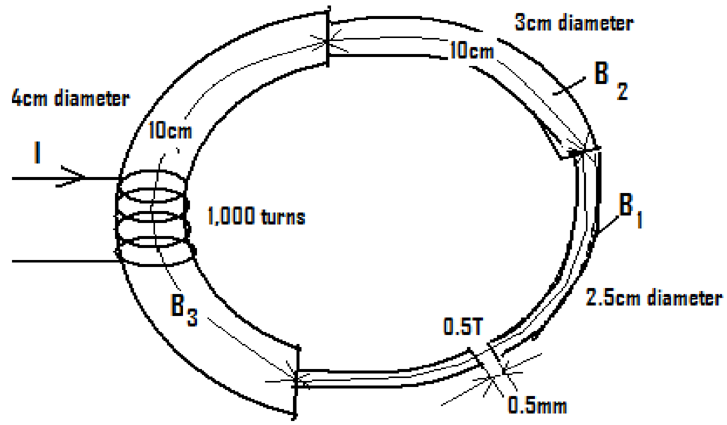

The magnetic flux in the air gap = The magnetic flux in all the three pieces = B x Area of air gap = 0.5 x π x $(1/2 \times 2.5/100)^2 = 2.454 \times 10^{-4}$ Wb.

Now to calculate the mmf in the pieces we need to calculate the flux density in each part. Hence

The flux density in the air gap, $B_g = 0.5T$

The flux density in the 2.5cm piece $B_1 = 0.5T$, i.e. it has the same cross section with the air gap.

Flux density in 3cm piece $B_2 = \dfrac{2.454 \times 10^{-4}}{\pi \times \left(\frac{1}{2} \times \frac{3}{100}\right)^2} = 0.11T$

Flux density in 4cm piece $B_3 = \dfrac{2.454 \times 10^{-4}}{\pi \times \left(\frac{1}{2} \times \frac{4}{100}\right)^2} = 0.06T$

Next is to plot the graph of B against H using the values in the table above, the values of the field strength, H corresponding to B_1, B_2 and B_3 will be taken from the graph. And the mmf is calculated using the field strength. The graph should be plotted using appropriate scales and it should be similar to the one below.

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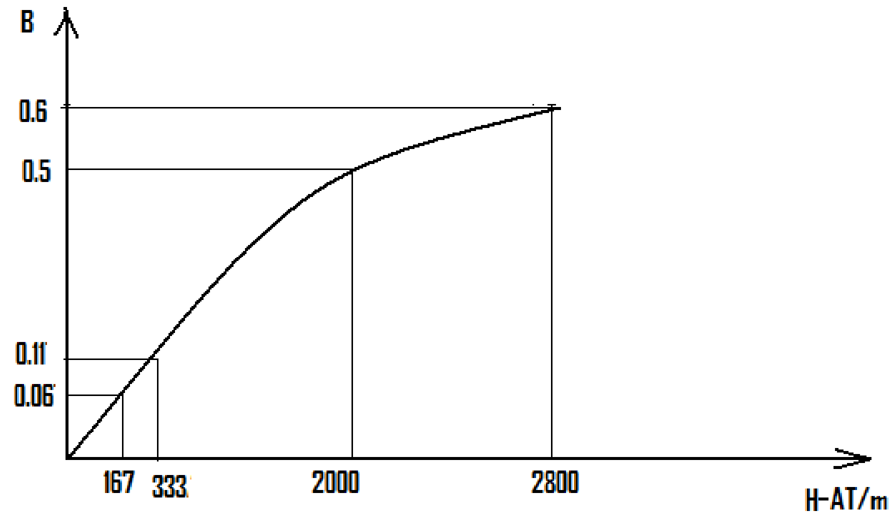

The corresponding values of the field strength from the graph are:

H_1 (corresponding to B_1 =0.5T) = 2,000AT/m

H_2 (corresponding to B2 =0.11T) = 333AT/m

H_3 (corresponding to B_3 =0.06T) = 167AT/m

The field strength of the air gap is given by $H_g = \frac{B_g}{\mu_o} = \frac{0.5}{4\pi \times 10^{-7}}$ = 3.98 x 10^5AT/m.

The total mmf in the ring is given by:

$\Im_T = H_1 L_1 + H_2 L_2 + H_3 L_3 + H_g L_g$ = 2000 x 0.0995 + 333 x 0.1 + 167 x 0.1 + 3.98 x 10^5 x 0.0005 = 447.94AT

And therefore the current is given by; $I = \frac{\Im_T}{N} = \frac{447.94}{1000}$ = 0.44795 $\approx$ 0.45A

EXERCISE I

Q1 A wire is bent into a plane to form a rectangle of 20cm by 10 cm side (figure Q1) and a current of 200A is passed through it. Calculate the field strength set up at the centre of the rectangle.

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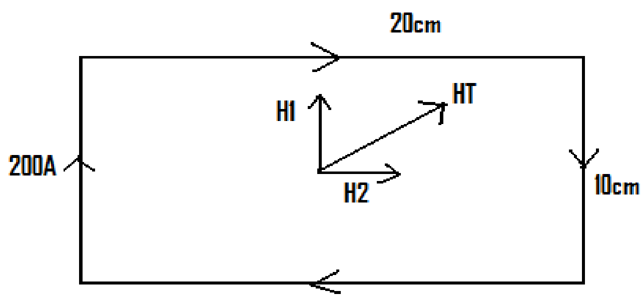

FIG. Q1

Q2 A cast steel magnetic structure is made up of a bar of cross sectional area 5cm x 4cm as shown in figure Q2 below.

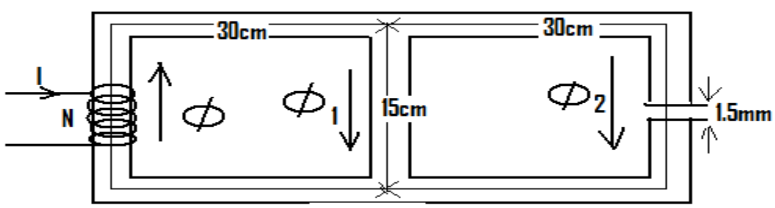

FIG. Q2

Calculate the **magnetic flux density** in each limb if the magnetization coil is of 600 turns and it carries a current of 7A. Take $\mu_r = 700$ and neglect leakage.

Q3 A coil of 650 turns having resistance of 30 ohms is wound uniformly on an iron ring of mean diameter of 16cm and cross sectional area of $2cm^2$. It is connected to a 30V dc supply.

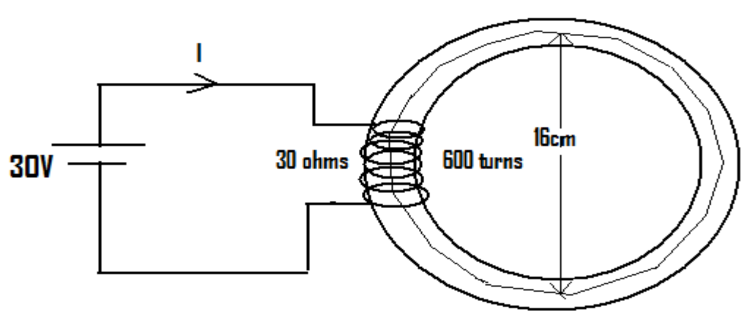

FIG. Q3

Taking $\mu_r = 600$ calculate the values of

(i) mmf in the coil.

(ii) The magnetizing force (field strength).

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(iii) The total flux in the iron core.

(iv) The reluctance of the ring.

Q4 In a magnetizing test on a certain iron bar of cross sectional area of $2cm^2$, the following values were obtained.

H (AT/m)	1,900	2,000	3,000	4,000	4,500	3,000	1,000	0	-1,000	-1,900
B (Tesla)	0	0.2	0.58	0.70	0.73	0.72	0.63	0.54	0.38	0

Draw the hysteresis loop. How do you obtain the area of the loop in cm^2 from the graph? Estimate the residual flux and coercive force of the bar. Find the loss in watt if the volume of the iron bar is $0.25dm^3$ and the frequency is 60Hz.

Q5 In a transformer, the hysteresis loss is 180W when the value of B_{max} = 1.1 Tesla and when supply frequency is 50Hz. What would be the loss when the value of B_{max} reduced to $0.9Wb/m^2$ and the supply frequency is reduced to 40Hz. Also calculate the corresponding eddy current loss if the core is given a lamination of 0.2mm thickness and the eddy current loss constant is 1.16×10^{15}, take the volume of the core to be $25,000cm^3$.

Q6 A magnetic core of uniform cross section of $15cm^2$ is casted into shell type with air gaps on both arms as shown in the following figure:

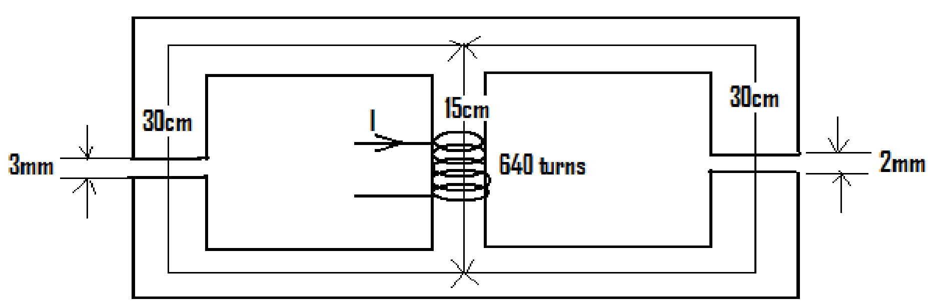

FIG. Q6

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Determine the current I to be supplied to the 640 turns coil in the central limb to set up a flux of 5mWb in the 2mm air gap. Take $\mu_r = 4\pi \times 10^{-7}$H/m and neglect leakage.

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CHAPTER TWO: TRANSFORMERS

2.1 INTRODUCTION:

A transformer is a device that transform electrical power from one electrical circuit to another electrical circuit through the medium of magnetic field (from one voltage level to another) without change in the frequency. The electric circuit that receives energy from the supply is called "primary winding" of the transformer, whereas the one that delivers energy to the load is called "secondary winding".

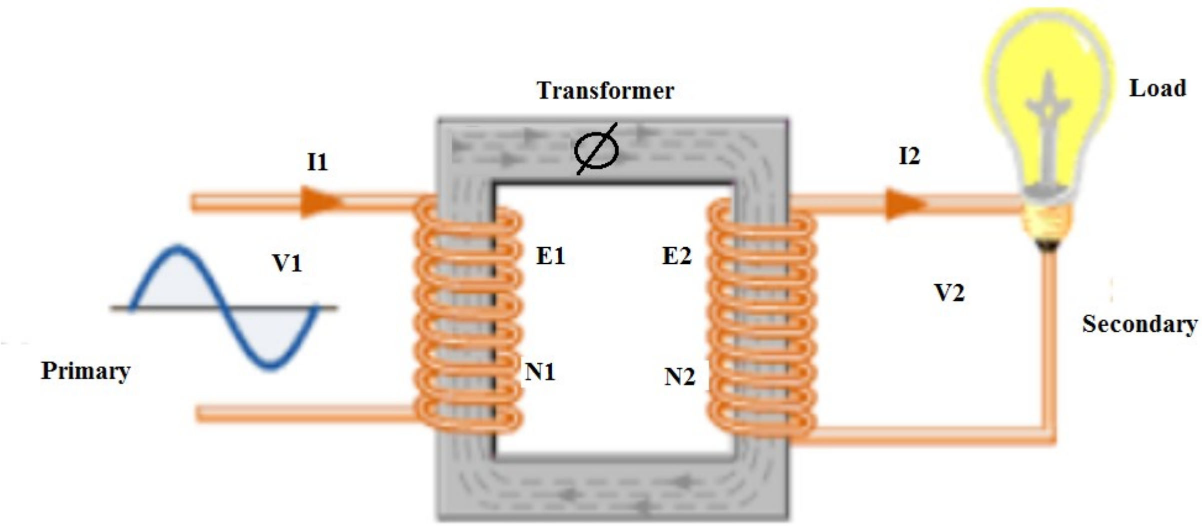

Figure 2.1 Transformer

In a transformer where the secondary windings has more turns than the primary windings, we have a **"step up transformer"** in case the primary has more turns than the secondary side then we will have a "**step down transformer**". However when referring to the windings of a particular transformer, the terms "**high voltage**" and "**low voltage**" windings are used to refer to the windings with higher and lower number of turns respectively. Hence a transformer can be step up or step down only when it has been put to service.

2.2 PRINCIPLE OF OPERATION OF A TRANSFORMER:

Generally the transformer works with ac supply. Therefore when an ac voltage is applied to the primary winding of the transformer in figure 2.1, it will set up a an alternating flux along the

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core, which depends on the number of turns in the primary and the reluctance of the path (section 1.2.6) . This alternating flux will induce emfs in both the primary winding and secondary winding (i.e. E_1 and E_2), depending on their number of turns N_1 and N_2.

Now, let us consider the case of an ideal transformer, that is a transformer with no winding loses and negligible leakage flux (no loses), having the voltage V_1 applied to the primary side such that V1 = $V_{max}\sin\omega t$. This will cause the primary current and hence the generated flux to be sinusoidal in nature: therefore

$\phi = \phi_{max}\sin\omega t$.

The emf induced in the primary winding is given by:

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \omega \phi_{max} \cos \omega t$$

$$= N_1 \omega \phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1)$$

Similarly: $e_2 = N_2 \omega \phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$ ……………………………………(2.2)

The rms values of the induced emfs =>

$$E_1 = \frac{N_1 \omega \phi_{max}}{\sqrt{2}} = \frac{N_1 \times 2\pi f \times \phi_{max}}{\sqrt{2}} = 4.44 N_1 f \phi_{max} \ldots\ldots\ldots\ldots(2.3)$$ and

$$E_2 = 4.44 N_2 f \phi_{max} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.4)$$

2.3 EQUIVALENT CIRCUIT OF A TRANSFORMER:

The equivalent circuit of a transformer is a circuit model of the transformer in which the transformer is represented by resistance, inductance, voltages and emfs for easy analysis by direct application of circuit theory.

The equivalent circuit of the transformer above can be drawn by representing the resistance and leakage reactance of both the primary and secondary windings with a series resistance and

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reactance. A parallel of resistance and reactance (shunt branch) in the primary side represents the magnetizing reactance and core loss resistance.

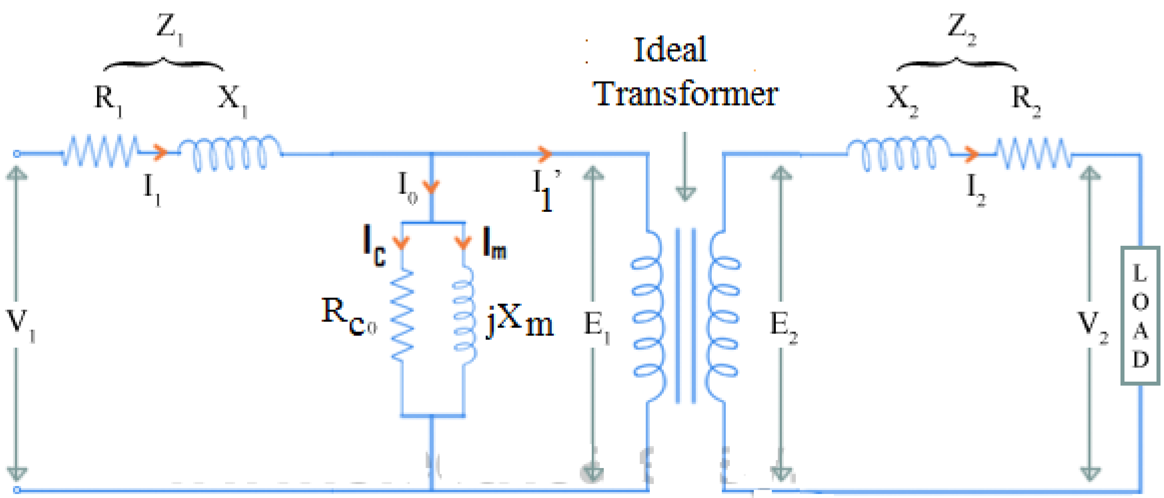

Figure 2.2 Transformer Equivalent Circuit

By analyzing the primary circuit we have:

$$V_1 = E_1 + I_1(R_1 + jX_1) = E_1 + I_1Z_1 \qquad (2.5)$$

And from the secondary side we have:

$$E_2 = V_2 + I_2(R_2 + jX_2) = V_2 + I_2Z_2$$

$$\Rightarrow V_2 = E_2 - I_2Z_2 \qquad (2.6)$$

Where R_1 & R_2 are the primary & secondary winding resistances

X_1 & X_2 are the primary & secondary leakage reactances

R_C is the core loss resistance and

X_m is the magnetizing reactance

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N/B I_O is the no load current of the transformer, it is the current that the transformer will draw when the secondary of the transformer is open circuited. It is made up of two component the core loss current I_C and magnetizing current I_μ ($I_O = I_C + I_m$). I_O is about 1% of the full load current of the transformer.

2.4 PHASOR DIAGRAM OF A TRANSFORMER:

When a transformer is operated on no load it only draw the no load current $I_O = I_C + I_m$. the core loss component of the current I_C has the same direction as the applied voltage and the magnetizing component I_m is lagging the voltage by 90°. Therefore I_O is the resultant of I_C and I_m as shown in the diagram below.

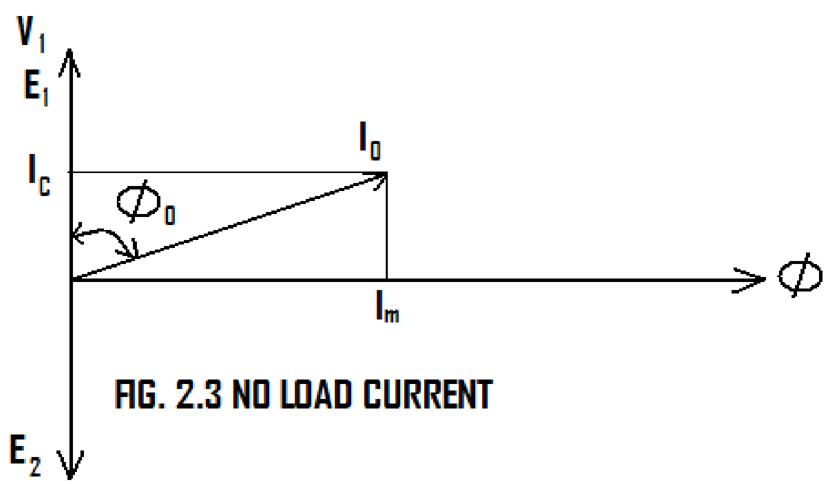

FIG. 2.3 NO LOAD CURRENT

The general phasor diagram of the transformer can be realized using equations (2.5) and (2.6)

$$V_1 = E_1 + I_1 R_1 + jI_1 X_1$$

$$E_2 = V_2 + I_2 R_2 + jI_2 X_2$$

Also $I_1 = I_o + I_1^{'}$ that is the vector sum.

$I_1^{'}$ is the load component of the primary current.

Using the rules for vectors addition the phasor diagram obtained from the above equations is shown in figure 2.4.

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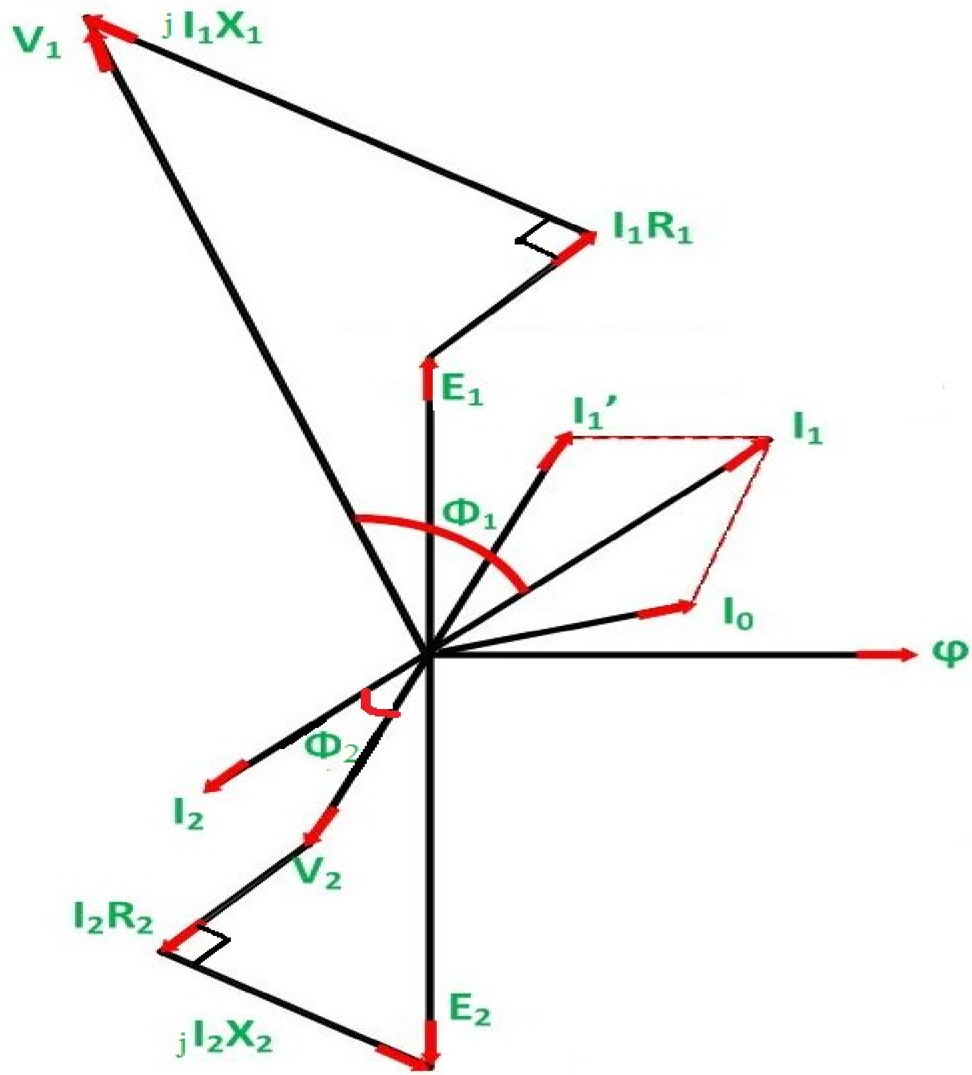

Figure 2.4 General Phasor Diagram of Transformer

N/B I_1R_1 is a vector parallel to I_1 and jI_1X_1 is a vector 90^o to the direction of I_1R_1 (because of j) the same thing is applied to I_2R_2 and jI_2X_2.

2.5 TRANSFORMER PARAMETERS FROM SHORT CIRCUIT AND OPEN CIRCUIT TESTS:

Open circuit and short circuit tests are the tests carried out on a transformer to find the transformer's parameters. (R_{e1}, X_{e1}, X_m, R_c and K).

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In the open circuit test the high voltage side of the transformer is open circuited and normal voltage (V_1) is applied to the low voltage side (figure 2.5). the wattmeter measures the no load power (core loss) and the Ammeter measures the no load current.

On the other hand in short circuit test the low voltage side is short circuited and a low voltage is applied to the high voltage side (the voltage that will produce rated current in the transformer) as shown in figure 2.6.

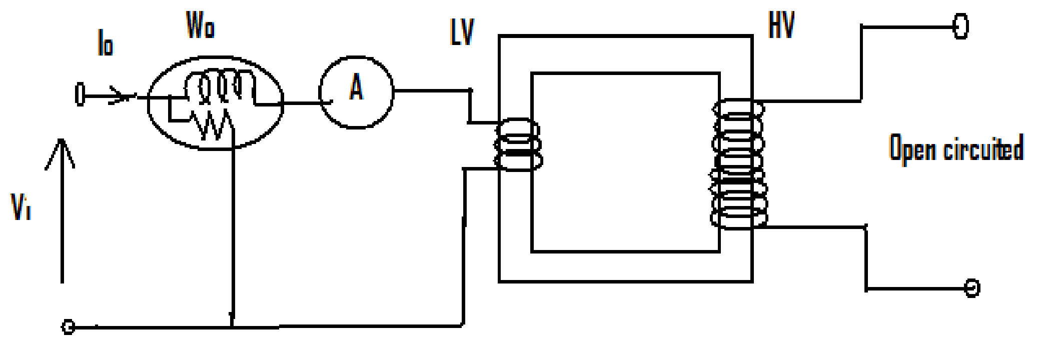

FIG. 2.5 OPEN CIRCUIT TEST

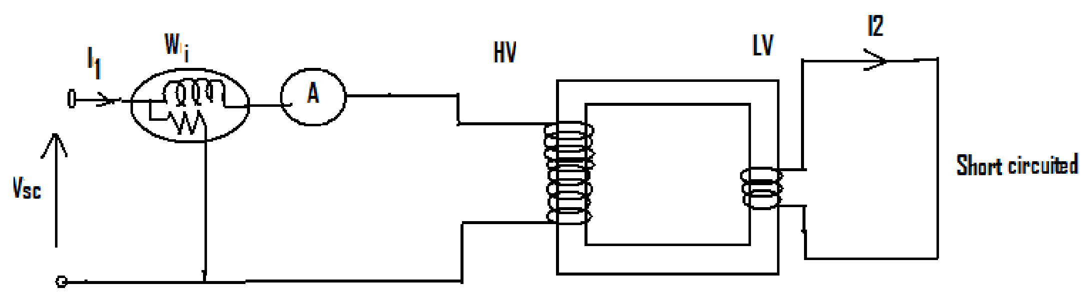

FIG. 2.6 SHORT CIRCUIT TEST

From the open circuit test we have the following analysis:

$$W_o = V_1 I_o \cos\phi_o \Rightarrow \cos\phi_o = \frac{W_o}{V_1 I_o} \quad \text{.....(2.7a)}$$

$$R_c = \frac{V_1}{I_o \cos\phi_o} \quad \text{.....(2.7b)}$$

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$$X_m = \frac{V_1}{I_o \sin\phi_o} \quad \text{……………(2.7c)}$$

$$K = \frac{V_2}{V_1} \quad \text{……………(2.7d)}$$

Where $\cos\phi_o$ is the no load power factor, R_c is the core loss resistance, X_m is the magnetizing reactance and V_2 is the secondary open circuit voltage.

And from the short circuit test we have:

$$Z_{e1} = \frac{V_{sc}}{I_1} \quad \text{……………(2.8a)}$$

$$R_{e1} = \frac{W_i}{I_1^2} \quad \text{……………(2.8b)}$$

$$X_{e1} = \sqrt{\left(Z_{e1}^2 - X_{e1}^2\right)} \quad \text{……………(2.8c)}$$

$R_{e1} = R_1 + R_2' = R_1 + R_2/K^2$ is the total resistance of the transformer referred to primary

$X_{e1} = X_1 + X_2' = X_1 + X_2/K^2$ the total leakage reactance of the transformer referred to primary.

2.6 EFFICIENCY OF A TRANSFOMER:

The efficiency of a transformer is given by the following expressions:

$$Efficiency = \frac{output}{input} \text{ or}$$

$$Efficiency = \frac{input - loses}{input}$$

$$= 1 - \frac{loses}{input}$$

Now the loses in a transformer are the copper loss and core loss.

Therefore:

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$$\eta = 1 - \frac{I_1^2 R_{e1} + W_i}{V_1 I_1 \cos\phi_1} \text{ x } 100\% \ldots\ldots(2.9)$$

$I_1^2 R_{e1}$ is the copper loss

W_i is the core loss or iron loss

$V_1 I_1 \cos\phi_1$ is the input power and

$\cos\phi_1$ is the primary power factor.

If the transformer quantities available are that of the secondary, the efficiency is given by:

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e2} + W_i} \text{ x } 100\% \ldots\ldots(2.10)$$

Where $\cos\phi_2$ is the secondary power factor.

2.7 VOLTAGE REGULATION OF A TRANSFORMER:

The voltage regulation of a transformer is the variation of the output voltage (V_2) of the transformer between no load and full load conditions, expressed as a percentage of no load voltage.

$$\therefore Voltage\ \text{Re}gulation = \frac{No \text{ load voltage - full load voltage}}{No \text{ load voltage}}$$

$$= \frac{{}_oV_2 - V_2}{{}_oV_2}$$

Now the voltage drop in the primary winding due to no load current is negligible we have:

$$Voltage\ \text{Re}gulation = \frac{V_1 \frac{N_2}{N_1} - V_2}{V_1 \frac{N_2}{N_1}}$$

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$$= \frac{V_1 - V_2 \frac{N_1}{N_2}}{V_1} \times 100\% \qquad (2.11)$$

EXAMPLE 2.1

The maximum flux density in the core of a 250/3000V, 50Hz single phase transformer is 1.2Wb/m^2. If the primary number of turns is 375. Determine:

(i) Secondary no. of turns.

(ii) EMF per turn and

(iii) Cross sectional Area of the core.

SOLUTION:

V_1 = 250V, V_2 = 3000V, f = 50Hz, N_1 = 375 turns and B_{max} = 1.2T.

(i) Using the relation $\frac{V_1}{V_2} = \frac{N_1}{N_2} \Rightarrow N_2 = N_1 \frac{V_2}{V_1}$ = <u>4,500 turns</u>

(ii) EMF per turn = $\frac{V_1}{N_1} = \frac{V_2}{N_2} = \frac{3000}{4500}$ = <u>0.67 V/turn</u>

(iii) Using the emf formula we have $E_1 = 4.44 N_1 f \phi_{max}$

$$\therefore \phi_{max} = \frac{E_1}{4.44 N_1 f} = 0.003Wb$$

And the cross sectional area of the core => $A = \frac{\phi_{max}}{B_{max}} = \frac{0.003}{1.2}$ = 0.0025m^3

EXAMPLE 2.2

The following figures were obtained from tests on a certain transformer:

OC TEST: W_o = 350W I_o = 0.5A V_1 = 3000V

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SC TEST: Wi = 500W I_1 = 10A V_{sc} = 150V

Calculate the parameters of the transformer.

SOLUTION:

From the OC TEST we have the following analysis:

$$\cos\phi_o = \frac{W_o}{I_o V_1} = \frac{350}{0.5 \times 3000} = 0.233$$

$$R_c = \frac{V_1}{I_o \cos\phi_o} = \frac{3000}{0.5 \times 0.23} = 25.7K\Omega$$

$$X_m = \frac{V_1}{I_o \sin\phi_o} = 6.17K\Omega$$

And from the SC TEST we have

$$Z_{e1} = \frac{V_{sc}}{I_1} = \frac{150}{10} = 15\Omega$$

$$R_{e1} = \frac{W_i}{I_1^2} = 5\Omega$$

$$X_{e1} = \sqrt{\left(Z_{e1}^2 - X_{e1}^2\right)} = \sqrt{\left(15^2 - 5^2\right)} = 14.14\Omega$$

EXERCISE II

Q1 A 30KVA, 50Hz single phase transformer is tested as follows:

OC TEST: 240V 5.2A 215W LV side

SC TEST: 110V 13.64A 430W HV side

The transformer has 250 turns in the primary side; Calculate:

(i) Transformer parameters

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(ii) The maximum flux density in the core of the transformer

(iii) The transformer efficiency at 0.88pf lagging and at unity pf lagging.

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CHAPTER THREE: ANALYSIS OF INDUCED EMF:

3.1 ELECTRICITY FROM MAGNETSM:

In our previous topics we mentioned that whenever an electric current flows through a conductor, a magnetic field is generated around the conductor (section 1.2.3). The process is the same if a conductor moves in a magnetic field (or vice versa), a current is established in the conductor. The phenomenon whereby an emf and hence current is induced in a any conductor which cut across or is cut by a magnetic flux is known as **"electromagnetic induction".**

The phenomenon can be illustrated using figure 3.1 below. The insulated coil N is stationary and it is found that the galvanometer is not showing any deflection when the magnet is stationary.

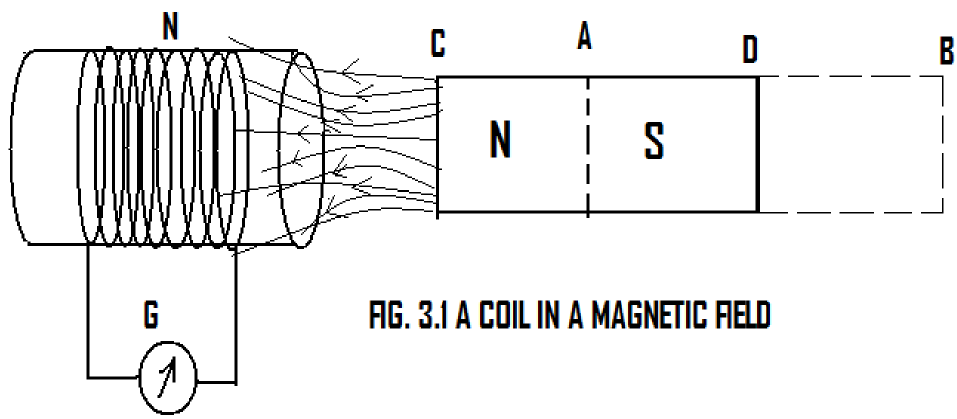

FIG. 3.1 A COIL IN A MAGNETIC FIELD

Suppose the magnet was previously in position AB (as shown) and it is suddenly moved to position CD, the galvanometer will show a deflection as long as the magnet is in motion. The galvanometer will also deflect in opposite direction as the magnet is moved from position CD back to AB. The deflection of the galvanometer indicates the production of emf in the coil and this induction occurs only when the magnet is in motion.

On the other hand a moving conductor in a stationary magnetic field will also produce an emf. In both cases the induce emf is as the result of conductor cutting the magnetic lines of force (flux). In figure 3.2 the emf is produced as long as the conductor is moving from AB to A'B' back to AB and so on.

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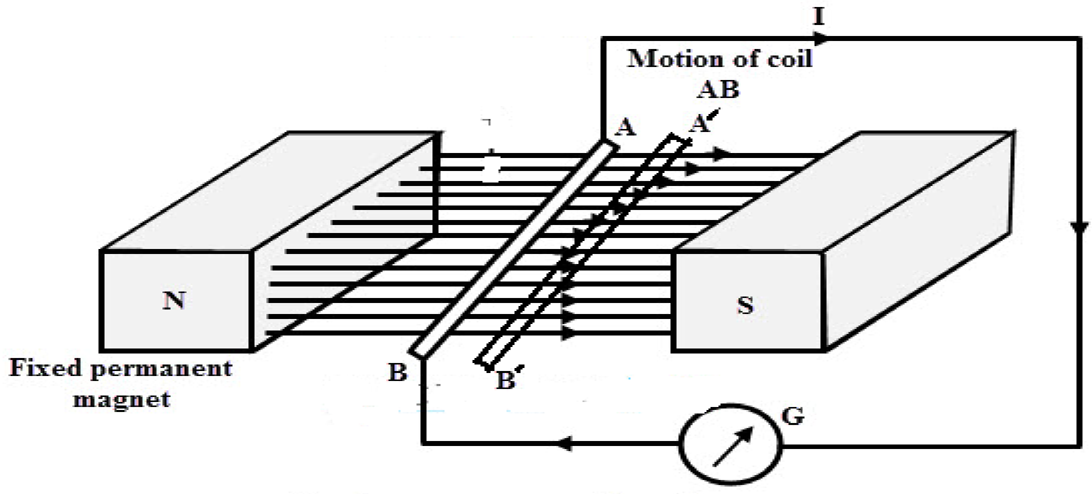

Figure 3.2 Moving Conductor in a Magnetic Field

3.2 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION:

Michael Faraday sum up the above facts into two laws known as Faraday's laws of electromagnetic induction

First law: **This law states that whenever the magnetic flux linked with a circuit changes an emf is induced in the circuit.** And

Second law: **States that the magnitude of the induced emf is directly proportional to the rate of change of the magnetic flux linkage.**

Now suppose a coil having N turns is placed in a changing flux that changes from initial value, ϕ_1 to the final value, ϕ_2 in time t seconds.

The initial flux linkage = $N\phi_1$ and final flux linkage = $N\phi_2$

The induced emf = $\dfrac{\text{Change in flux}}{Time}$

$$\Rightarrow e = \frac{N\phi_2 - N\phi_1}{t} = \frac{N(\phi_2 - \phi_1)}{t} \text{ Volts} \quad \ldots\ldots(3.1)$$

By putting it in differential form we have:

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$$e = -N\frac{d\phi}{dt} \text{ Volts} \qquad (3.2)$$

The negative sign is due to the fact that induce emf is in opposition to the direction of motion producing it.

3.3 DIRETION OF INDUCED EMF:

To determine the direction of induced current (or induced emf) one of the following three methods can be used.

(.i) **Fleming's Right Hand Rule:**

This rule states that **if we stretch out the thumb and first two fingers of our right hand such that they are perpendicular to one another and we placed the fore finger in the direction of field (North to South) and the thumb in the direction of motion of the conductor, then the second finger will point the direction of induced current (or emf)** (figure 3.3)

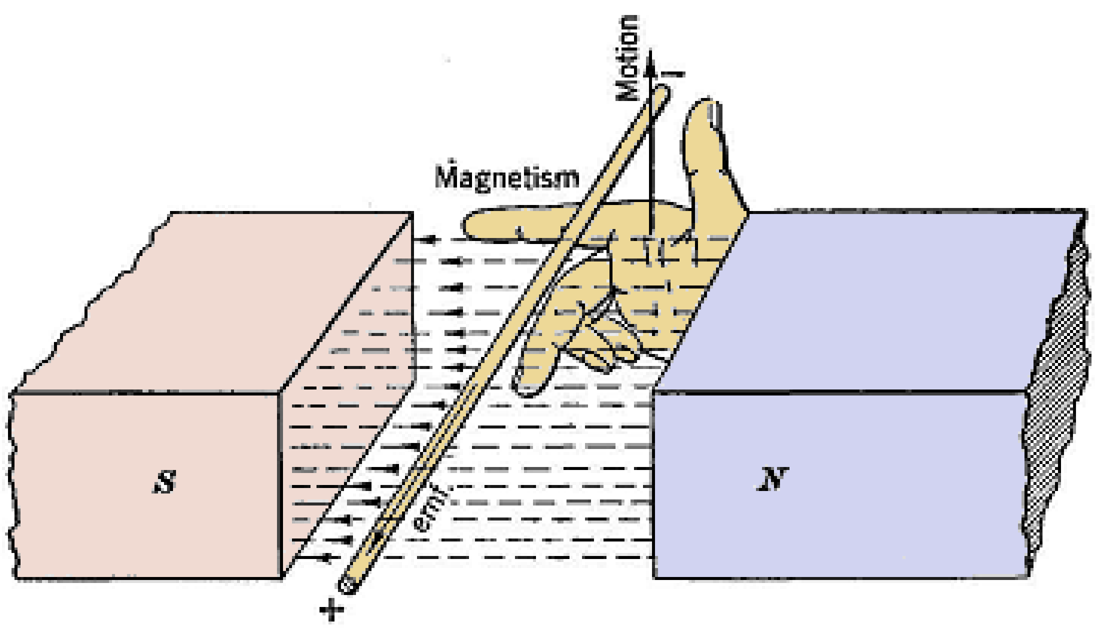

Figure 3.3 Fleming's Right Hand Rule

(ii)Right Flat Hand Rule:

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In this rule **we open our right hand such that the thumb is perpendicular to the group of the other fingers. The front side of the hand is held perpendicular to the incident flux and the thumb pointing the direction of motion of the conductor, then the direction of the other fingers will give the direction of the induced emf.** (figure 3.4).

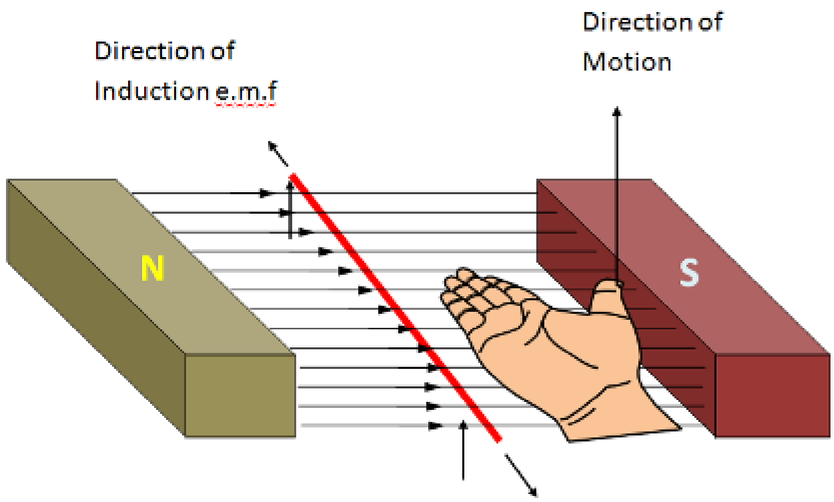

Figure 3.4 Right Flat Hand Rule

Fleming's rule and right flat hand rule are used for dynamically induced emf

(iii) **Lenz's law:**

This law **states that induced emf move in the direction as to oppose the motion producing it.** This law is applicable to a statically induced emf. That is the situation in which the conductor is not moving.

The stationary coil and moving magnet of figure 3.1 can be used to illustrate this law. It is found that the direction of the induced emf is anticlockwise when the magnet is approaching and the direction is clockwise when the magnet is leaving the coil.

3.4 INDUCED EMF:

There are two classification of induced emf these are (i) Dynamically induced emf and (ii) Statically induce emf.

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In the first case usually, the field is stationary and the conductor cut across it. But in the second case the conductor or the coil remain stationary and the flux linked with it changes. The dynamically induced emf can be illustrated using figure 3.5

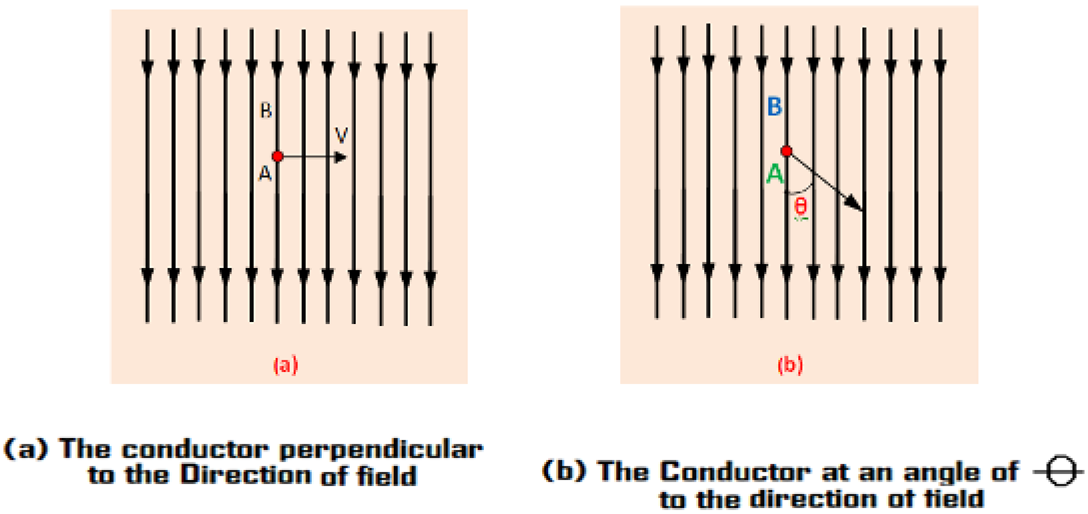

Figure 3.5 Dynamically Induced EMF

Let the flux density of the field be uniform with magnitude B and let the conductor A move a distance dx in the time dt along the directions as shown.

In case (a) the change in flux = BLdx, where L is the length of the conductor

The rate of change of the flux = $\frac{BLdx}{dt}$ = BLV, where V = $\frac{dx}{dt}$ is the velocity of the conductor.

$\therefore$ Dynamically induced emf = BLV ………………………………………………………(3.3)

And for case (b) e = BLVsinθ…………………………………………………………(3.4)

N/B if there are N conductors the above equations should be multiplied by N.

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Statically induced emf on the other hand can be subdivided into two (a) mutually induced emf and (b) self induced emf. Mutually induced emf is the emf induced as the result of the field from another coil whereas self induced emf is the emf induced in the coil as the result of the field created by the coil itself.

3.5 COEFFICIENTS OF SELF AND MUTUAL INDUCTION:

There exist a coefficient of induction for both types of inductions, these are

(.i) Coefficient of Self induction (or Self inductance), L: This coefficient can be defined in any of the following form of the equations

1. $L = \frac{N\phi}{I}$ Henry…..……………………………………………………(3.5)

   That is the flux linkage produced by the coil per Ampere

2. $L = \frac{\mu_o \mu_r A N^2}{L}$ Henry………………………………………………(3.6)

   This is the same as (3.5) with $\phi = \frac{NI}{L / \mu_o \mu_r A}$

3. $e_L = -L\frac{dI}{dt}$ …………………………………………………………..(3.7)

That is a coil is said to have an inductance (self-inductance) of 1 Henry if 1 Volt is induced in it when current through it changes at the rate of 1 Ampere/second.

(iii) **Coefficient of Mutual Induction (or Mutual Inductance), M:** This can also be expressed in three forms:

(1) $M = \frac{N_2 \phi_1}{I_1}$ ……………………………………………………………(3.8)

Where N_2 is the number of turns in the coil (the coil to be induced), ϕ_1 and I_1 are the flux and current in the other coil (coil producing the field).

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Therefore two coils are said to have a mutual inductance of 1 Henry if 1 Ampere of current in one coil produces flux linkage 1 Weber turn.

(2) Substituting $\phi_1 = \frac{N_1 I_1}{L / \mu_o \mu_r A}$ in (3.8) yield:

$$M = \frac{\mu_o \mu_r A N_1 N_2}{L} \quad \text{.......(3.9)}$$

(3) The mutually induced emf is given by:

$$e_M = -M \frac{dI_1}{dt} \quad \text{.......(3.10)}$$

Hence two coils are said to have a mutual inductance of 1 Henry if the current changing at the rate of 1 Ampere/second in one coil induces an emf of 1 Volt in the other coil.

EXAPLE 3.1

A coil of resistance 50 ohms is placed in a magnetic field of 1mWb. The coil has 100 turns and a galvanometer of 450 ohms resistance is connected in series with it. Find the induce emf in the coil and the current through it if the field changes to 0.2mWB in 1/10 second.

SOLUTION:

Induced emf $e = -N \frac{d\phi}{dt}$

Now N = 100 turns, $\phi_{innitial}$ = 1mWb, ϕ_{final} = 0.2mWb and time of change = 1/10 = 0.1second

The change in flux $d\phi = \phi_{innitial} - \phi_{final}$ =1 – 0.2 = 0.8mWb = 0.0008Wb

$$\therefore e = -N \frac{d\phi}{dt} = 100 \times \frac{0.0008}{0.1} = \underline{0.8\text{V}}$$

The total circuit resistance = 50 + 450 = 500Ω.

Therefore the current = $I = \frac{e}{R} = \frac{0.8}{500}$ = 0.0016A = 1.6mA

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EXAMPLE 3.2

A circuit has 1,000 turns enclosing a magnetic flux of 2mWb with a current of 4A. When the current becomes 9A the magnetic flux becomes 2.8mWb. Find the self inductance of the coil between these limits and the induced emf if the current changes in 0.05 second.

SOLUTION:

N = 1000, ϕ_1 = 2mWb, ϕ_2 = 2.8mWb

Change in fux = $\phi_2 - \phi_1$ = 0.8mWb = 0.0008Wb.

Change in current = 9 – 4 = 5A and the time dt = 0.05s

Self inductance $L = \frac{Nd\phi}{dI} = 1000 \times \frac{0.0008}{5}$ = <u>0.16H</u> and the induced emf =>

$$e_L = -L\frac{dI}{dt} = 0.16 \times \frac{5}{0.05} = 16\text{V}$$

The negative sign indicate the direction of the current (emf).

EXAMPLE 3.3

Two coils having 50 and 500 turns respectively are wound side by side on a closed iron circuit of cross sectional area of 50cm^2 and mean length of 120cm. Estimate the mutual inductance between the coils if the relative permeability of the iron is 1000. Also find the self inductance of each coil. If the current in one coil grow suddenly from zero to 5A in 0.01 second, find the induced e.m.f. in the other coil.

SOLUTION

N_1 = 50, N_2 = 5000, A = 50cm^2 = 0.005m^2, L = 1.2m, μ_r = 1000.

Chang in current dI = 5A, Time for Change dt = 0.01s

The mutual inductance, => $M = \frac{\mu_o \mu_r A N_1 N_2}{L} = \frac{4\pi \times 10^{-7} \times 1000 \times 5 \times 10^{-3} \times 50 \times 500}{1.2}$ = 0.131H

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The self inductance of the first coil=> $L_1 = \frac{\mu_o \mu_r A N_1^2}{L} = 0.0131\text{Henry}.$

The self inductance of the second coil=> $L_2 = \frac{\mu_o \mu_r A N_2^2}{L} = 1.31\text{Henry}.$

The mutually induced emf in the coil => $e_M = -M\frac{dI_1}{dt} = 0.131 \times \frac{5}{0.01} = 65.5\text{V}$

3.6 CONSTRUCTIONAL FEATURES OF ELECTRIC MACHINES:

In our previous topics we discovered that an emf is generated in a conductor when the conductor is cutting magnetic lines of force (Faraday's laws), and on the other hand a current carrying conductor in a magnetic field experience a force (F = BIL). The first case gives an idea on how to make a generator while the second case gives an idea of electric motor.

DC motors and DC generators are similar whereas AC motors and AC generators are most of the times different. The basic principle of generator can be illustrated using simple loop generator in figure 3.5.

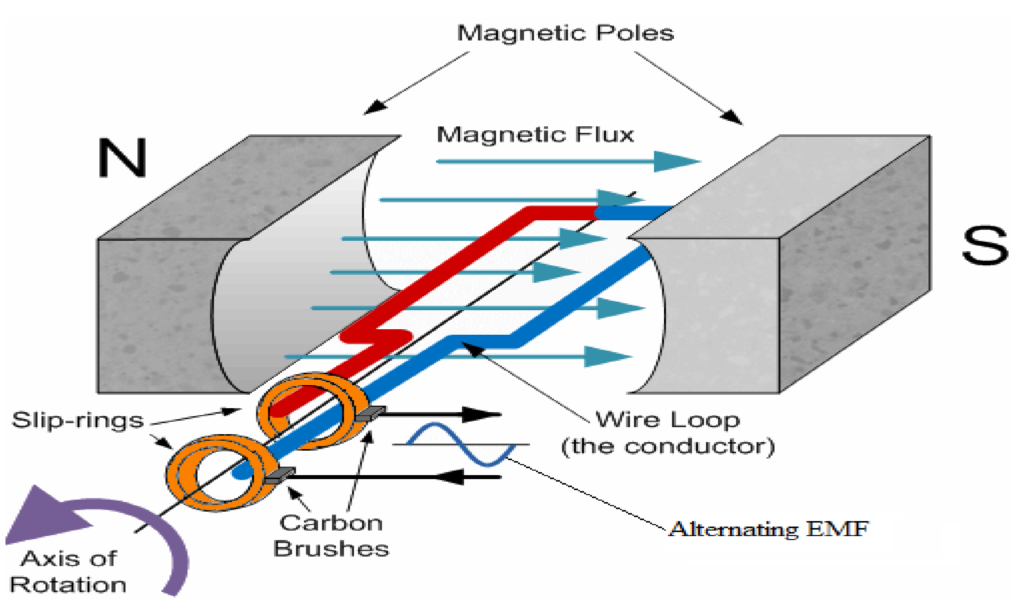

Figure 3.6: Simple Loop Generator

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As the coil rotate in between the magnetic field an emf is induced in it which is collected through the slip ring and brushes.

The actual DC generator (as shown in figure 3.7) has the following essential parts

(1) Yoke or Magnetic Frame .

(2) Pole coil or Field coil.

(3) Pole Core and pole shoe.

(4) Armature windings or conductors.

(5) Armature core

(6) Commutators

(7) Brushes and Bearings.

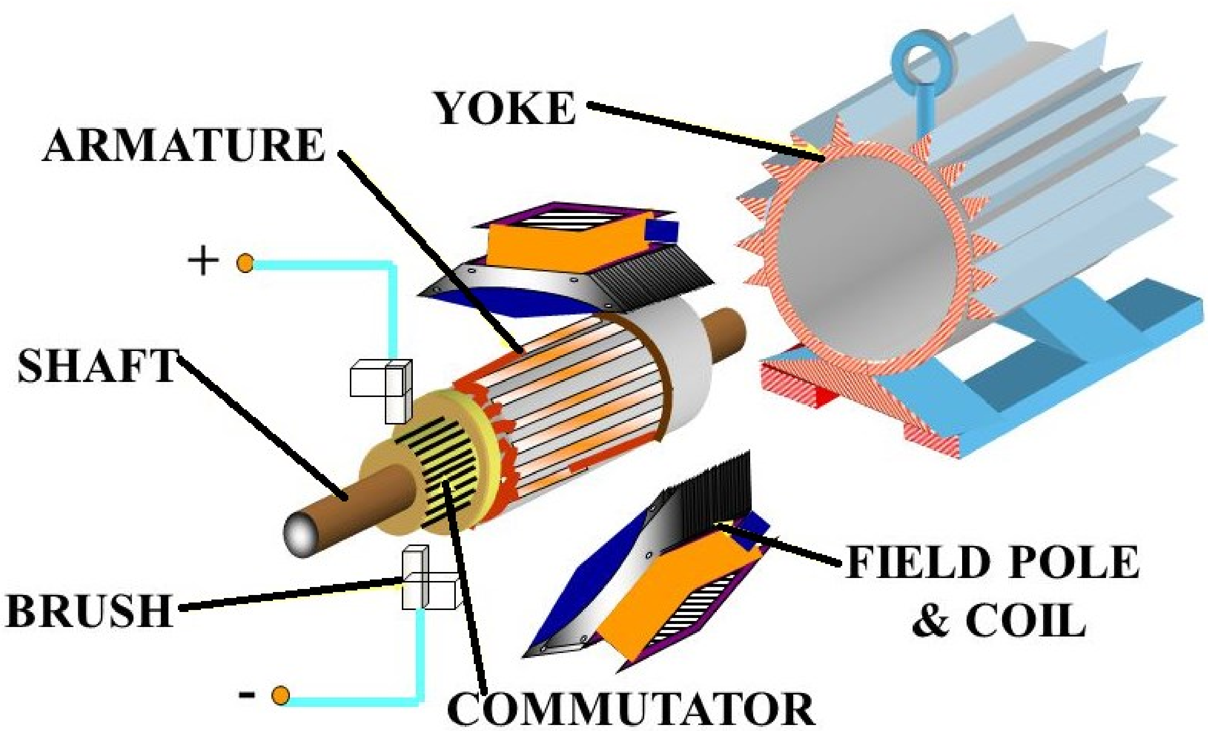

Figure 3.7 DC Generator

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(i) Yoke: this is the outer frame of the machine, it provide mechanical support for the machine.

(ii) Field coil or pole coil: This is the coil on the pole of the machine which when energized it will generate a field (magnetic flux).

(iii) Armature conductors or windings: The windings that generate emf when rotated in the field.

(iv) Commutators: They are the mechanical rectifiers that convert the generated emf to DC.

(v) Armature core: the part of the machine that houses the armature winding.

EXERCISE III

(1) A conductor of length 30cm carries a current of 100A and lies at right angle to a magnetic field of density 0.4Wb/m2. Calculate the force in Newton exerted on it. If the force causes the conductor to move with a velocity of 10m/s, calculate (a) the dynamically induced emf in the conductor and (b) The power developed by the conductor neglecting loses.

(2) A ferromagnetic ring of cross sectional area 16cm2 and of mean radius of 34cm has two windings connected in series, one of 500 turns and the other of 700turns. If the relative permeability of the material is 1200, calculate the self inductance of each coil and mutual inductance of the coils assuming that there is no flux leakage.

(3) What are the functions of bearings and pole shoe in an electric machines.

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CHAPTER FOUR: SINGLE PHASE MOTORS

4-0 INTRODUCTION:

Single phase motors are motor designed to operate from single phase (AC) supply. The working principle of an AC motor depend on the type of motor, but it generally involve the production of sinusoidally varying flux which produces rotation of the machine depending on the type of machine.

The single phase motors may be classified into the following categories depending on their construction and method of starting.

(1) Induction motors.

(2) Repulsion motors.

(3) Unexcited synchronous motors.

(4) AC series motors.

(5) Universal motor

4.1 SINGLE PHASE INDUCTION MOTOR:

As the name indicated, induction motors are AC motors that work based on the principles of induction. In this type of motors (unlike in DC motors) the rotating part of the machine is not connected to an external source. But when the stator is energized by a sinusoidal voltage, the sinusoidal flux produced in the stator will induce an emf in the short circuited rotor which as the result produces its own flux. The interaction between the stator flux and the rotor flux will cause the rotor to rotate.

Induction motors are classified into (i) Squirrel cage induction motor and (ii) Slip ring (external resistance) induction motor. In Slip ring type provision is made for the rotor to be connected (through slip rings) to external resistors for controlling the motor. Whereas the

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squirrel cage type has no external connection from the rotor, that is all controls are done from the stator only.

The single phase induction motor is not self-starting because the single phase does not produce a revolving magnetic flux. Therefore the single phase induction motor need some other means to start it. When the rotor is given an initial start, a unidirectional torque will immediately arise and the motor will accelerate till it's rated speed. In terms of starting method a single phase induction motor can be classified into Split-phase IM, Capacitor start IM and Shaded coil IM.

The induction motors (more especially the poly-phase type) form the majority of industrial derives. Figure 4.1 Shows a Single Phase induction.

Figure 4.1 Single Phase Induction Motor

4.2 REPULSION MOTOR:

These types of motors work on the repulsion principle (like pole repel and unlike pole attract). The armature of the machine is of DC construction that is with commutator and brushes. As shown in figure 4.1 the direction of the current in the poles and armature are such that similar poles are produced in them. With the poles being the same (S and S or N and N) a repulsion will occur which turn the armature.

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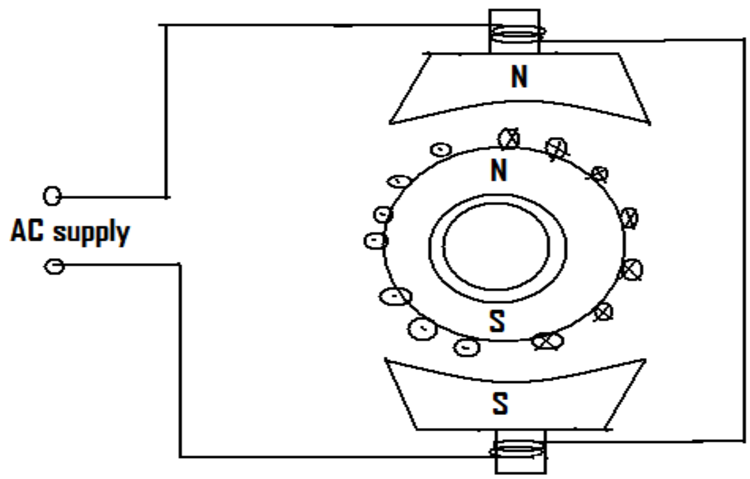

FIG. 4.1 REPULSION MOTOR

The repulsion motors are of three types:

- Compensated repulsion motor
- Repulsion start - induction run motors and
- Repulsion induction motor:

4.3 UNEXCITED SYNCHRONOUS MOTOR:

The unexcited single phase synchronous motors are motors that operate on single phase AC supply and they need no DC excitation for their rotors, that is why they are called unexcited. Moreover, these type of motors run at constant speed (equal to the synchronous speed of the revolving flux) and they are self starting.

The unexcited single phase synchronous motors are of two types (i) Reluctance motor and (ii) Hysteresis motors.

4.4 AC SERIES MOTORS:

When an ordinary DC series motor, (Figure 4.2) is connected to an AC supply, it will rotate with a unidirectional torque (though the torque will be a varying torque, unlike in DC operation).

The performance of such motor will not be satisfactory because of the alternating flux, sparking cause by the brushes and low power factor cause by high inductance of the field and

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armature windings. However, by making such modification as laminating the field core and yoke, improving the power factors of field and armature windings and reducing the mmf (flux) in the air gap, a good performance is obtained.

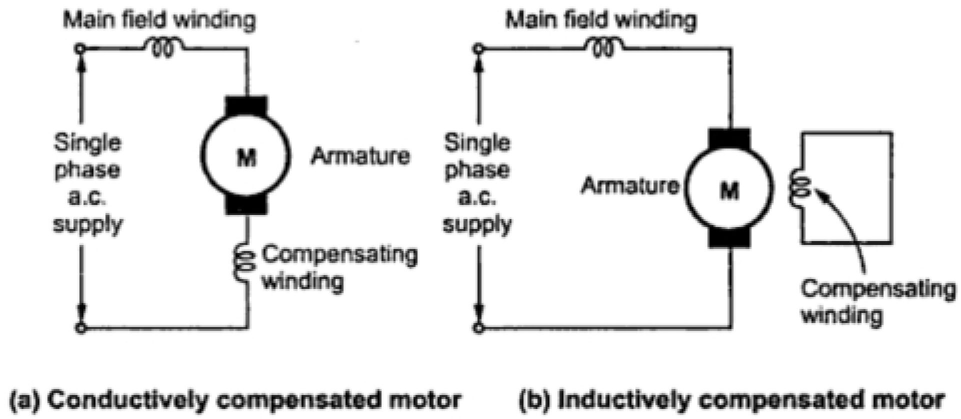

Figure 4.2 AC Series Motor with different compensating windings

As shown in figure 4.2 a compensating winding is added to the motor to reduce or neutralize the high armature mmf, the compensating winding can be **conductively** or **inductively** as shown

4.5 UNIVERSAL MOTOR:

A universal motor is the one which may be operated either on DC or a single phase AC supply at approximately the same speed and output power.

This motor is a smaller version (5 to 150W) of series motor, therefore it has high starting torque and variable speed characteristic. The universal motor runs at extremely high speed on no load, that is why they are usually built into the device they drive. They are of two types (i) concentrated pole, non-compensated type (low power) and (ii) Distributed field compensated type (high power rating).

EXERCISE IV

Briefly explain the following types of single phase Induction Motors:

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(1) Split-phase Induction Motors

(2) Capacitor start Induction Motors and

(3) Shaded coil Induction Motors.

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CHAPTER FIVE: THREE PHASE INDUCTION MOTOR

5.1 PRODUCTION OF REVOLVING MAGNETIC FIELD:

Three phase induction motor (rather a poly phase induction motor) works in similar principle with the single phase induction motor (section 4.1). While in a single phase induction motor a sinusoidal flux is produced, a uniform and rotational flux is produced in the case of poly phase motor.

We will now use the case of two phase supply and three phase supply to show that the uniform rotating flux is always of constant value.

(.i) Two Phase Supply:

In the case of two phase, 2 pole stator the windings are spaced 90^o apart. The flux due to this (2 phase) supply will also be spaced by 90^o (figure 5.1).

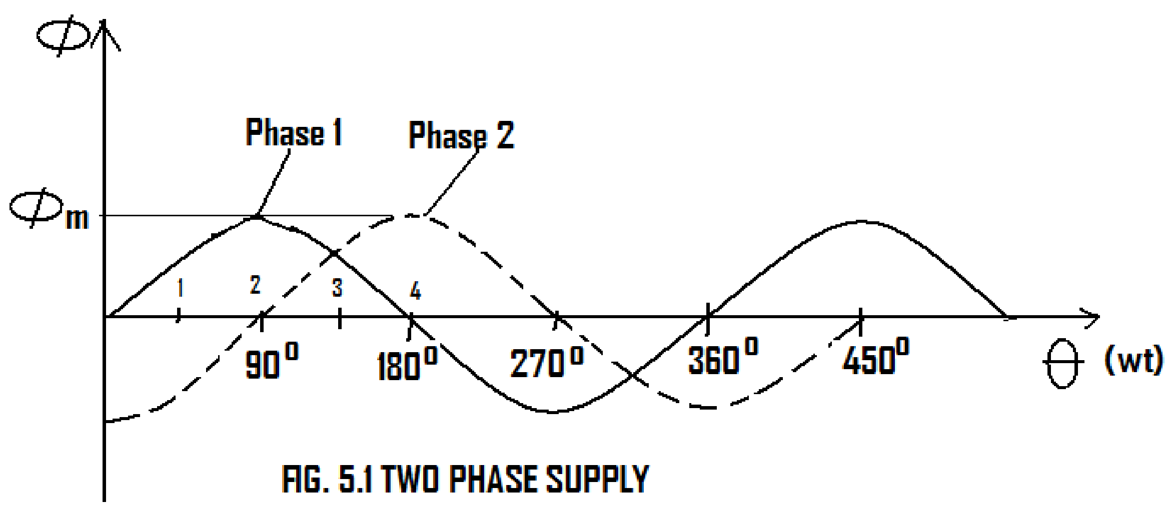

FIG. 5.1 TWO PHASE SUPPLY

Now if ϕ_m is the maximum flux density of each of the phase then:

$\phi_1 = \phi_m \sin wt$ ……………………………………………………(5.1)

$\phi_2 = \phi_m \sin(wt - 90^o)$ ……………………………………………..(5.2)

The resultant flux (according to method of addition of two vectors) is given by:

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$$\phi_r = \sqrt{\left(\phi_1^2 + \phi_2^2\right)} = \sqrt{\left(\phi_m^2 \sin^2(wt) + \phi_m^2 \sin^2(wt - 90^o\right)} \text{ ....(5.3)}$$

If we evaluate equation (5.3) at points 0, 1, 2, 3 and 4 corresponding to wt = 0, π/4, π/2, 3π/4 and π respectively we can see that the values of the resultant flux is always equal to ϕ_m in magnitude.

However if we use the following identity sin (90º – θ) = cosθ, we can say that sin(wt - 90º) = -sin(90 – wt) = -coswt. Substituting this in (5.3) we have:

$$\phi_r = \sqrt{\left(\phi_m^2 \sin^2(wt) + \phi_m^2 \cos^2 wt\right)} = \sqrt{\phi_m^2 (\sin^2 wt + c\,os^2 wt)} = \phi_m \text{ ....(5.4)}$$

The direction of this resultant flux is given by:

Direction =

$$\tan^{-1}\left(\frac{\phi_1}{\phi_2}\right) = \tan^{-1}\left(\frac{\sin wt}{\sin\left(wt - 90^o\right)}\right) = \tan^{-1}\left(\frac{\sin wt}{\sin(wt - 90^o)}\right) = \tan^{-1}\left(\frac{\sin wt}{-\cos wt}\right) = \tan^{-1}\left(-\tan wt\right)$$

= -wt =-θ…………………………………………………………………..(5.5)

Hence we can conclude that:

(i) The magnitude of the resultant flux is constant and is equal to ϕ_{max}, the maximum flux of either of the phases.

(ii) The resultant flux rotate as the θ is rotating.

(.ii) Three Phase Supply:

In a three phase supply, the phases are displaced by 120º, hence the fluxes will also be displaced by 120º. The stator windings of the motor are also displaced by 120º electrical depending on the number of poles on the rotor.

The three phase sinusoidal flux and the direction of the flux (with respect to each other) are shown in figure 5.2.

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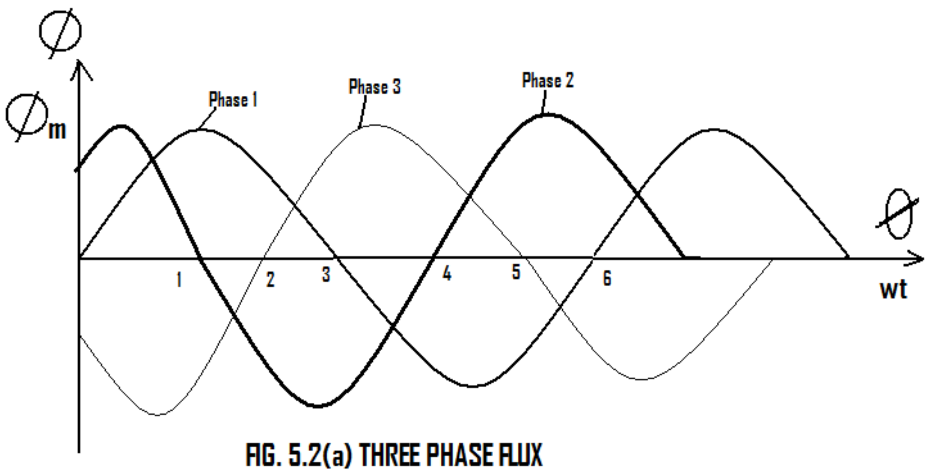

FIG. 5.2(a) THREE PHASE FLUX

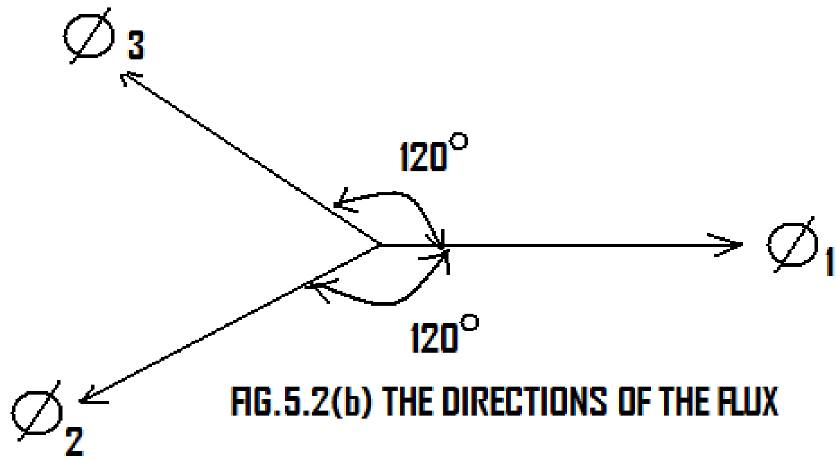

FIG.5.2(b) THE DIRECTIONS OF THE FLUX

With ϕ_m as the maximum flux of each phase we have the following:

$$\phi_1 = \phi_m \sin wt$$
$$\phi_2 = \phi_m \sin(wt - 120^o)$$
$$\phi_3 = \phi_m \sin(wt + 120^o)$$

Using the method of adding multiple vectors the resultant flux can be found as follows:

$$\phi_r = \sqrt{\phi_y^2 + \phi_x^2}$$

Where $\phi_y = \phi_{1y} + \phi_{2y} + \phi_{3y} = \phi_1 \sin\theta_1 + \phi_2 \sin\theta_2 + \phi_3 \sin\theta_3$

Here $\theta_1 = 0^o$, $\theta_2 = -120^o$ and $\theta_3 = 120^o$.

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$$\phi_y = \phi_m \sin wt \sin 0^o + \phi_m \sin(wt-120)\sin(-120^o) + \phi_m \sin(wt+120)\sin(120^o)$$

Now sin120º = 0.866, sin0º = 0 and sin(-120º) = -0.866. substituting these values in the above expression and the identity, sin(A +B) = sinAcosB + cosAsinB we have:

$$\phi_y = 1.5\phi_m \cos wt$$

Similarly; $\phi_x = \phi_{1x} + \phi_{2x} + \phi_{3x} = \phi_1 \cos\theta_1 + \phi_2 \cos\theta_2 + \phi_3 \cos\theta_3 = 1.5\phi_m \sin wt$

$$\phi_y = \sqrt{(1.5\phi_m \cos wt)^2 + (1.5\phi_m \sin wt)^2}$$
$$= \sqrt{1.5^2 \phi_m^2 \left(\cos^2 wt + \sin^2 wt\right)} = 1.5\phi_m \text{................(5.6)}$$

The direction =

$$\tan^{-1}\left(\frac{\phi_y}{\phi_x}\right) = \tan^{-1}\left(\frac{\phi_m \cos wt}{\phi_m \sin wt}\right) = \tan^{-1}\left(\frac{\cos wt}{\sin wt}\right) = \tan^{-1}\left(\frac{1}{\tan wt}\right) = \tan^{-1}\left(\tan(90^o - wt)\right)$$

$$= 90^o - wt = 90^o - \theta \text{ ....(5.7)}$$

Hence the magnitude of the resultant flux is always constant ($1.5\phi_m$) and the direction is rotational.

N/B the direction (90º –θ) is rotating since θ is changing from 0º to 360º, that is as θ goes through one revolution (90º –θ) will also go through one revolution.

5.2 THE INDUCTION MOTOR AS A TRANSFORMER:

Like in transformer the transfer of energy from stator to rotor of an induction motor takes place inductively (i.e. by mutual induction). Therefore an induction motor is very similar to a transformer with stator forming the primary and rotor forming the secondary (short circuited secondary). One of the differences between induction motor and transformer is that the flux generated in the stator of an induction will pass through the air gap (of high reluctance) before it reaches the rotor. This high reluctance of the air gap necessitates a very large magnetizing current and hence the induction motor has very large no load current (about 40 – 50% of the full load current).

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The equivalent circuit of an induction motor is similar to that of transformer (figure 2.2) only that the load in the case of transformer is here represented by a resistive load ($R_2(1/s - 1)$) known as the electrical equivalent of the mechanical load on the motor. The load in the equivalent circuit of an induction motor is purely resistive unlike a transformer which can have both resistive and reactive load. The complete equivalent circuit is shown in figure 5.3 below:

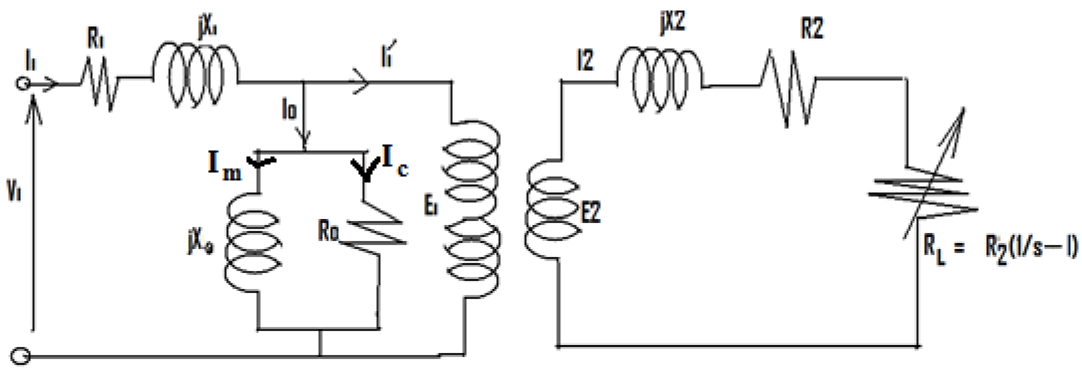

FIG. 5.3 EQUIVALENT CIRCUIT OF AN INDUCTION MOTOR

R_1 is the resistance of the stator windings.

X_1 is the leakage reactance of the stator windings.

R_2 is the resistance of the rotor windings.

X_2 is the leakage reactance of the rotor windings.

R_o and X_o are the core loss resistance and magnetizing reactance.

S is known as the slip (or slip ratio) of the motor, mathematically $s = \frac{N_s - N}{N_s}$ where N_s is the synchronous speed of the motor and N is the actual speed of the rotor.

For the equivalent circuit above, the following relations hold;

$$V_1 = E_1 + I_1\left(R_1 + jX_1\right) \quad \text{......(5.8)}$$

$$E_2 = KE_1 \quad \text{......(5.9)}$$

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$$E_2 = I_2\left(R_2 + jX_2 + R_2\left(\frac{1}{s} - 1\right)\right) = I_2\left(\frac{R_2}{s} + jX_2\right)$$

$$\Rightarrow sE_2 = I_2\left(R_2 + jsX_2\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.10)$$

N/B K is the turn ratio N_2/N_1 where N_1 is the number of turns in the stator and N_2 is the numbers of turns in the rotor.

To determine the parameters of the equivalent circuit of the induction motor laboratory experiments known as No Load Test and Block Rotor Tests are carried out on the induction motor.

EXERCISE V

(1) In a similar way to a transformer, define the equivalent circuit of induction motor. Calculate the slip ratio of a 6 poles induction motor working under a supply voltage of 50Hz frequency and running at a speed of 980 rev/min. (Hint: N = 120f/p).

(2) Briefly explain how **no load test** and **block rotor** test are carried out on an induction motor.

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CHAPTER SIX: THREE PHASE SYNCHRONOUS MACHINE:

6.1 GENERATION OF A THREE-PHASE VOLTAGE SYSTEM:

The generation of three phase voltage system can be illustrated using three similar loops (windings) fixed to one another at 120° out of phase to one another as shown in figure (6.1). Each loop as shown in figure (6.2) is terminating in a pair of slip ring carried on the shaft.

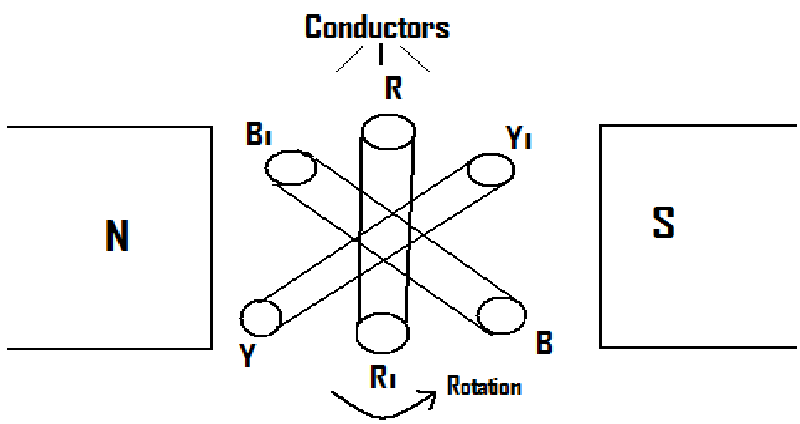

FIG. 6.1 GENERATION OF THREE PHASE VOLTAGE

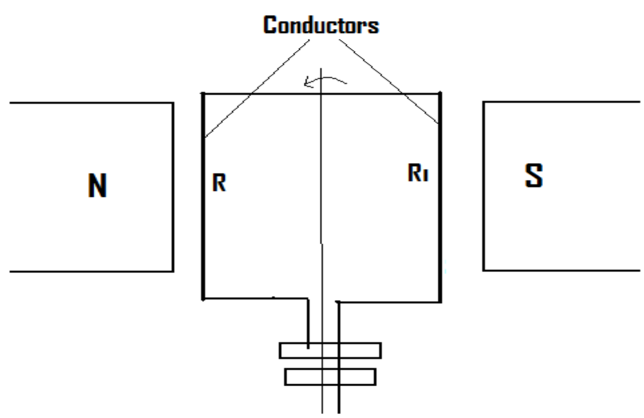

FIG. 6.2 LOOP RR1 WITH SLIP RINGS

Suppose the three coils are rotated anticlockwise at uniform speed in the magnetic field having N and S poles (as shown in figure 6.1). The emfs generated in the three loops RR_1, BB_1 and YY_1 will be 120° electrical in between. The curves of these emfs will have the same amplitude as shown in figure (6.3) below.

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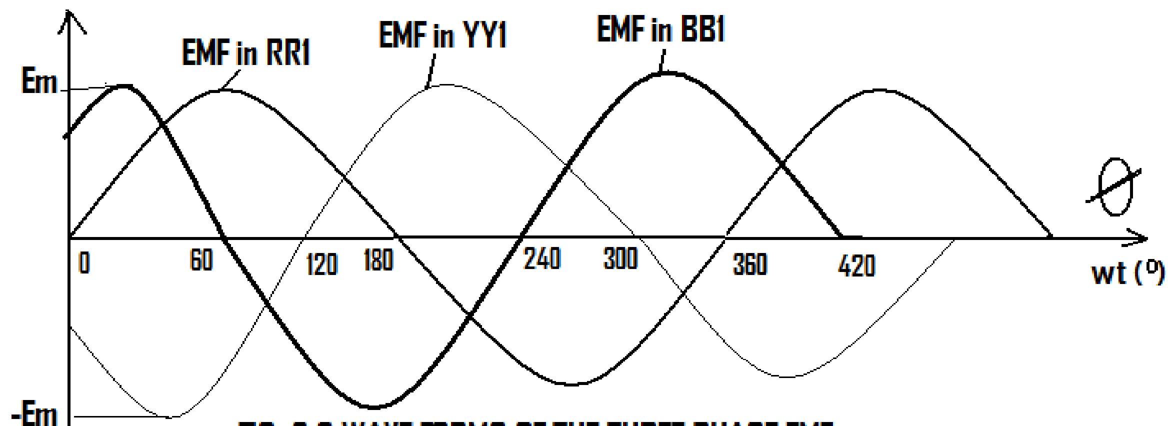

FIG. 6.3 WAVE FORMS OF THE THREE PHASE EMFs

Now let the emf generated in phase RR_1 be represented by:

$$e_R = E_m \sin\theta = E_m \sin wt \quad \text{.......(6.1)}$$

Then the emfs in BB_1 and YY_1 are given by:

$$e_Y = E_m\left(\sin\theta + 120^o\right) = E_m\left(\sin wt + 120^o\right) \quad \text{.......(6.2) and}$$

$$e_B = E_m\left(\sin\theta - 120^o\right) = E_m\left(\sin wt - 120^o\right) \quad \text{.......(6.3)}$$

6.2 MEASUREMENT OF THREE PHASE POWER

Basically, there are two main ways of measuring the power of a three phase power supply (the active power of the system). These methods according to the type of load connected are one-watt meter and two-watt meter methods. In some peculiar situations, three-watt meter method is employed. These methods are applied in accordance with the type of the three phase load:

(i) **Star connected balanced load, with neutral point accessible:** In the case where the loads on the individual phases are equal (equal impedance) one wattmeter W can be used. The wattmeter is connected such that its current coil is on the one line and the voltage circuit between that line and neutral (hence the system has to be star connected). The reading of the wattmeter gives the power of one phase (figure 6.4)

Therefore Total active power = 3 x Wattmeter reading……………(6.4)

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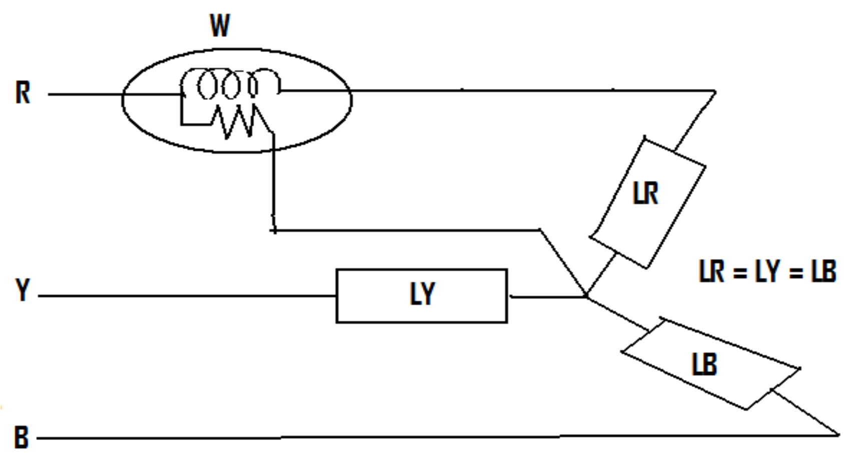

FIG. 6.4 STAR CONNECTED LOAD WITH NEUTRAL ACCESSIBLE

(ii) **Two Wattmeter method**: This method is used to measure the total power of a three phase system whether it is a balanced or unbalanced load, star or delta connected. The connection of the two wattmeter is shown in figure 6.5 below

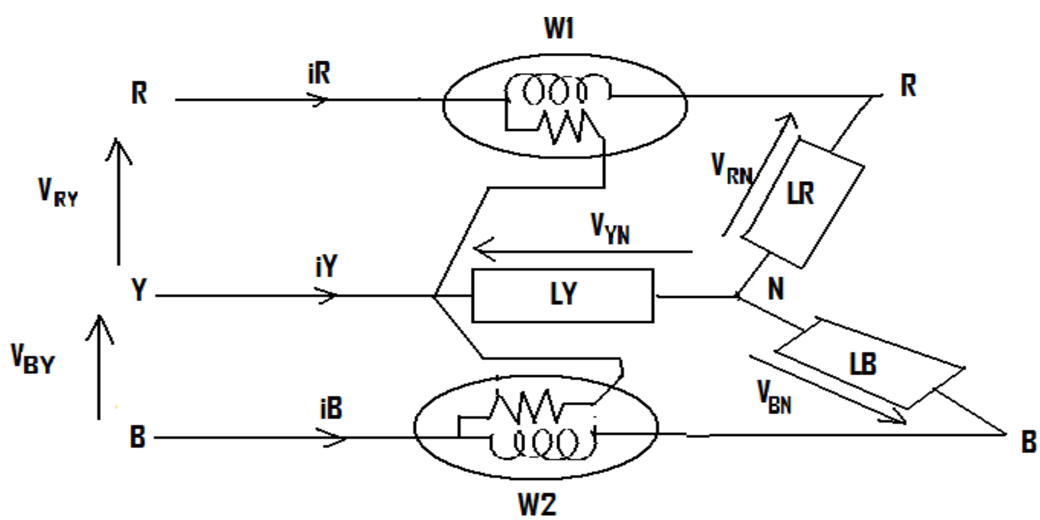

FIG. 6.5 MEASUREMENT OF THREE PHASE POWER BY TWO WATTMETER METHOD

As shown in the figure the current coils of the two watt meters are connected along two lines and their voltage circuits are connected between the lines and the third line, which show that a neutral is not required.

The analysis of the system is as follows:

The total instantaneous power = $i_R V_{RN} + i_Y V_{YN} + i_B V_{BN}$

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The current measured by $W_1 = i_R$

Current measured by $W_2 = i_B$

Power measured by $W_1 = i_R\left(V_{RN} - V_{YN}\right)$ and power measured by $W_2 = i_B\left(V_{BN} - V_{YN}\right)$

Now the total power measured by W_1 and W_2 is given by:

$W = W_1 + W_2 = i_R\left(V_{RN} - V_{YN}\right) + i_B\left(V_{BN} - V_{YN}\right) = i_R V_{RN} + i_B V_{BN} - \left(i_R + i_B\right)V_{YN}$ …..(6.5)

From Kirchhoff's law the algebraic sum of the instantaneous currents at N is zero.

That is $i_R + i_Y + i_B \Rightarrow i_R + i_B = -i_Y$

Substituting this in equation (6.5) we have:

$W = i_R V_{RN} + i_Y V_{YN} + i_B V_{BN}$ ……………………………………………….(6.6)

Hence the sum of the two wattmeter readings gives the average value of the total load absorbed by the three phases, that is the active power.

N/B The above proof was derived for a star connected load, but it is still holds for a delta connected loads.

(iii) **Three-watt meter Method**.

In some situation three watt meters are employed to measure the 3 phase power, this is especially in star connected unbalanced load and where it is interested to know the power consumption of each phase. Sometimes even the delta-connected loads can be measured using the 3-watt meter method in which a neutral point is created for the phases. Figure 6.5(b) illustrate the connection of the 3 watt meters.

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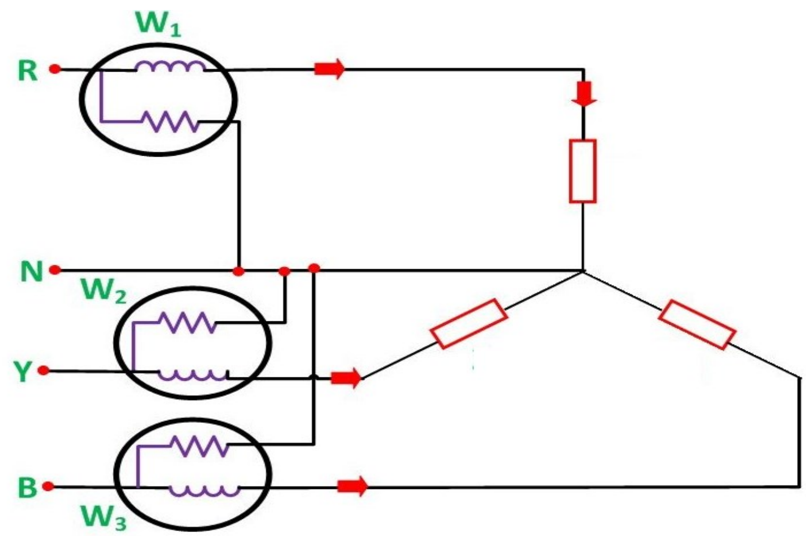

Figure 6.5(b) 3-watt meter method

6.3 SYNCHRONOUS GENERATOR:

The synchronous generators are AC generators that operate on the same fundamentals principles of electromagnetic induction as the DC generators. The major different between the two is that while DC generators have stationary fields and rotating armatures, the synchronous generator has stationary armature (mounted on the stator of the machine) and a rotating field as shown in figure 6.6

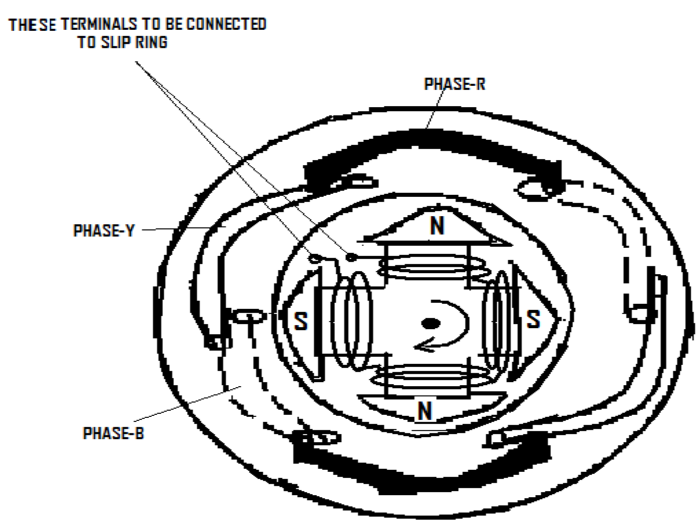

FIG. 6.6 SINGLE LAYER THREE PHASE SYNCHRONOUS GENERATOR

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The stator consists of an iron frame, which support the armature core. The rotor is like a flywheel having alternate N and S poles fixed on it. These alternate N and S poles are provided through excitation of the rotor from DC source (that is the excitation of the rotor windings).

When the rotor rotates, the stator conductors (being stationary) are cut by magnetic flux and therefore they will have an induced emf in them. Because of the alternate N and S poles of the rotor the induced emf is alternating. The frequency of the alternating emf depends on the number of poles of the rotor and the speed of rotation of the rotor. As such, if N is the rotor speed in rpm then:

Frequency, $f = \frac{PN}{120}$ Hz……………………………………………………………..(6.8)

Where P = number of poles.

The generator in figure 6.7 can be represented by the following equivalent circuit (that is the per phase equivalent circuit).

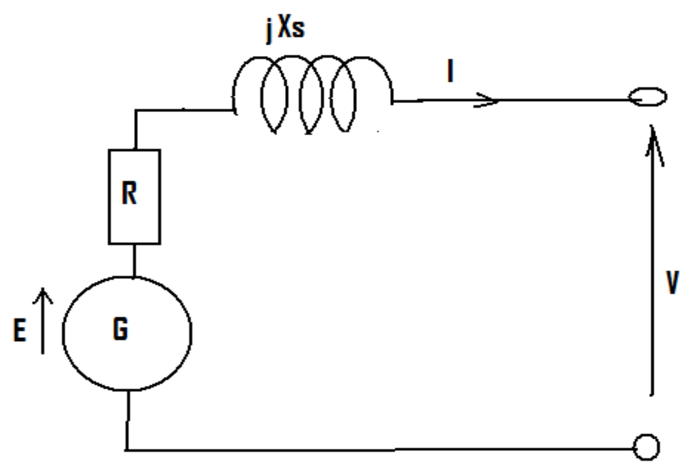

FIG. 6.7 EQUIVALENT CIRCUIT OF A SYNCHRONOUS GENERATOR

Where Xs is the synchronous reactance per phase.

R is armature resistance per phase

The emf equation can be derived from the equivalent circuit as follows:

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$$E = V + I(R + jX_s) = V + IR + jIX_s \quad \text{.....(6.9)}$$

And therefore the phasor diagram (per phase) is as follows:

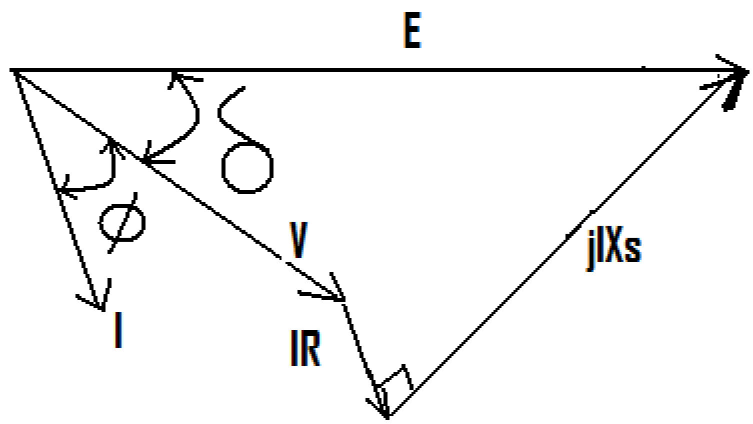

FIG. 6.8 PHASOR DIAGRAM OF SYNCHRONOUS GENERATOR.

In this case δ is phase the phase difference between E and V and ϕ is the phase angle between V and I

6.4 SYNCHRONOUS MOTOR:

A synchronous motor is very similar to a synchronous generator. In fact the generator in figure 6.6 may be used, at least theoretically as a motor and on the other hand as a generator. The synchronous motor has the following characteristics:

(.i) it runs either at synchronous speed or not at all, that is while running it remain at constant speed. The only way to change the speed is by changing the frequency of the supply (the synchronous speed is given by N = 120F/P, equation 6.9).

(.ii) It is not self starting. It has to be run up to synchronous speed (or nearly synchronous speed) by some external means, before it can be synchronized to the ac supply.

(.iii) it can be operated under wide range of power factor, both leading and lagging. It can therefore be used for power factor correction in addition to supplying torque to drive loads.

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The general principle of operation of a synchronous motor is that, when the stator is supplied by a three phase voltage system, a revolving magnetic flux is produced (section 5.1). this rotating magnetic flux is equivalent to having two poles (N and S) rotating on the stator. The rotor is run up by an external source (probably another motor) such that its speed in nearly or equal to the speed of the stator poles (the rotating flux). The motor is then excited by a DC source to produce alternate N and S poles. Once the exciter is removed (that is the external source that drive the rotor) the rotor will continue to rotate at the speed of the stator (rotating) flux. This is as the result of the rotor field locking with the stator field.

The equivalent circuit of synchronous motor (per phase) is similar to that of synchronous generator only that in this case voltage is supplied to the motor (figure 6.9)

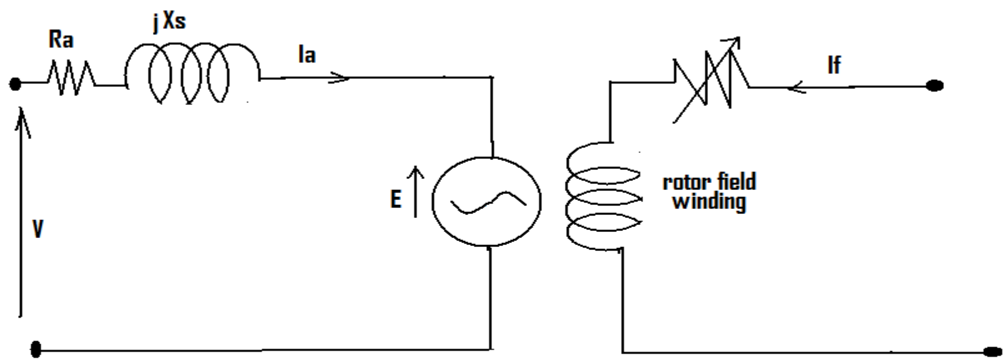

FIG. 6.9 EQUIVALENT CIRCUIT OF A SYNCHRONOUS MOTOR (With rotor windings)

The emf equation of the synchronous motor is as follows:

$$V = E + I_a R_a + jI_a X_a \quad \text{.......... (6.10)}$$

And therefore the Phasor diagram is as follow:

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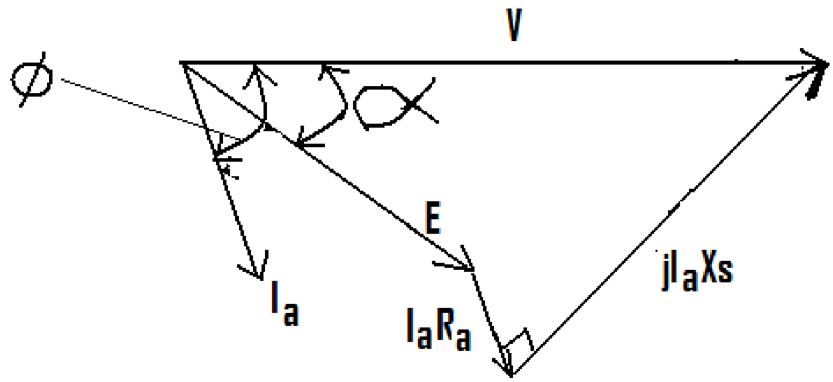

FIG. 6.8 PHASOR DIAGRAM OF SYNCHRONOUS MOTOR

In this case α is the phase angle between E and V and ϕ is the phase angle between V and I.

EXERCISE VI

(1) Draw a labeled diagram of a three phase synchronous generator and name the constructional features.

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CHAPTER SEVEN: DC MACHINES

7.1 DC GENERATOR:

In section 3.6 we described the constructional features of a DC generator (figure 3.7). In that case it was shown that the stator poles are provided by the windings on the pole core. The pole becomes an N pole if the current flowing through it started from back of the pole core to the front of it and it becomes an S pole if the current flows from the front of the pole to the back of the pole as shown in figure 7.1 below:

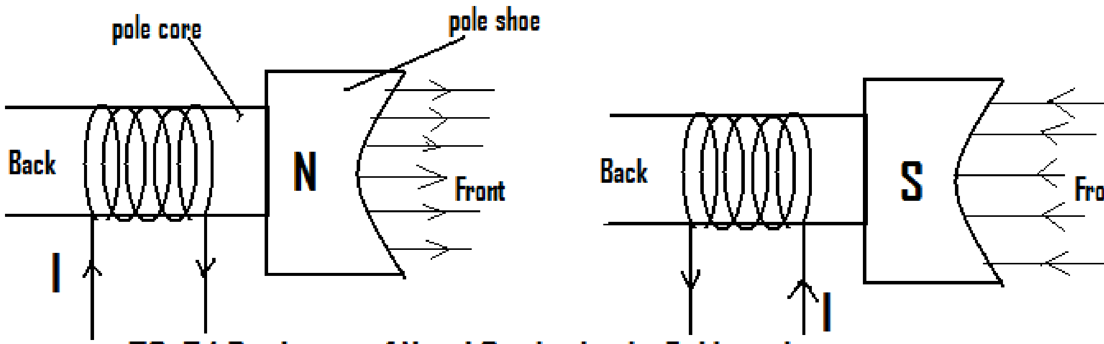

FIG. 7.1 Production of N and S poles by the Field windings
(Take note of the direction of current)

The number of poles in a generator is even, that is the number of poles exist in pair (e.g. 2, 4, 6, 8 etc).

DC generators can be classified according to the number of poles (2 poles, 4 poles, 6 poles etc), according to the type of armature winding (lap wound or wave wound) or according to the way their fields are excited. But they are generally classified according to excitation of their windings. They are divided into separately excited and self excited generators.

(a) Separately excited generator:

These are DC generators in which the field windings are energized from independent external DC current. It could be a battery or any other source (figure 7.2).

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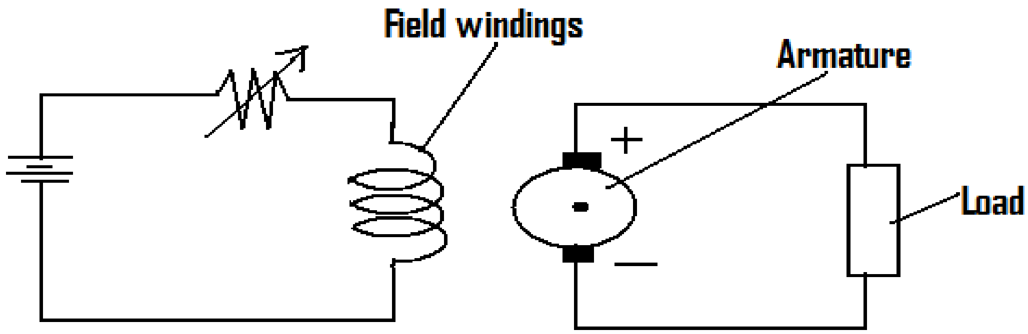

FIG. 7.2 SEPARATELY EXCITED GENERATOR

(.b) Self excited generator:

To avoid using external source in the generator, field winding in the case of self excited generators are energized by the current produced by the generator itself. Self excited generators are further classified into three, according to the way the field windings are connected to the armature, these are:

(i) Shunt wound (or shunt generator).

(ii) Series wound (or series generator) and

(iii) Compound wound (compound generator)

(.i) Shunt generator:

The generator is said to be a shunt generator if the field windings are connected in parallel with (or across) the armature so that it will have the full armature voltage across it. The load is also connected in parallel with the armature (as shown in figure 7.3).

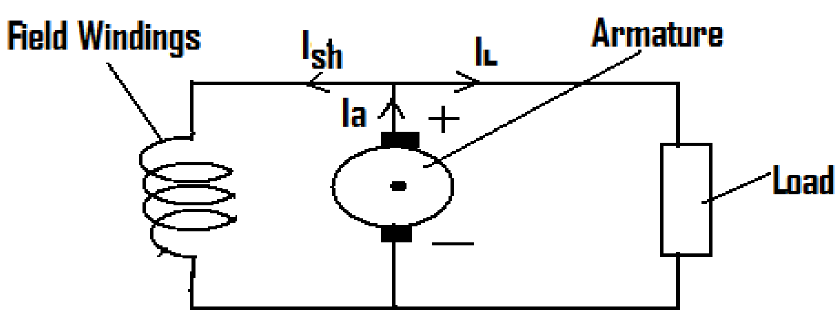

FIG. 7.3 SHUNT GENERATOR

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(.ii) Series generator:

In this case the field windings are connected in series with the armature. The field windings consist of few turns of thick copper wire as they carry the full armature current.

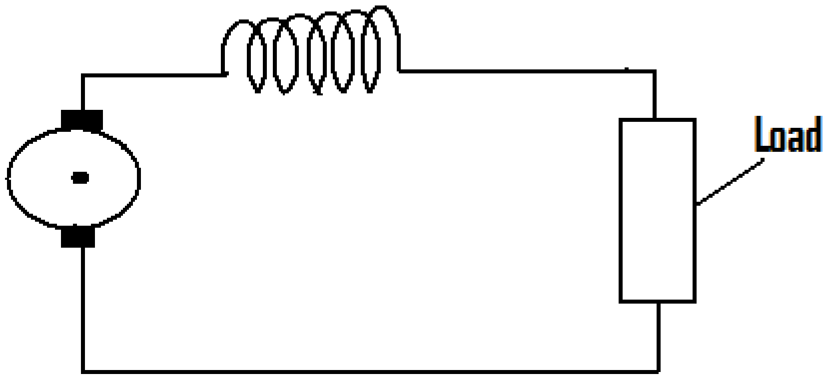

FIG. 7.4 SERIES GENERATOR

(.iii) Compound generator:

The field winding of a compound generator consists of a combination of few series and some shunt windings (figure 7.5).

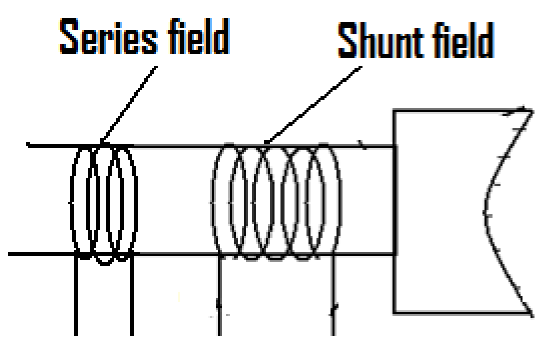

FIG. 7.5 COMPOUND GENERATOR

Usually the shunt field is stronger than the series field and when the series field aids the shunt field (producing N & N or S & S), the generator is said to be "cumulatively compounded" and when the series field opposes the shunt field the generator is said to be "differentially compounded".

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On the other hand a compound generator can be "long shunt" if the shunt winding is connected across the series of armature and series winding and it is referred to as "short shunt" if the shunt winding is connected across the armature alone.

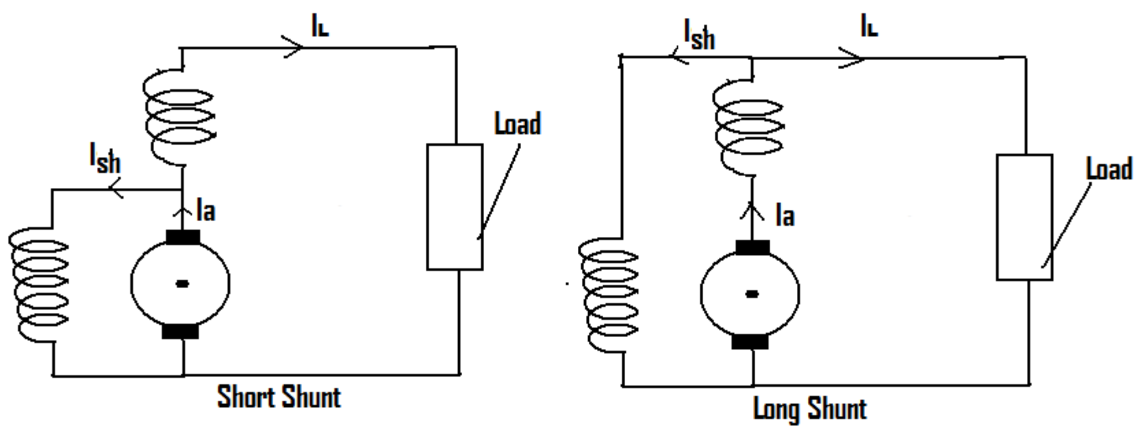

FIG. 7.6 COMPOUND GENERATOR

What normally happen in a self excited generator is that, there is always the presence of a residual flux in their poles, and when the armature started to rotate an emf is produced (due to the residual flux) which will pass current through the field windings and there by strengthen the poles. The generator is allowed to develop its output voltage (before loading) in the case of shunt generator whereas in the series wound the load has to be connected before the generator can developed its output voltage (the field current = the load current).

7.2 DC MOTORS:

The constructional features of a DC motor is the same as that of a DC generator. Most times a DC machine can act as both DC motor and generator. The only difference between the two is that while a DC generator converts the mechanical energy (from the shaft of the armature) to electrical energy at the armature terminals, the motor converts the electrical energy (voltage) supplied to the armature terminals to mechanical energy at the shaft.

The motor works on the **principle that a current carrying conductor in a magnetic field experienced a mechanical force whose direction is given by the Fleming's left hand rule and whose magnitude is given by the relation:**

F = BIL……………………………………………………………………………..(7.1)

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Where **B** is the magnetic flux density of the field, **I** is the current through the conductor and **L** is the length of the conductor.

DC motors are classified according to the way their field windings are connected to the armature. Just like in DC generator we have (i) Shunt motors (ii) Series motors and (iii) Compound motors as shown in figure 7.7 below:

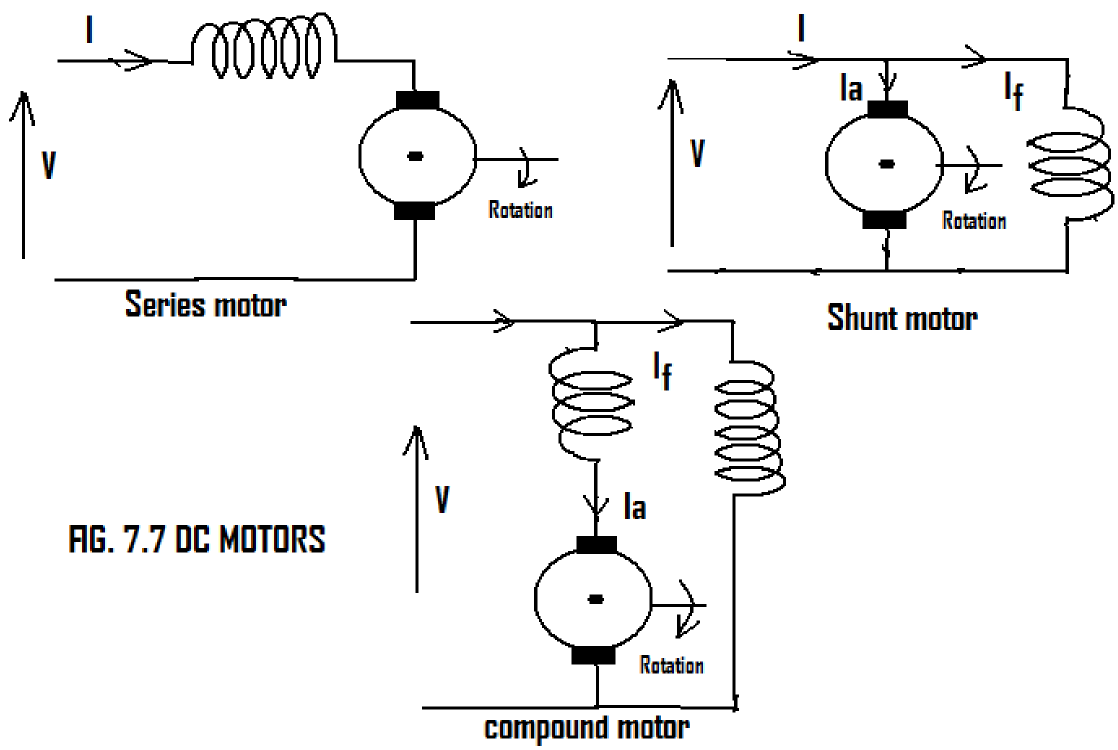

FIG. 7.7 DC MOTORS

7.3 MOTOR SPEED – TORQUE CHARACTERISTIC (N/Ta):

This characteristic is the one that shows the relationship between the motor speed and torque, that is why it is sometimes called the mechanical characteristic of the motor.

The expression for the torque in a DC motor is given by: $T_a = \frac{\phi Z P I_a}{2\pi A}$ ....(7.2)

Where ϕ is the flux per pole, Z is the total number of conductors, P is the number of poles, I_a is the armature current and A is the number of parallel path (2 for wave winding and P for lap winding). Sometimes the torque equation is written as:

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$$T_a = \frac{E_b I_a}{2\pi N} \quad \text{……………………………………………………………………………..(7.3)}$$

Where E_b is back emf of the motor and N is the speed in rev./sec. of the motor (this is because $\left. E_b = \frac{\phi ZNP}{A} \right)$.

Now for particular motor Z,P and A are constant and therefore equation (7.2) may be written as:

$$T_a = K\phi I_a \Rightarrow T_a \alpha \phi I_a \quad \text{…………………………………………………………………..(7.4)}$$

Equations (7.3) and (7.4) can be used to analyze N/T_a characteristic of a DC motor.

(.i) Series motor:

For a series motor the back emf E_b and I_a are approximately constant. Therefore using equation (7.3), the torque is inversely proportional to the speed i.e. $T_a \alpha \frac{1}{N}$. Hence the curve of T against N is shown in the figure below:

FIG. 7.8 N/Ta Characteristic of a series motor

(.ii) Shunt motor:

In a shunt motor the field current is independent on the armature current, therefore the flux is considered to be constant (even though it can change due to armature reaction). This therefore makes the equation (7.2) to be represented as:

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$T_a \alpha I_a$ ……………………………………………………………………(7.5)

Also from the back emf equation $\left(E_b = \frac{\phi ZNP}{A}\right)$ we can have

$N \alpha \frac{E_b}{\phi}$ ....(7.6)

Both E_b and ϕ (which is a function of I_f) decrease in a very small quantity with increase in load. However the decrease in E_b is slightly more than that of ϕ. Hence the N/I_a characteristic is a straight line with slight decrease in N as I_a is increased. The drop varies from 5% to 15% of full load speed.

Now from equation (7.5) $T_a \alpha I_a$, therefore N/T_a characteristic will be similar to N/I_a characteristic as shown in figure 7.9.

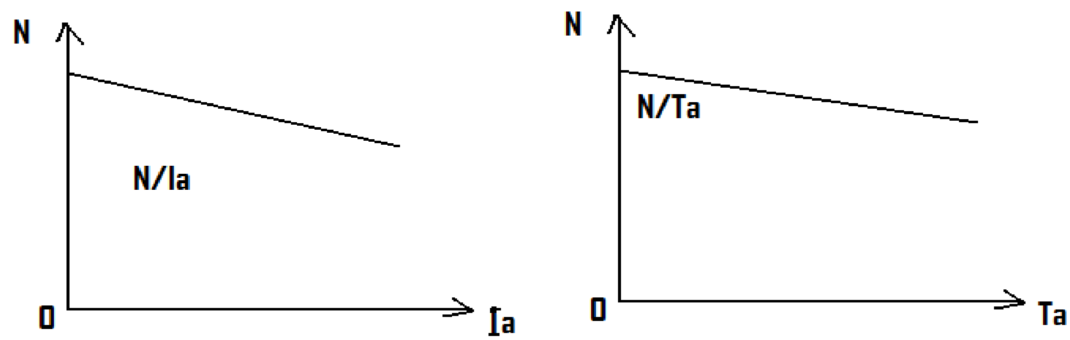

FIG. 7.9 CHARATERISTIC OF SHUNT MOTOR

7.4 SPEED CONTROL OF A DC MOTOR:

From the back emf equation we can have the speed of the motor being represented as:

$$N = \frac{E_b A}{\phi ZP} \Rightarrow N = \frac{K\left(V - I_a R_a\right)}{\phi} \quad \text{……………………………………………………..(7.7)}$$

Where K =A/ZP = constant for a particular motor.

Equation (7.6) shows that the speed can be controlled by varying:

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(.i) Flux per pole, ϕ (flux control)

(.ii) The armature resistance R_a (Rheostatic control).

(.iii) Applied voltage control.

(.i) Flux control:

At constant V, I_a and R_a the speed of the motor (from equation 7.6) is inversely proportional to the flux ($N \alpha 1/ \phi$). Hence by varying the field current (which means varying the flux) a variable speed is obtained.

For a shunt motor a variable resistor can be connected in series with the field windings and for a series motor a variable resistor is connected across the field windings as shown in figure 7.10.

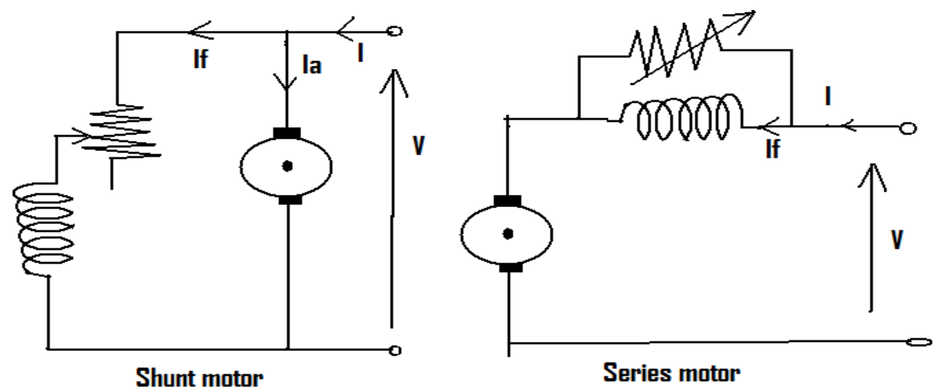

FIG. 7.10 FLUX CONTROL

(.ii) Armature resistance control.

At constant V and ϕ the speed of a DC motor can be controlled by adding a variable resistor in series with the armature. As the resistor increases the speed will be decreasing according to equation (7.6).

In the case of shunt motor a variable resistor is connected in series with the armature likewise in series motor (even though it also vary the field in the case of series motor). These are shown in figure 7.11

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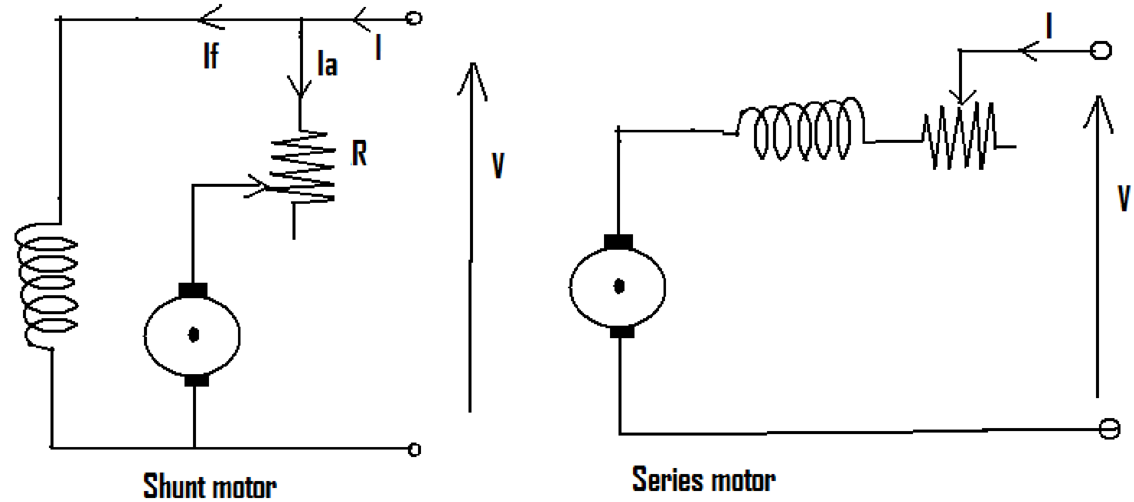

FIG. 7.11 ARMATURE RESISTANCE CONTROL

Other ways of controlling the speed of a DC motor include:

(i) Field diverter (another way of flux control).
(ii) Paralleling fields (flux control).
(iii) Tapped field (flux control). and
(iv) Armature diverter.

EXAMPLE 7.1

A four pole DC motor is connected to a 500V DC supply and takes an armature current of 80A. The resistance of the armature is 0.4 ohms. The armature is wave connected with 522 conductors and flux per pole is 25mWB. Calculate:

(a) Back e.m.f. of the motor.
(b) The speed of the motor. And
(c) The developed by the armature.

SOLUTION:

$P = 4$, $V = 500V$, $I_a = 80A$, $R_a = 0.4\Omega$, $A = 2$ (wave connected), $Z = 522$ and $\phi = 25mWb = 0.025Wb$.

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(.a) $E_b = V - I_a R_a$
= 500 – 80 x 0.4 = 468V

(.b) Using the relation

$E_b = \frac{\phi ZPN}{A}$ we have $N = \frac{E_b A}{\phi ZP} = \frac{468 \times 2}{0.025 \times 522 \times 4} = 17.93 rev/\sec = 1075.86 rpm$

(.c) $T_a = \frac{E_b I_a}{2\pi N} = \frac{468 \times 80}{2\pi \times 17.93} = 332.33 Nm$.

EXERCISE 7

(1) What is back EMF?

(2) Highlight the differences between DC motors and DC generators.

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