



COURSE CODE: ECO 122

COURSE TITLE:INTRODUCTION TO MATHEMATICS FOR ECONOMICS I1

NUMBER OF UNITS: 2 Units

COURSE DURATION: TWO HOURS PER WEEK

COURSE LECTURER: DR ODIWO WILLIAMS OMOKHUDU

INTENDED LEARNING OUTCOMES

At the completion of this course, students are expected to:

1. Student should have an understanding of mathematics concepts and their applications in real life situation.
2. Understand the stages and processes involved in identify business problems.
3. Understand what constitutes Business and Economics problems.
4. Analyse different business and Economics problems as a student
5. Generate, access and work with quantitative idea to solve Economics

COURSE DETAILS:

1. **Week 1-2:** Set Theory
2. **Week 3 :**Series, Sequence and progression
3. **Week 4 :** Elementary Geometry
4. **Week 5-6:** Intro to matrix Algebra
5. **Week 7:** General Arithmetic: Approximation and percentage errors
6. **Week 8 :** inequalities
7. **Week 9:** Intro. to probability and Trigonometric function and their inverse
8. **Week 10:**permutation and combination
9. **Week 11:** Differentiation ,calculus ,logarithmic ,implicit functions

Week 12 *Revision*

RESOURCES

- **Lecturer's Office Hours:**

- Dr. Odiwo Williams Oomkhudu, Mondays 2-4pm,

- **Course lecture Notes:** <http://www.edouniversity.edu.ng/oer/Bus/ECO122.pdf>

- **Books:**

- Agbadudu A.B. (1998) Mathematical methods in Business and Economics: United City press, Benin City, Nigeria.
- Eke,C.E.(2010) Solution Oriented on Business Mathematics :Justice Jeco Printing & publishing Global (recommended).
- Kenneth, . C.A,Nathaniel,O.O, & Raymond(1995) Revision course in Mathematics Space Matex publication ltd
- Adibe,O.R(1995) Introduction to management Mathematics for Business students :Model publishing ltd Aba.

Class work:

- Multiple parts (2 or 3).
- Must be done in class
- Home works + class work: ~ 30% of final grade.

- **Exams:**

- Final, comprehensive (according to university schedule): ~ 70% of final grade

Assignments & Grading

- **Academic Honesty:** All class work should be done independently, unless explicitly stated otherwise on the assignment handout.
- You may discuss general solution strategies, but must write up the solutions yourself.
- If you discuss any problem with anyone else, you must write their name at the top of your assignment, labeling them “collaborators”.
- **NO LATE HOMEWORKS ACCEPTED**
- Turn in what you have at the time it's due.
- All home works are due at the start of class.
- If you will be away, turn in the homework early.
- Late Home work (Home work) will not be accepted, but penalized according to the percentages given on the syllabus.

SET THEORY

INTRODUCTION: Now a day, the concept of sets and the algebra of sets is gaining ground in business and it related subjects. This is because money areas of business mathematics are based on this concept. For example, a business analysis involves such a set of data, sets of legal documents, sets of orders, and so on.

Besides, students use the notion of sets in the study of probability, statistics, mathematical programming, optimization etc.

Therefore, it will be useful to expose most students to the idea of elementary set theory and some selected applications. It is the aim of this topic to do so. Elementary notion of logic is also introduced.

THE CONCEPT OF SET AND NOTATION

A set is defined as a collection, or aggregate, of well-defined objects. Example, all the members of the business students are a set of students. The academic staff of the Faculty of Social Sciences is a set of Lecturers, set of students in Economic year one and the English alphabet is a set of letters.

There are certain requirements for the collection or aggregate of objects that constitute the set. They are;

- i. Collection of objects must be well-defined, i.e. we must be able to determine whether or not any object belongs to the set.
- ii. The object of a set must be distinct,
- iii. The order of the objects with the set must be immaterial i.e. a, b, c, is the same set as c, b, a.

SET NOTION AND DEFINITION

It is customary to designate a set by a capital letter. The individual objects of a set are called the element of the set. They are usually denoted by small letters or numbers. They are said to belong to, or to be members of, or to be in the set. The relationship between an object of the set and the set itself is expressed in the form as an element of or in a member of. The symbol, ε , is used often to denote this relationship. For example, $y \varepsilon A$ means y is element of A. non-members is indicated by $\notin$ and this is read “is not an element of”.

There are essentially two ways to specify a particular set, the roster method and the descriptive method the roster method involves listing within the braces all members of the set. It is sometimes called tabulation method. Thus {x, y, z} is read

‘the set whose members are x, y and z’. Also consider the set of digits. It can be written as $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Sometimes sets have so many elements that it is difficult or even impossible to tabulate them. In this case, we resort to the second method, the descriptive method, to specify such sets. For example, the following sets are well-specified without a tabulation of members; the counting numbers, the even counting numbers less than 1000, Vice-chancellor of all universities in a nation, the shareholders of Nigerian Brewery Limited.

EXAMPLE

If the descriptive method is used in specifying the set of digits that we gave above, the set would be;

$$D = \{x / x = 0, 1, 2, \dots, 9\}.$$

Two notations are to be noted here. One is the vertical line that may be read such that’ or ‘satisfying the condition that’. The second one is the three dots that mathematics use to indicate the omission of terms. So putting everything together, the above set read as D is equal to the set of elements x such that x equals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Example 1: Use either the roster or tabulation method or the descriptive method to develop the set notation for the English alphabet. Representing the set of letter in English alphabet by A, the set is $A = \{a, b, c \dots y, z\}$.

The letter or symbol that is used to denote any element of a specified set is called a variable over the set, and the set is referred to as the domain of the variable. A set may consist of no elements, a limited number of elements or unlimited number of elements. A set that contains no element is the *empty* or *null set* and is denoted by $\emptyset$ or sometimes by $[\]$, A *finite* set is a set that contains no element or a definite number of elements. The set D and A above are examples of finite sets. A set that contains an unlimited number of elements is called an *infinite set*. An example of an infinite set is set N where $N = \{n / n = 1, 2, 3\}$, the set of natural O counting number.

The totality of number under consideration is contained in one set, usually called the universal set. We shall denote it by U . that is, U is the umbrella set that includes all the items of particular interest for a specific situation. In most cases of infinite mathematics, one knows in advance the universal set about which discussion is centered. U varies from situation to situations.

Definition 1: Set X is said to be a subset of set Y , denoted by $X \subseteq Y$, if and only if each elements of X is an element of Y . for example, the set of all heads of departments in a university is a subset of all senior staff of the university. The set of all students taking business mathematics is a subset of all students enrolled in the university. The set of all people living in Kano or Warri is a subset of all people living in Nigeria.

For example, let $X = \{\text{table, spoon, chair}\}$ $Y = \{\text{comb, table, spoon, chair, tea}\}$. Then we see that any member found in X is also found in Y . thus $X \subseteq Y$.

Definition 2: Set X is said to be a proper subset of set Y , denoted by $X \subset Y$, if and only if each element of X is an element of Y and there is at least one element of Y that is not an element of X .

The examples given above are also good examples of proper subsets.

Definition 3: Two sets X and Y are said to be equal, denoted by $X = Y$, if and only if $X \subseteq Y$ and $Y \subseteq X$.

What we mean here is that every element of X is an element of Y and every member of Y is a member of X .

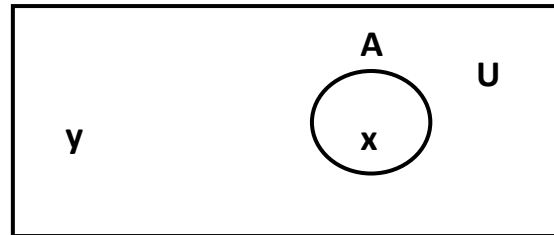
Example 2: Let $X = \{a, b, c, d, e\}$; $Y = \{b, c, a, e, d\}$, then $X = Y$.

Remarks: The empty set, is a subset of any set. The number of subset that can be obtained from a set is 2^n , where n is the number of elements in the original set. For instance, if

$A = \{a, b, c\}$; the subsets are $\{a,b,c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a\}$, $\{c\}$, $\emptyset$ eight of them.

SET OPERATION

We shall deal here with the relationships among a universal set and its subsets. The useful tool we are using in Venn Diagram which displays sets as regions of a place. A universal set, U , is represented by points inside a rectangle, and a subset A of the universal set is represented by points inside a circle that is in the rectangle.



An element of U is represented by a dot inside the rectangle from figure 1, we see the following;

- i. $A \subset U$
- ii. $x \in A$
- iii. $x \in U$
- iv. $y \in U$
- v. $y \notin A$

Definition 4: The Union of two sets, A and B , denoted by $A \cup B$ (reads ‘ A union B ’) is the set of all elements which belong to A or to B .

i.e. $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Definition 5: The intersection of two sets A and B , denoted by $A \cap B$ (reads ‘ A intersection B ’) is the set of elements which belong to both A and B .

i.e. $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Definition 7: The absolute complement or, simply, complement of a set A , denoted by A' , is the set of elements which do not belong to A .

i.e. $A' = \{x \mid x \in U, x \notin A\}$

Example 3: Let $U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$
 $A = \{1,2,5,7,8\}$
 $B = \{3,5,6,7,9,11,12\}$

- Then: (i) $A \cup B = \{1,2,3,5,6,7,8,9,11,12\}$
 (ii) $A \cap B = \{5,7\}$
 (iii) $A - B = \{1,2,8\}$
 (iv) $B - A = \{3,6,9,11,12\}$
 (v) $A' = \{3,4,6,9,10,11,12\}$
 (vi) $B' = \{1,2,4,10\}$

Generally, the following cases hold. You may verify it with

Example 3:

- (a) $A \cup A' = U$
 (b) $A \cap A = \emptyset$
 (c) $\emptyset \cap U = \emptyset$ and $\emptyset \cup U = U$
 (d) $(A - B) \cap B = \emptyset$ and $(B - A) \cap A = \emptyset$

SET OPERATIONS AND VENN DIAGRAM

Set operations can be illustrated with the aid of Venn Diagram the operations, union, intersection and complement are shown in figure 2a – 2e below:

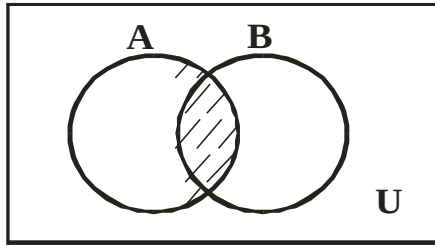


Fig. 2a
 $A \cup B$ is shaded

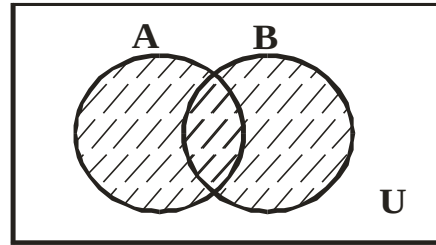


Fig. 2b
 $A \cap B$ is shaded

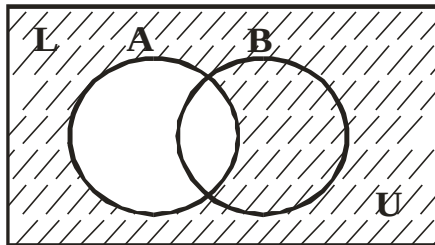


Fig. 2c
 A' is shaded

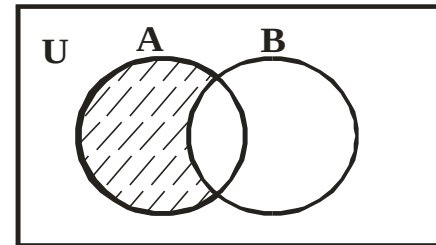


Fig. 2b
 $A - B$ is shaded

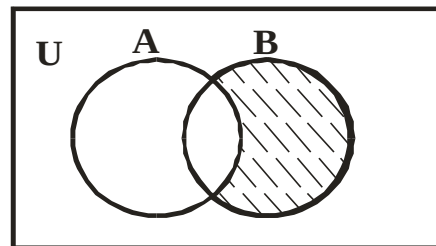
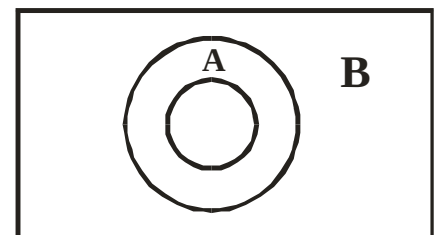
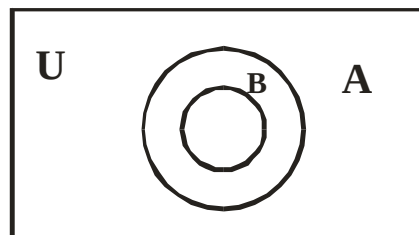
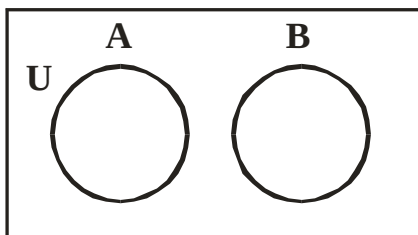


Fig. 2e
 $A - B$ is shaded

In the case of the operation Union and intersection, you may still want to illustrate them in three other conditions as shown in figure 3.



In Figure 3a, A and B are disjoint, that is, there is no element in common. The union of A and B is the interior of both circles. The intersection of A and B is the Null set; i.e. $A \cap B = \emptyset$, in figure 3b, $A \subset B$, the union of A and B is the interior of the circle representing B; i.e. here, $A \cup B = B$. the intersection of A and B is A; that is, $A \cap B = A$.

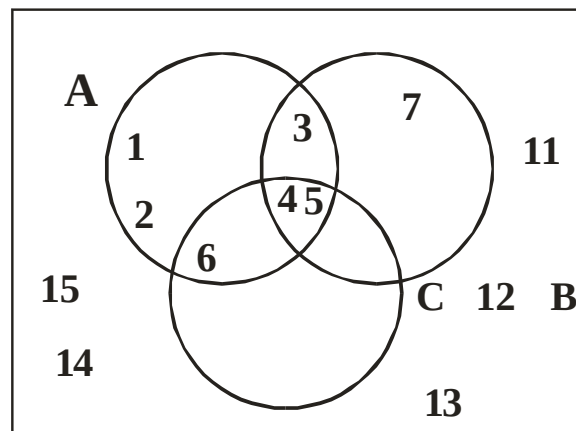
In figure 3c, $B \subset A$ so that $A \cap B = B$ and $A \cup B = A$.

CHARACTERISTICS OF OPERATIONS OF UNION AND INTERSECTION

We state below without proof, the important characteristics of the operations of union and intersection of sets. Let A, B and C be any subsets of the universal set U. the following are true:

- | | |
|------------------------------|--|
| 1. Idempotent properties: | $A \cup A = A$
$A \cap A = A$ |
| 2. Communicative properties: | $A \cap B = B \cap A$
$A \cup B = B \cup A$ |
| 3. Associative properties: | $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$ |
| 4. Distributive properties: | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| 5. Identity properties: | $A \cup \emptyset = A$
$A \cup U = U$
$A \cap \emptyset = \emptyset$
$A \cap U = A$ |
| 6. Complement properties: | $A \cup A' = U$
$A \cap A' = \emptyset$
$(A')' = A$
$U' = \emptyset$ and $\emptyset^1 = U$ |
| 7. Morgan's laws: | $(A \cap B)' = A' \cup B'$
$(A \cup B)' = A' \cap B'$ |

Example 4: Using the Venn diagram below, try to verify the above properties and law.



$U = 1, 2, \dots, 15$

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{3, 4, 5, 7\}$

$C = \{4, 5, 6, 8, 9\}$

As you will find out, most of these properties are trivial. Let us try numbers 4 and 7.

Properties 4 $A \cup (B \cap C)$:

Now $B \cap C$ = {4, 5}
 So that $A \cup B$ = {1, 2, 3, 4, 5, 6}
 Now $A \cup B$ = {1, 2, 3, 4, 5, 6, 7} and
 $A \cup C$ = {1, 2, 3, 4, 5, 6, 7, 8, 9}

$\therefore (A \cup B) \cap (A \cup C)$ = {1, 2, 3, 4, 5, 6}
 = $A \cup (B \cap C)$

Now verify $A \cap (B \cup C)$ = $(A \cap B) \cup (A \cap C)$

De Morgan's Laws:

$(A \cup B)' = \{1, 2, 4, 5, 6, 7\}$

$(A \cup B)' = \{8, 9, 10, 11, 12, 13, 14, 15\}$

$A' = \{7, 8, 9, 10, 11, 12, 13, 14, 15\}$ and

$B' = \{1, 2, 6, 8, 9, 10, 11, 12, 13, 14, 15\}$

So that $A' \cap B' = \{8, 9, 10, 11, 12, 13, 14, 15\}$

$$= \{A \cup B'\}$$

You can verify $\{A \cap B\}' = (A' \cup B')$

DETERMINATION OF NUMBER OF ELEMENTS IN A SET

In business or any social science discipline, one many resort to making use of a survey. It happens sometimes that one may not be able to obtain the information needed because something is missing. A knowledge of sets is very handy in this situation. In this section, we shall determine how many elements there are in a specified set.

For example, a researcher found in a survey of 200 customers that 80 drank beer brand A and brand B. but unfortunately, the researcher could not lay hands on a number of those who drank beer brand B. how many is this?

Take another example: suppose 5 students enroll in course A and nine enroll in a course B, then how many enrolled in either course A or course B?

Order to get a way of solving such problems, we introduce the concept of the number of a set.

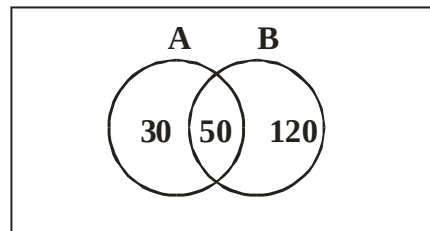
Definition 8: If S is any finite set then $n(S)$ read as ‘the number of S’ or ‘n or S’) is the number of elements in set S.

Example 5: If S = {a,b,c,d,e,f}
 Then $n(s)$ = 6
 If A = {l,t,m,k}
 Then $n(A)$ = 4 and
 $N\{7,21,45\}$ = 3
 $N\{1,2,4,5,10\}$ = 5

Note also that $n(\emptyset) = 0$

Now, we come back to our earlier example . let A be the set of people who drank beer brand A, and let B be the set of people who drank beer brand B. then the following information is given:

$$n(A) = 80; n(A \cap B) = 50; n(A \cup B) = 200$$



In figure 4, 50 has been placed in the intersection of A and B. since A contains 80 elements out of which 50 are common to both, A will have 30 elements that is not common to B. since A union B contains 200 elements, B alone will contain $200 - 30 - 50 = 120$.

$$\text{Hence } n(B) = 50 + 120 = 170$$

The researcher now knows that 170 people drank beer brand B.

Before attempting the second example let us take another simpler example after which we state a theorem.

Example 6 Let

$$\begin{aligned} \text{Let } A &= \{1,4,5\}; \\ B &= \{2,3,4,5\}; \\ C &= \{6,7\} \\ \text{Then } n(A) &= 3 \\ n(B) &= 4 \\ n(C) &= 2 \\ A \cup B &= \{1,2,3,4,5\} \\ \text{And } n(A \cup B) &= 5 \\ n(\emptyset) &= 0 \end{aligned}$$

In the last two examples here, you will notice that the set A and C are disjoint, in other words they are mutually exclusive. As a result, we see that

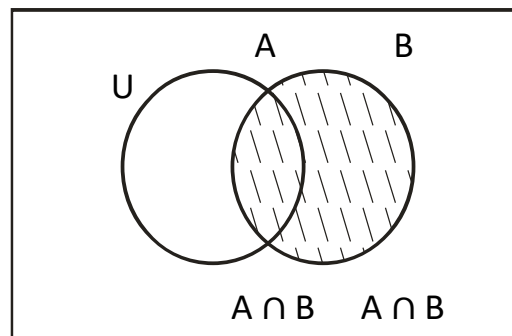
$$\begin{aligned} n(A \cup C) &= n(A) + n(C) \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

That is, if two sets X and Y are mutually exclusive

$$n(X \cup Y) = n(X) + n(Y) \text{ since } n(X \cap Y) = 0$$

Theorem 1: For any set A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$



Proof: By aid of Venn diagram, we decompose $A \cup B$ and B into mutually exclusive sets (see figure 5).

Thus, we can write

$$A \cap B = A \cup (A \cap B)$$

$$\text{And } B = (A \cap B) \cup (A' \cap B)$$

Note the following from the diagram.

A and $A \cap B$ are mutually exclusive set, so too are $A' \cap B$.

Therefore,

$$n(A \cup B) = n(A) + n(A' \cap B)$$

$$\text{and } n(B) = n(A \cap B) + n(B \cap A')$$

subtracting the second equation from the first one gives

$$n(A \cup B) - n(B) = n(A) - n(A \cap B)$$

$$\text{or } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

which we set out to prove.

Remarks: If A and B are disjoint or mutually exclusive, we note that $A \cap B = \emptyset$ and hence $n(A \cap B) = 0$.

$$\text{So that } n(A \cup B) = n(A) + n(B) - 0$$

$$= n(A) + n(B)$$

In solving that problem of enrolment in courses, we note that $n(A) = 5$ and $n(B) = 9$. If no one enrolls in course A and in course B, then the number of students that enroll in either A or B; that is;

$$n(A \cup B) = n(A) + n(B) = 5 + 9 = 14. \text{ But if there are 4 students that enroll in both course, then } n(A \cap B)$$

$$= 5 + 9 - 4 = 10$$

Theorem 2: For any sets A, B and C.

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Proof: Use repeated applications of theorem 1.

Example 7: Suppose that out of 120 year one students of Faculty of Social Sciences in a State University, 50 are taking Economics, E, 60 are taking Sociology, S, and 20 are taking both Economics and Sociology. How many are taking either Economics or Sociology?

Our problem is to find $N(E \cup S)$. Now

$$n(E) = 50; n(S) = 60; n(E \cap S) = 20$$

$$\text{Therefore, } n(E \cup S) = 50 + 60 - 20 = 90$$

Example 8: In a survey of 250 families in a wealthy part of a city in 1990, each family was asked the following questions:

- a. Do you have children in private school?
- b. Do you object to government taking over all private school?
- c. Do you object to removing your children from private to public school?

The answer to the questions were tabulated in set notation, where A, B, and C are sets of yes in answer to question (a), (b) and (c) respectively.

$$n(A) = 140$$

$$n(B) = 170$$

$$n(C) = 160$$

$$n(A \cap B \cap C) = 180$$

SOLUTION

- i. $n(B \cup C) = n(B) + n(C) - n(B \cap C)$
 $= 8 + 7 - 3$
 $= 12$
- ii. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
 $= 10 + 8 - 4$
 $= 14$
- iii. $n(B \cap C) = n(B' \cup C')$
 $= 20 - 3$
 $= 17$
- iv. $n(A \cup B \cup C) = 10 + 8 + 7 - 4 - 5 - 3 + 1$
 $= 14$
- v. $n(A' \cup C) = n(A') + n(C) - n(A' \cap C)$
 $= 10 + 7 - (7 - 5)$
 $= 15$
- vi. $n(A \cap B' \cap C') = 10 + 8 + 7 - 4 - 5 - 3 + 1 = 14$

Remarks: From Example 10, the following is apparent; for any subsets A, B, C of the universal set, U.

1. $n(A \cap B')$ = $n(A) - n(A \cap B)$
2. $n(A')$ = $n(U) - n(A)$
3. $n(B \cap C)$ = $n(U) - n(B \cap C)$
4. $n(A' \cup C)$ = $n(A') + n(A \cap C)$
5. $n(A \cap B' \cap C')$ = $n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

Example 11a

A company has 24 sportsmen, of whom 16 play football, 14 play tennis and 9 play both games.

- i. How many do not play either game?
- ii. How many play football as their only game?

Solution: Let A be the set of football players and B the set of tennis players

Then the following given: $n(A) = 16$; $n(B) = 14$ and $n(A \cap B) = 9$; and $n(U) = 24$.

- i. We are seeking $n(A \cup B)'$

But $n(A \cup B)' = n(U) - n(A \cup B)$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 16 + 14 - 9 = 21$

Therefore $n(A \cup B)' = 24 - 21 = 3$

Hence 3 do not play either game;

- ii. Here we find number of elements of $A \cap B'$ $n(A \cap B') = n(A) - n(A \cap B) = 16 - 9 = 7$

Hence seven have football as their only game.

Example 11b

A market research group conducted a survey of 500 consumers and reported that 360 consumers like product A and 225 consumers like product B. what is the least number that must have liked both products? Also determine the maximum number that must have like both products.

Solution: The following information is given: $n(U) = 500$; $n(A) = 360$; $n(B) = 225$

Now, $n(A \cup B) - n(A) + n(B) - n(A \cap B) = 360 + 225 - n(A \cap B)$ i.e. $n(A \cup B) = 585 - n(A \cap B)$.

But the intersection is least when the union is at maximum.

Since maximum $n(A \cup B) = 500 = u(U)$

Therefore least $n(A \cup B) = 585 - 500 = 85$.

Also note that maximum of intersection is the minimum either $n(A)$ or $n(B)$ that is minimum of 360 or 225.

Hence maximum $n(A \cap B) = 225$

Also that the range of values for the intersection is from 85 to 225.

Example 12:

In a survey of 400 families the number that read the most recent issues of various magazines were found to be as follows:

Business Times (T), 112; Business World (W) 120; Business Punch (P) 168;
Business Times and Business World 32; Business Time and Business Punch, 40;
Business World and Business Punch 20; all the three magazines 12.

Using set theory, find

- How many read none of the three magazines?
- How many Business Punch as their only magazine?
- How many read Business World if they read Business Punch?
- How many read exactly one magazines only?
- How many read exactly two magazines only?

SOLUTION

$n(U) = 400$; $n(T) = 112$; $n(W) = 120$; $n(P) = 168$; $n(T \cap W) = 32$; $N(T \cap P) = 40$;
 $n(W \cap P) = 20$;
 $n(T \cap W \cap P) = 12$.

(a) Are looking for $n(T \cup W)$ which equals $n(U)$, $n(T \cup P \cup W)$.

Now $n(T \cup W \cup P) = n(T) + n(W) + n(P) - n(T \cap W) - n(T \cap P) - n(W \cap P) + n(T \cap P \cap W)$
 $= 112 + 120 + 168 - 32 - 40 - 20 + 12$.

$$\therefore N(TUWUP) = 32.$$

Therefore $n(T \cap W \cap P)' = 400 - 320 = 80$.

Hence eighty read none of the 3 magazines.

(b) Punch only is $P \cap T' \cap W'$

$$\begin{aligned} N(P \cap T \cap W') &= n(P) - n(P \cap T) - n(P \cap W) + n(P \cap T \cap W) \\ &= 163 - 40 - 20 + 12 = 120. \end{aligned}$$

(c) World and Punch only is $W \cap P \cap T'$

$$\text{Now } n(W \cap P \cap T') = n(W \cap P) - n(P \cap T \cap W) = 20 - 12 = 8.$$

(d) Exactly one magazine only means either Punch alone, or World Alone or Times alone.

That is, $n(P \cap T' \cap W') + n(T \cap P \cap W') + n(W \cap P' \cap T')$.

Using the rate in part (b), we have this to be:

$$120 + 52 + 80 = 252$$

Hence 252 read exactly one magazine alone.

(e) Exactly two magazines means either; $n(P \cap T \cap W') + n(P \cap T' \cap W) + n(P' \cap T \cap W)$.

Again, using result in part ©, this equal to $28 + 8 + 20 = 56$.

CONCEPT OF LOGIC

Introduction

In the normal English Language, statements can be classified as declarative, exclamatory or interrogative. But in mathematics, a declarative sentence is called a statement. This is because this declaration is either true or false.

The analysis of Mathematical statements, together with the principles and methods employed in distinguishing valid arguments from those that are not valid, is the study of logic. In this section, we touch simple method involving the usage of logic as many examples as we can. But just let us give examples of statement; (by this we means mathematical statements).

Example 13: Consider the following expressions or statements;

- i. Profit equals total revenue minus total costs;
- ii. Three is an even number;
- iii. Where were you born
- iv. Accra is the capital of Nigeria;
- v. Five is an odd number;
- vi. God save us!

Sentences, i, ii, iv and v are statements because they are either true or false. For instance, statement (i) and (v) are true while statements (ii) and (iii) are false. Sentences (iii) and (vi) are not statements because neither of them is true or false.

TRUTH VALUE: If a statement is true, it is truth value and is then T and if it is false we say it is truth value F.

COMPOUND STATEMENT: The statements given in the above examples are simple statements. But if a statement is made up of two or more simple statements, you have what is called a compound statement. For example. Lagos is in Nigeria. $5+10 = 15$. These are two simple statements. However, a statement like, Lagos is in Nigeria and $5+10=15$ is a compound statement. A statement such as he is a musician or he likes music is also a compound statement.

In compound statements, the truth value is determined by the truth values of the simple statements that made it up and together with the way in which these simple statements are combined to form the compound statement. The words that connect the simple statements together are called connectives.

There are five connectives, which are used often. They are (i) and denoted by $\wedge$, (ii) or denoted by $\vee$ (iii) not denoted by $\neg$ (iv) implies denoted by $\Rightarrow$ (v)

if and only if denoted by $\leftrightarrow$. The first connective is also called conjunction. The second negation. (Readers/attention is drawn to the similarities between intersection, union and complement in the section of set, and conjunction dis-conjunction and negations as we shall treat them here). In logic, we use small letters such as p,q,r,s,t. to represent statements.

numerical. Much of the subject matter of Business and Economics has a structure that can be expressed mathematically as could be illustrated as follows:

- a. Quantity demanded depends on price. There are functional relationships between quantity and price.
- b. At the point of 'equilibrium' the price at which sellers are willing to sell equals the price at which buyers are willing to buy. This perhaps leads to the solution of simultaneous equations in two unknown variables.

Mathematics plays two basic roles in business and economics. There is a direct role and indirect role. The comparison of two or more equilibrium situations, which invariably involves relationships between increments of quantities and directions of change, is one area of the direct use of Mathematics. Another area of mathematical importance is in maximization and minimization. The use of mathematical calculus has helped a great deal.

Mathematics is not only a powerful tool for making logical deductions in business and economics, and specifically for facilitating the solution of problems of resources allocation. It also aid in making the distinction between the variables and constants of a problem. It also focuses attention on the data required for solving a particular problem.

Again, in the form of econometrics (defined here as the direct application of mathematics and statistics to economic theory), mathematical tools help in business and economics predictions. The mathematical use of tables, graphs, equations and elementary statistics has helped Bankers and other economic policy makers in their different level of operations.

SEQUENCES, SERIES AND PROGRESSION

A sequence is simply arrangement of number in order of descending or ascending order e.,g 1, 3, 5, 7, 9, 11. The order must follow a certain called the common differences between each successive terms.

$$X_1, X_2, X_3 \dots X_k \text{ ----- (1)}$$

The object X_k is called the k th element of term of the sequences.

If $n = 1, 2, 3, \dots, n$ which is called a finite sequences while

If $n = \infty$ such a sequence is called an infinite sequences

Example 1

Given 3, 7, 11, 15, 19 27

Type - Arithmetic of 7 terms

Rule - Addition of 4 to a given term to produce the next term

The first term is 3

The second term is $3+4 = 7$

The third term is $7+4 = 11$

Example 2

Given a sequence of 3, 6, 12, 24, 48, ... 768....

Type - infinite sequence

Rule - multiply a given term by 2 to produce the next term

If the 1st term = 3

2nd term = $3 \times 2 = 6$

3rd term = $6 \times 2 = 12$

4th term = $12 \times 2 = 24$

$$5^{\text{th}} \text{ term} \quad = 24 \times 2 = 48$$

$$6^{\text{th}} \text{ term} \quad = 48 \times 2 = 96$$

And so on until the last term is achieved. We have earlier defined A.P as a sequence in which each term after the first is formed by adding a fixed number, called the common difference to the preceding term is called an arithmetic progression (A.P).

Example 1: 3, 7, 11, 15, 19, ...

Is an AP with common difference of

$$7 = 3 + 4$$

$$11 = 7 + 4$$

$$15 = 11 + 4$$

$$19 = 15 + 4$$

Example 2

Consider the simple amount A_t , given by

$$A_t = p + ptr$$

Where $P =$ Principal

$T =$ Time (years)

$I =$ Interest

For $t = 1$

$$A_1 = p + pr$$

For $t = 2$

$$A_2 = p + pr$$

For $t = 3$

$$A_3 = p + pr$$

For $t = n$

$$A_n = p + pr$$

Hence each term increases by P_i

That is $A_2 = A_1 + O_i$

$$P + p_i + p_i$$

$$A_2 = p + 2P_i$$

Hence p_i is the common difference

FINDING THE NTH TERM OF AN A.P

	2 ND TERM	3 RD TERM	NTH TERM
A	$a + d$	$a + 2d$	$a + (n-1)d$

Given that $a = 1^{\text{st}}$ term

$D =$ common difference

Nth term of a sequence

$$N^{\text{th}} = a + (n-1)d$$

Example 1

Given $a = 1, d = 1$

Such that an AP is the sequence of the first n , natural numbers $1, 2, 4, 5, \dots n$

Example 2

Find the 25th term of the sequence

2, 4, 6, 8

$$+20 = a + (n-d)d$$

$$= 2 + (20 - 1)2$$

$$= 2 + (19)(2)$$

$$= 2 + 38$$

$$= \underline{40}$$

CLASS WORK

1. Find the 15 term of the sequence 28, 25, 22, 19 ...
2. Find the 8th term of the sequence 21, 10, -1, -12....

Example 3

Which term of the A.P 49, 44, 39 ... is 9?

General $tn = a + (n - 1) d$

$$49 + (n - 1) (-5)$$

$$9 = 49 - 5n + 5$$

$$9 = 54 - 5n$$

$$9 - 54 = -5n$$

$$\frac{-45}{-5} = \frac{5n}{5}$$

$$N = 9$$

Example 4

The 6th term of an AP is 35 and the 13th term is 77. Find 26th term.

$$a + (6-1) d = 35 \quad - \quad - \quad (1)$$

$$a + (13-1) d = 77 \quad - \quad - \quad (2)$$

$$a + 5d = 35 \quad - \quad - \quad (1)$$

$$a + 12d = 77 \quad - \quad - \quad (2)$$

Simple rule of simultaneous equation

1. Same number Same sign = -
2. Same number Different sign = +

We adopt rule (i) which is the subtraction sign

$$\frac{+7d}{7} = \frac{42}{7}$$

$$D = \underline{\underline{6}}$$

Put d into equation (1) or (2) to get (a)

$$a + 5 \times 6 = 35$$

$$a + 30 = 35$$

$$a = 35 - 30$$

$$a = \underline{\underline{5}}$$

To get the 20th term

$$t_n = a + (n-1) d$$

$$\begin{aligned} t_{20} &= 5 + (20 - 1) 6 \\ &= 5 + (19) 6 \\ &= 5 + 114 \\ &= \underline{\underline{119}} \end{aligned}$$

Example 5

A firm rents its premises and the rental agreement provides for a regular annual increase of N2,550.00, if the rent in the first year is N8,500.00 what is the rent of the tenth year?

Solution

$$T_n = a + (n - 1) d$$

$$T_n = 10 \quad a = \text{N}8,500.00 \quad d = \text{N}2,650.00$$

$$T_{10} = 8,500.00 + (10-1) 2,650$$

$$= 8,500.00 + (9) (2,650)$$

$$= 8,500 + 23,850$$

$$= \text{N}32,350.00$$

Class work

Find the 5th and 10th term of an A.P are -12 and - 27. Find the A.P and its 15th term.

SUM AND ARITHMETIC PROGRESSIONS

One way to find the sum of an arithmetic progression is to evaluate each of the successive terms and add them up. This could be lengthy if numerous terms are involved, so the following formula can be used.

$$S_n = \frac{n}{2} [2a + (n-1) d]$$

Where s - sum of the progression

N - no of term

D - common difference

Question

The salary scale of a seminar officer starts at N570,000 per annum a rise of N2,800 is given at the end of each year. Find the total amount of money that the officer will earn in 14 years.

Solu

$$\begin{aligned}\text{Given } a &= \text{N}570,000 & d &= \text{N}2,800.00 \\ & & &= \text{N}572,800.00\end{aligned}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned}S_{14} &= \frac{14}{2} [2 \times 570,000.00 + (14-1) 2,800] \\ &= 7 (1,140,000 + (13) (2,800)) \\ &= 7 (1,140,000 + 36,400) \\ &= 7 (1,176,400) = \text{N}8,234,800.00\end{aligned}$$

Example 2

An employee, who received fixed annual increments had a final salary of N9,000 P.A after 10 years. If his total salary was N65,000 over the 10 years, what was his initials salary?

Solu

$$S_n = \text{N}65,000 \quad n = 10 \quad a = ? \quad d = ?$$

$$\text{Thus } S_n = n [a + \frac{1}{2} (n-1) d]$$

$$65,000 = 10 [a + \frac{1}{2} (10 - 1) d]$$

$$65,000 = 10 (a + \frac{1}{2} (a) d)$$

$$65,000 = a + 4.5d \quad - \quad - \quad - \quad (1)$$

At the end of 10 years expected salary was ~~N~~9,000.00

$$9,000 = a + (10-1) d$$

$$9,000 = a + 9d \quad - \quad - \quad - \quad - \quad (2)$$

Solving (1) & (2) simultaneously

$$\text{We have } 2,500 = 4.5d$$

$$d = \text{N}555.53$$

Putting d into equation (1) or (2)

$$a = \text{N}4,000.00$$

$$\text{annual increment} = \text{N}555.55$$

$$\begin{aligned} 2^{\text{nd}} \text{ year} &= 4,000.00 \\ &\quad + 555.55 \\ &\quad \underline{\text{N}4,555.55} \end{aligned}$$

A. Let S_n be the sum of the first terms of an A.P

$$\text{Then } S_n = a + (a + d) + (a + 2d) + (a + 3d) + \dots + a + (n-1)d$$

The sum of this sequence can also be written in decreasing terms as follows:

$$S_n = a + (n-1)d + a + (n-2)d + a + (n-3)d + \dots +$$

Adding the equation terms by terms, we have

$$2S_n = n [2a + (n-1)d]$$

Divide through by 2

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Example

Find the 20th term and the sum of the first 20 terms of the A.P 4, 9, 14, 19, ...

Solu

$$1^{\text{st}} \text{ term } a = 4$$

$$\text{Common difference } d = 5$$

$$\text{Number of terms } n = 20$$

$$\begin{aligned} T_{20} &= 4 + (20 - 1)(5) \\ &= 4 + 19(5) \end{aligned}$$

$$= 4 + 95$$

$$= 99$$

Sum of the first 20 terms is given by

$$\begin{aligned} S_{20} &= \frac{20}{2} [4 + 99] \\ &= 10 (103) = 1030 \end{aligned}$$

HOME WORK

If -8, m, n 19 are in arithmetic progression find (m, n)

GEOMETRIC PROGRESSION G.P

A sequence can be geometric progression when each term after the first is formed by multiplying or dividing the proceeding term by a fixed number, called the common ratio, is called Geometric progression.

Example the sequence 3, 16, 12, 24, 48, 96, is a G.P with common ratio as 2

Find the nth term of GP

$$1^{\text{st}} \text{ term} = a$$

$$2^{\text{nd}} \text{ term} = ar$$

$$3^{\text{rd}} \text{ term} = ar^2$$

$$4^{\text{th}} \text{ term} = ar^{n-1} = ar^{4-1} = ar^3$$

Note r is always

$$r = \frac{2^{\text{nd}} \text{ term}}{1^{\text{st}} \text{ term}}$$

$$r = \frac{3^{\text{rd}} \text{ term}}{2^{\text{nd}} \text{ term}}$$

$$r = \frac{4^{\text{th}} \text{ term}}{3^{\text{rd}} \text{ term}}$$

GEOMETRIC SERIES

The series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots + a_n + \dots$$

Is called a geometric series if there exist a fixed constant r , such

That $a_{n+1} = r a_n$ for each $n > 0$

$\therefore r$ is the ratio of each term its predecessor is c

Generally $a_n = ar^{n-1}$

SUM OF GEOMETRIC PROGRESSION

$$\text{If } S_n = \sum_{k=1}^n ar^{k-1} = ar + ar^2 + ar^3 + \dots + ar^n \quad (1)$$

$$\text{And } S_n = \sum_{k=1}^n ar^k = ar + ar^2 + ar^3 + \dots$$

$$Ar^{n-1} + ar^n \quad (2)$$

Subtracting equation (2) from equation (1)

$$S_n - rS_n = a - ar^n$$

$$\frac{S_n(1-r)}{1-r} = \frac{a(1-r^n)}{1-r}$$

$$S_n = a \frac{(1-r^n)}{(1-r)} \quad (r < 1)$$

$$\text{Or } S_n = \frac{a(r^n - 1)}{(r-1)} \quad (r > 1)$$

When $(r) = 1$ the sum is not defined

Example

Find the sum of 10 terms of the G.P 2, 4, 8, 16, 32, ...

Solution

$$n = 10$$

$$r = \frac{4}{2} = \frac{8}{4} = \frac{16}{8} = \frac{32}{16} = 2$$

$$a = 2$$

$$r = 2$$

Since $r > 1$

$$\begin{aligned} S_n &= \frac{2(2^{10} - 1)}{(2 - 1)} \\ &= 2(1024 - 1) = 2(1023) \\ &= \underline{\underline{2046}} \end{aligned}$$

Example

Find the 9th term and the sum of 8 term of the G.P 8, 4, 2, 1, ...

Solution

$$a = 8$$

$$n = 9$$

$$r = \frac{4}{8}, \frac{2}{4}, \frac{1}{2} = \frac{1}{2}$$

$$r = \frac{1}{2}$$

since $r < 1$

$$\begin{aligned} \text{i. } L_n &= a(r)^{n-1} \\ &= 8(1/2)^{9-1} \\ &= 8(1/2)^8 \\ &= \cancel{8} \frac{1}{\cancel{128} \quad \cancel{64} \quad 32} = \frac{1}{32} \end{aligned}$$

$$\text{ii. } S_n = \frac{a(1-r^n)}{(1-r)} \text{ since } (r) < 1$$

$$\begin{aligned} S_n &= \frac{8[(1 - (\frac{1}{2}))]}{[1 - \frac{1}{2}]} \\ &= \frac{8[1 - \frac{1}{256}]}{[\frac{1 - \frac{1}{2}}{2}]} = 8 \frac{1 - \frac{1}{256}}{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned}
&= 8 \left[1 - \frac{1}{256} \right] \div \frac{1}{2} \\
&= 8 \left[1 - \frac{1}{256} \right] \times \frac{2}{1} \\
&= 8 \left[\frac{256-1}{256} \right] \times \frac{2}{1} \\
&= 9 \frac{[255]}{\cancel{256} 128} \times \frac{2}{1} \\
&= 8 \frac{[255]}{128} \\
&= \underline{\underline{15.97}}
\end{aligned}$$

APPLICATION OF G.P

If the production of a factory was 5,000,000 tons of fertilizer in 1990 and no quantity have increase by 10% per annum each year. Computer the output for the year 1990 x 1994 to the nearest ton. Find the rate of growth, assuming that the rate of growth is to be continued, estimate the production of the year 2000 to the nearest million.

Solution

1990	1991	1992	1993	1994
5,000,000	5,500,000	6,050,000	6,655,000	7,320,500

$$\text{i.e } r = \frac{5500,000}{5,000,000} = \frac{6050000}{5500000} = \frac{6655000}{5500000} = \frac{7320500}{5500000}$$

$$r = 1.1$$

$$t_n = ar^{n-1}$$

$$a = 5,000,000$$

$$n = \text{in the year 2000} = 11 \text{ yrs}$$

$$t_{11} = 5,000,000 (1.1)^{11-1}$$

$$= 5,000,000 (1.1)^{10}$$

$$t_{11} \cong 13,000 \text{ tons or } 12,968712.3$$

Example:

A building costs N500,000 and it is expected to depreciate at 10%P.A on the reducing balance method, what will its written down value be after 25 years.

Solu

$$t_n = ar^{n-1}$$

Where a = N500,000

$$n = 26$$

$r = (1-d)$, where d is depreciation rate.

$$\begin{aligned} \text{Value after 25 yrs} &= 500,000 (1-0.1)^{n-1} \\ &= 500,000 (1-0.1)^{26-1} \\ &= 500,000 (0.9)^{25} \\ &= \text{N}35,985.00 \end{aligned}$$

INTRODUCTION TO COORDINATE GEOMETRY

It is one of the most important and exciting ideals of mathematics. It is a connection between algebra and geometry through graph of lines and curves. These enable geometry problems to be solved algebraically and prove geometric insights into algebra. The invention of calculus was an extremely important development in mathematics that enables

Mathematicians and physicists to model the real world in ways that was previously impossible. It brought together nearly all of algebra and geometry using the coordinate plane. The invention of calculus depended on the development of coordinate geometry.

THE REAL NUMBER SYSTEM

- a. **Natural Numbers:** These are positive whole numbers or integers. They are the numbers used for counting the smallest natural number is 1 and there is no largest natural number. The positive integers 1, 2, 3, 4 ... may be obtained by adding the real number 1 successfully to itself.
- b. **Integers:** These are natural numbers, zero and negative whole numbers. The integers consist of all positive and negative whole numbers together with the real number 0. The set is infinite in both directions.
- c. **Rational Numbers:** Integers and fractions together make up the set of rational numbers. Numbers are described as rational if they can be expressed as a ratio of two integers such as $p/q = r$ for all integral values of p, q , except $q=0$. It is important that p, q do not have any common factor than 1 or else, further simplification of p/q may produce an integer. Numbers which do not have any common factor better than 1 are called 'relatively prime'.

Properties of Rational Numbers/Economic Application

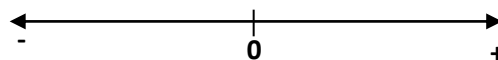
- i. **Complete Ordering:** If a, b and c are rational numbers, $a > b$ and $b > a$: then $a = b$. This property states that two entities are either unequal or equal. In economic terms, this property says that when a consumer is confronted with two situations a, b , he will either prefer one to the other or be indifferent between the two. In other words, he must not say "I cannot choose". Consumers for whom the above property does not hold are generally ruled out of consideration because their behaviour cannot be predicted.
- ii. **Transitivity:** If $a < b$ and $b < c$, then $a < c$. Rational behaviour in the sense of a consumer always preferring more of a good to less of it implies that transitivity should hold. If product A is preferred to product B and product B is preferred to product C, then product A is preferred to product C.

- iii. **Denseness:** If $a < b$, there exist C such that $a < c < b$. it means the set of all rational numbers is dense or without holes. Example ($\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{8} \dots$) since $\frac{1}{4} < \frac{1}{2} < \frac{3}{4}$. Note that 1, 2, 3, 4 ... does not satisfy the condition because there is no integer for example between 1 and 2. The economic interpretation of the denseness property is that the 'commodity space' (i.e. the plane in which indifference curves are drawn) contains an infinite amount of bundles; there is infinitely many commodity bundles.
- iv. (a) If $a > b$, then $a \pm c > b \pm c$ for any $c > 0$
 (b) If $a > b$ then $a.c > b.c$ for any $c > 0$

But for multiplication, the inequality sign is reversed for $c < 0$, and $a.c < b.c$.

A reasonable economic interpretation can be placed on this such as: if a, b represent two different commodity bundles so that ' a ' is preferred to ' b ', then the introduction of a third bundle ' c ' will not affect the existing relation between ' a ' and ' b '.

- (d) **Irrational Numbers:** These are numbers that "cannot" be expressed as ratio on pair of integers. Examples of irrational numbers include: $\sqrt{2}, \sqrt{3}, \sqrt{7}, \dots \pi$, etc. These are on repeating, non terminating decimal.
- (e) **Real Numbers:** When the set of rational numbers is extended to include irrational numbers, the results set constitutes the real numbers system. The set of real numbers thus combines the properties of both rational and irrational numbers. Real numbers can be represented geometrically by points on a straight line. The set R called the real numbers is represented by straight line called number line which is assumed to extend indefinitely in both directions. We use 0 to divide the number line into two. 0 is called the origin and numbers to the right of 0 called positive while those to the left of 0 are called negative.



Now, let a and b be real numbers with $a < b$. The following sets are called intervals from a to b .

- i. $(a,b) = (x \in \mathbb{R}: a \leq x \leq b)$
- ii. $(a,b) = (x \in \mathbb{R}: a < x \leq b)$
- iii. $(a,b) = (x \in \mathbb{R}: a \leq x < b)$
- iv. $(a,b) = (x \in \mathbb{R}: a < x < b)$

- i. Is called the closed interval from a to b
- ii. Is called half-open from a to b.
- iii. Is called half closed intervals from a to b
- iv. Is called the open interval from a to b

For each intervals, the points a,b are called the end points $\mathbb{R}$. The set of real numbers is sometimes written as $(-\infty, +\infty)$. The symbol ∞ is called infinity and is not an element of $\mathbb{R}$. $+\infty$ is bigger than any real number and $-\infty$ is less than any real number.

Exercise

1. Explain the following terms.
 - a. Quantity demanded depends on price.
 - b. Equilibrium point
 - c. Natural numbers
 - d. Integers
 - e. Rational numbers irrational numbers
 - f. Real numbers
2. State the properties of a real number, what are the economic importance
3. What is the relevance of mathematics to Business Accounting and Banking?

ELEMENTARY ALGEBRAIC

Algebraic statements are expressed with the signs of addition (+) subtraction (-) multiplication (x) division ($\div$) brackets ()

Basic algebraic operations

The symbols +, -, $\times$ and $\div$ have the same meanings in algebra as in arithmetic. The operations

- i) Multiply a number x by 6 is written as $6 \times x = 6x$
- ii) Add a number x to 2 is written as $x + 2 = x + 2$
(or $2 + x$) And so on.

Simplification

Expressions such as $6 + x$, $3x - 4$, $x + y$, etc are called algebraic expressions. This is because each of them contain letter(s) (i.e x or y)

1. Simplify the following expressions

- a) $x - 2y + 3x - 2x + 5y$
- b) $x^2 - 2xy + 8xy + 3x^2$

Solution:

$$\begin{aligned}\text{a) } & x - 2y + 3x - 2x + 5y \\ &= x + 3x - 2x - 2y + 5y \text{ (Like terms are grouped together)} \\ &= 4x - 2x + 3y \\ &= 2x + 3y\end{aligned}$$

Note: $2x + 3y$ cannot be added together to obtain a unique value because x and y are unlike terms.

$$\text{b) } x^2 - 2xy + 8xy + 3x^2$$

$$= x^2 + 3x^2 - 2xy + 8xy$$

$$= 4x^2 + 6xy$$

The result ' $4x^2 + 6xy$ ' can be further simplified to obtain $2x(2x+3y)$. The method used to simplify $4x^2 + 6xy$ to give $2x(2x+3y)$ is called FACTORISATION

Tutorial Questions

Simplify the following algebraic expressions

1. $2x^2 - 3y + x^2$
2. $m + n + 2m - n$
3. $2p - q + p + 3q$
4. $2x + 5y - (2y - 3x)$

Answers

1. $3x^2 - 3y$ (OR $3(x^2 - y)$)
2. $3m$
3. $3p + 2q$ (OR $2q + 3p$)
4. $5x + 3y$ (OR $3y + 5x$)

ALGEBRAIC FRACTIONS

The procedure for simplifying algebraic fractions with monomial denominations are $\frac{2}{x}$, $\frac{5}{x}$, $\frac{8}{3x}$, etc.

1. Simplify the following

a) $\frac{5}{x} + \frac{1}{2x}$

b) Express the sum of $\frac{x-2}{2x}$ and $\frac{3y-1}{4}$ as single fraction in its lowest term.

Solution

a) $\frac{5}{x} + \frac{1}{2x}$

L.C.M is x and $2x$

$$= \frac{(2x \times 5) + (x \times 1)}{2x}$$

$$= \frac{10x + 1x}{2x}$$

$$= \frac{11x}{(x)(2x)} = \frac{11}{(2x)}$$

b) $\frac{x-2}{2x} + \frac{3y-1}{4y}$

$2x$ and $4y$ can be taken as L.C M

$$= \frac{4y(x-2) + 2x(3y-1)}{(2x)(4y)}$$

$$= \frac{4xy - 8y + 6xy - 2x}{(2x)(4y)}$$

$$= \frac{10xy - 8y - 2x}{(2x)(4y)}$$

$$= \frac{10xy-8y-2x}{(2x)(4y)} \quad \text{OR}$$

$$= \frac{2x(5y-1)-8y}{(2x)(4y)}$$

ALGEBRAIC FRACTIONS WITH BINOMINAL DENOMINATIONS

Examples of algebraic fractions with binominal denominations are

$$\frac{2}{3xy}, \frac{x+2}{xy+x^2}, \frac{2}{ab^2}, \frac{1}{ab} \text{ etc}$$

1. Simplify the following

$$\text{a) } \frac{1}{u} + \frac{1}{v} + \frac{1}{u+v}$$

$$\text{b) } \frac{3}{x+y} + \frac{2}{xy}$$

$$\text{c) } \frac{xy}{x^2-y^2} - \frac{y}{x+y}$$

Solution

$$\text{a) } \frac{1}{u} + \frac{1}{v} - \frac{1}{u+v}$$

L.C.M of u, v and (u+v) is uv (u+v)

$$\begin{aligned}
&= \frac{v(u+v) + u(u+v) - uv}{uv(u+v)} \\
&= \frac{uv + v^2 + u^2 + uv - uv}{uv(u+v)} \\
&= \frac{u^2 + v^2 + uv}{uv(u+v)}
\end{aligned}$$

$$\text{b) } \frac{3}{x+y} + \frac{2}{xy}$$

L.C.M of $(x+y)$ and xy is $xy(x+y)$

$$\begin{aligned}
&= \frac{3xy + 2(x+y)}{xy(x+y)} \\
&= \frac{3xy + 2x + 2y}{xy(x+y)} \\
&= \frac{3xy + 2(x+y)}{xy(x+y)}
\end{aligned}$$

$$\text{c) } \frac{xy}{x^2 - y^2} + \frac{y}{x+y}$$

L.C.M of $x^2 - y^2$ and $x+y$ is $x^2 - y^2$

$$\begin{aligned}
&= \frac{xy + y(x-y)}{x^2 - y^2} \\
&= \frac{xy + xy - y^2}{x^2 - y^2} \\
&= \frac{xy + xy - y^2}{x^2 - y^2}
\end{aligned}$$

$$= \frac{y(2x-y)}{x^2-y^2}$$

Tutorial Questions

1. Express the following as single fractions in their lowest terms.

a) $\frac{1}{x} + \frac{1}{x-1}$ (b) $\frac{1}{x+y} + \frac{1}{x-y}$ (c) $\frac{xy}{x^2-y^2} - \frac{y}{x+y}$

2. Simplify the following:

a) $\frac{1-y}{x+y} \times \frac{m-n}{y-x} \times \frac{y+x}{n-m}$

b) $\frac{2y}{z+2y} \times \frac{z^2+2yz}{6y^2}$

c) $\frac{3}{x} + \frac{5}{2x}$

d) $\frac{5}{x-2} + \frac{3-x}{x-3}$

Answers

a) $\frac{2x-y}{x(x-y)}$ (OR) $\frac{2x-y}{x^2-xy}$

b) $\frac{x^2+x^2}{(x+y)(x-y)}$ (OR) $\frac{x^2+y^2}{x^2-y^2}$

c) $\frac{y^2}{x^2-y^2}$

(a) 1

(b) $z/3y$

(c) $11/2x$

(d) $\frac{7-x}{x-2}$ (OR) $\frac{7-x}{x-2}$