SOLUTIONS TO MATH 101 MOCK TEST

QUESTION 1

(a) Simplify $\log_2 y^7 + \log_4 y^3$

Solution

Recall that :- $\log_{a^c} b = \frac{1}{c}\log_a b$ [We will prove this in the next tutorial]

$$\therefore \log_2 y^7 + \log_4 y^3 = \log_2 y^7 + \log_{2^2} y^3$$

$$= \log_2 y^7 + \tfrac{1}{2}\log_2 y^3$$

$$= \log_2 y^7 + 3\left(\tfrac{1}{2}\right)\log_2 y$$

$$= \log_2 y^7 + \tfrac{3}{2}\log_2 y$$

$$= \log_2 y^7 + \log_2 y^{3/2}$$

$$= \log_2 \left(y^7 \times y^{3/2}\right)$$

$$= \log_2 \left(y^{7+3/2}\right)$$

$$= \log_2 y^{17/2} = \tfrac{17}{2}\log_2 y$$

$$= \tfrac{17}{2}\log_2 y \quad \text{OR} \quad 17\log_4 y$$

(b)

$$\left[\frac{2^n(9^{n+2}) - 36(2^n)(3^{2n})}{45(18^n)}\right]^{1/2}$$

$$= \left[\frac{2^n(9^n \times 9^2) - 36(2^n \times 3^{2n})}{45(9\times 2)^n}\right]^{1/2}$$

$$= \left[\frac{2^n(3^{2n} \times 81) - 36(2^n \times 3^{2n})}{45(3^2\times 2)^n}\right]^{1/2}$$

$$= \left[\frac{(2^n \times 3^{2n} \times 81) - 36\times(2^n \times 3^{2n})}{45(3^{2n}\times 2^n)}\right]^{1/2}$$

$$= \left[\frac{\cancel{2^n \times 3^{2n}}\,[81 - 36]}{45\,\cancel{(2^n \times 3^{2n})}}\right]^{1/2} = \left[\frac{45}{45}\right]^{1/2} = \left(\frac{1}{1}\right)^{1/2} = 1^{1/2}$$

$$= 1$$

QUESTION 2

Find the positive square root of $19 + 6\sqrt{2}$

Solution

$\sqrt{19 + 6\sqrt{2}} = \sqrt{a} + \sqrt{b}$ ... [questions of this nature is always solved this way..]

Square both sides :-

$(\sqrt{19 + 6\sqrt{2}})^2 = (\sqrt{a} + \sqrt{b})^2$

$19 + 6\sqrt{2} = (\sqrt{a})^2 + \sqrt{ab} + \sqrt{ab} + (\sqrt{b})^2$

$19 + 6\sqrt{2} = a + 2\sqrt{ab} + b$

$19 + 6\sqrt{2} = a + b + 2\sqrt{ab}$

Equating constants :-

$a + b = 19$ .... (1.)

Equating surds :-

$\frac{2\sqrt{ab}}{2} = \frac{6\sqrt{2}}{2}$

$\sqrt{ab} = 3\sqrt{2}$

Square both sides

$(\sqrt{ab})^2 = (3\sqrt{2})^2$

$ab = 9 \times 2$

$ab = 18$ .... (11.)

Now we have :-

$a + b = 19$ .... (1.)

$ab = 18$ .... (11.)

From eqt (1.) $a = 19 - b$ substitute into eqt (11.)

$(19 - b)b = 18$

$19b - b^2 = 18$

$b^2 - 19b + 18 = 0$

By Factorization :-

$b^2 - 18b - b + 18 = 0$

$b(b - 18) - 1(b - 18) = 0$

$(b - 1)(b - 18) = 0$

$b = 1$ or $b = 18$

Recall that ;

$a = 19 - b$

When $b = 1$: $a = 19 - 1$, $a = 18$

When $b = 18$: $a = 19 - 18$, $a = 1$

Hence; the positive square root is :- $\sqrt{1} + \sqrt{18}$ OR $\sqrt{18} + \sqrt{1}$

$= 1 + 3\sqrt{2}$ OR $3\sqrt{2} + 1$

QUESTION 3

(a) $^{100}C_{98} = \frac{100!}{(100-98)!\,98!} = \frac{100!}{2!\,98!}$

$= \frac{100 \times 99 \times \cancel{98!}}{2!\,\cancel{98!}}$

$= \frac{\cancel{9900}}{\cancel{2 \times 1}}$

$= 4950$

(b) Express $\frac{2+3i}{3+2i}$ in the form $a+ib$

Solution

To express in the form $a+ib$ we rationalize with the conjugate

$= \frac{2+3i}{3+2i} \times \left(\frac{3-2i}{3-2i}\right)$

$= \frac{(2+3i)(3-2i)}{(3+2i)(3-2i)}$

$= \frac{6-4i+9i-6(-1)}{3^2-(2i)^2}$

$= \frac{6+5i+6}{9-4(-1)} = \frac{12+5i}{9+4}$

$= \frac{12+5i}{13}$

$= \frac{12}{13} + \frac{5i}{13}$

QUESTION 4

(a) In the binomial expansion of $(x^2 - 1/x)^{1/2}$. Find the coefficient of x^6.

Solution

$(a+b)^n = {}^nC_r\, a^{n-r}\, b^r \rightarrow$ term in the binomial expansion of $(a+b)^n$

$(x^2 - 1/x)^{1/2}$

$a = x^2 \cdot b = -1/x \quad n = 1/2$

$= {}^{12}C_r\, a^{12-r}\, b^r = {}^{12}C_r\, (x^2)^{12-r}\, (-1/x)^r$

$= {}^{12}C_r\, x^{24-2r} \cdot (-1^r \times (1/x)^r)$

$= {}^{12}C_r\, x^{24-2r} \times -1^r \times x^{-r}$

$= {}^{12}C_r\, (-1) \cdot x^{24-2r-r}$

$= {}^{12}C_r\, (-1) \times x^{24-3r}$

Since we need to note the coefficient of x^6 ∴ To get the coefficient

$x^{24-3r} = x^6$

$24 - 3r = 6$

$24 - 6 = 3r$

$\frac{\cancel{18}^{6}}{\cancel{3}} = \frac{\cancel{3}r}{\cancel{3}}$

$r = 6$

$\therefore {}^{12}C_r\, a^{12-r}\, b^r = {}^{12}C_6\, (x^2)^{12-6}\, (1/x)^6$

$= {}^{12}C_6\, (x^2)^6 \cdot (x^{-1})^6 \cdot (-1)^6$

$= {}^{12}C_6\, (x^{12}) \cdot x^{-6} \cdot (-1)$

$= -924 \times x^{12-6}$

$= -924x^6$

∴ The coefficient of $x^6 = \underline{-924}$.

(b) Obtain the coefficient of x^7 in the expansion of $(x^2 - 2x)^5$.

Solution

$(x^2 - 2x)^5$

Let the term with x^7 be $= {}^nC_r\, a^{n-r}\, b^r$

$a = x^2 \quad b = -2x \cdot n = 5$

$= {}^5C_r\, (x^2)^{5-r}\, (-2x)^r$

$= {}^5C_r\, (x^{10-2r})\, (-2)^r\, (x)^r$

$= {}^5C_r\, (-2)^r\, (x^{10-2r+r}) = {}^5C_r\, (-2)^r\, x^{10-r} \ldots$ (a)

Since we are interested in the coefficient of x^7

$\therefore x^{10-r} = x^7 \; ; \; 10 - r = 7 \; ; \; r = 10 - 7 = 3$

Continuation of 4(b.)

$r = 3$

Substitute $r=3$ into equation (a.)

$= {}^5C_r (-2)^r x^{10-r}$

$= {}^5C_3 (-2)^3 x^{10-3}$

$= 10 \times -8 \times x^7$

$= -80x^7$

$\therefore$ The coefficient of x^7 is $= -80$

QUESTION 5

If the second term of a geometric progression is 24 and the fifth term is 81. ~~Fin~~ Find the common ratio and 7th term.

Solution

For a geometric progression (G.P) :-

$$T_n = ar^{n-1}$$

Given $T_2 = ar = 24 \quad \ldots\ldots \text{(i.)}$

$T_5 = ar^4 = 81 \quad \ldots\ldots \text{(ii)}$

· Divide (ii.) by (i.) to find common ratio, 'r'.

$$\frac{T_5}{T_2} = \frac{\cancel{a}r^{\cancel{4}\,3}}{\cancel{a}\cancel{r}} = \frac{81}{24}$$

$$r^3 = \frac{\cancel{81}\ 27}{\cancel{24}\ 8}$$

$$r^3 = \frac{27}{8}$$

$$r^3 = \frac{3^3}{2^3}$$

Take cubic root of both sides :-

$$\sqrt[3]{r^3} = \sqrt[3]{\left(\frac{3}{2}\right)^3}$$

$$r = 3/2$$

$\therefore$ Common ratio, $r = 3/2$

7th term, $T_7 = ar^6$

To find 'a' Recall that $T_2 = ar = 24$

$a \times 3/2 = 24$

$a = 24 \times 2/3$

$a = 48/3$

$a = 16$

$T_7 = ar^6$

$= 16 \times (3/2)^6$

$T_7 = 182.25$

$\therefore$ 7th term is 182.25

QUESTION 6

If α and β are the roots of the equation $2x^2-7x-3=0$, Find the equation whose roots are $2/\alpha^2$ and $2/\beta^2$

Solution

From $2x^2-7x-3=0$

Compared with $ax^2+bx+c=0$

$a=2 \quad b=-7 \quad c=-3$

When solving for equation with roots α & β

We know that :- $\alpha+\beta = -b/a$

$\alpha\beta = c/a$

$\therefore \alpha+\beta = -(-7)/2$

$= 7/2$

$\alpha\beta = -3/2$

Since we were asked for equation with roots $2/\alpha^2$ & $2/\beta^2$

The equation will be :-

$$\left(x - 2/\alpha^2\right)\left(x - 2/\beta^2\right)$$

By opening the brackets we'll have :-

$$= x^2 - x\times\left(\frac{2}{\beta^2}\right) - x\times\left(\frac{2}{\alpha^2}\right) + \left(\frac{2}{\alpha^2}\times\frac{2}{\beta^2}\right)$$

$$= x^2 - 2x\left(\frac{1}{\beta^2}\right) - 2x\left(\frac{1}{\alpha^2}\right) + \left(\frac{4}{\alpha^2\beta^2}\right)$$

$$= x^2 - 2x\left(\frac{1}{\beta^2}+\frac{1}{\alpha^2}\right) + \frac{4}{(\alpha\beta)^2}$$

$$= x^2 - 2x\left(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}\right) + \frac{4}{(\alpha\beta)^2}$$

Note that :- $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ so, we put it in :-

$$= x^2 - 2x\left(\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}\right) + \frac{4}{(\alpha\beta)^2}$$

Recall that $\alpha+\beta = 7/2$; $\alpha\beta = -3/2$

$\therefore$ The equation is thus :-

$$= x^2 - 2x\left(\frac{(7/2)^2 - 2(-3/2)}{(-3/2)^2}\right) + \frac{4}{(-3/2)^2}$$

$$= x^2 - 2x\left(\frac{(49/4 + 6/2)}{9/4}\right) + \frac{4}{9/4}$$

$$= x^2 - 2x\left(\frac{61}{9}\right) + \frac{16}{9}$$

Multiply through by '9'

$$= 9x^2 - (2x \times 61) + 16$$

$$= \underline{\underline{9x^2 - 122x + 16}}$$

Question 7

Find the value of K given that $3x^3 + 2x^2 + 6x + k$ leaves a remainder of 8 when divided by $(x-3)$

Solution

Let $P(x) = 3x^3 + 2x^2 + 6x + K$

$R = 8$

$D(x) = x - 3$

Using our Remainder theorem

$D(x) = 0$

$x - 3 = 0$

$x = 3$

$\therefore P(x) = R$

$P(3) = 8$

$P(3) = 3(3)^3 + 2(3)^2 + 6(3) + k = 8$

$3(27) + 2(9) + 18 + k = 8$

$81 + 18 + 18 + k = 8$

$117 + k = 8$

$k = 8 - 117$

$k = \underline{\underline{-109}}$

QUESTION 8

Solve the inequality $\frac{x^2+x-2}{x^2+4} > \frac{1}{2}$

Solution

$$\frac{x^2+x-2}{x^2+4} > \frac{1}{2}$$

Multiply both sides by $2(x^2+4)$

$$\cancel{2(x^2+4)}\left(\frac{x^2+x-2}{\cancel{x^2+4}}\right) > \frac{1}{\cancel{2}} \times \cancel{2}(x^2+4)$$

$$2(x^2+x-2) > x^2+4$$

$$2x^2+2x-4 > x^2+4 \quad \text{collect like terms;}$$

$$2x^2-x^2+2x-4-4 > 0$$

$$x^2+2x-8 > 0$$

Factorizing we have:

$$x^2+4x-2x-8 > 0$$

$$x(x+4)-2(x+4) > 0$$

$$(x-2)(x+4) > 0$$

Using Truth table: First we have to find our turning values

i.e $x-2=0$ & $x+4=0$

$x=2$ $\quad x=-4$

Thus, the truth table can be drawn

	$x<-4$	$-4<x<2$	$x>2$
$x+4$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$
$(x-2)(x+4)$	$+$	$-$	$+$

Since the Inequality has >0 then it should be positive

$\therefore$ The solution to the Inequality $\frac{x^2+x-2}{x^2+4} > \frac{1}{2}$

is $\Rightarrow x: \{x<-4 \cup x>2\}$

QUESTION 9

Write the partial fraction decomposition of $\frac{x+1}{(x-1)(x^2+1)}$

Solution

$$\frac{x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\frac{x+1}{(x-1)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(x-1)}{(x-1)(x^2+1)}$$

$$\Rightarrow x+1 = A(x^2+1)+(Bx+C)(x-1)$$

To get A let $x=1$

$$1+1 = A(1^2+1) + (B(1)+C)(1-1)$$

$$2 = A(1+1) + 0$$

$$2 = 2A$$

$$\frac{2A}{2} = \frac{2}{2}$$

$$A = 1$$

$$\Rightarrow x+1 = Ax^2 + A + Bx^2 - Bx + Cx - C$$

$$x+1 = (A+B)x^2 + (-B+C)x + A - C$$

Equating coefficient of x^2

$$A+B = 0$$

Since $A=1$

$$\therefore 1+B=0$$

$$B = -1$$

Equating coefficien of 'x'

$$1 = -B+C$$

since $B=-1$

$$1 = -(-1)+C$$

$$1 = 1+C$$

$$C = 1-1$$

$$C = 0$$

$\therefore$ The partial fraction decomposition is given as: $\frac{1}{x-1} + \frac{(-1)x+0}{x^2+1}$

$$\Rightarrow \frac{x+1}{(x-1)(x^2+1)} = \frac{1}{x-1} + \frac{(-1)x}{x^2+1} = \frac{1}{x-1} - \frac{x}{x^2+1}$$

Question 10

(a) Find k if $x-1$ is a factor x^3+kx^2-x-8

Solution

Let $f(x) = x^3+kx^2-x-8$, $D(x) = x-1$

Using the remainder theorem:

If $D(x)$ is a factor then R, remainder $= 0$

set $x-1=0$

$x=1$

$\therefore f(x) = R = 0$ since '$x-1$' is a factor

Therefore, $f(1) = 1^3 + k(1)^2 - 1 - 8 = 0$

$1+k-9=0$

$k = 9-1$

$k=8$

(b) Find the remainder when $8x^4-12x^2+7$ is divided by $2x-1$

Solution

1st Method

Using the remainder theorem

Let $f(x) = 8x^4-12x^2+7$

set $2x-1=0$

$x = \frac{1}{2}$

$f(\frac{1}{2}) =$ Remainder

$\therefore f(\frac{1}{2}) = 8(\frac{1}{2})^4 - 12(\frac{1}{2})^2 + 7 = R$

$R = \left(8 \times \frac{1}{16}\right) - \left(12 \times \frac{1}{4}\right) + 7$

$R = \frac{1}{2} - 3 + 7$

$R = \frac{1}{2} + 4$

$R = \frac{1+8}{2}$

$R = \frac{9}{2}$

2nd Method

Long Division

$2x-1 \overline{)\,8x^4-12x^2+7}$

Can be further expressed as:

$$
\begin{array}{r}
4x^3 + 2x^2 - 5x - \frac{5}{2} \\
2x-1 \overline{)\,8x^4+0x^3-12x^2+0x+7} \\
-(8x^4 - 4x^3) \\
\hline
4x^3 - 12x^2 \\
-(4x^3 - 2x^2) \\
\hline
-10x^2 + 0x \\
-(-10x^2 + 5x) \\
\hline
-5x + 7 \\
-(-5x + \frac{5}{2}) \\
\hline
\frac{9}{2}
\end{array}
$$

The Remainder

$\therefore$ The remainder when $8x^4-12x^2+7$ is divided by $2x-1$ is $= \frac{9}{2}$

Wishing Y'all Success!!!
All is well!

C. Sam
Academic Coordinator
@ 2leccF_ FUTO
+2340903627084 9