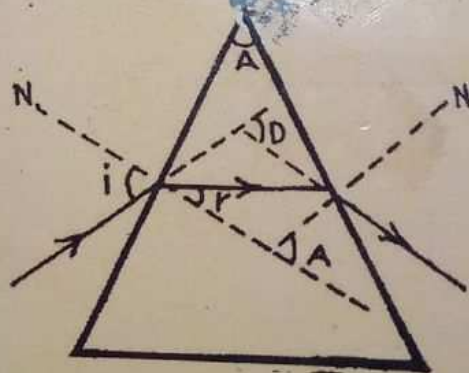
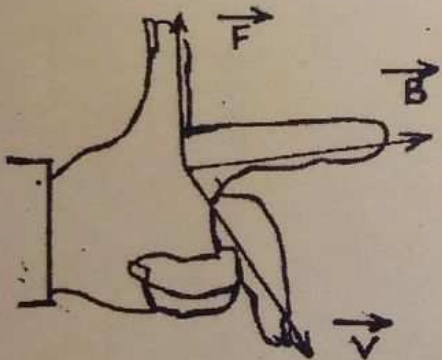
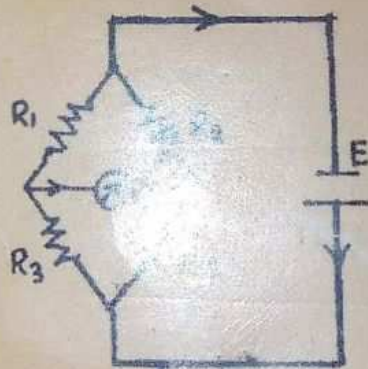
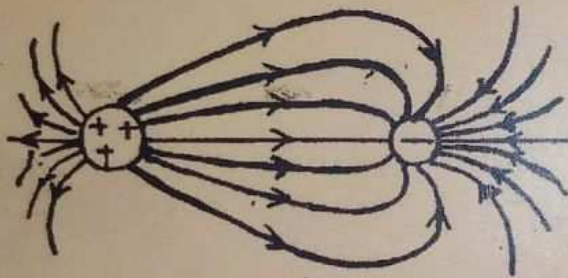


ADVANCE PHYSICS:

ELECTRICITY, ELECTROMAGNETISM AND OPTICS

FOR

SCIENCES, ENGINEERING AND MEDICAL STUDENTS



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PHYSICS 102

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ELECTROSTATICS

1.1 Electric Charges

Thales of Miletus (600 B.C.) discovered that when some dry materials are rubbed with others, they begin to exhibit attractive property. For example if amber (or glass rod) is rubbed with silk, it is found to begin to attract small pieces of dry paper, tin-foil and other light materials. On the other hand, when ebonite (or hard rubber) is rubbed with fur, this attractive property is again experienced, but the nature of this property is different in the two cases. For example if two rubbed glass rods are suspended with silk thread, and gradually brought close to each other, the two rods are found to repel each other. The same repulsion occurs if two rubbed ebonite rods are brought together. But if a rubbed glass rod is brought close to a rubbed ebonite rod, both will attract each other. Thus the attractive property of each rod is opposed in effect to the other.

The property responsible for this attractive or repulsive property is called a charge. Benjamin Franklin (1706-1790) was the first scientist to give the names positive and negative to the charges on the rubbed glass and ebonite rods respectively. The negative charge is called **electron**, taken from the Greek word, "**electron**" for amber.

Generally if two positively or negatively charged materials are brought close to each other, they repel, while if a positively charged material is brought near a negatively charged one, they attract. This is called the

LAW OF CHARGES and states that "**like charges repel and unlike charges attract**".

It is important also to note that charge can neither be created nor destroyed by any physical or chemical process. Thus the total charge of any matter is conserved.

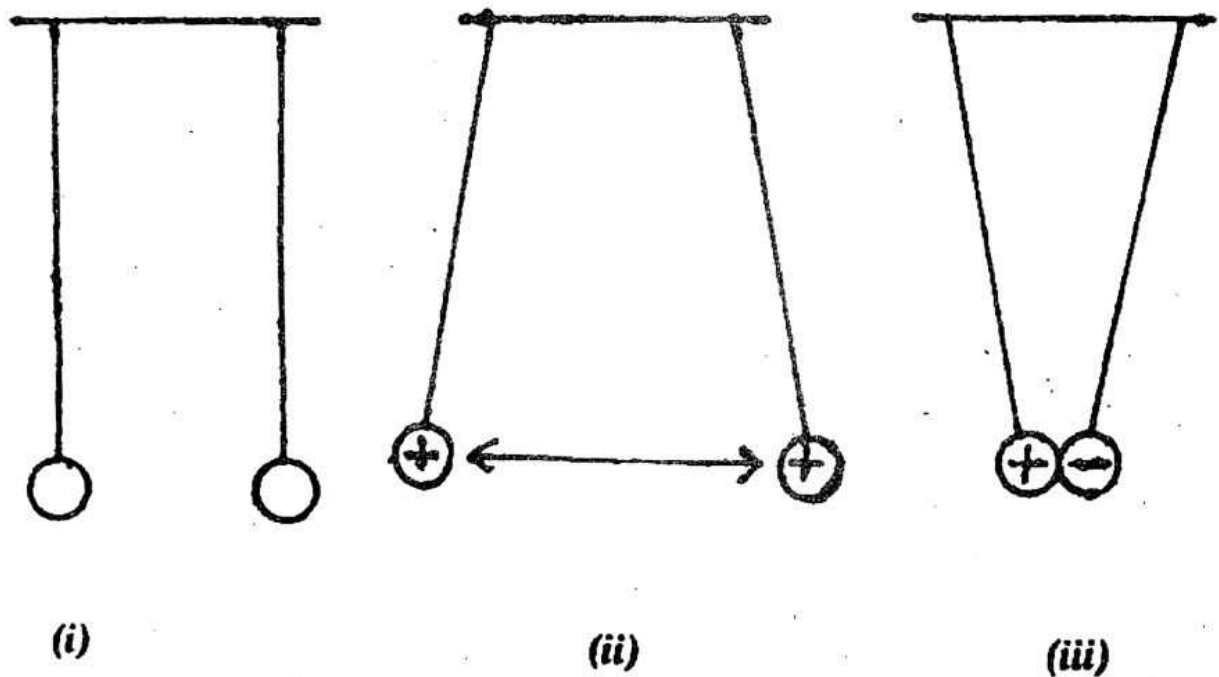


Figure 1

- (i) Uncharged balls suspended with silk.
- (ii) Like charges repel. The charges can be both positive or negative.
- (iii) Unlike charges attract.

Charges can be stationary or in motion. When they are stationary, they are called **static charges or static electricity**. The study of static electricity is called **electrostatics**. When charges on the other hand are in motion, they form electric current.

The smallest quantities of positive and negative charged are found in an atom. The smallest positive charge is called a **proton** and is located at the centre of the atom called the **nucleus**. The smallest negative charge on the other hand is called an **electron** and orbits the nucleus of the atom.

Hence charges fundamentally are properties of the electron and the proton. Specifically a charge can be described as that property of an electron (or proton) which causes it to experience a repulsive or attractive force when brought near another electron (or proton).

The charge on the electron is equal to that on the proton in magnitude, but opposite in effect.

When an atom contains equal number of electrons and protons, it becomes neutral in charge. The atom becomes charged when one charge is in excess of the other.

Small particles and bulk matter are made of atoms and therefore contain charges. If such bulk matter contains equal amount of positive and negative charges, it becomes neutral in state. It is said to be charged if it contains one charge in excess of the other. If it has excess positive charge, it is said to be positively charged, while if it contains excess negative charge, it is said to be negatively charged.

The proton is usually firmly bound to the nucleus of the atom while the electrons move in orbits round the nucleus as stated above. A normal hydrogen atom for example contains only one

proton located at its nucleus, and one orbiting electron. Such atoms with few electrons and protons like helium, lithium etc are said to be light. In heavy atoms like uranium, we have 92 protons and 92 electrons in its neutral state. The protons are all concentrated at the nucleus with 142 neutrons. The neutrons carry no charge and are said therefore to be neutral in charge. A neutron has approximately equal mass and size with a proton. The arrangement of the electrons in such atom is such that while some are in the inner orbits of the atom and are more strongly attracted to the nucleus, others are in outer orbits and are more loosely bound to the atom. The outermost electrons of such atoms are very loosely bound and can easily be robbed off. When amber (or glass) therefore is rubbed with silk, electrons are rubbed off from the amber and get attached to the silk so that the amber becomes positively charged while the silk becomes negatively charged. In the case of rubbing ebonite (or hard rubber) with fur, electrons are rubbed off the fur, and get transferred to the ebonite so that it becomes negatively charged while the fur becomes positively charged. This is called charging by friction.

1.2. Conductors, Insulators And Semiconductors.

In some materials like copper, silver, gold and metals in general, the human body, the earth etc, surface electrons of the atoms of such materials are loosely bound and free to move from one atom to the other, especially under certain potential conditions. Such materials are called **conductors**. In conductors, electrons flow (ie electric current) from one point to another if there is potential difference between the two points. This electronic motion is from the point of lower potential to that of higher potential. Usually this electrical conduction decreases as the temperature of the conductor increases.

In another class of materials like glass, ebonite, etc, the electrons associated with each atom are bound to the atom. Even though their surface electrons may be rubbed out of their atoms, they are strongly fixed to their individual atoms and cannot move from atom to atom.

Such materials are called insulators. Examples of insulators are *glass, amber, ebonite, silk, fur, wool, polytene, cellulose acetate, perspex, etc.*

In between conductors and insulators, we have semiconductors like *silicon, germanium, boron, arsenic, antimony* etc. In these materials electrical conduction increases as temperature increases.

1.3 Coulomb's Law.

In 1785, Charles Augustine de Coulomb (1736 - 1805) studied the electric forces between point charges responsible for their attraction or repulsion, and came out with Coulomb's law which states that whenever any two isolated point charges q_1 and q_2 are placed a small distance r apart, there exists always an electrostatic force between them which acts along the axis joining the two charges, and that this force is directly proportional to the product of the two charges, and inversely proportional to the square of their distance apart.

$$\text{Hence } F_e \propto \frac{q_1 q_2}{r^2}$$

where F_e is the electrostatic force, q_1 and q_2 are the magnitudes of the charges and r is the distance between the point charges.

If the two charges are unlike, the force between them is attractive, while if they are like, the force is repulsive. If q_1 and q_2 are equal in magnitude, the force on q_1 due to q_2 is equal to that on q_2 due to q_1 , but they point in opposite directions as shown in fig. 2.

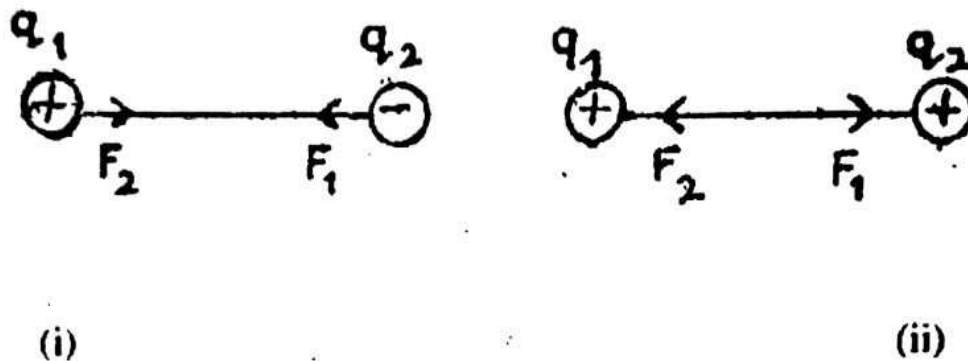


Fig. 2 (i): Attractive force between two unlike charges.

F_1 is due to q_1 and F_2 is due to q_2 .

(ii): Repulsive force between two like charges.

From the proportionality relation of Coulomb's law, we obtain,

$$F_e = \frac{k q_1 q_2}{r^2} \quad (1.1)$$

where K is a proportionality constant found to be equal to

$\frac{1}{4\pi\epsilon_0}$, where ϵ_0 is called the permittivity of free

space or vacuum, and has magnitude $8.85418 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

$$\text{Thus } F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad (1.2)$$

where $\frac{1}{4\pi\epsilon_0}$ has magnitude $8.99 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$.

Vectorially we write

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^3} \vec{r} \quad (1.3)$$

ϵ_0 can also be measured in farad per metre (Fm^{-1}), and its value is true for vacuum only. For any other denser medium, the permittivity increases as medium density increases, and is now generally represented by ϵ . Such mediums possess a property called dielectric constant K_r where

$$K_r = \frac{\epsilon}{\epsilon_0} \quad (1.4)$$

$$\text{Hence } \epsilon = K_r \epsilon_0. \quad (1.5)$$

For vacuum, $K_r = 1$ so that $\epsilon = \epsilon_0$.

For air medium at s.t.p., $K_r = 1.00054$.

Hence for air, $\epsilon = 1.00054 \epsilon_0$.

ϵ for air can therefore be approximated to ϵ_0 (for vacuum). For mica medium,

$K_r = 7$ while for pure water, $K_r = 80$.

Thus if two charges are kept the same distance apart in air and in water, the electrostatic force between them in water shall be reduced 80 times the magnitude of the force in air (or vacuum).

Generally we express the force in the form

$$F_e = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} \quad (1.6)$$

For air $\epsilon = \epsilon_0$, and for water $\epsilon = 80\epsilon_0$.

$$\text{Hence for air, } F_{ea} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad (1.7)$$

$$\text{while for water, } F_{ew} = \frac{1}{4\pi(80\epsilon_0)} \cdot \frac{q_1 q_2}{r^2} \quad (1.8)$$

$$= \frac{1}{80} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$= 1/80 F_{ea}$$

We note that the unit of charge is Coulomb (C) which is defined as the amount of charge that flows through a given cross-section of a conducting wire in one second due to a steady current of one ampere flowing in the wire.

$$\text{Thus } q = it \text{ (C)} \quad (1.9)$$

where i is the quantity of current flowing through the cross-section of the wire for t seconds.

1.4 Solved Examples:

(For all questions, $\frac{1}{4\pi\epsilon_0} = (9.0 \times 10^9 \text{ Nm}^2 \text{ C}^{-1})$)

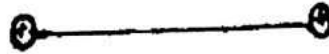
Example 1.1: Two charges of magnitude $+ 3.2 \times 10^{-19} \text{C}$ and $+ 4.8 \times 10^{-19} \text{C}$ are placed 10mm apart. Calculate the force between them when

they are placed in (i) air (ii) a liquid of dielectric constant, 20. Is the force attractive or repulsive?

Solution:

(i) For air medium

$$F_{\text{air}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad (1.10)$$



where $q_1 = +3.2 \times 10^{-19} \text{ C}$.

$q_2 = +4.8 \times 10^{-19} \text{ C}$

$r = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$.

$$\begin{aligned} \text{Hence } F_{\text{air}} &= \frac{9 \times 10^9 \times 3.2 \times 10^{-19} \times 4.8 \times 10^{-19}}{(10 \times 10^{-3})^2} \\ &= 9 \times 3.2 \times 4.8 \times 10^{-25} \\ &= 13.24 \times 10^{-25} \\ &= \underline{\underline{1.38 \times 10^{-23} \text{ N}}} \end{aligned}$$

(ii) For the given liquid medium.

$$\epsilon = K_r \epsilon_0 = 20\epsilon_0$$

$$\begin{aligned} \therefore F_{\text{d}} &= \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} \quad (1.11) \\ &= \frac{1}{4\pi(20\epsilon_0)} \cdot \frac{q_1 q_2}{r^2} \\ &= \frac{1}{20} \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \\ &= \frac{1}{20} F_{\text{air}} \end{aligned}$$

$$= \frac{1}{20} \times 1.38 \times 10^{-25}$$

$$= \underline{\underline{0.069 \times 10^{-23} \text{N}}}$$

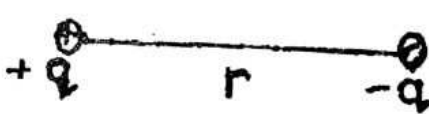
Note that the signs of the charges are not involved in the calculation. The signs determine whether the two charges are like or unlike, and therefore the force between them is repulsive or attractive respectively. If the signs of the charges are employed in the calculation, then a final positive force will imply repulsion, while a final negative force will imply an attraction.

In the case above, the two charges are like and therefore the force is repulsive.

Example 2: Two equal and opposite charges are placed 40mm apart in air. If the force between them is found to be 0.5N, calculate the magnitude of each charge.

Solution:

$$F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q \cdot q}{r^2} \quad (1.12)$$

$$= \frac{1}{4\pi \epsilon_0} \cdot \frac{q^2}{r^2}$$


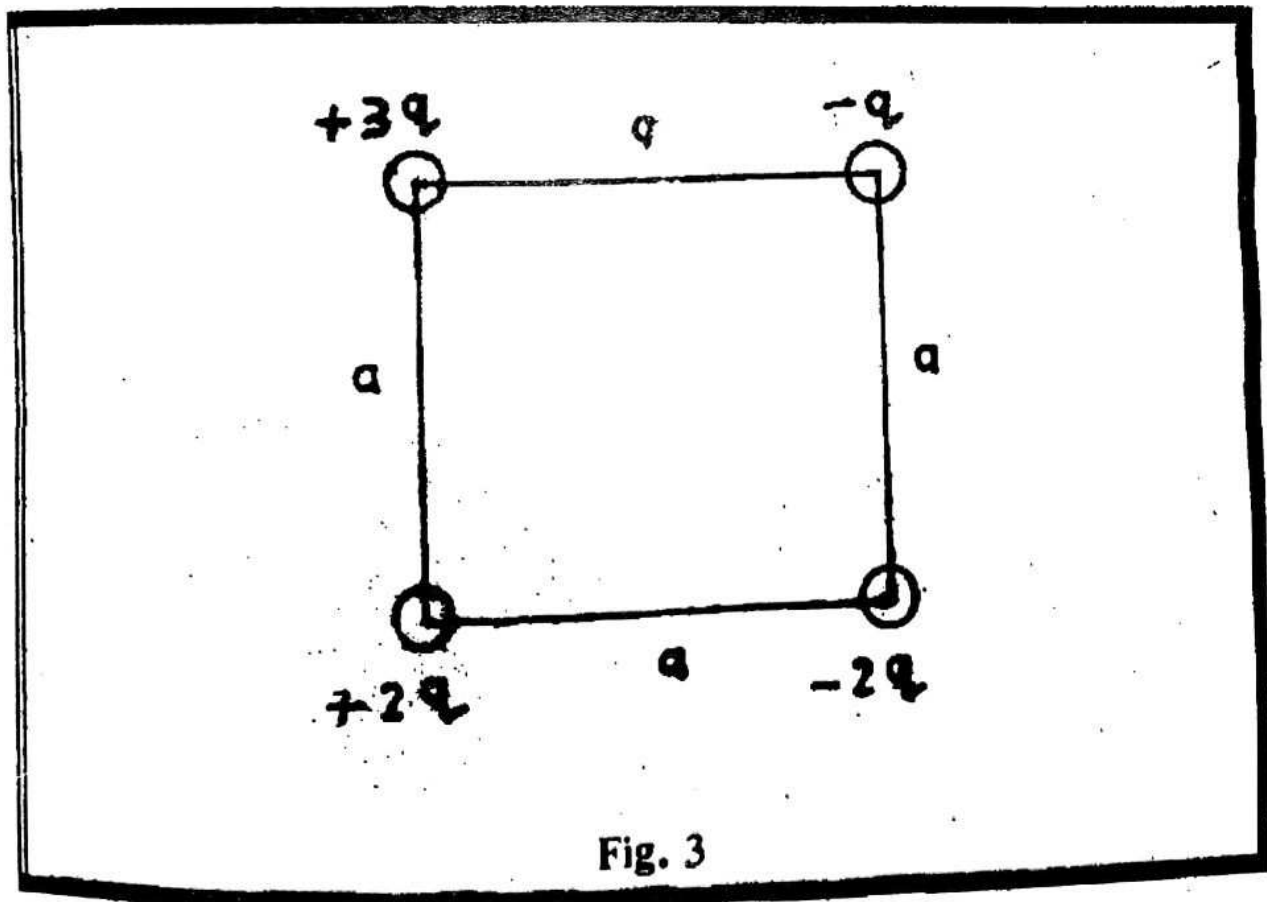
where $r = 40\text{mm} = 40 \times 10^{-3} \text{ m}$.

$$\therefore 0.5 = \frac{9 \times 10^9 \times q^2}{(40 \times 10^{-3})^2}$$

$$\therefore q^2 = \frac{0.5 \times (4 \times 10^{-2})^2}{9 \times 10^9}$$

$$\begin{aligned}
 &= \frac{0.5 \times 4 \times 4 \times 10^{-4}}{9 \times 10^9} \\
 &= \frac{8 \times 10^{-13}}{9} \\
 &= 0.89 \times 10^{-13} \\
 &= 8.9 \times 10^{-14} \\
 \therefore q &= \sqrt{8.9} \times 10^{-7} \\
 &= \underline{\underline{2.98 \times 10^{-7} \text{C}}}
 \end{aligned}$$

Examples 1.3: Charges are placed at the corners of a square of side a as shown in fig. 3. If $q = 1.6 \times 10^{-19} \text{C}$ and $a = 10 \text{mm}$, calculate the resultant force at the lower left corner of the square, and its direction.



Solution: This problem should be solved in steps.

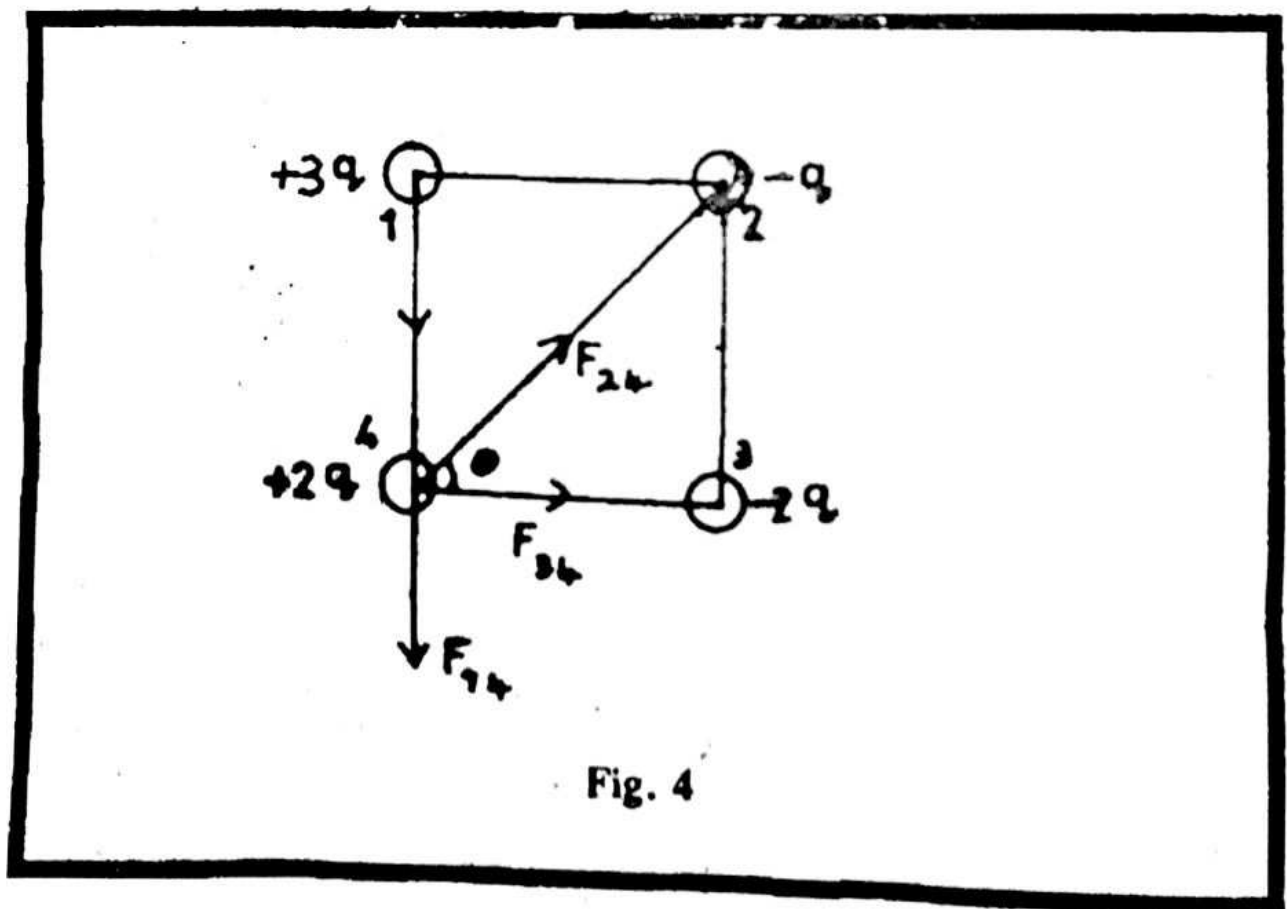
Step 1: Draw the system of charges and label the charges conveniently. Then draw the forces that act on the lower half corner with their directions indicated.

Step 2: Calculate the magnitude of these forces

Step 3: To find the resultant force at the lower left corner, resolve the forces into two convenient perpendicular directions (say F_x and F_y in x and y axes respectively).

Step 4: Draw F_x and F_y and hence calculate the magnitude and direction of F_R (the resultant force).

Step 1: Draw force diagram as shown in fig. 4.



From the force diagram, there are three forces acting on 4. These include F_{14} = a repulsion

F_{24} = an attraction

F_{34} = an attraction.

Step 2: - Calculate F_{14} , F_{24} and F_{34}

$$F_{14} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(3q)(2q)}{a^2} \quad (1.13)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{6q^2}{a^2}$$

$$= \frac{9 \times 10^9 \times 6 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(10 \times 10^{-3})^2}$$

$$= 9 \times 6 \times 1.6 \times 1.6 \times 10^{-25}$$

$$= 138.24 \times 10^{-25} \text{N}$$

$$F_{24} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(q)(2q)}{r^2} \quad (1.14)$$

where r is the diagonal of the square. Since side is given as a , then

$$r^2 = a^2 + a^2 = 2a^2.$$

$$\therefore r = a\sqrt{2}.$$

$$\text{Hence } F_{24} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(q)(2q)}{(2a^2)} \quad (1.15)$$

$$= \frac{9 \times 10^9 \times 2 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{2 \times (10 \times 10^{-3})^2}$$

$$= 9 \times 1.6 \times 1.6 \times 10^{-25}$$

$$= 23.04 \times 10^{-25} \text{ N.}$$

$$F_{34} = \frac{1}{4\pi \epsilon_0} \frac{(2q)(2q)}{a^2} \quad (1.16)$$

$$= \frac{9 \times 10^9 \times 4 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(10 \times 10^{-3})^2}$$

$$= 9 \times 4 \times 1.6 \times 1.6 \times 10^{-25}$$

$$= 92.16 \times 10^{-25} \text{ N}$$

Step 3: Suppose F_{24} makes an angle θ with x-axis as shown. Thus since the figure is a square, $\theta = 45^\circ$.

$$F_x = F_{34} + F_{24} \cos \theta \quad (1.17)$$

$$= 92.16 \times 10^{-25} + 23.04 \times 10^{-25} \cos 45.$$

$$= 92.16 \times 10^{-25} + 16.29 \times 10^{-25}$$

$$= (92.16 + 16.29) \times 10^{-25}$$

$$= 108.45 \times 10^{-25} \text{ N.}$$

$$\begin{aligned}
 F_y &= -F_{14} + F_{24} \sin 45 & (1.18) \\
 &= -138.24 \times 10^{-25} + 16.29 \times 10^{-25} \\
 &= -121.95 \times 10^{-25} \text{ N.}
 \end{aligned}$$

Step 4: F_x , F_y and F_R are shown in fig. 5

$$\text{Hence } F_R^2 = F_x^2 + F_y^2 \quad (1.19)$$

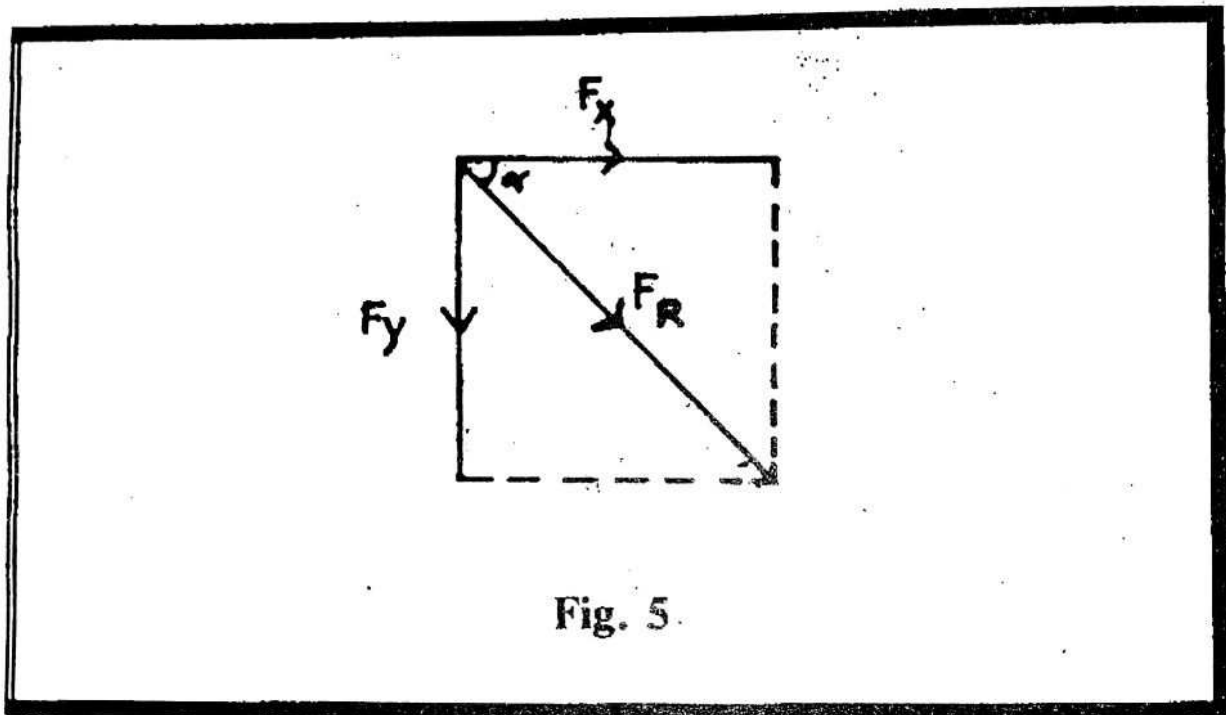


Fig. 5

$$\begin{aligned}
 &= (108.45 \times 10^{-25})^2 + (-121.95 \times 10^{-25})^2 \\
 &= 11761.40 \times 10^{-50} + 14871.80 \times 10^{-50} \\
 &= (11761.40 + 14871.80) \times 10^{-50} \\
 &= 26633.2 \times 10^{-50}
 \end{aligned}$$

$$\therefore F_R = \underline{\underline{-163.20 \times 10^{-25} \text{ N.}}}$$

If F_R makes an angle α with x-axis, then $\tan \alpha = F_y/F_x$

$$= \frac{121.95 \times 10^{-25}}{108.45 \times 10^{-25}}$$

$$= 1.1245.$$

$$\therefore \alpha = \tan^{-1} (1.1245)$$

$$\text{Example 1.4:} = \underline{48.35^\circ}$$

- (i) A neutral copper coin has mass of 20gm. If copper atom has positive nuclear charge of $46 \times 10^{-19}\text{C}$, calculate the total electronic charge in the coin. (The atomic weight of copper is 64gm per mole while Avogadro number is 6.0×10^{23} atoms per mole).
- (ii) Suppose a current of 0.9A flows through an electric bulb, for how long shall all the electrons in this coin supply current to the bulb?

Solution:

First of all, we need to find the total number of copper atoms in the coin. To do this, we employ the relation

$$\frac{N}{N_a} = \frac{m}{M} \quad (1.20)$$

where N = number of atoms in the coin
 N_a = Avo. No
 m = mass of the coin
 M = Atomic weight of copper

$$\therefore \frac{N}{6 \times 10^{23}} = \frac{20 \times 10^{-3}}{64 \times 10^{-3}}$$

$$N = 20/64 \times 6 \times 10^{23}$$

$$= 1.875 \times 10^{23} \text{ atoms.}$$

Since the coin is neutral, then total quantity of positive charge = total quantity of electronic charge.

Hence electronic charge per atom

$$= 46 \times 10^{-19} \text{C.}$$

Therefore total quantity of electronic charge in the coin, Q, is given by

$$Q = 1.875 \times 10^{23} \times 46 \times 10^{-19}$$

$$= \underline{86.25 \times 10^4 \text{C.}}$$

(ii) Since $q = it$,

$$\text{Then } 86.25 \times 10^4 = 0.9 \times t.$$

$$\therefore t = \frac{86.25 \times 10^4}{0.9} \text{ sec.}$$

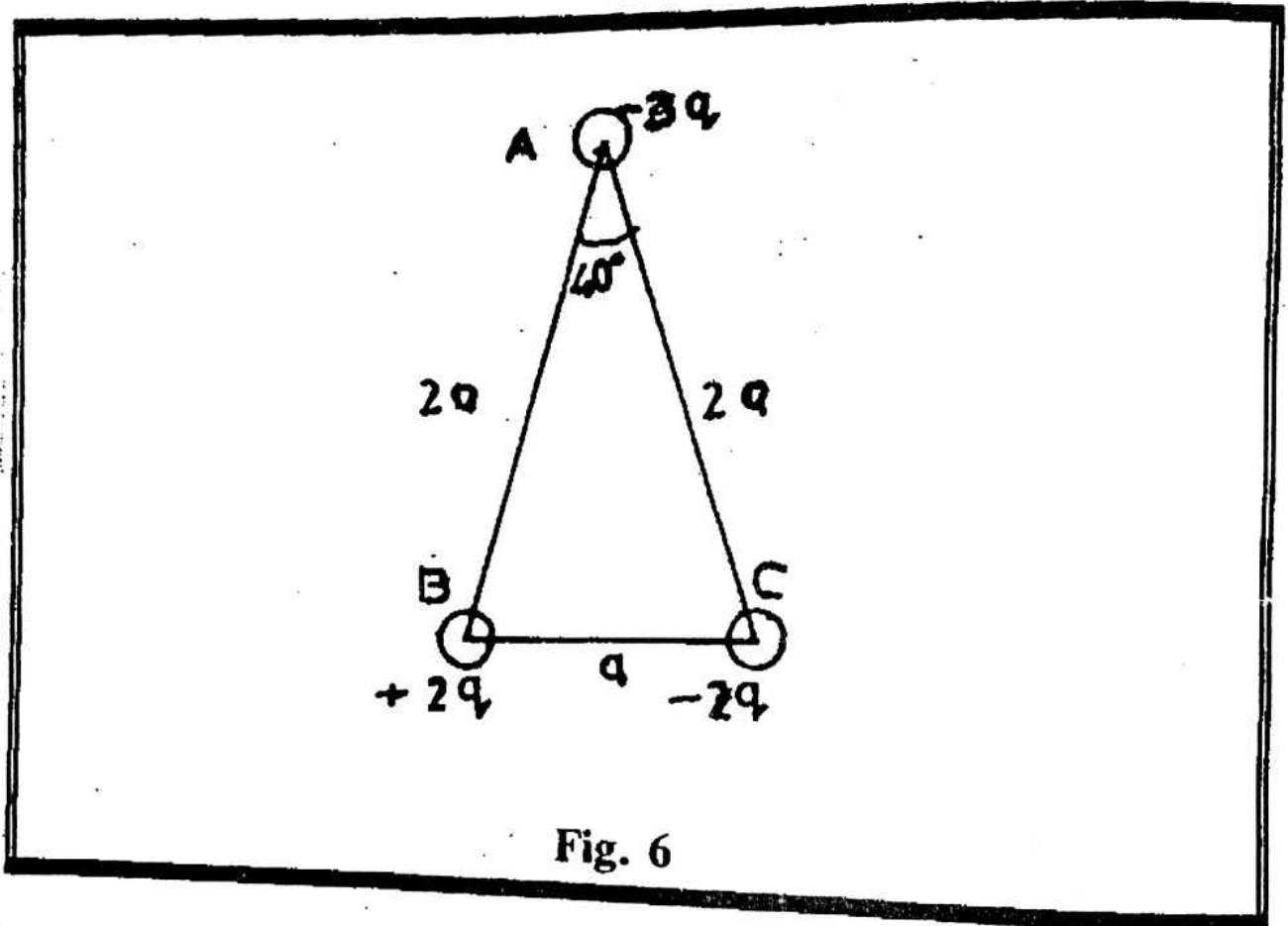
$$= \frac{86.25 \times 10^4}{0.9 \times 60 \times 60} \text{ Hrs}$$

$$= 0.0266 \times 10^4 \text{ Hrs.}$$

$$= 266 \text{ Hrs.}$$

$$= \underline{11.08 \text{ days.}}$$

Example 1.5: Calculate the magnitude and direction of the resultant force at the lower right corner of the given system of triangular charges shown in fig. 6, where $q = 1.6 \times 10^{-6} \text{C}$, a is 4cm and angle A is 40° .



Solution:

Step I: Force diagram (shown in fig. 7).

The force diagram is a little complicated involving the location of the charges, the indication of the forces and the inclusion of the angles necessary for the calculation. The two forces acting at the lower right corner include F_{12} = a repulsion, and

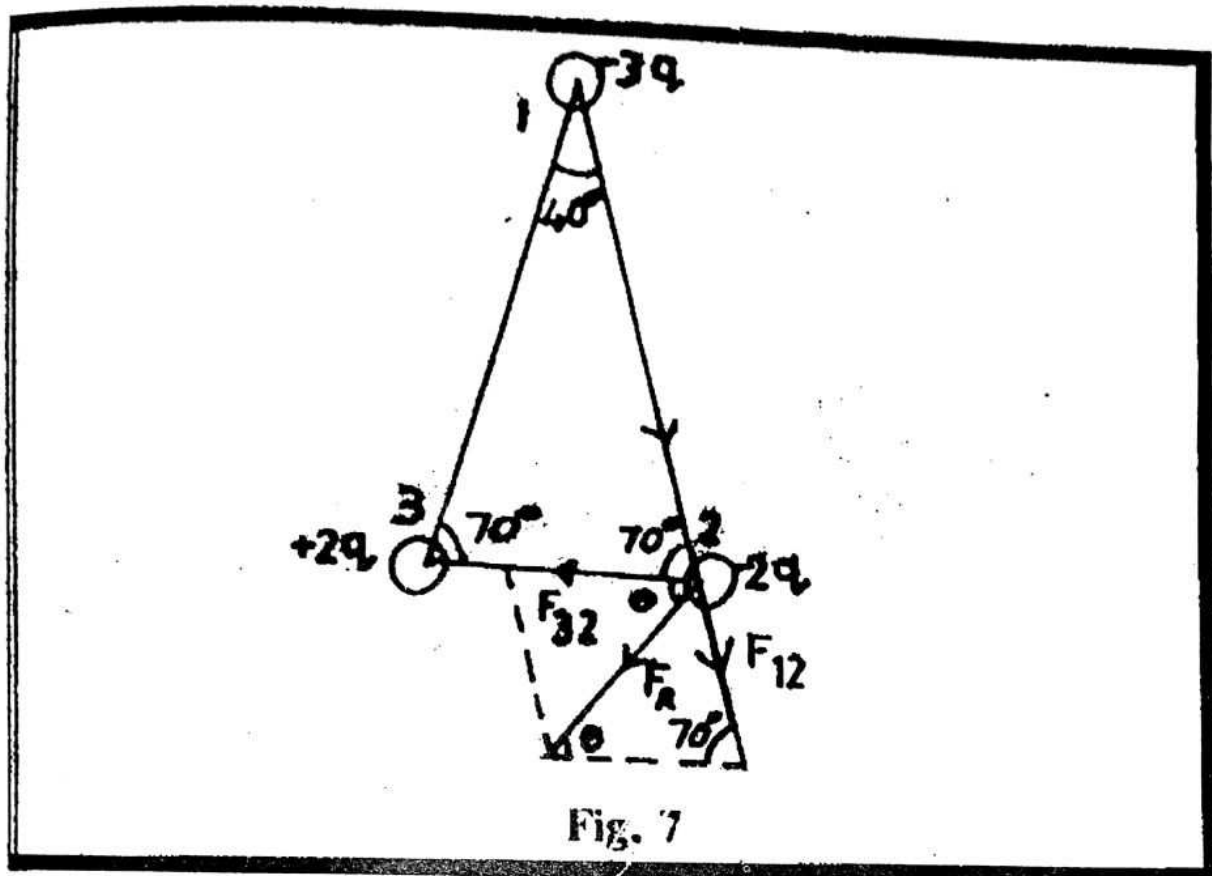


Fig. 7

F_{32} = an attraction.

Step 2: Calculation of F_{12} and F_{32} .

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{(3q)(2q)}{(2a)^2} \quad (1.21)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{6q^2}{4a^2}$$

$$= \frac{9 \times 10^9 \times 6 \times 1.6 \times 10^{-6} \times 1.6 \times 10^{-6}}{4 \times (4 \times 10^{-2})^2}$$

$$= \frac{9 \times 6 \times 1.6 \times 1.6 \times 10}{64}$$

$$= 21.6\text{N.}$$

$$F_{12} = \frac{1}{4\pi \epsilon_0} \cdot \frac{(2q)(2q)}{a^2} \quad (1.22)$$

$$= \frac{1}{4\pi \epsilon_0} \cdot \frac{4q^2}{a^2}$$

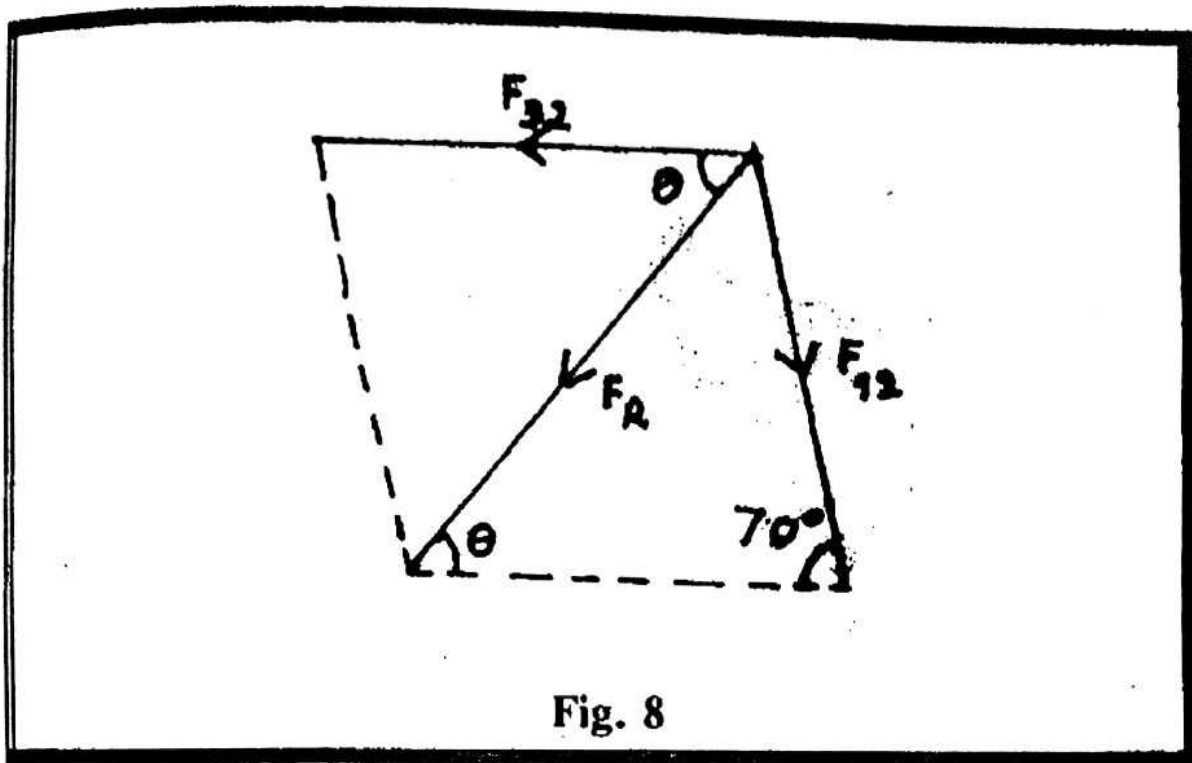
$$= \frac{9 \times 10^9 \times 4 \times 1.6 \times 10^{-6} \times 1.6 \times 10^{-6}}{(4 \times 10^{-2})^2}$$

$$= \frac{9 \times 1.6 \times 1.6 \times 10}{4}$$

$$= 57.6\text{N}$$

Step 3. Find F_R .

To find F_R , we show F_{12} and F_{34} in a parallelogram of forces, with F_R as a resultant diagonal as shown in fig. 8. We then apply cosine rule to determine F_R .



Hence from the diagram,

$$F_R^2 = F_{12}^2 + F_{32}^2 - 2 F_{12} \cdot F_{32} \cos 70.$$

$$\begin{aligned} &= 21.6^2 + 57.6^2 - 2 \times 21.6 \times 57.6 \times 0.3420 \\ &= 466.46 + 3317.76 - 851.01 \\ &= 2933.31 \end{aligned}$$

$$\therefore F_R = \sqrt{2933.31}$$

$$= \underline{\underline{54.16\text{N}}}$$

Step 4: Finding the direction of F_R . To determine the direction of F_R , we find θ from sine rule.

$$\frac{\sin \theta}{F_{12}} = \frac{\sin 70}{F_R} \quad (1.23)$$

Hence $\sin \theta = \frac{F_{12} \sin 70^\circ}{F_R}$ Download more from Learnclax.com

$$= \frac{21.6 \times 0.9397}{54.16}$$

$$= 0.3748$$

Therefore, $\theta = \sin^{-1}(0.3748)$

$$= \underline{\underline{22^\circ}}$$

1.5 PROBLEMS.

(For all questions, $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ Nm}^2\text{C}^{-2}$)

1.5.1 Two point charges of magnitude $+1.6 \times 10^{-6}\text{C}$ and $-6.4 \times 10^{-6}\text{C}$ are separated in air by a distance of 4cm. calculate the magnitude of the force between them.

1.5.2. Two metal balls of mass m each are suspended from the same point with silk threads of the same length, l . If the two balls are now given equal like charges such that the charge on each ball has magnitude $+q$, and the two balls settle at a separation x after repulsion, show that

$$X = \left(\frac{q^2 l}{2\pi \epsilon_0 mg} \right)^{1/3}$$

(Assume small angle deflection θ of each ball from the centre such that $\tan \theta = \sin \theta$)

- 1.5.3. Each of two spheres are charged negatively, the combined charge being $-3.2 \times 10^6 \text{ C}$. If each sphere is repelled from the other by a force of 0.45 N when they are 20 cm apart, calculate the charge on each sphere.
- 1.5.4. A square of side a has two like charges each of magnitude $+Q$ on opposite sides of the square, and two unlike charges of magnitude $+q$ and $-q$ on the other opposite corners. Determine expression for the resultant force on Q .
- 1.5.5. Three point charges of magnitudes $+48 \times 10^9 \text{ C}$, $-3.2 \times 10^9 \text{ C}$ and $-1.6 \times 10^9 \text{ C}$ are placed in order on the three corners of an equilateral triangle of side 2 cm . Calculate the magnitude and direction of the resultant force on the smallest charge.
- 1.5.6. Two charged water molecules carry charges $+6.4 \times 10^{19} \text{ C}$ and $+1.6 \times 10^{18} \text{ C}$ respectively. Calculate the electric force between them if they are 5 mm apart.

$$(\epsilon_w = 80 \epsilon_o).$$

2

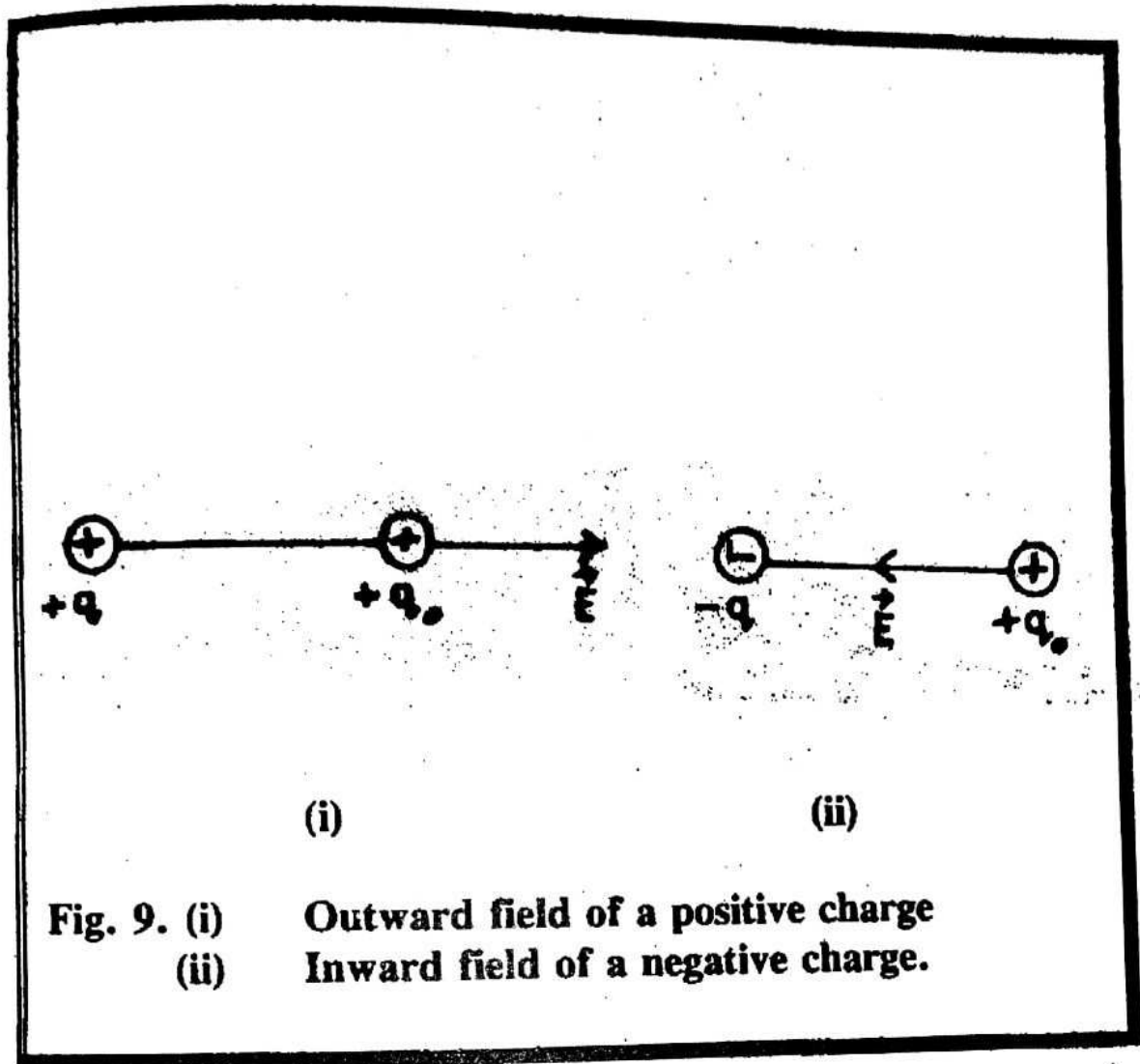
ELECTRIC FIELD INTENSITY, $\vec{E}$

2.1 Definition

Electric field is defined as *the region within which electric force is experienced*. It is usually created by the presence of electric charge. The magnitude of the electric field intensity (or simply electric field) E , at a point, is defined as the electric force per unit charge at that point. Thus if the magnitude of the charge at the point is q_0 , then

$$E = \frac{F_e}{q_0} \quad (2.1)$$

Hence the unit of E is Newton per Coulomb (NC^{-1}). E is a vector quantity, and its direction at any point is determined by placing a positive test charge there. The direction of motion of the positive test charge is the same as the direction of the field at that point. Thus if a field is due to a positive charge, then when a positive test charge is placed at any point near it, it will be repelled away. Thus the direction of the field due to a positive charge is outward, away from the charge. On the other hand, for the case of the field due to a negative charge, a positive test charge placed near it will be attracted towards the negative charge. Thus the direction of the field due to a negative charge will be inward, towards the charge.



2.2. Electric Field Due To A Point Charge.

Suppose we want to determine the electric field at a point, distance r from a charge $+q$. Then if we place a positive test charge $+q_0$ at that point, the force between the two charges is given by

$$F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{r^2} \quad (2.2)$$

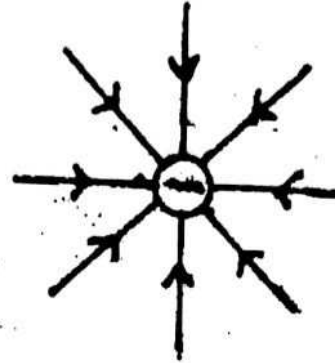
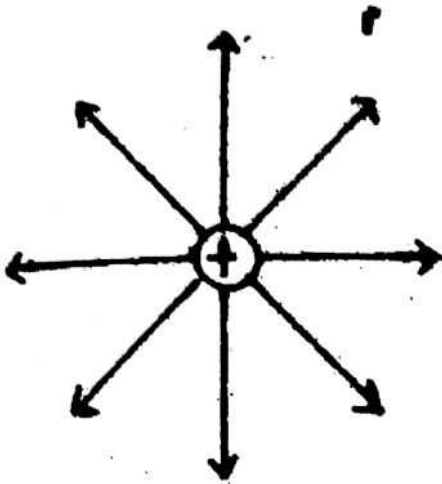
The magnitude of the electric field at that point is therefore given by

$$\begin{aligned}
 E &= \frac{F_e}{q_0} \\
 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q q_0}{r^2} \cdot \frac{1}{q_0} \\
 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad (2.3)
 \end{aligned}$$

2.3. Electric Lines Of Force

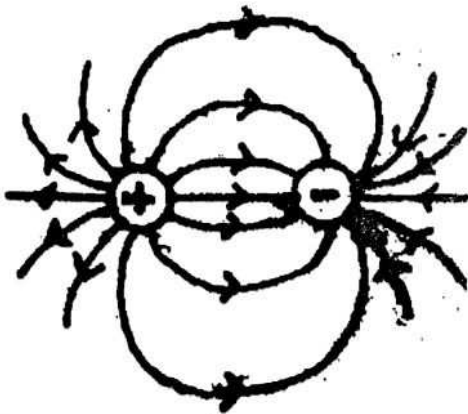
An electric field is usually mapped out with electric lines of force. An electric line of force is the line or curve which a positive test charge placed in an electric field will trace out if allowed to move. The tangent to a line of force at any point defines the direction of the electric force on the positive test charge at that point, and hence the direction of the electric field there.

The direction of a line of force is usually indicated with an arrow. All the lines of force in a field make up the electric flux pattern for the field. The lines in such case are called flux lines. For a positive charge, the flux lines diverge outward from the charge, while for a negative charge, they converge on the charge. The electric flux pattern for various charge configurations are shown in fig 10.

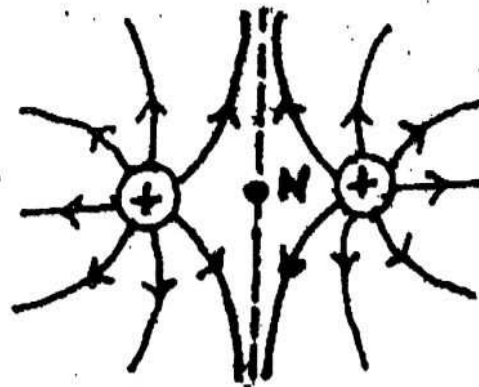
**Fig. 10**

(i) *Isolated +ve charge.*
Flux is symmetrically outward (diverging from the charge).

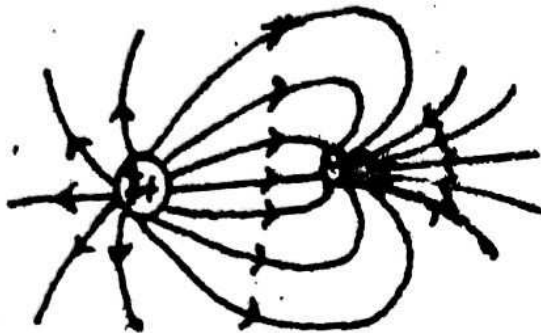
(ii) *Isolated -ve charge.*
Flux is symmetrically inward (converging on the charge).



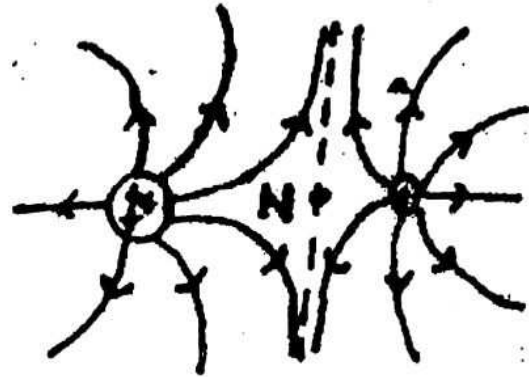
(iii) *Two unlike charges of equal magnitudes.* Flux flows from +ve to -ve charge symmetrically.



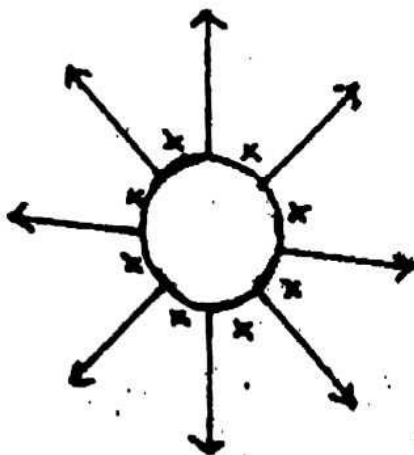
(iv) *Two like charges (+ve) of equal magnitudes.* Flux shows repulsion of lines of force with neutral point at the middle. (For two -ve charges, flux has the same pattern but direction of lines of force are reversed).



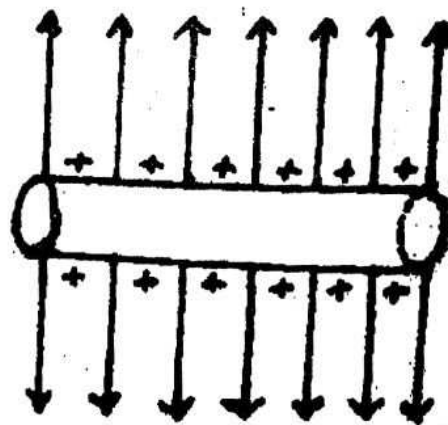
(v) Two unlike charges of unequal magnitudes. Flux pattern is skewed towards charge of smaller magnitude.



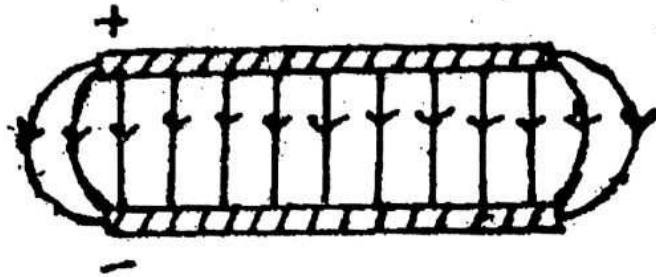
(vi) Two like charges of unequal magnitudes. Flux is also skewed with the neutral point nearer the smaller charge.



(vii) A uniformly +vely charged hollow sphere. Flux is uniformly outward as though diverging from the centre of the sphere. No flux appears within the sphere.



(viii) A uniformly +vely charged hollow cylinder. Flux is symmetrically outward. No flux appears within the cylinder.



*(ix) Two parallel plates carrying opposite charges.
Flux is uniform and parallel in between the
plates but show distortion at the ends.*

2.4. Properties Of Electric Flux

There are a number of properties associated with electric flux and these include;

- (i) *They do not intersect at points in space where there are no charges.*
- (ii) *A flux line begins at a positive charge and ends at a negative charge or at infinity.*
- (iii) *The number of lines of force (electric flux) N per unit area, crossing a surface area placed at right angles to the direction of a uniform electric field in which the surface is immersed, is at every point numerically equal to the magnitude of the electric field intensity at that point.*

$$\text{Thus } E_N = N/A$$

(2.4)

Hence the total electric flux crossing a surface is given by

$$\phi_E = E_N A \quad (2.5)$$

The unit of electric flux ϕ_E is $\text{N m}^2 \text{C}^{-1}$.

- (iv) *The greater the density of flux lines in a region, the larger the magnitude of the electric field in that region. Thus the field is larger where the flux are more closely packed than where they are spaced out. In fig.11, for example, E_B is greater than E_A .*

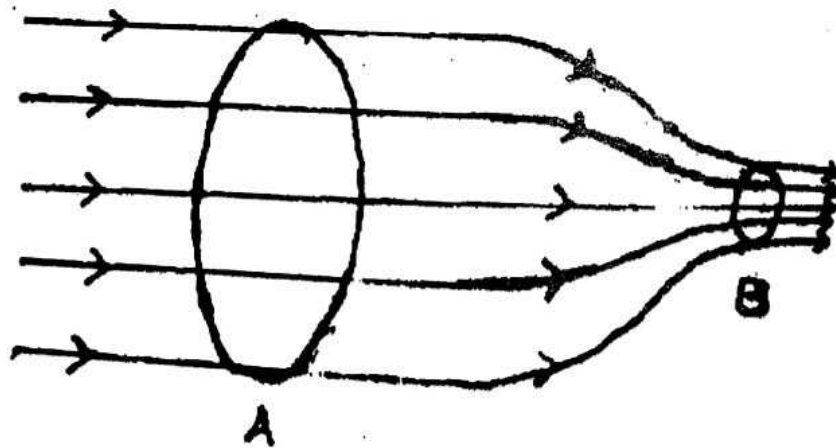


Fig. 11. Flux pattern narrow at B. $E_B > E_A$

2.5 Calculation Of Flux Density, ϕ_E

Consider a uniform field E flowing across a sheet of surface area A as shown in fig. 12.

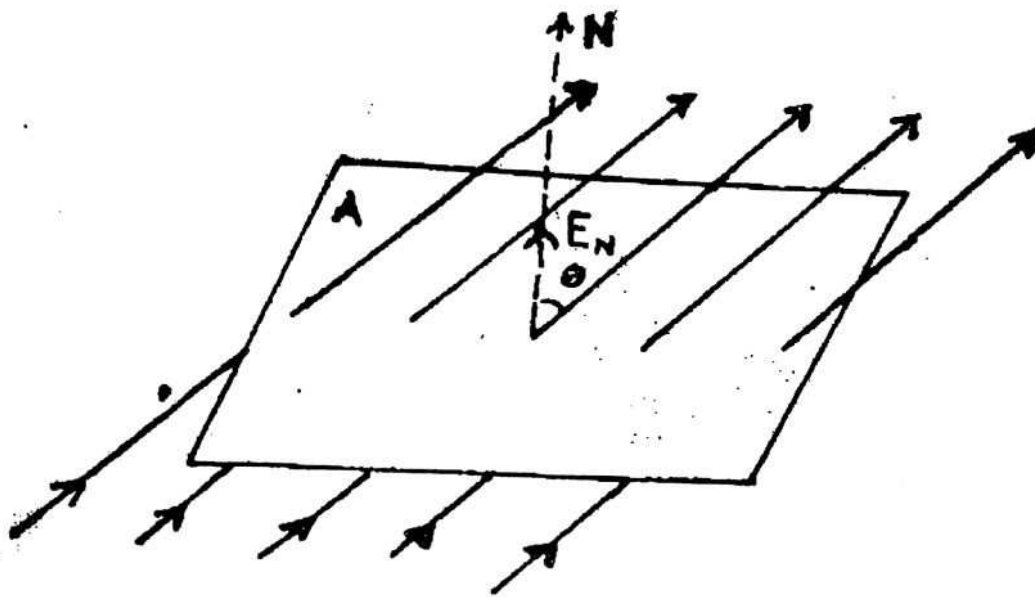


Fig. 12. A uniform electric field E crossing a sheet of surface area A .

Suppose N is an outward normal drawn perpendicular to the surface and makes an angle θ with the direction of the electric field. Then the component of E normal to the surface is given by

$$E_N = E \cos \theta \quad (2.6)$$

Thus the total flux flowing through the sheet is given by

$$\phi_E = E_N A = EA \cos \theta. \quad (2.7)$$

Thus when $\theta = 0$, ϕ_E is +ve and equal to EA .

$\theta < 90^\circ$, ϕ_E is +ve

$\theta = 90^\circ$, $\phi_E = 0$ and lying on the surface

$\theta > 90^\circ$, ϕ is -ve

We note that when $\theta < 90^\circ$, $\vec{E}$ is outward,

when $\theta > 90^\circ$, $\vec{E}$ is inward.

If we consider a hollow sphere immersed in an electric field as shown in fig. 13, at S_1 where the flux enters the sphere, the outward normal is in opposite direction to the field so that $\theta = 180^\circ$. Hence the flux ϕ_{s1} is negative and inward. On the other hand, at S_2 , the outward normal and the field are in the same direction so that $\theta = 0$.

Hence ϕ_{s2} is positive and outward.

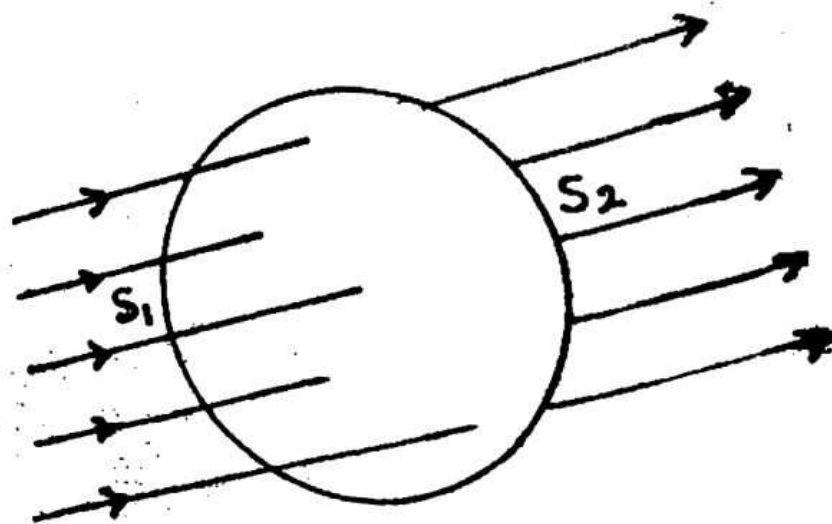


Fig. 13 (i): Uniform Electric Field flowing across a hollow sphere immersed in it.

If a surface is closed, and the total incoming flux equals the total outgoing flux, then the total flux crossing the surface is zero. For example if we consider a flux line that enters a hollow sphere normally and leaves also normally, at point of entry, $\theta = 180^\circ$.

Therefore, $\phi_1 = EA \cos 180 = -EA$

At point of leaving the sphere, $\theta = 0$.

$$\therefore \phi_2 = EA \cos 0 = EA.$$

$$\therefore \text{Total flux } \phi = \phi_1 + \phi_2 = -EA + EA = 0.$$

To generalize, let us consider a field which varies from point to point across a surface which is not everywhere at right angles to the field, as shown in fig. 13.

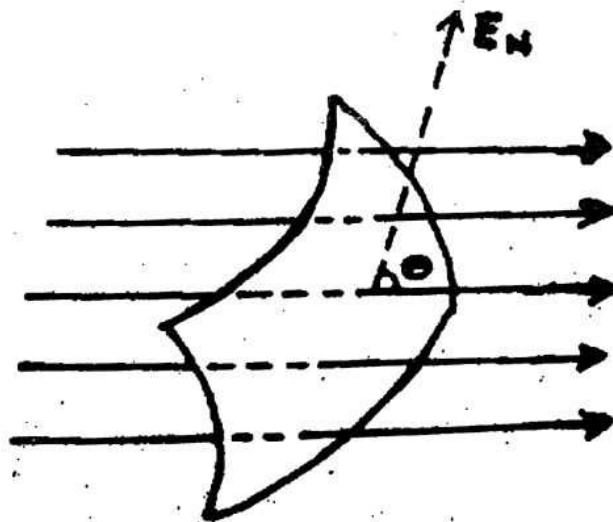


Fig. 13 (ii): A surface element, dA , immersed in a uniform field E .

Consider a surface A with surface element dA . Let a uniform field E , which makes an angle θ with the outward drawn normal, cross this surface element.

Thus the total flux crossing dA is given by,

$$\begin{aligned} d\phi_E &= E_N dA = E dA \cos\theta \\ \phi_E &= \int E dA \cos\theta \\ &= \int \vec{E} \cdot \vec{dA} \quad (2.8) \end{aligned}$$

Generally, for a closed surface, we have

$$\begin{aligned} \phi_E &= \oint E ds \cos\theta \\ &= \oint \vec{E} \cdot \vec{ds} \quad (2.9) \end{aligned}$$

where $d\vec{S}$ is called an area vector in the direction of the outward normal.

Let us apply the above equation in the case of determination of the magnitude of the electric flux due to a positive charge $+q$, at a point distance r from the charge.

To do this, we assume a hypothetical or imaginary sphere of radius r , with the charge $+q$ at its centre. (The hypothetical sphere defines the locus of all points distance r from the charge and hence represents all such points).

The total flux crossing the hypothetical sphere is therefore given by

$$\phi_E = \oint \vec{E} \cdot d\vec{s} = \oint E ds \cos \theta$$

Since E is uniformly distributed and radially outward, $\theta = 0$. Thus $\cos \theta = 1$.

$$\begin{aligned} \therefore \phi_E &= E \oint ds \\ &= E \cdot 4\pi r^2 \end{aligned} \quad (2.10)$$

$$\begin{aligned} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2 \\ &= q/\epsilon_0 \end{aligned} \quad (2.11)$$

2.6 Gauss Law

This law states that the surface integral of the normal component of the electric field (due to some charge immersed in a closed surface) over the closed surface is equal to the ratio of the charge enclosed within the surface to the permittivity of free space.

Thus

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\text{or } \epsilon_0 \oint \vec{E} \cdot d\vec{s} = q$$

In another form, the law can be stated as saying that the total electric flux crossing a closed surface is equal to the ratio of the charge enclosed within the surface to the permittivity of free space irrespective of the location or distribution of the charge enclosed, or the configuration of the surface.

$$\text{Hence } \phi_E = q/\epsilon_0$$

$$\text{Thus } \phi_E = \oint \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

If more than one charge is enclosed, then the total charge enclosed becomes the algebraic sum of all the charges i.e $\sum q$.

Hence Gauss Law becomes

$$\phi = \oint \vec{E} \cdot d\vec{s} = \sum q / \epsilon_0 \quad (2.12)$$

2.7 Implications Of Gauss Law.

- (1) *The law does not depend on the shape or contour of the surface provided it is closed.*
- (2) *It does not depend on the arrangement of charges enclosed or their locations within the closed surface.*
- (3) *If the charge is placed outside the closed surface, its flux contribution will be zero. This is because the flux due to the charge will cross the closed surface even number of times, once going in (which is -ve) and the other coming out (which is +ve) and their net contribution is zero. Thus charges outside the closed surface make no contribution to the total outward (or inward) flux due to some charge (or charges) enclosed by the surface, or to the value of the charge enclosed.*
- (4) *If no charge is enclosed, $\phi_E = 0$.*
- (5) *If equal and opposite charges are enclosed, $\phi_E = 0$*
- (6) *The net charge enclosed is the algebraic sum of all the charges enclosed irrespective of their locations within the closed surface.*

- (7) ϕ_E will be positive if the net charge enclosed is +ve, and -ve if the net charge enclosed is -ve.

LIMITATION OF GAUSS LAW.

Though Gauss Law is defined for arbitrary surface, it can only be evaluated easily for surfaces and fields of high geometric symmetry. For non-symmetric systems, the integration is non-trivial and difficult to evaluate.

2.8 Applications Of Gauss Law

To apply Gauss Law to any charge system, an appropriate hypothetical (or imaginary) surface called Gauss surface must be constructed. For example, for a point charge or point charges, and a charged hollow or solid sphere, the Gauss surface is a sphere. For a line of charge and a hollow or solid cylinder, the Gauss surface is a cylinder.

Let us now employ Gauss Law to determine the electric field due to some charge systems.

1. An Infinite Line of Uniform Charge Distribution.

Consider the infinite line of charge of linear charge density (charge per unit length) λ . If we draw a cylindrical Gauss surface of length l and radius r such that the line of charge lies along the axis of the Gauss surface as shown in fig. 14.

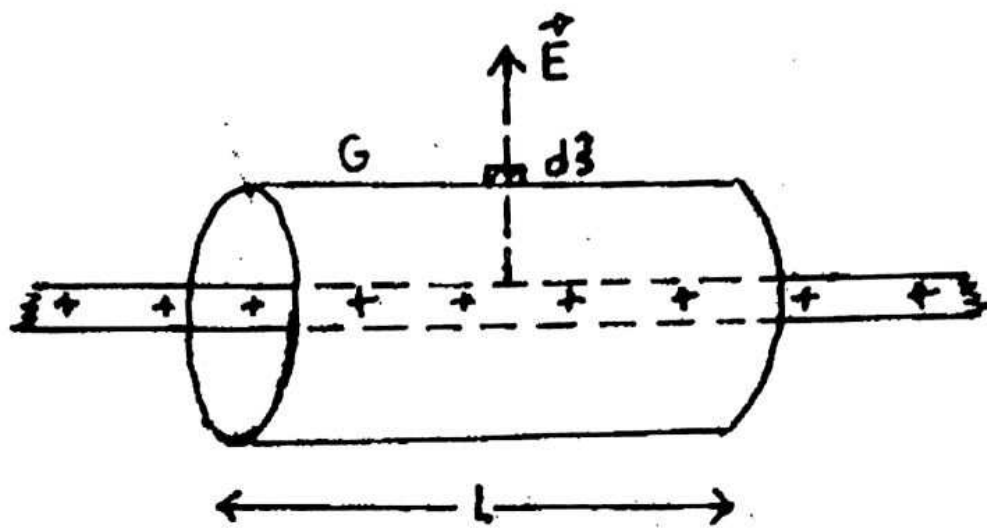


Fig. 14: A linear charge distribution with cylindrical Gaussian surface.

Since the charge distribution is uniform, then the field is symmetrical and radially outward as shown. For -ve charge distribution, the field will be radially inward.

Since the field is radially outward, no flux will flow out of the ends of the cylinder.

The ends of the closed cylinder do not therefore contribute in the calculation.

Applying Gauss Law to the cylinder,

$$\phi_E = q/\epsilon_0.$$

$$\text{Hence } q = \epsilon_0 \phi_E = \epsilon_0 \oint \vec{E} \cdot \vec{ds} \quad (2.13)$$

$$\therefore q = \epsilon_0 \oint E ds \cos \theta$$

But $\theta = 0$ since E is radially outward.

$$\therefore q = \epsilon_0 E \oint ds$$

$$\text{But } q = \lambda l \quad (2.14)$$

$$\therefore \lambda l = \epsilon_0 E \cdot 2 \pi r l$$

$$\therefore E = \frac{\lambda l}{2 \pi \epsilon_0 r l} \quad (2.15)$$

$$= \frac{\lambda}{2 \pi \epsilon_0 r}$$

$$= \frac{1}{4 \pi \epsilon_0} \cdot \frac{2 \lambda}{r} \quad (2.16)$$

=====

2. A uniformly charged hollow cylinder.

Consider a uniformly charged hollow cylinder S of length l and radius a . Let its linear charge density be λ .

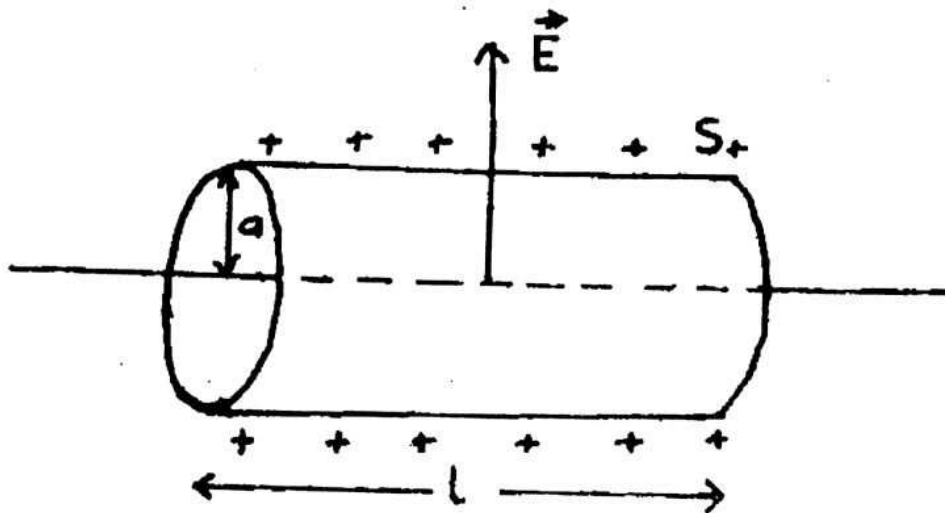


Fig. 15 (i): A positively charged hollow cylinder, S .

Again the field is symmetrical and radially outward, and there is no flux through the ends.

To find the electric field at locations outside the cylinder, say at a point distance r_1 from the axis of the charged hollow cylinder such that $r_1 > a$, we construct a cylindrical Gaussian surface G coaxial with the charged hollow cylinder S , having the same length l and radius r_1 as shown in fig. 15 (ii).

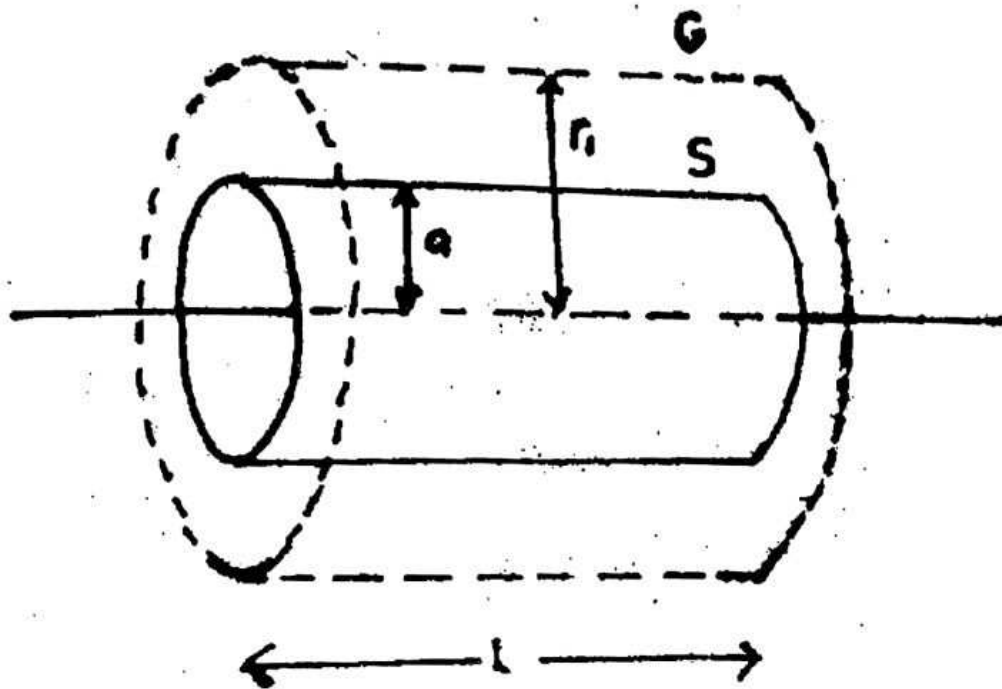


Fig. 15 (ii): Charged hollow cylinder S with Gaussian surface G where $r_1 > a$.

Applying Gauss Law,

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = q = \lambda l \quad (2.17)$$

$$\therefore \epsilon_0 E \oint ds = \epsilon_0 \cdot E \cdot 2\pi r_1 l = \lambda l.$$

$$\begin{aligned} \therefore E &= \frac{\lambda l}{2\pi \epsilon_0 r_1 l} \\ &= \frac{\lambda}{2\pi \epsilon_0 r_1} \\ &= \frac{1}{4\pi \epsilon_0} \cdot \frac{2\lambda}{r_1} \end{aligned} \quad (2.18)$$

Thus for locations outside the charged hollow cylinder, it behaves as though its charge lies along its axis, ie like a line of charge.

On the surface of the cylinder, $r_1 = a$

$$\begin{aligned} \text{Thus } E &= \frac{\lambda}{2\pi \epsilon_0 a} \\ &= \frac{1}{4\pi \epsilon_0} \cdot \frac{2\lambda}{a} \end{aligned} \quad (2.19)$$

For locations within the cylinder, we consider a coaxial cylindrical Gaussian surface of radius r_2 where $r_2 < a$. However since no charge is enclosed within the cylinder, $q = 0$

$$\phi_E = \epsilon_0 \oint \vec{E} \cdot d\vec{s} = 0$$

Hence $\phi_E = 0$ and $E = 0$.

Thus electric flux and field are everywhere zero within a charged hollow cylinder.

3. Uniformly Charged Solid Cylinder.

For finding the field outside and on the surface of the charged solid cylinder, the procedure and results are the same as in charged hollow cylinder.

At a location of distance r_1 from the axis of the charged solid cylinder of radius a , where $r_1 > a$,

$$E = \frac{\lambda}{2\pi \epsilon_0 r_1}$$

$$= \frac{1}{4\pi \epsilon_0} \frac{2\lambda}{r_1}$$

Thus a uniformly charged solid cylinder also behaves as if its charge is concentrated along its axis, ie as a line of charge.

Also when $r_1 = a$,

$$E = \frac{\lambda}{2\pi \epsilon_0 a}$$

$$= \frac{1}{4\pi \epsilon_0} \frac{2\lambda}{a} \quad (2.20)$$

In the case of field at $r_2 < a$, ie within the charged solid cylinder, a coaxial cylindrical Gaussian surface G of radius r_2 will enclose some charge since there is uniform charge distribution throughout the solid. Since the charge distribution is uniform, then the volume charge density (ie. charge per unit volume) of the solid is constant.

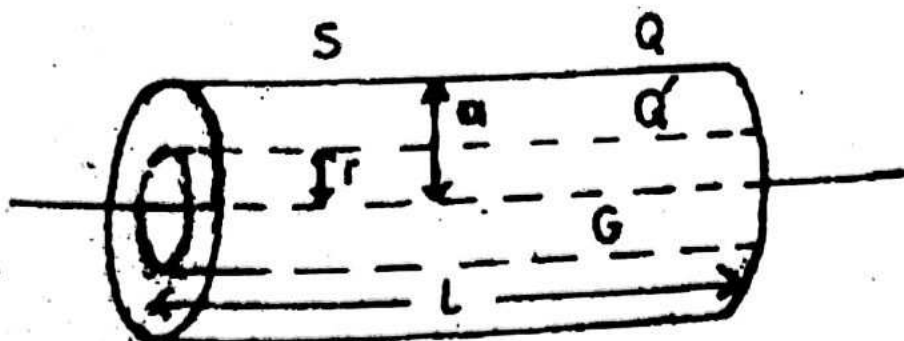


Fig. 15 (III): Charged solid cylinder S with Gaussian surface G of radius $r_2 < a$.

Suppose the total charge of the solid is Q , and that the Gaussian surface encloses a charge Q^1 . Then since the volume charge density of the solid is constant,

$$\frac{Q}{V} = \frac{Q^1}{n_j V^1} \quad (2.21)$$

where V is the volume of the solid and V^1 is the volume enclosed by the Gaussian surface.

$$\begin{aligned} \therefore Q^1 &= \frac{V^1}{V} \cdot Q \\ &= \frac{\pi r_2^2 l}{\pi a^2 l} \cdot Q \\ &= \frac{r_2^2}{a^2} \cdot Q \end{aligned} \quad (2.22)$$

Thus applying Gauss Law, we obtain,

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = Q^1 \quad (2.23)$$

$$\therefore \epsilon_0 E \oint ds = \frac{r_2^2}{a^2} \cdot Q \quad (2.24)$$

$$= \frac{r_2^2}{a^2} \cdot \lambda l$$

$$\therefore \epsilon_0 E \cdot 2\pi r_2 l = \frac{r_2^2}{a^2} \cdot \lambda l$$

$$\therefore E = \frac{1}{2\pi\epsilon_0 r_2 l} \cdot \frac{r_2^2}{a^2} \cdot \lambda l \quad (2.25)$$

$$= \frac{1}{2\pi\epsilon_0} \cdot \frac{r_2 \cdot \lambda}{a^2}$$

$$= \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{r_2}{a^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2\lambda r_2}{a^2} \quad (2.26)$$

=====

4. A uniformly charged hollow sphere

Consider a uniformly charged hollow sphere of radius a and charge q . We want to first determine the electric field at a location of distance r_1 from the centre of the charged hollow sphere, where $r_1 > a$. To do this we construct a spherical Gaussian surface of radius r_1 and concentric with the charged hollow sphere as shown in fig. 16.

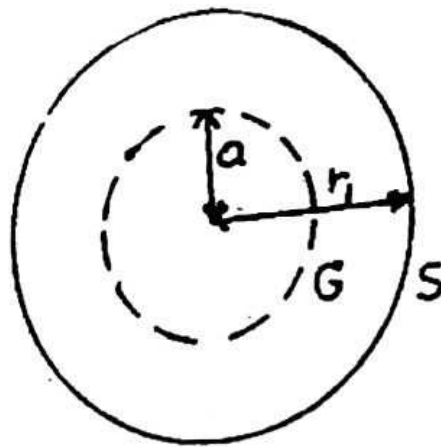


Fig. 16 **Charged solid sphere S with Gaussan surface G.**

Since the charge distribution is uniform, then E is uniform and radially outward.

Thus applying Gauss law,

Thus at locations outside a charged hollow sphere, it behaves as if its charge is concentrated at its centre, ie as if it is a point charge.

For the field on the surface of the sphere, $r_1 = a$

$$\text{Thus } E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2} \quad (2.28)$$

=====

For the field inside the charged hollow sphere, we draw a surface of radius $r_2 < a$. However, since no charge is enclosed within the surface, (ie. $q = 0$), then $E = 0$.

Thus the field is everywhere zero within a charged hollow sphere.

5. **A Uniformly charged solid sphere.**

Consider a uniformly charged solid sphere S of radius a and charge q .

To determine its electric field outside the sphere at a location distance r_1 from the centre of the charged sphere, where $r_1 > a$, we proceed exactly in the same way as in the case of charged hollow sphere and obtain identical result.

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1^2}$$

Thus again, at locations outside a charged solid sphere, it behaves like a point charge, as if its charge is concentrated at its centre.

Also at its surface where $r_1 = a$,

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2} \quad (2.29)$$

In the case of the interior of the solid sphere, charge is uniformly distributed through it. Thus a Gaussian surface constructed within the solid will enclose some charge. Suppose a Gaussian surface of radius r_2 (where $r_2 < a$) is constructed concentric with the solid sphere S as shown in fig. 17.

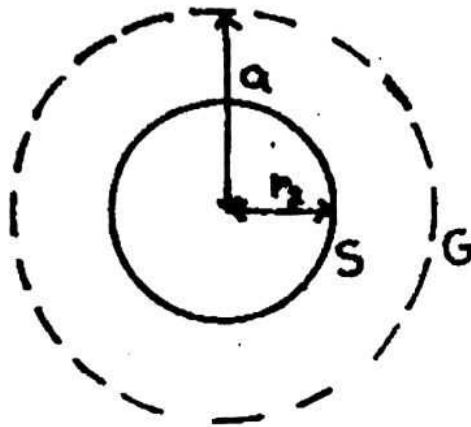


Fig. 17: A charged solid sphere S of radius a , with a Gaussian surface G of radius r_2 , where $r_2 < a$.

Suppose the Gaussian surface of volume v^1 encloses a charge q^1 . Then since there is uniform charge distribution, the volume charge density of the sphere is constant. Hence if the volume of the solid sphere is v , then

$$\frac{q^1}{v^1} = \frac{q}{V} \quad (2.30)$$

$$\therefore q^1 = \frac{v^1}{V} \cdot q = \frac{4/3 \pi r_2^3}{4/3 \pi a^3} \cdot q \quad (2.31)$$

$$\therefore q^1 = \frac{r_2^3}{a^3} \cdot q \quad (2.32)$$

Applying Gauss Law,

$$\epsilon_0 \oint \vec{E} \cdot \vec{ds} = q \quad (2.33)$$

$$\therefore \epsilon_0 \oint E ds \cos \theta = q$$

$$\therefore \epsilon_0 E \oint ds = \frac{r_2^3}{a_3} q \quad (\text{since } \theta = 0) \quad (2.34)$$

$$\therefore \epsilon_0 E \cdot 4\pi r_2^2 = \frac{r_2^3}{a_3} q$$

$$\therefore E = \frac{1}{4\pi\epsilon_0 r_2^2} \frac{r_2^3}{a_3} q$$

$$= \frac{q}{4\pi\epsilon_0} \cdot \frac{r_2}{a_3} \quad (2.35)$$

6. A sheet of uniform charge distribution.

Suppose a sheet of charge has uniform charge density, σ .

To find the electric field at a point distance r from the sheet, we construct a "pill - box" of cross-sectional area A and length $2r$ across the sheet as the Gaussian surface, as shown in fig. 18.

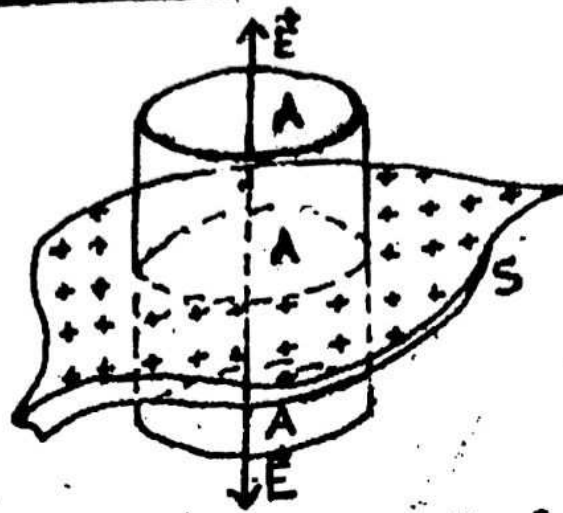


Fig. 18: Pill-Box Gaussian Surface for a sheet of charge.

The charge enclosed in the area A is σA . The field due to this charge is usually outward and uniform.

Thus applying Gauss Law.

For accurate results, we consider points not near the edges of the sheet, and whose distance from the sheet is small compared with dimensions of the sheet.

2.9 Solved Examples.

$$\left(\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ Nm}^2 \text{ C}^{-2} \right)$$

Example 2.1: Calculate the electric field at a point 5 cm from a charge of magnitude $-3.2 \times 10^{19} \text{ C}$.

Solution:

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

where q is the charge, and r is the distance from the charge.

$$\begin{aligned} \therefore E &= \frac{9 \times 10^9 \times 3.2 \times 10^{-19}}{(5 \times 10^{-2})^2} \\ &= \frac{9 \times 3.2 \times 10^{-6}}{25} \\ &= \underline{\underline{1.15 \times 10^{-6} \text{ NC}^{-1}}} \end{aligned}$$

Example 2.2: A straight wire of length 500m carries a charge of $+6.4 \times 10^{-6} \text{ C}$. Calculate the electric field at a point 2m from the centre of the wire, assuming uniform distribution of charge along the wire.

Solution: Let linear charge density of the wire be λ . Let us also draw a cylindrical Gaussian surface of length l , radius r such that the wire lies on the axis of the cylinder as shown in fig. 19.

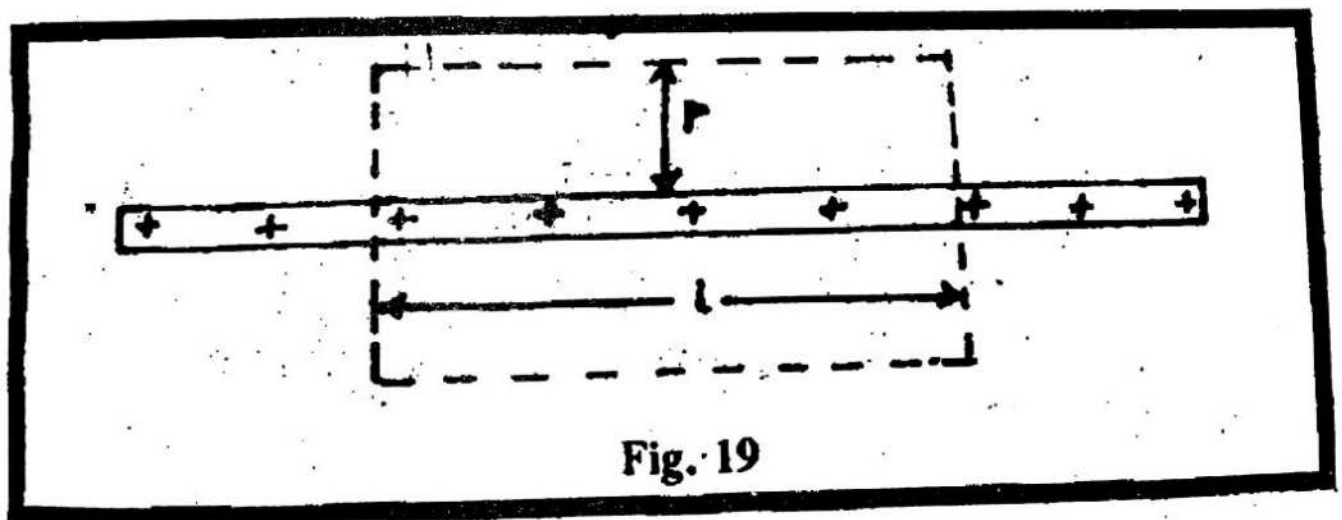


Fig. 19

Applying Gauss Law, we obtain

$$\therefore E = \frac{\lambda}{2\pi \epsilon_0 r} \quad (2.39)$$

$$= \frac{1}{4\pi \epsilon_0} \cdot \frac{2\lambda}{r}$$

$$\text{But } \lambda = \frac{6.4 \times 10^{-6}}{500}$$

$$= 1.28 \times 10^{-8} \text{ Cm}^{-1}$$

$$\therefore E = \frac{9 \times 10^9 \times 2 \times 1.28 \times 10^{-8}}{2}$$

$$= \underline{\underline{115.2 \text{ NC}^{-1}}}$$

Example 2.3: A uniformly charged hollow sphere of radius 10cm carries a charge of $+ 3.2 \times 10^{-10}\text{C}$. Calculate the electric field at locations 20cm, 10cm, 5cm and 0cm from the centre of the sphere. Hence plot a graph of E against distance (r) from +20cm to -20cm from the centre of the sphere.

Solution: Let the radius of the sphere be a. It has been shown for the case $r > a$ that

$$E = \frac{1}{4\pi \epsilon_0} \cdot \frac{q}{r^2} \quad (2.40)$$

where r is distance from centre of the sphere.

(i) when $r = 20\text{cm}$, then $r > a$

$$\therefore E = \frac{9 \times 10^9 \times 3.2 \times 10^{-10}}{(20 \times 10^{-2})^2}$$

$$\begin{aligned}
 &= \frac{28.8 \times 10^{-1}}{400 \times 10^{-4}} \\
 &= 7.2 \times 10 \\
 &= \underline{72 \text{ NC}^{-1}}
 \end{aligned}$$

(ii) When $r = 10\text{cm}$, then $r = a$

$$E = \frac{9 \times 10^9 \times 3.2 \times 10^{-10}}{(10 \times 10^{-2})^2}$$

$$= 28.8 \times 10$$

$$= \underline{288 \text{ NC}^{-1}}$$

(iii) When $r = 5\text{cm}$, $r < a$.

Hence $q = 0$

$\therefore E = 0$.

(iv) When $r = 0$, $r < a$

Hence $q = 0$.

$\therefore E = 0$.

The graph of E versus r is shown in fig. 20.

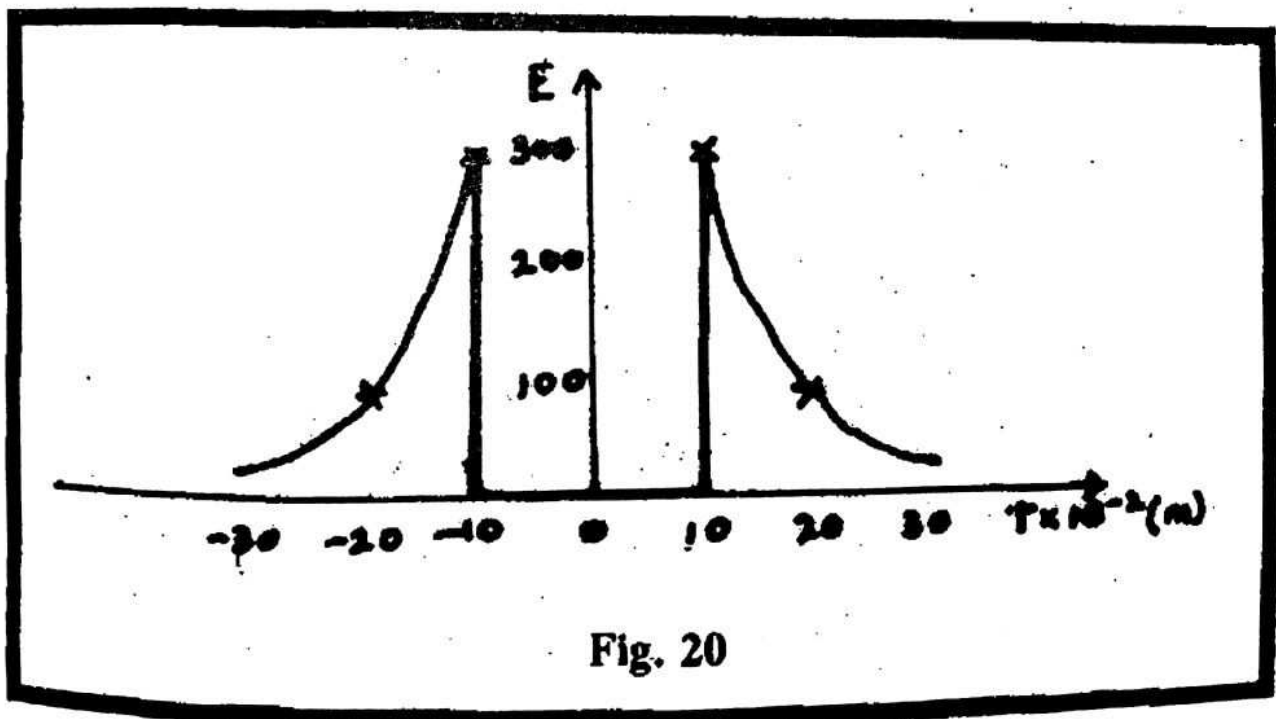


Fig. 20

Example 2.4: Repeat Q.3 for uniformly charged solid sphere carrying the same charge.

Solution: It has also been shown for the case $r > a$. that

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad (2.41)$$

(i) Hence for $r = 20\text{cm}$, $r > a$

$$\begin{aligned} \therefore E &= \frac{9 \times 10^9 \times 3.2 \times 10^{-10}}{(20 \times 10^{-2})^2} \\ &= \underline{72 \text{ NC}^{-1}} \end{aligned}$$

(ii) For $r = 10\text{cm}$, $r = a$

$$\begin{aligned} \therefore E &= \frac{9 \times 10^9 \times 3.2 \times 10^{-10}}{(10 \times 10^{-2})^2} \\ &= \underline{288 \text{ NC}^{-1}} \end{aligned}$$

(iii) For $r = 5\text{cm}$, $r < a$.

Therefore, suppose a Gaussian surface of radius r encloses a charge q' , then

$$\frac{q'}{q} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} = \frac{r^3}{a^3} \quad (2.42)$$

$$\begin{aligned} \therefore q' &= r^3/a^3 \cdot q \\ &= \frac{(5 \times 10^{-2})^3}{(10 \times 10^{-2})^3} \times 3.2 \times 10^{-10} \\ &= \frac{125 \times 10^{-6} \times 3.2 \times 10^{-10}}{1000 \times 10^{-6}} \\ &= 0.125 \times 3.2 \times 10^{-10} \\ &= \underline{0.4 \times 10^{-10} \text{C.}} \end{aligned}$$

Applying Gauss Law

$$\epsilon_0 \oint \vec{E} \cdot \vec{ds} = q^1$$

$$\therefore \epsilon_0 \oint E ds \cos \theta = q^1$$

Since field is radially outward, $\theta = 0$.

$$\therefore \epsilon_0 E \oint ds = q^1$$

$$\therefore \epsilon_0 E \cdot 4\pi r^2 = q^1$$

$$\therefore E = \frac{1}{4\pi \epsilon_0} \cdot \frac{q^1}{r^2} \quad (2.43)$$

$$= \frac{9 \times 10^9 \times 0.4 \times 10^{-10}}{(5 \times 10^{-2})^2}$$

$$= \frac{3.6 \times 10^{-1}}{25 \times 10^{-4}}$$

$$= 0.144 \times 10^3$$

$$= \underline{\underline{144 \text{ NC}^{-1}}}$$

(iv) For $r = 0, r < a$

$$\text{But } q^1 = r^3/a^3 \cdot q = 0$$

$$\text{Hence } E = 0$$

The graph of E versus r in this case is shown in fig. 21.

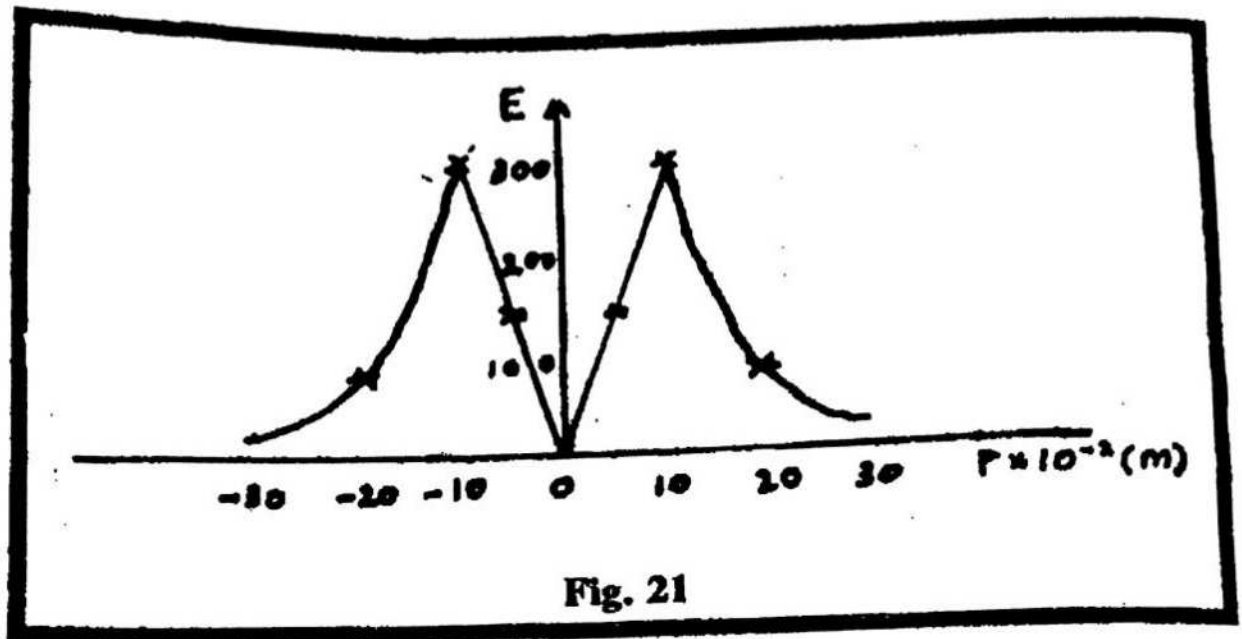


Fig. 21

(You can verify the curve by calculating values for E at $r = 3\text{cm}$, 8cm and 30cm , and compare with the curve).

We have verified that within the uniformly charged sphere,

$$E = \frac{q}{4\pi\epsilon_0} \cdot \frac{r}{a^2} \quad (2.44)$$

Thus since a is constant

$E \propto r$ within the sphere

This accounts for the linear relationship between E and v from the centre of the charged sphere to its surface. From the surface onwards the relationship changes. It is now determined by the relation,

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad (2.45)$$

Hence $E \propto \frac{1}{r^2}$ as expressed in the curve.

Example 2.5: A solid cylinder of infinite length has diameter of 10cm and carries a uniform charge of charge density $6.0 \times 10^{-10} \text{ Cm}^{-3}$. Calculate the electric field at position 10cm and 3cm from the axis of the cylinder.

Solution: Consider a section of the cylinder, length l , and carrying a charge q .

Let the volume charge density and radius of the cylinder be ρ and R respectively.

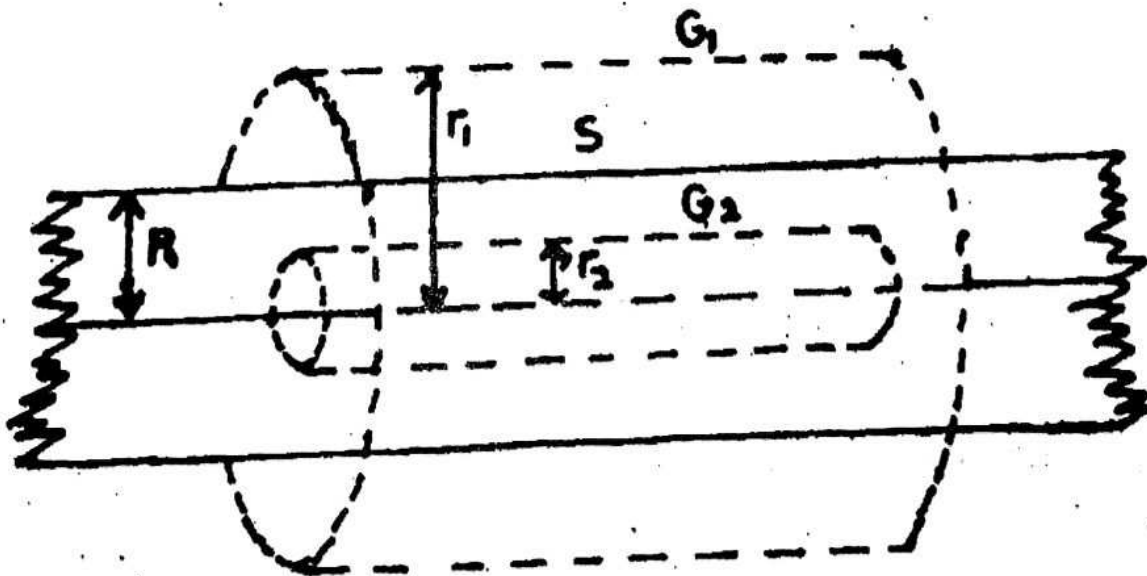


Fig. 22

We now draw two Gaussian surfaces G_1 and G_2 with radii r_1 and r_2 respectively where $r_1 > R$ and $r_2 < R$, which are co-axial with the charged cylinder as shown in fig. 22. Applying Gauss Law to G_1 , we obtain

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = q = \rho v \text{ where } V \text{ is the volume of the}$$

section of the cylinder in consideration.

$$\therefore \epsilon_0 \oint E \, ds \cos \theta = \rho V \quad (2.46)$$

But E is radially outward, so that $\theta = 0$.

$$\therefore \epsilon_0 \oint ds = \rho V$$

$$\therefore \epsilon_0 E 2\pi \cdot r_1 l = \rho \cdot \pi R^2 l$$

$$\therefore E = \frac{\rho \cdot \pi R^2 l}{2\pi \epsilon_0 r_1 l} = \frac{\rho R^2}{2\epsilon_0 r_1} \quad (2.47)$$

$$\text{OR } E = \frac{1}{4\pi \epsilon_0} \cdot \frac{2\pi \rho R^2}{r_1} \quad (2.48)$$

Substituting, we obtain value for E at $r_1 = 10\text{cm}$ ($r_1 > R$), where $R = 5\text{cm} = 5 \times 10^{-2}\text{m}$.

$$\begin{aligned}
 \therefore E &= \frac{9 \times 10^9 \times 2 \times 3.142 \times 6 \times 10^{-10} \times (5 \times 10^{-3})^2}{(10 \times 10^{-3})^2} \\
 &= 108 \times 3.142 \times 25 \times 10^{-3} \\
 &= 8483.4 \times 10^{-3} \\
 &= \underline{8.48 \text{ NC}^{-1}}
 \end{aligned}$$

For the second case of when $r_2 = 3\text{cm}$ ($r_2 < R$), we use the Gaussian surface, G_2 .

Suppose the charge q^1 is enclosed by G_2 .

Then $q^1 = \rho V^1$ where V^1 is the volume enclosed by G_2 .

$$\therefore q^1 = \rho \cdot \pi r_2^2 l. \quad (2.49)$$

Applying Gauss law, we obtain

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = q^1$$

$$\therefore \epsilon_0 E \cdot 2\pi r_2 l = \rho \pi r_2^2 l.$$

$$\therefore E = \frac{1}{2\pi \epsilon_0 r_2 l} \cdot \rho \pi r_2^2 l$$

$$= \frac{\rho r_2}{2 \epsilon_0} \quad (2.50)$$

$$\text{or } E = \frac{2\pi \rho r_2}{4\pi \epsilon_0} = \frac{1}{4\pi \epsilon_0} \cdot 2\pi \rho r_2. \quad (2.51)$$

Substituting we obtain

$$\begin{aligned}
 E &= 9 \times 10^9 \times 2 \times 3.142 \times 6 \times 10^{-10} \times 3 \times 10^{-2} \\
 &= 108 \times 3.142 \times 3 \times 10^{-3} \\
 &= 1018 \times 10^{-3} \\
 &= \underline{\underline{1.02 \text{ NC}^{-1}}}
 \end{aligned}$$

Example 2.6: Four charged particles carrying charges of magnitudes $+2q$, $+4q$, $-q$ and $+2q$ are placed one after the other respectively at the four corners of a square of side 2cm . Calculate the magnitude and direction of the resultant electric field at the centre of the square, given that $\epsilon = 1.0 \times 10^{-12}\text{C}$.

Solution:

Step 1: Draw and label the square charge system, and indicate the various fields due to the charges at the centre of the square, P , as shown in fig. 23a.

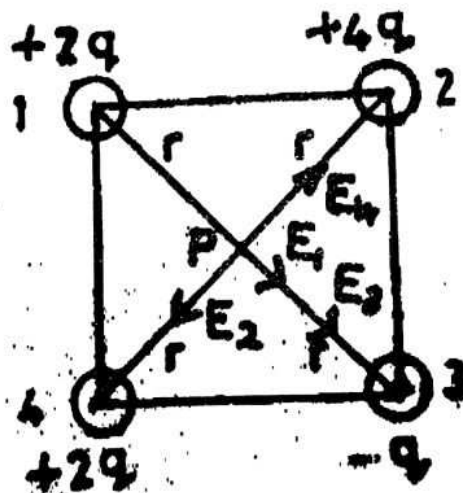


Fig. 23a.

Step 2: Calculate the fields

$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{r^2} \quad (2.52)$$

where r is the distance of each charge from the centre P and equals $\frac{1}{2}$ the square diagonal.

Hence since the square side is 2cm,

$$(2r)^2 = 2^2 + 2^2 = 4 + 4 = 8.$$

$$\therefore 4r^2 = 8$$

$$\therefore r^2 = 8/4 = 2$$

$$\therefore r = \sqrt{2}\text{cm} = \sqrt{2} \times 10^{-2}\text{m}.$$

$$\therefore E_1 = \frac{9 \times 10^9 \times 2 \times 10^{-12}}{(\sqrt{2} \times 10^{-2})^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 10^{-12}}{2 \times 10^{-4}}$$

$$= \underline{90 \text{ NC}^{-1}}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{4q}{r^2}$$

(2.53)

$$= \frac{9 \times 10^9 \times 4 \times 10^{-12}}{2 \times 10^{-4}}$$

$$= \underline{1.80 \text{ NC}^{-1}}$$

(2.54)

$$\begin{aligned}
 E_3 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \\
 &= \frac{9 \times 10^9 \times 10^{-12}}{2 \times 10^{-4}} \\
 &= \underline{45 \text{ NC}^{-1}} \\
 E_4 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{r^2} \\
 &= \frac{9 \times 10^9 \times 2 \times 10^{-2}}{2 \times 10^{-4}} \\
 &= \underline{90 \text{ NC}^{-1}}
 \end{aligned}$$

Step 3: Resolve the fields into two perpendicular directions, say E_x and E_y .

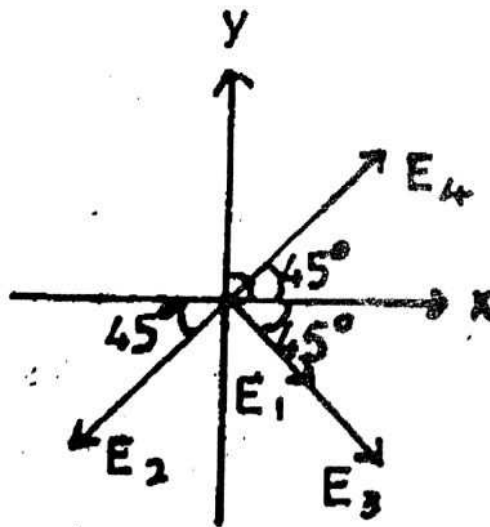


Fig. 23b.

From fig. 23b,

$$\begin{aligned}
 E_x &= E_1 \cos 45 + E_3 \cos 45 + E_4 \cos 45 - E_2 \cos 45. \quad (2.55) \\
 &= (E_1 + E_3 + E_4 - E_2) \cos 45. \\
 &= (90 + 45 + 90 - 180) 0.707 \\
 &= 45 \times 0.707 \\
 &= \underline{\underline{31.82 \text{ NC}^{-1}}}
 \end{aligned}$$

$$\begin{aligned}
 E_y &= E_4 \sin 45 - E_1 \sin 45 - E_3 \sin 45 - E_2 \sin 45 \quad (2.56) \\
 &= (E_4 - E_1 - E_3 - E_2) \sin 45 \\
 &= (90 - 90 - 45 - 180) 0.707 \\
 &= -225 \times 0.707 \\
 &= \underline{\underline{-159.08 \text{ NC}^{-1}}}
 \end{aligned}$$

Step 4: Represent E_x , E_y and E_R in a diagram (fig. 23c) and hence calculate E_R and θ where θ is the angle made by E_R with x-direction.

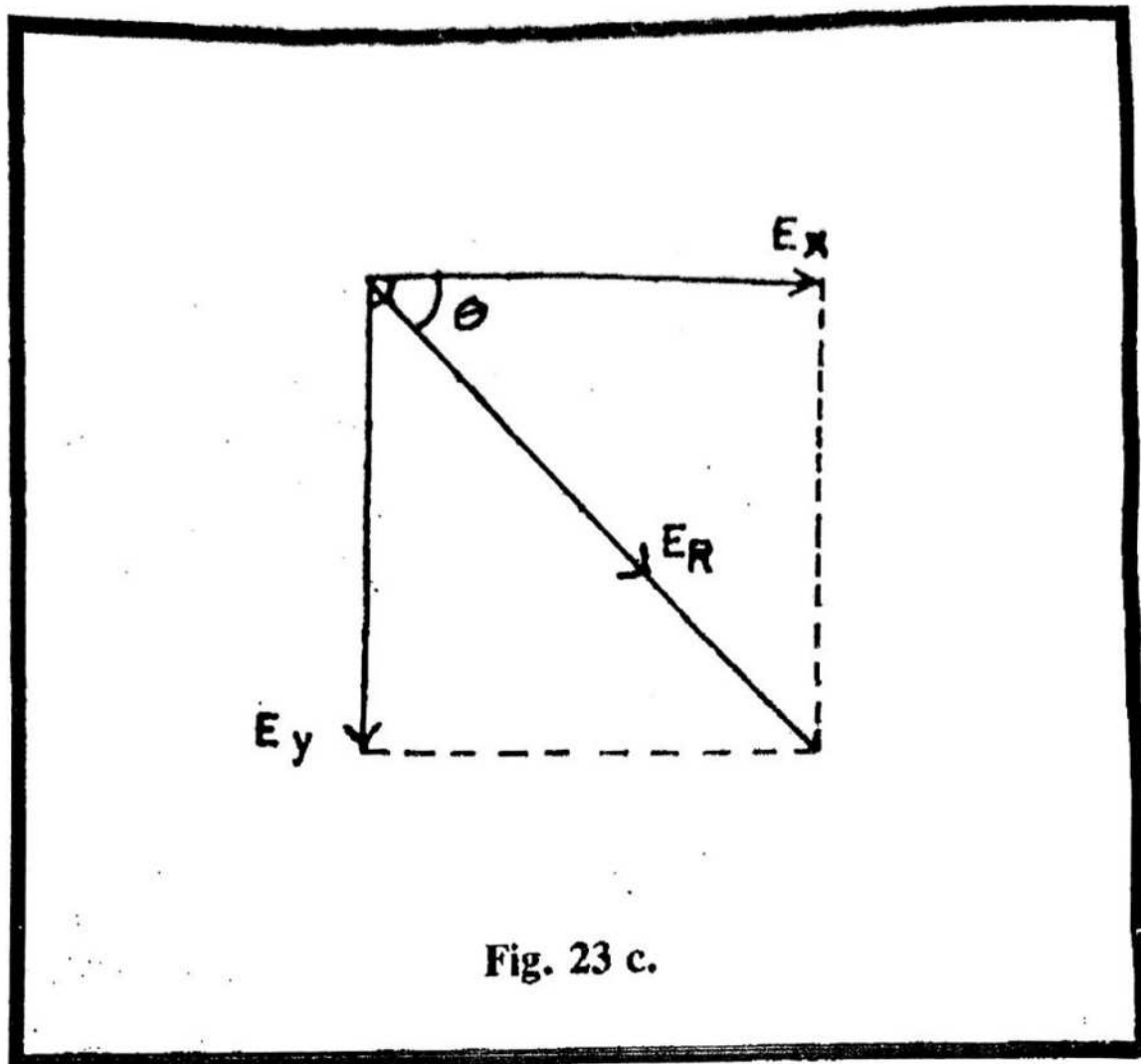


Fig. 23 c.

From fig. 23c,

$$\begin{aligned}
 E_R^2 &= E_x^2 + E_y^2 & (2.57) \\
 &= 31.82^2 + 159.08^2 \\
 &= 1012.52 + 25306.45 \\
 &= 26318.96
 \end{aligned}$$

$$\therefore E_R = \underline{\underline{162.23 \text{ NC}^{-1}}}$$

To find θ ,

$$\tan \theta = \frac{E_y}{E_x} \quad (2.58)$$

$$= \frac{159.08}{31.82}$$

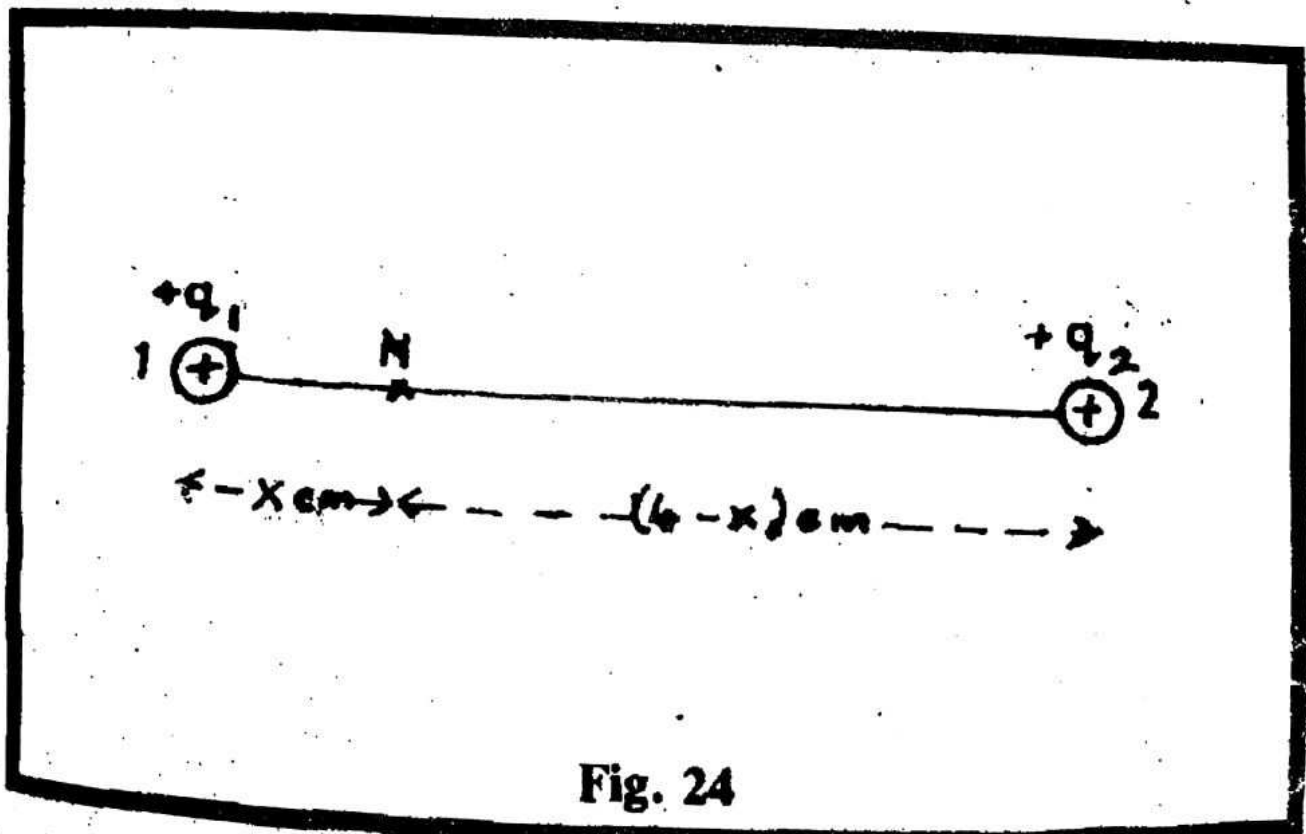
$$= 4.9994$$

$$\therefore \theta = \tan^{-1} (4.9994)$$

$$= \underline{78.69^\circ}$$

Example 2.7: Two like charges of magnitudes $+1.6 \times 10^{-16}\text{C}$ and $+6.4 \times 10^{-16}\text{C}$ are placed 4cm apart. Determine the location of the neutral point between them.

Solution: Let the charges be represented respectively by $+q_1$ and $+q_2$, and let the neutral point be distance x from the smaller charge as shown in fig. 24.



$$\therefore E_1 = E_2$$

$$\therefore \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{(4-x)^2} \quad (2.59)$$

$$\therefore \frac{1.6 \times 10^{-16}}{x^2} = \frac{6.4 \times 10^{-16}}{(4-x)^2}$$

$$\therefore (4-x)^2 = \frac{6.4}{1.6} x^2 = 4x^2$$

$$\therefore 16 - 8x + x^2 = 4x^2$$

$$\therefore 3x^2 + 8x - 16 = 0$$

$$\therefore (3x - 4)(x + 4) = 0$$

$$\therefore x = 4/3 \text{ cm or } -4 \text{ cm.}$$

But x cannot be -ve, because that will imply that the neutral point is not located in between the charges, but 4cm away to the left of the smaller charge ($+q_1$).

Thus $x = 4/3 = 1.33\text{cm}$, which implies that the neutral point is 1.33cm away from q_1 (or 2.67cm from q_2) along the axis joining q_1 and q_2 .

2.10 PROBLEMS.

2.10.1 Two electrons and two protons are kept 10^{-8} cm apart. Calculate the resultant electric field midway between them (Electronic charge = $-1.6 \times 10^{-19}\text{C}$ while protonic charge = $+1.6 \times 10^{-19}\text{C}$).

2.10.2 Calculate the magnitude and direction of the resultant electric field at the centroid of an equilateral triangle of side 2cm, due to three charges, $+q$, $+3q$ and $-2q$ placed in order at its vertices, where $q = 1.6 \times 10^{-10} \text{ C}$.

2.10.3

A proton is kept 3mm away from another group of three protons. Determine the neutral point of the system if protonic charge is given as $+ 1.6 \times 10^{-19}\text{C}$.

2.10.4

A uniformly charged solid sphere of radius 5 cm and carrying a charge of $+3.2 \times 10^{-12}\text{C}$ is placed concentric with a hollow sphere of radius 10 cm. Calculate the electric field due to this arrangement at locations 20cm, 10cm, 8cm, 5cm and 3cm from the centre of the spheres.

2.10.5

An electric dipole of separation 4cm carries charges $\pm 4.8 \times 10^{-16}\text{C}$. Calculate the resultant electric field at a location,

(i) *1m away along its dipole axis.*

(ii) *1m away along its median axis.*

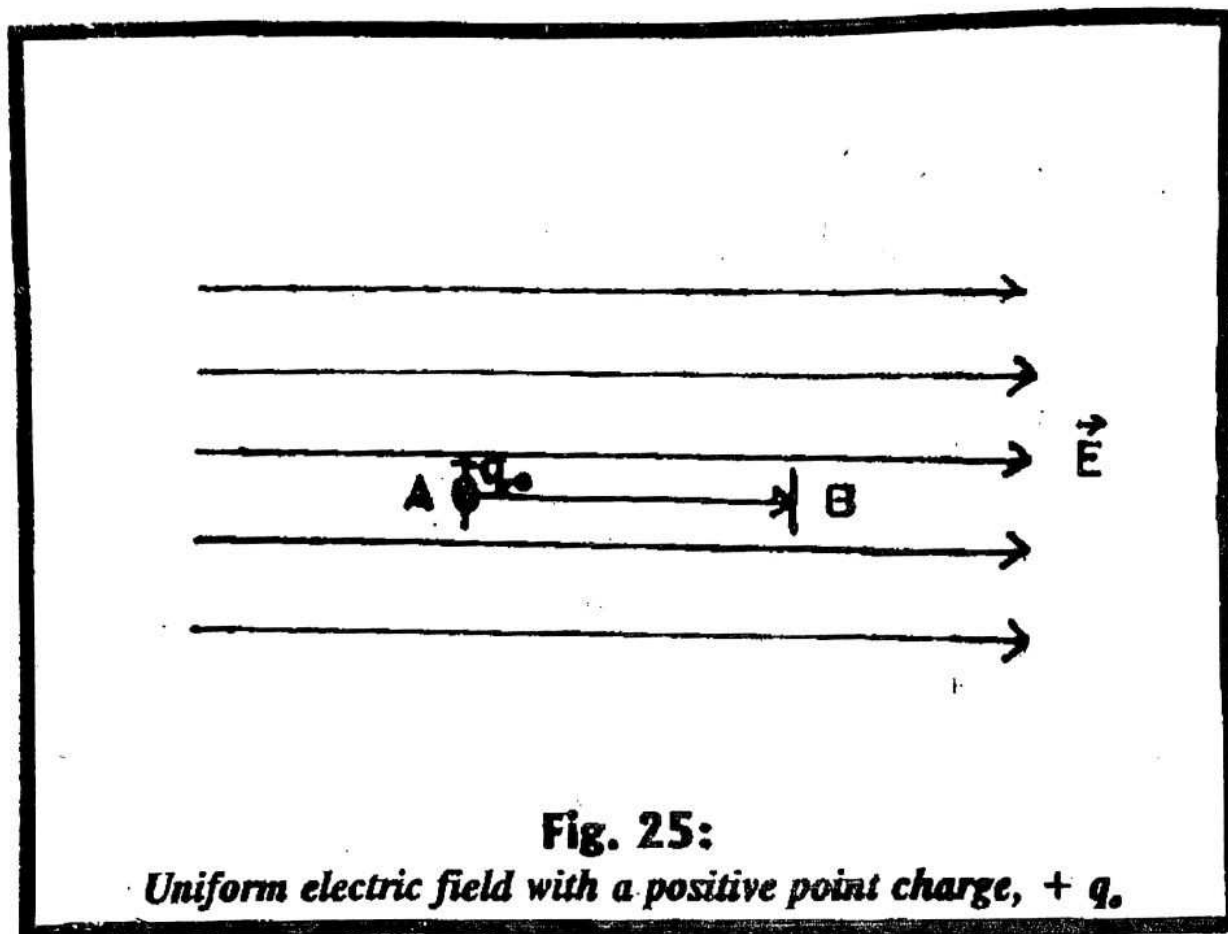
2.10.6

Two equal and opposite charges of magnitude $4.0 \times 10^{-8}\text{C}$ are 12cm apart. What is the magnitude of E midway between them? What force will therefore act on an electron placed at that position. If the electron is released to move, what will be its acceleration, and with what velocity and kinetic energy will it hit the positive charge. (electronic charge = $-1.6 \times 10^{-19}\text{C}$; electronic mass = $9.1 \times 10^{-31}\text{Kg}$).

ELECTRICAL POTENTIAL

3.1. Electrical Potential Energy

Consider a uniform field due to a positive charge, as shown in fig. 25.



If we place a positive point charge $+q_0$ at the point A, it will move in the direction of the field, say to the point B, due to the electric force of the field. To move the charge therefore from B to A against this electric force, an external force must be employed and work is done. Thus this movement of the charge from B back

to A by an external force increases its electrical potential energy much the same way as the lifting of a mass to a height in earth's gravitational field will increase its gravitational potential energy.

The work done by the external force is the measure of the increase in electrical potential energy of the charge.

Since work done = Force (F_e) x displacement(s), the electrical potential energy = $F_e \times s$

$$= q_0 E \times BA \quad (3.1)$$

Thus the electrical potential energy associated with the charge q_0 at a point in the electric field is dependent on

- (i) *the magnitude of the field*
- (ii) *the magnitude of the charge*
- (iii) *the position of the charge in the field.*

3.2. Electrical Potential

The potential at a point in an electric field is defined as the potential energy or work done per unit charge in moving a positive point charge from infinity to that point (provided that the point charge does not affect the field). The potential at infinity is arbitrarily chosen to be zero.

In fig. 25, the potential at A exceeds that at B by the work which must be done against electric force to take a unit positive charge from B to A. Potential is therefore a property of a point in the field and is therefore a scalar quantity. It is measured in JC^{-1} or volt (V).

Just as mass moves from a point of higher gravitational potential to another of lower potential, (that is falling from a height to a lower level), so does a positive charge move in an electric field from a point of higher electrical potential to one of lower potential (that is, it climbs down potential gradient). The negative charges in a positive field moves in the opposite direction (that is, it climbs up potential gradient).

3.3. Potential Due To An Isolated Point Charge.

Consider the potential due to an isolated positive charge $+q$ at a position A, distance r from the charge as shown in fig. 26.

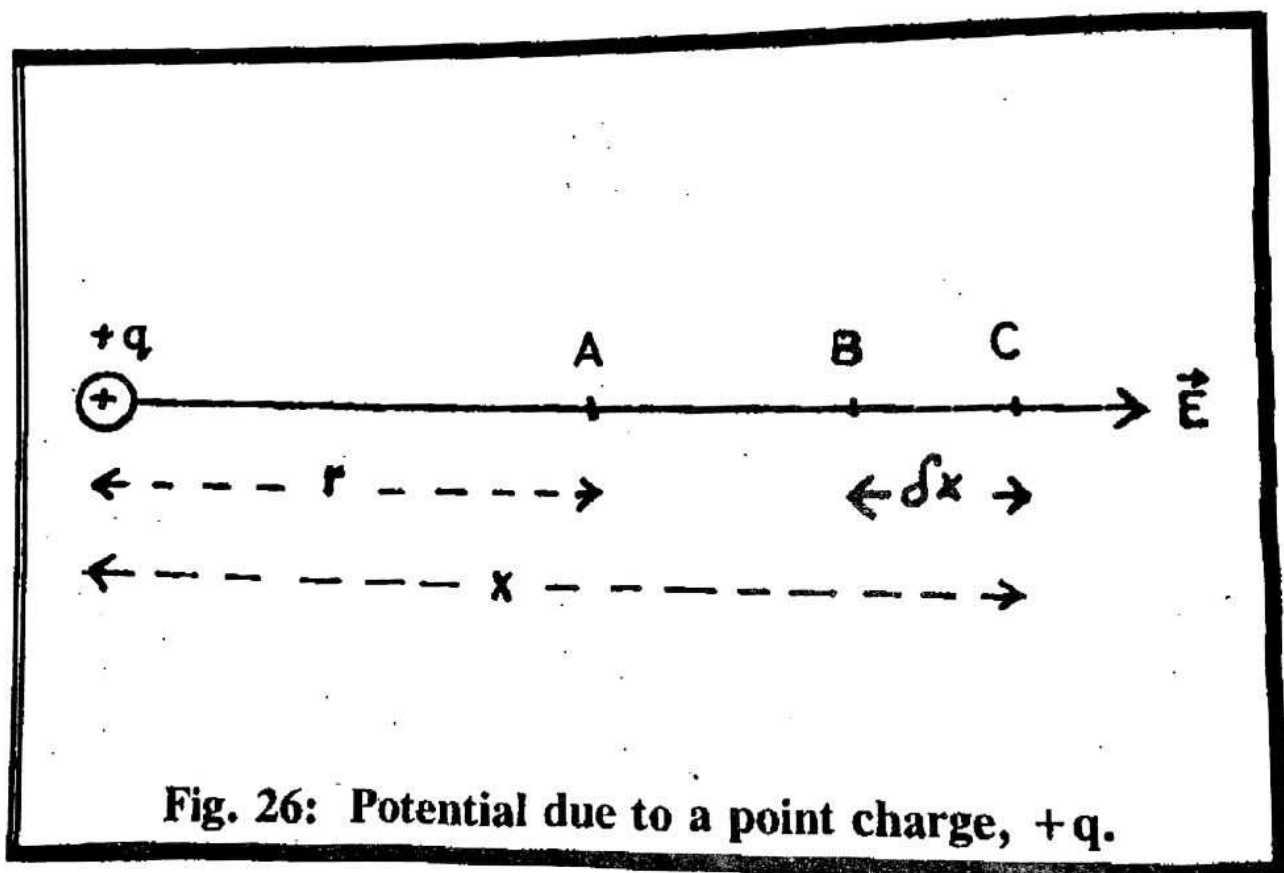


Fig. 26: Potential due to a point charge, $+q$.

Let a positive test charge $+q_0$ be placed at the position C, distance x from the isolated charge. The electric force between the two charges is given from Coulomb's law by

$$F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{x^2} \quad (3.2)$$

Suppose we now move the test charge from the position C to B, through a small distance δx , without affecting the field due to $+q$. Assuming that the force on $+q_0$ remains constant during this very small movement; then the work done δW by the external force over the distance δx against the force of the field, is given by

$$\delta W = F (-\delta x) \quad (3.3)$$

The negative sign is inserted to show that the displacement δx is in opposite direction to the direction of the field or to show that x increases in the direction opposite to that of increase in work done or potential energy.

$$\text{Hence } \delta W = -F\delta x \quad (3.4)$$

$$= - \frac{qq_0}{4\pi\epsilon_0} \cdot \frac{\delta x}{x^2}$$

Thus the total work done in bringing $+q_0$ from infinity to A is given by

$$W = \int \delta W = \int_{\infty}^r - \frac{qq_0}{4\pi\epsilon_0} \cdot \frac{\delta x}{X^2} \quad (3.5)$$

$$= \frac{-qq_0}{4\pi\epsilon_0} \int_{\infty}^r \frac{\delta x}{X^2}$$

$$= \frac{-qq_0}{4\pi\epsilon_0} \left(-\frac{1}{x}\right) \Big|_{\infty}^r$$

$$= \frac{q q_0}{4\pi \epsilon_0} \frac{1}{r} \quad (3.6)$$

Thus the potential V at A is given by

$$V = W/q_0$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{q_0 r}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad (3.7)$$

=====

3.4 Potential Difference.

Again consider a field due to an isolated positive charge $+q$. Suppose we have two points A and B , distance a and b respectively from the charge as shown in fig. 27.

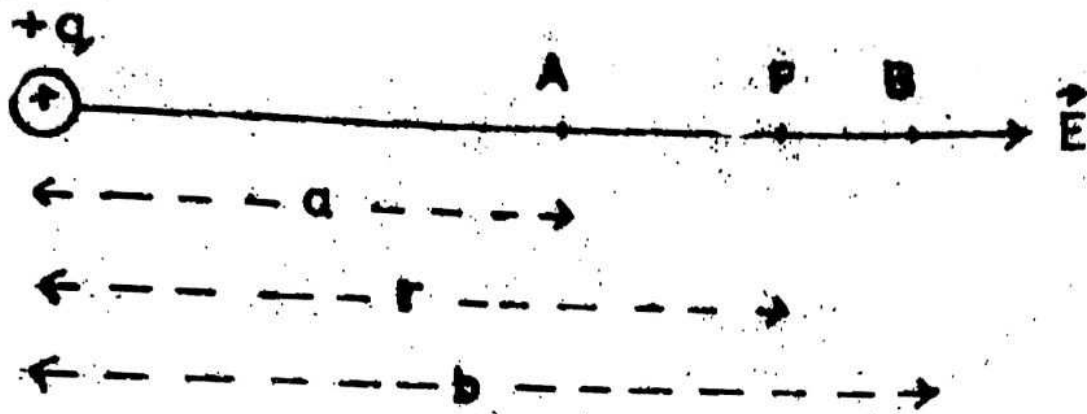


Fig. 27. Potential difference between two points.

If a positive test charge $+q_0$ is placed at the point P, distance r from the isolated charge, an external force shall be required to move the charge from P to A, causing increase in potential energy, while if the that charge is simply released at P it will move towards B under the electric force of the field causing decrease in potential energy. The points A and B are therefore said to be at different potentials. A is at a higher potential than B.

We want now to find the potential difference between A and B. Consider a small movement of the test charge towards A through a distance δr .

Thus the work done,

$$\delta w = - F \delta r \quad (3.8)$$

But when the test charge is at P,

$$F = \frac{1}{4\pi \epsilon_0} \cdot \frac{qq_0}{r^2} \quad (3.9)$$

$$\therefore \delta w = - \frac{qq_0}{4\pi \epsilon_0} \cdot \frac{\delta r}{r^2}$$

$$W = \int \delta W = - \frac{qq_0}{4\pi \epsilon_0} \int_b^a \frac{\delta r}{r^2} \quad (3.10)$$

$$= - \frac{qq_0}{4\pi \epsilon_0} \left(-\frac{1}{r} \right) \Big|_b^a$$

$$= \frac{qq_0}{4\pi \epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) \quad (3.11)$$

Therefore, the potential between A and B is given by V_{AB} , where

$$\begin{aligned} V_{AB} &= \frac{W_{BA}}{q_0} \\ &= \frac{qq_0}{4\pi \epsilon_0 q_0} (1/a - 1/b) \end{aligned} \quad (3.12)$$

$$= \frac{q}{4\pi\epsilon_0} (1/a - 1/b) \quad (3.12)$$

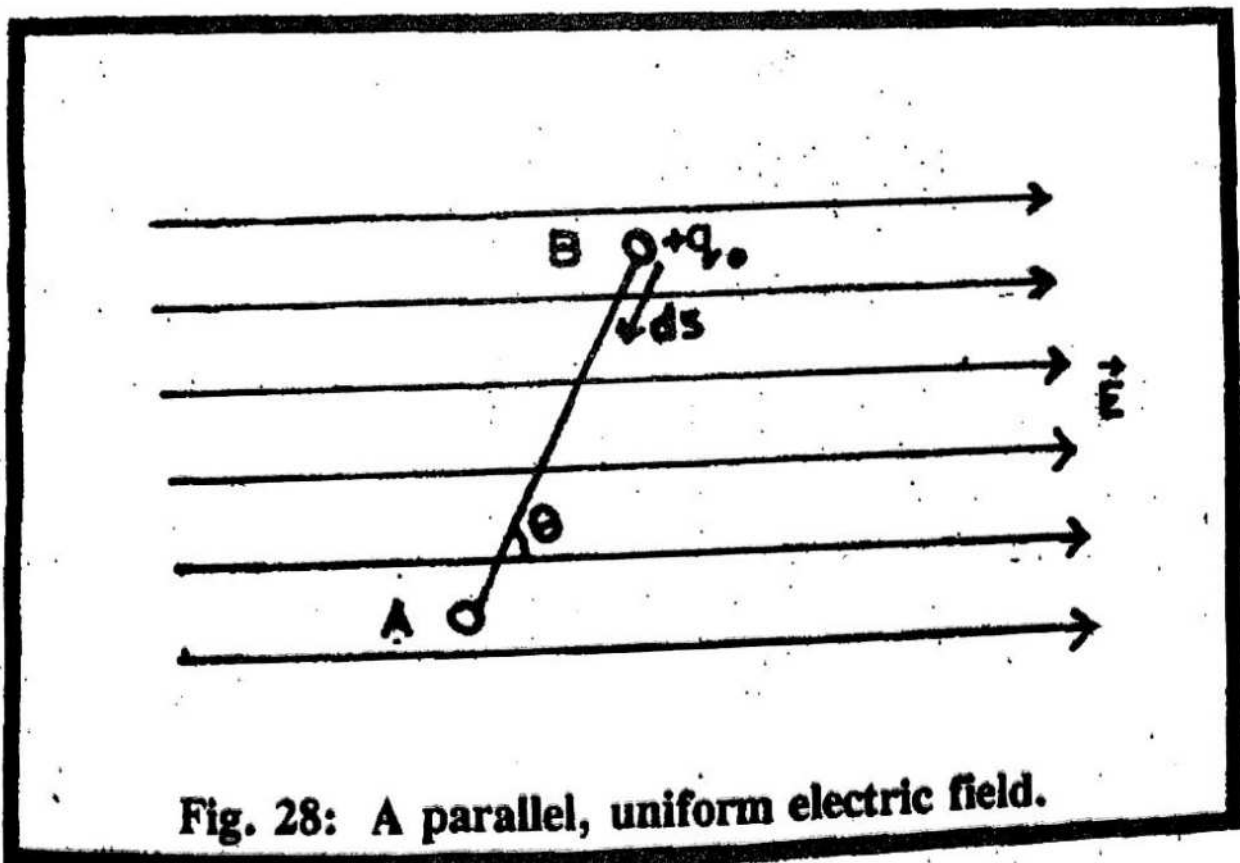
We observe that when b is removed to infinity, then the potential at A is given by

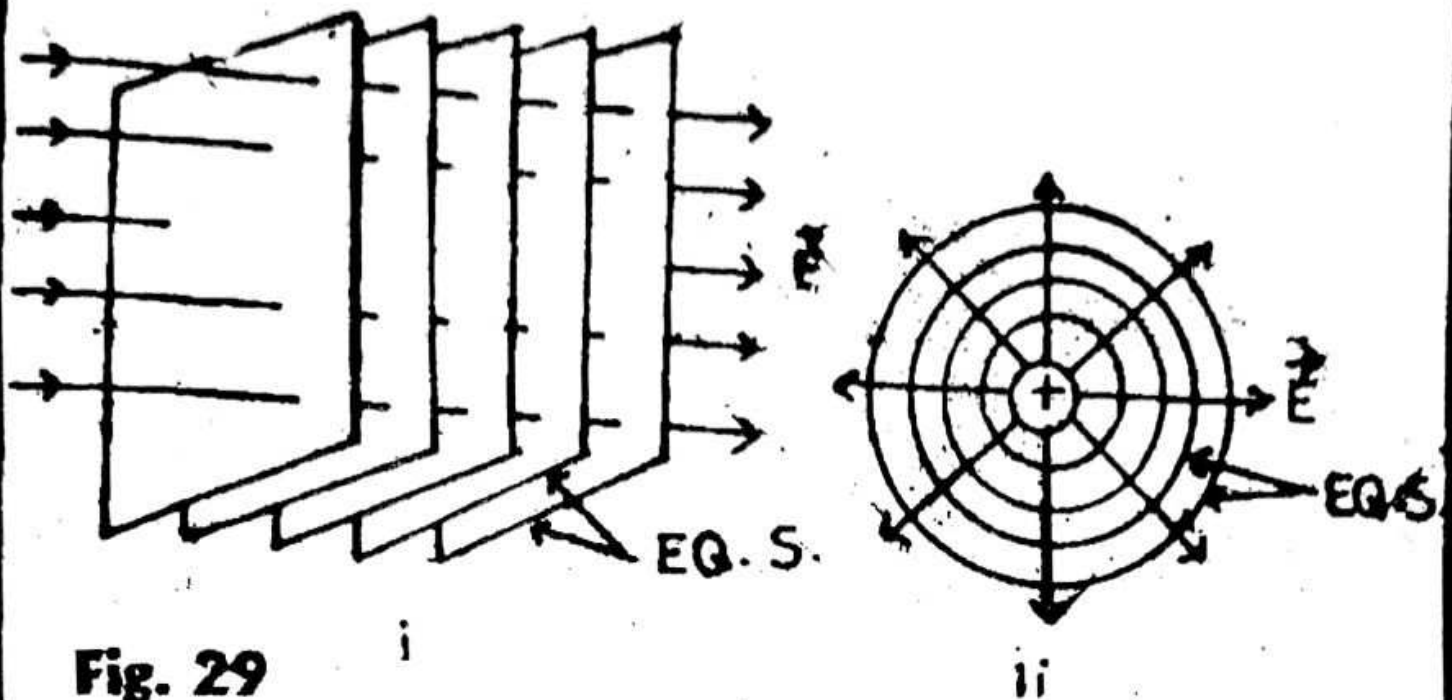
$$V_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a} \quad (3.14)$$

The distance b is said to be infinite if $1/b$ is negligible compared with $1/a$.

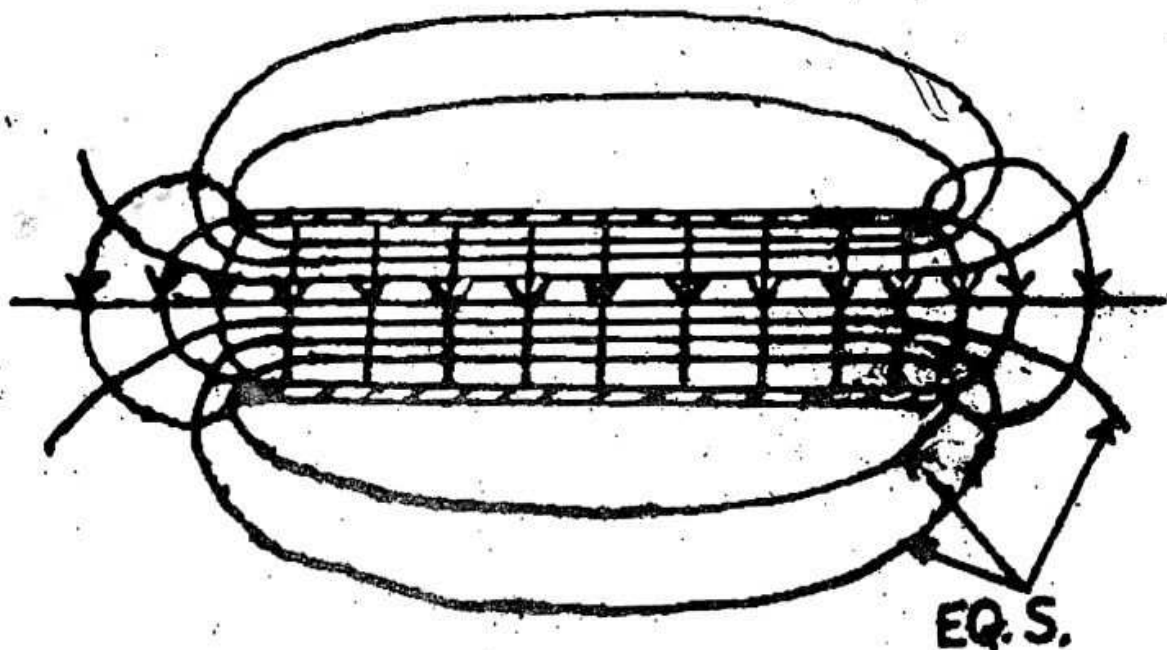
3.5 Equipotential Surfaces.

Consider the parallel and uniform electric field shown in fig. 28.

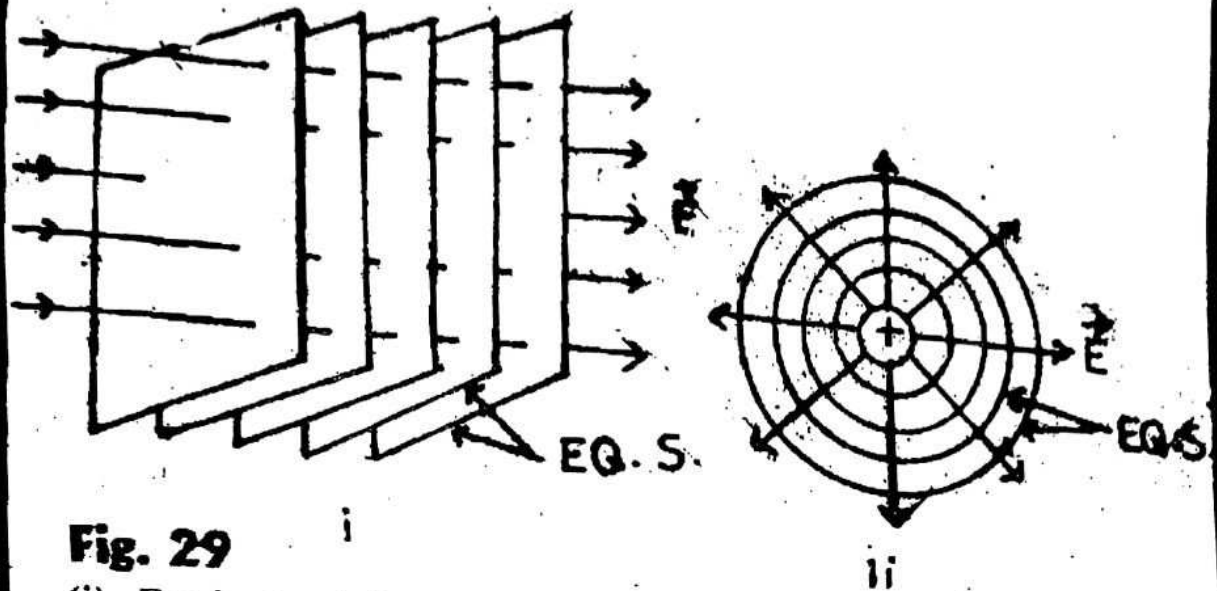


**Fig. 29**

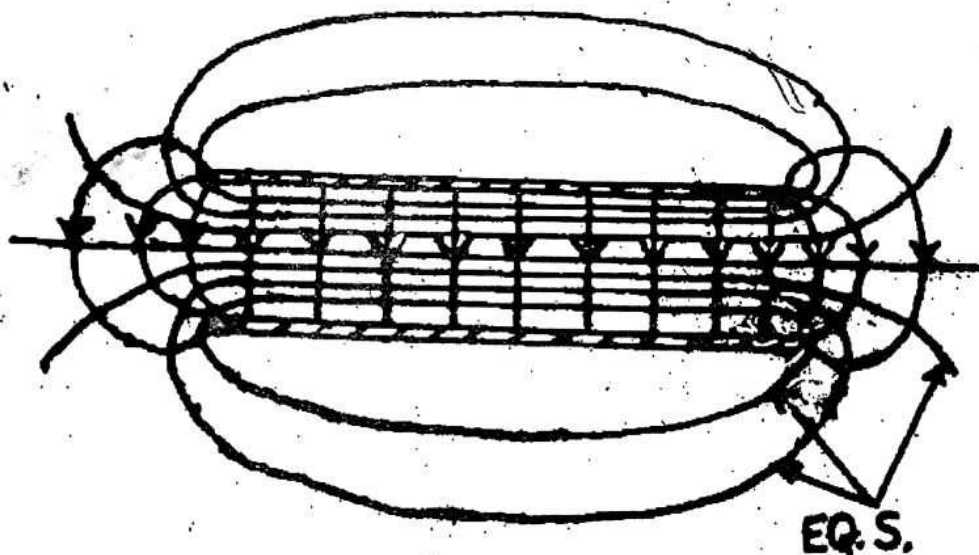
(i) Equipotential surfaces for a parallel field, and
 (ii) for a point charge.



(iii) Equipotential surfaces for field between two charged parallel plates.

**Fig. 29**

(i) Equipotential surfaces for a parallel field, and
 (ii) for a point charge.



(iii) Equipotential surfaces for field between two charged parallel plates.

3.6 Relation Between E And V.

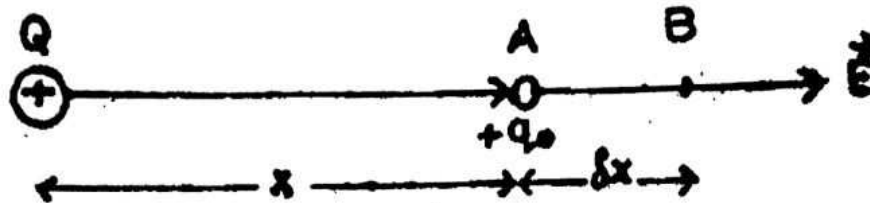


Fig. 30. Electric field due to a point charge, Q

Consider the electric field E due to the charge Q shown in fig. 30. Suppose we place a test charge $+q_0$ at the point A, distance x from the charge Q as shown. Then the electric force on the point charge is given by

$$F = q_0 E.$$

If $+q_0$ is moved a very small distance δx from A to B in the direction of E , then assuming that E is constant over this distance, the work done δw by the electric force F on $+q_0$ is given by

$$\delta W = F \delta x = q_0 E \delta x.$$

If p.d between B and A is δV , then

$$\delta V = - \frac{\delta w}{q_0} = - \frac{q_0 E \delta x}{q_0} = - E \delta x \quad (3.18)$$

The -ve sign is inserted to show that if displacement δx in the direction of E is taken to be positive, then δV is -ve (that is, potential energy is decreasing). On the other hand if the displacement is in the opposite direction to E , then δV is positive (that is, potential energy is increasing).

In the limits,

$$E = - \lim_{\delta x \rightarrow 0} \frac{\delta V}{\delta x} = - \frac{dV}{dx} \quad (3.19)$$

Here dV/dx is called potential gradient in x -direction. Thus the electric field at any point in this direction is the potential gradient at that point. It is a vector quantity and is measured in volts m^{-1} , which introduces another unit for E .

From the relationship in a uniform field, E is constant in magnitude and direction at all points.

$$\therefore \frac{dV}{dx} = \text{Const} \quad (3.20)$$

Thus $V = kx$ where k is the magnitude of the constant field.

Thus in such field, V changes steadily with x . Near the middle of oppositely charged parallel plates, E is uniform. If the separation between the plates is d , then

$$E = - \frac{V}{d} \text{ where } V \text{ is the p.d across the plates.}$$

We also note that when $E = 0$,

$$\frac{dV}{dx} = 0 \quad (3.21)$$

Hence $V = \text{Constant}$

This is the case for equipotential surfaces.

3.7 Potential Due To A Uniformly Charged Hollow Sphere.

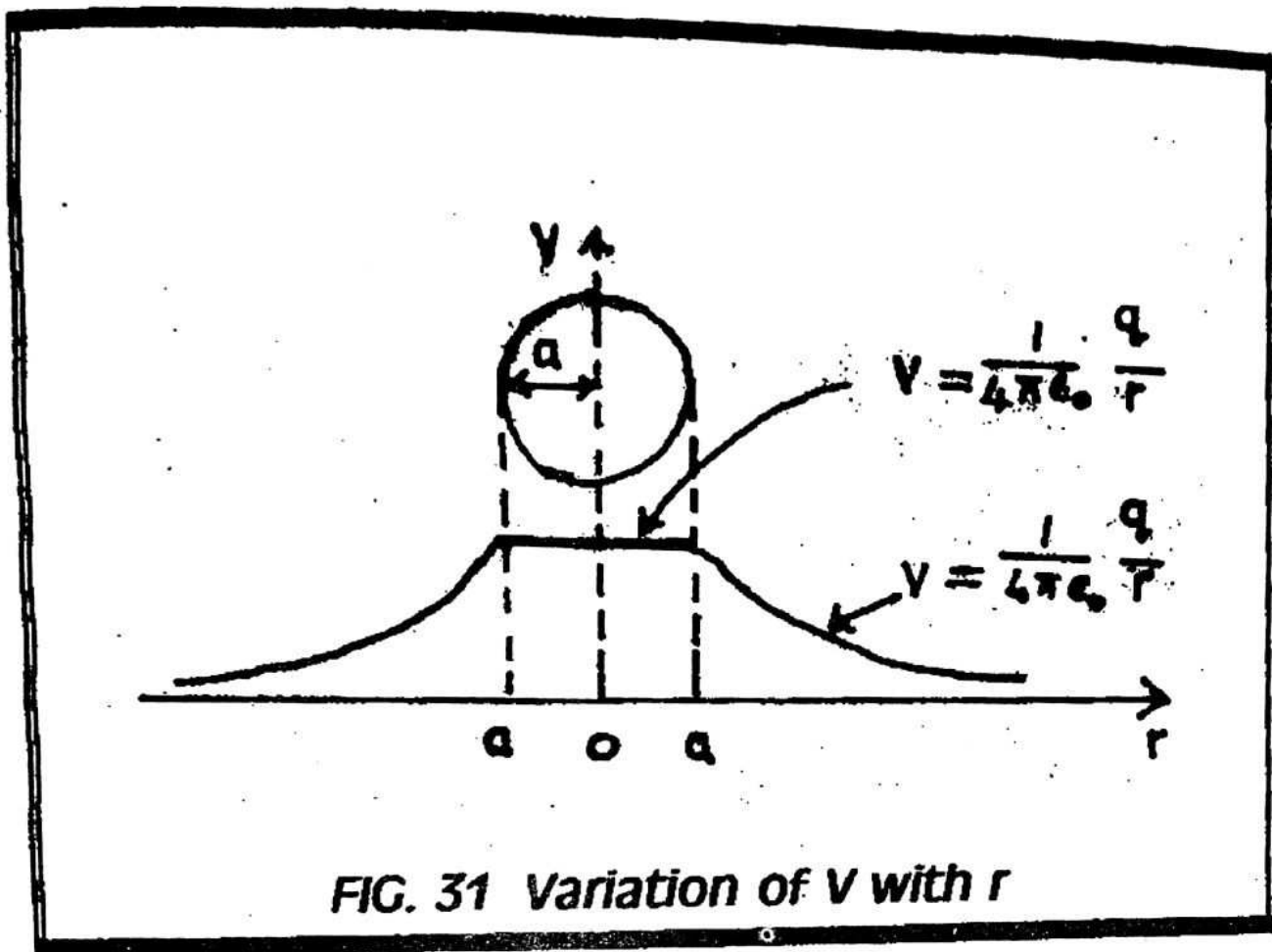
We have shown in 2.27 that for all points outside the hollow sphere, the electric field behaves exactly the same as if the whole charge were concentrated at the centre of the sphere, that is, the sphere behaves like a point charge. To find the electric potential therefore at any point outside the hollow sphere, we treat the sphere as if it is a point charge. Thus if a hollow sphere of radius a carries a charge $+q$ for example, then its potential at a point, distance r from its centre (where $r > a$), is given by

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad (3.22)$$

On the surface of the sphere,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a} \quad (3.23)$$

while inside the hollow sphere, V is constant (since $E = 0$) as shown before. Thus if we plot a graph of V against r we will obtain fig 31.



3.8 Effect Of Neighbouring Bodies On V .

The electric potential at a point in an electric field due to a charged body is not usually determined by consideration of the charge alone unless it is isolated as assumed in theory. In practice the potential is affected by the presence of other bodies, charged or uncharged, and the surrounding material. Thus the potential at a point near a positively charged body increases when another positively charged body approaches, and decreases when a negatively charged body approaches. For example let us consider bringing a neutral body BC near a positively charged body A as shown in fig 32. The variation of v with r for the positive charge both before and after an uncharged conductor BC is brought near shows how V is affected. Initially each point on BC has its own potential and BC is neutral. As it is brought near A, the potential

of B is raised and since BC is a conductor, electrons will flow from C to B (that is from lower to higher potential) and electrostatic induction has occurred. The induced negative charge at B then lowers the potential there, as well as at all points between B and A (including A), while the induced positive charge at C raises the potential at C and at points beyond C. Between B and C the potential is constant as shown.

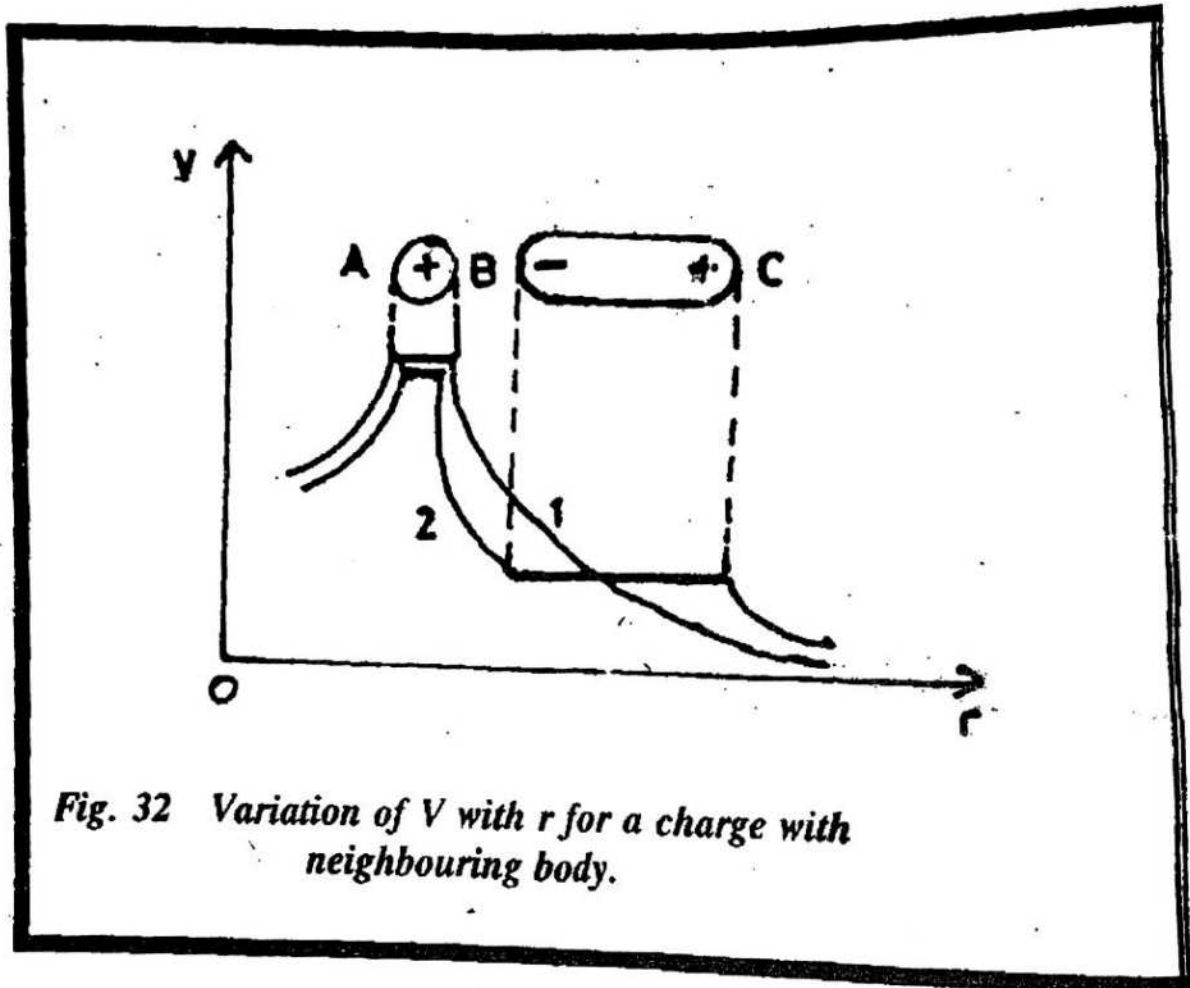


Fig. 32 Variation of V with r for a charge with neighbouring body.

3.9 SOLVED EXAMPLES

Example 3.1

Calculate the potential midway between two charges of magnitude $+1.6 \times 10^{-16}\text{C}$ and $-3.2 \times 10^{-16}\text{C}$, if the two charges are separated by a distance of 1.0m.

SOLUTION:

Let the charges be q_1 and q_2 , and their separation $2r$ as shown in fig. 33, and let P be the midpoint between the two charges as shown in fig. 33.

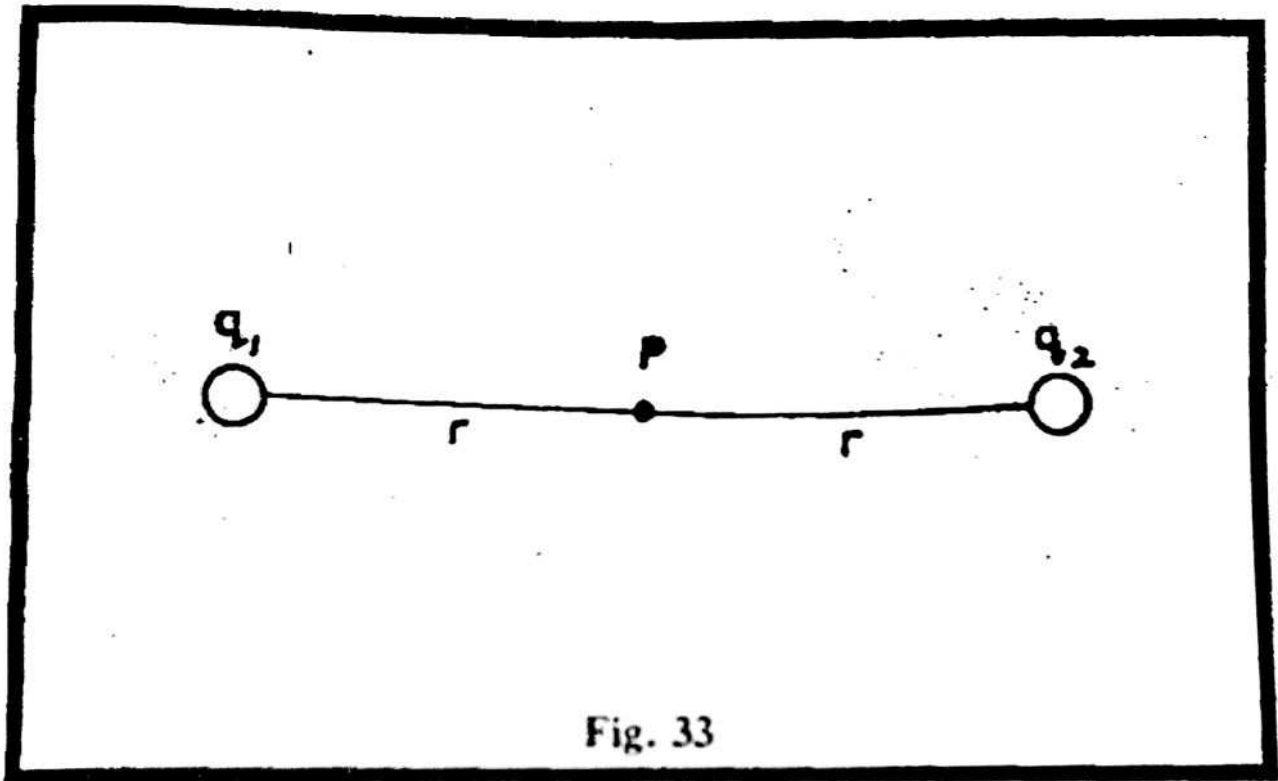


Fig. 33

$$\therefore V_P = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{r} \quad (3.32)$$

Substituting values we obtain,

$$\begin{aligned} V_P &= 9 \times 10^9 \times \left\{ \frac{(1.6 \times 10^{-16})}{0.5} \right\} + \left\{ \frac{(-3.2 \times 10^{-16})}{0.5} \right\} \\ &= \frac{9 \times 10^9 \times 10^{-16}}{0.5} \{ 1.6 - 3.2 \} \end{aligned}$$

$$= 9 \times 2 \times (-1.6) \times 10^{-7}$$

$$= -28.8 \times 10^{-7}$$

$$= \underline{-2.88 \times 10^{-6} \text{ volts}}$$

Example 3.2.

Calculate the resultant potential at the centre of the square of charge shown in fig. 34 where

$$a = 10\text{cm and}$$

$$q = 1.6 \times 10^{-19}\text{C.}$$

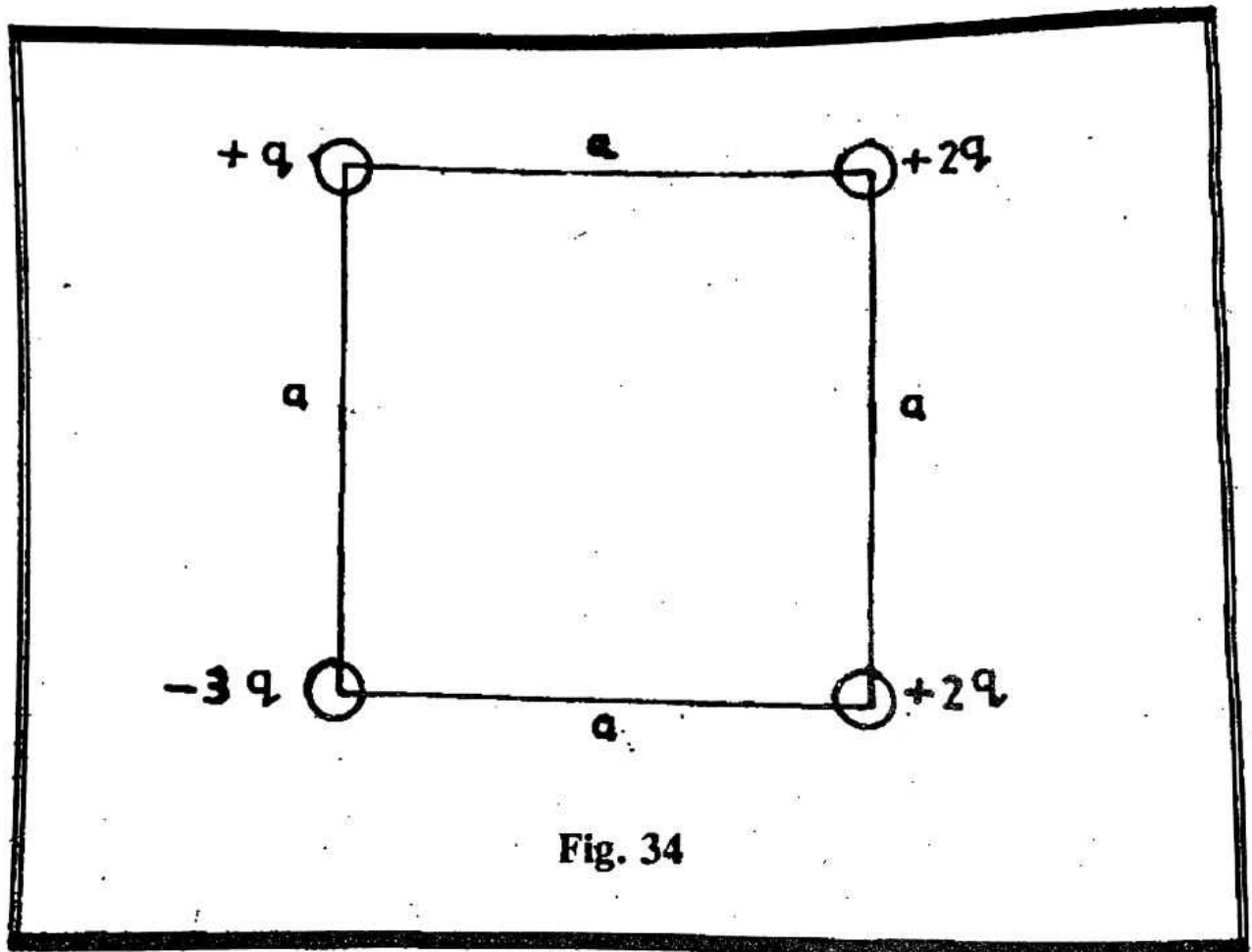


Fig. 34

SOLUTION.

As shown in fig. 35, let the diagonals intersect at P. Thus P is the centre of the square. If the distance from each charge to P is given by r , then

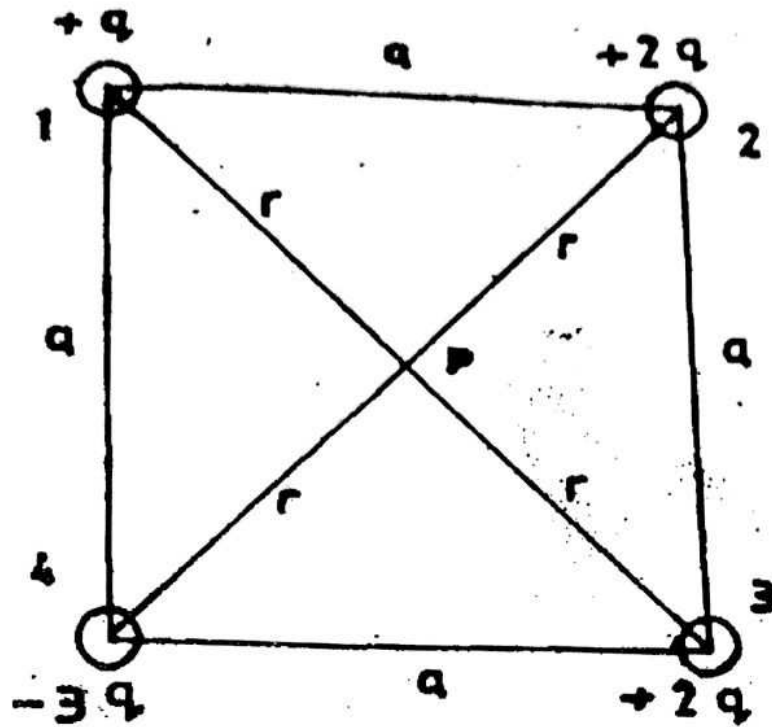


Fig. 35.

$$V_p = V_1 + V_2 + V_3 + V_4 \quad (3.25)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(+q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(+2q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(+2q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(-3q)}{r}$$

$$= \frac{1}{4\pi\epsilon_0 r} (q + 2q + 2q - 3q)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{r} \quad (3.26)$$

To substitute, we note that

$$(2r)^2 = a^2 + a^2$$

$$\therefore r^2 = \frac{2a^2}{4} = \frac{a}{2}$$

$$\therefore r = \sqrt{\frac{a}{2}} = \frac{a\sqrt{2}}{2}$$

$$= \frac{10 \times 1.414}{2}$$

$$= 7.07 \text{ cm}$$

$$\therefore V_p = \frac{9 \times 10^9 \times 2 \times 1.6 \times 10^{-19}}{7.07}$$

$$= \frac{9 \times 2 \times 1.6 \times 10^{-10}}{7.07}$$

$$= \underline{\underline{4.07 \times 10^{-10} \text{ volts.}}}$$

Example 3.3

The potential due to a point charge is given by

$$V = \frac{1}{4 \pi \epsilon_0} \cdot \frac{1}{r} \quad (3.27)$$

Find the electric field E_x .

SOLUTION.

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r}$$

$$\therefore E = -\frac{dv}{dr} = -\frac{d}{dr} \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r} \right) \quad (3.28)$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{d}{dr} \left(\frac{1}{r} \right)$$

$$\text{But } r^2 = x^2 + y^2 + z^2 \quad (3.29)$$

$$\therefore r = (x^2 + y^2 + z^2)^{1/2} \quad (3.30)$$

$$\therefore E_x = -\frac{1}{4\pi\epsilon_0} \frac{d}{dx} \left[\frac{1}{(x^2 + y^2 + z^2)^{1/2}} \right] \quad (3.31)$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{d}{dx} \left[(x^2 + y^2 + z^2)^{-1/2} \right] \quad (3.32)$$

$$= -\frac{1}{4\pi\epsilon_0} \left(-\frac{1}{2} \right) (x^2 + y^2 + z^2)^{-3/2} 2x \quad (3.33)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot x (x^2 + y^2 + z^2)^{-3/2} \quad (3.34)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{x}{(x^2 + y^2 + z^2)^{3/2}} \quad (3.35)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{x}{(r^2)^{3/2}} \quad (3.36)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{x}{r^3} \quad (3.37)$$

Example 3.4

Two equal and opposite charges each of magnitude $1.6 \times 10^{-16} \text{C}$ are kept 10cm apart. calculate the electric field and potential at the centre of the dipole, and at points 10cm away from the centre, along the dipole axis and the medium axis.

SOLUTION.

Let the charges of the dipole be $+q$ and $-q$, with separation $2a$ as shown in fig. 36.

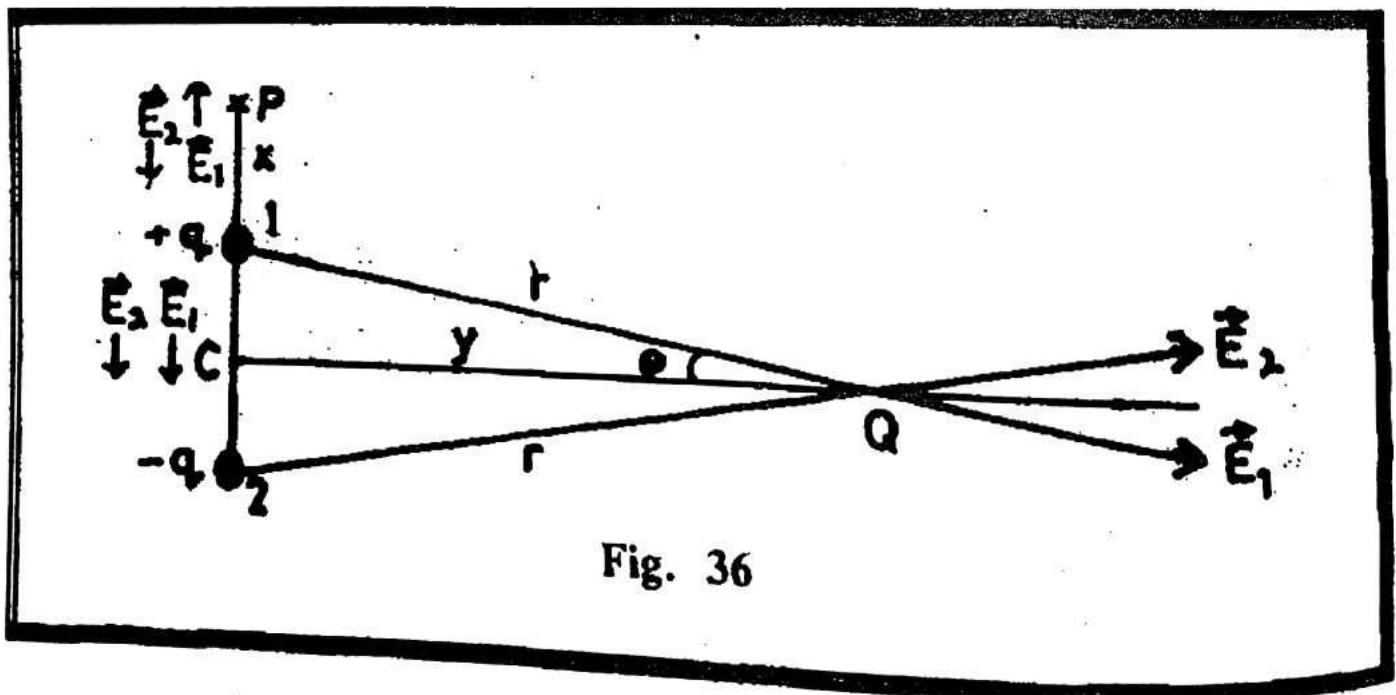


Fig. 36

Let us now consider the midpoint C of the dipole; a point P, distance x from the charge $+q$ along the dipole axis, and a point Q, distance y along the medium axis.

(i) *Midpoint of the dipole.*

At C, the electric field due to the charges will be in the same direction as shown.

Thus the resultant field at C is given by

$$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 \quad (3.38)$$

$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2} \quad (3.39)$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-16}}{(5 \times 10^{-2})^2}$$

$$= \frac{14.4 \times 10^{-1}}{25 \times 10^{-4}}$$

$$= 0.576 \times 10^{-3}$$

$$= \underline{5.76 \times 10^{-4} \text{ NC}^{-1}}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2} \quad (3.40)$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-16}}{(5 \times 10^{-2})^2}$$

$$= \underline{5.76 \times 10^{-4} \text{ NC}^{-1}}$$

Since E_1 and E_2 are along the same line and direction, then the magnitude of their resultant is given by

$$E_o = E_1 + E_2$$

$$= 5.76 \times 10^{-4} \text{ NC}^{-1} + 5.76 \times 10^{-4} \text{ NC}^{-1}$$

$$= 11.52 \times 10^{-4}$$

$$= \underline{1.152 \times 10^{-3} \text{ NC}^{-1}}$$

Resultant Potential at O.

$$V_o = V_1 + V_2 \quad (3.41)$$

$$= \frac{1}{4\pi\epsilon_o} \frac{(+q)}{a} + \frac{1}{4\pi\epsilon_o} \frac{(-q)}{a} \quad (3.42)$$

$$= \frac{1}{4\pi\epsilon_o a} (q - q) \quad (3.43)$$

$$= \underline{0 \text{ volts.}}$$

(ii) Point along the dipole axis.

At P, the field E_1 due to $+q$ points away from the charge, while E_2 due to $-q$ points towards the charge. Therefore resultant field at that point, E_p is given by

$$\vec{E}_p = \vec{E}_1 - \vec{E}_2 \quad (3.44)$$

$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2} \quad (3.45)$$

$$E_2 = \frac{1}{4\pi\epsilon_2} \cdot \frac{q}{(x+2a)^2} \quad (3.46)$$

Again since E_1 and E_2 are along the same line but opposite in directions the magnitude of their resultant is given by

$$E_p = E_1 - E_2. \quad (3.47)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(x+2a)^2} \quad (3.48)$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{x^2} - \frac{1}{(x+2a)^2} \right] \quad (3.49)$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{x^2 + 4xa + 4a^2 - x^2}{x^2 (x^2 + 4xa + 4a^2)} \right] \quad (3.50)$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{4xa + 4a^2}{x^4 + 4x^3a + 4x^2a^2} \right) \quad (3.51)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4a(x+a)}{x^2 (x^2 + 4xa + 4a^2)} \quad (3.52)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4a(x+a)}{x^2 (x+2a)^2} \quad (3.53)$$

Substituting values $a = 5\text{cm}$ and $r = 5\text{cm}$, we obtain,

$$\begin{aligned}
 E_p &= \frac{9 \times 10^9 \times 1.6 \times 10^{-16} \times 4 \times 5 \times 10^{-2} (5 \times 10^{-2} + 5 \times 10^{-2})}{(5 \times 10^{-2})^2 (5 \times 10^{-2} + 2 \times 5 \times 10^{-2})^2} \\
 &= \frac{9 \times 10^9 \times 1.6 \times 10^{-16} \times 4 \times 5 \times 10^{-2} \times 10 \times 10^{-2}}{25 \times 10^{-4} [(5 + 10)^2 \times 10^{-4}] } \\
 &= \frac{9 \times 10^9 \times 1.6 \times 10^{-16} \times 200 \times 10^{-4}}{25 \times 10^{-4} \times 225 \times 10^{-4}} \\
 &= 0.512 \times 10^{-3} \\
 &= \underline{5.12 \times 10^{-4} \text{ NC}^{-1}}
 \end{aligned}$$

To calculate V_p ,

$$V_p = V_1 + V_2 \quad (3.54)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{(+q)}{x} + \frac{1}{4\pi\epsilon_0} \frac{(-q)}{(x+2a)} \quad (3.55)$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{x} - \frac{1}{(x+2a)} \right] \quad (3.56)$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{x + 2a - x}{x(x + 2a)} \right] \quad (3.57)$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{2a}{x^2 + 2xa} \right) \quad (3.58)$$

$$= \frac{2aq}{4\pi\epsilon_0} \left(\frac{1}{x^2 + 2xa} \right) \quad (3.59)$$

Substituting we obtain,

$$V_p = \frac{2 \times 9 \times 10^9 \times 5 \times 10^{-3} \times 1.6 \times 10^{-18}}{(5 \times 10^{-2})^2 + 2 \times 5 \times 10^{-2} \times 5 \times 10^{-2}}$$

$$= \frac{2 \times 9 \times 10^9 \times 5 \times 10^{-3} \times 1.6 \times 10^{-18}}{25 \times 10^{-4} + 50 \times 10^{-4}}$$

$$= \frac{90 \times 1.6 \times 10^{-9}}{75 \times 10^{-4}}$$

$$= \underline{1.92 \times 10^{-4} \text{ volts.}}$$

(iii) Point Q along the median axis.

Let the distance of Q from each charge be r.

$$\therefore r^2 = y^2 + a^2 \quad (3.60)$$

$$= (10 \times 10^{-2})^2 + (5 \times 10^{-2})^2$$

$$= 100 \times 10^{-4} + 25 \times 10^{-4}$$

$$= 125 \times 10^{-4}$$

$$\therefore r = 5\sqrt{5} \times 10^{-2} \text{ cm.}$$

Also, let the angle between r and the median axis be θ .

Then $\cos \theta = y/r$ while $\sin \theta = a/r$. The electric field contributions at Q of the dipole charges are E_1 and E_2 as shown, where each field makes angle θ with the median axis.

$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad (3.61)$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-16}}{125 \times 10^{-4}}$$

$$= 0.1152 \times 10^{-3}$$

$$= \underline{1.152 \times 10^{-4} \text{ NC}^{-1}}$$

$$\text{Also } E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad (3.62)$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-16}}{125 \times 10^{-4}}$$

$$= 1.152 \times 10^{-4} \text{ NC}^{-1}$$

To find the resultant field, E_0 , we resolve the fields into x and y directions and apply the relation

$$E_0^2 = E_x^2 + E_y^2 \quad (3.63)$$

$$\text{Thus } E_x = E_1 \cos \theta + E_2 \cos \theta \quad (3.64)$$

$$\text{But } E_1 = E_2$$

$$\therefore E_x = 2 E_1 \cos \theta \quad (3.65)$$

$$= 2 E_1 \cdot y/r$$

$$\frac{2 \times 1.152 \times 10^{-4} \times 10 \times 10^{-2}}{5\sqrt{5} \times 10^{-2}}$$

$$= \frac{2 \times 11.52 \times 10^{-4}}{11.18}$$

$$= \underline{2.06 \times 10^{-4} \text{ NC}^{-1}}$$

$$E_y = E_2 \sin \theta - E_1 \sin \theta \quad (3.66)$$

$$= 0.$$

$$\therefore E_p^2 = E_x^2 + E_y^2 \quad (3.67)$$

$$= E_x^2 + 0$$

$$= E_x^2$$

$$\therefore E_p = E_x = \underline{2.06 \times 10^{-4} \text{ NC}^{-1}}$$

Calculation of V_o

$$V_o = V_1 + V_2 \quad (3.68)$$

$$= \frac{1}{4\pi\epsilon_o} \cdot \frac{(+q)}{r} + \frac{1}{4\pi\epsilon_o} \frac{(-q)}{r} \quad (3.69)$$

$$= \frac{1}{4\pi\epsilon_o r} (q - q) \quad (3.70)$$

$$= 0$$

Hence V is zero along the median axis of a dipole. This is generally true of the median plane of a dipole. Hence the median plane of a dipole is an equipotential surface.

3.10 PROBLEMS

3.10.1

Two horizontal metal plates of separation 4cm are placed in a vacuum, with the upper plate maintained at a positive potential relative to the lower plate. If the field strength between the plates is $2.5 \times 10^5 \text{ Vm}^{-1}$ calculate the p.d between them. If an electron of charge $1.6 \times 10^{-19} \text{ C}$ and mass $9.1 \times 10^{-31} \text{ Kg}$ is liberated from the lower plate, what will be its speed and kinetic energy on reaching the upper plate?

3.10.2

A cube of side 2cm has an electron placed on each of its six corners. Calculate the resultant potential at its centre (electronic charge $= -1.6 \times 10^{-19} \text{ C}$).

3.10.3

A hydrogen atom consists of a proton and an electron. Suppose their separation is about $0.5 \times 10^{-8} \text{ cm}$, calculate:

- (i) *the electric potential at distance $0.5 \times 10^{-8} \text{ cm}$ from the proton.*
- (ii) *the potential energy of the electron*
- (iii) *the amount of energy required to remove the electron from the atom assuming that the electron is at rest. (electronic charge $= -1.6 \times 10^{-19} \text{ C}$; protonic charge $= + 1.6 \times 10^{-19} \text{ C}$).*

3.10.4

The potential at an axial point for a charged disc is found to be

$$V = \frac{\sigma}{2\epsilon_0} [(a^2 + r^2)^{1/2} - r]$$

Show that E at axial points is given by

$$E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{r}{(a^2 + r^2)^{1/2}} \right)$$

3.10.5 Three charges $-2\mu\text{C}$, $1.6\mu\text{C}$ and $4.8\mu\text{C}$ are placed respectively at the corners A, B, C of an equilateral triangles of side 10 cm. Calculate the electric field and potential at a point midway along BC. What is the direction of the field?

3.10.6 A charge of $20\mu\text{C}$ is uniformly distributed across a solid spherical conductor of radius, 15 cm. Calculate the field and potential at a point 10 cm from the centre of the sphere.

3.10.7 A thin nonconducting rod of length l carries a charge Q uniformly distributed along it. Show that E at a point P distance y along the perpendicular bisector of the rod is given by

$$E = \frac{Q}{2\pi\epsilon_0 y} \frac{1}{(l^2 + 4y^2)^{1/2}}$$

Show that as $l \rightarrow \infty$, E reduces to

$$E = \frac{\lambda}{2\pi\epsilon_0 y} \text{ where } \lambda \text{ is the charge per unit length.}$$

3.10.8

Show that the components of $\mathbf{E}$ due to a dipole at an arbitrary distant point are given by

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{3Pxy}{(x^2 + y^2)^{5/2}}$$

$$E_y = \frac{1}{4\pi\epsilon_0} \frac{P(2y^2 - x^2)}{(x^2 + y^2)^{5/2}}$$

where x and y are the co-ordinates of the arbitrary point, and P is the dipole moment given as $P = 2aq$ where a is half of the dipole separation. Hence determine expressions for V_x and V_y of the dipole.

CAPACITORS AND DIELECTRICS

4.1. The Capacitor

A capacitor is a device for storing electric charge. Basically it is made up of two parallel conductors called plates which are usually separated by an insulator like air, oil, mica, glass etc. The insulator is called a dielectric and plays a very important part in the determination of the capacitor parameters. The general symbol for a capacitor is $\text{—}| \quad | \text{—}$ whatever its form, though the symbol $\text{—}| \text{—}$ for electrolytic capacitor carries +ve and -ve signs to differentiate the electrodes

4.2. Capacitance

A conductor, eg a hollow conducting sphere, can be mounted on an insulator and charges added and stored in it. Any such system that is able to store charges is called a capacitor.

The capacitance C of such capacitor is defined as the charge required to cause a unit change in its potential. Alternatively, it can simply be defined as the ratio of the charge on a capacitor to the p.d. across its plates. Thus $C = Q/V$ where Q is the charge on the conductor or capacitor, and V is its potential. If Q is in Coulombs (C) and V is in volts (V), then C is in coulomb per volt (CV^{-1}) or farad (F). The smaller units of the farad are microfarad ($\mu\text{F} = 10^{-6} \text{ F}$); Picofarad or Micro farad (IPF = $1\mu\mu\text{F} = 10^{-12} \text{ F}$)



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N400 only

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or simply search starberecharge
or email
starberecharge@gmail.com

4.3. Capacitance Of An Isolated Conducting Sphere.

The p.d. for a sphere of radius a , which carries a charge Q , in a medium of permittivity ϵ , is given by

$$V = \frac{1}{4\pi\epsilon} \frac{Q}{a} \quad (4.1)$$

$$\text{But } C = Q/V$$

$$\therefore C = 4\pi\epsilon a. \quad (4.2)$$

The capacitance of a sphere is therefore directly proportional to its radius. We note from the last equation that another unit of permittivity ϵ is farad per meter

$$(\text{F m}^{-1}), \text{ and } \epsilon = \frac{C}{4\pi a} \text{ for the sphere.}$$

4.4. Parallel Plate Capacitor

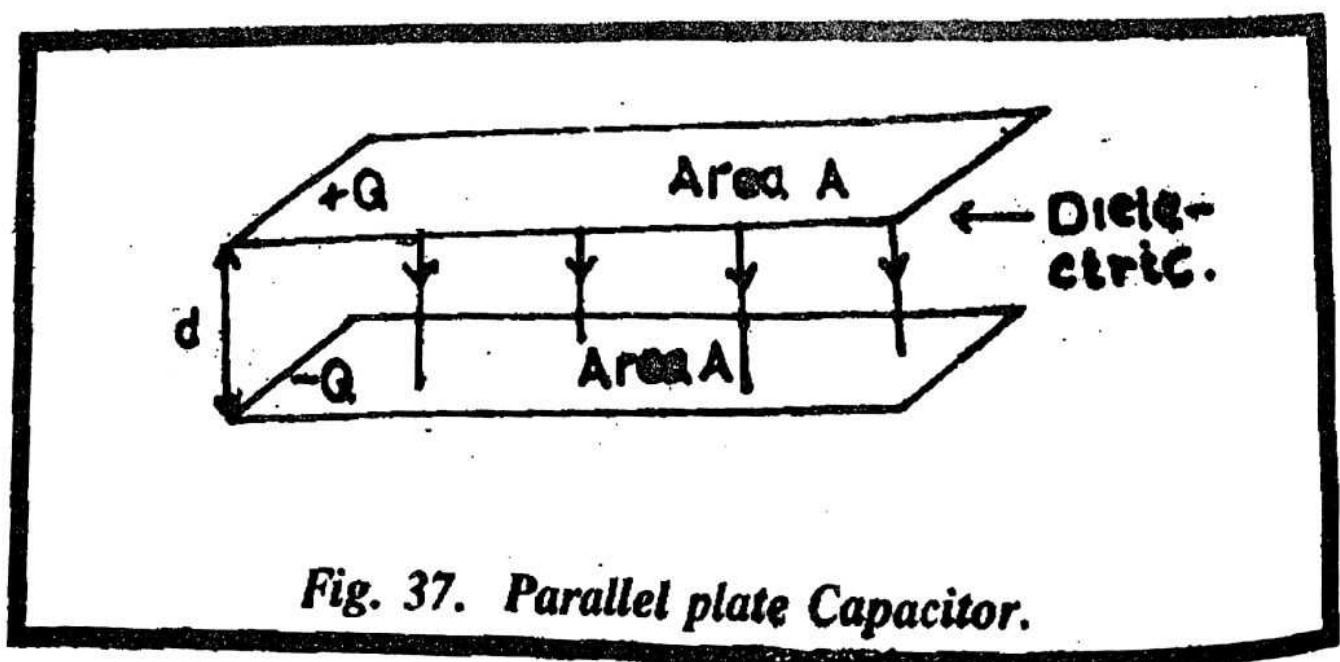


Fig. 37. Parallel plate Capacitor.

Consider Fig. 37. The upper plate carries charge $+Q$ and the lower plate carries charge $-Q$, and the area of each plate is A . Suppose the separation between the plates is d , and contains a medium of permittivity ϵ . The charge density σ of each plate is given by

$$\sigma = \frac{Q}{A} \text{ (Cm}^{-2}\text{)} \quad (4.3)$$

Assuming that the electric field E is uniform between the plates.

$$\text{Then } E = \frac{\sigma}{\epsilon} = \frac{Q}{\epsilon A} \quad (4.4)$$

But if V is the p.d. between the plates,

$$\text{then } E = V/d \text{ (Vm}^{-1}\text{)} \quad (4.5)$$

$$\therefore Q/\epsilon A = V/d$$

$$\therefore C = Q/V = \epsilon A/d \quad (4.6)$$

We note that this equation assumes uniformity of E between the plates. At the edges therefore where there are distortions, the equation does not strictly apply.

4.5 Factors That Affect Capacitance Of A Parallel Plate Capacitor.

The capacitance of a parallel plate capacitor depends on three main factors:

- (i) The distance between the plates. From the last equation, we see that the capacitance of a parallel plate capacitor is inversely proportional to the plate separation. Thus when the separation increases, the capacitance decreases.

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If we connect one plate of the capacitor to earth, and connect the other to an electroscope, then the introduction of some charge to the plate will cause the leave of the electroscope to diverge in proportion to the p.d. across the plates. If this charge is kept constant and the separation between the plates is increased, the divergence of the leave will increase indicating an increase in p.d. since $C = Q/V$, then the capacitance of the system has correspondingly decreased. This confirms that capacitance is inversely proportional to the plate separation.

(ii) The dielectric between the plates. If again the charge in the plate is kept constant, it is found that if a denser dielectric than air like mica or glass is inserted inbetween the plates, the divergence of the leave will reduce indicating decrease in p.d. which implies increase in capacitance. Thus the capacitance of a parallel plate capacitor is directly proportional to the dielectric density of the medium between the plates.

(iii) Area of plates. If the separation between the plate is kept constant and the plate area increased, the divergence of the leave will decrease indicating decrease in p.d. which implies increase in capacitance. In fact the capacitance is directly proportional to the area of the plate as seen from the last equation.

4.6 Dielectric Constant Or Relative Permittivity.

We have seen that the capacitance of a capacitor with air dielectric increases when a denser dielectric is inserted inbetween the plates. If its capacitance when the separation between the plates is vacuum is C_0 , and that when it is filled with a dielectric material is C then the dielectric constant or relative permittivity when dielectric

material is inserted is given as

$$\epsilon_r = C/C_0$$

For the parallel plate capacitor, (4.7)

$$\epsilon_r = \frac{C}{C_0} = \frac{\epsilon A/d}{\epsilon_0 A/d} = \frac{\epsilon}{\epsilon_0},$$
(4.8)

$$\therefore \epsilon = \epsilon_r \epsilon_0$$
(4.9)

where ϵ is the permittivity of the dielectric material and ϵ_0 is that of vacuum. Thus if dielectric constant ϵ_r is known for a parallel plate capacitor, then

$$C = \frac{\epsilon A}{d} \quad \text{where } \epsilon \text{ is the permittivity of the dielectric.}$$

$$= \frac{\epsilon_0 \epsilon_r A}{d}$$
(4.10)

Generally the dielectric constant or relative permittivity of a capacitor with a dielectric is the ratio of the capacitance of the capacitor with the dielectric to the capacitor without the dielectric (that is with vacuum separation). Alternatively, it is the ratio of the permittivity of the dielectric to that of vacuum (or air approximately).

Relative permittivity has no units, and the relative permittivity of some substances are listed below.

Table 1: Relative Permittivity of some dielectric substances

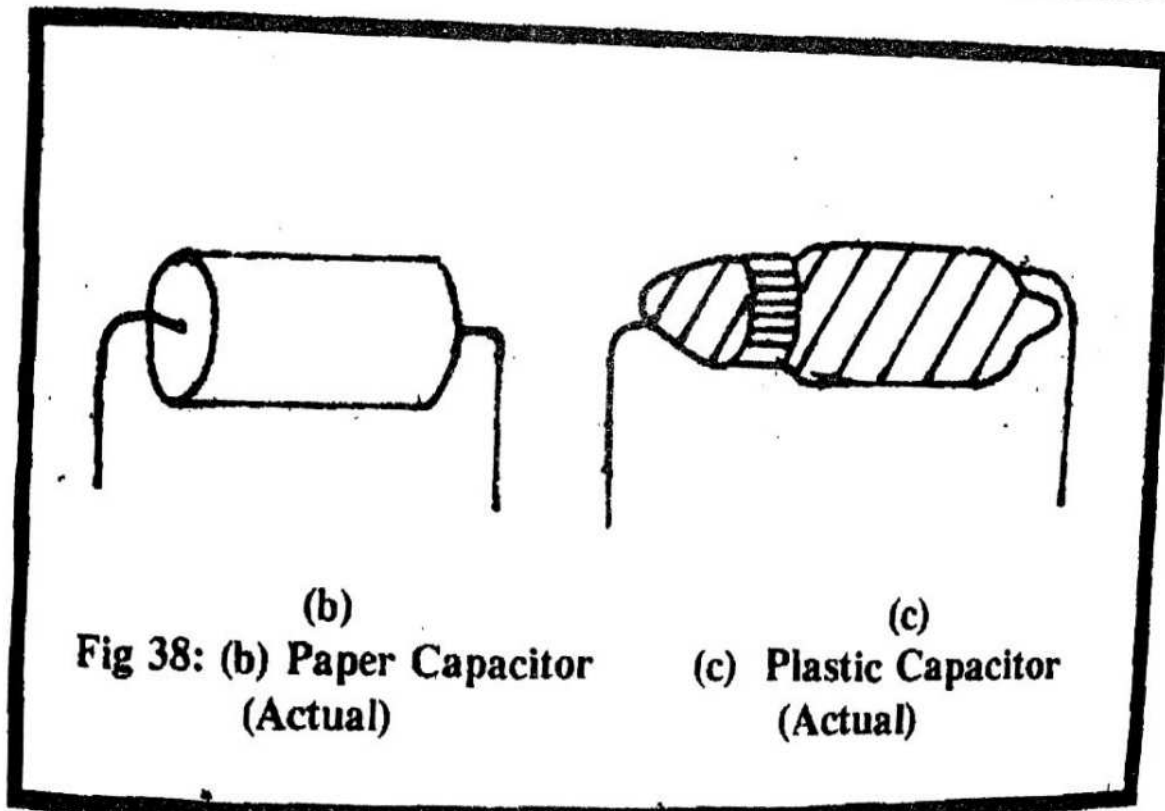
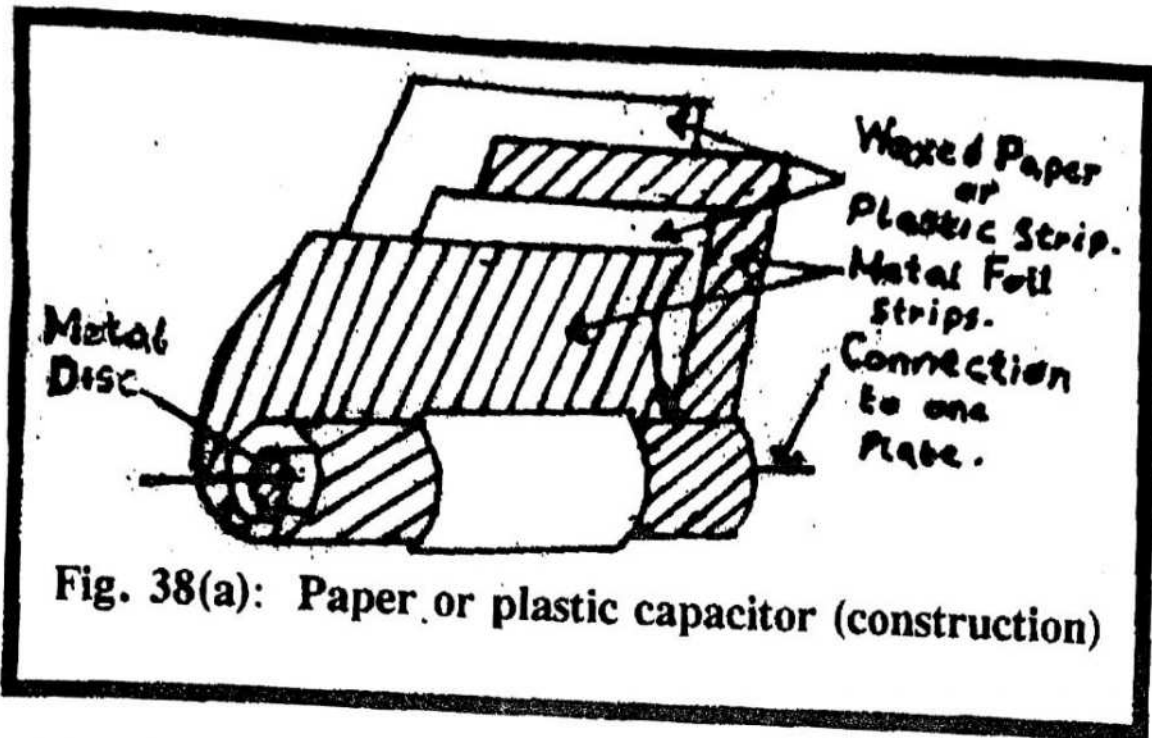
Dielectric	Rel. Permittivity ϵ_r
Vacuum	1.00000
Air (at STP)	1.00054
Teflon	2.1
Polytene	2.3
Perspex	2.6
Polystyrene	2.6
Amber	2.7
Paper (waxed)	2.7
Paper	3.5
Fused quartz	3.8
Pyranol oil	4.5
Pyrex glass	4.5
Bakelite	4.8
Porcelain	6.5
Mica	7.0
Water (pure)	80.0
Barium Titanate	1200.0

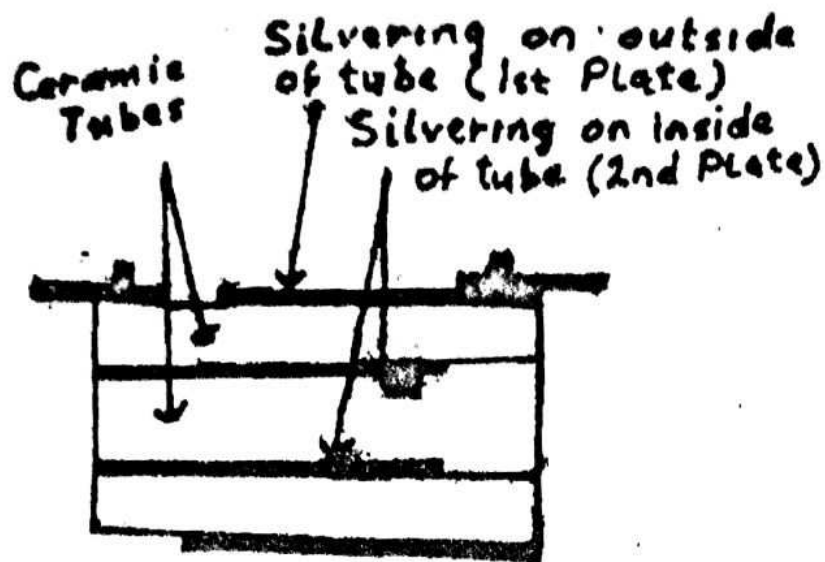
It is important to note from this table the very high value for dielectric constant of Barium Titanate. Since capacitance is directly proportional to permittivity, this material is therefore quite

adequate for the construction of high value capacitors.

4.7 Types Of Capacitors.

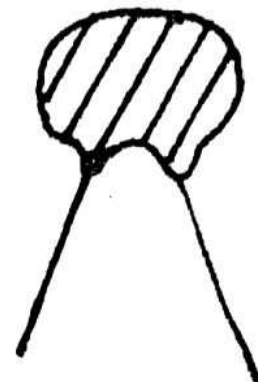
There are many types of capacitors some of which are shown from fig. 38 to 44.





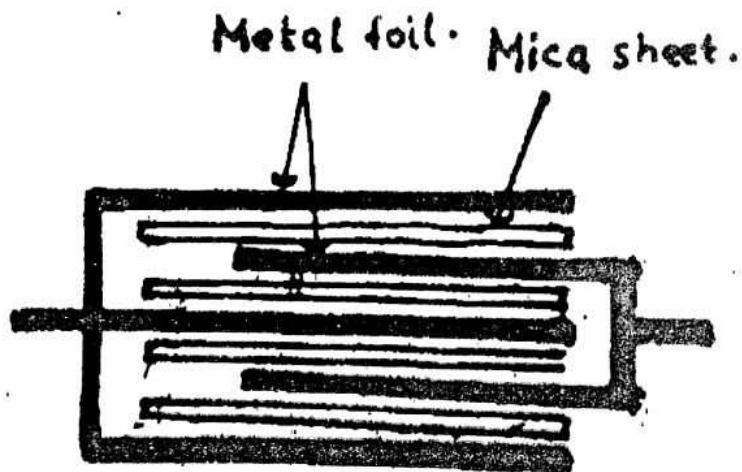
(a)

Fig 39: (a) Ceramic Capacitor (Construction).



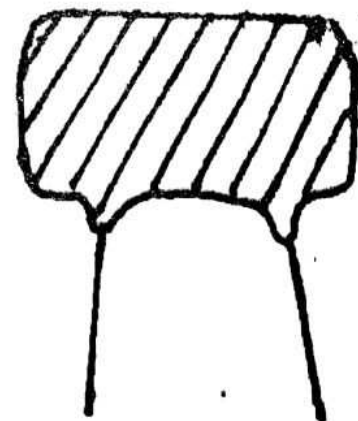
(b)

(b) Actual.



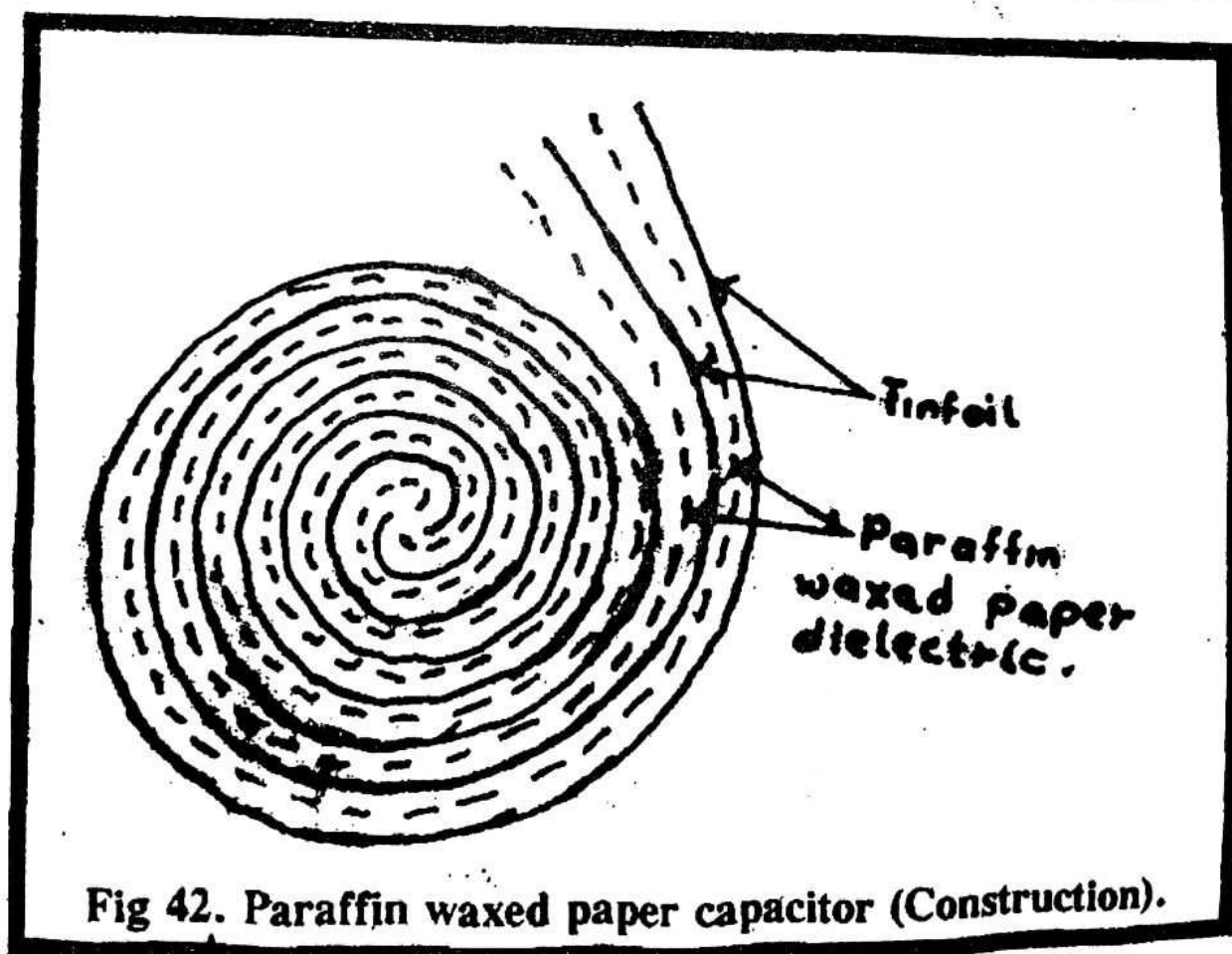
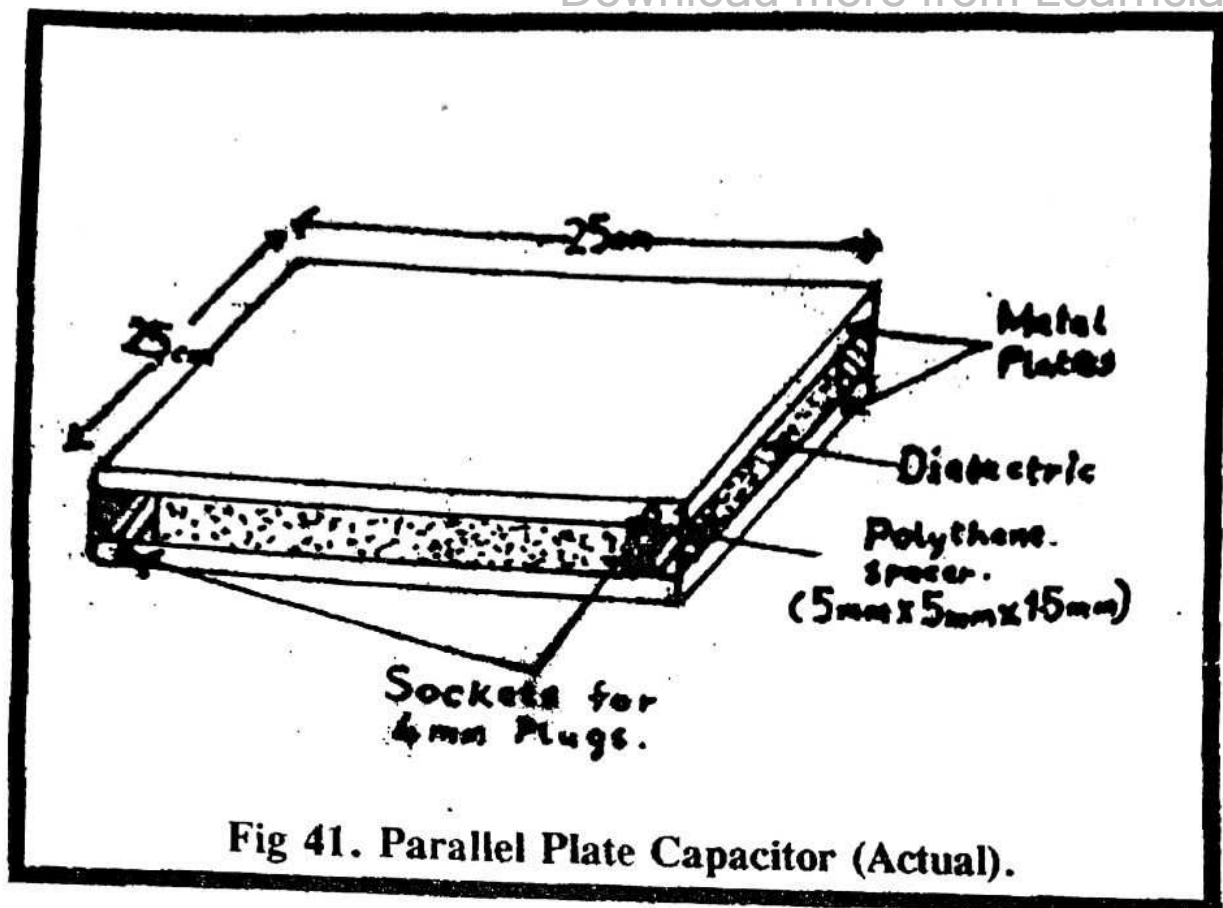
(a)

Fig 40: (a) Mica Capacitor (Construction)



(b)

(b) Mica Capacitor (Actual).



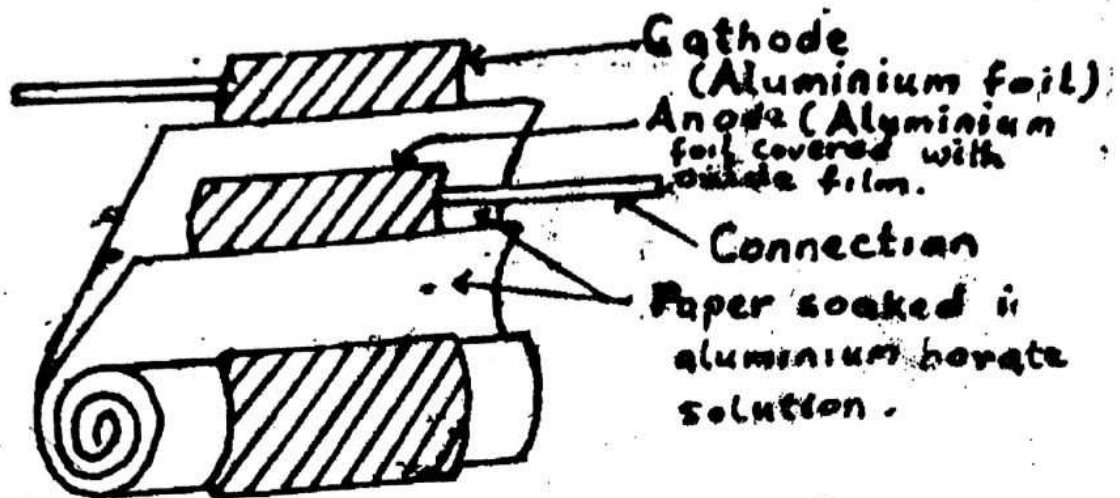


Fig. 43. Electrolytic Capacitor (Construction).

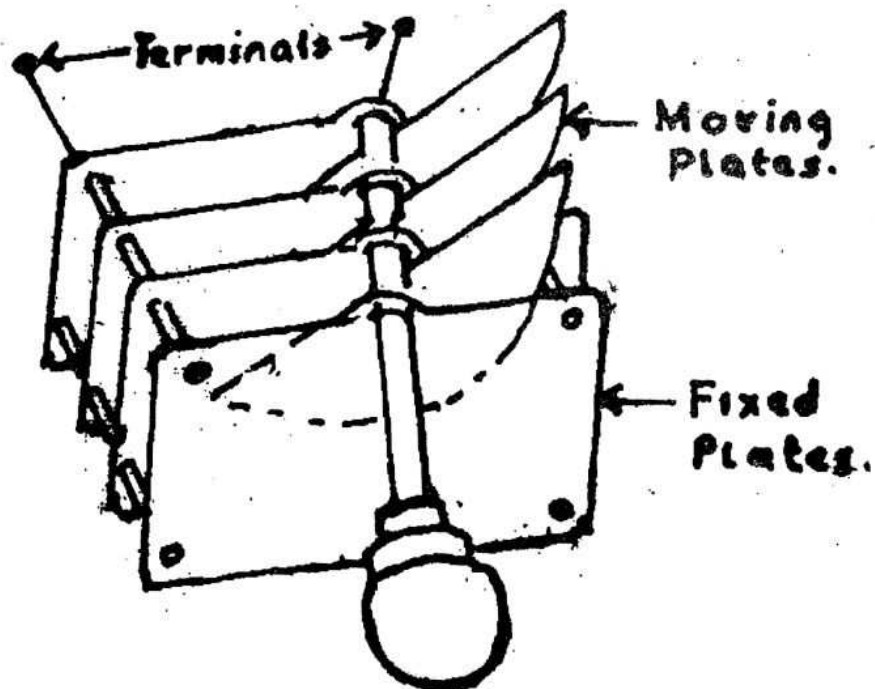


Fig. 44. Variable air capacitor.

The quality of a capacitor depends on its stability; that is its ability to keep the same value with age, temperature change, etc. The type to be employed in any circuit will depend also on its stability, and the frequency of the a.c. that can flow through it. The working p.d. of a capacitor depends on the thickness of its dielectric. For every dielectric, there is a maximum p.d. at which it breaks down and spark occurs. The operation and use of any capacitor must therefore be at p.d. below its breakdown value.

In paper, plastic, ceramic and mica capacitors, waxed paper, plastic (eg polystyrene), ceramics (eg tale with barium titanate added) and mica are used as dielectrics respectively. In paper capacitor, the losses limit their frequencies to less than **1 MHz**, and their stability is poor (up to 10% with time). Plastic, ceramic and mica capacitors have better stability (about 1%) and can be used at higher frequencies. These four capacitors have capacitance of a few μF . The mica capacitor has limit capacitance of about $0.01\mu\text{F}$ (that is 10000 pF).

The parallel plate capacitor is made up of two conducting plates separated by polytene spacers. One plate is connected to a positive p.d. source while the other is earthed. This is a very useful laboratory capacitor and different types of dielectric materials can be inserted inbetween the plates and their qualities studied.

The electrolytic capacitor has capacitance of up to 0.1 F (ie 100000 μF) and do not break down for applied p.d. of a few hundred volts. The dielectric of this capacitor is a film of aluminium oxide formed by passing current through a strip of paper soaked with aluminium borate solution and sandwiched between two aluminium foil electrodes. As current flows through the borate solution (which is an electrolyte), an oxide is formed on the anode. The other plate becomes the cathode. The electrolytic capacitor is usually used in d.c. circuits. For a.c. circuit, it cannot

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be used when the a.c. frequency is over 10 kHz. The stability of the electrolytic capacitor is poor (10 - 20%).

In the case of variable capacitors, a dielectric is usually employed. The capacitor is made up of two sets of parallel metal plates, one set fixed while the other set are moveable but not touching the fixed set. Inbetween each plate and its neighbour is air dielectric. The variation of the interleaved area of the capacitor varies its capacitance. Losses in variable capacitor are very low at all frequencies, and breakdown p.d. value is also quite low. It is used to tune radio and TV receivers.

4.8 Charging And Discharging Of A Capacitor.

The circuit for verification of this is shown in fig 45. A high impedance voltmeter V is connected in parallel across the capacitor to measure the voltage across its plates. The circuit is then connected to a battery V_0 through a 2-way key, k as shown.

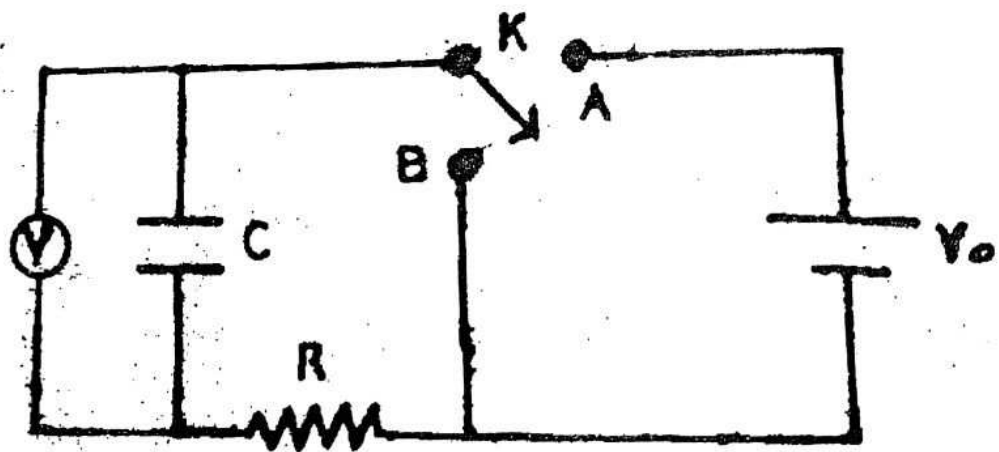


Fig. 45. Capacitor charging circuit.

A high resistance R is also connected in one arm of the circuit as shown, for slowing down the charging process. If we connect the key switch to A, the capacitor will become connected to the battery and will begin to get charged. The voltmeter V will begin to rise until it is equal to V_0 when it becomes steady. The capacitor is now said to be charged to the battery voltage.

If we now switch off the key, V will remain steady unless the capacitor leaks. But if we switch on to B, the capacitor will discharge gradually through the resistor until the voltmeter returns to zero. The voltage-time graph of the charging and discharging processes is shown in fig.46.

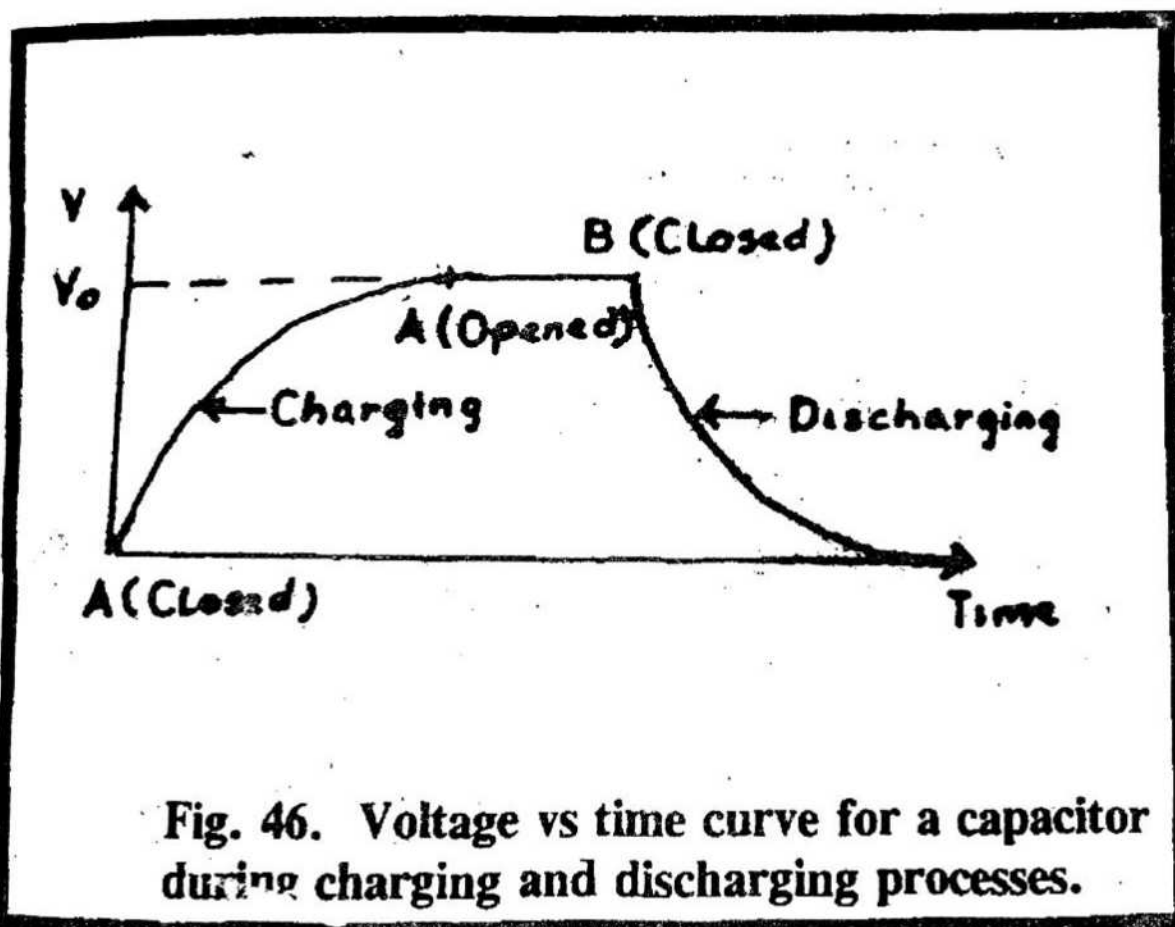


Fig. 46. Voltage vs time curve for a capacitor during charging and discharging processes.

We note that after charging the capacitor, if a piece of wire is used to close its terminals directly (cutting off R), a big spark will show and the voltmeter will quickly return to zero indicating that the capacitor has discharged.

Explanation of the charging and discharging processes.

As shown in Fig. 47., when the key is switched to A, electrons flow from the -ve terminal of the battery to the plate X of the capacitor. At the same time electrons flow from the plate Y to the positive terminal of the battery. Positive and negative charges therefore appear at Y and X respectively, and this opposes more flow of electrons causing the charge build up. Thus the build up slows down gradually until V equals V_0 , when the build up reaches its maximum value and stops. When the key is switched to B, the electrons flow in the opposite direction until the positive charge on Y is completely neutralized and the capacitor is said to be discharged.

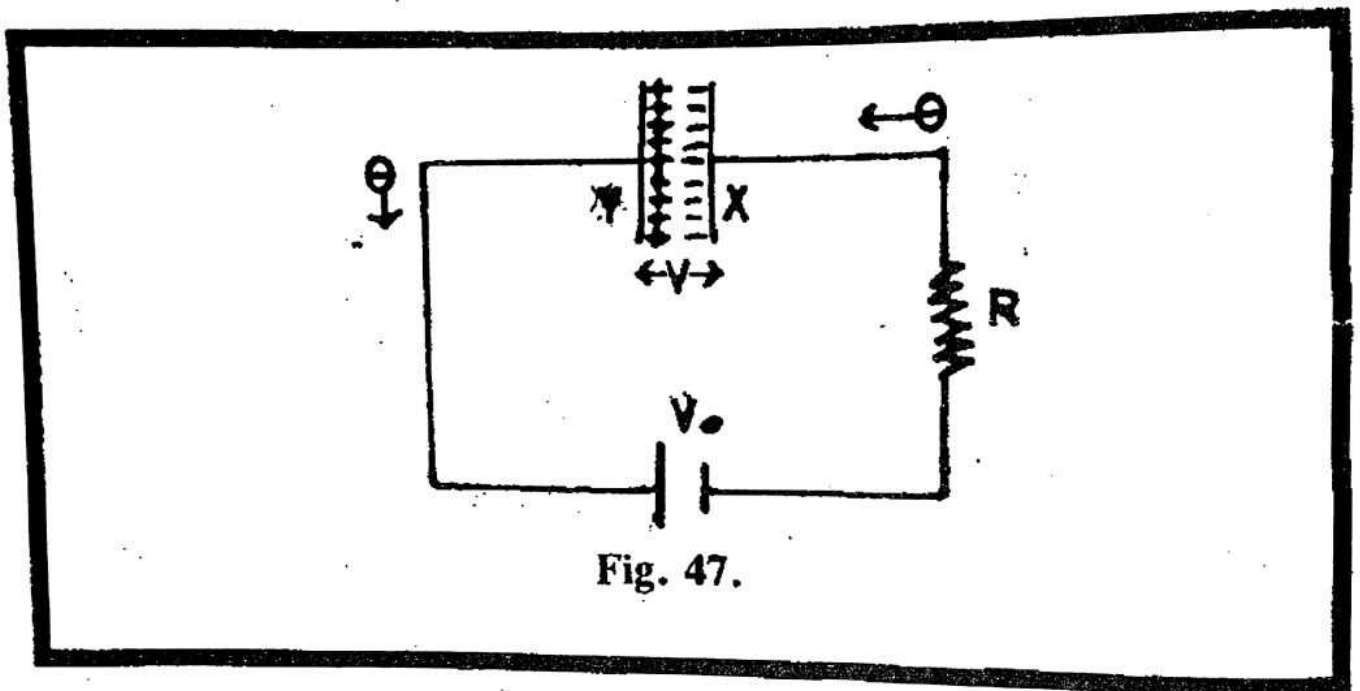


Fig. 47.

The discharging of a capacitor is exponential as shown in fig. 48.

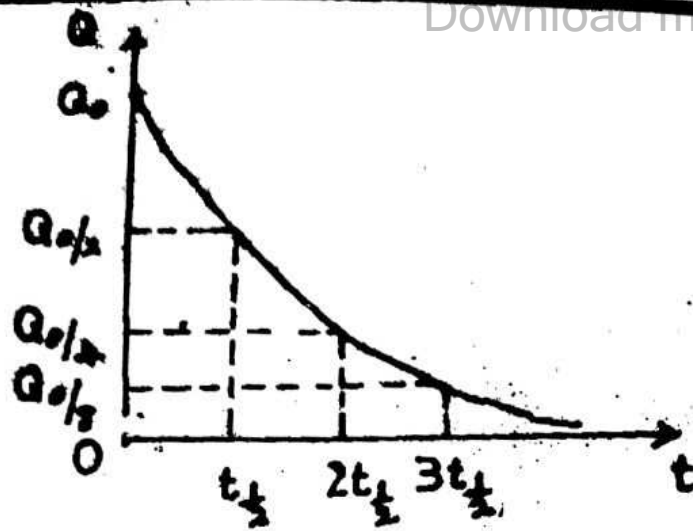


Fig. 48. The discharging of a capacitor.

Usually, the larger the values of C and R , the slower the discharge. The Q versus t decay curve shown in fig. 48 will have the same shape as I versus t curve, since $V = IR$ (V causes current flow through the resistor, R). If we substitute for V in $Q = VC$, we obtain that

$Q = IRC$, which implies that $Q \propto I$ since R and C are constant. From calculus notation, the discharge current I can be expressed in the form

$$I = -dQ/dt \quad (4.11)$$

where the negative sign shows that Q decreases as t increases in the decay process.

$$\text{Hence } Q = -CR dQ/dt. \quad (4.12)$$

Rearranging and integrating, we obtain

$$\int_0^t \frac{dt}{CR} = - \int_{Q_0}^Q \frac{dQ}{Q} \quad (4.13)$$

where Q_0 is the initial charge at $t = 0$.

$$= -(\ln Q - \ln Q_0) \quad (4.15)$$

$$\therefore \frac{t}{CR} = -\ln Q/Q_0 \quad (4.14)$$

$$= -\ln Q/Q_0 \quad (4.16)$$

$$\therefore \ln \frac{Q}{Q_0} = -\frac{t}{CR} \quad (4.17)$$

$$\therefore \frac{Q}{Q_0} = e^{-t/CR} \quad (4.18)$$

$$\therefore Q = Q_0 e^{-t/CR} \quad (4.19)$$

This is the equation of the exponential decay of the discharge process.

Suppose it take the time $t_{1/2}$ for half the charges Q_0 on the plate to discharge, then

$$\frac{Q_0}{2} = Q_0 e^{-t_{1/2}/CR} \quad (4.20)$$

$$\therefore e^{-t_{1/2}/CR} = \frac{1}{2} \quad (4.21)$$

$$\therefore e^{t_{1/2}/CR} = 2. \quad (4.22)$$

$$\therefore t_{1/2}/CR = \ln 2.$$

$$= 0.693$$

$$\therefore t_{1/2} = 0.693 CR. \quad (4.23)$$

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Thus if $C = 500 \mu\text{F}$ and $R = 100\text{k} \Omega$, then

$$t_{\text{cs}} = 0.693 \times 500 \times 10^{-6} \times 100 \times 1000$$

$$= \underline{34.655}$$

CR has unit of time and is called the time constant of the circuit.

Where $t = CR$, we have

$$Q = Q_0 e^{-t/CR} = Q_0 e^{-1} \quad (4.24)$$

$$= \frac{Q_0}{e^1} \quad (4.25)$$

$$= \frac{Q_0}{2.718}$$

$$= 0.368 Q_0$$

$$= 0.37 Q_0 \quad (4.26)$$

Thus CR is the time it will take for Q to fall to 37% of its initial value.

It should be noted that when the charging process of a capacitor is through a resistor, the charge build up is also exponential.

4.9 Finding The Unknown Capacitance Of A Capacitor.

To do this, we use a capacitor of known capacitance and a ballistic galvanometer, and the circuit is shown in fig. 49.

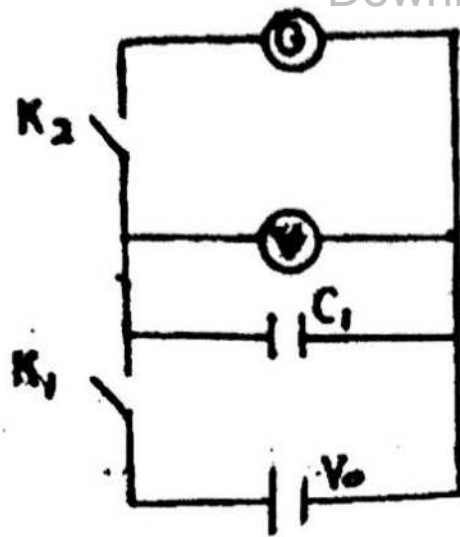


Fig. 49.

V is a high resistance voltmeter to measure the p.d across the capacitor. First K_2 is kept open while K_1 is closed to charge the capacitor. After charging to specific value of V, K_1 is disconnected and K_2 is then connected so that the capacitor discharges through the ballistic galvanometer. Let the deflection of the galvanometer be θ_1 .

The second capacitor C_2 is then used in replacing C_1 and the process is repeated for the same voltage V, producing a deflection, θ_2 .

Since the quantity of charge flowing through the galvanometer is directly proportional to its deflection, then

$$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2} \quad (4.27)$$

$$\text{But } Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

$$\therefore \frac{C_1 V}{C_2 V} = \frac{\theta_1}{\theta_2}$$

$$\therefore \frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

Thus if we know the value of one of the capacitors, say C_1 , we can find the value of the second, C_2 , from this equation.

4.10 Capacitor Network.

Where we have more than one capacitor in a circuit, we will require an equivalent or combined capacitance of the capacitors for circuit calculations. Capacitor arrangement in circuits can either be in parallel or in series.

1. *Capacitors in Parallel.*

Consider three capacitors in parallel as shown in fig. 50. Since the LH plates of the three capacitors are connected together to one junction, and the RH plates are connected together also, then if these junctions are connected to a voltage source, they must be at the same p.d. V with the source.

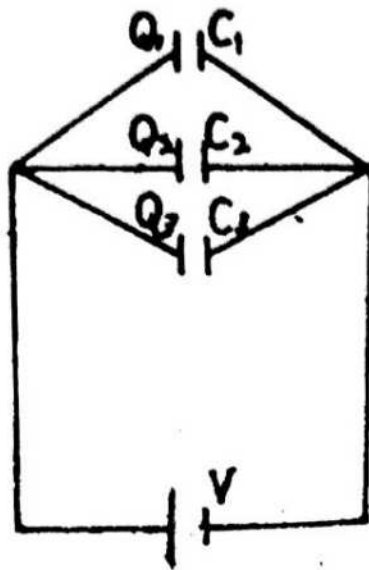


Fig. 50: Capacitors in parallel

Thus the charges of the capacitors are given as follows:

$$\begin{aligned} Q_1 &= C_1 V \\ Q_2 &= C_2 V \\ Q_3 &= C_3 V. \end{aligned}$$

Therefore, if the total charge of the system is Q , the

$$\begin{aligned} Q &= Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V \\ &= V (C_1 + C_2 + C_3) \end{aligned} \quad (4.29)$$

$$\therefore \frac{Q}{V} = C_1 + C_2 + C_3 \quad (4.30)$$

Therefore, if the combined capacitance of the system is given by C , then

$$C = Q/V = C_1 + C_2 + C_3 \quad (4.31)$$

Thus for n capacitors in parallel, the combined capacitance C is given by the algebraic sum of all the individual capacitances.

$$\text{Thus, } C = C_1 + C_2 + C_3 + \dots + C_n \quad (4.32)$$

$$= \sum_{i=1}^n C_i \quad (4.33)$$

We note that

$$\frac{Q_1}{C_1} = V$$

$$\frac{Q_2}{C_2} = V$$

$$\frac{Q_3}{C_3} = V$$

$$\therefore \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3} \quad (4.34)$$

$$\therefore Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3 \quad (4.35)$$

2. Capacitors in Series.

For capacitors in series, the circuit is as shown in fig. 51. The applied p.d. V , is distributed over the capacitors.

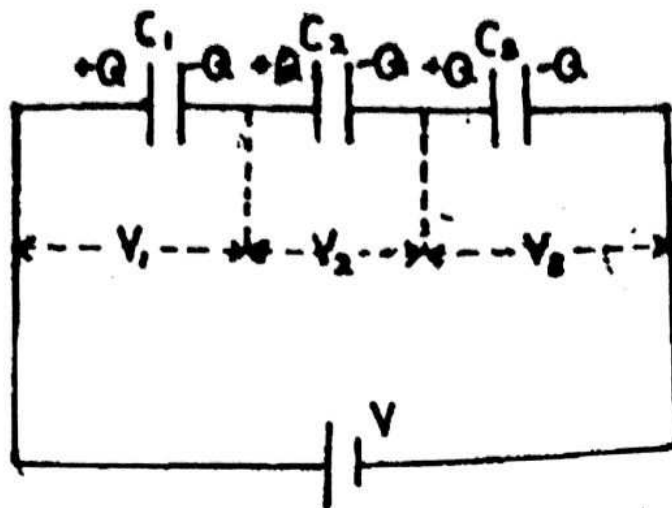


Fig. 51: Capacitors in Series

However, since the same charge will flow through all the capacitors, then the magnitude of the charges on the plates must be equal, though the plates on the LH will be +ve while the ones on the RH will be -ve as shown.

Thus again,

$$V_1 = \frac{Q}{C_1}$$

$$V_2 = \frac{Q}{C_2}$$

$$V_3 = \frac{Q}{C_3}$$

$$\text{But } V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad (4.36)$$

$$= Q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \quad (4.37)$$

$$\therefore \frac{V}{Q} = \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (4.38)$$

where C is the combined capacitance.
Again for n capacitors,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n} \quad (4.39)$$

$$= \sum_{i=1}^n 1/C_i \quad (4.40)$$

4.11 Energy Stored In A Capacitor.

A charged capacitor is a store of electrical energy. This is easily seen from the great spark that results if a charged capacitor is short - circuited.

Now consider a capacitor C charged to a point when the charge on it is Q and p.d. across it is V . If we wish to increase the charge by a very small quantity δQ , then the p.d. must simultaneously increase by a small quantity δV , where

$$\delta V = \frac{\delta Q}{C} \quad (4.41)$$

From the definition of p.d.,

$$V = \frac{\delta W}{\delta Q} \quad \text{where } \delta W \text{ is the small work that must be done to increase } Q \text{ by } \delta Q.$$

$$\begin{aligned}\text{Hence } \delta W &= V\delta Q \\ &= CV\delta V.\end{aligned}\quad (4.42)$$

Integrating we obtain

$$W = \frac{1}{2} CV^2 \quad (4.43)$$

where W is the measure of the energy stored in the capacitor.

$$\text{Since } C = Q/V,$$

$$\text{then } W = \frac{1}{2} CV^2$$

$$= \frac{1}{2} QV$$

$$= \frac{1}{2} Q^2/C$$

(4.44)

4.12 Solved Examples ($1 \mu\text{f} = 10^{-6}$ farads).

Example 4.1:

Two concentric spheres of radii 5cm and 8cm carry charges $+ 3.2 \times 10^{-16}\text{C}$ and -3.2×10^{-16} respectively.

Find the capacitance of the system.

Solution:

Let the smaller sphere have radius a and carry charge $+Q$ while the larger sphere have radius b and carry charge $-Q$ as shown in fig. 52.

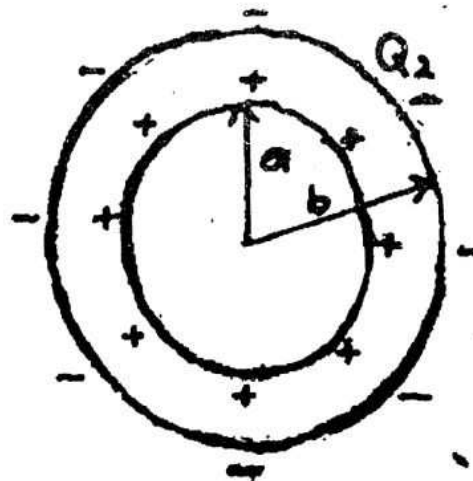


Fig. 52

Therefore, the potential difference between the two sphere V is given by

$$V = V_1 - V_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{a} - \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{b} \quad (4.45)$$

$$= \frac{Q}{4\pi\epsilon_0} (1/a - 1/b) \quad (4.46)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{b-a}{ab} \right) \quad (4.47)$$

$$\therefore V/Q = \frac{1}{4\pi\epsilon_0} \left(\frac{b-a}{ab} \right) \quad (4.48)$$

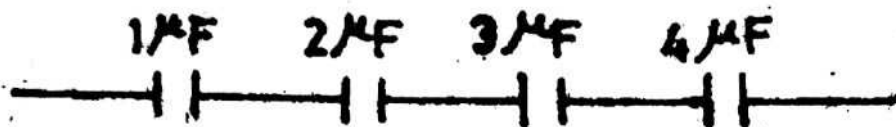
$$\therefore C = \frac{Q}{V} = 4\pi\epsilon_0 \left(\frac{ab}{b-a} \right). \quad (4.49)$$

Substituting we obtain

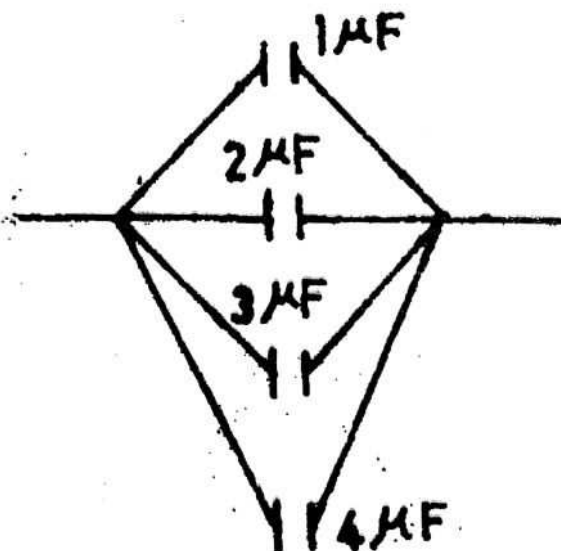
$$\begin{aligned} C &= \frac{4 \times 3.142 \times 8.85 \times 10^{-12} \times 5 \times 10^{-2} \times 8 \times 10^{-2}}{(8-5) \times 10^{-2}} \\ &= \frac{62.84 \times 8.85 \times 8 \times 10^{-14}}{3} = 62.84 \times 2.95 \times 8 \times 10^{-14} \\ &= 1482 \times 10^{-14} = 1.48 \times 10^{-11} \text{ F.} \\ &\approx \underline{1.48 \times 10^{-5} \mu\text{F.}} \\ &= \underline{14.8 \text{ PF.}} \end{aligned}$$

Example 4.2

Calculate the combined capacitance of the capacitor arrangements in fig. 53.



(i)



(ii)

Fig. 53.

Solution.

- (i) Let the combined capacitance be C . (μF). Since the capacitors are in series, then

$$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$$

$$\therefore C = 12/25 = \underline{0.48 \mu\text{F}}$$

- (ii) Let the combined capacitance be C (μF). Since the capacitors are in parallel, then

$$C = 1 + 2 + 3 + 4 = \underline{10 \mu\text{F}}$$

Example 4.3.

Calculate the combined capacitance of the arrangement shown in fig. 54

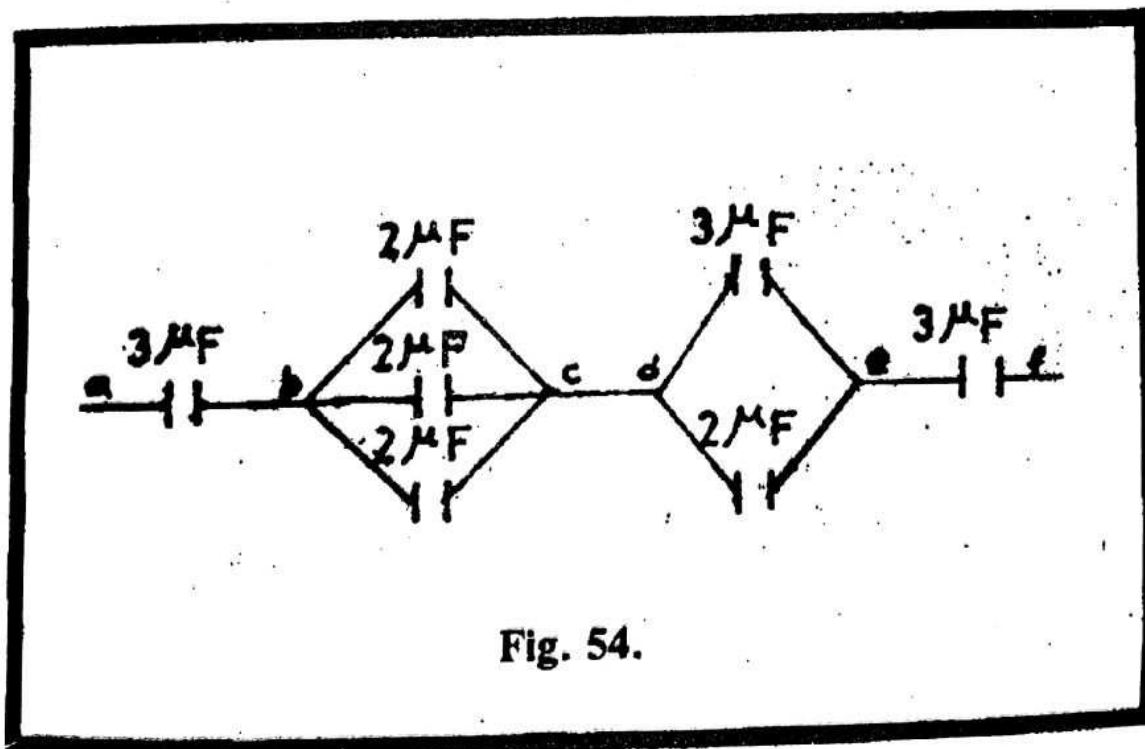


Fig. 54.

Solution.

$$C_{bc} = 2 + 2 + 2 = 6 \mu\text{F}$$

$$C_{de} = 3 + 2 = 5 \mu\text{F}$$

Let the combined capacitance be C_{af} (μF).

$$\begin{aligned} \therefore \frac{1}{C_{af}} &= \frac{1}{3} + \frac{1}{6} + \frac{1}{5} + \frac{1}{3} \\ &= \frac{10 + 5 + 6 + 10}{30} = \frac{31}{30} \end{aligned}$$

Therefore, $C_{af} = 30/31 = \underline{0.97\mu F}$.

Example 4.4

In the circuit shown in fig. 55,

$C_1 = 4\mu F$; $C_2 = 2\mu F$; $C_2 = 2\mu F$; $C_3 = 1\mu F$, and E is 12 volts

Calculate the charge and p.d. across each capacitor.

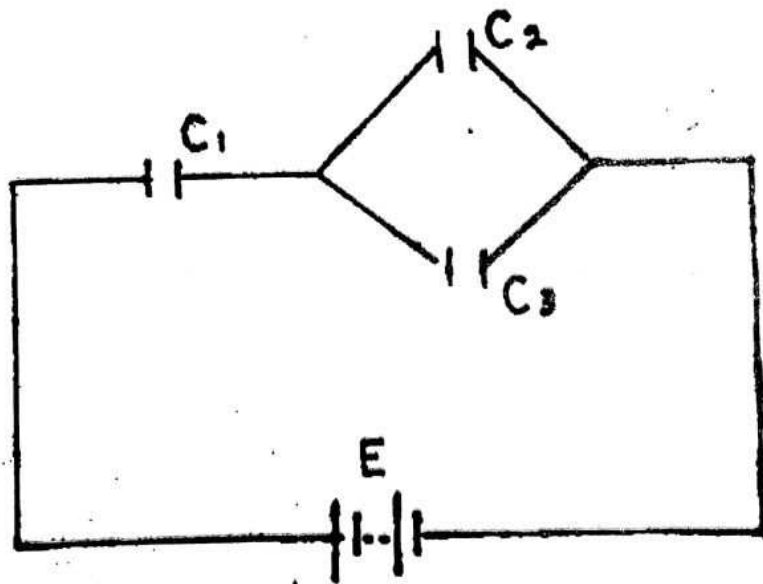


Fig. 55.

Solution:

Let the combined capacitance of C_2 and C_3 be C_4 .

$$\text{Hence } C_4 = C_2 + C_3 = 2 + 1 = 3\mu\text{F.}$$

Since C_1 and C_4 are in series, then their charges are equal.

$$\therefore q_1 = q_4.$$

$$\text{But } q_1 = C_1 V_1 \text{ and } q_4 = C_4 V_4.$$

$$\therefore \frac{q_1}{q_4} = \frac{C_1 V_1}{C_4 V_4} = 1$$

$$\therefore \frac{V_1}{V_4} = \frac{C_4}{C_1} = \frac{3}{4}$$

$$\therefore V_4 = \frac{4}{3} V_1.$$

$$\text{But } V_1 + V_4 = 12 \text{ volts.}$$

$$\therefore V_1 + \frac{4}{3} V_1 = 12$$

$$\therefore 3V_1 + 4V_1 = 36$$

$$\therefore 7V_1 = 36$$

$$\therefore V_1 = \frac{36}{7} = \underline{5.14 \text{ volts}}$$

$$\therefore V_4 = 12 - 5.14 = \underline{6.86 \text{ volts}}$$

Hence the p.d. across C_1 , $V_1 = \underline{5.14 \text{ volts}}$.

And the p.d. across $C_4 =$ p.d. across $C_2 =$ p.d. across C_3 .

$$\text{Hence } V_4 = V_2 = V_3 = \underline{6.86 \text{ volts.}}$$

To find the charges.

Since $C_1 = q_1/v_1$,

Then $q_1 = C_1 V_1 = 4 \times 10^{-6} \times 5.14 = 20.56 \times 10^{-6} \text{C}$
 $= \underline{20.56 \mu\text{C}}$

Also $q_4 = C_4 V_4 = 3 \times 10^{-6} \times 6.86 = 20.58 \times 10^{-6} \text{C}$
 $= \underline{20.58 \mu\text{C}}$

Since $V_4 = V_3 = V_2 = 9.86 \text{ volts}$

Then $q_2 = C_2 V_2 = 2 \times 10^{-6} \times 6.86 = 13.72 \times 10^{-6} \text{C}$
 $= \underline{13.72 \mu\text{C}}$

Also $q_3 = C_3 V_3 = 1 \times 10^{-6} \times 6.86 = 6.86 \times 10^{-6} \text{C}$
 $= \underline{6.86 \mu\text{C}}$

Example 4.5

A $20\mu\text{F}$ capacitor is charged from a 50 volt supply and then connected to two capacitors of magnitude $40\mu\text{F}$ and $60\mu\text{F}$ in series. Calculate

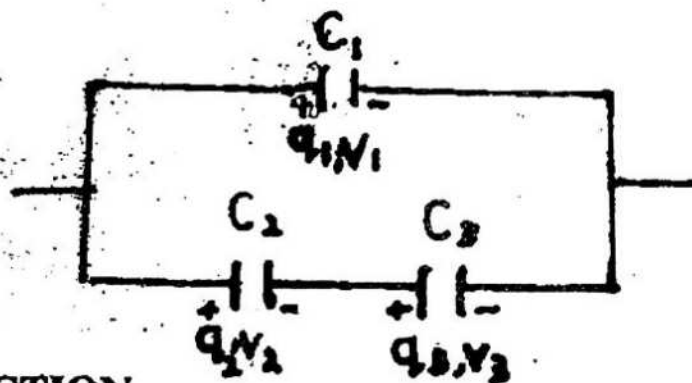
- the final p.d. across the combined capacitors.
- the final charge on each capacitor
- the initial and final energies on the capacitors.

Solution.

BEFORE CONNECTION:

$V = 50 \text{ volts}$

$C_1 = 20 \mu\text{F}$



AFTER CONNECTION

FIG. 56

Let the initial charge on C_1 when it is alone connected to the battery be q . If p.d. across C_1 is $V = 50$ volts,

$$\text{then } C_1 = \frac{q}{V}$$

$$\begin{aligned}\therefore q &= C_1 V = 20 \times 10^{-6} \times 50 = 1000 \times 10^{-6} \text{C.} \\ &= \underline{1000 \mu\text{C}}\end{aligned}$$

Let the combined capacitance of C_2 and C_3 be C_4 .

When C_1 is connected to C_4 , the charge q on C_1 will get distributed between C_1 and C_4 until the p.d. across $C_1 =$ p.d. across C_4 (since C_1 and C_4 are in parallel).

$$\text{Hence } V_1 = V_4.$$

$$\text{But } V_1 = q_1/c_1 \text{ and } V_4 = q_4/c_4.$$

$$\text{But } q_1 + q_4 = 1000 \mu\text{C.}$$

If final p.d. across C_1 and C_4 is V , then

$$V = V_1 = V_4.$$

$$\therefore V = \frac{q_1}{C_1} = \frac{q_4}{C_4}$$

$$\text{Thus } V = \frac{454.55 \times 10^{-6}}{20 \times 10^{-6}} = 22.7 \text{ volts.}$$

$$\text{Thus } V_1 = \underline{22.7 \text{ volts.}}$$

Since C_2 and C_3 are in series, p.d. V is distributed over them to give V_2 and V_3 ,

$$\text{while } q_2 = q_3$$

$$\text{Hence } q_2 = C_2 V_2$$

$$\text{and } q_3 = C_3 V_3.$$

$$\therefore \frac{q_2}{q_3} = \frac{C_2 V_2}{C_3 V_3} = 1. \quad (4.53)$$

$$\therefore \frac{V_2}{V_3} = \frac{C_3}{C_2} = \frac{60 \mu\text{F}}{40 \mu\text{F}} = \frac{3}{2}$$

$$\therefore V_3 = \frac{2}{3} V_2. \quad (4.54)$$

$$\text{But } V_2 + V_3 = 22.7$$

$$\therefore V_2 + \frac{2}{3} V_2 = 22.7$$

$$\begin{aligned} \text{Hence } q_2 = q_3 &= C_2 V_2 = 40 \times 10^{-6} \times 13.62 \\ &= 544.80 \times 10^{-6} \\ &= \underline{545 \mu\text{C}} \end{aligned}$$

$$\therefore 3V_2 + 2V_2 = 68.1$$

$$\therefore 5V_2 = 68.1$$

$$\therefore V_2 = \frac{68.1}{5} = \underline{13.62 \text{ volts}}$$

$$\therefore V_3 = \frac{2}{3} \times 13.62 = \frac{27.24}{3} = \underline{9.08 \text{ volts.}}$$

Energy distribution.

$$\begin{aligned} \text{Initial energy} &= \frac{1}{2} qV = \frac{1}{2} \times 1000 \times 10^{-6} \times 50 \\ &= 25000 \times 10^{-6} \\ &= \underline{2.5 \times 10^{-2} \text{ J.}} \end{aligned} \quad (4.55)$$

After charge distribution, the final energy stored is given by,

$$\text{Final Energy} = \frac{1}{2} q_1 V_1 + \frac{1}{2} q_4 v_4 \quad (4.56)$$

$$\text{But } v_1 = v_4 = 22.7 \text{ volts.}$$

$$\text{Therefore final energy} = \frac{1}{2} \times 22.7 (q_1 + q_4)$$

$$\begin{aligned} &= \frac{1}{2} \times 22.7 \times 1000 \times 10^{-6} \\ &= 11.35 \times 10^{-3} \\ &= \underline{1.14 \times 10^{-2} \text{ J.}} \end{aligned}$$

Energy stored in C_1 , C_2 , and C_3 respectively are given by

$$E_1 = \frac{1}{2} q_1 v_1 \quad (4.57)$$

$$\begin{aligned} &= \frac{1}{2} \times 454.6 \times 10^{-6} \times 22.7 \\ &= 227.3 \times 22.7 \times 10^{-6} \\ &= 5159.71 \times 10^{-6} \\ &= 5.16 \times 10^{-3} \text{ J.} \end{aligned}$$

$$E_2 = \frac{1}{2} q_2 v_2$$

$$= \frac{1}{2} \times 545 \times 10^{-6} \times 13.62$$

$$= 545 \times 6.81 \times 10^{-6}$$

$$= 3711.45 \times 10^{-6}$$

$$= \underline{\underline{3.71 \times 10^{-3} \text{ J.}}}$$

(4.58)

$$E_3 = \frac{1}{2} q_3 v_3$$

$$= \frac{1}{2} \times 545 \times 10^{-6} \times 9.08$$

$$= 545 \times 4.54 \times 10^{-6}$$

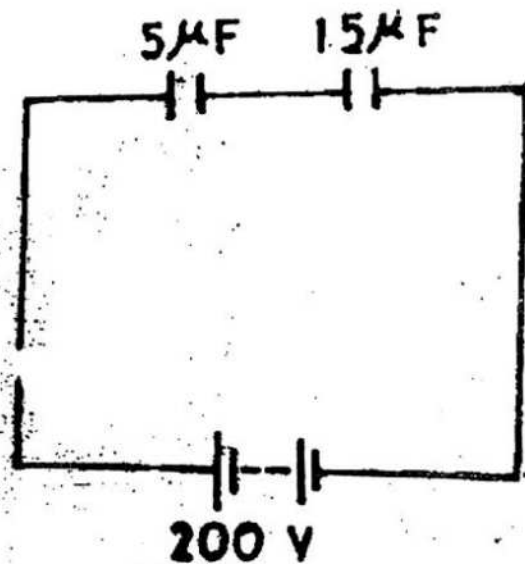
$$= 2474.30 \times 10^{-6}$$

$$= \underline{\underline{2.47 \times 10^{-3} \text{ J.}}}$$

Example 4.6

In the given circuit, calculate

- (i) the combined capacitance of the capacitors;
- (ii) the p.d. and charge on each capacitor;
- (iii) the energy on each capacitor;
- (iv) the combined energy of the capacitors.

**Fig. 57**

SOLUTION:

(i) Let the combined capacitance be C .

$$\therefore \frac{1}{C} = \frac{1}{5} + \frac{1}{15} = \frac{4}{15}$$

$$\therefore C = \frac{15}{4} = \underline{3.75\mu F}$$

(ii) Let the charge and p.d on the $5\mu F$ and $15\mu F$ capacitors be q_1, V_1 and q_2, V_2 respectively

Therefore, $q_1 = C_1 V_1$ and $q_2 = C_2 V_2$.

But the capacitors are in series.

$$\therefore q_1 = q_2$$

$$\therefore \frac{q_1}{q_2} = \frac{C_1 V_1}{C_2 V_2} = 1$$

$$\therefore \frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{15\mu F}{5\mu F} = 3.$$

$$\therefore V_1 = 3V_2.$$

But $V_1 + V_2 = 200$ volts

$$\therefore 3V_2 + V_2 = 200$$

$$\therefore 4V_2 = 200$$

$$\therefore V_2 = \frac{200}{4} = \underline{50 \text{ volts}}$$

$$V_1 = 3V_2 = \underline{150 \text{ volts.}}$$

$$\begin{aligned} \therefore q_1 &= q_2 = C_1 V_1 \\ &= 5 \times 10^{-6} \times 150 \\ &= 750 \times 10^{-6} \\ &= \underline{750 \mu \text{ C.}} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad E_1 &= \frac{1}{2} q_1 V_1 = \frac{1}{2} \times 750 \times 10^{-6} \times 150 \\ &= 750 \times 75 \times 10^{-6} \\ &= 56250 \times 10^{-6} \\ &= \underline{5.6 \times 10^{-2} \text{ J}} \end{aligned}$$

$$\begin{aligned} E_2 &= \frac{1}{2} q_2 V_2 = \frac{1}{2} \times 750 \times 10^{-6} \times 50 \\ &= 750 \times 25 \times 10^{-6} \\ &= 18750 \times 10^{-6} \\ &= \underline{1.9 \times 10^{-2} \text{ J.}} \end{aligned}$$

$$\text{Thus } E_c = E_1 + E_2 = 5.6 \times 10^{-2} + 1.9 \times 10^{-2}$$

$$\begin{aligned} &= (5.6 + 1.9) 10^{-2} \\ &= \underline{7.5 \times 10^{-2} \text{ J.}} \end{aligned}$$

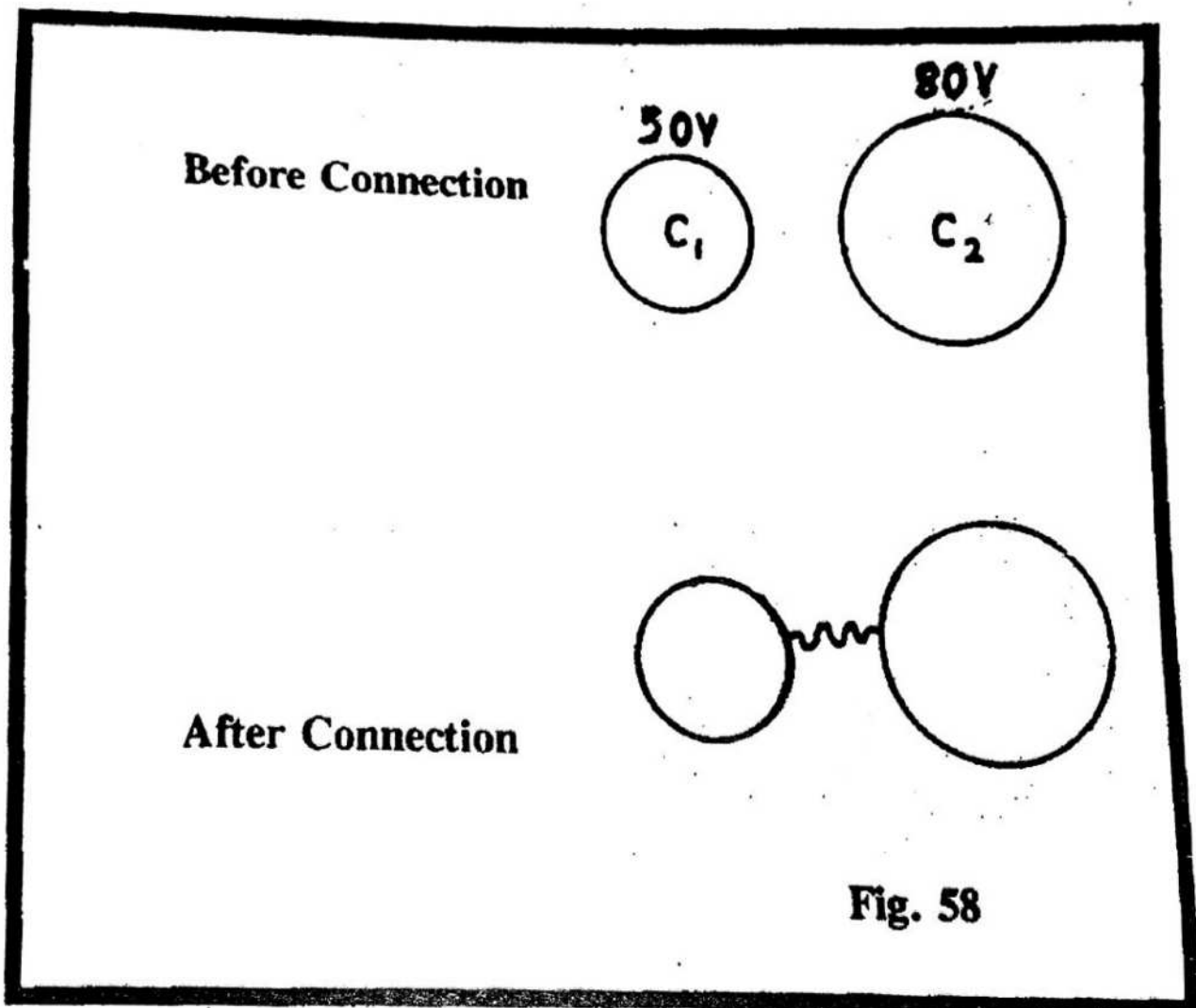
Alternatively, let the charge on the combined capacitor be q .

$$\text{Then } q = q_1 = q_2$$

$$\begin{aligned} \therefore E_c &= \frac{1}{2} qV \\ &= \frac{1}{2} \times 750 \times 10^{-6} \times 200 \\ &= 75000 \times 10^{-6} \\ &= \underline{7.5 \times 10^{-2} \text{ J.}} \end{aligned}$$

EXAMPLE 4.7

Two insulated spherical conductors of radii 10cm and 15cm are charged to potentials 50 V and 80 V respectively. calculate the capacitance and energy of each sphere. What shall be the resultant energy on each sphere if both spheres are connected with a wire?

SOLUTION:

For the sphere of radius 10cm,

$$V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r_1}$$

$$\therefore C_1 = \frac{q_1}{V_1} = 4\pi\epsilon_0 \cdot r_1 = \frac{1}{9.0 \times 10^9} \times 10 \times 10^{-2}$$

where $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

and $r_1 = 10\text{cm} = 10 \times 10^{-2}\text{m}$

$$\therefore C_1 = \frac{10^{-1}}{9 \times 10^9} = \underline{0.11 \times 10^{-10} \text{ F}}$$

Similarly, $C_2 = 4\pi\epsilon_0 r_2$

$$= \frac{1}{9 \times 10^9} \times 15 \times 10^{-2}$$

$$= \frac{1.5}{9} \times 10^{-10} = \underline{0.17 \times 10^{-10} \text{ F.}}$$

Hence $E_1 = \frac{1}{2} C_1 V_1^2$

$$= \frac{1}{2} \times 0.11 \times 10^{-10} \times 50 \times 50$$

$$= 0.055 \times 2500 \times 10^{-10}$$

$$= 137.5 \times 10^{-10}$$

$$= \underline{1.38 \times 10^{-8} \text{ J.}}$$

$$E_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} \times 0.17 \times 10^{-10} \times 80 \times 80$$

$$= 0.085 \times 6400 \times 10^{-10}$$

$$= 544 \times 10^{-10}$$

$$= \underline{5.4 \times 10^{-8} \text{ J}}$$

When the two spheres are connected, their potentials equalize. Let this final potential be V . Hence since the total charge on the two spheres remain unchanged after connection,

$$\text{then } C_1 V_1 + C_2 V_2 = C_1 V + C_2 V.$$

$$\therefore 4\pi\epsilon_0 r_1 V_1 + 4\pi\epsilon_0 r_2 V_2 = 4\pi\epsilon_0 r_1 V + 4\pi\epsilon_0 r_2 V.$$

$$\therefore r_1 V_1 + r_2 V_2 = (r_1 + r_2) V.$$

$$(10 \times 10^{-2}) 50 + (15 \times 10^{-2}) 80 = (10 + 15) \times 10^{-2} V.$$

$$= 5 + 12 = 25 \times 10^{-2} V$$

$$\therefore V = \frac{17}{25 \times 10^{-2}} = \frac{17}{25} \times 10^2 = \underline{\underline{68 \text{ volts.}}}$$

Therefore, total energy after connection is given by

$$E = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

$$= \frac{1}{2} \times 0.11 \times 10^{-10} \times 68^2 + \frac{1}{2} \times 0.17 \times 10^{-10} \times 68^2$$

$$= (0.055 + 0.085) 68^2 \times 10^{-10}$$

$$= 0.14 \times 68^2 \times 10^{-10}$$

$$= 647.36 \times 10^{-10}$$

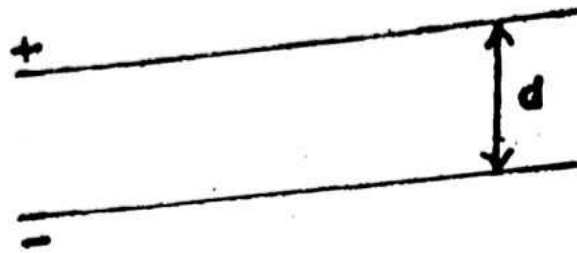
$$= \underline{\underline{6.5 \times 10^{-8} \text{ J.}}}$$

EXAMPLE 48.

A parallel plate capacitor of surface area $5 \times 10^{-4} \text{ m}^2$ is connected to an external voltage of 240 volts. If the separation of the plates is 4cm, calculate the capacitance and charge of the capacitor, and the total energy stored in it.

If the separation is then filled with mica of dielectric constant 7, calculate the new capacitance and energy of the capacitor.

$$(\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2})$$

SOLUTION:**Fig. 59**

The capacitance of a parallel plate capacitor is given by

$$C = \frac{\epsilon_0 A}{d}$$

where A is the surface area of one plate,
 d is the separation between the plates, and ϵ_0 is
 permittivity of free space.

$$\text{Hence } C = \frac{8.85 \times 10^{-12} \times 5 \times 10^{-4}}{4 \times 10^{-3}}$$

$$\begin{aligned} &= 8.85 \times 5 \times 10^{-13} \\ &= 44.25 \times 10^{-13} \\ &= \underline{4.4 \times 10^{-12} \text{ F}} \end{aligned}$$

The charge on each plate is given by

$$\begin{aligned} Q &= CV = 4.4 \times 10^{-12} \times 240 \\ &= 1056 \times 10^{-12} \\ &= \underline{1.06 \times 10^{-9} \text{ C}} = \underline{1.06 \times 10^{-3} \mu\text{C}}. \end{aligned}$$

The energy stored E is given by

$$\begin{aligned}
 E &= \frac{1}{2} QV \\
 &= \frac{1}{2} \times 1.056 \times 10^{-9} \times 240 \\
 &= 120 \times 1.056 \times 10^{-9} \\
 &= 126.72 \times 10^{-9} \\
 &= \underline{\underline{1.27 \times 10^{-7} \text{ J}}}
 \end{aligned}$$

With the dielectric, then

$$\epsilon = \epsilon_r \epsilon_0$$

where ϵ_r is dielectric constant.

$$\therefore \epsilon = 7\epsilon_0$$

Thus if the new capacitance of the capacitor is C^1 ,

$$\begin{aligned}
 \text{then } C^1 &= \frac{\epsilon A}{d} = \frac{7 \epsilon_0 A}{d} \\
 &= 7C \\
 &= 7 \times 4.4 \times 10^{-12} \\
 &= \underline{\underline{30.8 \times 10^{-12} \text{ F}}}
 \end{aligned}$$

$$\begin{aligned}
 E^1 &= \frac{1}{2} C^1 V^2 \\
 &= \frac{1}{2} \times 30.8 \times 10^{-12} \times 240^2 \\
 &= 15.4 \times 240^2 \times 10^{-12} \\
 &= 887040 \times 10^{-12} \\
 &= \underline{\underline{8.9 \times 10^{-7} \text{ J}}}
 \end{aligned}$$

4.13. PROBLEMS

- 4.13.1 Two capacitors of value $1\mu\text{F}$ and $2\mu\text{F}$ are connected first in parallel, then in series, across a 3v battery. Calculate the charge and p.d of each capacitor in each arrangement. Which arrangement has higher value of charge in the $1\mu\text{F}$ capacitor?
- 4.13.2 A $4\mu\text{F}$ capacitor is connected in series with an unknown capacitance across a 4V battery source. If a charge of $8\mu\text{C}$ flows in the circuit, determine the unknown capacitance and the p.d. across it. Hence calculate the combined capacitance of the circuit?
- 4.13.3 A hollow sphere of radius 2cm carries a charge of $+4.8 \times 10^{-12}\text{C}$. Calculate the capacitance of the sphere and hence the p.d. on its surface.
- 4.13.4 A p.d. of 100v is maintained across two capacitors of value $5\mu\text{F}$ and $10\mu\text{F}$ connected in series. Calculate (i) the p.d. (ii) the charge (iii) the energy stored, in each capacitor.
- 4.13.5 A square parallel plate capacitor of side 25cm and separation 1.5mm is maintained at a p.d. of 300v. Calculate the capacitance of the system and the charge on its plate. Suppose the separation is filled with mica dielectric of dielectric constant 7, calculate its new capacitance and charge ($\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$).
- 4.13.6 Repeat Q.5 for a disc parallel plate capacitor of diameter 25cm.

4.13.7 A spherical capacitor consists of two concentric spherical shells of radii x and y , where $y > x$. Show that its capacitance $C = 4\pi\epsilon_0$.

4.13.8 A dielectric slab of thickness h and dielectric constant k is inserted exactly inbetween a parallel plate capacitor of area A and separation d . Show that its capacitance after the insertion is given by

$$C = \frac{k\epsilon_0 A}{kd - h(k-1)}$$

If $A = 80 \text{ cm}^2$, $d = 1.5 \text{ cm}$, $h = 0.5 \text{ cm}$, $k = 8.0$ and p.d across the plates is 100 volts, calculate the capacitance before and after the insertion of the slab.

4.13.9 If the separation of a parallel plate capacitor is filled with two dielectrics as shown in fig. 59(b) and (c), show that the capacitance of b is given by

$$C = \frac{\epsilon_0 A}{d} \left(\frac{k_1 + k_2}{2} \right)$$

and that of C is given by

$$C = \frac{2 \epsilon_0 A}{d} \left(\frac{k_1 k_2}{k_1 + k_2} \right)$$

5

CURRENT ELECTRICITY

5.1. Electric Current

The flow of any fluid in a particular direction is called a current, eg water and air current. Similarly electric current I is the flow of electrons (or charges generally) in a particular direction. It is defined as the quantity of charge q that flows across a cross-section of a conductor per unit time. Hence

$I = q/t$ (CS^{-1}) or Ampere (amp). If the rate of flow of charge with time is not constant, then the current varies with time. In such case, we express I as a differential equation.

$$I = \frac{dq}{dt} \quad (5.1)$$

In isolated metallic conductors, outermost electrons of atoms (called free electrons) are free to drift randomly in different directions from atom to atom, just as gas or water molecules are free to move at random. If we consider the cross-section of an isolated copper wire for example, at a particular location along the wire, the net flow of electrons across the cross-section is zero. This means that the rate at which the electrons pass the cross-section from left to right is the same as the rate from right to left.

If the ends of the wire are set at different potentials by connection to the positive and negative terminals of a source of emf (say a battery), electric field will be created at every point along the wire,

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which will cause all the electrons to begin to drift from location of lower potential to that of higher potential, that is in a direction opposite to that of the electric field, hence generating electric current. The direction of the electric field is from the point of higher potential to that of lower potential. Hence the direction of the electrons in electric current is opposite to that of the electric field moving them. However, the electronic motion in electric current is not smooth. The electrons are accelerated as they move, but keep colliding with atoms of the metal as they move. Each time an electron collides with an atom, it loses part of its energy to the atom and slows down, while the vibrational energy of the atom increases. Hence the overall effect of these collisions is a transfer of kinetic energy of the accelerating electrons to increase in vibrational energy of the lattice, and hence increase in temperature of the wire.

However, there is a net forward movement of the electrons in a direction opposite that of the positive field, with a constant average drift speed, V_d .

The conventional direction of electric current however is opposite to the actual direction of motion of the electrons generating the current. Hence the direction of the electric current is from higher potential to lower potential (eg from positive terminal of a battery to the negative terminal).

5.2. Electronic Drift Speed, V_d

Consider a section of a conducting wire shown in fig 60.

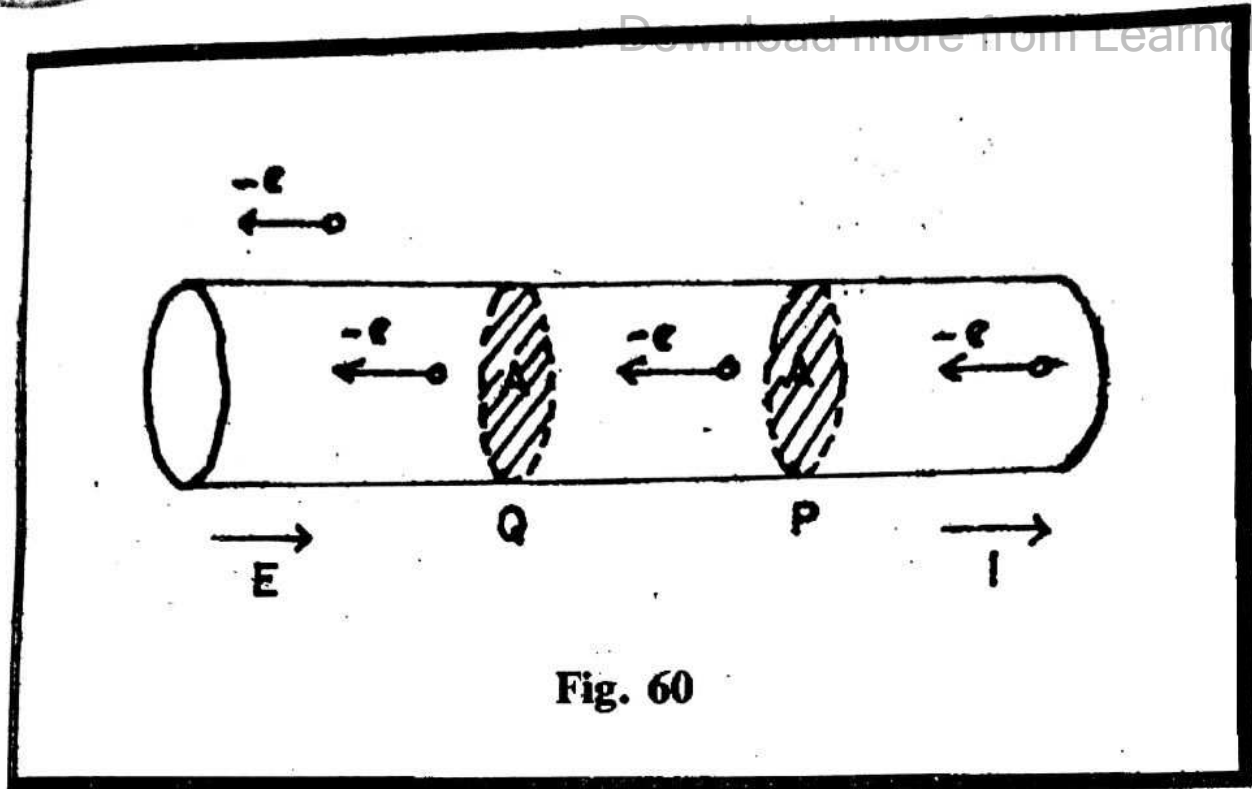


Fig. 60

Consider a cross-sectional of the wire at P. Suppose the electrons that pass P reach Q in 1 sec. Suppose also that there are n drifting electrons per unit volume of the wire, and that each electron carries a charge e . If the drift speed of the electrons is V_d , then in 1 sec, the electrons crossing P will cover a distance, V_d . Hence if the cross-sectional area of the wire is A , then the section PQ will have volume AV_d .

Hence the number of electrons within PQ is nAV_d , and carry a total charge, nAV_de .

Thus the total charge per second (that is the current) that crosses P is nAv_de .

$$\therefore I = nA V_d e. \quad (5.2)$$

$$\therefore V_d = \frac{I}{nAe} \quad (5.3)$$

Generally for any charge carrier in a conductor, carrying a charge q ,

$$V_d = \frac{I}{nAq} \quad (5.4)$$

Current density j is defined as the amount of current per unit area at all points across a cross-section of a conductor. It is a vector quantity and has direction of electric current. In magnitude,

$$j = \frac{I}{A} \quad (5.5)$$

Thus in terms of current density,

$$V_d = \frac{j}{ne} \quad (5.6)$$

5.3. Resistance, Resistivity And Conductivity

1. Resistance, R

This is one of the electrical properties of materials. It is defined as the ratio of the potential difference between two points to the current flowing between them.

$$\text{Thus } R = \frac{V}{I} \quad (5.7)$$

When p.d. is in volts, m and current is in amps, resistance R is measured in ohms.

2. Resistivity, ρ

This is another electrical property of materials and depends on resistance. For isotropic materials (ie material in which its property does not depend on direction within the material), resistivity is generally defined as

$$\rho = \frac{E}{j} \quad (5.8)$$

where E is the electric field in the material, and j is current density.

Let us now consider a conducting wire of length l and cross sectional area A . Suppose we apply a p.d., V between its ends.

Then if the cross-sectional surface of the ends of the wire are equipotential surface, then E and j will each be constant at every point along the wire, and will have the values

$$E = \frac{V}{l} \text{ and } j = \frac{i}{A} \text{ where } i \text{ is the current flowing}$$

along the wire.

$$\begin{aligned} \text{Hence } \rho &= \frac{E}{j} = \frac{V/l}{i/A} = V/l \cdot A/i \\ &= \frac{V}{i} \cdot \frac{A}{l} = R \frac{A}{l} \end{aligned} \quad (5.9)$$

$$\therefore R = \rho l/A$$

(5.10)

From this relation, we derive the unit of resistivity, which is ohm - m.

3. *Temperature Dependence of Resistivity.*

In conductors, resistance increases as temperature increases. If ρ_0 is the resistivity of a conductor at T_0 (0°C), then its resistivity ρ at temperature T is given by

$$\rho = \rho_0 \{1 + \alpha (T - T_0)\} \quad (5.11)$$

where α is the mean temperature coefficient of resistivity between two temperatures.

$$\alpha = \frac{1}{\rho_0} \frac{(\rho - \rho_0)}{(T - T_0)} \quad (5.12)$$

Generally the temperature coefficient of resistivity is defined as the fractional change in resistivity ($d\rho/\rho$) per unit change in T .

Hence

$$\alpha = \frac{1}{\rho} \frac{d\rho}{dT} \quad (5.13)$$

4 *Conductivity, σ .*

This is defined as the inverse of resistivity;

$$\text{Hence } \sigma = 1/\rho. \quad (5.14)$$

The unit of conductivity is mho.

Table 2: Resistivity of some substances

Substance	Resistivity ρ (ohm-m) at $20^\circ\text{C} \times 10^{-8}$	Temperature coefficient of Resistivity α ($^\circ\text{K}^{-1}$) $\times 10^{-3}$
Aluminium	2.82	3.9
Brass	8.0	1.5
Constantan	49.0	0.01
Copper	1.72	4.3
Carbon (amorphous)	3500.0	-0.5
Carbon (graphite)	33 to 185	(-0.6) to (-1.2)
Iron	9.8	5.6
Manganin	44.0	0.01
Mercury	95.77	0.91
Nickel	6.8	6.0
Nichrome	100.0	0.4
Silver	1.62	3.9
Steel	18.0	3.0
Tungsten	5.5	5.8

5.4 Ohm's Law

Robert Ohm in 1826 discovered a very important relationship between the p.d. applied to a conductor and the resultant current. This is called Ohm's law and states that the p.d. applied across a conductor is directly proportional to the current flowing through it, provided temperature and other physical conditions remain constant.

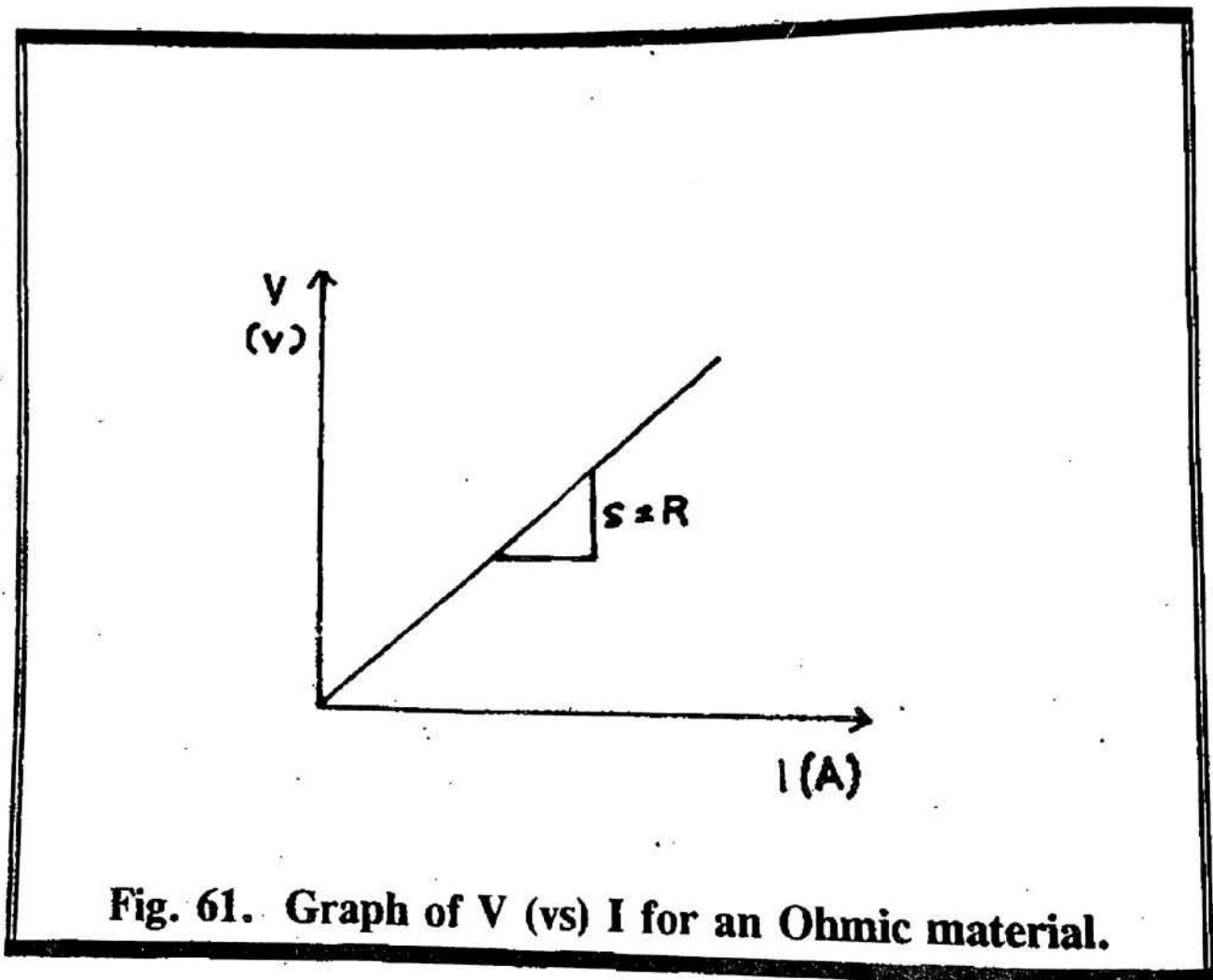
$$\text{Thus } V \propto I$$

$$\therefore V = IR$$

(5.15)

where the proportionality constant R is called the resistance of the conductor.

If a graph of V is plotted against I for an ohmic material (that is a material that obeys Ohm's law), as shown in fig. 61, it will be a straight line that passes through the origin. The slope of the linear curve is a measure of the resistance, of the material.



For non-ohmic material (that is materials that do not obey Ohm's law), the graph of V (Vs) I is non-linear, but a curve. For example, for a thermistor, the V (vs) I curve is shown in fig. 62.

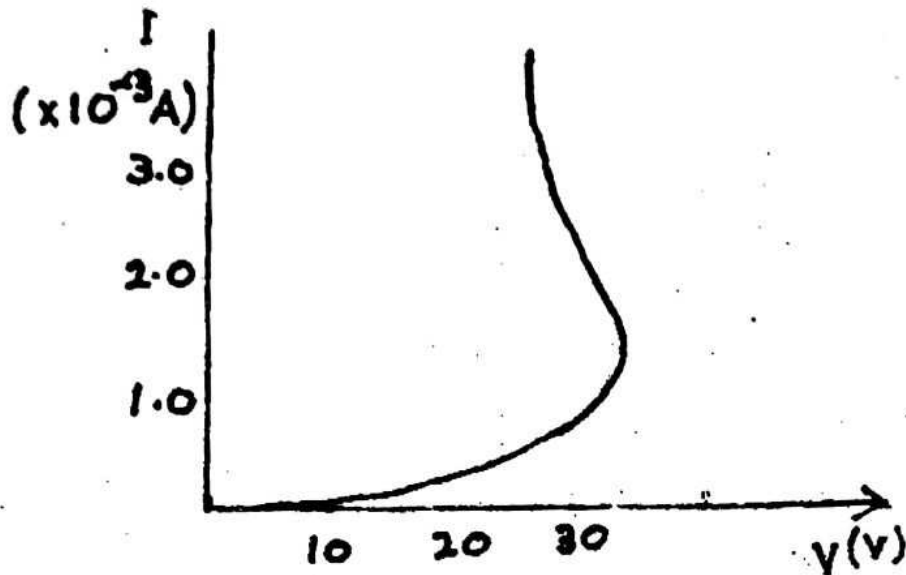


Fig. 62. I (vs) V curve for thermistor.

Other non-ohmic materials include: vacuum tube (Type 2A3), semi-conductors (eg. silicon and germanium etc), transistors, etc

I vs V curve for a diode valve is shown in fig 63.

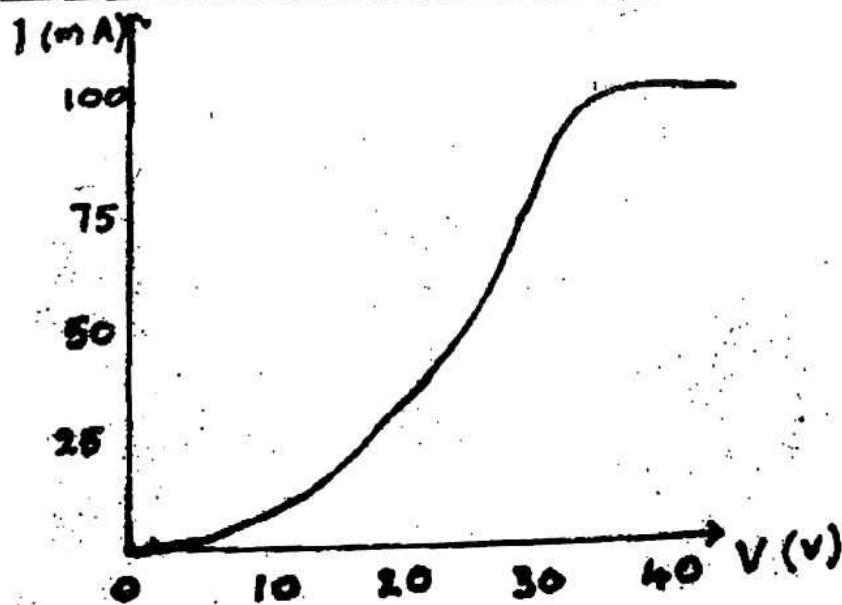


Fig. 63 I (vs) V curve for a diode valve.

5.5. Verification Of Ohms Law.

The circuit for this experiment is shown in fig. 64. It consists of a cell or battery, an ammeter, a voltmeter, a resistor and a variable resistance (like rheostat or resistance box).

In the experiment, the invariable resistance is used to vary the current in the circuit by using the resistance. For each current value, the p.d. across the resistance R is measured with a voltmeter. When graph of I (vs) V is plotted, it is found to be a straight line through the origin, confirming Ohm's law.

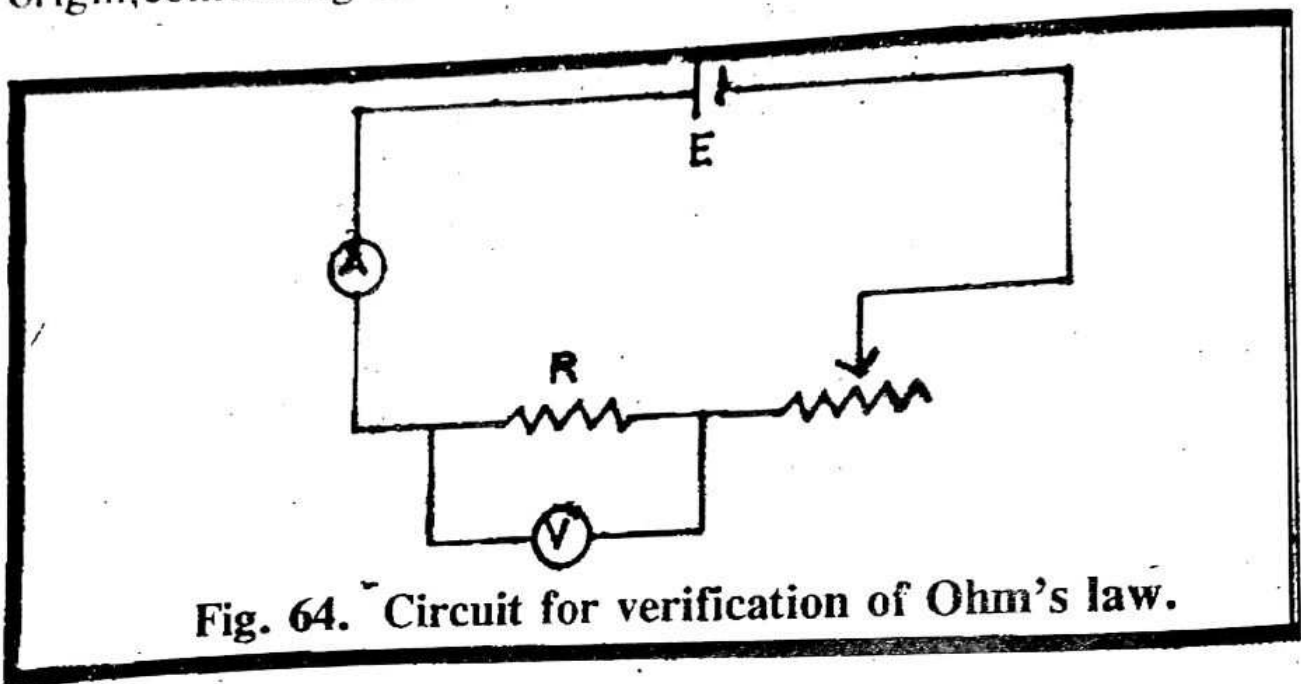


Fig. 64. Circuit for verification of Ohm's law.

The Potentiometer.

This is one of the high accuracy devices for electrical measurements. Pointer instruments like ammeters and voltmeters have intrinsic errors (about 1% of full scale) which is absent with potentiometer.

As shown in fig. 65 below, a potentiometer consists of a uniform wire AB (about one metre long), and an accumulator 'E' which maintains a steady current I through the potentiometer wire. The

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terminals of the accumulator are connected to the ends A and B of the wire as shown.

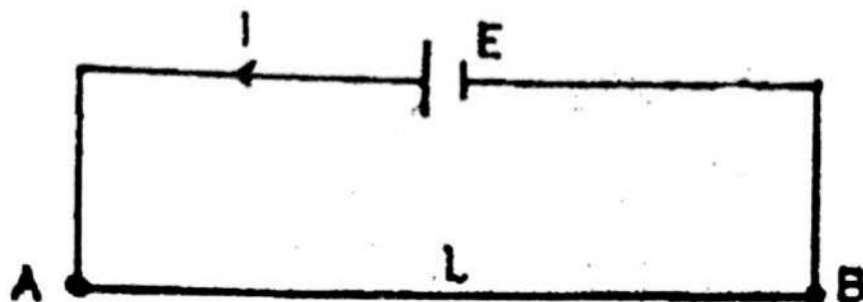


Fig. 65. A Potentiometer.

Usually, a meter rule is mounted below the potentiometer wire to determine its length during measurement. If the resistance of the wire is R , then since the wire is uniform, its resistance per unit length

($r = R/l$) is constant. Hence its voltage drop per unit length ($V = V_{AB}/l = Ir$) along the wire is also constant. (neglecting internal resistance of the cell).

5.7 Measurement Of emf Of A Cell By Use Of A Potentiometer.

To perform the experiment, the +ve terminal of the unknown emf E_0 is connected to the point A of the potentiometer, while the -ve terminal is connected through a sensitive galvanometer G to a slider, S , as shown in fig. 66.

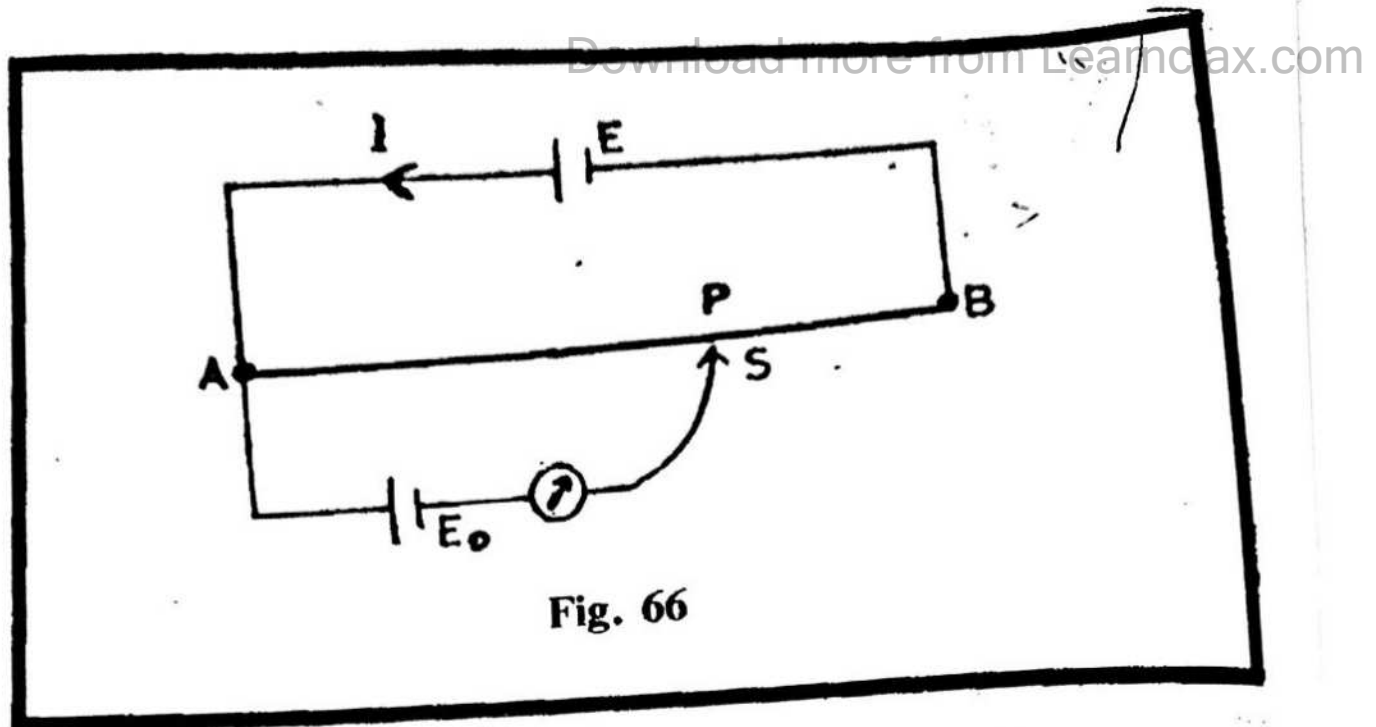


Fig. 66

When the slider is placed at A, the cell E_0 will be connected straight across its terminals and current I_1 will flow in the loop in the direction of E_0 causing the galvo to deflect say to the left. When the slider is moved to B, current I_2 will flow from the potentiometer circuit into the loop in opposite direction to E_0 (if V_{AB} is greater than E_0) causing the galvo to deflect to the right. If the galvo deflects to the same side in both cases, then E_0 is greater than V_{AB} or both of the positive terminals of E and E_0 are not connected to A.

The deflection of the galvo to the left and then to the right in the two arrangements implies that the currents I_1 and I_2 are in opposite directions and that there is a point between A and B where $I_1 = I_2$ and hence where the galvo deflection shall be zero.

The slider is then moved (by trial and error) to different positions until the balance point P is located. (Care must be taken not to slide the slider along the wire to avoid scraping its surface which will destroy its uniformity).

At the balance point, $V_{AP} = E_0$. The voltage E_0 fully appears on AP and there is no voltage drop due to its internal resistance or the galvo resistance because no current is flowing through the loop under this condition. On the other hand all the current due to E will flow under this condition through AB hence creating the p.d. V_{AB} . Comparing the loop and the potentiometer circuit, since voltage drop per unit length along the wire is constant, then

$$\frac{V_{AB}}{AB} = \frac{E_0}{AP} \quad (5.16)$$

Suppose $AB = l$ and $AP = l_0$, then

$$\frac{V_{AB}}{l} = \frac{E_0}{l_0} \quad (5.17)$$

$$\therefore E_0 = \frac{V_{AB}}{l} \cdot l_0 = \frac{l_0}{l} V_{AB} \quad (5.18)$$

If the internal resistance of the cell E is r_0 , then

$$E = V_{AB} + Ir_0 \quad (5.19)$$

$$\therefore V_{AB} = E - Ir_0 \quad (5.20)$$

Hence if E , r_0 and I are known, we can then easily determine V_{AB} .

5.8 Comparison Of emfs Of Two Cells

As in the last case, the cells E_1 and E_2 are connected one after the other and their respective balance points l_1 and l_2 located.

$$\text{Then, } E_1/I_1 = E_2/I_2 \quad (5.21)$$

$$\therefore E_1/E_2 = I_1/I_2 \quad (5.22)$$

Hence if we know the value of one emf, we can easily determine the other. The known emf is usually called a standard cell.

In the use of a potentiometer, the following precautions must be taken.

- (i) ensure that the wire AB is uniform
- (ii) ensure that constant current flows through AB
- (iii) ensure that the positive terminals of E and the loop emf E_o are connected to A.
- (iv) ensure that the loop emf is smaller than E.

5.9 Comparison Of Two Resistances.

In this case, the two resistances R_1 and R_2 are first connected to external voltage source as shown in fig 67. before connection to potentiometer wire. The connection to external voltage creates p.d. across the resistances which can then be used to balance the p.d. across the potentiometer. The resistance are therefore connected in turn to the potentiometer and the balance points of the two p.d.s., l_1 and l_2 respectively are determined as before. The rheostat in the external circuit is used to keep the current I' , flowing through the resistances, constant in the two measurements.

Hence as before,

$$\frac{V_1}{V_2} = \frac{l_1}{l_2} \quad (5.23)$$

But $V_1 = I' R_1$
and $V_2 = I' R_2$

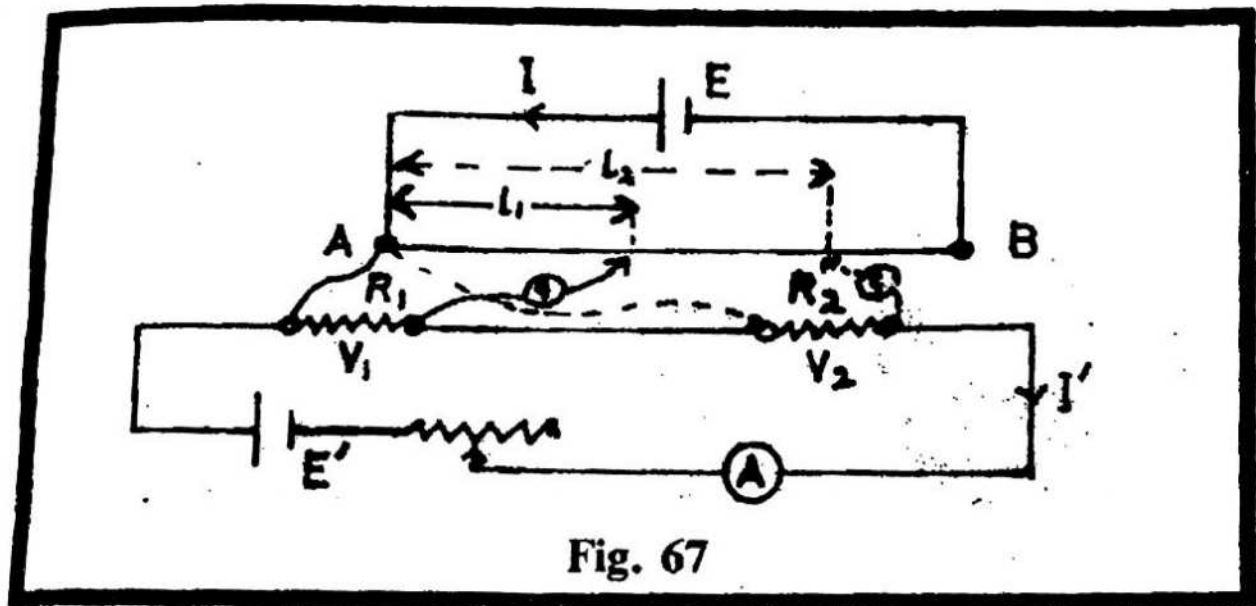


Fig. 67

$$\therefore \frac{I' R_1}{I' R_2} = \frac{l_1}{l_2}$$

$$\therefore \frac{R_1}{R_2} = \frac{l_1}{l_2} \quad (5.15)$$

5.10 Kirchhoff's Laws

Kirchhoff (1824 - 87) extended Ohm's law to networks. There are two basic laws.

The first law. This law refers to current flowing into and out of a junction in a network. It states that the total current flowing into a junction is equal to the total current leaving the junction. This if i_2 in fig 68 is current flowing into the junction P, and i_1, i_3 are current leaving the junction, then,

$$i_1 = i_2 + i_3$$

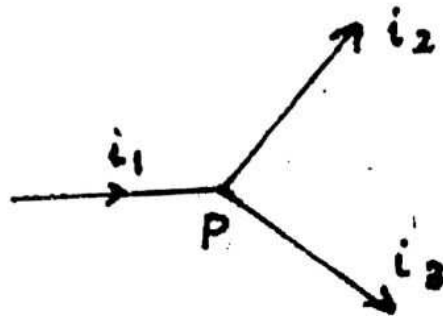


Fig. 68. Kirchhoff's 1st law.

$$\text{Hence } i_1 - i_2 - i_3 = 0$$

(5.25)

The law can therefore be stated as follows; that the algebraic sum of the current at a junction in a circuit is zero (that is $\sum I = 0$) where the current is taken to be positive if it is flowing towards the junction and negative if it is leaving it.

The Second Law: This refers to potential changes in traversing a circuit and states that, the algebraic sum of the potential changes encountered in a complete traversal of a circuit loop is zero, where

- (i) if a resistor R is traversed in the direction of the current i , the change in potential is $-iR$, while in the opposite direction, the change is $+iR$.

- (ii) if a seat of emf E is traversed in the direction of the emf, the change is $+E$, while in the opposite direction, the change is $-E$.

The choice in sign of the change depends on whether the change is from a lower to higher potential (+ve) or from a higher to lower potential (-ve).

Let us now consider some cases of circuit loops.

Case 1. A single circuit loop having one seat of emf E , and an external resistance, R . Suppose a current i flows in the circuit as shown in fig. 69

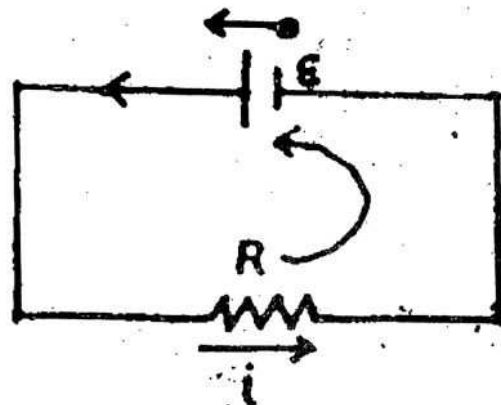


Fig. 69. A single circuit loop.

Then applying the second law, if we traverse the loop in anticlockwise direction, then

$$E - iR = 0$$

$$E = iR$$

(5.26)

Case 2. If we consider the internal resistance of the cell in case 1, then we can draw the circuit as shown in fig 70, where r is the internal resistance of the cell and is included in the circuit separated for clarity. In this case, the algebraic sum of the potential charges will become,

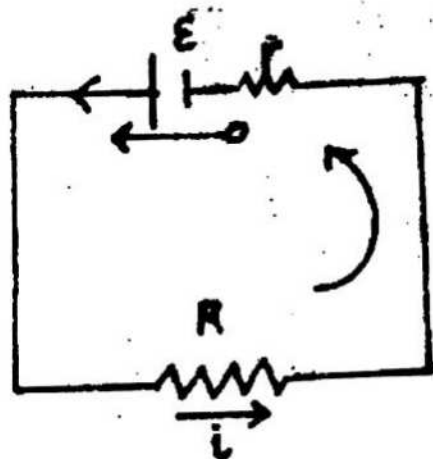


Fig. 70: Case I with the internal resistance of the cell considered.

$$E - iR - ir = 0$$

$$\text{Hence } E = i(R + r) \quad (5.27)$$

Case 3. Suppose we have two seats of emf, ϵ_1 and ϵ_2 arranged as shown in fig 71, with say two external resistances, R_1 and R_2 . Then if $\epsilon_1 > \epsilon_2$, and the internal resistances of the cells are r_1 and r_2 respectively, then a current i will flow in the circuit as shown. Thus applying the second law, we will obtain by traversing the circuit in clockwise direction that

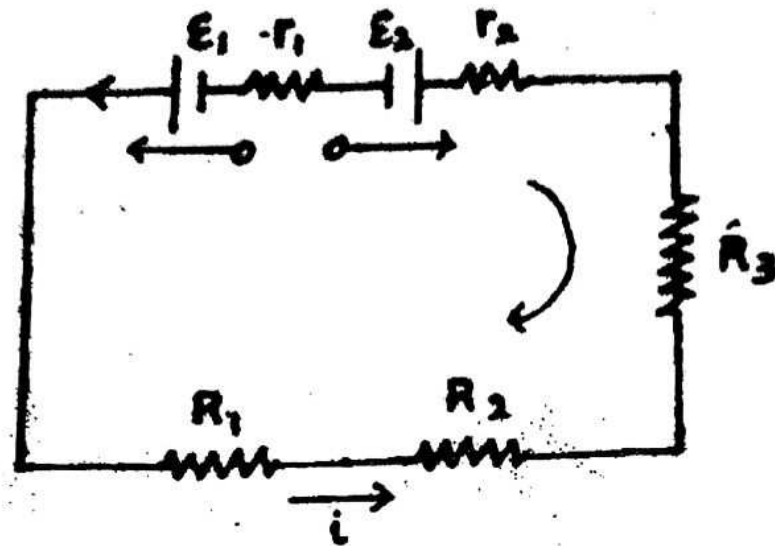


Fig. 71: Two cells and resistances in a single loop circuit.

$$- \epsilon_1 + ir_1 + \epsilon_2 + ir_2 + iR_1 + iR_2 = 0 \quad (5.28)$$

$$\therefore \epsilon_2 - \epsilon_1 + i(r_1 + r_2 + R_1 + R_2) = 0 \quad (5.29)$$

$$\therefore i = \frac{\epsilon_1 - \epsilon_2}{(r_1 + r_2 + R_1 + R_2)} \quad (5.30)$$

Case 4: Consider a double loop circuit as shown in fig 72.

If we traverse both loops in clockwise direction, then from loop 1,

$$- \epsilon_1 + (i_1 + i_2) R_3 + i_1 R_1 = 0 \quad (5.31)$$

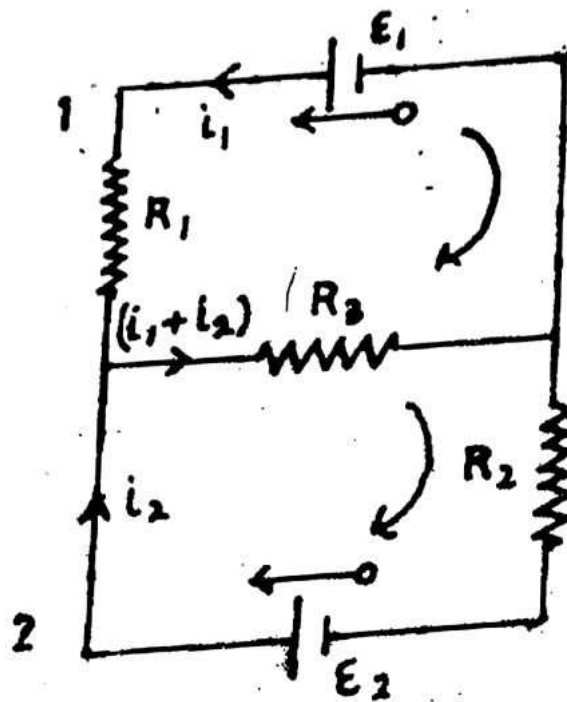


Fig. 72: A double loop circuit.

From loop 2,

$$\epsilon_2 + (i_1 + i_2) R_3 - i_2 R_2 = 0 \quad (5.32)$$

If we solve these two equations simultaneously, we obtain by adding the two that

$$\epsilon_2 - \epsilon_1 + i_1 R_1 - i_2 R_2 = 0 \quad (5.33)$$

$$\text{Thus } i_1 R_1 = i_2 R_2 + \epsilon_1 - \epsilon_2 \quad (5.34)$$

$$\therefore i_1 = \frac{(i_2 R_2 + \epsilon_1 - \epsilon_2)}{R_1} \quad (5.35)$$

f we substitute this in equation 5.31 we obtain,

$$\epsilon_1 + \left(\frac{i_2 R_2 + \epsilon_1 - \epsilon_2}{R_1} \right) R_3 + i_2 R_3 + \left(\frac{i_2 R_2 + \epsilon_1 - \epsilon_2}{R_1} \right) R_1 = 0 \quad (5.36)$$

$$\therefore -\epsilon_1 R_1 + (i_2 R_2 + \epsilon_1 - \epsilon_2) R_3 + i_2 R_1 R_3 + (i_2 R_2 + \epsilon_1 - \epsilon_2) R_1 = 0$$

$$\therefore -\epsilon_1 R_1 + i_2 R_2 R_3 + \epsilon_1 R_3 - \epsilon_2 R_3 + i_2 R_1 R_3 + i_2 R_1 R_2 + \epsilon_1 R_1 - \epsilon_2 R_1 = 0$$

$$\therefore i_2 (R_1 R_2 + R_2 R_3 + R_3 R_1) = \epsilon_2 R_1 + \epsilon_2 R_3 - \epsilon_1 R_3 = \epsilon_2 (R_1 + R_3) - \epsilon_1 R_3$$

$$\therefore i_2 = \frac{\epsilon_2 (R_1 + R_3) - \epsilon_1 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} \quad (5.40)$$

Substituting in equation 5.35 we obtain,

$$i_1 = \frac{\epsilon_1 (R_2 + R_3) - \epsilon_2 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} \quad (5.41)$$

5.11 The Wheatstone Bridge.

This bridge circuit was designed by Wheatstone about 1943, for accurate determination of a resistance. In the arrangement, three known resistances P, Q, R are arranged in four branches with the unknown resistance X as shown in fig 73. A galvo is connected between A and C with resistance R_g , while B and D are connected to a seat of emf.

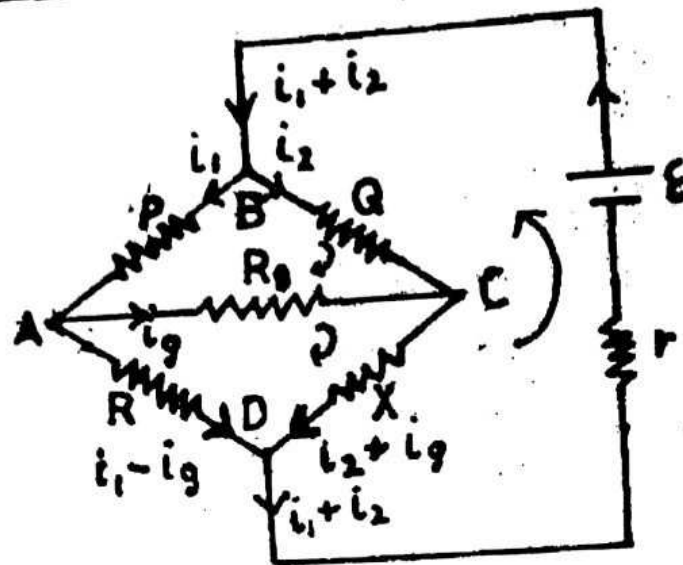


Fig. 73. The Wheatstone bridge circuit.

To determine the resistance x , R is adjusted until the galvo at R_g is balanced (that is shows no deflection). If we consider the loops, ABCA and ACDA and traverse the two loops in clockwise direction, then from ABCA,

$$i_g R_g + i_1 P - i_2 Q = 0 \quad (5.42)$$

From

ACDA,

$$-i_g R_g - (i_2 + i_g) X + (i_1 - i_g) R = 0 \quad (5.43)$$

$$\therefore i_g (R_g + X + Q) - i_1 R + i_2 X = 0 \quad (5.44)$$

Since $i_g = 0$, then from equation 5.42,

$$i_1 P - i_2 Q = 0$$

$$\therefore i_1 P = i_2 Q$$

$$\therefore \frac{i_1}{i_2} = \frac{Q}{P} \quad (5.45)$$

From 5.45

$$-i_1 R + i_2 X = 0 \quad (5.46)$$

$$\therefore i_1 R = i_2 X \quad (5.47)$$

$$\therefore \frac{i_1}{i_2} = \frac{X}{R} \quad (5.48)$$

$$\therefore \frac{Q}{P} = \frac{X}{R} \quad (5.49)$$

$$\text{Hence } X = \frac{Q}{P} \cdot R. \quad (5.50)$$

If X is known and I_g is to be determined, then one more loop must be considered, say BDCB. If the loop is traversed in anticlockwise direction, then

$$\epsilon - i_2 Q - (i_2 + i_g) X = 0 \quad (5.51)$$

(neglecting the internal resistance of the cell).

$$\therefore \epsilon - i_2 Q - i_2 X - i_g X = 0$$

$$\therefore \epsilon - i_2 (Q + X) - i_g X = 0 \quad (5.52)$$

To eliminate i_1 from equations (5.42) and (5.44), we multiply (5.42) by R and (5.44) by P and add the results. Hence we obtain,

$$i_g R_g R + i_1 PR - i_2 QR = 0 \quad (5.53)$$

$$i_g (R_g + X + Q) P - i_1 PR + i_2 PX = 0 \quad (5.54)$$

Therefore, adding (5.53) and (5.54) we obtain,

$$i_g R_g R + i_g (R_g + X + Q) P - i_2 QR + i_2 XP = 0 \quad (5.55)$$

$$\therefore (R_g R + R_g P + XP + QP) i_g + (XP - QR) i_2 = 0 \quad (5.56)$$

We now eliminate i_2 from equations (5.52) and (5.56) by multiplying (5.52) by $(XP - QR)$ and (5.56) by $(Q + X)$ and add the results. Hence we obtain

$$\epsilon (XP - QR) - (Q + X) (XP - QR) i_2 - X (XP - QR) i_g = 0 \quad (5.57)$$

$$(R_g R + R_g P + XP + QP)(Q + X) i_g + (XP - QR) (Q + X) i_2 = 0 \quad (5.58)$$

Adding (5.57) and (5.58), we obtain

$$\epsilon (XP - QR) - X (XP - QR) i_g + (R_g R + R_g P + XP + QP) (Q + X) i_g = 0 \quad (5.59)$$

$$\therefore \epsilon (XP - QR) - \{X(XP - QR) - (R_g R + R_g P + XP + QP) (Q + X)\} i_g = 0 \quad (5.60)$$

$$\therefore i_g = \frac{\epsilon (XP - QR)}{X(XP - QR) - (R_g R + R_g P + XP + QP) (Q + X)} \quad (5.61)$$

$$= \frac{\epsilon (QR - XP)}{(R_g + Q) (PQ + PX + RX) + Q (R_g R + PX)} \quad (5.62)$$

5.12 Electrical Heating And Power.

Consider current i flowing through a conductor from point A to B as shown in fig 7.4

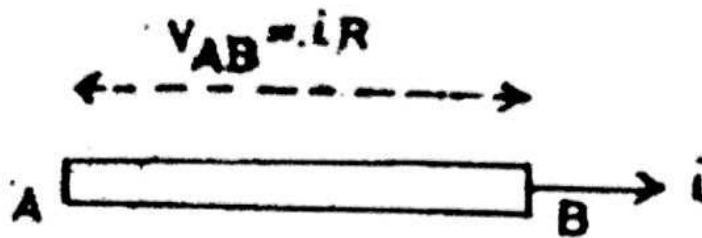


Fig. 74: Heating affect of electric current

In time dt ,

$$i = \frac{dq}{dt} \quad \text{where}$$

dq is the amount of charges flowing across a point in the conductor in time dt .

Hence $dq = idt$.

The work done in moving these charges $dw = Vdq = Vidt$.

Thus if it takes time t for the charges to cross AB, then the work done

$$W = \int dw = Vi \int dt = iV_{AB} t. \quad (5.63)$$

w is the measure of the amount of electrical energy E given up by the charge q in passing from A to B. Generally $E = iVt$.

Thus since $V = iR$, $E = i^2 R t =$

$$\frac{V^2 t}{R}$$

Usually this work done or electrical energy released appears as heat in the conductor. Joule investigated this heat in 1841 and found that the heat produced per second in the conductor by passage of a current I is directly proportional to the square of the current, and the resistance of the conductor.

Separately, $H \propto I^2$.

and $H \propto R$

Thus combining the two relationships, we have

$$H \propto I^2 R.$$

This is called Joules law while the heat produced is called Joules heat.

$$\text{Generally, } H = I^2 R$$

(5.64)

Since H is the heat energy liberated from the conductor per second (E/t), it is the measure of electrical power P supplied.

$$\begin{aligned} \text{Thus } P &= I^2 R \\ &= IV \\ &= V^2/R \end{aligned}$$

(5.65)

(5.66)

(5.67)

The electrical power is measured in watts (W)

$$1 \text{ Kilowatt (KW)} = 10^3 \text{ watts}$$

$$1 \text{ Gigawatt (GW)} = 10^{12} \text{ watts.}$$

5.13 Solved Examples Download more from Learnclax.com

Example 5.1

A current of 5A flows along a copper wire of radius 0.5mm. If the wire contains 10^{27} electrons per unit volume, calculate the drift velocity of the electrons ($e = 1.6 \times 10^{-19}\text{C}$).

SOLUTION:

Let V_d be the drift velocity of the electron. In one second therefore, an electron will cover a distance of V_d (m). The volume in this distance = AV_d (m^3) where A is the area of cross-section of the wire.

Therefore, the number of electrons in this volume = $nA V_d$ where n is number of electrons per unit volume.

Therefore, total charge in this volume = $nA V_d e$ where e is electronic charge.

$$\text{But } I = \frac{\text{total charge } (q)}{\text{Time } (t)} = \frac{nAv_d e}{t} = nAv_d e$$

(since $t=1\text{s}$)

$$\therefore V_d = \frac{I}{nAe}$$

The radius of the wire, $r = 0.5\text{mm} = (0.5 \times 10^{-3})\text{m}$

$$\therefore A = \pi r^2 = 3.142 \times 0.25 \times 10^{-6} = 0.7855 \times 10^{-6} \text{ m}^2$$

$$\therefore V_d = \frac{5}{10^{27} \times 0.7855 \times 10^{-6} \times 1.6 \times 10^{-19}}$$

$$\begin{aligned}
 &= \frac{5}{0.7855 \times 1.6 \times 10^2} = \frac{5}{1.2568 \times 10^2} \text{ (ms}^{-1}\text{)} \\
 &= \frac{5 \times 10^2}{1.2568 \times 10^2} \text{ (cm s}^{-1}\text{)} \\
 &= \frac{5}{1.2568} \text{ (cm s}^{-1}\text{)} \\
 &= 3.9 \text{ cm s}^{-1}.
 \end{aligned}$$

EXAMPLE 5.2

- (i) A steel rod has length 80cm and radius 5mm. If the resistivity of steel at 20°C is 18×10^{-8} ohm, calculate the resistance of the steel rod.
- (ii) If the mean coefficient of resistivity of steel is $3 \times 10^{-3} \text{ K}^{-1}$, calculate the resistivity of the steel at 90°C.

SOLUTION:

(i) $\rho_{\text{steel}} = 18 \times 10^{-8} \text{ ohm} \cdot \text{m}$

$l = 80\text{cm} = 0.8\text{m}$

$r = 5\text{mm} = 5 \times 10^{-3}\text{m}$

Therefore, Area of cross-section of the steel rod

$$= \pi r^2 = 3.142 \times (5 \times 10^{-3})^2$$

$$= 3.142 \times 25 \times 10^{-6} \text{ m}^2.$$

$$\therefore \text{Resistance } R = \rho \frac{l}{A} = \frac{18 \times 10^{-8} \times 0.8}{3.142 \times 25 \times 10^{-6}}$$

$$= \underline{1.83 \times 10^{-3} \text{ ohms.}}$$

$$(ii) \quad \alpha = 3 \times 10^{-3} \text{ K}^{-1}$$

$$T - T_0 = 90 - 20 = 70\text{K}$$

$$\therefore \rho = \rho_0 \{1 + \alpha (T - T_0)\}$$

$$= 18 \times 10^{-8} \{1 + 3 \times 10^{-3} \times 70\}$$

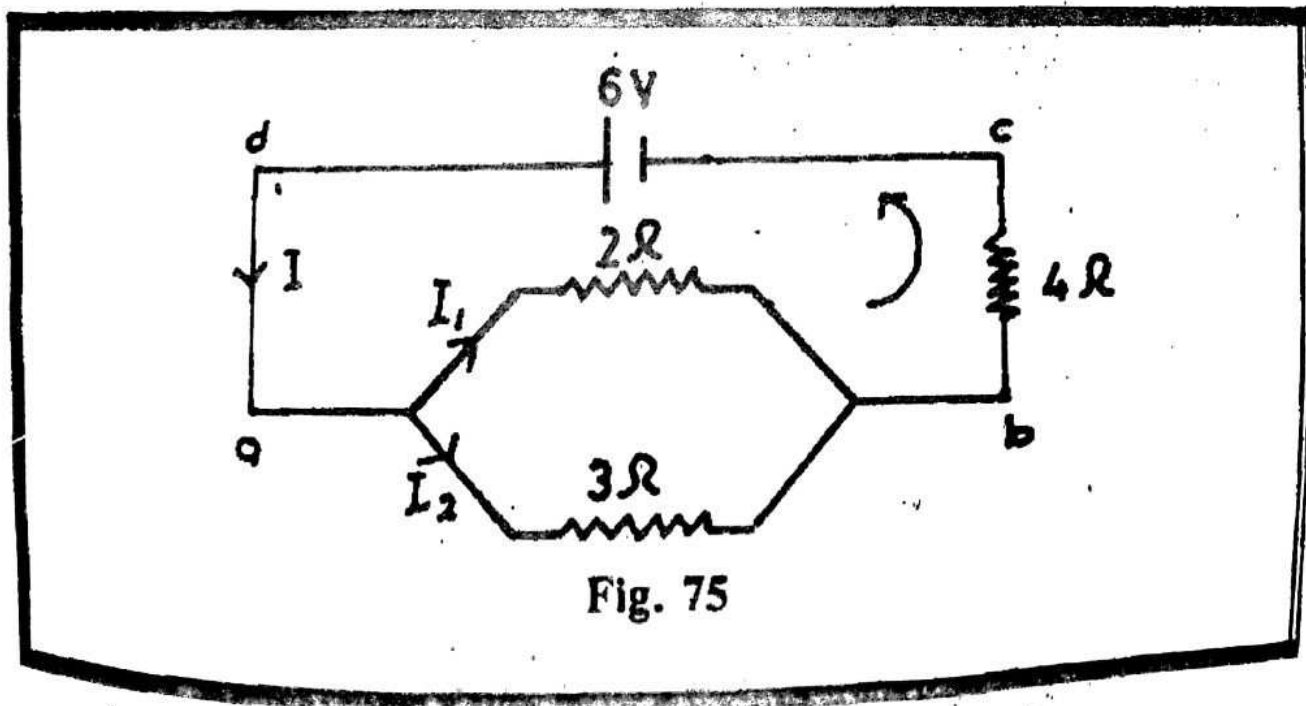
$$= 18 \times 10^{-8} (1 + 0.210)$$

$$= 18 \times 10^{-8} \times 1.210$$

$$= \underline{21.78 \times 10^{-8} \text{ Ohm m.}}$$

EXAMPLE 5.3

A battery of p.d 6 volts is connected in a circuit to two resistances of 2 ohms and 3 ohms through a resistance of 4 ohms as shown in fig. 75. Calculate the current flowing through each resistance (neglect internal resistance of the battery).



SOLUTION.

(i) To find I (the current flowing through the 4Ω - resistor) we first find R_{ab} which is the combined resistance of the resistances 2Ω and 3Ω in parallel.

$$\frac{1}{R_{ab}} = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

$$\therefore R_{ab} = \frac{6}{5} = 1.2\Omega$$

Therefore, combined resistance of the circuit,

$$R = R_{ab} + 4 = 1.2 + 4 = 5.2\Omega \text{ since } R_{ab} \text{ is now in series with the } 4\Omega \text{ resistance.}$$

But from ohm's law,

$$V = IR.$$

$$\therefore 6 = I \times 5.2$$

$$\therefore I = 6/5.2 = \underline{\underline{1.15 \text{ amp.}}}$$

(ii) Let the p.d. between a and b be V_{ab} .

$$\therefore V_{ab} = I_1 \times 2 = 2I_1$$

Also, $V_{ab} = 3I_2$

$$\therefore \frac{V_{ab}}{V_{ab}} = 1 = \frac{2I_1}{3I_2}$$

$$\therefore \frac{I_1}{I_2} = \frac{3}{2}$$

$$\therefore I_1 = \frac{3}{2} I_2$$

But $I_1 + I_2 = I = 1.15$ amp. Therefore substituting for I_1 we obtain

$$\therefore \frac{3}{2} I_2 + I_2 = 1.15$$

$$\therefore I_2 \left(\frac{3}{2} + 1 \right) = 1.15$$

$$\therefore \frac{5}{2} I_2 = 1.15$$

$$\therefore I_2 = 1.15 \times \frac{2}{5}$$

$$= 0.23 \times 2$$

$$= \underline{0.46 \text{ amp.}}$$

Also $I_1 = I - I_2$

$$= 1.15 - 0.46$$

$$= \underline{0.69 \text{ amp.}}$$

Alternatively, we can use Kirchhoff's Law. To do this, we note that $I = I_1 + I_2$. Let us traverse the two loops, aPbcd and aQbcd both in anticlockwise direction. We will obtain the equations

$$\begin{aligned}
 6 - 2I_1 - 4(I_1 + I_2) &= 0 & (1) \\
 \text{ie } 6 - 6I_1 - 4I_2 &= 0 & (2) \\
 \text{And } 6 - 3I_2 - 4(I_1 + I_2) &= 0 & (3) \\
 \text{ie } 6 - 4I_1 - 7I_2 &= 0 & (4) \\
 \text{Equation (2)} \times 7 \Rightarrow 42 - 42I_1 - 28I_2 &= 0 & (5) \\
 \text{Equation (4)} \times 4 \Rightarrow 24 - 16I_1 - 28I_2 &= 0 & (6) \\
 \text{Equation (5)} - (6) \Rightarrow 18 - 26I_1 &= 0. & (7)
 \end{aligned}$$

$$\therefore I_1 = 18/26 = \underline{\underline{0.69 \text{ amp.}}}$$

Substituting in (2) we obtain

$$6 - 6 \times 0.69 - 4I_2 = 0$$

$$\therefore 6 - 4.14 - 4I_2 = 0$$

$$\therefore 1.86 - 4I_2 = 0$$

$$\therefore I_2 = \frac{1.86}{4} = \underline{\underline{0.46 \text{ amp.}}}$$

$$\begin{aligned}
 \therefore I &= I_1 + I_2 = 0.69 + 0.46 \\
 &= \underline{\underline{1.15 \text{ amp.}}}
 \end{aligned}$$

EXAMPLE 5.4

In the given circuit (fig. 76) calculate the current flowing through the branches AC, AB, DA, DC, CB, and BD.

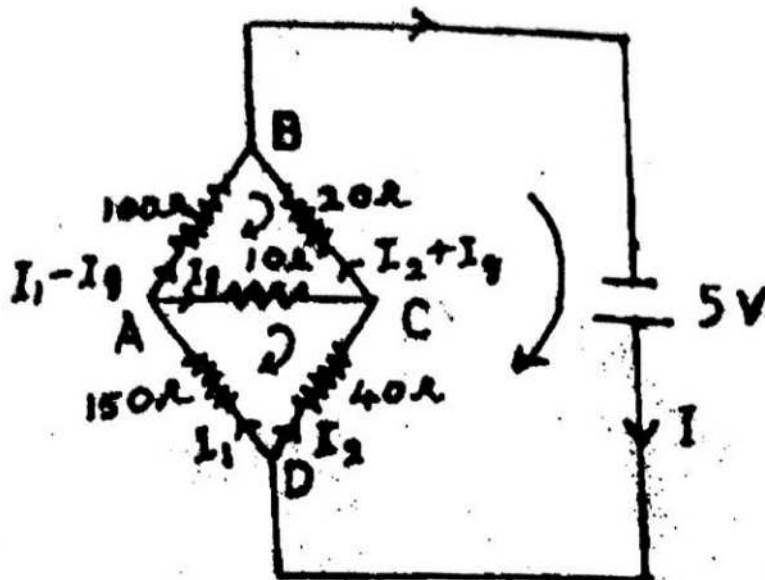


Fig. 76

SOLUTION:

Traversing the loop ABCA in clockwise direction, we obtain,

$$20(I_2 - I_3) + 10I_3 - 100(I_1 - I_3) = 0$$

$$\therefore 20I_2 - 20I_3 + 10I_3 - 100I_1 + 100I_3 = 0$$

$$\therefore 20I_2 - 100I_1 + 90I_3 = 0 \quad (1)$$

Traversing ADCA in clockwise direction we get,

$$40I_2 - 150I_1 - 10I_3 = 0 \quad (2)$$

$$\text{Equation (1)} \times 40 \Rightarrow 800I_2 - 4000I_1 + 3600I_3 = 0 \quad (3)$$

$$\text{Equation (2)} \times 20 \Rightarrow 800I_2 - 3000I_1 - 200I_3 = 0 \quad (4)$$

$$(3) - (4) \Rightarrow -1000I_1 + 3800I_3 = 0$$

$$\therefore I_1 = \frac{3800 I_3}{1000} = \underline{\underline{3.8 I_3}} \quad (5)$$

Substituting in equation (1) we obtain,

$$20I_2 - 100 \times 3.8I_g + 90I_g = 0$$

$$\therefore 20I_2 - 380I_g + 90I_g = 0$$

$$\therefore 20I_2 - 290I_g = 0$$

$$\therefore I_2 = \frac{290I_g}{20} = \underline{14.5I_g} \quad (6)$$

Traversing loop BCDB, in clockwise direction, we obtain,

$$5 - 40I_2 - 20(I_2 + I_g) = 0$$

$$\therefore 5 - 60I_2 - 20I_g = 0 \quad (7)$$

Substituting for I_2 in (7), we obtain

$$5 - 60 \times 14.5I_g - 20I_g = 0$$

$$\therefore 5 - 870I_g - 20I_g = 0$$

$$\therefore 5 - 890I_g = 0$$

$$\therefore I_g = 5/890 = \underline{0.0056 \text{ amp}}$$

$$\therefore I_1 = 3.8 \times 0.0056 = \underline{0.0213 \text{ amp}}$$

$$\text{And } I_2 = 14.5 \times 0.0056 = \underline{0.0812 \text{ amp}}$$

$$\therefore I_{AC} = I_g = \underline{0.0056 \text{ amp}}$$

$$I_{AB} = I_1 - I_g = 0.0213 - 0.0056$$

$$= \underline{0.0157 \text{ amp}}$$

$$I_{DA} = I_1 = \underline{0.0213 \text{ amp}}$$

$$I_{DC} = I_2 = \underline{0.0812 \text{ amp.}}$$

$$I_{CB} = I_2 + I_g = 0.0812 + 0.0056$$

$$= \underline{0.0868 \text{ amp.}}$$

$$I_{BD} = I = I_1 + I_2 = 0.0213 + 0.0812$$

$$= \underline{0.1025 \text{ amp.}}$$

EXAMPLE 5.5

- (i) A heating coil has resistance of 1.5×10^2 ohms. If it is plugged to a 240 volt supply, what amount of heat will it deliver in 5 mins.
- (ii) If the coil is immersed into a liquid of mass 1.2kg, temperature 25°C and s.h.c. $3.2 \times 10^3 \text{ J Kg}^{-1} \text{ K}^{-1}$, calculate the temperature of the liquid after 5 mins.

SOLUTION.

- (i) $V = 240$ volts
 $R = 1.5 \times 10^2 \Omega$, time = 5 min = 5×60 s.

Heat energy produced, $E = IVt$

But $V = IR$

$$\therefore I = \frac{V}{R} = \frac{240}{1.5 \times 10^2} = 1.6 \text{ amp}$$

$$\therefore E = 1.6 \times 240 \times 5 \times 60$$

$$= \underline{115200 \text{ J}}$$

$$E = 115200 = mst.$$

- (ii) where m is mass of liquid, s is its s.h.c. and t is its increase in temperature.

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Therefore, if its temperature after 5 min is $T^{\circ}\text{C}$, then

$$T - 25 = \frac{115200}{1.2 \times 3.2 \times 10^3} = \frac{115.2}{1.2 \times 3.2}$$
$$= \underline{30}.$$

$$\therefore T = 30 + 25 = 55^{\circ}\text{C}$$

5.14 Solved Examples

5.14.1

Two resistance of value 2Ω and 3Ω are connected in parallel before a 6-v battery is connected across them. If a current of 2 amps flows in the circuit, calculate the internal resistance of the battery and the amount of current that flows through each resistance.

5.14.2

A resistance of 2000Ω is connected in series with another resistance of 1000Ω , and both are connected to a 120-volt source. Calculate the current in the circuit and the p.d. across each resistance.

5.14.3

Four resistors AB, BC, CD and DA of resistances 2Ω , 4Ω , 2Ω and 4Ω respectively are connected to form a closed loop, and a 4-v battery of negligible internal resistance is connected externally to A and C. Calculate:

- (i) the p.d. between BD
- (ii) the current flowing through each resistance
- (iii) the value of the additional resistance which

must be connected between A and D so that no current flows through a galvo connected between B and D.

- 5.14.4 A thin carbon wire of diameter 0.8mm has length of 1.5m. If its resistivity at room temperature (27°C) is 3.8×10^{-5} ohm metre, calculate the resistance of the wire. If the temperature of the wire rises to 80°C , calculate its resistivity and hence the new resistance of the wire.
- 5.14.5 A wire with resistance of 20Ω is drawn out so that its length doubles. If the resistivity and density are not changed during the process, then calculate the new resistance of the wire.
- 5.14.6 A small current of 10^{-8} amp flows in a copper wire of diameter 0.72mm. Calculate the electron drift speed.
- 5.14.7 A battery of emf 6 volts and internal resistance 1.5Ω is connected across a Wheatstone Bridge as shown in fig. 77. Calculate the current flowing through each branch of the circuit for the given resistances.

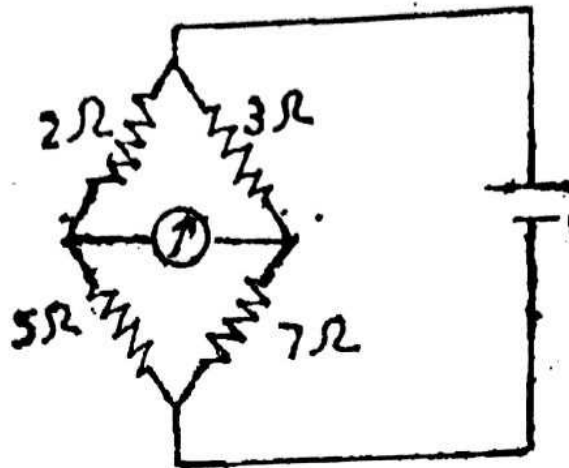


Fig. 77

- 5.14.8 A three loop circuit is arranged as shown in fig. 78. If the internal resistances of the two cells A and B are 1.0Ω and 1.5Ω respectively, calculate the current flowing through each branch of the circuit.

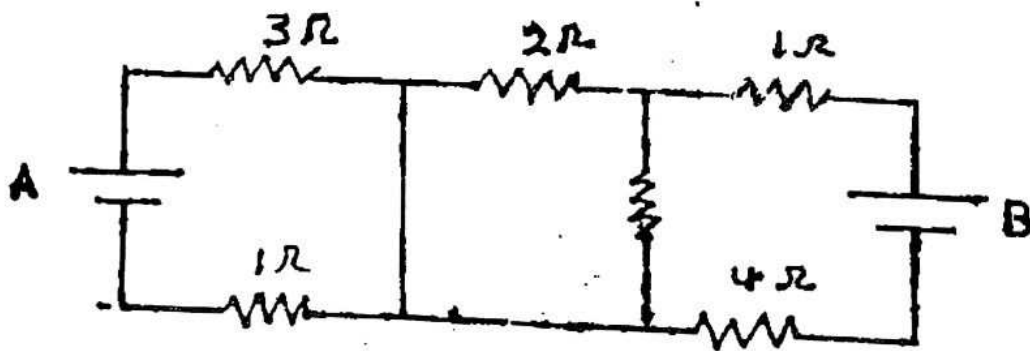


Fig. 78.



MAGNETIC FIELD AND INDUCTION

The region around a magnet or a current-carrying conductor where a magnetic force is experienced is called a magnetic field. The magnetic field exists around a moving charge in addition to the electrostatic field which always surrounds the charge whether it is in motion or not. The electrostatic field or simply electric field, set up by moving charges is comparatively very small, such that the electrostatic force on a moving charge is negligible in comparison with the magnetic force.

A magnetic field can be represented by magnetic field lines in the same way an electric field is represented by electric field lines. The magnitude, sense and direction of the magnetic field is represented by a vector quantity, $\mathbf{B}$, called the **magnetic flux density** or simply **magnetic induction**.

By convention, magnetic field lines are drawn such that the tangent to the field line at a point gives the direction of $\mathbf{B}$ at that point; and the number of these lines per unit area normal to their direction is made equal to the magnitude of the induction, $\mathbf{B}$. Hence the induction at a point can be expressed in "**lines per unit area**". A line of induction is called a **weber** and the magnetic induction $\mathbf{B}$ is expressed in **weber per square meter** (W/m^2). The total number of lines of induction threading through a surface is called the **magnetic flux** through the surface and is denoted by Φ . If $\mathbf{B}$ is uniform and normal to a surface of area A then

$$\Phi = BA$$

(6.1)

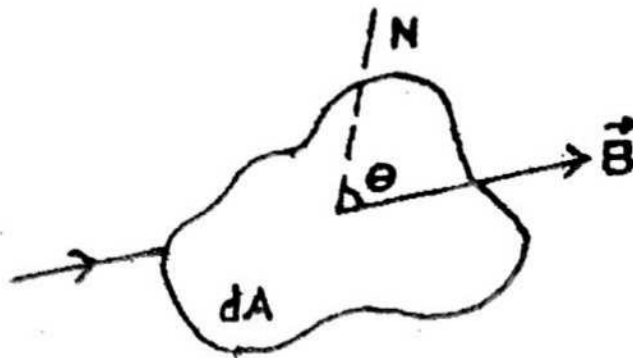


Fig. 77a.

If $\vec{B}$ varies from point to point on a surface and if the surface is not everywhere at right angles to the field, then consider an infinitesimal element of area dA whose normal, N , makes an angle θ with $\vec{B}$. The projected area of dA perpendicular to the field is $dA \cos \theta$ and the flux through it is $B \cos \theta dA$. The magnetic flux through the whole surface is expressed in webers.

$$\Phi = (B \cos \theta dA) \quad (6.2)$$

6.1 Magnetic Force On A Moving Charge

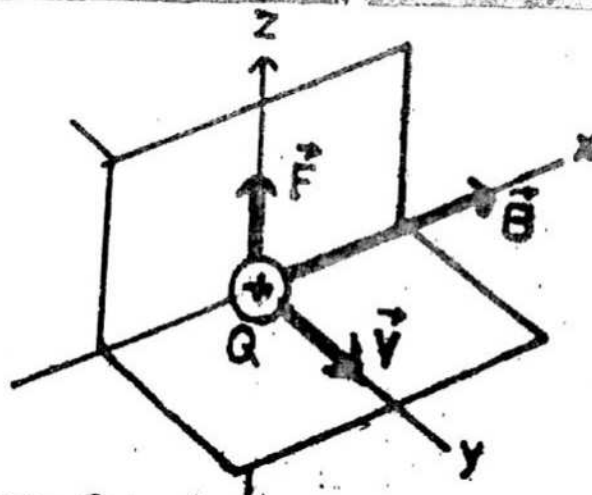


Fig. 77 b. Charge Q moving perpendicular to a uniform magnetic field.

Fig. 77 b. shows a region throughout which the magnetic lines of induction are straight, parallel to the X-axis and uniformly spaced. A positive charge Q , moving with velocity $\mathbf{v}$ perpendicular to $\mathbf{B}$ experiences a force $\mathbf{F}$ in the direction shown, perpendicular both to $\mathbf{v}$ and $\mathbf{B}$. The magnitude of this force is

$$F = QvB \quad (6.3)$$

in vector notation

$$\mathbf{F} = Q \mathbf{v} \times \mathbf{B} \quad (6.4)$$

The relation between the directions of $\mathbf{F}$, $\mathbf{v}$ and $\mathbf{B}$, which are mutually perpendicular, is obtained using the left-hand rule, fig. 78

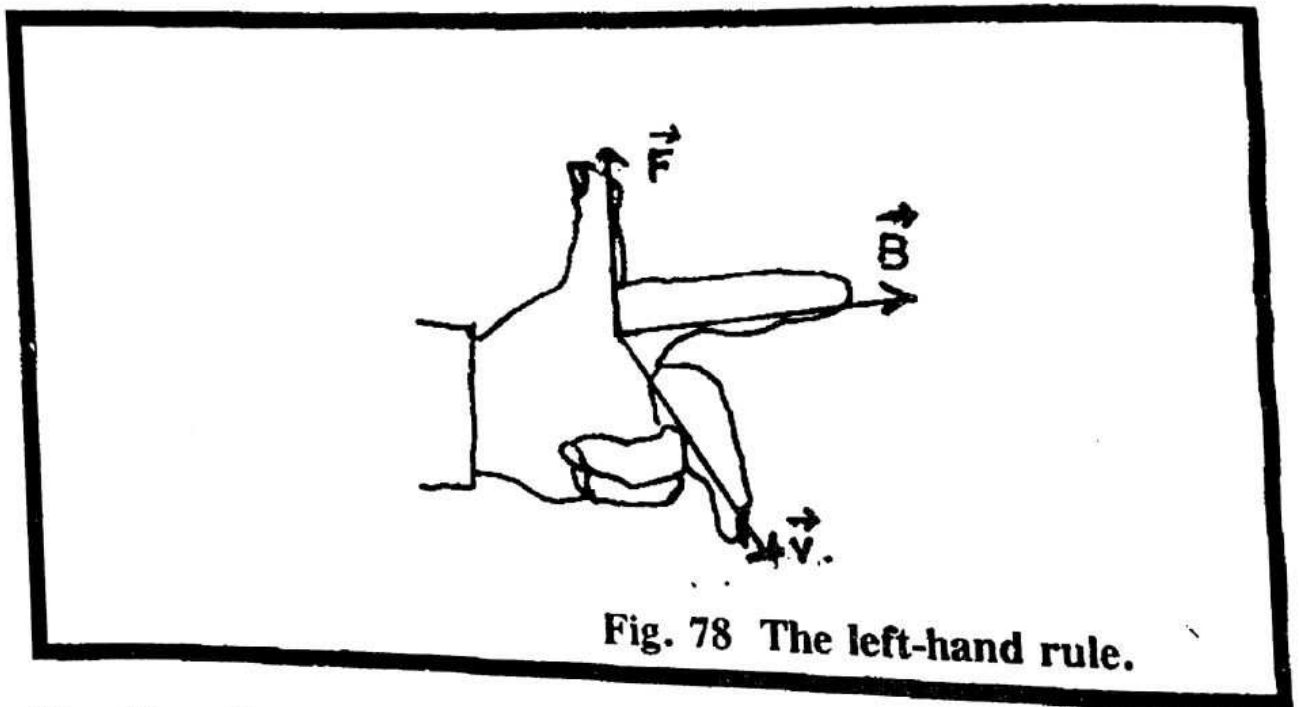


Fig. 78 The left-hand rule.

The First finger points in the direction of the Field; the Second finger in the direction of the velocity (or speed); and the Thumb points in the direction of the force or (Thrust).

If the velocity of the moving charge is not perpendicular to the direction of the field, say, it makes an angle θ with the induction vector as shown in fig. 79, the magnitude

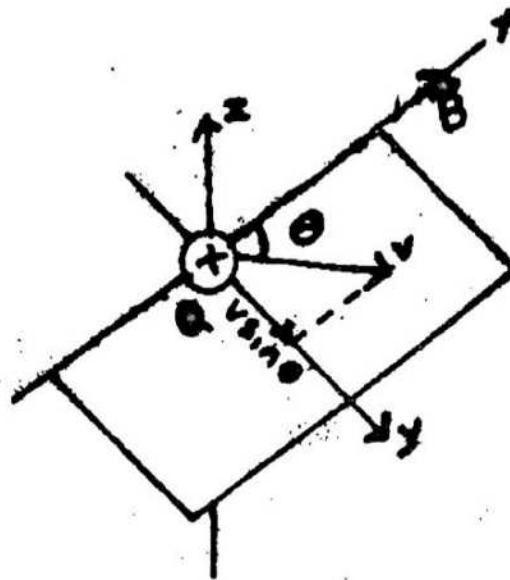


Fig. 79: Charge moving at an angle to a uniform magnetic field.

of the force exerted on the charge is given by

$$F = BQv\sin\theta \quad (6.5)$$

Hence the force is proportional to the component of the velocity ($v \sin\theta$) perpendicular to the direction of the magnetic induction. The direction of the force is always perpendicular to the plane determined by v and B . When the angle θ between v and B , is zero then V is either parallel or antiparallel to the direction of B , and the magnetic force vanishes. Equation (6.5) may be used to define the magnetic induction as

$$B = \frac{F}{Qv\sin\theta}$$

which in words can be stated thus: The magnetic of the magnetic induction at a point is the quotient obtained when the force on a moving charge at the point is divided by the product of the charge and its component of velocity normal to the induction.

Note: (a) The unit of **B** can be deduced

$$B = \frac{F}{Qv\sin\theta} = \frac{N}{Cms^{-1}} = \frac{N}{AM} = NA^{-1} M^{-1}$$

The unit of **B** is thus $1NA^{-1} m^{-1}$ but for convenience we define a new unit called the weber (Wb) such that

$$1 Wbm^{-2} = 1NA^{-1} m^{-1}$$

or $1 Wb = 1NmA^{-1} = 1Vs$

The unit Wbm^{-2} is consistent with the idea that **B** is a flux density.

(b) The S.I. unit of **B** is called the tesla (T), defined so that

$$1T = 1Wbm^{-2} = 1NA^{-1} m^{-1}$$

(c) **Orders of magnitude**

In the earth's field **B** is about $5 \times 10^{-5}T$. At the centre of a circular coil of 1000 turns each of radius $10^{-1}m$ and carrying a current of 1.5A, **B** is about $10^{-2}T$. Between the poles of a large electromagnet, **B** is about 2T.

(d) If a charged particle moves through a region in which both an electric field and a magnetic field are present the resultant force is

$$F = QE + Qv \times B$$

$$F = Q(E + V \times B)$$

Equation (6.6) is sometimes called the Lorentz force.

(6.6)

(e) An earlier unit for **B** still in common use is the Gauss (G) such that $1T = 10^4G$.

6.2 Magnetic Force On A Current-Carrying Conductor

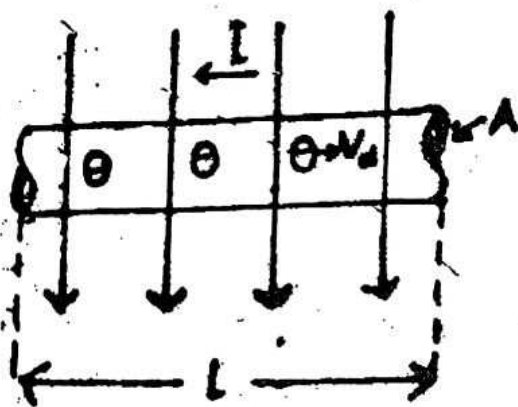


Fig. 80 (a): A current-carrying wire placed at right angles to a uniform Magnetic Field, B .

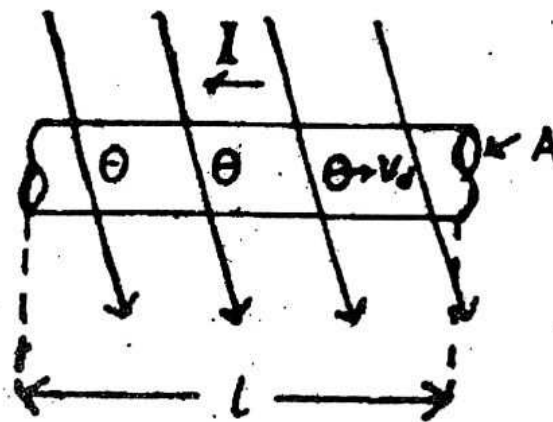


Fig. 80 (b): Uniform Magnetic field B acting at an angle θ to the current-carrying wire.

Fig. 80 (a) shows a wire carrying a current I , cross-sectional area A , and the velocity drift of the conduction electrons v_d . The conductor will experience a magnetic force directed out of the paper from the agent responsible for establishing the uniform magnetic field round the conductor. The current, I , in a metal is carried by conduction electrons, n being the number of such electrons per unit volume of the wire. The magnitude of the average force on one such electron is given by

$$F_e = Bev_d$$

where V_d is the velocity drift of the electron

$$V_d = \frac{I}{nAe}$$

Therefore, $F_e = Be (I/nAe)$ and it is directed out of the paper. If a length, L , of the conductor is available there are nAL conduction electrons the electrons experience a total force F given by

$$F = (nAL).Be \left(\frac{I}{nAe} \right)$$

$$F = BIL$$

If the field makes an angle θ with the wire

$$F = \mathbf{IL} \times \mathbf{B}$$

$$F = BI L \sin \theta$$

(6.7)

where L is a vector whose magnitude is the length of the wire and which points along the (straight) wire in the direction of the current.

6.3 Motion Of Charged Particles In A Magnetic Field

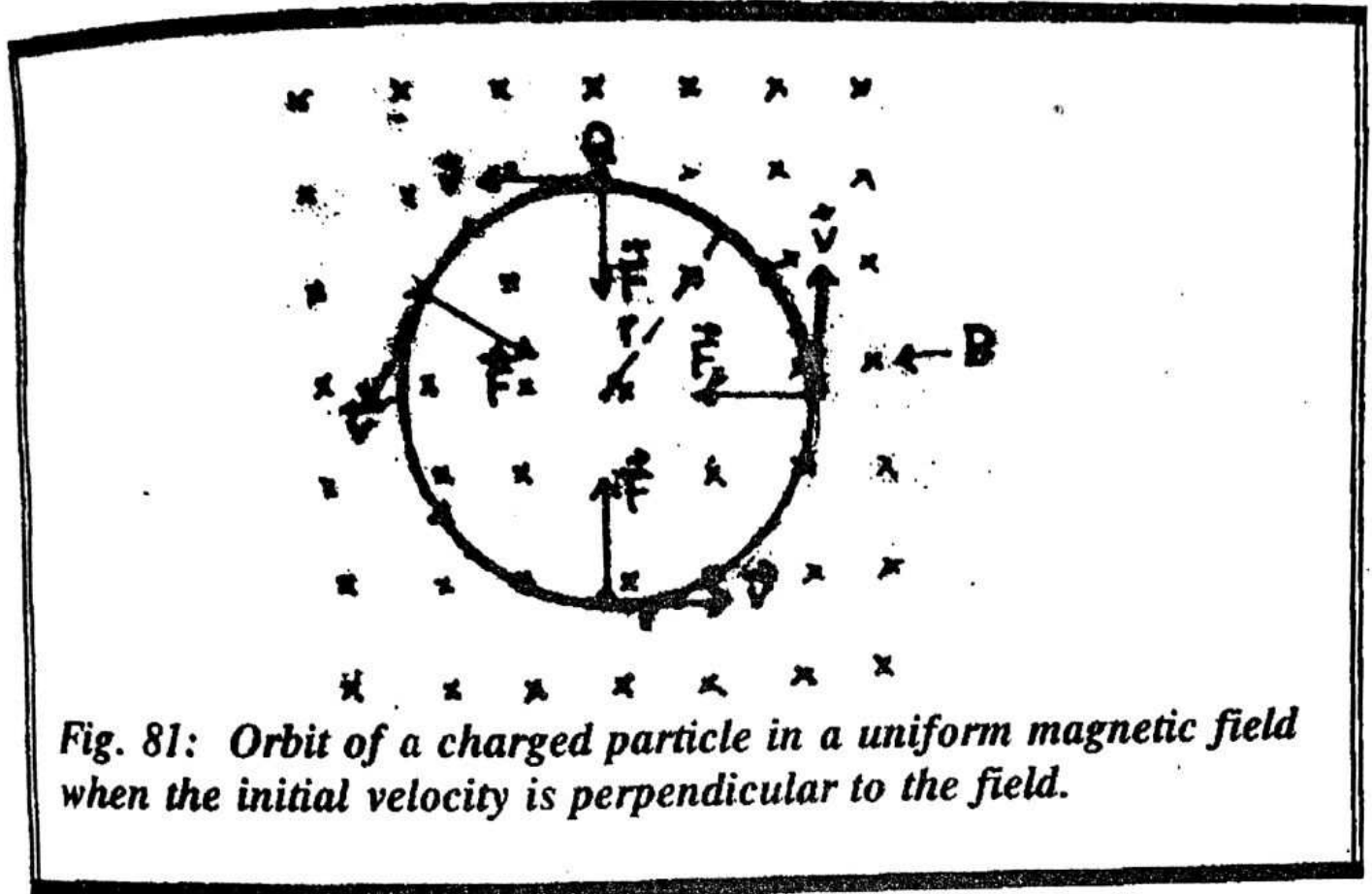


Fig. 81: Orbit of a charged particle in a uniform magnetic field when the initial velocity is perpendicular to the field.

Let a positively charged particle of charge Q , at point O , in a uniform magnetic field B , be given a velocity V in a direction at right angles to the field. An upward constant force F defined by (6.3) is exerted on the particle at this point. Since the force is at right angles to the velocity, it does not change the magnitude of this velocity but merely alters its direction. The particle therefore moves under the influence of a force whose magnitude is constant but whose direction is always at right angles to the velocity of the particle. The orbit of the particle is, therefore, a circle described with constant tangential speed, the force F being the centripetal force.

Since centripetal force
$$\frac{mv^2}{r}$$

$$QvB = \frac{mv^2}{r}$$

and the radius of the circular orbit is

$$r = \frac{mv}{BQ} \quad (6.8)$$

The angular velocity, ω , is given by

$$\omega = \frac{v}{r} = \frac{BQ}{m} \quad (6.9)$$

The frequency, f , is given by

$$f = \frac{\omega}{2\pi} = \frac{BQ}{2\pi m} \quad (6.10)$$

The frequency, f , in (6.10) is a characteristic frequency for the charged particle in the magnetic field and it is called the **Cyclotron Frequency** of the particle in the field because the particle circulates at this frequency in a cyclotron. The angular velocity, ω (and hence the period $T = 2\pi/\omega$) is independent of the speed of the particle, while (6.8) shows that the radius of the orbit is determined by the speed.

If the initial velocity makes an angle θ with $\mathbf{B}$, then the particle moves in a helix, the axis of which is parallel to $\mathbf{B}$. In this case, v in (6.9) is the component of velocity perpendicular to $\mathbf{B}$ (i.e. $v \sin \theta$) and the particle moves with a steady axial speed $v \cos \theta$ along the direction of $\mathbf{B}$.

6.4 Worked Examples

Example 6.1: The simultaneous influence of an electric field of intensity $3.4 \times 10^5 \text{ V/m}$ and a magnetic field of flux density $2.0 \times 10^{-3} \text{ Wb/m}^2$, both fields being normal to the beam and to each other, produces no deflection of a beam of electrons moving with velocity, v . Calculate the value of v and deduce the radius of the electron orbit when the electric field is removed.

Solution: When there is no deflection of the beam the electrostatic force, $F_E = QE$ equals the magnetic force $F_m = BQv$.

$$QE = BQv$$

$$v = \frac{E}{B} = \frac{3.4 \times 10^5}{2.0 \times 10^{-3}}$$

$$v = 1.7 \times 10^8 \text{ m/s}$$

Second part: When the electric field is removed, the beam of electrons move in circular orbit whose radius is given by (8)

$$r = \frac{mv}{BQ}$$

$$r = \frac{9.11 \times 10^{-31} \times 1.7 \times 10^8}{2.0 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

$$r = 0.48 \text{ m}$$

Example 6.2: Determine the cyclotron frequency of an electron circulating in an orbit of radius 11cm at right angles to a magnetic field of induction $1.0 \times 10^{-4} \text{ w/m}^2$

Solution: The cyclotron frequency is given by (10).

$$F = \frac{BQ}{2\pi m}$$

$$F = \frac{1.0 \times 10^{-4} \times 1.6 \times 10^{-19}}{2\pi \times 9.11 \times 10^{-31}}$$

$$F = 2.8 \times 10^6 \text{ rev/sec.}$$

Example 6.3: A wire 50cm long and carrying an electron current of 2.0 amps is located in a uniform magnetic field in which $B = 10^{-2} \text{ w/m}^2$. Calculate the force on the wire when it makes an angle of 60° with the field direction.

Solution: Since θ is measured in the direction of B , $\theta = 60^\circ$ substituting in (7), $F = BIL \sin \theta$

$$F = 10^{-2} \times 2.0 \times 0.5 \times 0.866$$

$$F = 8.66 \times 10^{-3} \text{ N}$$



6.5 PROBLEMS.

- 6.5.1. Calculate the length of a wire placed at 45° to a magnetic field of flux density 0.15 w/m^2 if the current in the wire is 10.0 A when it experiences a force of 2.12 N .
- 6.5.2. Write down the expression for the magnetic force, F , on a charge, Q , moving with velocity, v , in a magnetic field of flux density B . Hence show that the magnitude of the magnetic force F on a straight conductor of length, L , carrying a current I in a uniform field of flux density B is given by
 $F = BIL \sin\theta$ where θ is the angle the field makes with the conductor
- A straight conductor of length 6.0 cm , carries a current of 1.5 A and experiences a force of $4.5 \times 10^{-3} \text{ N}$ when placed in a uniform magnetic field of flux density $4.0 \times 10^{-2} \text{ w/m}^2$. What is the angle between the direction of the magnetic field and that of the current.
- 6.5.3. Determine the ratio of the magnetic force to the weight for an electron projected into a magnetic field of flux density 9.8 w/m^2 moving with a velocity of $3.0 \times 10^7 \text{ m/s}$ in a direction at right angles to the field.
- 6.5.4. Compute the radius of the orbit and the period of revolution for an electron projected with a velocity of $1.9 \times 10^6 \text{ m/s}$ into a uniform magnetic field of magnetic induction $5.0 \times 10^{-3} \text{ w/m}^2$.
- 6.5.5. A positively charged particle is shot with a speed of $2.5 \times 10^7 \text{ m/s}$ through a region where the magnetic induction is 0.05 w/m^2 . If the particle moves along a direction that makes an angle of 30° with the magnetic field

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and experiences a force of $9.6 \times 10^{-12} \text{ N}$. What is the charge of the particle.

6.5.6. An electron with its charge of $1.6 \times 10^{-19} \text{ C}$ moves with one-tenth the speed of light across a magnetic field of $2.0 \times 10^{-2} \text{ W/m}^2$. Calculate the magnitude of the mechanical force acting on the electron.

6.5.7. A current of 30 amps flows through a straight wire 12 cm long. If this wire is placed at right angles to a field of 0.02 W/m^2 what is the force on the wire.

7

THE MAGNETIC FIELD RELATED TO ITS CAUSE.

7.1 Magnetic Field Created By Moving Charges.

In chapter 6 the effect of a magnetic field on a moving charge was discussed. In this chapter we discuss the cause of the magnetic fields. Other charges moving in these fields experience forces. More commonly the moving charges setting up a field are those constituting current in a conductor.

7.2 The Biot-Savart Law.

7.2.1. The Magnetic Field of a Moving Charge

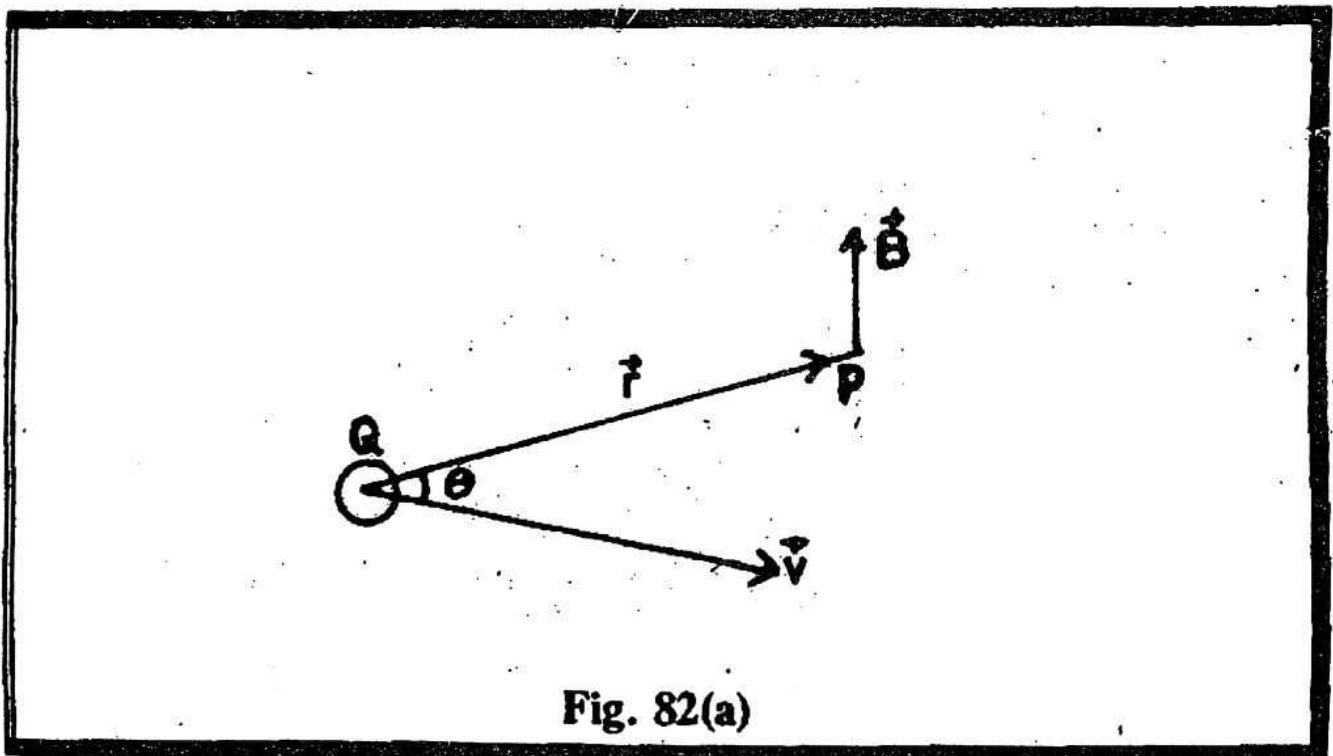


Fig. 82(a)

Consider the field $\vec{B}$ produced at the point of observation P by a single point positive charge Q moving with a velocity $\vec{v}$ relative to the observer. Point P has a position vector $\vec{r}$ with respect to Q .

The magnitude of the magnetic field at P is found experimentally to be proportional to

$$\frac{Qv\sin\theta}{r^2}$$

B is found experimentally to be perpendicular to the plane determined by **r** and **v**. We can write the constant of proportionality as $\mu_0/4\pi$ leading to the equation

$$B = \left(\frac{\mu_0}{4\pi}\right) \frac{Qv\sin\theta}{r^2} \quad (7.1)$$

μ_0 is called the permeability of free space, the permeability constant or simply the magnetic constant. It has the assigned value, in SI units, of exactly

$$\mu_0 = 4\pi \times 10^{-7} \text{ wbA}^{-1} \text{ (or TA}^{-1}\text{m)}$$

μ_0 can have the unit $\text{wbA}^{-1} \text{ m}^{-1}$, but is more commonly expressed in the equivalent unit henry per metre (Hm^{-1}).

Expressed in vector form equation (7.1) becomes

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right) Q \frac{\vec{v} \times \vec{r}}{r^3} \quad (7.2)$$

- (i) If $\theta = 0$, $B = 0$. Hence **B** is zero at all points on a line through the charge in the direction of **v**.
- (ii) For a given value of r , **B** is a maximum for $\theta = 90^\circ$. Hence at a given distance r from the charge, **B** is a maximum in the plane through the charge perpendicular to **v**.

- (iii) The lines of induction of the B field are circles in planes perpendicular to the velocity vector with centres on a line through the vector.

7.2.2. The magnetic Field of a Current Element

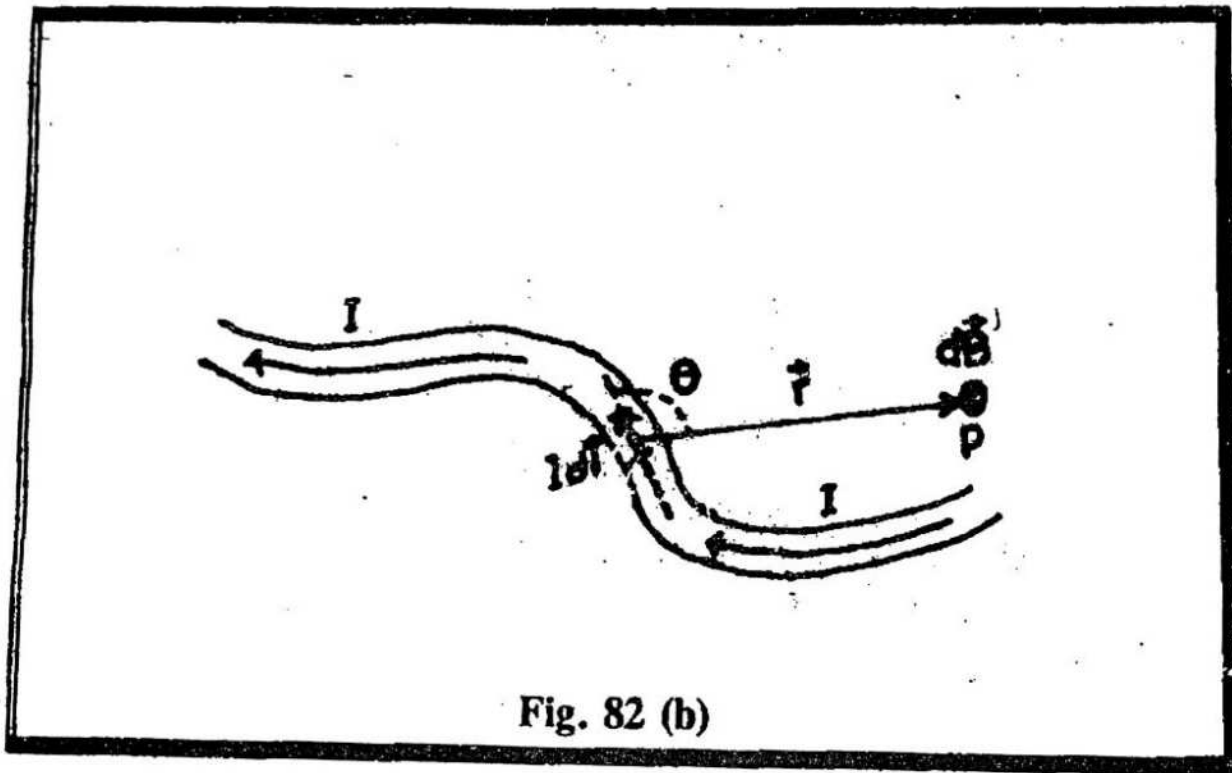


Fig. 82 (b)

The magnetic field set up at any point P by the current I in a circuit is the resultant (vector sum) of the fields due to all the moving charges in the circuit. The circuit is to be divided, in imagination, into short elements of length dl . Volume of element is Adl where A is the cross sectional area.

Let the number of charge carriers per unit volume be n and let q be the charge carried by each carrier. Total charge dQ in the element is

$$dQ = nqAdl.$$

The moving charges are, therefore, equivalent to a single charge dQ travelling with the drift velocity $\mathbf{v}$.

$$\begin{aligned} \text{Hence } dB &= \left(\frac{\mu_o}{4\pi} \right) \frac{dQv \sin\theta}{r^2} \\ &= \frac{\mu_o}{4\pi} \cdot \frac{nqv A dl \sin\theta}{r^2} \end{aligned}$$

But $nqvA$ = the current I in the element. Thus

$$dB = \frac{\mu_o}{4\pi} \frac{Idl \sin\theta}{r^2} \quad (7.3)$$

or in vector form

$$d\mathbf{B} = \frac{\mu_o}{4\pi} \frac{Idl \times \mathbf{r}}{r^3} \quad (7.4)$$

where $d\mathbf{l}$ is a differential element of the length of the wire pointing along the tangent to the wire in the direction of I .

The direction of $d\mathbf{B}$ is that of a vector $d\mathbf{l} \times \mathbf{r}$, namely into the plane of the figure. We find the resultant field $\mathbf{B}$ at point P by integrating the contributions $d\mathbf{B}$ from all current elements Idl that make up the arbitrary current distribution.

Thus

$$\vec{B} = \frac{\mu_o}{4\pi} \int \frac{Id\vec{l} \times \vec{r}}{r^3} \quad (7.5)$$

Equation (7.3) or (7.4) is called the **Biot-Savart Law**, a magnetic equivalent of Coulombs law. All the expressions above hold strictly only for conductors in vacuum or in the vicinity of non-ferromagnetic materials.

7.3 The Magnetic Fields Established By Complete Circuits.

The Biot-Savart law can be used to find B for various current configurations.

7.3.1. The Field B due to a long straight wire.

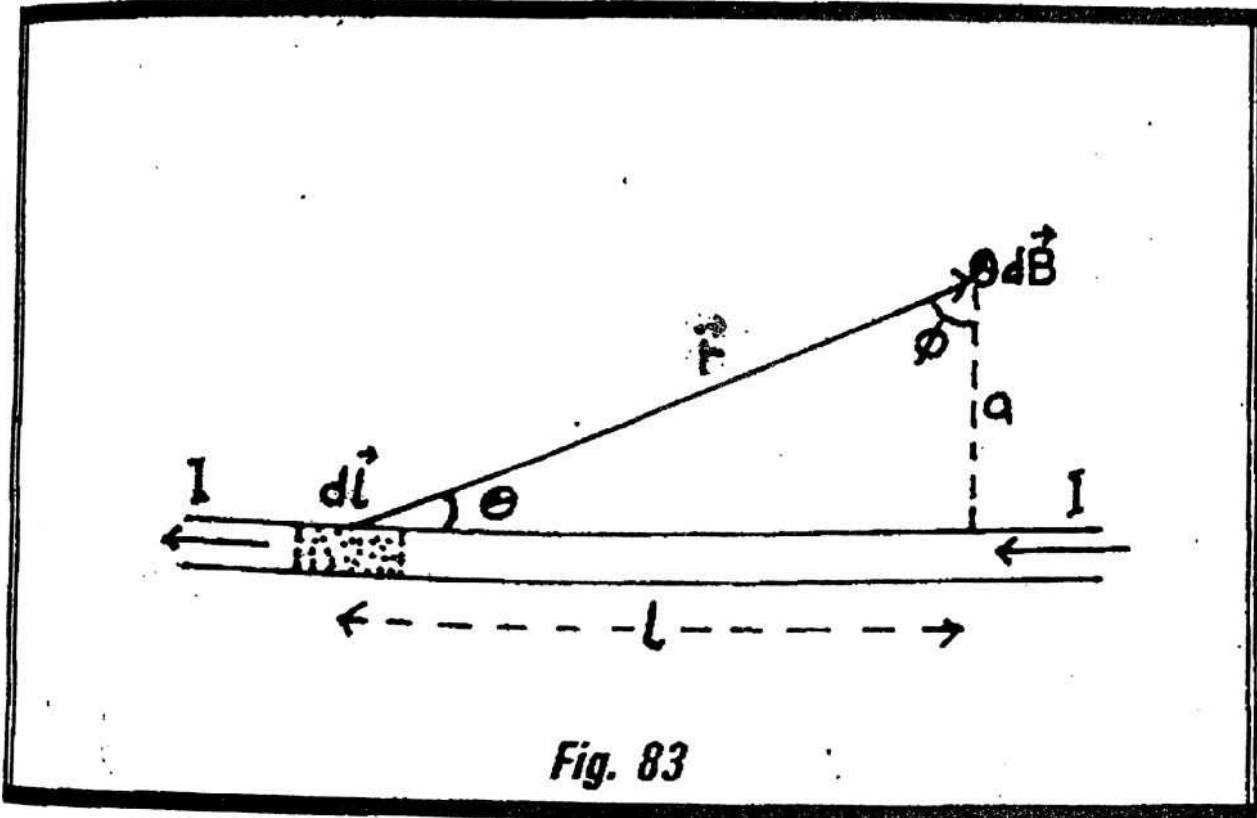


Fig. 83

The magnitude of $d\vec{B}$ set up at P by the current element Idl is

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2} \quad (7.6)$$

The direction of $d\vec{B}$ is that of $dl \times r$, i.e into the plane of the figure at right angles. We note that $d\vec{B}$ for every current element has the same direction. Thus to find $\vec{B}$ at point P we simply integrate equation (7.6) above

To proceed we write all variables in terms of ϕ :

$$\begin{aligned} l &= a \tan \phi \\ dl &= a \sec^2 \phi d\phi \\ r &= a \sec \phi, \sin \theta = \cos \phi \end{aligned}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I a \sec^2 \phi d\phi \cos \phi}{a^2 \sec^2 \phi}$$

or

$$dB = \frac{\mu_0}{4\pi} \frac{I}{a} \cos \phi d\phi$$

$$B = \frac{(\mu_0)}{4\pi} \frac{I}{a} \int_{\phi_1}^{\phi_2} \cos \phi d\phi$$

$$B = \frac{(\mu_0)}{4\pi} \frac{I}{a} (\sin \phi_2 - \sin \phi_1)$$

for the special case of infinitely long wire,

$\phi_2 = \pi/2$ radians, $\phi_1 = -\pi/2$ radians, and

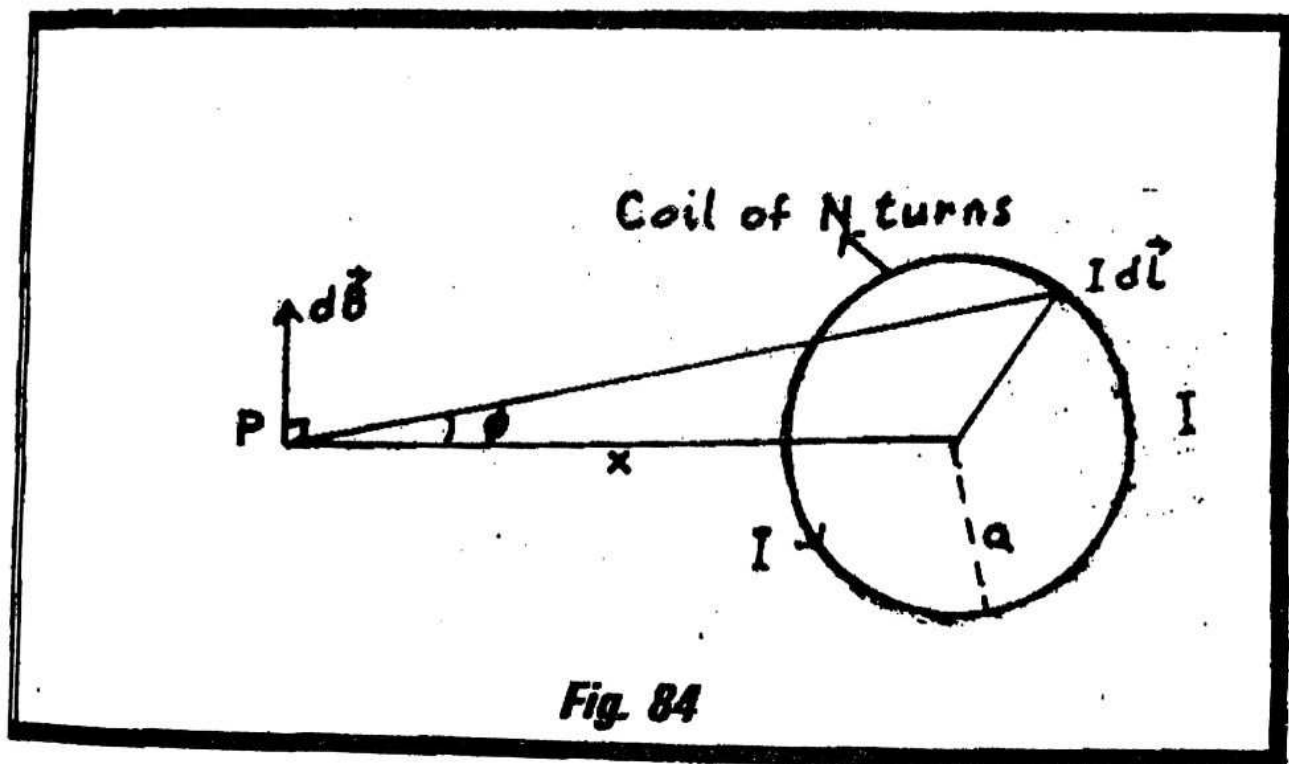
$$B = \frac{\mu_0 I}{2\pi a} \quad (7.7)$$

7.3.2. B at the centre of circular coil.

Suppose the coil has N turn's each of radius r and carries a current I . The field at the centre is given by

$$\begin{aligned}
 B &= \int dB \\
 &= \frac{(\mu_0)}{4\pi} \frac{I \sin 90^\circ}{r^2} \int dl \\
 &= \frac{\mu_0 IN}{2} \cdot \frac{1}{r} \quad (7.8)
 \end{aligned}$$

7.3.3. B at any point on the axis of a circular coil.



We sum $d\vec{B}$, taking its direction into account

- (i) By symmetry those components of $d\vec{B}$ perpendicular to the axis sum to zero.
- (ii) Each $d\vec{B}$ has a component parallel to the axis, of magnitude

$$dB \cos (90 - \phi) = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^\circ \sin \phi}{r^2}$$

Hence the magnetic field at P is given by

$$B = \int \frac{(\mu_0)}{4\pi} \frac{Idl}{r^2} \sin \phi$$

$$\text{or } B = \frac{(\mu_0)}{4\pi} = \frac{2\pi a N I \sin \phi}{r^2}$$

$$= \frac{\mu_0 I N}{2} \cdot \frac{a^2}{(a^2 + x^2)^{3/2}} \quad (7.9)$$

If we put $x \gg a$ so that points close to the coil are not considered, equation (7.9) reduces to

$$B = \frac{\mu_0 I N a^2}{2 x^3} \quad (7.10)$$

Recalling that πa^2 is the area A of the coil, we can write equation (7.10) as

$$B = \frac{\mu_0}{2\pi} \cdot \frac{(NIA)}{x^3} = \frac{\mu_0}{2\pi} \cdot \frac{\mu}{x^3} \quad (7.11)$$

where μ is the magnetic dipole moment of the current loop.

7.3.4. B along the axis of a Solenoid.

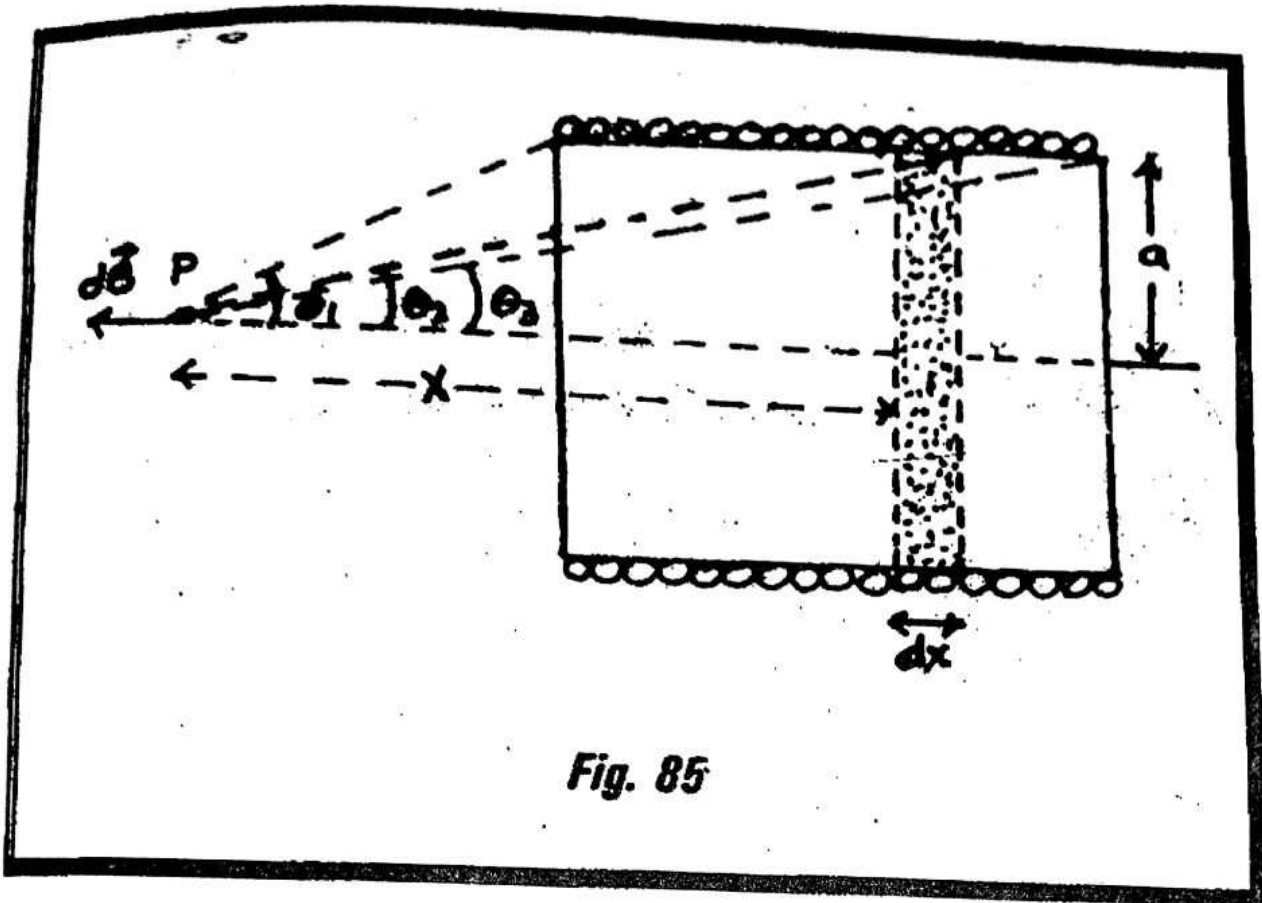


Fig. 85

Let n be the number of turns per unit length of solenoid. Consider an element of the solenoid of length dx . Using the result of section 7.3.3. i.e. eqn. (7.9), the contribution $d\mathbf{B}$ of the infinitesimal element dx is given by

$$dB = \frac{\mu_0 a^2 (ndx)I}{2(a^2 + x^2)^{3/2}}$$

where ndx is the number of turns in the element.

Write all the variables in terms of θ :

$$\begin{aligned} x &= a \cot \theta \\ dx &= -\operatorname{Cosec}^2 \theta d\theta \\ (x^2 + a^2) &= a^2 \operatorname{Cosec}^2 \theta \end{aligned}$$

$$\text{Then } dB = \frac{\mu_0 a^2 n (-a \operatorname{Cosec}^2 \theta d\theta) I}{2 a^3 \operatorname{Cosec}^3 \theta}$$

$$= \frac{-\mu_0}{2} nI \sin \theta d\theta$$

$$\text{Hence } B = \int_{\theta_1}^{\theta_2} -\frac{\mu_0}{2} nI \sin \theta d\theta$$

$$= \frac{\mu_0}{2} nI (\cos \theta_2 - \cos \theta_1) \quad (7.12)$$

SPECIAL CASES:

- (i) When P is inside an infinitely long solenoid, $\theta_2 = 0$ and $\theta_1 = \pi_{\text{rad}}$ and so $B = \mu_0 nI$.
- (ii) For P inside a toroidal solenoid, $B = \mu_0 nI$.
- (iii) For P at one end of a long solenoid, $\theta_2 = 0$, $\theta_1 = \pi/2$ rad and

$$B = \frac{\mu_0 nI}{2}$$

7.4 Force Between Two Parallel Current Carrying Conductors.

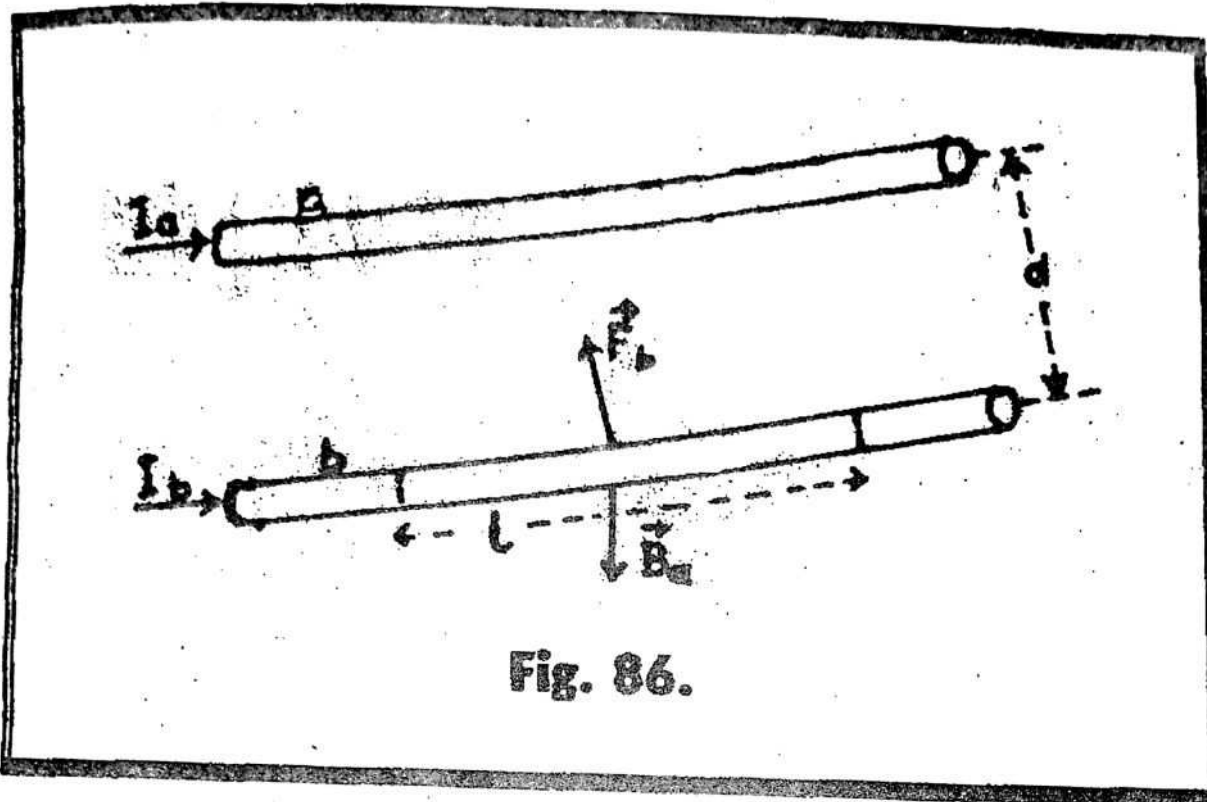


Fig. 86.

At all point along the wire b, the field created by the wire a is given by

$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

The right hand rule shows that the direction of $\vec{B}_a$ at wire b is down as shown in the figure. Wire b is immersed in an external field $\vec{B}_a$. Using $\vec{F} = I_b \vec{l} \times \vec{B}_a$, the magnitude of the force F experienced by a length l of wire b is

$$F_b = I_b l B_a = \frac{\mu_0 I_a I_b l}{2\pi d} \quad (7.13)$$

$\mathbf{F}_b$ lies in the plane of the wires and points towards wire a.

Similarly wire a finds itself immersed in a magnetic field $\mathbf{B}_b$ created by current I_b in wire b. The force $\mathbf{F}_a$ on wire a is equal and opposite to $\mathbf{F}_b$, i.e. directed towards wire b. If the direction of either current is reversed, the force reverses also.

Hence when the currents are in the same direction, the conductors attract each other. When they are in opposite directions they repel each other.

CORRELATION WITH THE AMPERE.

The fact that two straight parallel conductors exert forces of attraction or repulsion on each other is made the basis of the definition of the ampere in the SI system. The ampere is defined as follows:

One ampere is that steady current which, flowing in two infinitely long straight parallel conductors of negligible circular cross sectional area placed 1 m apart in vacuum causes each wire to exert a force of $2 \times 10^{-7} \text{ N}$ on each metre of the other wire.

$$\text{Then in } F = \frac{\mu_0}{2\pi} \cdot \frac{I_b I_a l}{d}$$

we have $F = 2 \times 10^{-7} \text{ N}$, $I_a = I_b = 1 \text{ A}$, $d = l = 1 \text{ m}$.
Substituting, we find

$$\mu_0 = 4\pi \times 10^{-7} \text{ wb A}^{-1} \text{ m}^{-1} \quad (7.14)$$

Having defined the ampere, we can now define the coulomb as the quantity of charge that in one second crosses a section of a circuit in which there is a constant current of one ampere.

7.5 Ampere's Circuital Law.

Ampere's law is an alternative to Biot-Savart law just as Gauss's Law is an alternative to Coulomb's law. Ampere's law helps us to solve certain magnetic problems of high geometrical symmetry with ease and elegance. The present discussion will be limited to constant conduction currents and to non-magnetic materials.

We have shown that the magnetic field $\mathbf{B}$ near a long straight wire carrying a current I has a magnitude given by

$$B = \frac{\mu_0 I}{2\pi r}$$

where r is the distance from the wire to the point considered.

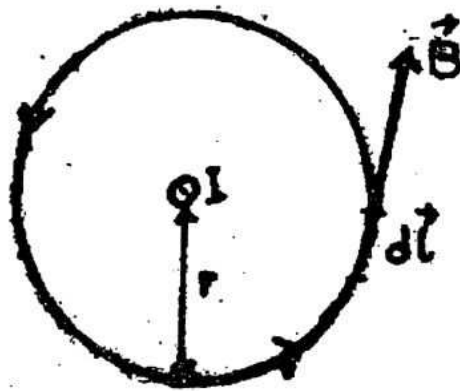


Fig. 87

Thus over a circle of radius r centered on the wire as shown above,

$$\oint \vec{B} \cdot d\vec{l} = \frac{\mu_0 I}{2\pi r} \times 2\pi r = \mu_0 I$$

This is a general result and, for any closed curve C,

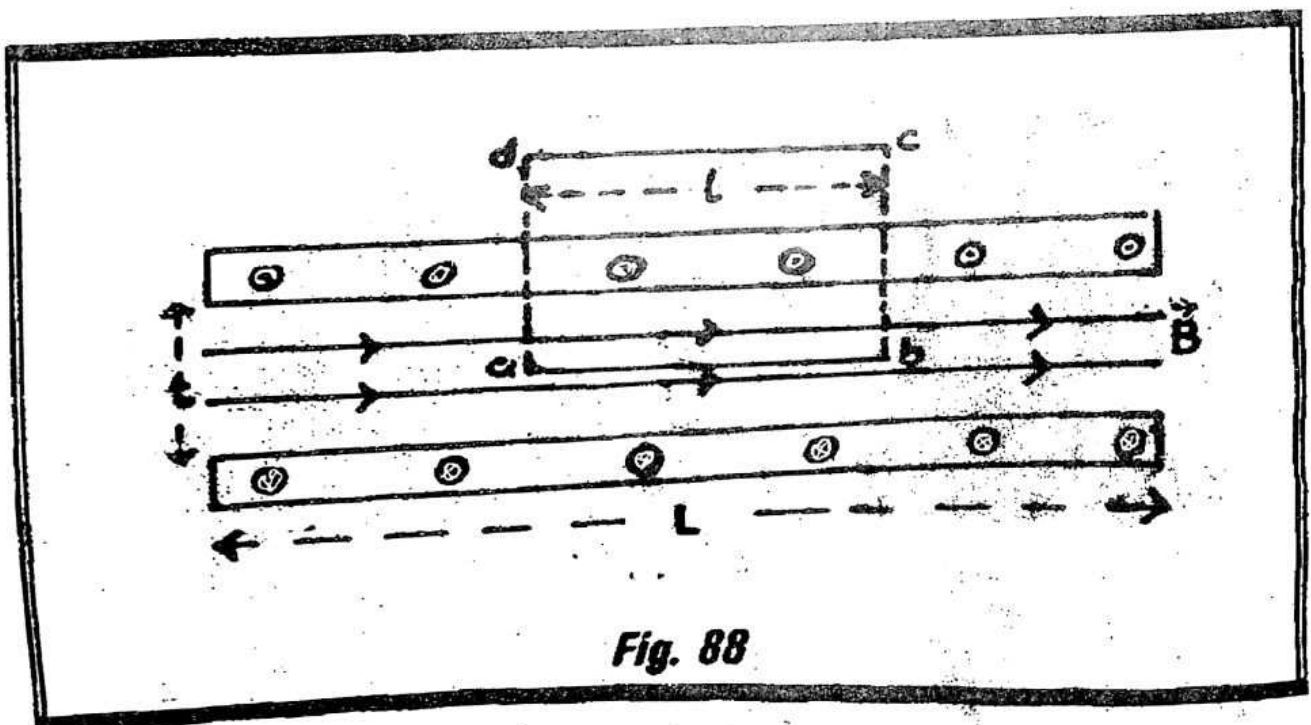
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad (7.15)$$

where I is the current enclosed by C, and $(\vec{B} \cdot d\vec{l})$ is the line integral round the closed curve. Equation (7.15) is Ampere's circuital law.

7.6 Application Of Ampere's Law

This law can be used to find the magnetic field in situations which possess the necessary geometrical symmetry, for example for the long straight wire, the solenoid and the toroid.

7.6.1. Magnetic Field of a Solenoid



Consider a long solenoid ($L \gg t$) with n turns of wire per unit length and carrying a current I . To a first approximation, one may consider the field outside the solenoid to be zero.

Let us apply Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

to the rectangular path abcd in the ideal solenoid. By symmetry, the $\vec{B}$ field along side ab is parallel to this side and is constant so that for this side

$$\oint \vec{B} \cdot d\vec{l} = Bl$$

Along sides bd and da the line integrals are zero because $\vec{B}$ and $d\vec{l}$ are at right angles. Along cd the line integral is zero because we have assumed $\vec{B} = 0$ outside the solenoid.

Thus the line integral for the entire rectangular path is Bl . The total current through the rectangle is then nIl and from Ampere's law,

$$\begin{aligned} Bl &= \mu_0 nIl \\ \text{or } B &= \mu_0 nI \end{aligned} \quad (7.16)$$

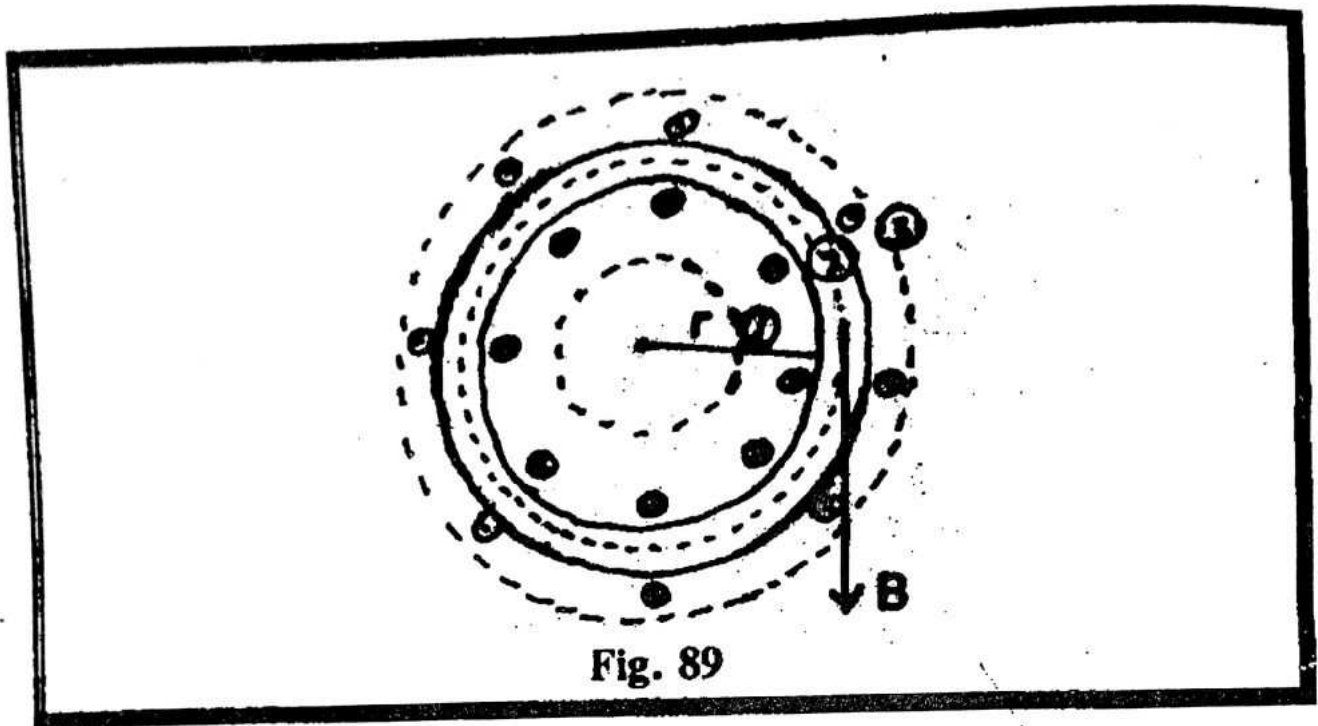
Since ab need not lie on the axis of the solenoid, the field is uniform over the entire section.

7.6.2. The Field of a Toroid.

The figure represents a toroid, wound with wire carrying a current I . For path (1)

$$\oint B \cdot dl = 2\pi r B = \mu_0 I$$

But $I = 0$ and hence $B = 0$.



For path (3) each turn of the winding passes twice through the area bounded by this path, carrying equal currents in opposite directions. Thus the net current through this area is zero and hence $B = 0$ at all points of this path.

The field of the toroid is therefore confined wholly to the space enclosed by the windings.

For path 2, by symmetry B is tangent to the path at a radius r ,

$$\text{Thus } \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Each turn of the winding passes once through the area bounded by path 2, and the total current through the area is NI where N is the total number of turns in the winding. Then from Ampere's law,

$$2\pi rB = \mu_0 NI, \text{ and}$$

$$B = \frac{\mu_0}{2\pi} \frac{NI}{r}$$

$$\text{or } B = \mu_0 nI \quad (7.17)$$

where n is the number of turns per unit length.

Equation (7.17) is the same result as for the field at the centre of a long straight solenoid.

7.7 Worked Examples

Example 7.1: Two long parallel conductors carrying currents of 8A and 12A respectively in the same direction are placed 10cm apart. Find where a third parallel wire also carrying a current must be placed so that the force experienced by it shall be zero.

Solution: If the two conductors carry equal currents then the third wire would experience a zero force at the centre, 5cm from either conductor. Since the currents are 8A and 12A respectively, assume that the neutral point is a distance x from the 8A wire, i.e., $10-x$ from the 12A wire.

$$\frac{12}{10-x} = \frac{8}{x}$$

$$12x = 80 - 8x$$

$$20x = 80$$

$$x = 4\text{cm}$$

Example 7.2: At what radial distance from a long straight wire carrying a current of 75 amps would the magnetic induction be $1.5 \times 10^{-3} \text{ w/m}^2$.

Solution:

$$B = \frac{\mu_0 I}{2\pi a}$$

$$a = \frac{\mu_0 I}{2\pi B} = \frac{4\pi \times 10^{-7} \times 75}{2\pi \times 1.5 \times 10^{-3}}$$

$$a = 1\text{cm}$$

Example 7.3: Two parallel wires in air each 10m long and carrying equal currents of 200A are placed 10cm apart. Calculate the force acting on each.

Solution:

$$\begin{aligned} F &= \frac{\mu_0}{2\pi r} \cdot \frac{I_1 I_2 l}{1} \\ &= \frac{2 \times 4\pi \times 10^{-7} \times (200)^2 \times 10}{2\pi \times 0.1} \\ &= 8.0 \times 10^{-1} \text{N} \end{aligned}$$

Example 7.4: A ring of non-magnetic material of mean circumference 100cm and cross section 10cm^2 has a uniform winding of 2,500 turns which carries a current of 10A. Calculate the flux through it.

Solution:

$$B = \frac{\mu_0 N}{l}$$

$$B = \frac{4\pi \times 10^{-7} \times 2500}{1} = \pi \times 10^{-2} \text{ w/m}^2$$

flux $\Phi = BA$ where $A = 10\text{cm}^2 = 10^{-3}\text{m}^2$
 Therefore $\Phi = \pi \times 10^{-2} \times 10^{-3}$
 $\Phi = \pi \times 10^{-5} \text{ weber.}$

Example 7.5: A current of 5 amps flows along a straight wire. Calculate the magnetic field at a point 5cm from the wire.

Solution:

from Ampere's Law

$$B = \frac{\mu_0 I}{2\pi a} \text{ where } a \text{ is the distance}$$

of the point from the wire.

Therefore

$$B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 5 \times 10^{-2}}$$

$$B = \underline{2.0 \times 10^{-3} \text{ w/m}^2}$$

Example 7.6: Two parallel wires each carrying current 30A are separated by a distance of 80cm. Calculate the force per meter between them.

Solution:

$$B_1 = \frac{\mu_0 i_1}{2\pi d}$$

$$F = i_2 B_1 l$$

$$F = \frac{\mu_0 i_1 i_2 l}{2\pi d}$$

Therefore force per meter

$$= \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$= \frac{4\pi \times 10^{-7} \times 30 \times 30}{2\pi \times 80 \times 10^{-2}}$$

$$= \underline{\underline{2.25 \times 10^{-4} \text{ N}}}$$

FIG. 90

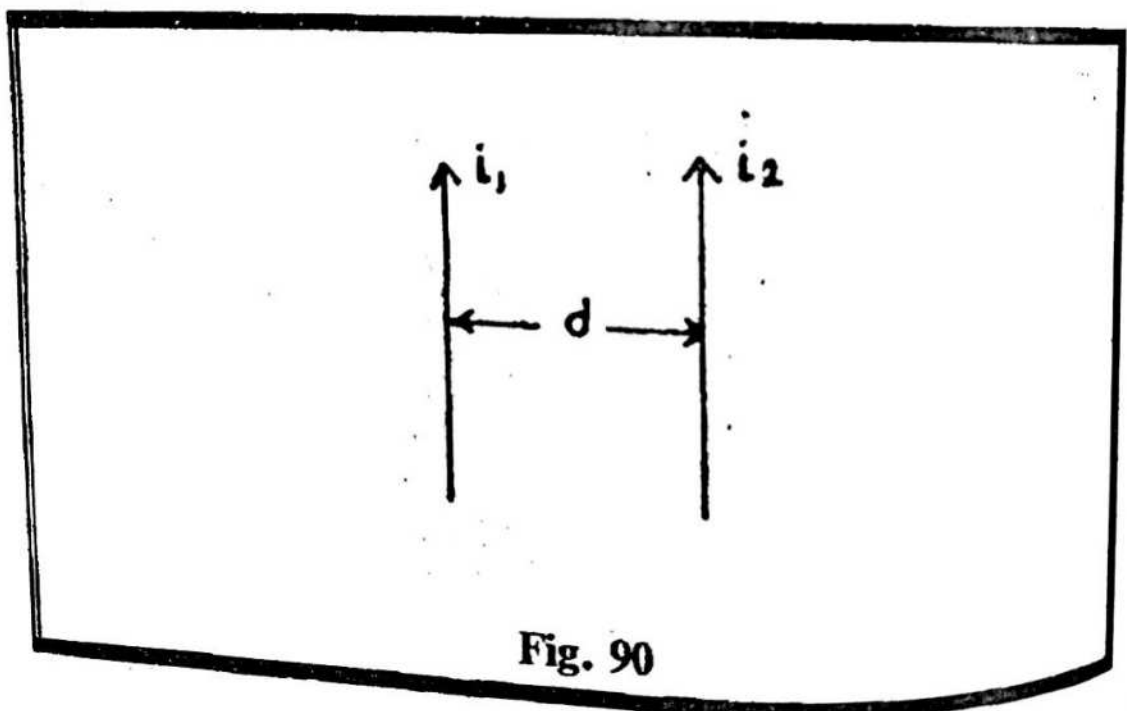


Fig. 90

7.8 PROBLEMS

7.8.1 Two long straight parallel wires, A, and B, separated by a distance, d , carry currents I_a and I_b respectively in the same direction in air. Deduce an expression for the magnetic force exerted on a length, l , of wire B by the wire A.

Two parallel cables of a high voltage powerline carry opposite currents of $1.8 \times 10^3 \text{ A}$. The distance between the cables is 3.0m. What is the magnetic force pushing on a 50m segment of one of these cables.

7.8.2 Two long straight parallel wires 8cm apart each carry an electron current of 40 amp. Calculate the magnetic induction at a point between the wires 3cm from one and 5cm from the other when the currents are

- (a) in opposite directions
- (b) in the same direction

7.8.3 Two long straight wires A and B are stretched parallel to each other 20cm apart. Wire A carries 45 amp and wire B carries 30 amp. In both wires the flow of the current is in the same direction.

At what place between the wires will the magnetic induction due to one of the currents neutralize the magnetic induction due to the other?

7.8.4 A toroid having a square cross section 4.0 cm on edge and an inner radius of 12 cm has 400 turns and carries a current of 0.6A. What is the magnetic field at

- (a) *the inner radius*
- (b) *the outer radius of the toroid?*

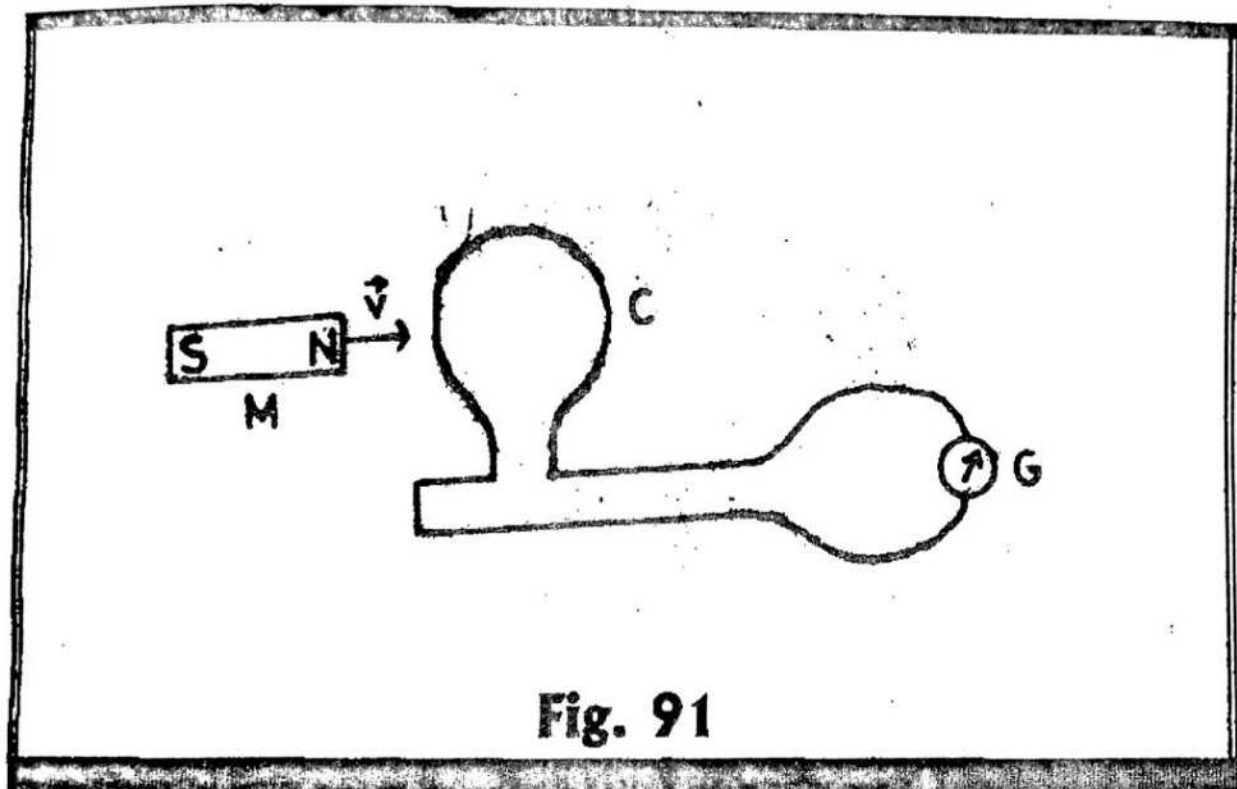
7.3.5 What is the magnitude of the magnetic field at the centre of a solenoid of 200 turns and having a length of 25 cm, diameter of 10cm and carrying a current of 0.4A.



8

ELECTRO MAGNETIC INDUCTION

Faraday's law of electromagnetic induction was deduced to explain a number of simple experiments carried out mostly by Michael Faraday in 1831.

8.1 The Experiments**Fig. 91****(a) THE MAGNET AND COIL EXPERIMENT.**

Consider a closed coil C connected to the terminals of a sensitive galvanometer G. When there is relative motion between the coil and the bar magnet M, the galvanometer deflects showing there is an **induced current** in the coil. The induced current is said to be set up by an **induced electromotive force (emf)**.

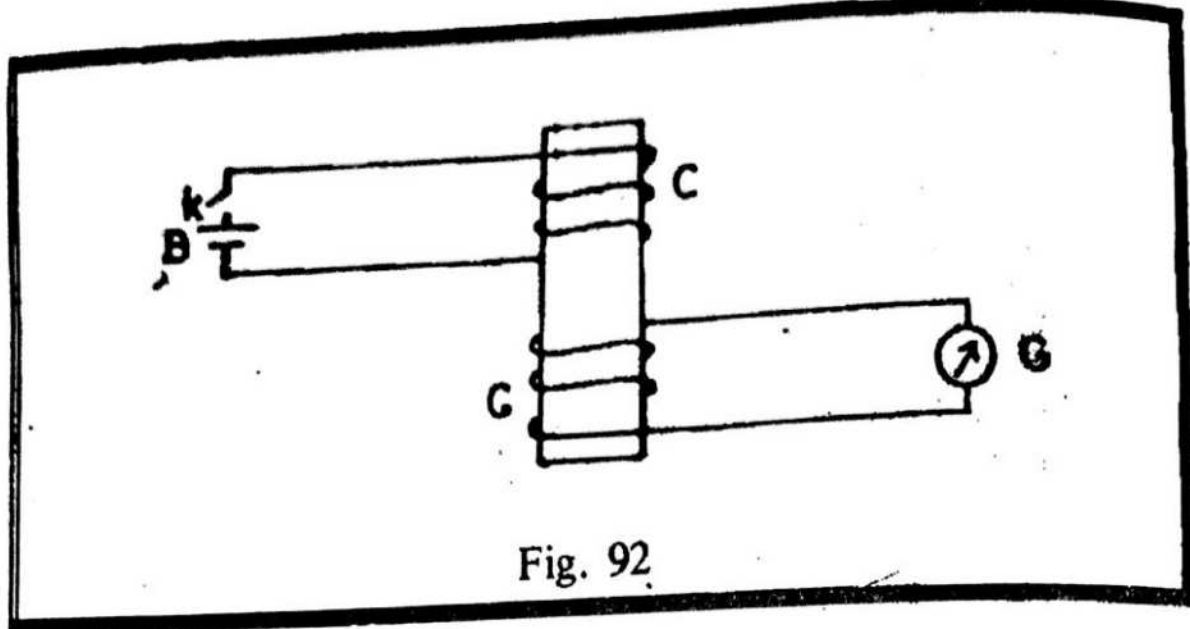


Fig. 92

(b) **THE COIL AND COIL EXPERIMENT.**

When two separate coils C_1 and C_2 are connected as shown in fig. 92 the galvanometer is found to deflect any time the key k is opened or closed. This experiment shows that there will be an induced emf in C_2 whenever the current in C_1 is changing.

(c) **CURRENT PRODUCED BY THE MOTION OF A CONDUCTOR IN A MAGNETIC FIELD.**

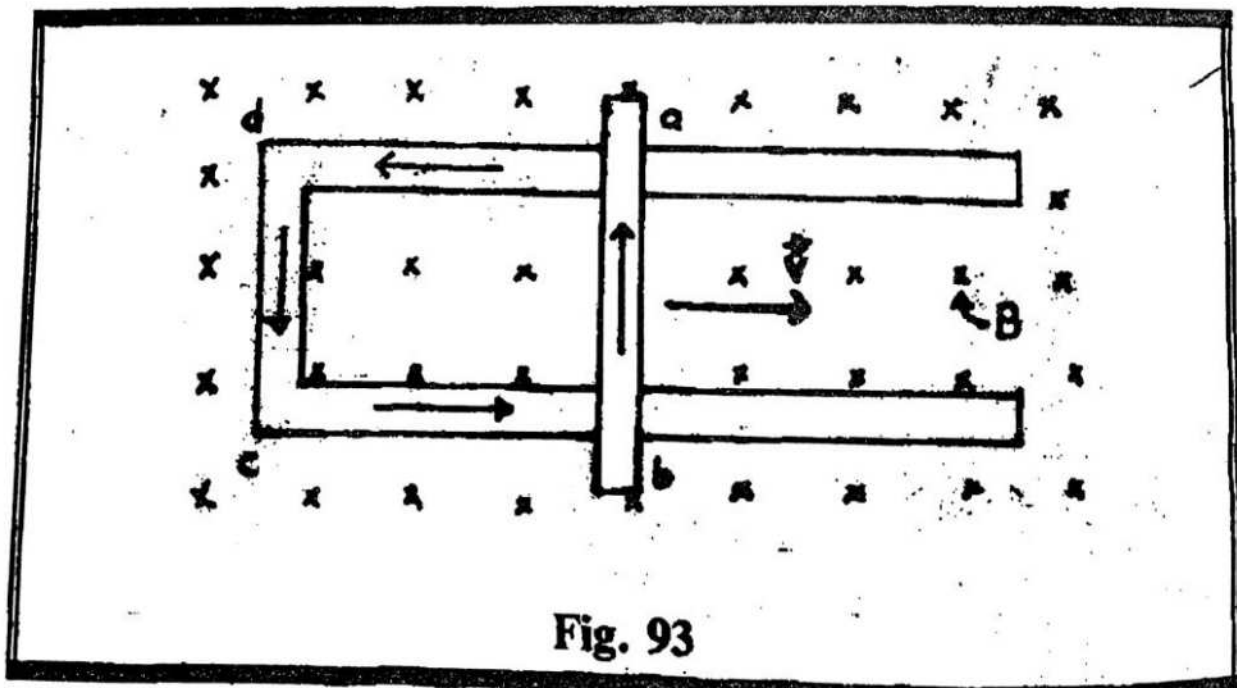


Fig. 93

A moving conductor ab slides (with velocity v) along a stationary U-shaped conductor $bcd a$ (as in fig. 93) immersed in a steady magnetic field $\mathbf{B}$ at right angles to the plane of $bcd a$. An induced current is observed in the loop $abcd$ as long as ab is moving. The moving conductor ab acts as the seat of motional emf.

8.2 The Laws Of Electromagnetic Induction.

The induced emf in the experiments above appeared only when the number of lines of $\mathbf{B}$ through the coil (in which current was induced) was changing. The rate at which these lines are changing is related to the appearance of the induced emf. We measured the number of magnetic field lines that passes through a given surface a by the magnetic flux Φ_B for that surface where, by definition,

$$\Phi_B = \int \vec{B} \cdot d\vec{s} \quad (8.1)$$

The SI unit for magnetic flux is $T \cdot m^2$ to which the name weber (W_b) is given. i.e. $1 \text{ wb} = 1 T \cdot m^2$. The magnetic flux passing through a loop or coil is said to **thread** or **link** the coil or loop. Suppose a coil has N turns and the flux linking each turn is Φ . Then we define the flux-linkage to be $N\Phi$. The unit of flux linkage is weber turn, but $N\Phi$ has the same dimensions as the flux Φ .

Faraday's first law:

When the magnetic flux threading a circuit is changing, an emf is induced in the circuit.

Faraday's second law:

The magnitude of an induced emf is proportional to the rate of change in flux-linkage.

This law is sometimes expressed in Neumann's equation as

$$\epsilon = - \frac{Nd\Phi}{dt} = - \frac{d(N\Phi)}{dt} \quad (8.2)$$

If the rate of ~~change~~ of flux is in wbs^{-1} , the emf ϵ will be in volts (V). The minus sign is discussed below.

Lenz's law:

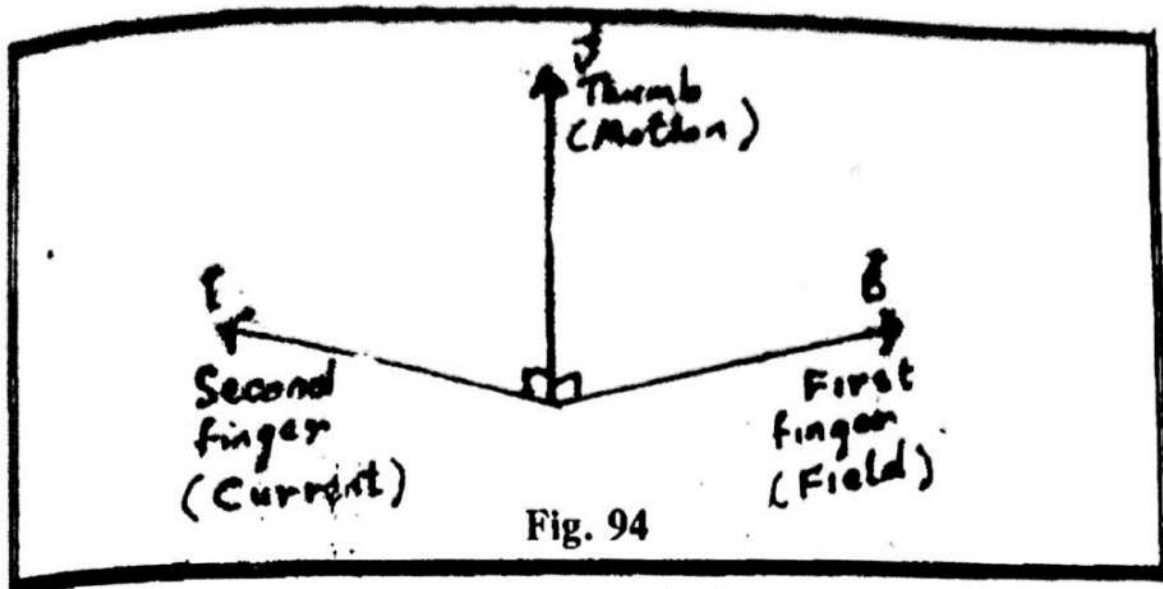
The direction of the induced emf must be such that the law of conservation of energy is obeyed. This direction is given by Lenz's law which states:

"The direction of an induced current (if one were to flow) is such that its effect would oppose the change in magnetic flux which gave rise to the current."

Or "the induced current will appear in such a direction that it opposes the change that produced it."

This is achieved by exerting forces of some kind, or by establishing a magnetic field.

The direction of induced current can also be found (particularly for straight conductors) through **Fleming's Right hand Dynamo Rule:**



Lenz's law is an example of conservation of energy. For example in the coil and magnet experiment discussed above, the induced current sets up a force on the magnet, which the mover of the magnet must overcome. The work done in overcoming this force provides the electrical energy of the current. This electrical energy is dissipated as heat in the coil.

8.3 Calculation Of The Magnitude Of Induced Motional Emf.

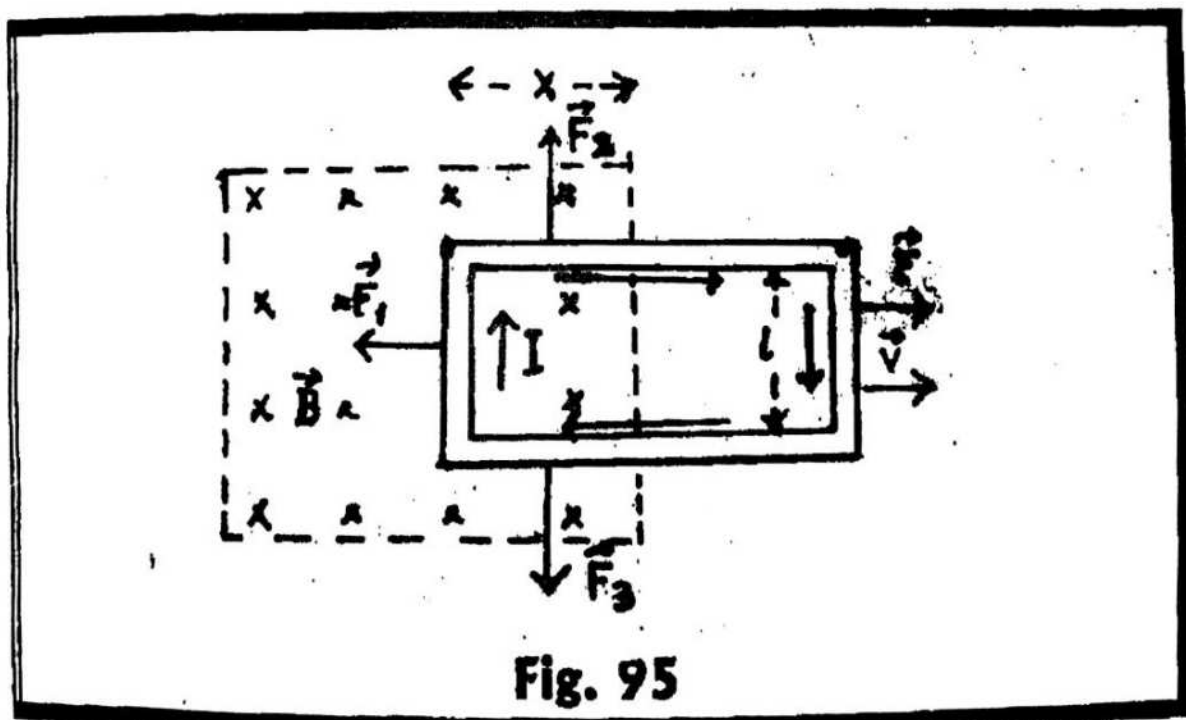


Fig. 95 shows a rectangular loop of wire of width l , one end of which is in a uniform field $\mathbf{B}$ pointing at right angles to the plane of the loop. An external agent exerts a force while pulling the loop to the right at a constant velocity $\mathbf{v}$.

The flux Φ enclosed by the loop is

$$\Phi = Blx$$

By Faraday's law, the emf is

$$\epsilon = - \frac{d\Phi}{dt} = - \frac{d}{dt} (Blx) = Blv \quad (8.3)$$

where we have set $-dx/dt$ equal to the speed v at which the loop is pulled out of the magnetic field.

The emf Blv sets up a current I in the loop given by

$$I = \epsilon/R = Blv/R \quad (8.4)$$

where R is the loop resistance. From Lenz's law, this current (and, thus ϵ) must be clockwise as shown in fig. 95 since it opposes the change (the decrease in Φ) by setting up a field that is parallel to the external field within the loop.

The current in the loop will cause force F_1 , F_2 and F_3 to act on the three conductors in accordance with the equation

$$\mathbf{F} = I \mathbf{l} \times \mathbf{B}.$$

F_2 and F_3 are equal and opposite and have the same line of action, and so cancel out. F_1 which opposes our effort to move the loop has a magnitude given by

$$F_1 = I l B \sin 90^\circ = \frac{Blv}{R} l B = \frac{B^2 l^2 v}{R}$$

The agent that pulls the loop must do work at a steady rate of

$$P = F_1 v = \frac{B^2 l^2 v^2}{R} \quad (8.5)$$

While the agent is pulling the loop, a current given by eqn. (8.4) is set up. Thus thermal energy must appear in the loop because of Joule effect.

$$P = I^2 R = \left[\frac{Blv}{R} \right]^2 R = \frac{B^2 l^2 v^2}{R}$$

which is the same as the power supplied by the external agent. Thus mechanical energy (work done by the agent) is transformed into electrical energy and finally into thermal energy (manifested by a small temperature rise of the loop).

8.4 Maxwell's Equation And Electromagnetic Oscillations

Maxwell's equations, summarised in Table 3, arise from the basic nature of an electromagnetic wave. These equations together with the Lorentz force (equation 8.10) give the complete equation of electromagnetic interactions.

Table 3: Basic equations of Electromagnetic interactions.

$\oint \mathbf{E} \cdot d\mathbf{s} = q/\epsilon_0$	(8.6)	Gauss's Law for electricity.
$\oint \mathbf{B} \cdot d\mathbf{s} = 0$	(8.7)	Gauss Law for magnetism.
$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(i + \epsilon_0 \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{s} \right)$	(8.8)	Ampere's Law as extended by Maxwell with the introduction of the displacement current.
$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s}$	(8.9)	Faradays law of electromagnetic induction describes the electrical effect a changing magnetic field.
$\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$	(8.10)	Lorentz Force

The Lorentz force is the force on an electric charge moving with speed v in an electromagnetic field. When the charge is at rest, $\mathbf{F} = q\mathbf{E}$. A stationary electric charge produces only an electric field just as a steady current in a conductor produces only a magnetic field. In a practical circuit, electromagnetic oscillations are obtained from a capacitor and from an inductor (coil) only.

Electric energy is stored in the electrostatic field between the plates of a charged capacitor while magnetic energy is stored in the magnetic field of the inductor. The potential difference across the plates of the capacitor causes a transfer of energy from electric to magnetic and then back to electric, giving rise to oscillations.

The quantities μ_0 and ϵ_0 have great impact in the study of physics and are important physical properties. It is known that the speed of light, C , is related to these two quantities as

$$C^2 = \frac{1}{\mu_0 \epsilon_0} \quad \text{where } \epsilon_0 \text{ is the permittivity of free space.} \quad (8.11)$$

Light is an electromagnetic wave and its speed, c , is related to the wavelength, λ , and frequency, f , as

$$c = f\lambda$$

(8.12)

The position of visible light in the electromagnetic spectrum is shown in fig. 96 below:

Table 4: The electromagnetic spectrum

f (Hz)	λ (m)	Type of ray y of wave	Detection Instrument
10^{19}	10^{-11}	S - rays	Geiger and scintillation counters
10^{17}	10^{-8}	x-rays ultra violet	ionization chamber photoelectric photomultiplier
10^{14}	10^{-6}	visible light infrared	EYE Bolometer Thermopile
10^{11}	10^{-3}	microwaves	radar
10^9	10^1	U.H.F. V.H.F. T.V. Broadcast	F.M. Radio T.V. Antenna
10^3	10^5	Radio freq. Power waves	

8.5 Worked Examples

Example 8.1: A wheel with metal spokes 120 cm long is rotated in a magnetic field of flux density $5 \times 10^{-5} \text{ W/m}^2$ normal to the plane of the wheel. An emf. of 10^{-2} V is induced between the rim and the axle. Find the rate of rotation of the wheel.

Solution: The rate of rotation of the wheel is the angular speed $\omega = v/r$

$$\text{i.e. } v = r\omega$$

$$\text{induced emf, } E = Blv$$

$$E = Blr\omega$$

$$\text{but } lr = \text{Area} \quad A = \pi r^2$$

$$E = B A \omega$$

$$\omega = E/BA$$

$$\omega = \frac{10^{-2}}{5 \times 10^{-5} \times \pi \times (1.2)^2}$$

$$\omega = 44.2 \text{ S}^{-1}$$

Example 8.2: Find the emf. induced in a conductor of length $l = 80 \text{ cm}$ moving with a velocity $v = 5 \text{ m/s}$ at an angle $\theta = 30^\circ$ to the lines of force of a uniform magnetic field of flux density 1.2 W/m^2 .

Solution: At an angle $\theta = 30^\circ$ between the magnetic lines of force and the moving conductor, the induced emf. will be

$$\begin{aligned} E &= Blv \sin \theta \\ &= 1.2 \times 5 \times 0.8 \sin 30 \\ &= 2.4 \text{ Volts} \end{aligned}$$

Example 8.3: The magnetic flux, Φ , through a loop perpendicular to the plane of a coil is varying according to the relation $\Phi = 6t^2 + 7t + 4$ where Φ is in milliwebers and t is in seconds.

Determine the magnitude of the e.m.f. induced in the loop when $t = 2$ seconds.

Solution:

The e.m.f. $\epsilon = d\Phi/dt$

$$\Phi = 6t^2 + 7t + 4$$

$$d\Phi/dt = 12t + 7$$

when $t = 2$ sec., $\epsilon = 24 + 7$

$$\epsilon = 31 \text{ milliweber/sec}$$

$$\epsilon = 3.1 \times 10^{-2} \text{ wb/s}$$

Example 8.4: A coil of 5 turns and cross-sectional area 5cm^2 is immersed at right angles into a magnetic flux of density $4.0 \times 10^{-2}\text{T}$. If the flux is then gradually reduced to zero in 8 seconds, calculate the induced e.m.f. during the operation.

Solution: The magnetic flux $B = 4.0 \times 10^{-2}$ and the flux linkage $= N\Phi$. The total flux $\Phi = BA$ where A = area of cross-section of the coil.

$$\text{Therefore, } \Phi = 4.0 \times 10^{-2} \times 5.0 \times 10^{-4} = 20 \times 10^{-6} \text{ T}$$

$$\text{Therefore, Change in flux linkage} = d(N\Phi)$$

$$\begin{aligned} \text{Therefore induced emf, } \epsilon &= \frac{d(N\Phi)}{dt} \\ &= \frac{5 \times 20 \times 10^{-6}}{8} \end{aligned}$$

$$= \frac{100}{8} \times 10^{-6}$$

$$= \frac{100}{8} \times 10^{-6}$$

$$= \underline{\underline{1.25 \times 10^{-5} \text{V}}}$$

8.6 PROBLEMS.

- 8.6.1 A conducting wire 60cm long is moved at a constant speed of 5 m/s perpendicular to a uniform magnetic field. If the e.m.f induced in the wire is 2 volts deduce the value of the magnetic induction.
- 8.6.2 A wire 20cm long moves with a speed of 25/ms through a uniform magnetic field where the magnetic induction is 0.3 w/m^2 . If the field, wire and motion are all mutually perpendicular, calculate the e.m.f. induced in the wire.
- 8.6.3 At what speed should a wire 40cm long be moved through a magnetic field of intensity 0.5 w/m^2 . if it is to produce an emf of 1 volt?
- 8.6.4 The magnetic induction in the region between the pole faces of an electromagnet is 0.5 w/m^2 . Calculate the induced e.m.f. in a straight conductor 8cm long, perpendicular to the magnetic field and moving perpendicular both to the field and its own length with a velocity of 1m/s.

9

OPTICS**9.1 Prisms And Lenses**

There are two common types of prisms - **rectangular prism** and **triangular prism** - available in most laboratories for practical studies.

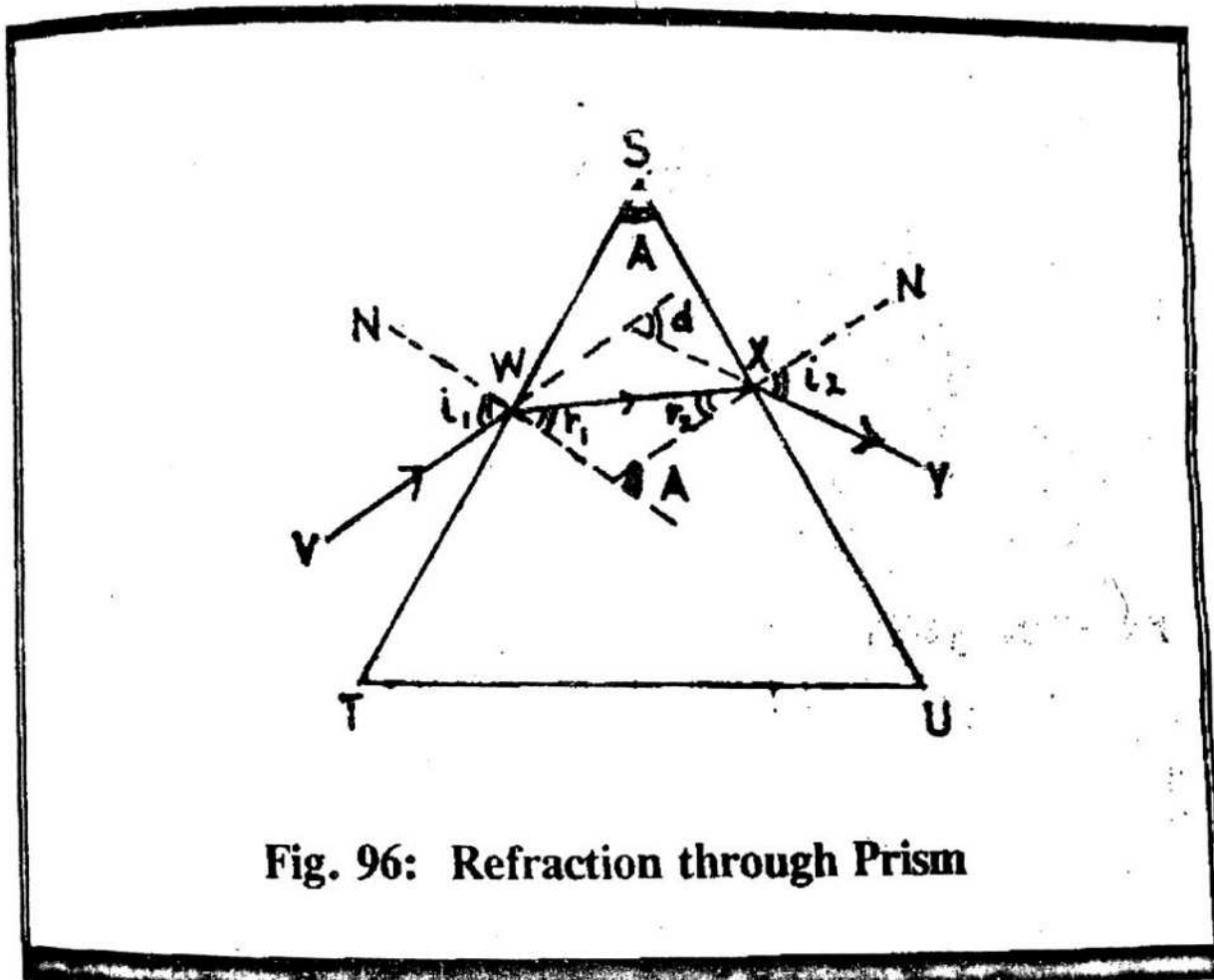


Fig. 96: Refraction through Prism

S T U in fig.4.1 is a prism of refracting angle A and refractive index n . A ray VW incident in air on the surface ST emerges as XY after suffering two refractions at W and x where the angles of

incidence and refraction are respectively i_1, r_1 and i_2, r_2 .

According to Snells Laws of refraction

$$\sin i_1 = n \sin r_1 \quad (9.1)$$

$$\sin i_2 = n \sin r_2 \quad (9.2)$$

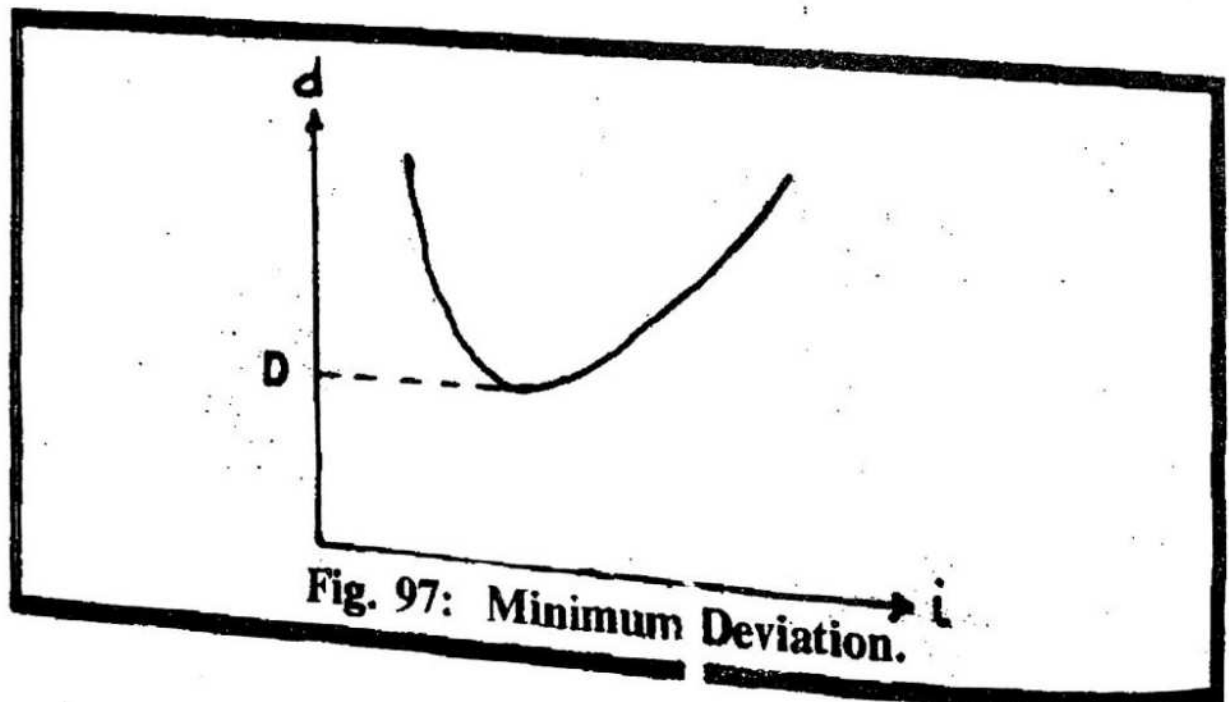
From simple geometry

$$r_1 + r_2 = A \quad (9.3)$$

The prism has deviated the incident ray VW to emerge as XY as a result of two deviations at W and X, therefore the total deviation, d , will be the sum of these two deviations thus

$$d = (i_1 - r_1) + (i_2 - r_2) \quad (9.4)$$

If a new value of the angle of incidence i_1 , is chosen, the values of r_1, r_2 and d will vary. Experiment shows that as the angle of incidence, i_1 , is increased from zero, the deviation, d , begins to decrease continuously to some value D , and then increases to a maximum as i is increased further to 90° .



It is found that the **minimum deviation**, D , of the light occurs when the ray passes symmetrically through the prism and at this particular case, the angle of incidence and angle of emergence are equal as shown in fig. 98

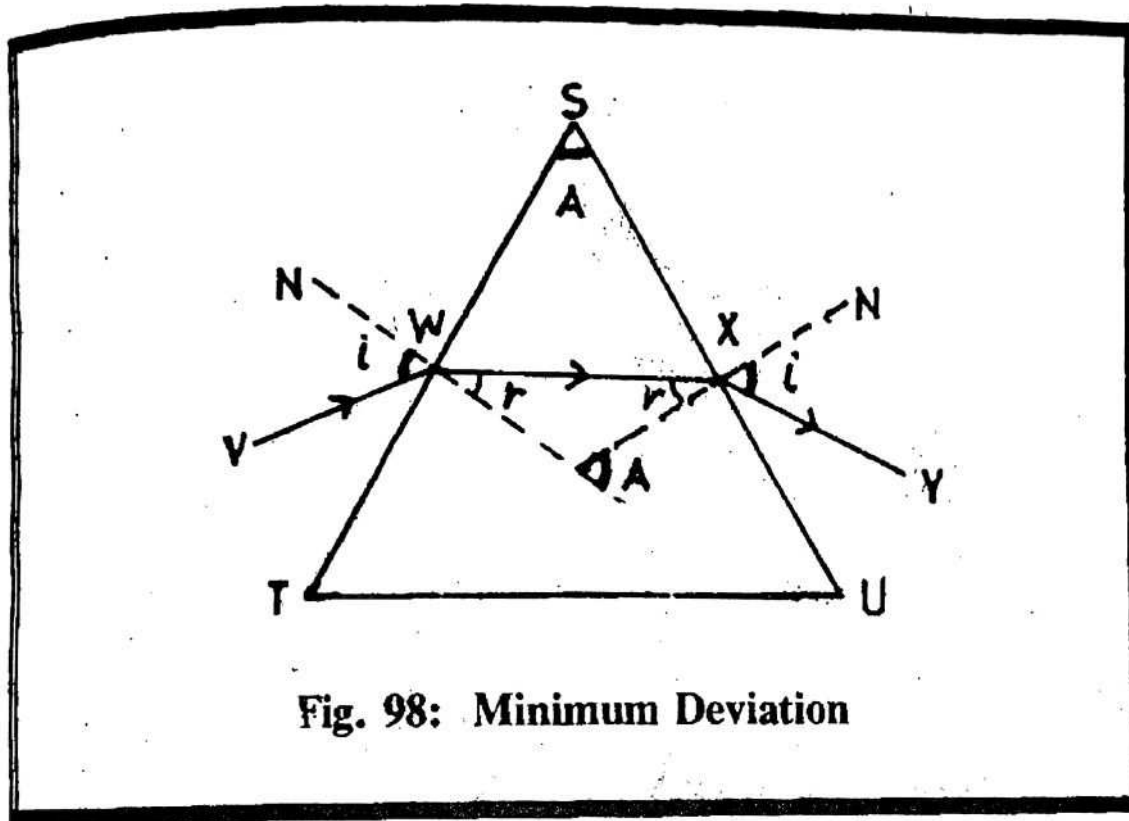


Fig. 98: Minimum Deviation

For the case of minimum deviation equations (9.3) and (9.4) become

$$r + r = A$$

$$r = A/2 \quad \text{---- } (\alpha)$$

$$i - r + i - r = D \quad \text{---- } (\beta)$$

Substituting (α) in (β)

$$2i = A + D$$

$$i = \frac{A + D}{2}$$

$$n = \frac{\sin i}{\sin r}$$

$$= n \frac{\sin \frac{A+D}{2}}{\sin A/2} \quad (9.5)$$

9.2 Refraction Of Light At A Curved Surface

A lens is a piece of glass bounded by one or two curved surfaces. It is necessary to commence the study of lenses by discussing the concept of "power of the surface".

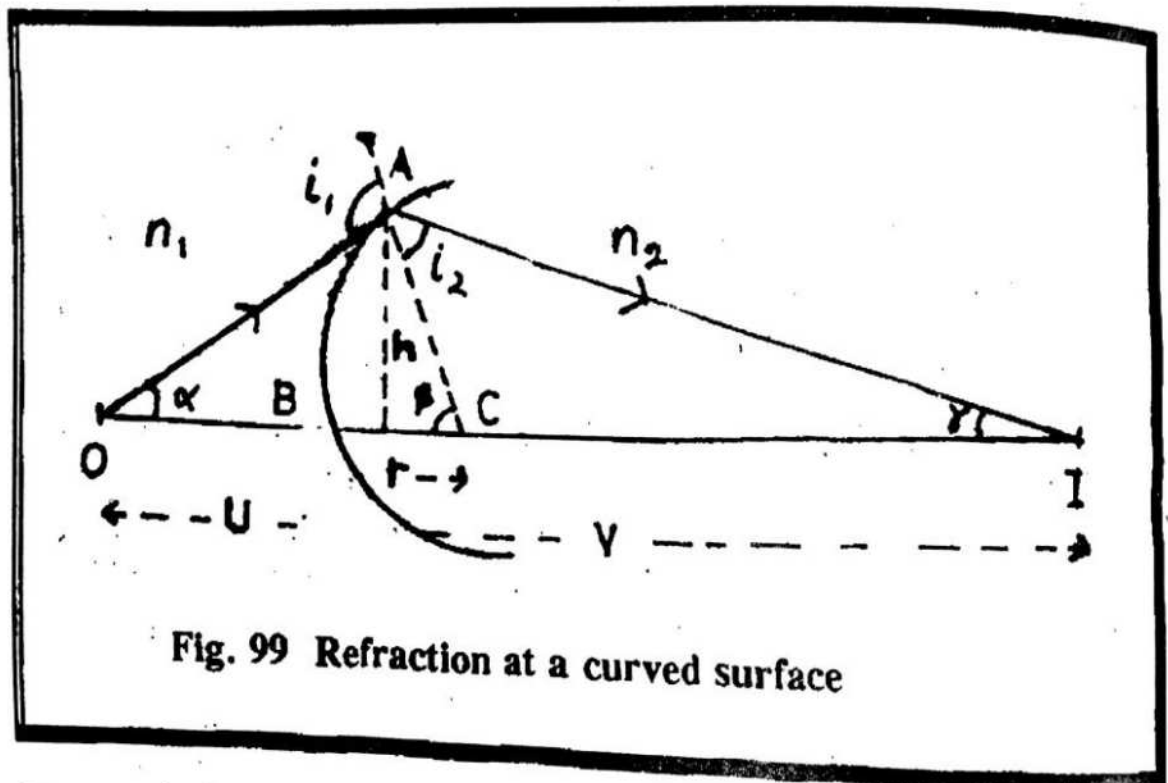


Fig. 99 Refraction at a curved surface

The curved surface AB bounds two media of refractive indices n_1 , n_2 as shown in fig. 99. Let us assume that $n_2 > n_1$ and that OA is very close to the axis. The arrows shown how the rays from the object O form a real image, I .

Using the principle that " $n \sin i = \text{constant}$ "

$$n_1 \sin i_1 = n_2 \sin i_2 \quad (9.6)$$

Since A is very close to B, i_1 is small and hence $\sin i_1 = i_1$ in radians. Similarly $\sin i_2 = i_2$ in radians.

From (9.6) it follows that

$$n_1 i_1 = n_2 i_2 \quad (9.7)$$

From the geometry of triangle OAC, $i_1 = \alpha + \beta$ and from the geometry of triangle CAI, $i_2 = \beta - \gamma$. Substituting for i_1 , i_2 in (9.7) we have

$$n_1 (\alpha + \beta) = n_2 (\beta - \gamma)$$

$$\text{Therefore, } n_1 \alpha + n_2 \gamma = (n_2 - n_1) \beta \quad (9.8)$$

Since AB is small, it is approximately perpendicular to OB, Therefore, $\alpha = h/OB$, $\gamma = h/BI$, $\beta = h/BC$

we have rewrite (8) as

$$\frac{n_1}{OB} + \frac{n_2}{BI} = \frac{n_2 - n_1}{BC}$$

that is

$$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{r} \quad (9.9)$$

where r is the radius of curvature of the surface.

$\frac{n_2 - n_1}{r}$ is known as the "power of the surface".

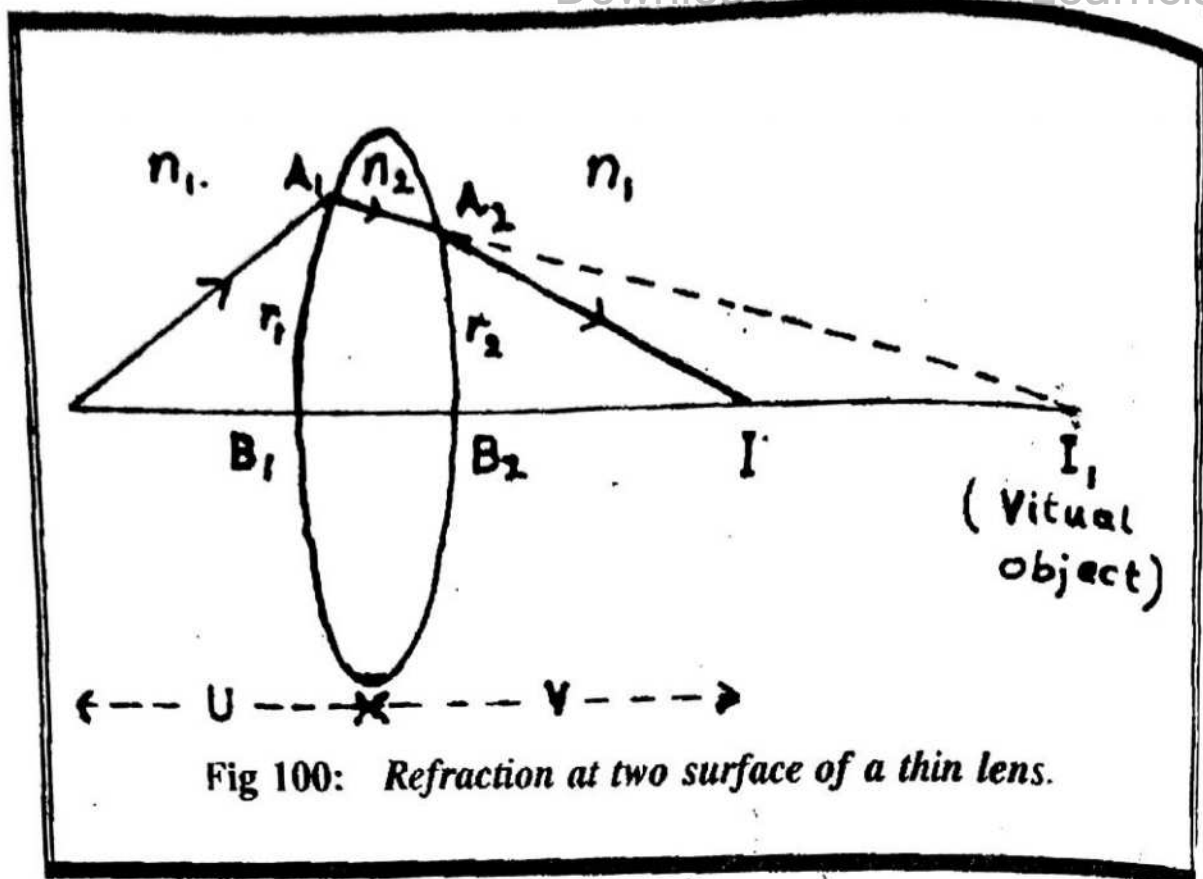


Fig 100: Refraction at two surface of a thin lens.

We now extend the results of refraction at a curved surface to two curved surfaces such as a thin lens whose thickness is small compared with its other dimensions as shown in fig. 100. Suppose narrow beams of light are incident on the central portion of a thin lens of refractive index n_2 placed in a medium of refractive index n_1 with r_1 and r_2 as the radii of curvatures of the surfaces. The image I is formed from the rays OB_1B_2I and OA_1A_2I where the latter is refracted at the first surface at A_1 , and then refracted again at A_2 at the second surface.

Refraction at first surface A_1B_1 : Suppose $u = OB_1$, then $v^1 = B_1I^1$ is the distance of the image I^1 by refraction at this surface of the lens. Since I^1 is situated in the medium of refractive index n_2 (assume $n_2 > n_1$)

$$\frac{n_2}{v^1} + \frac{n_1}{u} = \frac{n_2 - n_1}{r_1} \quad (9.10)$$

Refractive at second surface $A_2 B_2$: $A_1 A_2$ and $B_1 B_2$ are the incident rays on this surface, therefore I^1 is a virtual object for refraction at this surface. The object distance $I_1 P_2$ is negative = $-V^1$. Since I is a real image the corresponding image distance IP_2 is positive = V

substituting in (9)

$$\frac{n_2}{-v^1} + \frac{n_1}{v} = \frac{n_2 - n_1}{r_2} \quad (9.11)$$

Solving (9.10) and (9.11) simultaneously to eliminate v^1 , we obtain

$$\begin{aligned} \frac{n_1}{v} + \frac{n_1}{u} &= (n_2 - n_1) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \\ \frac{1}{v} + \frac{1}{u} &= \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \end{aligned} \quad (9.12)$$

Objects at infinity generate parallel rays and when incident on the lens, are brought to a focus i.e. $u = \infty$, $v = f$

$$\begin{aligned} (9.12) \rightarrow \frac{1}{f} + \frac{1}{\infty} &= \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \\ \therefore \frac{1}{f} &= \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \end{aligned} \quad (9.13)$$

Substitute (9.13) in (9.12) to obtain

$$1/v + 1/u = 1/f \quad (9.14)$$

(9.13) is the "Lens equation" which applies equally to concave and convex lens.

9.3 Worked Examples

Example 9.1: A 60° glass prism has a refractive index of 1.5. Determine:

- (i) *the angle of incidence for minimum deviation*
- (ii) *the minimum deviation*
- (iii) *the angle of emergence of light at maximum deviation.*

Solution:

- (i) At minimum deviation the ray traverses the prism symmetrically.

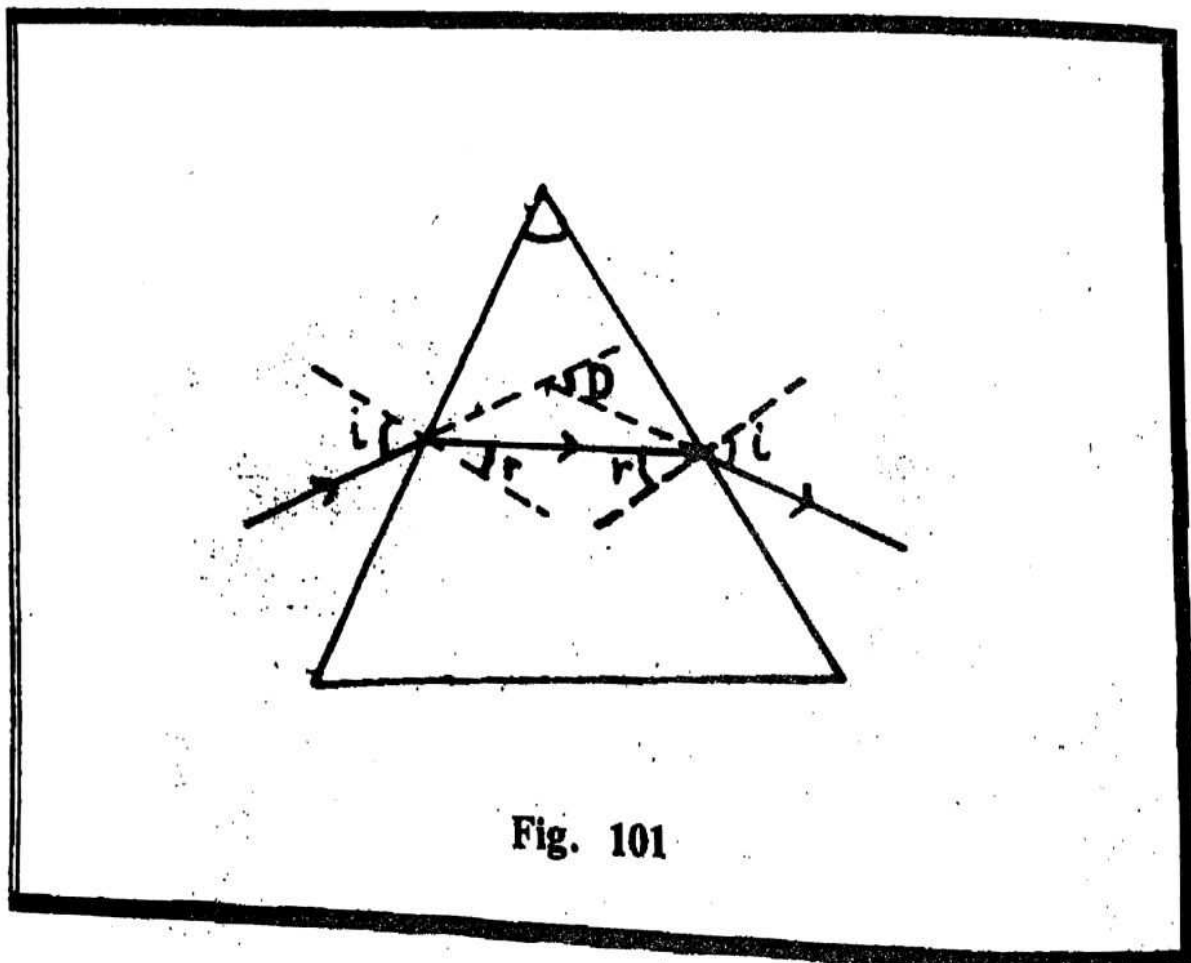


Fig. 101

Therefore, $r + r = 60^\circ$, or $2r = 60^\circ$ and $r = 30^\circ$

From the snells law,

$$\frac{\sin i}{\sin r} = n$$

We deduce that $\sin i = n \sin r = 1.5 \sin 30^\circ$

$$\therefore \sin i = 0.75$$

$$i = \sin^{-1}(0.75) = 49^\circ$$

$$\therefore i = 49^\circ$$

(ii) Similarly the minimum deviation can be deduced thus: $D = i - r + i - r = 2(i - r)$

$$= 2(49^\circ - 30^\circ)$$

$$\text{minimum deviation} = 38^\circ$$

Alternatively, recall that $n = \sin \frac{(A + D)/2}{(\sin (A/2))}$

$$\therefore 1.5 = \sin \frac{(60^\circ + D)}{2} \div \sin 60/2$$

$$= \sin \frac{(60^\circ + D)}{2} \div \sin 30^\circ$$

$$\therefore 1.5 \sin 30^\circ = \sin \frac{(60^\circ + D)}{2}$$

$$0.75 = \sin \frac{(60^\circ + D)}{2}$$

$$\sin (0.75) = \frac{60 + D}{2}$$

$$49 = \frac{60 + D}{2}$$

$$98 = 60 + D$$

$$D = 98 - 60 = 38$$

$$D = 38$$

- iii) Maximum deviation occurs at grazing incidence, that is, when the light is incidence at 90° on one face of the prism.

Maximum deviation may also occur at any angle of incidence, i , corresponding to grazing emergence.

$$\therefore \frac{\sin 90^\circ}{\sin r_1} = n = 1.5$$

$$\therefore \frac{1}{\sin r_1} = 1.5 \text{ since } \sin 90^\circ = 1$$

$$\sin r_1 = \frac{1}{1.5} = 0.6667$$

$$r_1 = \sin^{-1}(0.6667) = 42^\circ$$

$$r_1 = 42^\circ$$

$$\therefore r_2 = A - r_1 = 60 - 42^\circ = 18^\circ$$

$$r_2 = 18^\circ$$

$$\therefore \frac{\sin i_2}{\sin r_2} = n = 1.5$$

$$\sin i_2 = 1.5 \sin r_2$$

$$= 1.5 \sin 18^\circ = 0.4635$$

$$\sin i_2 = 0.4635$$

$$i_2 = \sin^{-1}(0.4635)$$

$$= 28^\circ$$

$$\therefore i_2 = \text{angle of Emergence}$$

$$= 28^\circ$$

Example 9.2: Determine the least value of the refracting angle of a prism made of glass of refractive index $7/4$ so that no rays incident on one of the faces containing this angle can emerge from the other.

Solution: Note that the least value of the refracting angle of a prism corresponds to a ray at grazing incidence and grazing emergence.

Therefore, the minimum angle of the prism $= 2c$
where c is the critical angle.

Since $i = 90^\circ$, then $r = c =$ the critical angle

$$\therefore \frac{\sin i}{\sin r} = \frac{\sin 90}{\sin c} = n = 7/4$$

$$\frac{\sin 90}{\sin c} = \frac{7}{4}$$

$$\frac{1}{\sin c} = \frac{7}{4} \quad \text{since } \sin 90^\circ = 1$$

$$\therefore \sin c = 4/7 = 0.5714$$

$$c = \sin^{-1}(0.5714) = 34^\circ 30'$$

$$\therefore \text{minimum angle} = 2c = 69^\circ$$

Example 9.3: A small piece of paper is stuck on a glass sphere of 5cm radius and viewed through the glass from a position directly opposite.

- Find:** (i) the position of the image formed.
 (ii) the position of the image if the piece of paper is removed to infinity.

Solution:
 (i)

Since the paper is stuck on the first surface of the glass sphere, its object position is therefore the second surface and hence $u = 10\text{cm}$. We then apply the formula;

$$\frac{n_2}{v} + \frac{n_1}{u} = \frac{n_2 - n_1}{r}$$

to the second refracting surface, giving that

$$n_1 = 1.5, n_2 = 1, r = 5\text{cm} \quad v = \text{the image distance.}$$

Substituting:

$$\frac{1}{v} + \frac{1.5}{10} = \frac{1.5-1}{5}$$

Solving therefore $v = 20\text{cm}$

Thus the image is virtual and is located 20cm from this refracting surface on the same side as the object.

- (ii) Since the object is at infinity, refraction will take place at the first and second refracting surfaces of the sphere.

Suppose that I is the position of the image after refraction at the first surface. We again apply the usual formula to this surface;

where $u = \infty$ (at infinity)

$$\begin{aligned} n_1 &= 1 \\ n_2 &= 1.5 \\ r &= +5 \end{aligned}$$

Substituting:

$$\frac{1.5}{v} + \frac{1}{\infty} = \frac{1.5 - 1}{5}$$

$$\therefore \frac{1.5}{v} = \frac{1.5 - 1}{5}$$

Solving therefore, $v = 15\text{cm}$

This result implies that the image I is located 5cm from the second curved surface of the glass sphere since the diameter of the sphere is 10cm. This image is considered a virtual object for refraction at the second curved surface of the glass sphere: hence

$$U = 5\text{cm}, n_1 = 1.5, n_2 = 1, r = +5\text{cm}$$

It therefore follows that

$$\frac{n_2}{v} + \frac{n_1}{u} = \frac{n_2 - n_1}{r}$$

$$\frac{1}{v} + \frac{1.5}{(-5)} = \frac{1.5 - 1}{5}$$

Solving therefore $v = 2.5\text{cm}$

the final image is hence located 2.5cm from the second refracting surface.

Example 9.4: A certain prism is found to produce a minimum deviation of 51° while it produces a deviation of 62° for two values

of the angle of incidence namely 40° and 82° respectively. Determine the refracting angle of the prism, the angle of incidence at minimum deviation and the refractive index of the materials of the prism.

Solution:

$$(i_1 - r_1) + (i_2 - r_2) = D$$

$$(40 - r_1) + (82 - r_2) = 62$$

$$122 - r_1 - r_2 = 62$$

$$r_1 + r_2 = 60 = \text{angle of prism } A.$$

At minimum deviation,

$$i = \frac{A + D}{2} \text{ and } r = \frac{A}{2}$$

$$i = \frac{60 + 51}{2} = 55.5^\circ$$

$$n = \frac{\sin i}{\sin r} = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}}$$

$$n = \frac{\sin 55.5}{\sin 30}$$

$$n = \underline{1.648}$$

9.4 PROBLEMS

9.4.1 The prism of a spectrometer has a refractive angle of 60° and is made of glass whose refractive index for red and violet are respectively 1.514 and 1.530 respectively.

A white source is used and the instrument is set to give minimum deviation for red light.

Determine:

- (i) *the angle of incidence of the light on the prism*
- (ii) *the angle of emergence of the violet light*
- (iii) *the angular width of the spectrum.*

9.4.2 A glass prism of angle 72° and index of refraction 1.66 is immersed in a liquid of refractive index 1.33. What is the angle of minimum deviation for a parallel beam of light passing through the prism.

9.4.3 An illuminated object O, a converging lens A, and a diverging lens B are all placed in a line with each other. The converging lens A has a focal length of 10cm and is located 12 cm from the object O, the diverging lens has a focal length of 20 cm and is located 35cm from the converging lens A.

Determine:

- (i) *the final position of the image after refraction of light through both lenses.*
- (ii) *the final magnification.*

9.4.4 In a Physics 102 class, a young male undergraduate observes with her right eye two small illuminated objects placed 15cm apart in a line perpendicular to the visual axis of her eye and 60cm away.

Determine the position and the distance between the images reflected from the surface of her cornea which has radius of curvature of 8cm

9.4.5 The range of distinct vision of a short-sighted female Biology student of NAU Awka extends between the distances of 10cm and 30cm from her eyes. Calculate the focal length of the spectacle lenses which she must wear to see distant objects at her far point. With these spectacles what would be the least distance of her distinct vision?

9.4.6 What is meant by total internal reflection of light? State the condition necessary for its occurrence. A thin bi-convex lens rests on a plane mirror. It is found that a point object placed 20cm above the lens coincides with its own image.

Determine the position and nature of the image when the object is placed:

- (i) 8 cm
- (ii) 12 cm from the lens - mirror combination.

