QUESTION ONE

(i) Distinguish between a field and a σ-field.

(ii) Let (Ω, f, p) be any given probability space. If $X, Y, Z \in \Omega$ then show that

$$P(X \cup Y \cup Z) = P(X) + P(Y) + P(Z) - P(X \cap Y) - P(X \cap Z) - P(Y \cap Z) + P(X \cap Y \cap Z)$$

QUESTION TWO

(i) Distinguish between the difference and equality of two sets.

(ii) Let V and R be two sets such that $V \subset R$, show that $P(V) \leq P(R)$.

(iii) The probability that a fourth year student will fail STA 321 Examination is 0.61. What is the probability that out of seven students, at least two students will pass the Examination.

QUESTION THREE

(i) Distinguish between measurable space and probability measure.

(ii) Show that if $Y_1, Y_2, \ldots, Y_n$ is a partition of the sample space Ω and B is any event defined on Ω then,

$$P(B) = \sum_{i=1}^{n} P(B/Y_i) P(Y_i)$$

Hence, determine the probability that a chip selected from a Bowl is yellow; given that Bowl I contains 3 red chips, 3 yellow chips and 5 blue chips, Bowl II contains 3 red chips, 5 yellow chips and 4 blue chips while Bowl III contains 4 red chips, 5 yellow chips and 3 blue chips

QUESTION FOUR

(i) Distinguish between the Central Limit Theorem and Chebyshev's Inequality.

(ii) Suppose X has the distribution

$$f(X) = \begin{cases} \dfrac{1}{2}; & 1 < X < 3 \\ 0; & \text{otherwise} \end{cases}$$

Obtain the chebyshev's bound with the exact value of the probability;

$$P(|X - \mu| \leq k\sigma) \text{ for any } k = 1, 2, 3, 4, 5$$

QUESTION FIVE

(i) When is a sequence $\langle X_n, n = 1, 2, \ldots \rangle$ said to be monotone.

(ii) Two breeds of chickens kept in two adjacent Poultries are recorded as follows:

Poultry	Breeds of Chickens		Total
	Type A	Type B	
Poultry I	13	47	60
Poultry II	10	31	41
Total	23	78	101

If a chicken is selected at random, what is the probability that:

(i) The chicken selected is of Type B breed.

(ii) Type A chicken selected is taken from Poultry 1.

QUESTION SIX

(i) Distinguish between convergence in probability and convergence in the kth mean for a sequence of random variables $\langle X_1, X_2, \ldots, X_n \rangle$.

(ii) Let $\langle X_1, X_2, \ldots, X_n \rangle$ be a sequence of random variables that follow the Bernoulli distribution with parameter $\dfrac{1}{n}; n \in N$. Show that X_n converges in probability to the random variable $\bar{X}$.