Chapter Three

3.0 D.C. CIRCUIT ANALYSIS

3.1 Kirchhoff's Laws

Question 1

Use Kirchhoff's current law to find the value of the unknown currents indicated in figure 3.1 below:

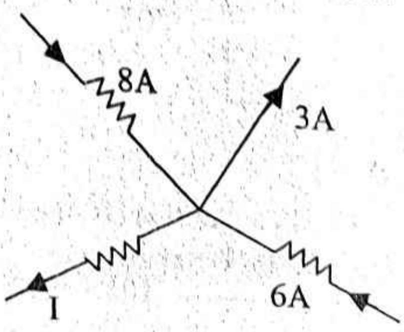


Fig. 3.1a

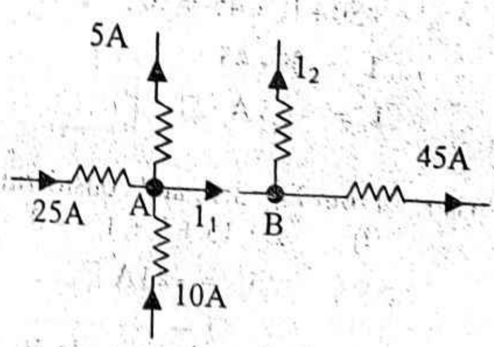


Fig. 3.1b

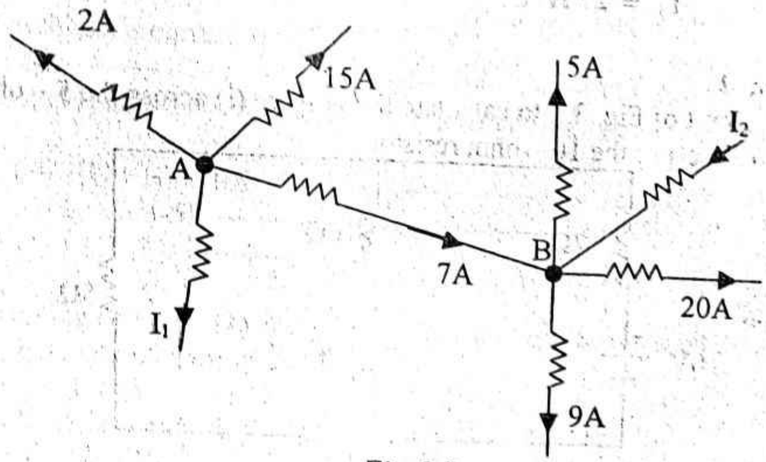


Fig. 3.1c

Answer 1

(1a) From Fig. 3.1a, we assume that current into a node is negative while otherwise positive.

$\therefore -8-6+3+I = 0$

$3+I = 14$

$I = 14-3$

$I = 11\text{ A}$ □

(1b) From fig. 3.1b, considering node A,

$-25-10+5+I_1 = 0$

$-35+5+I_1 = 0$

$I_1 = 30\text{ A}$ □

Considering node B,

$-I_1+I_2+45 = 0$

$-30+I_2+45 = 0$

$I_2 = 30-45$

$I_2 = -15\text{ A}$ □

(1c) From Fig. 3.1c and considering node A,

(i) $2+15+7+I_1 = 0$

$\therefore I_1 = -24\text{ A}$ □

(ii) Considering node B,

$-7-I_2+9+5+20 = 0$

$27-I_2 = 0$

$I_2 = 27\text{ A}$ □

Question 2

Use the circuit of Fig. 3.2 to calculate the voltage (i) across the 5 – ohm resistor (ii) across the 10 – ohm resistor.

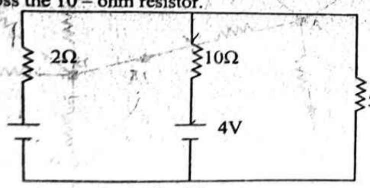


Fig. 3.2

Answer 2

Let the currents flowing in the network be as shown in fig. 3.3

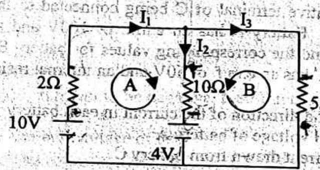


Fig. 3.3

In loop (A) and in the direction shown:

$10-2I_1-10I_2-4=0$ ---------- (1)

$6=2I_1+10I_2$ ---------- (2)

In loop (B) and in the direction shown:

$-10I_2-4+5I_3=0$ ---------- (3)

$-10I_2+5I_3=4$ ---------- (4)

From Kirchhoff's current law, KCL

$I_1=I_2+I_3=$ ---------- (5)

Since I_2 and I_3 are of interest to us, we put equation (5) into equation (2) to get,

$6=2(I_2+I_3)+10I_2$ ---------- (6)

$3=I_2+I_3+5I_2$ ---------- (7)

$6I_2+I_3=3$ ---------- (8)

Now, solving equation (4) and equation (8) simultaneously we have,

$I_2=0.275\text{A}$ and

$I_3=1.35\text{A}$

(i) Voltage across the 5 – Ω resistor is V_5

$V_5=I_3 \times R_5$

$V_5 = $ [illegible] $\times 5$

V [illegible]

(ii)

Question 3

Two batteries A and B are connected in parallel. Connected across the bettery terminals is a circuit consisting of a battery C in series with a 13 ohm resistor, the negative terminal of C being connected to the positive terminals of A and B. Battery A has an e.m.f. of 100V and an internal resistance of 2 ohm, and the corresponding values for battery B are 150V and 1.0 ohm. Battery C has an e.m.f. of 50V and an internal resistance of 7 ohm. Determine:

(i) the value and direction of the current in each battery
(ii) the terminal voltage of battery A
(iii) the total current drawn from battery C

Using kirchhoff's laws.

Answer 3

The circuit is as shown in fig. 3.4 with the current directions.

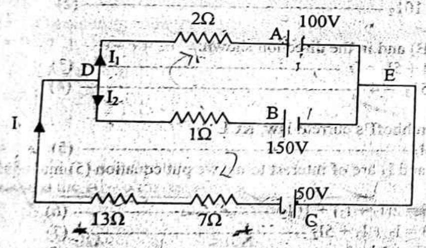


Fig. 3.4

From KCL,

$I = I_1 + I_2$ ---------------------------- (1)

In loop D A E B D,

$-2I_1 - 100 + 150 + I_2 = 0$ ---------------- (2)

$-2I_1 + I_2 = -50$ ---------------------------- (3)

In loop D A E C D,

$-2I_1 - 100 - 50 - 20I = 0$ ------------------ (4)

$-2I_1 - 150 - 20(I_1 + I_2) = 0$ ------------- (5)

$-2I_1 - 20I_1 - 20I_2 - 150 = 0$ ------------ (6)

$-22I_1 - 20I_2 = 150$ ------------------------- (7)

Solving equation (3) and equation (7)
Simultaneously we have,
$I_1 = 13.71$ A and $I_2 = -22.58$A

(i) The value and direction of the current in each battery

$I_A = 13.71A$ □
$I_B = -22.58A$ □
$I_C = I_A + I_B$
$I_C = 13.71 - 22.58$
$I_C = -8.87A$ □

Since I_B and I_C turn out to be negative, their direction of flow is opposite to that shown in Fig. 3.4.

(ii) the terminal voltage of battery A is

$V_A = E_A + I_1 r_A$
$V_A = 100 + 13.71 \times 2$
$V_A = 127.42V$ □

(iii) The total current drawn from battery C is I

$I = I_1 + I_2$
$I = 13.71 - 22.58$
$I = I_C = -8.87$ A □

Question 4

Determine the value of the unknown Voltage in the circuit shown in Figure 3.5 given that 3 A current flows through the 6 – ohm resistor.

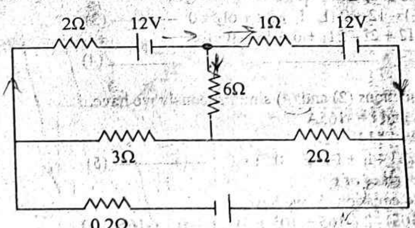


Fig. 3.5

Answer 4

We now redraw the circuit showing the current directions as shown in Figure 3.6

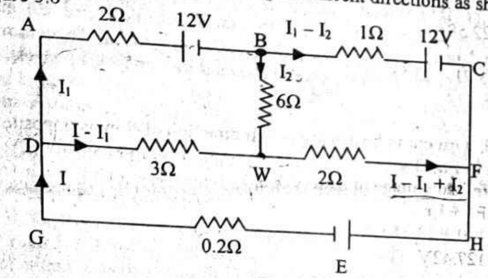


Fig. 3.6

In loop A B W D A,

$-2I_1 - 12 - 6I_2 + 3(I - I_1) = 0$ ------------------------ (1)

$-5I_1 - 6I_2 - 12 + 3I = 0$

But $I_2 = 3A$ (Given)

$-5I_1 - 6 \times 3 - 12 + 3I = 0$

$-5I_1 + 3I = 30$ --- (2)

In loop B C F W B,

$-1(I_1 - I_2) - 12 + 2(I - I_1 + I_2) + 6I_2 = 0$ ---------- (3)

$-I_1 + 3 - 12 + 2I - 2I_1 + 6 + 18 = 0$

$-3I_1 + 2I = -15$ ________________________________ (4)

Solving equations (2) and (4) simultaneously we have,

$I_1 = -105A$ and $I = -165A$

In loop D W F H E G D,

$-3(I - I_1) - 2(I - I_1 + I_2) - E - 0.2I = 0$ -------------- (5)

Putting the values of I,

I_1 and I_2 into equation (5) we have,

$-3(-165 + 105) - 2(-165 + 105 + 3) - E - 0.2(-165) = 0$

$180 + 114 - E + 33 = 0$

$E = 327V$ □

Question 5

In the bridge network shown in figure 3.7 below, the resistance of the galvanometer is 5Ω and the internal resistance of the cell is 2Ω. Determine:

(i) the magnitude of the branch currents

(ii) the p.d across AC

(iii) the resistance from A to C.

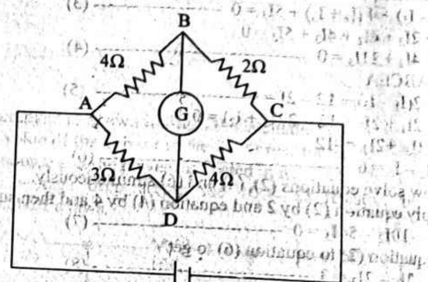


Fig. 3.7

Answer 5

We redraw Fig. 3.7 as shown in Fig. 3.8 showing all the branch currents.

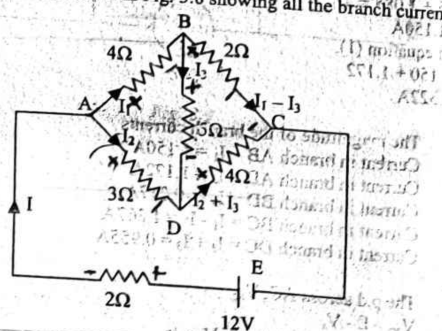


Fig. 3.8

Assume current directions as shown above.

Applying KCL

$I = I_1 + I_2$ -- (1)

We now apply the kirchhoff's voltage law, KVL to get, Loop ABDA,

$-4I_1 - 5I_3 + 3I_2 = 0$ ---------------------------- (2)

Loop BCDB,

$-2(I_1 - I_3) + 4(I_2 + I_3) + 5I_3 = 0$ ------------------ (3)

$-2I_1 + 2I_3 + 4I_2 + 4I_3 + 5I_3 = 0$

$-2I_1 + 4I_2 + 11I_3 = 0$ ---------------------------- (4)

Loop ABCEA

$-4I_1 - 2(I_1 - I_3) + 12 - 2I = 0$ ------------------ (5)

$-4I_1 - 2I_1 + 2I_3 + 12 - 2(I_1 + I_2) = 0$

$-8I_1 - 2I_2 + 2I_3 = -12$

$4I_1 + I_2 - I_3 = 6$ ---------------------------- (6)

We now solve equations (2), (4) and (6) simultaneously.

Multiply equation (2) by 2 and equation (4) by 4 and then subtract to get

$-10I_2 - 54I_3 = 0$ ---------------------------- (7)

Add equation (2) to equation (6) to get

$2I_2 - 3I_3 = 3$ ---------------------------- (8)

Solving equation (7) and equation (8) we get,

$I_2 = 1.172A$

$I_3 = -0.217A$

From equation (2),

$-4I_1 + 3 \times 1.172 + 5 \times 0.217 = 0$

$3.516 + 1.085 = 4I_1$

$I_1 = 1.150A$

From equation (1),

$I = 1.150 + 1.172$

$I = 2.322A$

(i) The magnitude of the branch currents

Current in branch AB = I_1 = 1.150A

Current in branch AD = I_2 = 1.172A

Current in branch BD = I_3 = -0.217A

Current in branch BC = $I_1 - I_3$ = 1.367A

Current in branch DC = $I_2 + I_3$ = 0.955A

(ii) The p.d across AC, V_{ac}

$V_{ac} = E - V_r$

Voltage drop in the cell, $V_r = Ir$

$V_r = 2.322 \times 2$

$V_r = 4.644V$

$\therefore V_{ac} = 12 - 4.644$

$V_{ac} = 7.356V$ □

(iii) The resistance of the bridge between points A and C, r_{ac}

$$r_{ac} = \frac{V_{ac}}{I}$$

$$r_{ac} = \frac{7.356}{2.322}$$

$r_{ac} = 3.168\Omega$ □

Question 6

A network is arranged as shown in Fig. 3.9. Calculate

(i) the value of the current in the 2 – Ω resistor connected near the 5V source and the power dissipated in it.

(ii) The voltage drop across the 10 – Ω resistor, by using Kirchhoff's laws.

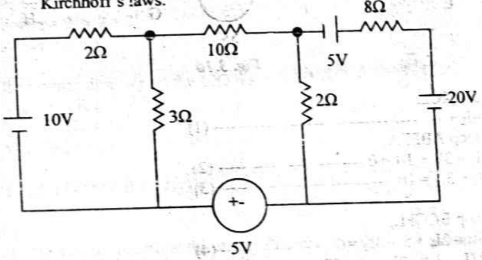


Fig. 3.9

Answer 6

Figure 3.9 is redrawn as shown in Fig. 3.10 with the assumed current directions.

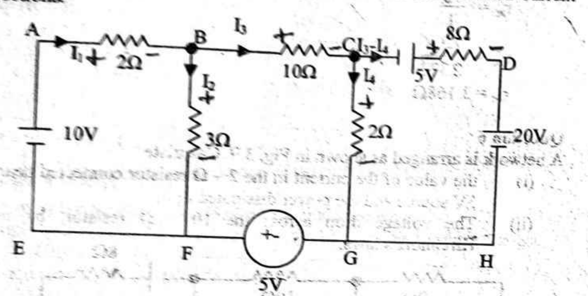


Fig. 3.10

From KCL,

$I_1 = I_2 + I_3$ ———————— (1)

In loop ABFEA,

$-2I_1 - 3I_2 + 10 = 0$ ———————— (2)

$2I_1 + 3I_2 = 10$ ———————— (3)

In loop BCGFB,

$-10I_3 - 2I_4 + 5 + 3I_2 = 0$ ———————— (4)

$-10(I_1 - I_2) - 2I_4 + 5 + 3I_2 = 0$

$10I_1 - 13I_2 + 2I_4 = 5$ ———————— (5)

In loop CDHGC,

$5 - 8(I_3 - I_4) - 20 + 2I_4 = 0$

$5 - 8I_3 + 8I_4 - 20 + 2I_4 = 0$

$-8(I_1 - I_2) + 10I_4 = 15$

$-8I_1 + 8I_2 + 10I_4 = 15$ ———————— (6)

We now solve equations (3), (5) and (6) simultaneously.
Multiply equation (5) by 10 and equation (6) by 2, then subtract to get

$116I_1 - 146I_2 = 20$ ———————— (7)

Now solve equation (3) and (7) simultaneously to get

$I_1 = 2.375A$ and $I_2 = 1.75A$

But $I_3 = I_1 - I_2$

$I_3 = 2.375 - 1.75$

$I_3 = 0.625A$

From equation (5)

$10 \times 2.375 - 13 \times 1.75 + 2I_4 = 5$

$23.75 - 22.75 + 2I_4 = 5$

$2I_4 = 5 - 1$

$I_4 = \frac{4}{2}$

$I_4 = 2.0A$

(i) the value of the current in the 2Ω resistor is I_4.

$\therefore I_4 = 2.0A$ □

Power = I^2R

Power = $2^2 \times 2 = 2^3$

Power = 8.0W □

(ii) the voltage drop across the 10Ω resistor is V_{10}

$V_{10} = I_3 \times R_{10}$

$V_{10} = 0.625 \times 10$

$V_{10} = 6.25$ V □

3.2 Mesh and Nodal Analysis

Question 1

Calculate the Voltage drop and the energy dissipated in the 4 - Ω resistor for 10 minutes using

(a) loop – current (Mesh) method

(b) Node – Voltage method. See figure 1 below.

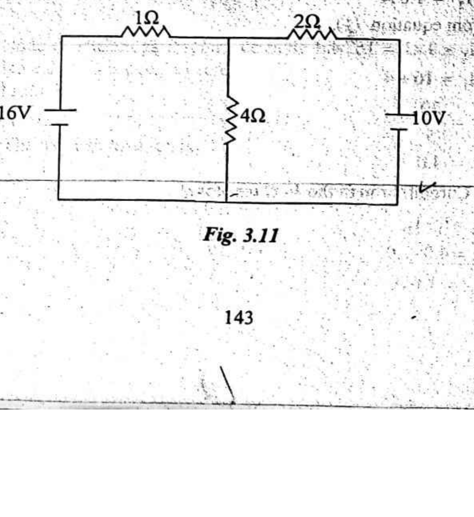


Fig. 3.11

Answer 1

(a) Applying the loop-current method, we assume loop-current directions as shown in Figure 3.12.

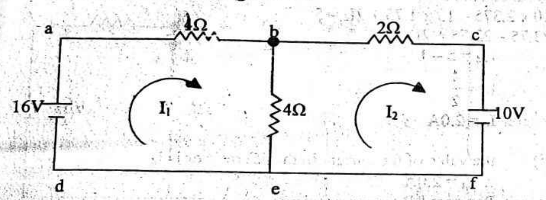


Fig. 3.12

In Loop abeda,

$I_1 + 4(I_1 - I_2) - 16 = 0$

$5I_1 - 4I_2 = 16$ ---------- (1)

From loop bcfeb,

$2I_2 + 10 + 4(I_2 - I_1) = 0$

$-4I_1 + 6I_2 = -10$

$-2I_1 + 3I_2 = -5$ ----------- (2)

Multiply equation (1) by 2 and equation (2) by 5 to get,

$10I_1 - 8I_2 = 32$ --------------- (3)

$-10I_1 + 15I_2 = -25$ --------------(4)

Add equation (4) and equation (3),

$7I_2 = 7$

$\therefore I_2 = 1.0\,A$

From equation (1),

$5I_1 - 4 \times 1 = 16$

$5I_1 = 16 + 4$

$I_1 = \frac{20}{5}$

$I_1 = 4.0\ A$

$\therefore$ Current through the 4 - Ω resistor is, I_4

$I_4 = I_1 - I_2$

$I_4 = 4.0 - 1.0$

$I_4 = 3.0\,A$

(i) Voltage across the 4Ω resistor, V_4

$V_4 = I_4 R_4$

$V_4 = 3 \times 4$

$V_4 = 12.0\,V$ □

(ii) Energy dissipated in the 4-Ω resistor

$= I_4^2 R_4 t$

where :

t = time in seconds

Energy $= 3^2 \times 4 \times 10 \times 60$

Energy = 21600W

Energy = 21.6KW □

(b) We now apply the node – voltage method.

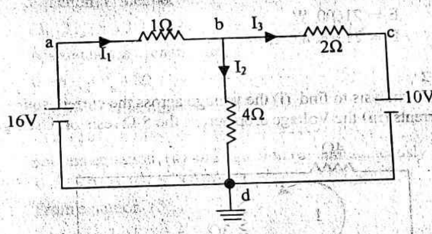


Fig. 3.13

Node d is taken as a reference node having grounded it to earth. Note that V_d is equal to zero.

From KCL,

$I_1 = I_2 + I_3$ ---------------- (1)

The voltage equation is,

$\frac{V_a - V_b}{R_{ab}} = \frac{V_b - V_d}{R_{bd}} + \frac{V_b - V_c}{R_{bc}}$ (Considering node b)

$\frac{16 - V_b}{1} = \frac{V_b - 0}{4} + \frac{V_b - 10}{2}$

$4(16 - V_b) = V_b + 2(V_b - 10)$

$64 - 4V_b = V_b + 2V_b - 20$

$64 + 20 = V_b + 2V_b + 4V_b$

$7V_b = 84$

$V_b = \frac{84}{7}$

$V_b = 12V$

(i) the voltage across the 4Ω resistor is

$V_b = 12.0V$ □

(ii) Energy dissipated in the 4-Ω resistor, E

$E = \frac{V^2 t}{R}$

$E = \frac{12^2 \times 10 \times 60}{4}$

$E = 21600\ W$

$E = 21.6\ KW$ □

Question 2

Use the Mesh analysis to find (i) the voltage across the current source (ii) the loop currents (iii) the Voltage drop across the 5-Ω resistor. Use figure 3.14 below.

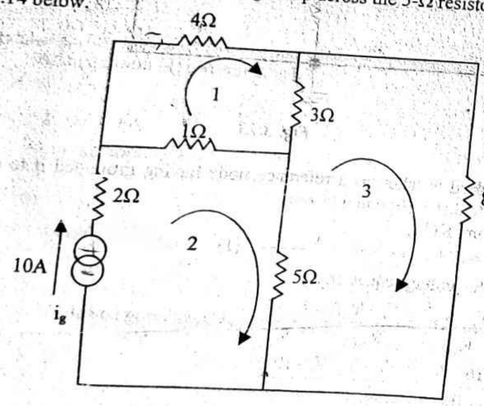


Fig 3.14

Answer **2**

From figure 3.14 and using mesh analysis we have,

Loop 1,

$4I_1 + 3(I_1 - I_3) + 1(I_1 - I_2) = 0$

$8I_1 - I_2 - 3I_3 = 0$ ---------------- (1)

loop 2,

$1(I_2 - I_1) + 5(I_2 - I_3) + 2I_2 + V_g = 0$

$-I_1 + 8I_2 - 5I_3 = -V_g$ ------------- (2)

loop 3,

$3(I_3 - I_1) + 5(I_3 - I_2) + 8I_3 = 0$

$-3I_1 - 5I_2 + 16I_3 = 0$

But $i_g = I_2 = 10A$ (see figure 3.14)

Equation (1) then becomes,

$8I_1 - 3I_3 = 10$ ------------------------ (4)

Also, equation (2) changes to,

$I_1 + 5I_3 = V_g + 80$ -------------------- (5)

Equation (3) becomes,

$-3I_1 + 16I_3 = 50$ ------------------------ (6)

Solving equation (4) and equation (6) simultaneously we get,

$I_1 = 2.61A,\ I_3 = 3.61A$

From equation (5),

$2.61 + 5 \times 3.61 - 80 = V_g$

$V_g = 20.66 - 80$

$V_g = -59.34\ V$ □

Note that the minus sign indicates that the polarity of the voltage source should be reversed.

(ii) the loop Currents

$I_1 = 2.61\ A$ □

$I_2 = i_g = 10\ A$ □

$I_3 = 3.61A$ □

(iii) the voltage drop across the 5 - Ω resistor, V_5

$V_5 = R_5 (I_2 - I_3)$

$V_5 = 5(10 - 3.61) = 5 \times 6.39$

$V_5 = 31.95\ V$ □

Question 3

Use Nodal analysis to find the Voltage across the 10 - Ω resistor in figure 3.15 below. The magnitude of the source current, i_s is 5A

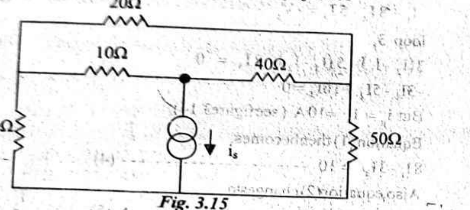


Fig. 3.15

Answer 3

The figure above is redrawn as shown in figure 3.16

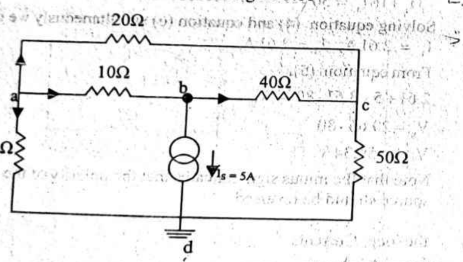


Fig. 3.16

Node a :

$$\frac{V_a - V_c}{r_{ac}} + \frac{V_a - V_b}{r_{ab}} + \frac{V_a - V_d}{r_{ad}} = 0$$

$$\frac{V_a - V_c}{20} + \frac{V_a - V_b}{10} + \frac{V_a - 0}{30} = 0$$

$$3(V_a - V_c) + 6(V_a - V_b) + 2V_a = 0$$

$$11V_a - 6V_b - 3V_c = 0 \quad \text{------------} \quad (1)$$

Node b

$$\frac{V_b - V_a}{r_{ab}} + \frac{V_b - V_c}{r_{bc}} + i_s = 0$$

$$\frac{V_b - V_a}{10} + \frac{V_b - V_c}{40} + 5 = 0$$

$$4(V_b - V_a) + V_b - V_c + 200 = 0$$

$$5V_b - 4V_a - V_c = -200$$

$$+4V_a - 5V_b + V_c = 200 \quad \text{------------} \quad (2)$$

Node c:

$$\frac{V_c - V_a}{r_{ac}} + \frac{V_c - V_b}{r_{bc}} + \frac{V_c - V_d}{r_{cd}}$$

$$\frac{V_c - V_a}{20} + \frac{V_c - V_b}{40} + \frac{V_c}{50} = 0$$

$$10(V_C - V_a) + 5(V_c - V_b) + 4V_c = 0$$

$$-10V_a - 5V_b + 19V_c = 0$$

$$10V_a + 5V_b - 19V_c = 0 \quad \text{----------------} \quad (3)$$

Solving equations (1), (2) and (3) Simultaneously we have,

$V_a = -97.7V$, $V_b = -135.6V$

$V_c = -87.2\ V$

∴ Voltage across the 10 - Ω resistor is

$V_{ab} = V_a - V_b$

$V_{ab} = -97.7 - (-135.6)$

$\therefore V_{ab} = 37.9\ V$ □

Question 4

Use Mesh analysis to find the voltage across the 8 ohm resistor in fig. 4 below

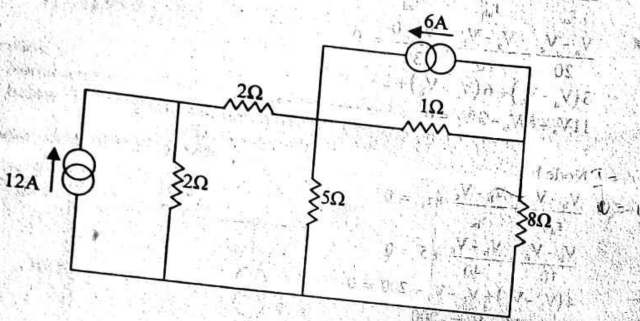


Fig. 3.17

Answer 4

Applying mesh analysis, we first of all convert all current sources to Voltage sources as shown in figure 3.18

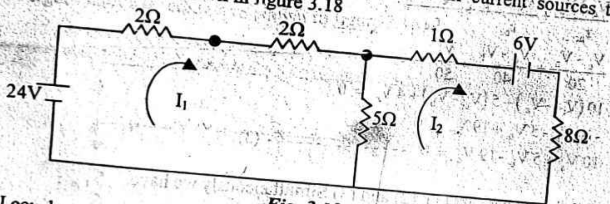


Fig. 3.18

Loop 1:

$-4I_1 - 5(I_1 - I_2) + 24 = 0$

$-9I_1 + 5I_2 = -24$

$9I_1 - 5I_2 = 24$ ---------------------- (1)

loop 2,

$-I_2 - 6 - 8I_2 - 5(I_2 - I_1) = 0$

$+5I_1 - 14I_2 = 6$ ------------------- (2)

Solving equations (1) and (2) simultaneously, we have,

$I_1 = 3.028\,A,\ I_2 = 0.653\,A$

Let V_8 be the Voltage across the 8 - Ω resistor.

$\therefore V_8 = I_2 \times R_8$

$V_8 = 0.653 \times 8$

$V_8 = 5.224\,V$ □

Question 5

Apply Nodal analysis to Calculate:

(i) the current supplied by the sources

(ii) the Voltage drop across the 1 - Ω resistor

(iii) the power consumed by the 4 - Ω resistor. See figure 3.19 below

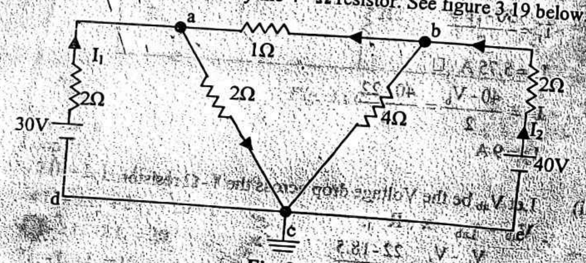


Fig. 3.19

Answer 5

Node a:

$$\frac{V_a - V_b}{r_{ab}} + \frac{V_a - 30}{r_{ad}} + \frac{V_a - V_c}{r_{ac}} = 0$$

$$\frac{V_a - V_b}{1} + \frac{V_a - 30}{2} + \frac{V_a}{2} = 0$$

$2(V_a - V_b) + V_a - 30 + V_a = 0$

$4V_a - 2V_b = 30$

$2V_a - V_b = 15$ --------------- (1)

Node b:

$$\frac{V_b - V_a}{r_{ab}} + \frac{V_b - 40}{r_{bc}} + \frac{V_b - V_c}{r_{bc}} = 0$$

$$\frac{V_b - V_a}{1} + \frac{V_b - 40}{2} + \frac{V_b - 0}{4} = 0$$

$$4(V_b - V_a) + 2(V_b - 40) + V_b = 0$$

$$-4V_a + 7V_b = 80 \quad \text{-----------------} \quad (2)$$

Solving equations (1) and (2) we have,

$V_a = 18.5\,V$, $V_b = 22\,V$

(i) Let the currents supplied by the sources be I_1 and I_2 as indicated in figure 3.19

$$\therefore I_1 = \frac{30 - V_a}{2}$$

$$I_1 = \frac{30 - 18.5}{2}$$

$$I_1 = 5.75\,A \quad \square$$

$$I_2 = \frac{40 - V_b}{2} = \frac{40 - 22}{2}$$

$$I_2 = 9\,A$$

(ii) Let V_{ab} be the Voltage drop across the 1 - Ω resistor.

$$V_{ab} = I_{ab} \times R$$

$$I_{ab} = \frac{V_b - V_a}{R_{ab}} = \frac{22 - 18.5}{1}$$

$$I_{ab} = 3.5\,A$$

$$\therefore V_{ab} = 3.5 \times 1$$

$$V_{ab} = 3.5\,V \quad \square$$

(iii) Power consumed by the 4 - Ω resistor,

$$P_4 = \frac{V_4^2}{R} = \frac{V_b^2}{R}$$

$$P_4 = \frac{22^2}{4}$$

$$P_4 = 121\,W \quad \square$$

Question 6

Using the network shown in figure 3.20 and applying mesh Analysis, Calculate:

(i) the loop currents

(ii) the resistance seen by the 20V source

(iii) the voltage drop across R_o.

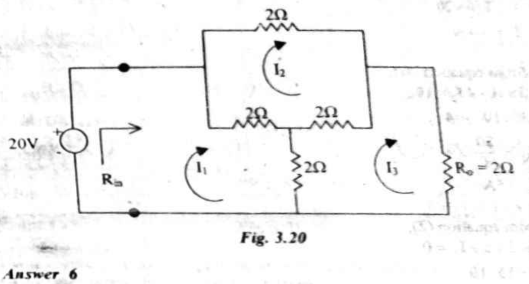


Fig. 3.20

Answer 6

Loop 1:

$$-2(I_1 - I_2) - 2(I_1 - I_3) + 20 = 0$$

$$-4I_1 + 2I_2 + 2I_3 + 20 = 0$$

$$4I_1 - 2I_2 - 2I_3 = 20$$

$$2I_1 - I_2 - I_3 = 10 \quad \text{-------------} \quad (1)$$

Loop 2:

$$-2I_2 - 2(I_2 - I_1) - 2(I_2 - I_3) = 0$$

$$2I_1 - 6I_2 + 2I_3 = 0$$

$$I_1 - 3I_2 + I_3 = 0 \quad \text{-------------} \quad (2)$$

Loop 3:

$$-2I_3 + 2I_2 - 2I_3 - 2(I_3 - I_1) = 0$$

$$2I_1 + 2I_2 - 6I_3 = 0$$

$$I_1 + I_2 - 3I_3 = 0 \quad \text{-----------} \quad (3)$$

We now solve the three equations Simultaneously.

Add equation (1) and equation (2)

$$3I_1 - 4I_2 = 10 \quad \text{-----------------} \quad (4)$$

Multiply equation (1) by – 3 and equation (3) by I and add, to get

$-5I_1 + 4I_2 = -30$ ------------------ (5)

Add equation (4) and equation (5) to eliminate I_2

$-2I_1 = -20$

$I_1 = 10\,A$

From equation (4),

$3\times 10 - 4I_2 = 10$

$30 - 10 = 4I_2$

$I_2 = \frac{20}{4}$

$I_2 = 5A$

From equation (2),

$10 - 3\times 5 + I_3 = 0$

$I_3 = 15 - 10$

$I_3 = 5\,A$

(i) **the loop currents are**

$I_1 = 10\,A$ □

$I_2 = 5A$ and

$I_3 = 5A$ □

(ii) $R_{\blacksquare} = \frac{V}{I_1}$

$R_{\blacksquare} = \frac{20}{10}$

$R_{\blacksquare} = 2.0\Omega$ □

(iii) Let V_o be the Voltage drop across R_o

$\therefore V_o = I_3 R_o$

$V_o = 5\times 2$

$V_o = 10V$ □

Question 7

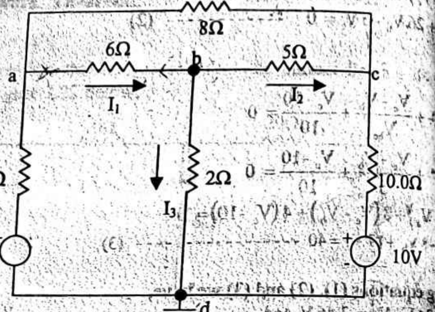


Fig. 3.21

Using Figure 3.21 above, apply Nodal Analysis to Calculate:

(i) the current through the $2-\Omega$ resistor

(ii) the Voltage drop across the $6-\Omega$ resistor

(iii) the heat wasted in the $5-\Omega$ resistor if current flows through it for 20 minutes

Answer 7

From Figure 3.21 we have,

Node a :

$$\frac{V_a - V_b}{R_{ab}} + \frac{V_a - V_c}{R_{ac}} + \frac{V_a - 14}{10.0} = 0$$

$$\frac{V_a - V_b}{6} + \frac{V_a - V_C}{8} + \frac{V_a - 14}{10} = 0$$

$$20(V_a - V_b) + 15(V_a - V_c) + 12(V_a - 14) = 0$$

$47V_a - 20V_b - 15V_c = 168$ ------------------------ (1)

Node b :

$$\frac{V_b - V_c}{R_{bc}} + \frac{V_b - V_d}{R_{bd}} + \frac{V_b - V_a}{R_{ab}} = 0$$

$$\frac{V_b - V_c}{5} + \frac{V_b - 0}{2} + \frac{V_b - V_a}{6} = 0$$

$$6(V_b - V_c) + 15V_b + 5(V_b - V_a) = 0$$

$$-5V_a + 26V_b - 6V_c = 0 \quad \text{-----------------} \quad (2)$$

Node c:

$$\frac{V_c - V_a}{R_{ac}} + \frac{V_c - V_b}{V_{bc}} + \frac{V_c - 10}{10} = 0$$

$$\frac{V_c - V_a}{8} + \frac{V_c - V_b}{5} + \frac{V_c - 10}{10} = 0$$

$$5(V_c - V_a) + 8(V_c - V_b) + 4(V_c - 10) = 0$$

$$-5V_a - 8V_b + 17V_c = 40 \quad \text{-----------------} \quad (3)$$

Solving equations (1), (2) and (3) we have,
$V_a = 6.34\,V$, $V_b = 2.46\,V$ and
$V_c = 5.38\,V$

(i) Current through the 2 – Ω resistor, I_3

$$I_3 = \frac{V_b}{2}$$

$$I_3 = \frac{2.46}{2}$$

$I_3 = 1.23\,A$ □

(ii) Voltage drop across the 6 – Ω resistor, V_6

$V_6 = V_a - V_b$

$V_6 = 6.34 - 2.46$

$V_6 = 3.88\,V$ □

(iii) To find the heat dissipated in the 5 - Ω resistor.

$$I_2 = \frac{V_c - V_b}{5} = \frac{5.38 - 2.46}{5}$$

$I_2 = 0.584\,A$

$\therefore H_5 = I_2^2 Rt$

$H_5 = 0.584^2 \times 5 \times 20 \times 60$

$H_5 = 2046.34\,J$

$H_5 = 2.05\,KJ$ □

3.3 Network Theorems.

Question 1

Use Superposition theorem to calculate the current flowing in the 6Ω Resistor in figure 3.22 below.

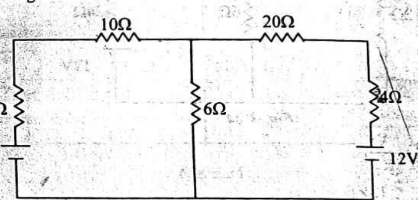


Fig. 3.22

Answer 1

1. Suppress the 12V – Source.

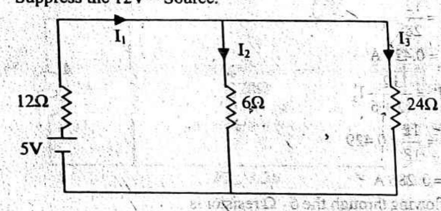


Fig. 3.23

From KCL,

$I_1 = I_2 + I_3$

$$I_1 = \frac{5}{12 + \frac{6 \times 24}{30}} = \frac{5}{12 + 4.8}$$

$I_1 = 0.298\,A$

$$\therefore I_2 = \frac{24}{24 + 6} \times I_1$$

$$I_2 = \frac{24}{30} \times 0.298$$

$I_2 = 0.238\,A$

2. Suppress the 5 V – source

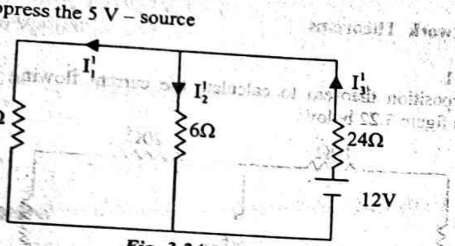


Fig. 3.24

$$I_3^1 = \frac{V}{R_T}$$

$$R_T = \frac{12 \times 6}{12 + 6} + 24$$

$$R_T = 28\,\Omega$$

$$I_3^1 = \frac{12}{28}$$

$$I_3^1 = 0.429\,A$$

$$\therefore I_2^1 = \frac{12}{12 + 6} I_3^1$$

$$I_2^1 = \frac{12}{18} \times 0.429$$

$$I_2^1 = 0.286\,A$$

$\therefore$ Current flowing through the 6 - Ω resistor is,

$$I_6 = I_2 + I_2^1$$

$$I_6 = 0.238 + 0.286$$

$$I_6 = 0.524\,A$$ □

Question 2
Calculate the current and the power in the 20 Ω resistor in the circuit in figure 3.25

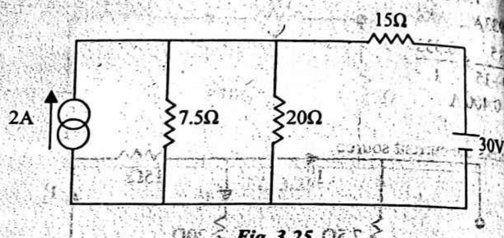


Fig. 3.25

Answer 2

1. Suppress the 30V - Source.

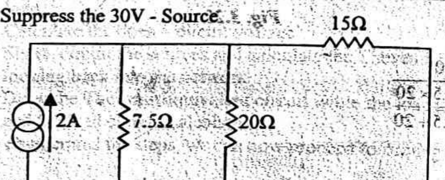


Fig. 3.26

The current source of figure 3.26 can be replaced by a voltage source shown in figure 3.27

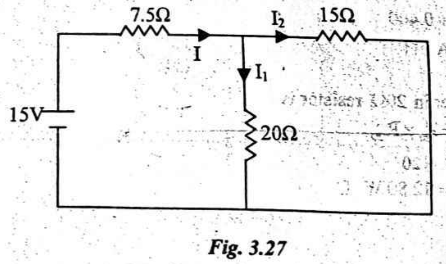


Fig. 3.27

$$I = \frac{15}{7.5 + \frac{15 \times 20}{15 + 20}}$$

$I = 0.933A$

$$I_1 = \frac{15}{20 + 15} \times \frac{0.933}{1}$$

$\therefore I_1 = 0.400\,A$

2. Suppress the current source

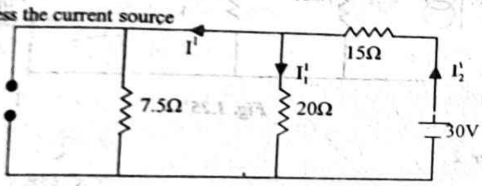


Fig. 3.28

$$I_2^1 = \frac{30}{15 + \frac{7.5 \times 20}{7.5 + 20}}$$

$I_2^1 = 1.467\,A$

$$\therefore I_1^1 = \frac{7.5}{7.5 + 20} \times 1.467$$

$I_1^1 = 0.400\,A$

$\therefore$ **Total current in 20Ω resistor is**

$I_{20} = I_1 + I_1^1$

$I_{20} = 0.400 + 0.400$

$I_{20} = 0.800\,A$ □

(ii) Power in 20Ω resistor is

$P_{20} = I_{20}^2 \times R_{20}$

$= 0.8^2 \times 20$

$P_{20} = 12.80\,W$ □

Question 3

Calculate the current in the 8Ω resistor using Thevenin's theorem.

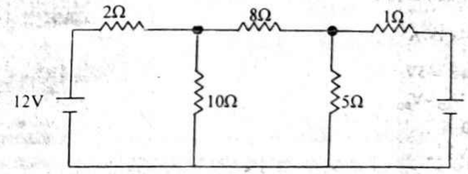


Fig 3.29

Answer 3

It is important to enumerate lucidly the steps to be taken when applying Thevenin's theorem;

(i) Remove that portion of the original network considered as the load.

(ii) Calculate the open – circuit voltage

(iii) Short- circuit the sources and calculate the Thevenin resistance by looking back into the network

(iv) Draw the Thevenin equivalent circuit while the load is reconnected and the load current calculated.

Having enumerated the steps, we can now proceed to solving the problem.

(i) Open circuit the branch containing the 8Ω resistor and then calculate the open – circuit voltage.

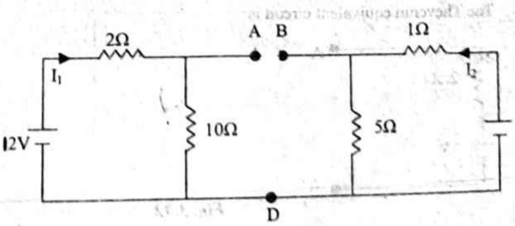


Fig 3.30

$I_1 = \frac{12}{2+10} = 1.0\text{ A}$

$V_{AD} = 1.0 \times 10 = 10\text{ V}$

$I_2 = \frac{6}{5+1} = 1.0\text{ A}$

$V_{BD} = 1 \times 5 = 5\text{V}$

$\therefore V_{AB} = V_{AD} - V_{BD}$

$V_{AB} = 10 - 5$

$V_{AB} = 5\text{V}$

(iii) Short – circuit the sources and calculate the Thevenin resistance, R_T

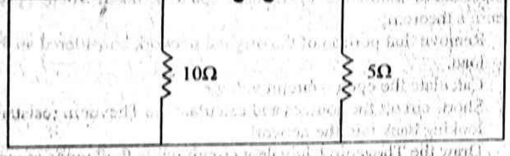


Fig. 3.31

$R_T = \frac{2\times10}{12} + \frac{5\times1}{6}$

$R_T = 2.5\Omega$

(iv) The Thevenin equivalent circuit is

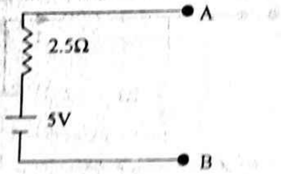


Fig. 3.32

∴ The current in the 8-Ω resistor is

$I_8 = \frac{V_{AB}}{R_T + R_8}$

$I_8 = \frac{5}{2.5+8}$

$I_8 = 0.476\text{ A}$ □

Question 4

Apply the Thevenin theorem on the bridge circuit of figure 3.33 to find the deflection of a galvanometer connected as shown with a resistance of 44 ohms and a sensitivity of 2.5 μA/mm

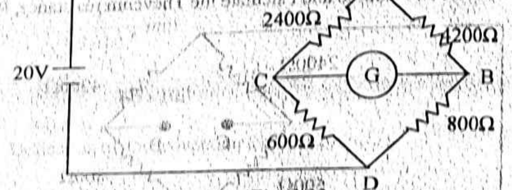


Fig. 3.33

Answer 4

(i) Open circuit the branch containing the galvanometer and then calculate the open circuit voltage, V_{CB}.

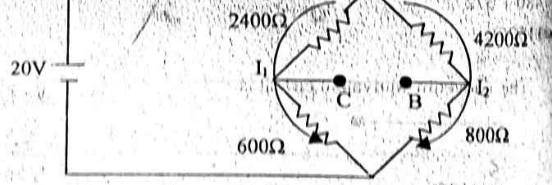


Fig. 3.34

2.8

6.7

$$I_1 = \frac{20}{2400+600}$$

$$I_1 = \frac{2}{300}A$$

$$I_2 = \frac{20}{4200+800} = \frac{2}{500}A$$

Assuming that the potential at C is greater than B, we have,

$$V_{CB} = \frac{2}{300} \times \frac{600}{1} - \frac{2}{500} \times \frac{800}{1}$$

$$V_{CB} = 4 - 3.2 = 0.8V$$

(iii) Short-circuit 20V source and calculate the Thevenin resistance, R_T

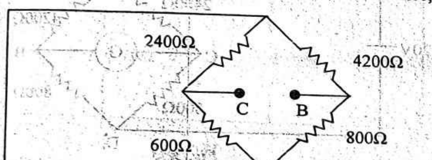


Fig.3.35

$$R_T = \frac{2400 \times 600}{2400+600} + \frac{4200 \times 800}{4200+800}$$

$$R_T = 480 + 672$$

$$R_T = 1152\Omega$$

(iv) The Thevenin equivalent circuit is

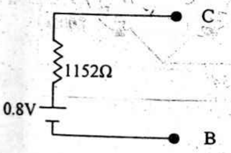


Fig. 3.36

∴ the current in the galvanometer having a resistance of 448 ohms, I_G

$$I_G = \frac{V_{CB}}{R_T + 448}$$

$$I_G = \frac{0.8}{1152+448}$$

$$I_G = 5 \times 10^{-4} A$$

Now to calculate the deflection of the galvanometer given that $2.5 \times 10^{-6} A = 1mm$

$$1A = \frac{1}{2.5 \times 10^{-6}} mm$$

$$\therefore 5 \times 10^{-4} A = \frac{1}{2.5 \times 10^{-6}} \times 5 \times 10^{-4} mm$$

$$= 200 mm$$

∴ Deflection of the galvanometer = 0.20 m □

Question 5

Use Norton's theorem to calculate the potential difference across the 15Ω resistor in the circuit shown in figure 3.37

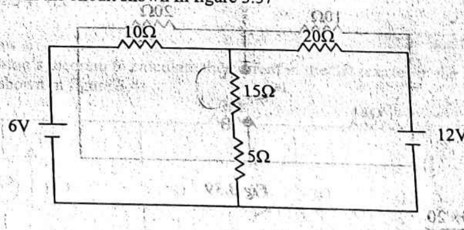


Fig. 3.37

Answer 5.

In order to apply Norton's theorem, one follows the general approach of thevenin's theorem as outlined in Question 3 above, except that when the load is removed the two terminals of the network are shorted so that I_{sc} can be calculated.

(i) Short – circuit the branch containing the 15-Ω resistor and find the short-circuit current, I_{sc}.

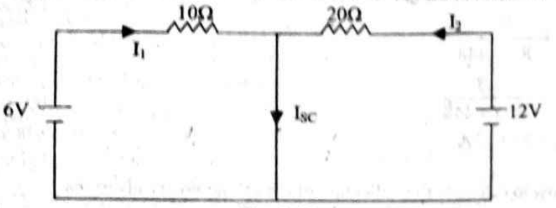


Fig. 3.38

$6 = 10I_1$

$I_1 = 0.6\text{ A}$

Also, $20I_2 = 12$

$I_2 = 0.6\text{ A}$

$\therefore I_{SC} = I_1 + I_2$

$I_{SC} = 1.2\text{A}$

(ii) Short – circuit the sources and find the open – circuit resistance, r_o

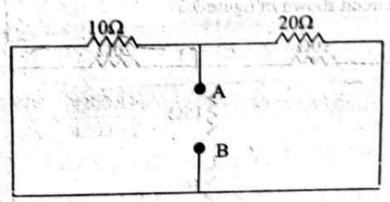


Fig 3.39

$$r_o = \frac{10 \times 20}{10 + 20}$$

$r_o = 6.67\Omega$

(iii) **Norton's equivalent circuit is**

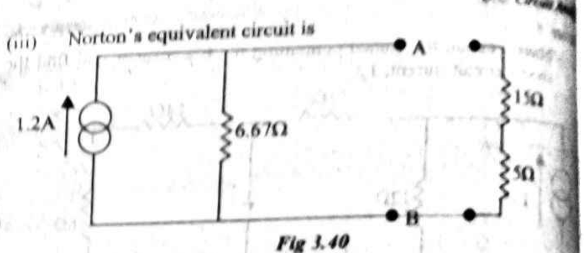


Fig 3.40

∴ current in the 15Ω resistor is

$$I_{15} = \frac{r_o}{r_o + 20} \times I_{SC}$$

$$I_{15} = \frac{6.67}{6.67 + 20} \times 1.2$$

$I_{15} = 0.30\text{ A}$

∴ potential difference across the 15 Ω resistor is V_{15}

$V_{15} = 0.30 \times 15$

$V_{15} = 4.5\text{ V}$ □

Question 6

Use Norton's theorem to calculate the current in the 8Ω resistor in the circuit shown in figure 3.41

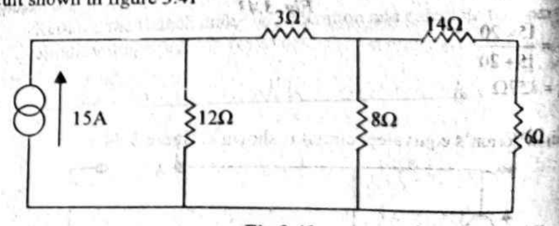


Fig 3.41

Answer 6

(i) Short – circuit the branch containing the 8Ω resistor and find the short – circuit current, I_{SC}

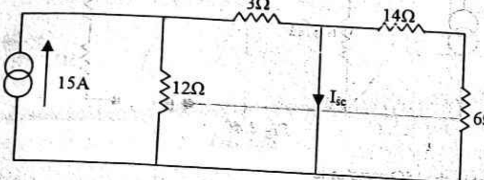


Fig 3.42

$$I_{SC} = \frac{12}{12+3} \times \frac{15}{1}$$

$$I_{SC} = 12\text{ A}$$

(ii) Open - circuit resistance, r_0

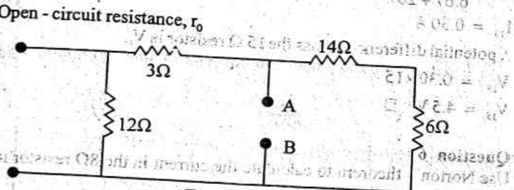


Fig. 3.43

$$r_o = \frac{15 \times 20}{15+20}$$

$$r_o = 8.57\Omega$$

(iii) Norton's equivalent circuit is shown in figure 3.44

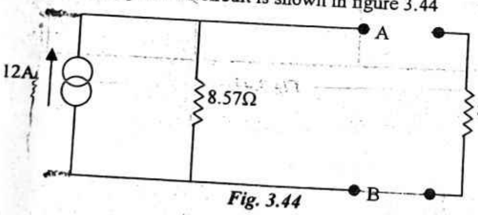


Fig. 3.44

∴ the current in the 8Ω resistor is

$$I_8 = \frac{r_o}{r_o + 8} \times I_{SC}$$

$$I_8 = \frac{8.57}{8.57+8} \times \frac{12}{1}$$

$$I_8 = 6.21\text{ A} \quad \square$$

Question 7

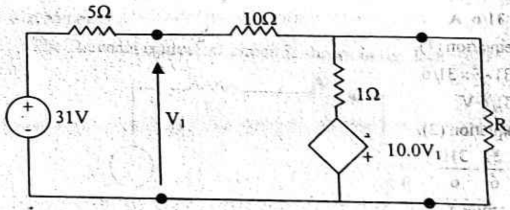


Fig. 3.45

(a) Determine the Thevenin equivalent circuit as seen by the Resistor R:

(b) What value of R is required if the power dissipated by R is to be a maximum.

(c) What is the value of the power in (b)

Answer 7

(i) Remove the branch under consideration and calculate the open – circuit voltage V_{AB}

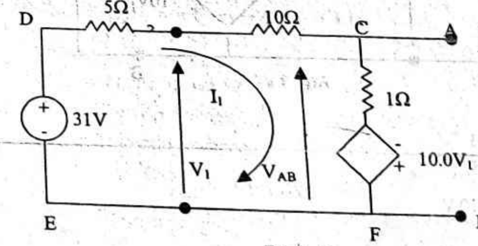


Fig 3.46

From Figure 3.46

$31 - 5I_1 = V_1$ -------------- (1)

Also, $V_{AB} = I_1 - 10.0V_1$ -------------- (2)

Taking the KVL of loop DCFED,

$31 - 16I_1 + 10.0V_1 = 0$ -------------------- (3)

Put equation (1) into equation (3),

$31 - 16I_1 + 10.0(31 - 5I_1) = 0$

$341 - 66I_1 = 0$

$I_1 = 31/6 \text{ A}$

From equation (1),

$V_1 = 31 - 5 \times 31/6$

$V_1 = 31/6 \text{ V}$

From equation (2),

$V_{AB} = \frac{31}{6} - \frac{310}{6}$

$V_{AB} = -46.5 \text{ V}$

(ii) We now calculate the Thevenin resistance, R_T

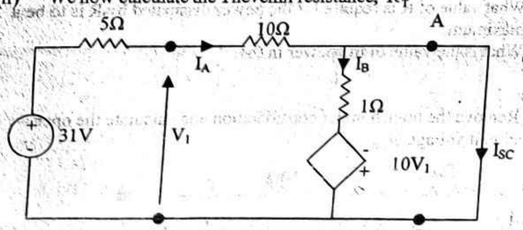


Fig. 3.47

$I_{SC} = I_A - I_B$

$I_A = \frac{31}{5+10} = \frac{31}{15}$

$I_B = \frac{10V_1}{1} = 10V_1$

But $V_1 = 31 - \frac{5 \times 31}{15}$

$\therefore I_B = 10\left(31 - \frac{5 \times 31}{15}\right)$

$\therefore I_{SC} = \left(\frac{31}{15} - 310 + \frac{310}{3}\right)$

$I_{SC} = -204.6\text{A}$

$\therefore R_T = \frac{V_{AB}}{I_{SC}} = \frac{-46.5}{-204.6}$

$R_T = 0.227\,\Omega$

(a) **The Thevenin equivalent circuit is shown in fig. 3.48**

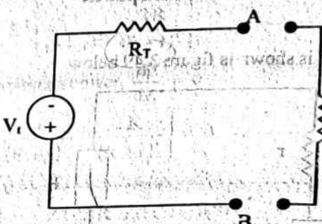


Fig. 3.48

(b) **The value of R for maximum power transfer must be equal to R**

$\therefore R = 0.227\Omega$ □

(c) **The value of the power in (b) is**

$$P_{max} = \frac{V_S^2}{4R_S} \quad \text{--------------- 3.3.1}$$

Where,

P_{max} = **Maximum power**

V_S = **Thevenin voltage**

R_S = **Thevenin resistance**

$\therefore P_{max} = \frac{46.5^2}{4 \times 0.227} = 2381.3\text{W}$

$P_{max} = 2.38 \text{ KW}$ □

Question 8

A battery of e. m. f, E and internal resistance, r are connected to a load of equivalent resistance, R.

(a) Show that the power dissipated in the load resistance is

$$P = \frac{(E^2R)}{(R+r)^2}$$

(b) Show that maximum power is given by

$$P_{max} = \frac{E^2}{4R}$$

(c) If E = 10 V and r = 2Ω, find the value of R which gives the greatest possible power output when connected to the cell.

(d) Calculate the value of the maximum power.

Answer 8

The circuit connection is shown is figure 3.49 below

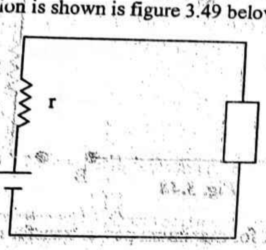


Fig. 3.49

To show that

$$P = \frac{E^2 R}{(R+r)^2}$$

Recall that Electrical power, P is given as

$$P = I^2 R \text{ ------------- (1)}$$

Also for a cell with internal resistance, r, Ohm's law modifies to

$$I = \frac{E}{R+r} \text{ -------------- (2)}$$

Putting equation (2) into equation (1) we have,

$$P = \left(\frac{E}{R+r}\right)^2 \times R \text{ ----------------- (3)}$$

$$\therefore P = \frac{(E^2R)}{(R+r)^2} \text{ ------------------- (4)}$$

(b) To show that

$$P_{max} = \frac{E^2}{4R} \text{ ------------------------ (5)}$$

For equation (4) to be maximum, $\frac{dp}{dR}$ must be equal to zero.

$$\therefore \frac{dp}{dR} = E^2\left[\frac{1}{(R+r)^2} - \frac{2R}{(R+r)^3}\right] \text{ ------------ (6)}$$

For maximum power, $\frac{dp}{dR} = 0$

$$\therefore E^2\left[\frac{1}{(R+r)^2} - \frac{2R}{(R+r)^3}\right] = 0 \text{ ------------ (7)}$$

$$E^2 \neq 0 \text{ ------------------------------ (8)}$$

$$\therefore \frac{1}{(R+r)^2} = \frac{2R}{(R+r)^3} \text{ ----------------- (9)}$$

$$1 = \frac{2R}{R+r} \text{ ----------------------- (10)}$$

$$\therefore R + r = 2R \text{ ----------------------- (11)}$$

$$\therefore R = r \text{ -------------------------- (12)}$$

Equation (12) shows that for power to be transferred from the source to the load maximally, the load resistance must be equal to the resistance of the sources. From equation (4),

$$P_{max} = \frac{(E^2 R)}{(R+R)^2} = \frac{E^2 R}{4R^2}$$

$$\therefore P_{max} = \frac{E^2}{4R} \text{ ----------- (13)}$$

(c) The value of R which gives the greatest possible power output is 2 ohms.

(d) $P_{max} = \frac{10^2}{4 \times 2}$ From equation (13)

P_{max} = 12.5 W □

Question 9

In order to verify the maximum power transfer theorem in the Electrical Engineering Laboratory, a student recorded the following readings tabulated in table 1. The equivalent circuit is shown in figure 3.50.

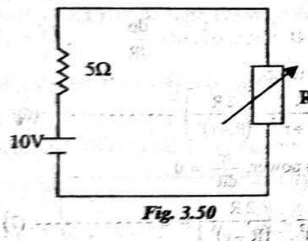


Fig. 3.50

Table 1

R_L (Ω)	0	1	2	3	4	5	6	7	8	10
P_L (W)	0	2.78	4.08	4.70	4.95	5.00	4.96	4.85	4.75	4.45
η(%)	0	16.6	28.5	37.6	43.0	50.0	54.5	58.2	61.8	67.0

(a) Use table 1 to plot the graph of Efficiency (η) and Load power (P_L) against the Load resistance (R_L) on the same graph.

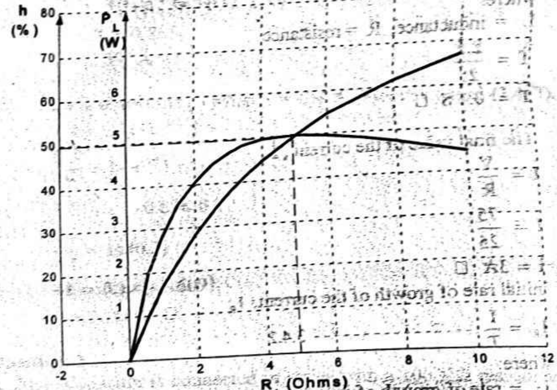
(b) From your graph find,

(i) the value of R_L for maximum power and the corresponding value of P_L

(ii) the efficiency at the maximum power.

Answer 9

(a) The graph is plotted as shown in graph 4.



(b) From the graph,

(i) the value of R_L for maximum power is

$R_{L_{max}}$ = 5.0 Ω □

Also, $P_{L_{max}}$ = 5.0 W □

(ii) the efficiency at the maximum power is

η_{max} = 50% □

3.4 Transients in RL and RC Circuits

Question 1

A Coil having a resistance of 25Ω and an inductance of 2.5 H is connected across a 75 – V d.c supply. Determine:

(a) the time constant (b) the final value of the current (c) the initial rate of growth of the current (d) the value of the current after 0.20s, and (e) the time required for the current to grow to 2.4A.

Answer 1

(a) Time constant, T

$T = \frac{L}{R}$ ------------------ 3.4.1

where:

L = inductance, R = resistance

$T = \frac{2.5}{25}$

T = 0.1 S □

(b) The final value of the current, I

$I = \frac{V}{R}$

$I = \frac{75}{25}$

I = 3A □

(c) Initial rate of growth of the current, I_R

$I_R = \frac{I}{T}$ ---------------- 3.4.2

where:

I_R = rate of growth of current

I = Final value of current

T = time constant

$I_R = \frac{3}{0.1}$

$I_R = 30 A/S$ □

(d) the value of the current when

t = 0.2 s

$i = I\left(1 - e^{-t/T}\right)$ ----------------- 3.4.3

where:

i = instantaneous current

t = time in seconds

T = time constant

I = Final value of current

$\therefore i = 3\left(1 - e^{-\frac{0.2}{0.1}}\right)$

$i = 3\left(1 - e^{-2}\right) = 3(1 - 0.135)$

$i = 3 \times 0.865$

i = 2.595 A □

(e) To find the time (t) when i = 2.4 A. From equation (3.4. 3),

$2.4 = 3\left(1 - e^{-t/0.1}\right)$

$\frac{2.4}{3} = 1 - e^{-t/0.1}$

$e^{-t/0.1} = 1 - 0.8 = 0.2$

$\frac{-t}{0.1} = \ln(0.2)$

$-t = 0.1 \times (-1.609)$

t = 0.161S □

Question 2

A 20μF Capacitor is connected in series with a 100 KΩ resistor across a 250 V d. c supply. Calculate (a) the time constant (b) the initial charging current (c) the time taken for the p.d across the capacitor to grow to 200V, and (d)the current and the p.d across the capacitor 2s after it is connected to the supply.

Answer 2

(a) Time constant, T

T = RC ------------------- 3. 4.4

where :

R = resistance

C = capacitance

$\therefore T = 100 \times 10^3 \times 20 \times 10^{-6}$

T = 2s □

(b) Initial charging current,

$I = \frac{V}{R} = \frac{250}{100 \times 10^3}$

I = 2.5 mA □

(c) To calculate the time given that
$v = 200V$
By Formula,

$v = V\left(1 - e^{-t/T}\right)$ ------------ 3.4.5

where :
v = instantaneous voltage
V = final Voltage
t = time
T = time constant

$\therefore 200 = 250\left(1 - e^{-t/2}\right)$

$\frac{200}{250} = 1 - e^{-t/2}$

$e^{-t/2} = 1 - 0.8$

$-\frac{t}{2} = \ln(0.2)$

$-\frac{t}{2} = -1.609$

$t = 3.22\ S$ □

(d) (i) To find the p.d across the capacitor at t = 2s
Recall equation (3. 4. 5.)

$v = V\left(1 - e^{-t/T}\right)$

$v = 250\left(1 - e^{-2/2}\right)$

$v = 250\left(1 - e^{-1}\right)$

$= 250(1 - 0.368) = 250 \times 0.632$

$v = 158\ V$ □

(ii) To find the current at t = 2s

$i = I\ e^{-t/T}$ --------------- 3.4.6

where:
i = instantaneous current
I = Final value of current
t = time
T = time constant

$\therefore i = 2.5\times10^{-3}\ e^{-2/2}$

$= 2.5\times10^{-3}\ e^{-1}$

$= 2.5\times10^{-3} \times 0.368$

$i = 9.2\times10^{-4}\ A$

$i = 0.92 m\ A$ □

Question 3
A d.c Voltage of 100V is applied to a circuit containing a resistance of 50Ω in series with an inductance of 20H. Calculate the growth of current at the instant
(i) of starting the circuit
(ii) when the current is 0.5A
(iii) when the current is 1A and
(iv) when the current is 2 A.

Answer 3
The Voltage equation for an R – L circuit is

$V = iR + L\frac{di}{dt}$ ------------------- 3.4.7

or $\frac{di}{dt} = \frac{1}{L}(V - iR)$ ---------------------- 3.4.8

where :
$\frac{di}{dt}$ = rate of growth of current
L = inductance, R = resistance
V = applied Voltage

(i) when $i = 0$

$\therefore \frac{di}{dt} = \frac{1}{20}(100 - 0\times50) = \frac{100}{20}$

$\frac{di}{dt} = 5\ A/s$ □

(ii) when $i = 0.5\ A$

$\frac{di}{dt} = \frac{1}{20}(100 - 0.5\times50) = \frac{75}{20}$

$\therefore \frac{di}{dt} = 3.75\ A/s$

(iii) When i = 1A

$$\frac{di}{dt} = \frac{1}{20}(100 - 1 \times 50) = \frac{50}{20}$$

$$\frac{di}{dt} = 2.5\ A/s\ \square$$

(iv) When i = 2A

$$\frac{di}{dt} = \frac{1}{20}(100 - 2\times 50) = \frac{0}{20}$$

$$\frac{di}{dt} = 0\ A/s\ \square$$

Question 4

The time constant of an inductive coil was observed to be 100ms. When a resistance of 120 ohm is added in series to the coil, a new time constant of 25 ms was obtained. Calculate the inductance and resistance of the coil. Determine the maximum value of the current and the time taken by the current to attain 63.2% of the maximum value when the coil alone is connected across a constant voltage source of 200V.

Answer 4

By formula, time constant, T is given by

$$T = \frac{L}{R}$$

For case one:

$$\frac{L}{R} = 100\times10^{-3} \text{ ------------------ (1)}$$

For second case:

$$\frac{L}{R+120} = 25\times10^{-3} \text{ ------------- (2)}$$

Equation (1) ÷ Equation (2) we have,

$$\frac{R+120}{R} = \frac{100\times10^{-3}}{25\times10^{-3}}$$

$$R + 120 = 4R$$

$$4R - R = 120$$

$$R = 40\Omega\ \square$$

From equation (1),

$$\frac{L}{R} = 100\times10^{-3}$$

$$L = 40\times100\times10^{-3}$$

$$L = 4H\ \square$$

When the coil is connected alone, the current will attain 63.2% of its maximum value in a time equal to its time constant (T = 100ms). The maximum current is

$$I = \frac{V}{R}$$

$$I = \frac{200}{40}$$

$$I = 5A\ \square$$

Question 5

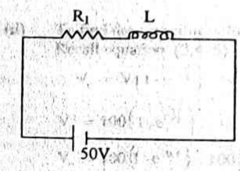


Fig. 3.51

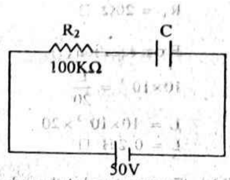


Fig. 3.52

The two circuits have the same time constant of 10ms. When connected as shown above, it is found that the steady state current of circuit figure 3.51 is 5000 times the initial current of circuit figure 3.52. Calculate the values of R_1, L_1 and C. (ii) find the energy stored in the inductor.

Answer 5

In figure 3.51,

$$T = \frac{L}{R_1} \text{ ------------ (1)}$$

In figure 3.52,

$T = CR_2$ ------- (2)

$\therefore 10\times10^{-3} = C \times 100 \times 10^3$

$$C = \frac{10\times10^{-3}}{100\times10^3} = 1\times10^{-7}\ F$$

$C = 0.1\mu F$ □

Let I_1 be the steady state current of circuit figure 3.51

$$\therefore I_1 = \frac{V}{R_1}$$

But $I_1 = 5000\ \frac{V}{R_2}$

Thus, $\frac{V}{R_1} = \frac{5000\ V}{R_2}$

$$\frac{1}{R_1} = \frac{5000}{100\times10^3}$$

$R_1 = 20\Omega$ □

From equation (1),

$$10\times10^{-3} = \frac{L}{20}$$

$L = 10\times10^{-3}\times20$

$L = 0.2H$ □

(ii) Energy stored in the inductor is

$E = ½ L I_1^2$

$I_1 = \frac{50}{20} = \frac{5}{2}\ A$

$$E = ½\times0.2\times\left(\frac{5}{2}\right)^2$$

$E = 0.625 J$ □

Question 6

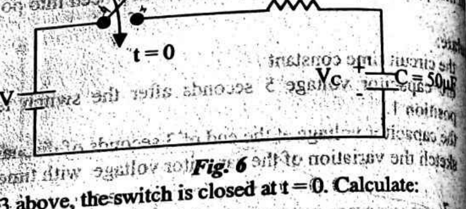


Fig. 6

In figure 3.53 above, the switch is closed at t = 0. Calculate:

(i) the circuit time constant

(ii) the capacitor voltage 2 seconds after the switch is closed

Answer 6

(i) Time constant, T

$T = RC$

$T = 100\times10^3\times50\times10^{-6}$

$T = 5s$ □

(ii) To find the capacitor voltage, V_c at t = 2s

Recall equation (3.4 .5)

$$\therefore V_c = V\left(1 - e^{-t/T}\right)$$

$$V_c = 100\left(1 - e^{-2/5}\right)$$

$V_c = 100\left(1 - e^{-0.4}\right) = 100\left(1 - 0.670\right)$

$V_c = 33V$ □

Question 7

Fig 3.54

The capacitor is initially uncharged as shown in figure 3.54. The switch is placed into position 1 for 5 seconds and is then placed into position 2.

Calculate:

(i) the circuit time constant

(ii) the capacitor voltage 5 seconds after the switch is placed into position 1.

(iii) the capacitor voltage at the end of 3 seconds of discharge.

(iv) sketch the variation of the capacitor voltage with time.

Answer 7

(i) The circuit time constant for both charging and discharging is

$T = RC$

$T = 1000 \times 10^3 \times 5 \times 10^{-6}$

$T = 5s$ □

(ii) At position 1, the capacitor will charge to a voltage of magnitude, V_c as indicated in figure 3.54. This voltage is given as

$V_c = 120\left(1 - e^{-5/5}\right)$ see equation (3.4.5)

$V_c = 120(1 - 0.368)$

$V_c = 75.84\text{ V}$ □

(iii) At position 2, the capacitor discharges through the resistor and $V_o = 75.84\text{ V}$

$\therefore V_c = V_o e^{-t/T}$ 3.4.9

$V_c = 75.84 e^{-3/5}$; V_o = initial voltage.

$V_c = 75.84 e^{-0.6} = 75.84 \times 0.5488$

$V_c = 41.62\text{ V}$ □

(iv) The variation of V_c with time is shown in figure 3.55

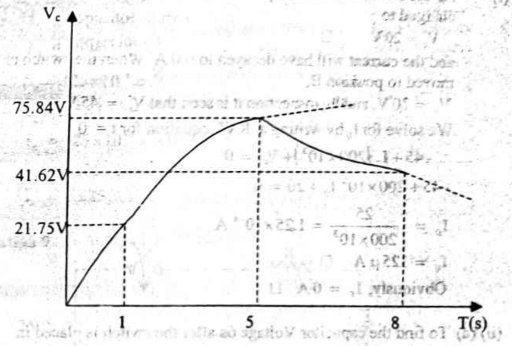


Fig. 3.55

Question 8

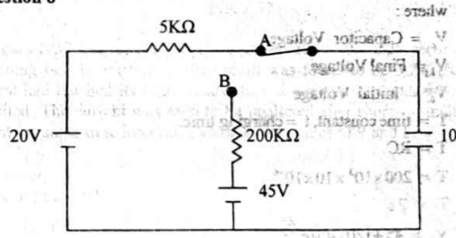


Fig. 3.56

After being in position A, the switch is moved to position B.

(i) Calculate the initial and final values of voltage and current for the capacitor.

(ii) Determine the capacitor voltage and current 6 seconds after the switch is placed in position B

Answer 8

(i) As the switch has been in position A, the capacitor would have charged to

$V_c = 20\,V$ □

and the current will have decayed to 0.0 A. When the switch is moved to position B,

$V_o = 20\,V$, and by inspection it is seen that $V_f = 45\,V$.

We solve for I_o by writing a KVL equation for $t = 0$

$\therefore -45 + I_o(200\times10^3) + V_o = 0$

$-45 + 200\times10^3 I_o + 20 = 0$

$$I_o = \frac{25}{200\times10^3} = 1.25\times10^{-4}\,A$$

$I_o = 125\,\mu A$ □

Obviously, $I_f = 0\,A$ □

(ii) (a) To find the capacitor Voltage 6s after the switch is placed in position B

$V_C = V_f + (V_o - V_f)e^{-t/T}$ ------------------- 3.4.10

where :

V_c = Capacitor Voltage

V_f = Final Voltage

V_o = Initial Voltage

T = time constant, t = charging time.

$T = RC$

$T = 200\times10^3 \times 10\times10^{-6}$

$T = 2\,s$

$V_c = 45 + (20-45)e^{-6/2}$

$V_c = 45 - 25e^{-3} = 45 - 25\times0.0498$

$V_c = 45 - 1.245$

$V_c = 43.76\,V$ □

(b) The capacitor current at $t = 6s$

$i_c = I_o e^{-t/T}$ ------------------ 3.4.11

where :

i_c = capacitor current

I_o = initial current

$i_c = 125\times10^{-6} e^{-6/2}$

$i_c = 125\times10^{-6} e^{-3}$

$i_c = 125\times10^{-6}\times0.0498$

$i_c = 6.225\times10^{-6}\,A$

$i_c = 6.225\,\mu A$ □

Question 9

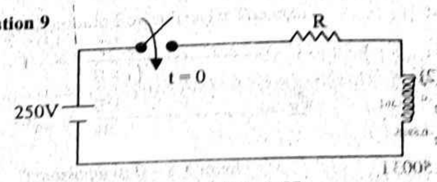


Fig. 3.57

In figure 3.57 above, the switch is closed at $t = 0$. 0.25 seconds after switching on, the current in the circuit was found to be 3.2A. After the current had reached its final steady state, the circuit was suddenly short circuited. The current was seen to be unaltered after short circuiting the coil at the same time interval. Calculate the values of R and L.

Answer 9

For current growth,

$i = I\left(1 - e^{-t/T}\right)$

$\therefore 3.2 = I\left(1 - e^{-0.25/T}\right)$ ---------------- (1)

For current decay,

$i = I e^{-t/T}$

$3.2 = I e^{-0.25/T}$ ----------- (2)

Equating equation (1) and (2) we have,

$$\left(1-e^{-0.25/T}\right) = e^{-0.25/T}$$

$$1 = 2e^{-0.25/T}$$

$$e^{-0.25/T} = 0.5$$

$$\frac{-0.25}{T} = \ln(0.5)$$

$$-\frac{0.25}{T} = -0.693$$

$$T = \frac{0.25}{0.693}$$

$$T = 0.361\,s$$

From equation (2)

$$3.2 = I e^{-0.25/0.361}$$

$$3.2 = I e^{-0.6925}$$

$$3.2 = 0.5003\,I$$

$$I = \frac{3.2}{0.5003}$$

$$I = 6.396\,A \;\square$$

But at steady state,

$$I = \frac{V}{R}$$

$$\therefore 6.396 = \frac{250}{R}$$

$$R = \frac{250}{6.396}$$

$$R = 39.09\,\Omega \;\square$$

$$\text{Recall, } T = \frac{L}{R}$$

$$0.361 = \frac{L}{39.09}$$

$$\therefore L = 14.11\,H \;\square$$

3.5 Summary of Equations Used.

1. Thevenin's Theorem

$$I_L = \frac{V_T}{R_T + R_L} \text{ ---------- } 3.1$$

$$R_T = \frac{V_T}{I_{SC}} \text{ ---------- } 3.2$$

2. Norton's Theorem

$$I_L = \left(\frac{r_0}{r_0 + R_L}\right) I_{SC} \text{ ---------- } 3.3$$

3. Maximum Power Trasfer Theorem

$$P = \frac{E^2 R}{(R+r)^2} \text{ ---------- } 3.4$$

$$P_{max} = \frac{E^2}{4R} \text{ ---------- } 3.5$$

4. Transients in R – L Circuit

$$T = \frac{L}{R} \text{ ---------- } 3.6$$

$$\frac{di}{dt} = \frac{I}{T} \text{ ---------- } 3.7$$

$$i = I\left(1-e^{-t/T}\right) \text{ ---------- } 3.8$$

$$i = I e^{-t/T} \text{ ---------- } 3.9$$

$$\frac{di}{dt} = \frac{1}{L}(V - iR) \text{ ---------- } 3.10$$

5. Transients in R – C Circuit

$$T = RC \text{ ---------- } 3.11$$

$$V_C = V\left(1-e^{-t/T}\right) \text{ ---------- } 3.12$$

$$i_C = I_0 e^{-t/T} \text{ ---------- } 3.13$$

$$V_C = V_0 e^{-t/T} \text{ ---------- } 3.14$$

$$V_C = V_f + (V_0 - V_f)e^{-t/T} \text{ ---------- } 3.15$$

3.6 Tutorial Problems Three

1. Two batteries, A and B, are connected in parallel, and an 80 Ohm resistor is connected across the battery terminals. The e.m.f. and the internal resistance of battery A are 100V and 5Ω respectively, and the corresponding values for battery B are 95V and 3Ω respectively. Calculate using kirchhoff's laws (a) the value and direction of the current in each battery and (b) the terminal voltage.

2. Find the unknown currents for the networks in figure 3.58 and figure 3.59

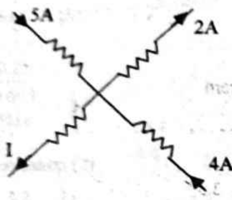


Fig. 3.58

Fig. 3.59

3. Use the network in fig. 3 to solve for current in the 1 - Ω resistor using
   (a) Branch current method
   (b) Loop current method
   (c) Node Voltage method

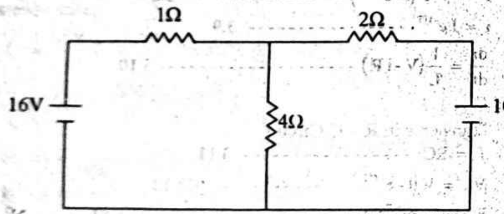


Fig. 3.60

4. Calculate the Voltage across the 2Ω resistor in figure 3.61 using the node voltage method.

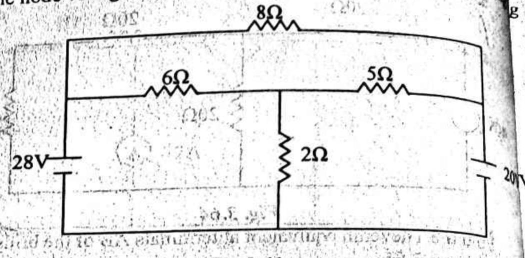


Fig 3.61

5. Determine the value of R in figure 3.62 using Thevenin's theorem.

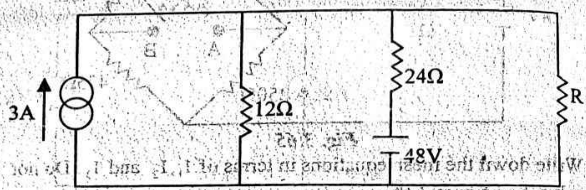


Fig. 3.62

6. Determine the current in the 6Ω, 8Ω and 10Ω resistances in figure 3.63 using mesh analysis

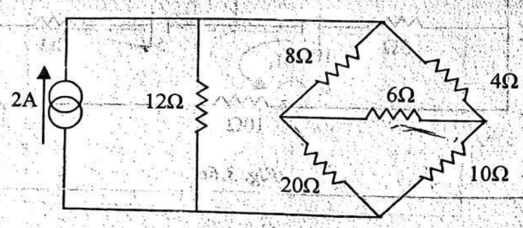


Fig. 3.63

7. Use Norton's theorem to solve for the current in the 10Ω resistor shown in figure 3.64

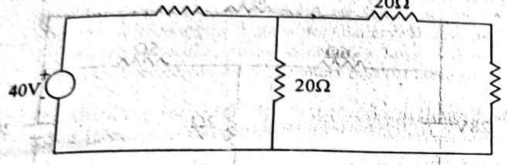


Fig. 3.64

8. Find the Thevenin equivalent at terminals AB of the bridge circuit of figure 3.65.

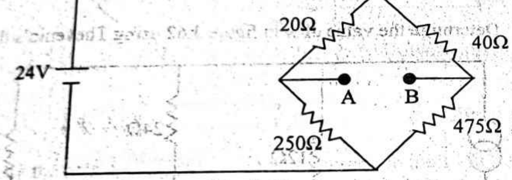


Fig. 3.65

9. Write down the mesh equations in terms of I_1, I_2 and I_3. Do not solve. See figure 3.66.

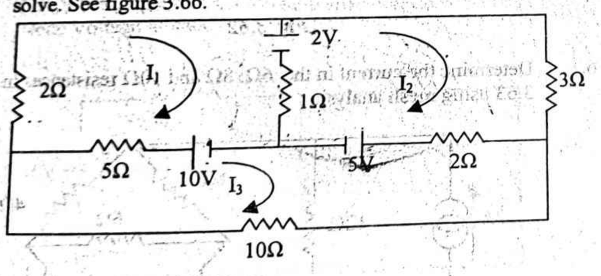


Fig. 3.66

10. Determine the value of R_L for maximum power transfer in the network of figure 3.67. Find also the value of the maximum power.

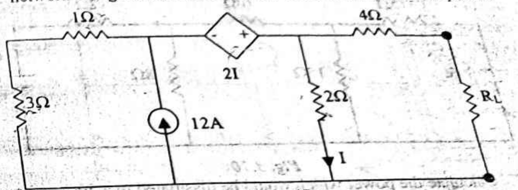


Fig. 3.67

11. Find R_L and the maximum power absorbed by the load using 3.68

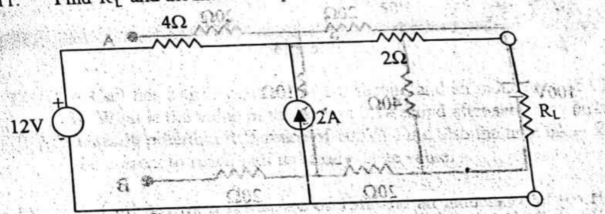


Fig. 3.68

12. Determine the current flowing in the 4Ω resistor in figure 3.69 using superposition theorem.

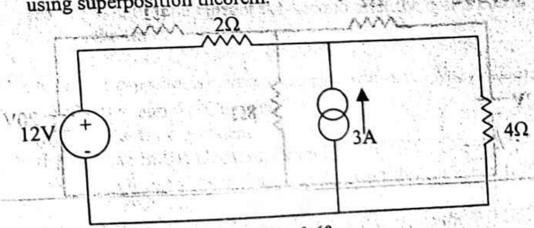


Fig. 3.69

13 Calculate the current in the 10Ω resistor using superposition theorem. Use figure 3.70.

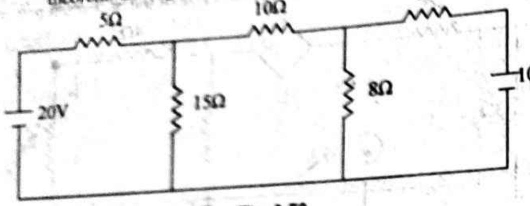


Fig. 3.70

14. Calculate the power which would be dissipated in a 50Ω resistor connected across AB in the network of figure 3.71. Use Thevenin's theorem

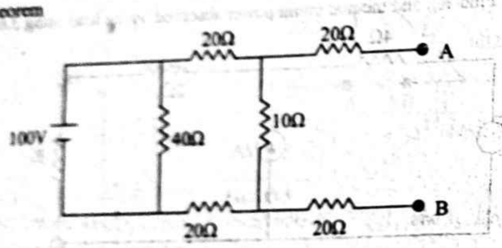


Fig. 3.71

15 Use Norton's theorem to calculate the current in the 8Ω resistor. Use figure 3.72

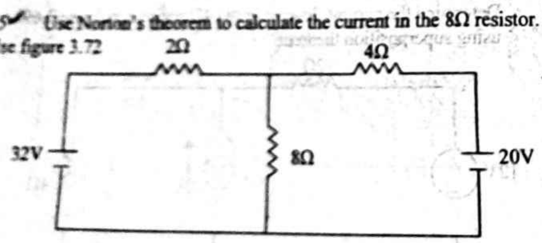


Fig. 3.72

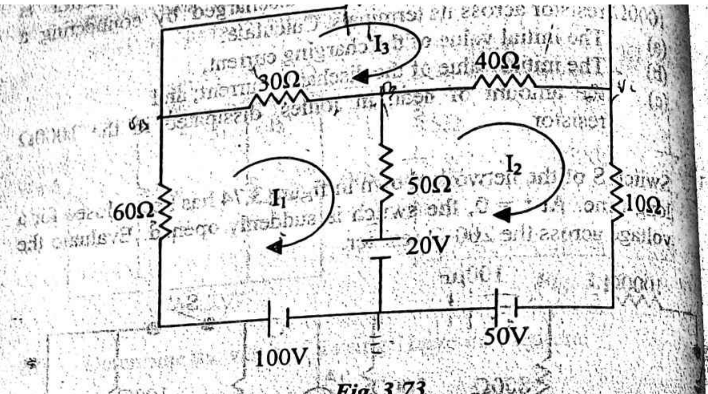


Fig. 3.73

17 A Coil has a time constant of 1.0 second and an inductance of H. What is the value of the current 0.1-second after switching on a steady potential difference of 100V? Find also the time taken the current to reach half its steady- state value.

18. A Coil having a resistance of 15Ω and an inductance of 0.6 connected to a 120V line.

(a) What is the rate at which the current is increasing at instant the Coil is connected to the line?

(b) What is the maximum current that can flow?

(c) What is the current 0.1-second after the coil is connecte the line?

19 An 8 - μF capacitor is conneted in series with a 0.5 MΩ resistor a a 200V d.c. supply. Calculate:

(a) The time constant,

(b) The initial charging current

(c) The time taken for the p.d across the capacitor to gro 160V, and

(d) the current and the p.d across the capacitor 4 seconds it is connected to the supply.

20. A 10μF capacitor in series with a 10kΩ resistor is connected across a 500V d.c supply. The fully charged capacitor is disconnected from the supply and discharged by connecting a 1000Ω resistor across its terminals. Calculate:
(a) The initial value of the charging current,
(b) The initial value of the discharge current; and
(c) the amount of heat, in Joules, dissipated in the 1000Ω resistor.

21. Switch S of the network shown in figure 3.74 has been closed for a long time. At t = 0, the switch is suddenly opened. Evaluate the voltage across the 200-Ω resistor.

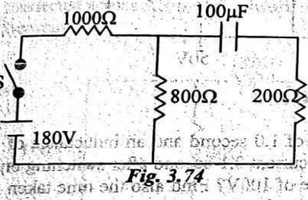


Fig. 3.74

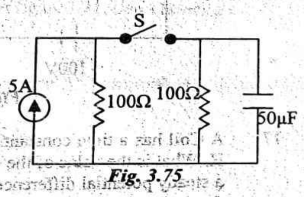


Fig. 3.75

22. The capacitor of the circuit shown in figure 3.75 has no charge when the switch is closed at t = 0. Calculate: (a) the current through and (b) the voltage across the capacitor.

23. Obtain the source voltage V for the circuit of figure 3.76

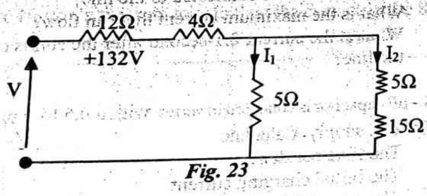


Fig. 23

24. If the networks figure 3.77 and figure 3.78 are equivalent with regard to the 20Ω resistor, determine the value of E. Also solve for the currents I_1 and I_2. Calculate the current in the 20Ω resistor by applying Norton's theorem.

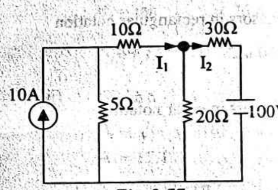


Fig. 3.77

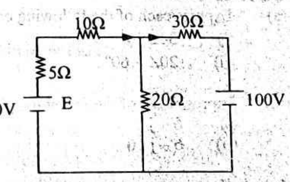


Fig. 3.78

25. Determine the value of I using Thevenin theorem

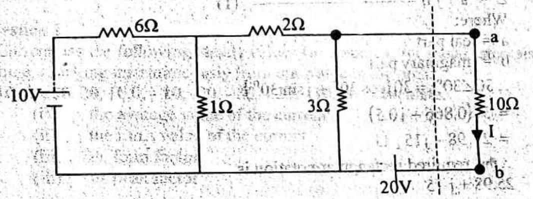


Fig. 3.79

Chapter Four

4.0 A. C. CIRCUIT ANALYSIS
4.1 Alternating Quantities

Question 1

(a) Express each of the following phasors in rectangular notation
- (i) $30\angle 30°$
- (ii) $20\angle -60°$

(b) Express each of the following phasors in polar notation
- (i) $6 + j3$
- (ii) $6 - j16$

Answer 1

(a) (i) To express $30\angle 30°$ in rectangular notation

General rectangular notation is

$Z = a + jb$ ---------------------- (1)

Where:

a = real part

b = imaginary part

$\therefore 30\angle 30° = 30\left(\cos 30° + j\sin 30°\right)$

$= 30(0.866 + j0.5)$

$= 25.98 + j15$ □

$\therefore$ the required rectangular notation is

$25.98 + j15$

(ii) $20\angle -60° = 20\left[\cos(-60°) + j\sin(-60°)\right]$

$= 20\left[0.5 - j0.866\right]$

$= 10 - j17.32$ □

(b) Generally polar notation is represented as

$Z = A\angle\theta$ ------------------------------- (2)

where :

A = magnitude

θ = phase angle

(i) Given $6 + j3$

By formula,

$A = \sqrt{a^2 + b^2}$ ---- ----------- 4.1.1

$A = \sqrt{6^2 + 3^2}$

$A = 6.71$

$\theta = \tan^{-1}\left[\frac{b}{a}\right]$ ----------- 4.1.2

$\theta = \tan^{-1}\left[\frac{3}{6}\right] = 26.6°$

$\therefore 6 + j3 = 6.71\ \angle 26.6°$ in polar form. □

(ii) Given $6 - j16$

$A = \sqrt{6^2 + (-16)^2} = \sqrt{292}$

$A = 17.1$

$\theta = \tan^{-1}\left[\frac{-16}{6}\right] = -69.4°$

$\therefore 6 - j16 = 17.1\angle -69.4°$ in polar form □

Question 2

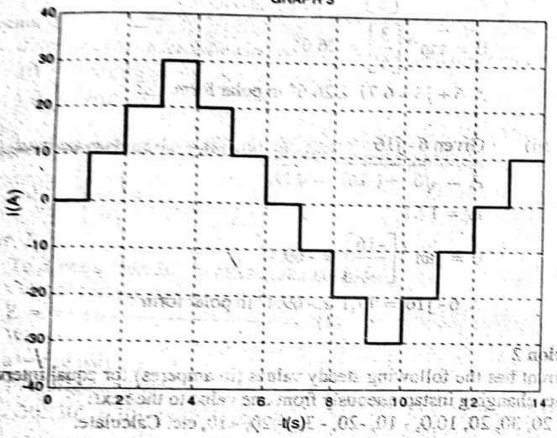
A Current has the following steady values (in amperes) for equal intervals of time, changing instantaneously from one value to the next:

0, 10, 20, 30, 20, 10, 0, - 10, -20, - 30, - 20, - 10, etc. Calculate:
- (i) the average value of the current
- (ii) the r.m.s value of the current
- (iii) its form factor
- (iii) its peak factor

Answer 2

The waveform of the alternating current is shown in Graph 5 below

GRAPH 5



(i) To calculate the average value of the current, I_{av}. Since the waveform is symmetrical, it is then important to calculate the values over the first half cycle.

$$\therefore I_{av} = \frac{\text{Area under curve}}{\text{Length of base}} \quad \text{------------} \; 4.1.3$$

$$I_{av} = \frac{0(1-0)+10(2-1)+20(3-2)+30(4-3)+20(5-4)+10(6-5)}{6-0}$$

$$I_{av} = \frac{10+20+30+20+10}{6} = \frac{90}{6}$$

I_{av} = 15.0 A □

Alternatively,

$$I_{av} = \frac{i_1+i_2+i_3+\ldots\ldots\ldots+i_n}{n} \quad \text{------------} \; 4.1.4$$

$$\therefore I_{av} = \frac{0+10+20+30+20+10}{6} = \frac{90}{6}$$

I_{av} = 15.0 A □

(ii) To find the root mean square (r.m.s) value of the current, $I_{r.m.s}$ we have,

$$I^2r.ms = \frac{0^2(1-0)+10^2(2-1)+20^2(3-2)+30^2(4-3)+20^2(5-4)+10^2(6-5)}{6-0}$$

$$I^2r.m.s = \frac{100+400+900+400+100}{6} = \frac{1900}{6}$$

$$I_{r.m.s} = \sqrt{\frac{1900}{6}}$$

$I_{r.m.s}$ = 17.8 A □

Alternatively,

$$I_{r.m.s} = \sqrt{\frac{i_1^2+i_2^2+i_3^2+\ldots\ldots+i_n^2}{n}} \quad \text{------------} \; 4.1.5$$

$$\therefore I_{r.m.s} = \sqrt{\frac{0^2+10^2+20^2+30^2+20^2+10^2}{6}}$$

$$I_{r.m.s} = \sqrt{\frac{1900}{6}}$$

$I_{r.m.s}$ = 17.8 A □

(iii) Form factor, k_F

$$K_F = \frac{\text{r.m.s Value}}{\text{average value}} \quad \text{------------------} \; 4.1.6$$

$$K_F = \frac{17.8}{15.0}$$

K_F = 1.19 □

(iv) Peak factor or crest factor, k_a

$$K_a = \frac{\text{Maximum value}}{\text{r.m.s. value}} \quad \text{--------} \; 4.1.7$$

$$K_a = \frac{30}{17.8}$$

K_a = 1.69 □

Question 3

Plot the half wave of voltage corresponding to the values in the table below. Draw a smooth curve through the points and use the mid-ordinate method to determine:

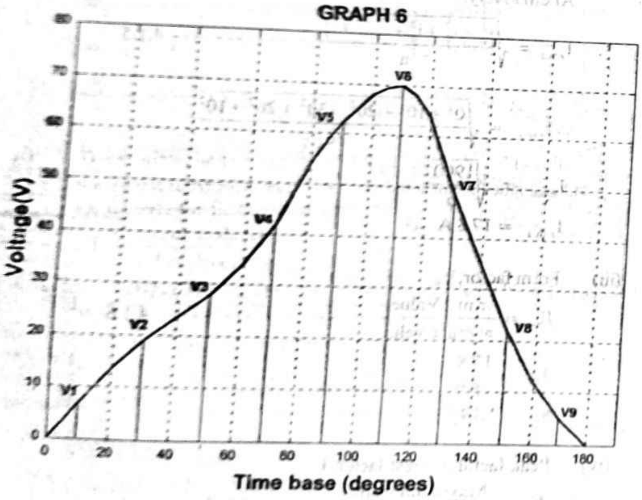
(a) the average voltage
(b) the r.m.s voltage
(c) its form factor.

Voltage in volts	0	15	24	35	54	68	70	64	35	12	0
Time base in degrees	0	20	40	60	80	100	110	120	140	160	180

ANSWER 3

The voltage waveform curve is shown in Graph 6

GRAPH 6



Applying the mid- ordinate method, we divide the curve of Graph 6 into 9 equal parts as shown by the undotted vertical lines.

(a) $V_{av} = \frac{v_1 + v_2 + ... + V_n}{n}$ ---------- 4.1.8

$\therefore V_{av} = \frac{v_1 + v_2 + v_3 + v_4 + v_5 + v_6 + v_7 + v_8 + v_9}{9}$

$V_{av} = \frac{6 + 18.5 + 28.5 + 42 + 61 + 70 + 49 + 22 + 4}{9}$

$V_{av} = \frac{301}{9}$

$V_{av} = 33.44V$ □

(b) $V r.m.s = \sqrt{\left(\frac{V_1^2 + V_2^2 + ... + V_n^2}{n}\right)}$ ------------- 4.1.9

$\therefore V r.m.s = \sqrt{\left(\frac{6^2 + 18.5^2 + 28.5^2 + 42^2 + 61^2 + 70^2 + 49^2 + 22^2 + 4^2}{9}\right)}$

$= \sqrt{\frac{14476.5}{9}}$

$Vr.m.s = 40.11V$ □

(c) Form factor $= \frac{V r.m.s}{V_{av}}$

$\therefore K_F = \frac{40.11}{33.44}$

$K_F = 1.20$ □

Question 4

In each of the waveforms shown in fig. 4.1, fig. 4.2 and fig. 4.3, calculate the waveform's:

(i) Period
(ii) Average value
(iii) r.m.s value
(iv) Form factor.

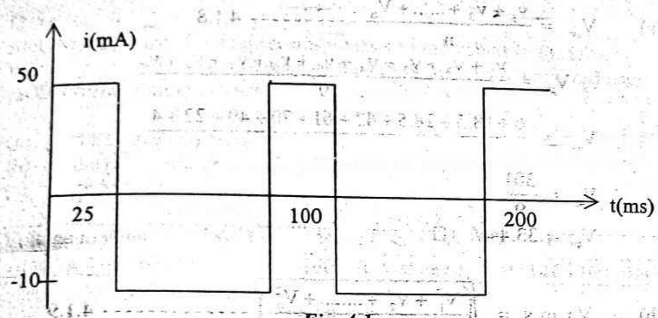


Fig. 4.1

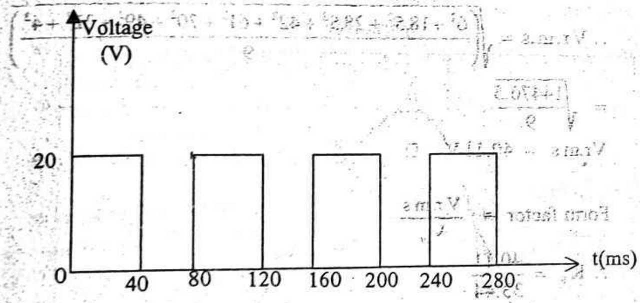


Fig. 4.2

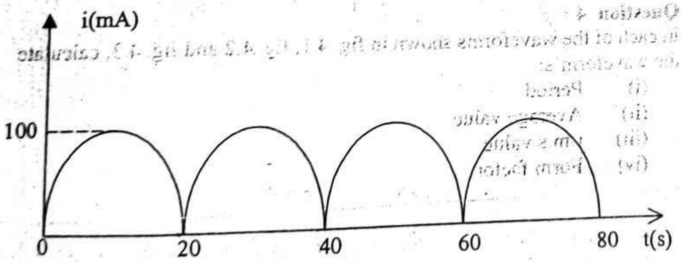


Fig. 4.3

(a) For Figure 4.1

(i) Period.

Generally, period of an alternating quantity is the time taken to complete one cycle. Therefore, for waveform of figure 4.1

Period, T = 100ms □

(ii) Average value, i_{av}

$$i_{av} = \frac{\text{Area under curve for one cycle}}{\text{Period}} \quad \text{--- 4.1.10}$$

$$i_{av} = \frac{(50)(25)+(-10)(75)}{100} = \frac{1250-750}{100}$$

$i_{av} = 5\,mA$ □

(iii) r.m.s value, I r.m.s

$$I^2\,r.m.s. = \left(\frac{50^2(25)+(-10)^2(75)}{100}\right) = \frac{62500+7500}{100}$$

I^2 r.m.s = 700

I r.m.s = $\sqrt{700}$

I r.m.s = 26.46 m A □

(iv) Form Factor, K_F

$$K_F = \frac{I\,r.m.s}{i_{av}}$$

$$K_F = \frac{26.46}{5}$$

$K_F = 5.29$

(b) For waveform of figure 4b

(i) Period, T

T = 80 m s □

(ii) $V_{av} = \frac{20 \times 40}{80}$

$V_{av} = 10\,V$ □

(iii) $V^2 r.m.s = \frac{20^2 \times 40}{80} = 200$

$V r.m.s = \sqrt{200}$

$V r.m.s = 14.14 V$ □

(iv) $K_f = \frac{V r.m.s}{V_{av}}$

$K_f = \frac{14.14}{10}$

$K_f = 1.41$ □

(c) For waveform of figure 4.3, it can be seen that it is a full-wave rectified sine wave.

(i) $T' = 20 s$ □

(ii) $i_{av} = 0.637 I_m$ ------------ 4.1.11

where:

I_m = maximum or peak current

$i_{av} = 0.637 \times 100$

$i_{av} = 63.7$ mA □

(iii) $I r.m.s = 0.707 Im$ ------------- 4.1.12

$I r.m.s = 0.707 \times 100$

$I r.m.s = 70.7$ mA □

(iv) $K_f = \frac{I r.m.s}{i_{av}}$

$K_f = \frac{70.7}{63.7}$

$K_f = 1.11$ □

Question 5

An alternating current i is represented by:

$i = 100 \sin 314 t$

where the current is measured in amperes and the time in seconds.

Calculate:

(a) the r.m.s current

(b) the frequency

(c) the period

(d) the instantaneous value of the current when t is 9ms

(e) the energy dissipated when the current flows through a 5Ω resistor for 20 minutes.

Answer 5

The general expression for an alternating current is

$i = I_m \sin \omega t$ --------------- (1)

where:

i. = instantaneous value of the current

I_m = Maximum or peak value of current

ω = angular velocity in rad/s

t = time in second

But the given alternating current is

$i = 100 \sin 314 t$ ----------------- (2)

Comparing equation (1) and equation (2) we have

$I_m = 100A$

$\omega = 314$ rad/s

(a) r.m.s. current, I r.m.s

$I r.m.s = \frac{I_m}{\sqrt{2}}$ ------------ 4.1.13

$\therefore I r.m.s = \frac{100}{\sqrt{2}}$

$I r.m.s = 70.71$ A □

(b) To find the Frequency, f

$\omega = 2\pi f$ ------------------ 4.1.14

where:

ω = angular velocity, f = frequency

$\therefore f = \frac{314}{2\pi}$

f = 50 HZ □

(c) Period, T

$T = \frac{1}{f}$ -------------- 4.1.15

$T = \frac{1}{50}$

T = 20 m s □

(d) **To find i when t = 9 ms**

From equation (2),

$i = 100 \text{ Sin } 314 \times 9 \times 10^{-3}$

$i = 100 \text{ Sin } (2.826)$

Note that angle 2.826 is in radians.

$i = 100 \times 0.3104$

$i = 31.04 \text{ A}$ □

(e) **To find the energy dissipated, E**

$E = I^2_{r.m.s} R t$ ---------------------- 4.1.16

where:

R = resistance

$E = 70.71^2 \times 5 \times 20 \times 60$

$E = 3.0 \times 10^7 \text{ J}$

$E = 30 \text{ MJ}$ □

Question 6

In an a.c circuit supplied from 60 HZ mains, the Voltage has a maximum value of 660V and the current has a maximum value of 15 A. At some instant, taken as t = 0, the instantaneous values of the voltage and the current are 484 V and 6A respectively, both increasing positively. Assuming sinusoidal variation,

(a) derive relations for the instantaneous values of voltage and current

(b) Calculate the instantaneous values of the voltage and current at the instant t = 6ms.

(c) find the angle of phase difference between the voltage and the current.

Answer 6

Since we assumed that the alternating quantities are sinusoidal, then

$v = V_m \text{ Sin } (\omega t + \theta)$ ---------------- (1)

$i = I_m \text{ Sin } (\omega t + \phi)$ -------------------- (2)

where :

i, v = instantaneous values of current and voltage

I_m, V_m = peak values of current and voltage

ω = angular velocity

θ = Voltage phase angle

Φ = current phase angle

Recall, $\omega = 2 \pi f$

$\omega = 2\pi \times 60$

$\therefore \omega = 377 \text{ rad/s}$

From equation (1),

$v = V_m \text{ Sin } (377t + \theta)$ -------------- (3)

Given :

$V_m = 660\text{V}$

$\therefore v = 660 \text{ Sin } (377t + \theta)$ ------------ (4)

When t = 0, v = 484 V, then

$484 = 660 \text{ Sin } (377 \times 0 + \theta)$

$484 = 660 \text{ Sin } (0 + \theta)$

$\text{Sin } \theta = \frac{484}{660} = 0.7333$

$\theta = \text{Sin}^{-1} [0.7333]$

$\theta = 47.2^\circ$ or

$\theta = 0.824$ rad since $180^\circ = \pi$ rad

$\therefore$ the required instantaneous voltage expression is

$v = 660 \text{ Sin } (377t + 0.824) \text{V}$ □

We now find the instantaneous current expression by rewriting equation (2),

$i = I_m \text{Sin} (\omega t + \Phi)$

Given: $I_m = 15 \text{ A}$

$\therefore i = 15 \text{ Sin} (\omega t + \Phi)$ ---------- (5)

Also, $\omega = 377$ rad/s

$i = 15 \text{ Sin} (377t \times 0 + \Phi)$ ------------- (6)

When t = 0, i = 6 A, then

$6 = 15 \text{ Sin} (377t + \Phi)$

$6 = 15 \text{ Sin } \Phi$

$\text{Sin } \Phi = \frac{6}{15} = 0.4000$

$\Phi = \text{Sin}^{-1} [0.4000]$

$\Phi = 23.6^\circ$ or

$\Phi = 0.412$ rad

∴ the required instantaneous current expression is

$i = 15\,\text{Sin}\,(377t + 0.412)$ A □

(b) (i) To Calculate the instantaneous value of the voltage at $t = 6$ms

$v = 660\,\text{Sin}\,(377t + 0.824)$

$v = 660\,\text{Sin}\,(377 \times 6 \times 10^{-3} + 0.824)$

$v = 660\,\text{Sin}\,(3.086)$

$v = 36.67$ V □

(i) Similarly, the instantaneous value of the current at $t = 6$ms is

$i = 15\,\text{Sin}\,(377t + 0.412)$

$i = 15\,\text{Sin}\,(377 \times 6 \times 10^{-3} + 0.412)$

$I = 15\,\text{Sin}\,(2.674)$

$i = 6.76$ A □

(c) the phase angle between the voltage and current, β is given by

$\beta = \theta - \Phi$

$= 47.2^0 - 23.6^0$

$\beta = 23.6^0$, current lagging. □

Question 7

Four e.m.fs $e_a = 100\,\text{Sin}\omega t$

$e_b = 250\cos\omega t$

$$V_C = 150\sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_d = 200\,\text{Sin}\left(\omega t - \frac{\pi}{4}\right)$$

Volts are induced in four coils connected in series so as to give the phasor sum of the four e.m.fs. (a) Calculate:

(i) the resultant e.m.f. and its phase relative to e.m.f, e_a.

(ii) the r.m.s value and form factor of the resultant e.m.f.

(iii) the frequency of the resultant waveform when ω = 754 rad/s

(iv) the value of the resultant wave form at time, t = 10ms.

(b) if the connections to coil 'b' were reversed, what would be the resultant e.m.f. and its phase relative to e_a ?

Answer 7

We are going to treat the e.m.fs as vectors. First express all the quantities as sine functions.

Therefore, $e_b = 250\cos\omega t = 250\,\text{Sin}\,(\omega t + \pi/2)$

Since $\text{Cos}\,\theta = \text{Sin}\,(\pi/2 + \theta)$

We now resolve the quantities into vertical and Horizontal components as shown in the table below:

Quantity	Max Value	Phase (rad).	Horizontal Component (V)	Vertical Component (V)
$100\,\text{Sin}\omega t$	100	0	$100\cos(0) = 100$	$100\,\text{Sin}(0) = 0$
$250\,\text{Sin}(\omega t + \pi/2)$	250	$\pi/2$	$250\cos(\pi/2) = 0$	$250\,\text{Sin}(\pi/2) = 250$
$150\,\text{Sin}(\omega t + \pi/6)$	150	$\pi/6$	$150\cos(\pi/6) = (129.9)$	$150\,\text{Sin}(\pi/6) = 75$
$200\,\text{Sin}(\omega t - \pi/4)$	200	$-\pi/4$	$200\cos(-\pi/4) = (141.4)$	$200\,\text{Sin}(-\pi/4) = (-141.4)$
Total			371.3	183.6

The net horizontal and vertical components are shown in figure 4.4 below with e_r as the resultant.

(i) From figure 4.4, the resultant e.m.f, e_r is

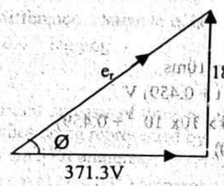


Fig. 4.4

$e_r = \sqrt{183.6^2 + 371.3^2}$

$e_r = 414.2$ V

$$\Phi = \tan^{-1}\left[\frac{183.6}{371.3}\right] = 26.3^0 \text{ or}$$

$\Phi = 0.459$ rad

The equation of the resultant e.m.f. e_r is $e_r = 414.2\,\text{Sin}\,(\omega t + 0.459)$ V □

The phase difference relative to e_a is $= 26.3^0$ leading. □

(ii) (a) $$\text{r.m.s. value} = \frac{\text{max imum value}}{\sqrt{2}}$$

r.m.s value = $\frac{414.2}{\sqrt{2}}$

= 292.88V □

(b) Form Factor = $\frac{\text{r.m.s. Value}}{\text{Average value}}$

Average value = 0.637 x maximum value

= 0.637 x 414.2

= 263.85 V

Form Factor = $\frac{292.88}{263.85}$

Form Factor = 1.11 □

iii) Frequency, f

$f = \frac{\omega}{2\pi}$ where ω = 754 rad/s (given)

$f = \frac{754}{2\pi}$

f = 120 Hz □

(iv) To find e_r when t = 10ms

e_r = 414.2 Sin (754 t + 0.459) V

e_r = 414 .2 Sin (754 x 10x 10^{-3} + 0.459)

e_r = 414 .2 Sin (8.0)

e_r = 409.8 V □

(b) If the connections to coil 'b' were reversed, then

e_b = - 250 Cos ωt

e_b = - 250 Sin (ωt + $\pi/2$)

Resolving e_b into components,

Horizontal component = -250 cos ($\pi/2$) = 0

Vertical component = -250 Sin ($\pi/2$) = -250V

∴ Net Horizontal component = 100 + 0 + 129.9 + 141.4

= 371.3 V

Net Vertical component = 0 - 250 + 75 - 141. 4

= - 316. 4 V

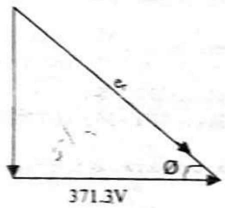


Fig. 4.5

$e_r = \sqrt{316.4^2 + 371.3^2}$

e_r = 487.8 V

$\phi = \tan^{-1}\left[\frac{-316.4}{371.3}\right] = -40.4^\circ$

or ϕ = -0.706 rad

∴ e_r = 487.8 Sin (ωt – 0.706) V □

The phase difference relative to e_a is

= 40.4° lagging □

Question 8

A single - phase circuit consists of three parallel branches, the currents in the respective branches being represented by:

i_a = 40 sin 377t amperes,

i_b = 60 sin (377t - 45°) amperes,

i_c = 36 cos 377t amperes.

(a) Represent the three currents by appropriate vectors.

(b) By resolving the vectors of (a), determine the resultant current and its phase angle relative to i_a

(c) Use a suitable scale and draw a phasor diagram and find the total maximum value of the current taken from the supply and the overall phase angle.

(d) If the supply voltage is represented by 280 sin 377t Volts, find the impedance, resistance and reactance of the circuit.

Answer 4.

The given currents are

i_a = 40 sin 377t

i_b = 60 sin (377t - 45°)

i_c = 36 cos 377t = 36 sin (377t +90°)

(a) The vectors are drawn as shown in figure 4.6.

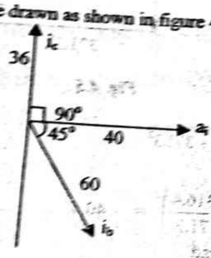


Fig. 4.6

(b) We now resolve figure 4.6 into horizontal and vertical components.

Quantity (A)	Horizontal component (A)	Vertical component (A)
40	40	0
36	0	36
60	60 cos 45° = 42.43	-60 sin 45° - 42.43
Total	82.43	-6.43

The net horizontal and vertical components are shown in figure 4.7.

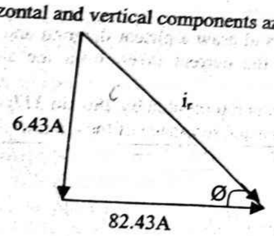


Fig. 4.7

$i_r = \sqrt{6.43^2 + 82.43^2}$

i_r = 82.68 A

$\phi = \tan^{-1}\left[\frac{-6.43}{82.43}\right] = -4.5°$ or

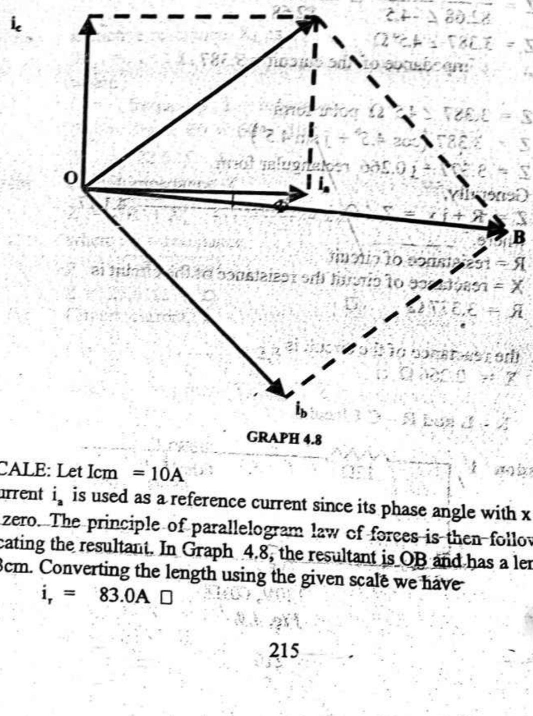
ϕ = 0.0785 rad

∴ the resultant current is i_r

i_r = 82.68 sin (377t - 0.0785) A □

The phase difference relative to i_a is 4.5° lagging □

(c) The phasor diagram is shown in Graph 4.8 below.



GRAPH 4.8

SCALE: Let Icm = 10A

Current i_a is used as a reference current since its phase angle with x - axis is zero. The principle of parallelogram law of forces is then followed in locating the resultant. In Graph 4.8, the resultant is OB and has a length of 8.3cm. Converting the length using the given scale we have

i_r = 83.0A □

The phase angle is as indicated in 4.8 and is measured as 4.5°

(d) Given: $v = 280 \sin 377t$ V

$i_r = 82.68 \sin(377t - 0.0785)$ calculated

(i) To calculate the impedance of the circuit, Z

$$Z = \frac{v}{i_r}$$

$$Z = \frac{280 \sin 377t}{82.68 \sin(377t - 4.5^\circ)}$$

In polar form,

$$Z = \frac{280 \angle 0^\circ}{82.68 \angle -4.5^\circ} = \frac{280 \angle 4.5^\circ}{82.68}$$

$$Z = 3.387 \angle 4.5^\circ \,\Omega$$

$\therefore$ Impedance of the curcuit = 3.387 Ω

(ii) $Z = 3.387 \angle 4.5^\circ \,\Omega$ polar form

$$Z = 3.387(\cos 4.5^\circ + j \sin 4.5^\circ)$$

$$Z = 3.377 + j\,0.266 \text{ rectangular form}$$

Generally,

$$Z = R + jx = Z\angle \emptyset \quad \text{------------ 4.1.17}$$

where,

R = resistance of circuit

X = reactance of circuit the resistance of the circuit is

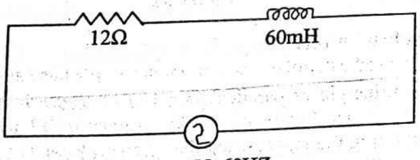
$R = 3.377\,\Omega$ □

(iii) the reactance of the circuit is

$X = 0.266\,\Omega$ □

4.2 R – L and R – C Circuits.

Question 1



120V, 60HZ

Fig. 4.9

Figure 4.9 shows an R-L circuit. Use the circuit to calculate:

(a) the reactance of the inductor
(b) the impedance of the circuit
(c) the circuit current
(d) the phase angle between the current and the applied Voltage
(e) the power factor (f) the apparent power
(g) the active power (h) the reactive power (i) Draw the phasor diagram.

Answer 1

Given:

R = 12 Ω, L = 60mH = 60×10^{-3} H

V = 120V, f = 60 H Z

(a) Inductive reactance, X_L is

$$X_L = 2\pi f L \quad \text{------------ 4.2.1}$$

where :

f = frequency, L = inductance

$$X_L = 2\pi \times 60 \times 60 \times 10^{-3}$$

$$X_L = 22.62\,\Omega \quad □$$

(b) Circuit impedance, Z

$$Z = \sqrt{R^2 + X_L^2} \quad \text{--------- 4.2.2}$$

where : R = resistance

$$Z = \sqrt{12^2 + 22.62^2}$$

$$Z = 25.61\,\Omega \quad □$$

(c) Circuit current, I

$$I = \frac{V}{Z} \quad \text{--------- 4.2.3}$$

where: V = r.m.s. Voltage, Z = impedance

$$I = \frac{120}{25.61}$$

$$I = 4.69 \text{ A} \quad □$$

(d) Phase angle, ϕ

$$\Phi = \tan^{-1}\left[\frac{X_L}{R}\right] \quad \text{---------- 4.2.4}$$

$$\Phi = \tan^{-1}\left[\frac{22.62}{12}\right]$$

$$\Phi = 62.1^\circ \quad □$$

(e) Power factor, p.f

$P.f. = \cos \Phi$ ---------------- 4.2.5

$P.f. = \cos (62.1^0)$

$P.f = 0.468$ Lag □

(f) Apparent power, S

$S = IV$ ---------------------- 4.2.6

Where: I = r.m.s. Current

V = r. m.s. Voltage

$S = 4.69 \times 120$

$S = 562.8$ VA □

(g) Active power or real power, P

$P = IV \cos \Phi$ ------------------- 4.2.7

$P = 4.69 \times 120 \cos 62.1^0$

$P = 263.4$ W □

(b) Reactive power, Q

$Q = IV \sin Ø$ ----------------------- 4.2.8

$Q = 4.69 \times 120 \sin 62.1^0$

$Q = 497.4$ Var □

Alternatively,

$Q = (S^2 - P^2)^{½}$ ---------------------- 4.2.9

$Q = (562.8^2 - 263.4^2)^{½}$

$Q = 497.4$ Var □

(i) Phasor diagram.

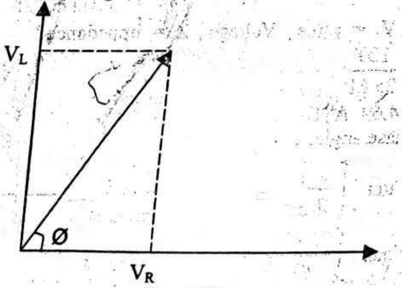


Fig. 4.10

Where V_R = Voltage across the resistor

V_L = Voltage across the inductor

Ø = Phase difference

Question 2

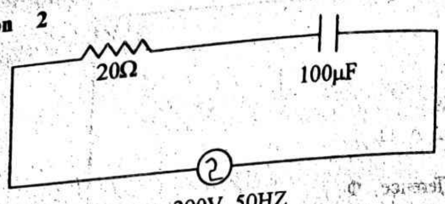


Fig. 4.11

Figure 4.11 shows an R-C circuit. Use the circuit to calculate:

(a) the reactance of the capacitor

(b) the impedance of the circuit

(c) the circuit current (d) the phase difference

(e) the power factor

(f) the Voltage across the resistor

(g) the Voltage across the capacitor

(h) Draw the phasor diagram.

Answer 2

Given:

$R = 20\Omega$, $C = 100\mu F = 100 \times 10^{-6}$ F

$V = 200$ V, $F = 50$ H Z

(a) Capacitive reactance, X_c

$$X_c = \frac{1}{2\pi fC} \text{ ------------------- } 4.2.10$$

where: f = frequency, C = Capacitance

$$X_c = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$X_c = 31.83\Omega$ □

(b) Impedance, Z

$Z = \sqrt{R^2 + X_C^2}$ ----------- 4.2.11

$Z = \sqrt{20^2 + 31.83^2}$

$Z = 37.59\Omega$ □

(c) Circuit current, I

$I = \frac{V}{Z}$

$I = \frac{200}{37.59}$

$I = 5.32$ A □

(d) Phase difference, Φ

$\Phi = -\tan^{-1}\left[\frac{X_c}{R}\right]$ ------------ 4.2.12

$\Phi = -\tan^{-1}\left[\frac{31.83}{20}\right]$

$\Phi = -57.9^o$ □

(e) Power factor, p.f

$P.f = \cos \Phi$

$P.f = \cos\left(-57.9^o\right)$

$P.f = 0.531$ Lead □

(f) Let V_R be the Voltage across the resistor,

V_R = IR from ohm's law

$V_R = 5.32 \times 20$

$V_R = 106.4$ V □

(g) Let V_c be the voltage across the capacitor

$V_C = I X_c$ ------------------------ 4.2.13

$V_C = 5.32 \times 31.83$

$V_C = 169.34$ V □

Alternatively,

$V_C = \sqrt{V^2 - V_R^2}$ -------- 4. 2. 14

$V_C = \left(200^2 - 106.4^2\right)^{1/2}$

$V_C = 169.34$ V □

(h) Phasor diagram

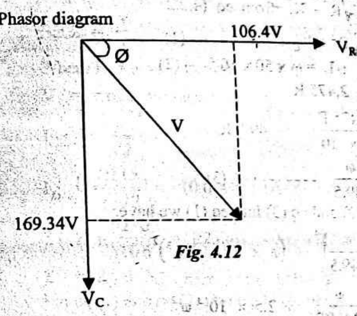


Fig. 4.12

Question 3

(a) A series R- L circuit has L = 50 mH and an impedance of 24.5 ohms. When an alternating Voltage is applied, the current lags the Voltage by 68°. Calculate:

(i) the mains frequency

(ii) the circuit resistance.

(b) A coil connected to a 240V, 60HZ Sinusoidal supply takes a peak current of 15 A at a phase angle of 40°. Calculate: (i) the resistance of the coil (ii) the inductance of the coil (iii) the power taken by the coil.

Answer 3

(a) Given: $L = 50\text{mH} = 50 \times 10^{-3}$H

$Z = 24.5\Omega$

$\text{Ø} = 68^o$

(i) To calculate the mains frequency, f

Recall equation (4.2.4)

$\Phi = \tan^{-1}\left[\frac{X_L}{R}\right]$

$\therefore 68 = \tan^{-1}\left[\frac{X_L}{R}\right]$

$\frac{X_L}{R} = \tan 68^0 = 2.475$

$X_L = 2.475R$

Also $Z = \sqrt{R^2 + X_L^2}$ from eq (4.2.2)

$24.5 = \sqrt{R^2 + X_L^2}$ ---------- (1)

But $x_L = \omega L = \omega \times 50 \times 10^{-3}$ -- (2)

$\therefore \omega L = 2.475\,R$

$\omega = \frac{2.475\,R}{50 \times 10^{-3}} = 49.5\,R$

$\therefore R = \frac{\omega}{49.5}$ ---------- (3)

Putting eq (2) and eq (3) into eq (1) we have,

$24.5^2 = \left(\frac{\omega}{49.5}\right)^2 + (\omega \times 50 \times 10^{-3})^2$

$600.25 = \frac{\omega^2}{2450.25} + 2.5 \times 10^{-3}\,\omega^2$

$1470762.6 = \omega^2 + 6.126\,\omega^2$

$\omega^2 = 206393.85$

$\omega = 454.31\ rad/s$

Recall, $\omega = 2\pi f$

$f = \frac{454.31}{2\pi}$

$f = 72.3HZ$ □

(ii) the circuit resistance, R From equation (3),

$R = \frac{\omega}{49.5}$

$R = \frac{454.31}{49.5}$

$R = 9.18\Omega$ □

(b) Given: $V = 240\ V$, $f = 60HZ$

$I_{peak} = 15A$, $\Phi = 40^0$

(i) Coil resistance, R

$\tan\Phi = \frac{X_L}{R}$

$\tan 40^0 = \frac{X_L}{R}$ $\therefore X_L = 0.8391\,R$ --- (1)

But $Z = \frac{V}{I}$

Where I = r.m.s. current

V = r.m.s. Voltage

$I_{r.m.s} = \frac{I_{peak}}{\sqrt{2}}$ ---------- 4.2.15

$I_{r.m.s} = \frac{15}{\sqrt{2}} = 10.6\ A$

$\therefore Z = \frac{240}{10.6} = 22.64\Omega$ ------ (2)

$Z^2 = R^2 + X_L^2$ ---------- (3)

Put eq (1) and eq (2) into eq (3) to get

$22.64^2 = R^2 + (0.8391\,R)^2$

$1.7041\,R^2 = 512.5696$, $R^2 = 300.786$

$R = 17.34\Omega$ □

(ii) Inductance of the coil, L

$X_L = 2\pi f L$

$L = \frac{X_L}{2\pi f}$

$X_L = 0.8391\,R$ from eq (1)

$X_L = 0.8391 \times 17.34$

$X_L = 14.55\Omega$

$\therefore L = \frac{14.55}{2\pi \times 60} = 0.0386\,H$

$L = 38.6\ mH$ □

(iii) Power taken by the coil, P

$P = I^2 R$

$P = 10.6^2 \times 17.34 = 1948.32\ W$

$P = 1.95\ KW$ □

Question 4

(a) Two pure circuit elements in a series connection have the following current and applied voltage

i = 12.5 sin (314t - 40°) A

V = 240 sin (314t + 23.4°) V

Find the elements comprising the circuit.

(b) A series circuit containing two pure elements has the following current and applied Voltage:

i = 10 cos (942t + 10°) A

V = 120 sin (942t + 58°)V

Find the elements comprising the circuit.

Answer 4

(a) By inspection, the current lags the Voltage by 23.40° + 40° = 63.40° hence the circuit must contain Resistor and inductor.

$$\tan \Phi = \frac{\omega L}{R}$$

$$\tan 63.4^\circ = \frac{\omega L}{R} = 2$$

$$2R = \omega L \quad \text{------------------------ (1)}$$

$$Z = \frac{V_m}{I_m} = \frac{240}{12.5}$$

$$Z = 19.2\Omega$$

$$Z = (R^2 + \omega^2 L^2)^{1/2}$$

$$19.2 = (R^2 + (2R)^2)^{1/2}$$

$$368.64 = 5R^2$$

$$R^2 = 73.728$$

$$R = 8.59\Omega$$

From the given expression, ω = 314 rad/s

∴ 2R = 314L from eq (1)

2 x 8.59 = 314L

L = 0.055 H

L = 55mH □

(b) Given: i = 10 cos (942t + 10°) A or

i = 10 sin (942t + 100°) A

V = 120 sin (942t + 58°) V

$$Z = \frac{120\angle 58^\circ}{10\angle 100^\circ}$$

$Z = 12\angle -42°\Omega$ polar form

$Z = 12(\cos 42° - j\sin 42°)$

$Z = 8.92 - j\,8.03$ rectangular form. From the rectangular form of the circuit impedance, the circuit must contain R and C.

Since for R – C circuit

$Z = R - j X_C$

(i) the circuit resistance, R

$R = 8.92\ \Omega$ □

(ii) the circuit capacitance, C

$$X_C = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi f \times c} = \frac{1}{\omega\, xc}$$

But X_C = 8.03Ω, ω = 942 rad / s

$$C = \frac{1}{942 \times 8.03} = \frac{1}{7564.26}$$

$$C = 1.32 \times 10^{-4}\ F$$

$$C = 132\ \mu F \quad □$$

Note that the method adopted for (4a) can also be used for (4b) and vice versa. By inspection, it can be seen that the current leads the Voltage by 42°, hence the circuit must contain R and C. This alternative method results in the same solution for C and R as above.

Question 5

(a) A capacitor has a capacitance of 30 μF and a phase difference of 25°. It is inserted in series with a 80Ω resistor across a 250V, 60HZ line. Find

(i) The increase in resistance due to the insertion of this capacitor

(ii) Power dissipated in the capacitor and

(iii) Circuit power factor.

(b) The data obtained from a series R-C Circuit are:

V = 180 V, I = 3.2A, P = 42.5W, f = 50HZ

Calculate

(i) Power factor
(ii) Effective resistance
(iii) Capacitive reactance and
(iv) Capacitance.

Answer 5

(a) $x_c = \frac{1}{2\pi fC}$

$x_c = \frac{1}{2\pi \times 60 \times 30 \times 10^{-6}}$

$x_c = 88.42\Omega$

The equivalent series resistance of the capacitor as shown in figure 4.13 is

$R_e = \frac{x_c}{\tan\Phi}$

Now $\Phi = 90^0 - 25^0 = 65^0$

$R_e = \frac{88.42}{\tan 65^0}$

$R_e = 41.23\Omega$

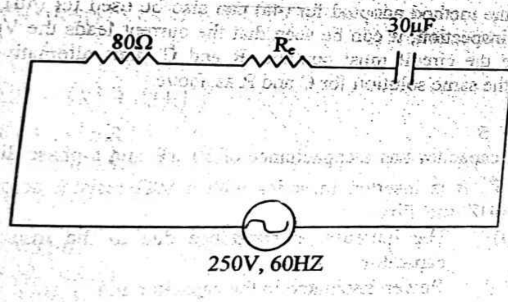


Fig. 4.13

(i) Therefore, resistance of the circuit increases by 41.23Ω □

(ii) $Z = \left[(R + R_e)^2 + x_c^2\right]^{1/2}$

$Z = \sqrt{(80 + 41.23)^2 + 88.42^2}$

$Z = 150\Omega$

$I = \frac{V}{Z} = \frac{250}{150} = 1.67\text{ A}$

Power dissipated in the capacitor is

$= I^2 R_e$

$= 1.67^2 \times 41.23$

$= 115W$ □

(iii) Circuit power factor is

$P.f = \left(\frac{R + R_e}{z}\right)$

$= \frac{121.23}{150}$

p.f = 0.808 Lead □

(b) Power factor, p.f,
Real power, P = IV CosØ

$42.5 = 180 \times 3.2 \cos Ø$

$\cos Ø = \frac{42.5}{180 \times 3.2} = 0.0738$

p.f = Cos Ø

p.f = 0.0738 □

(ii) Effective resistance, R_T

$p = I^2 R_T$

$42.5 = 3.2^2 R_T$

$R_T = 4.15\Omega$ □

(iii) Capacitive reactance, X_c

$Z = (R^2 + x_c^2)^{1/2} = \frac{V}{I}$

$\frac{180}{3.2} = (4.15^2 + x_c^2)^{1/2}$

$56.25^2 = 4.15^2 + x_c^2$

$x_c = 3146.84^{1/2}$

$x_c = 56.1\Omega$ □

(iv) Capacitance, C

$$x_c = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi \times 50 \times 56.1}$$

$$C = 56.7\mu F$$ □

Question 6

A Capacitor and a non- inductive resistor are connected in series across the output terminals of an a. c. Source of potential difference of r.m.s. value 50V alternating at $\frac{500}{\pi}$ HZ. The r.m.s value of the current in the circuit is 0.1A and the capacitance of the capacitor is 2.5 μF Draw a circuit diagram of the arrangement and calculate the

(i) r.m.s. potential difference across the capacitor
(ii) resistance of the resistor
(iii) impedance of the circuit
(iv) average power dissipated in the circuit.

Answer 6

The circuit diagram for the arrangement is shown in figure 4.14

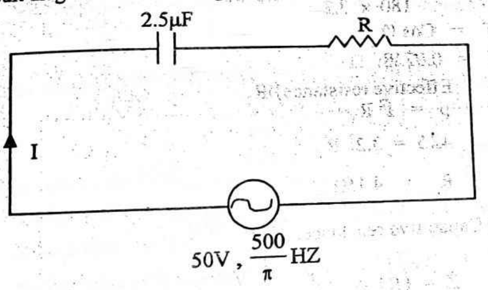


Fig. 4.14

Given:

$$I = 0.1A, V = 50V, f = \frac{500}{\pi} Hz$$

$$C = 2.5\mu F = 2.5 \times 10^{-6} F$$

$$x_c = \frac{1}{2\pi fC}$$

$$x_c = \frac{1}{2\pi \times \frac{500}{\pi} \times 2.5 \times 10^{-6}}$$

$$x_c = 400\Omega$$

(i) Let V_C be the voltage across the capacitor

$$V_C = I X_C$$

$$V_C = 0.1 \times 400$$

$$V_C = 40 V$$ □

(ii) $R = \frac{V_R}{I}$ where : V_R = Voltage across the resistor

$$V_R = \sqrt{V^2 - V_C^2}$$

$$V_R = \sqrt{50^2 - 40^2}$$

$$V_R = 30 V$$

$$R = \frac{30}{0.1}$$

$$R = 300 \Omega$$ □

(iii) Impedance of the circuit, Z

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = (300^2 + 400^2)^{1/2}$$

$$Z = 500 \Omega$$ □

(iv) Average power, $P = I^2 R$

$$P = (0.1)^2 \times 300$$

$$P = 3.0 W$$ □

4.3 Series R – L – C Circuit

Question 1

A Circuit having a resistance of 10Ω, an inductance of 0.2 H and a capacitance of 120 μF in series, is connected across an 80V, 50 Hz supply. Calculate:

(a) The Impedance g
(b) The current
(c) The Voltages across R, L and C
(d) The phase difference between the current and the supply voltage
(e) The power factor
(f) Average power dissipated in the circuit.
(g) Draw the phasor diagram.

Answer 1

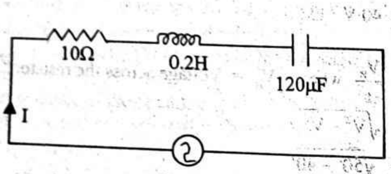


Fig. 4.15

Figure 4.15 shows the circuit arrangement

Given:

$R = 10\Omega$, $L = 0.2H$, $C = 120\mu f = 120 \times 10^{-6}$ F

$V = 80$ V, $f = 50$Hz

$X_L = 2\pi fL$

$X_L = 2\pi \times 50 \times 0.2 = 62.83\Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}}$

$X_C = 26.53\ \Omega$

(a) Impedance, Z

$$Z = \left[R^2 + (X_L - X_C)^2\right]^{1/2} \quad \text{-------------- 4.3.1}$$

$Z = [10^2 + (62.83 - 26.53)^2]^{1/2}$

$Z = [1417.69]^{1/2}$

$Z = 37.65\ \Omega$ □

(b) Current, I

$I = \frac{V}{Z}$

$= \frac{80}{37.65}$

$I = 2.12$A □

(c) (i) Let V_R be the Voltage across the resistor

$V_R = IR$

$V_R = 2.12 \times 10$

$V_R = 21.2$ V □

(ii) Let V_L be the Voltage across the inductor

$V_L = I X_L$

$V_L = 2.12 \times 62.83$

$V_L = 133.2$ V □

(iii) Let V_C be the Voltage across the capacitor

$V_C = I X_C$

$V_C = 2.12 \times 26.53$

$V_C = 56.24$ V □

Note that the current, I is the same since the circuit components connected in series.

(d) Let Φ be the phase difference,

$$\Phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] \quad \text{---------- 4.3.2}$$

$\Phi = \tan^{-1}\left[\frac{62.83 - 26.53}{10}\right] = \tan^{-1}[3.631]$

$\Phi = 74.6°$ □

(e) Power factor, p.f

P.F = cos Φ

P.F = cos 74.6°

P.F. = 0.266 □

(f) Average power = $I^2 R$ (Since power dissipated in L and C is

∴ Average power = $2.12^2 \times 10$

Average power = 44.94 W □

(g) **Phasor diagram**

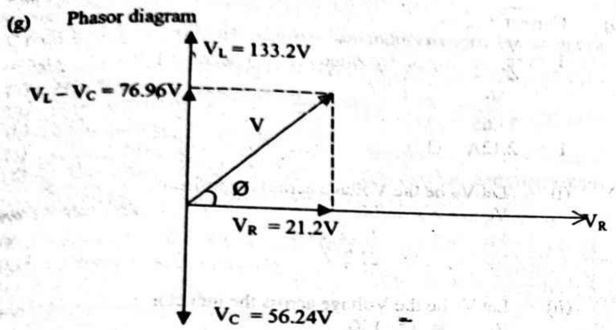


Fig. 4.16

Question 2

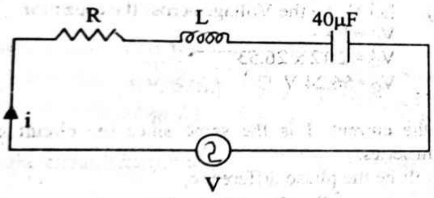


Fig. 4. 17

If in Figure 4.17 the voltage and current are expressed thus:

$V = 280.4\cos(2000t - 25^\circ)V$

$i = 9.6\cos(2000t - 70^\circ)A$,

Calculate the value of R and L.

Answer 2

By inspection, the current lags the voltage by $70^\circ - 25^\circ = 45^\circ$

$$\tan\Phi = \left[\frac{\omega L - \frac{1}{\omega C}}{R}\right]$$

$$\tan 45^\circ = \left[\frac{\omega L - \frac{1}{\omega C}}{R}\right] = 1 \text{ ------------(1)}$$

The circuit impedance is, Z

$$Z = \frac{280.4\angle -25^\circ}{9.6\angle -70^\circ} \text{ In polar form}$$

$$Z = \frac{280.4}{9.6}\angle 70^\circ - 25^\circ = 29.2\angle 45^\circ\ \Omega$$

Expressing the Impedance in rectangular form we have

$Z = 29.2(\cos 45^\circ + j\sin 45^\circ)$

$Z = 20.65 + j\,20.65\ \Omega$

Recall that $Z = R + jx$

Where: x = reactance of the Circuit

$\therefore R = 20.65\ \Omega$ ▯

From equation (1),

$$1 = \frac{\omega L - \frac{1}{\omega c}}{R}$$

But ω = 2000 rad/s from the given expressions

$$\therefore 1 = \left(2000\,L - \frac{1}{2000\times 40\times 10^{-6}}\right)/(20.65)$$

$$\therefore 20.65 = 2000\,L - \frac{1}{0.08}$$

$1.652 = 160\,L - 1$

$$L = \frac{2.652}{160} = 0.0166\,H$$

$L = 16.6m\ H$ ▯

Question 3

A Pure inductor, a non – inductive resistor and a capacitor are connected in series to form L-R-C circuit. The supply voltage is 100V at 60 H Z, the Potential difference across the inductor is 48 V and the p.d across the resistor and capacitor together is 120V. The current is 8 A. Calculate the value of all the components and the power factor of the circuit.

Answer 3

The circuit arrangement is shown in figure 4. 18 R C

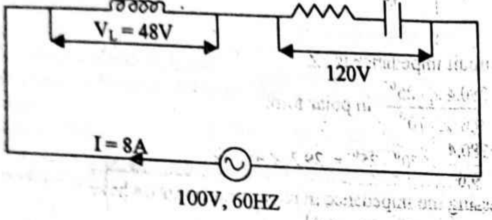


Fig. 4. 18

Circuit impedance, Z is

$Z = \frac{V}{I} = \frac{100}{8}$

$Z = 12.5\ \Omega$

$X_L = 2\pi fL = \frac{V_L}{I}$

where: V_L = Voltage across the inductor

$\therefore \frac{48}{8} = 2\pi \times 60 \times L$

$L = \frac{6}{2\pi \times 60} = 0.01592H$

$L = 15.92\ mH$ □

But $Z = \sqrt{R^2 + (x_L - x_c)^2}$

$12.5^2 = R^2 + X_L^2 - 2X_CX_L + X_C^2$ ------------- (1)

Let Z_C be the impedance of the R - C branch

$Z_C = \frac{V_c}{I}$

$Z_C = \frac{120}{8} = 15\Omega$

But $Z_C^2 = R^2 + X_C^2$

$15^2 = R^2 + X_C^2$ ------------ (2)

Substituting equation (2) into equation (1) we have

$12.5^2 = 15^2 + X_L^2 - 2X_CX_L$

$156.25 = 225 + X_L^2 - 2X_CX_L$ --------- (3)

But $X_L = \frac{V_L}{I} = \frac{48}{8} = 6\Omega$

Equation (3) now becomes,

$156.25 = 225 + 6^2 - 2 \times 6 X_C$

$156.25 = 261 - 12 X_C$

$12 X_C = 261 - 156.25$

$X_C = \frac{104.75}{12} = 8.73\ \Omega$

$X_C = \frac{1}{2\pi fC}$

$\therefore C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi x60x8.73}$

$C = 304\mu f$ □

From equation (2),

$15^2 = R^2 + 8.73^2$

$R^2 = 225 - 76.21$

$R = \sqrt{148.79}$

$R = 12.2\ \Omega$ □

$\tan\phi = \frac{X_L - X_C}{R} = \frac{6 - 8.73}{12.2}$

$\Phi = \tan^{-1}[-0.2238]$

$\Phi = -12.6^\circ$

$\therefore$ the circuit power factor P.F is

$P.F = \cos(12.6^\circ)$

$P.F = 0.976$ lead □

Question 4

Three impedances are connected in series across a $110\angle 45^\circ$ V supply. The first impedance is a pure resistive load of 5Ω, the second coil of 15Ω inductive reactance and 2Ω resistance, and the third con of a capacitive reactance of 5Ω and 3Ω resistance. Calculate usin Notation:

(a) the current through the circuit

(b) the voltage across each impedance
(c) the total power dissipated in the circuit in watts.

Answer 4

Figure 4. 19 shows the circuit arrangement.

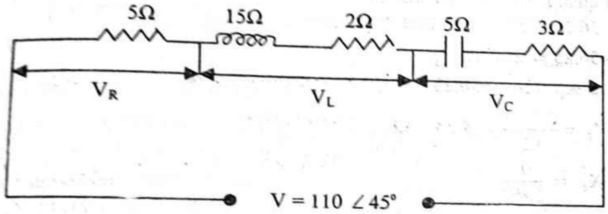


Fig. 4. 19

$Z_R = 5 + j0\ \Omega$

$Z_L = 2 + j15\Omega$

$Z_C = 3 - j5\ \Omega$

$\therefore$ Total impedance, Z_T is

$Z_T = 5 + j0 + 2 + j15 + 3 - j5$

$Z_T = 10 + j10$ rectangular form

$$Z_T = \sqrt{10^2 + 10^2}\angle \tan^{-1}\left(\frac{10}{10}\right)$$

$Z_T = 14.14\angle 45°\Omega$ polar form

$Z_T = 14.14 \angle 45°\ \Omega$

(a) Circuit Current, I

$$I = \frac{V}{Z_T}$$

$$I = \frac{110 \angle 45°}{14.14 \angle 45°} = \frac{110}{14.14} \angle 45° - 45°$$

$I = 7.78 \angle 0°$ A □

(b) Voltage across each impedance

(i) $V_R = IR$

$V_R = 7.78\angle 0° \times 5$

$V_R = 38.9\angle 0°$ V □

(ii) $V_L = I Z_L$

$Z_L = 2 + j12\Omega$

$Z_L = 12.17 \angle 80.50°\ \Omega$

$V_L = 7.78\angle 0° \times 12.17\angle 80.5°$

$V_L = 94.68 \angle 80.5°$ V □

(iii) $V_C = I Z_C$

$Z_C = 3 - j5\ \Omega$

$Z_C = 5.83 \angle -59°$

$V_C = 7.78 \angle 0° \times 5.83\angle -59°$

$V_C = 45.36 \angle -59°$ V □

(c) Total power, P

$S = VI^*$ ---------------------- 4.3.3

Where:

S = Apparent power in VA

I^* = Conjugate of current, I

Also, $S = P + jQ$ ---------------------- 4.3.4

Where:

P = real power in Watts

Q = reactive power in Var

$V = 110\angle 45°$ V polar form

$V = 77.78 + j\,77.78$ V rectangular form

$I = 7.78 \angle 0°$ A polar form

$I = 7.78 + j0$ rectangular form

$I^* = 7.78 - j0$

$S = (77.78 + j\,77.78)(7.78 - j0)$

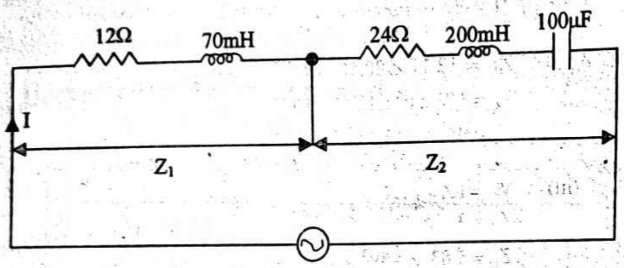
$S = 605.13 + j\,605.13 + 0$

$S = 605.13 + j\,605.13 = P + jQ$

$\therefore$ total power dissipated in the circuit,

P = 605.13 W □

Question 5



250∠30°V, 50HZ

Fig. 4.20

For the circuit in figure 4.20, calculate using j (complex) notation:

(a) Z_1 and Z_2
(b) the total impedance of the circuit
(c) circuit current
(d) the Voltage across Z_1 and across Z_2, and the phase angle between the voltage across Z_1 and that across Z_2
(e) the power dissipated in the circuit.

Answer 5

(a) $Z_1 = R + jX_L$

$X_L = 2\pi fL$

$= 2\pi \times 50 \times 70 \times 10^{-3}$

$X_L = 22\,\Omega$

$Z_1 = 12 + j22\,\Omega$ or

$Z_1 = 25.1\angle 61.4^\circ\,\Omega$ □

(ii) $Z_2 = R_2 + j\,X_L - jX_C$

$X_L = 2\pi fL$

$= 2\pi \times 50 \times 200 \times 10^{-3}$

$X_L = 62.83\,\Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$X_C = 31.83\ \Omega$

$\therefore Z_2 = 24 + j62.83 - j31.83$

$Z_2 = 24 + j31\,\Omega$

$Z_2 = 39.2\angle 52.3^\circ\,\Omega$ □

(b) Let Z_T be the total impedance of the circuit

$Z_T = Z_1 + Z_2$

$= 12 + j22 + 24 + j31$

$Z_T = 36 + j53\,\Omega$

$Z_T = 64.1\angle 55.8^\circ\ \Omega$ □

(c) Circuit Current, I

$I = \dfrac{V}{Z_T}$

$I = \dfrac{250\angle 30^\circ}{64.1\angle 55.8^\circ} = 3.9\angle 30^\circ - 55.8^\circ$

$I = 3.9\angle -25.8^\circ$ A □

(d)(i) Let V_1 be the voltage across Z_1

$V_1 = IZ_1$

$= 3.9\angle -25.8^\circ \times 25.1\angle 61.4^\circ$

$V_1 = (3.9)(25.1)\angle -25.8^\circ + 61.4^\circ$

$V_1 = 97.89\angle 35.6^\circ$ V □

(ii) Let V_2 be the Voltage across Z_2

$V_2 = IZ_2$

$V_2 = 3.9\angle -25.8^\circ \times 39.2\angle 52.3^\circ$

$V_2 = (3.9)(39.2)\angle -25.8^\circ + 52.3^\circ$

$V_2 = 152.88\angle 26.5^\circ$ V □

(iii) V_1 leads V_2 by $35.6^\circ - 26.5^\circ$

$= 9.1^\circ$ □

(e) Power dissipated in the circuit, P

$S = VI^*$

Where: $V = 250 \angle 30^\circ$ V

$I = 3.9 \angle -25.8^\circ$ A

$\therefore I^* = 3.9 \angle 25.8^\circ$ A

$S = 250 \angle 30^\circ \times 3.9 \angle 25.8^\circ$

$S = 975 \angle 55.8^\circ$ V A

But $S = P + jQ$

$\therefore S = 975 (\cos 55.8^\circ + j \sin 55.8^\circ)$ rectangular form

$S = 548 + j\,806$

$\therefore$ Total power dissipated in the circuit is

$P = 548$W □

Alternatively,

$P = I^2 R_1 + I^2 R_2$ (Since power dissipated in L and C is zero)

$P = 3.9^2 \times 12 + 3.9^2 \times 24$

$P = 182.52 + 365.04$

$P = 547.56$ W

$P = 548$W □

Question 6

The circuit shown in figure 4.21 is connected to a 240V, 60 HZ supply. Calculate:

(a) total impedance of the circuit
(b) current in the circuit
(c) the angle of phase difference
(d) the potential drops V_1 and V_2
(e) the power factor
(f) the real, apparent and reactive power.

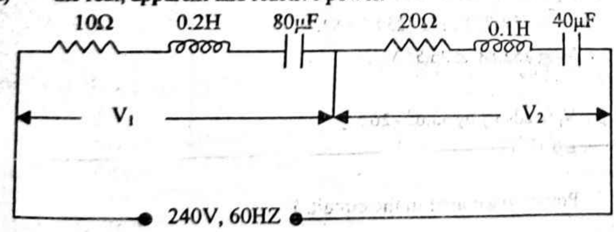


Fig. 4.21

Answer 6

$X_{L_1} = 2\pi f L_1$

$= 2\pi \times 60 \times 0.2$

$X_{L_1} = 75.4\,\Omega$

$X_{C_1} = \dfrac{1}{2\pi f C_1} = \dfrac{1}{2\pi \times 60 \times 80 \times 10^{-6}}$

$X_{C_1} = 33.2\,\Omega$

$R_1 = 10\,\Omega$

Also,

$X_{L_2} = 2\pi \times 60 \times 0.1 = 37.7\,\Omega$

$X_{C_2} = \dfrac{1}{2\pi \times 60 \times 40 \times 10^{-6}} = 66.3\,\Omega$

$R_2 = 20\,\Omega$

(a) Let Z be the total impedance of the circuit.

$Z = (R_T^2 + X_T^2)^{1/2}$

$R_T = R_1 + R_2 = 10 + 20$

$R_T = 30\,\Omega$

$X_T = X_1 + X_2$

$X_1 = X_{L_1} - X_{C_1} = 75.4 - 33.2$

$X_1 = 42.2\,\Omega$

$X_2 = X_{L_2} - X_{C_2} = 37.7 - 66.3$

$X_2 = -28.6\,\Omega$

$X_T = 42.2 - 28.6$

$X_T = 13.6\,\Omega$

$\therefore Z = (30^2 + 13.6^2)^{1/2}$

$Z = 32.94\,\Omega$ □

(b) Circuit Current, I

$I = \dfrac{V}{Z}$

$I = \dfrac{240}{32.94}$

$I = 7.29$ A □

(c) Angle of phase difference, Φ

$$\tan\Phi = \frac{X_{L_T} - X_{C_T}}{R_T} = \frac{X_T}{R_T}$$

where:

X_{L_T} = total inductive reactance in the circuit

X_{C_T} = total capacitive reactance in the circuit

R_T = total resistance in the circuit

$R_T = 10 + 20 = 30\,\Omega$

$\therefore X_{L_T} = 75.4 + 37.7 = 113.1\,\Omega$

$X_{C_T} = 33.2 + 66.3 = 99.5\,\Omega$

$$\therefore \tan\Phi = \frac{113.1 - 99.5}{30} = \frac{13.6}{30}$$

$\Phi = \tan^{-1}[0.4533]$

$\Phi = 24.4^\circ$ **Lagging** □

(d) (i) $V_1 = I Z_1$

$Z_1 = (R_1^2 + X_1^2)^{1/2}$

$Z_1 = (10^2 + 42.2^2)^{1/2} = 43.37\,\Omega$

$V_1 = 7.29 \times 43.37$

$V_1 = 316.2$ V □

(ii) $V_2 = I Z_2$

$Z_2 = (R_2^2 + X_2^2)^{1/2}$

$Z_2 = [20^2 + (-28.6)^2]^{1/2}$

$Z_2 = 34.9\,\Omega$

$V_2 = 7.29 \times 34.9$

$V_2 = 254.4$ V □

(e) Power factor, p.f

P.f = cos Φ

P.f = cos 24.4°

P.f = 0.911 □

(f) (i) Apparent power, S = VI

S = 240 x 7.29

S = 1749.6 VA

S = 1.75 KVA □ 4.3.4

(ii) Real power, p = S cos φ

P = 1749.6 cos 24.4°

P = 1593.3 W

P = 1.59KW □

(iii) Reactive power, Q = S sin Φ 4.3.5

Q = 1749.6 sin 24.4°

Q = 722.77 Var

Q = 0.72 K Var □

4.4 Parallel R-L-C- Circuit

Question 1

A circuit consists of a 25 Ω resistor in parallel with a 50 m H inductor and is connected to a 220V, 50 HZ supply. Determine:

(a) the branch currents and the supply current

(b) the circuit phase angle

(c) the circuit impedance.

Answer 1

The Circuit diagram is shown in figure 4.22

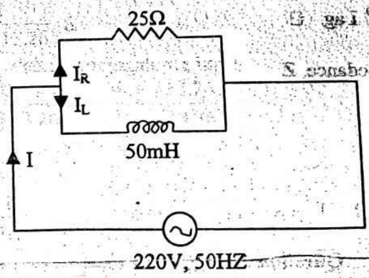


Fig. 4.22

Since the circuit is connected in parallel, $V_R = V_L = 220V = V$

(a) (i) Branch currents.

$$I_R = \frac{V}{R} = \frac{220}{25}$$

$I_R = 8.8\text{ A}$ □

$I_L = \dfrac{V}{X_L}$

$X_L = 2\pi fL = 2\pi \times 50 \times 50 \times 10^{-3}$

$X_L = 15.7\,\Omega$

$I_L = \dfrac{220}{15.7}$

$I_L = 14.0\text{ A}$ □

(ii) Supply current, I

$I = (I_R^2 + I_L^2)^{1/2}$ ------------------ 4.4.1

$I = (8.8^2 + 14.0^2)^{1/2}$

$I = 16.5\text{ A}$ □

(b) Circuit phase angle, Φ

$\Phi = \tan^{-1}\left[\dfrac{R}{X_L}\right]$ ------------ 4.4.2

$\Phi = \tan^{-1}\left[\dfrac{25}{15.7}\right]$

$\Phi = 57.9^\circ$ Lag □

(c) Circuit impedance, Z

$Z = \dfrac{V}{I}$

$Z = \dfrac{220}{16.5}$

$Z = 13.3\,\Omega$ □

Question 2

Use Circuit Figure 4.23 to calculate:

(a) the circuit impedance
(b) the supply current and its phase relative to the supply voltage
(c) the circuit resistance
(d) the circuit reactance

$I = \sqrt{I_R^2 + I_L^2}$

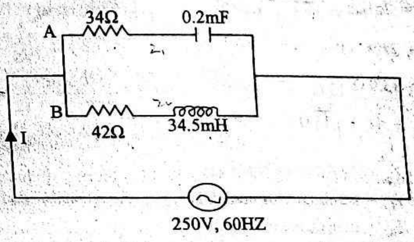


Fig. 4.23

Answer 2

$X_C = \dfrac{1}{2\pi fC}$

$X_C = \dfrac{1}{2\pi \times 60 \times 0.2 \times 10^{-3}} = 13.26\,\Omega$

$Z_A = 34 - j\,13.26\,\Omega$

$Y_A = \dfrac{1}{Z_A}$ ---------------- 4.4.3

Where: Y_A = admittance of branch A
Z_A = impedance of branch A

$Y_A = \dfrac{1}{34 - j\,13.26}$ S

Rationalizing with its conjugate we have

$Y_A = \dfrac{1}{34 - j\,13.26} \times \dfrac{34 + j\,13.26}{34 + j\,13.26}$

$Y_A = \dfrac{34 + j\,13.26}{34^2 + 13.26^2}$

$Y_A = \dfrac{34 + j\,13.26}{1331.83} = 0.0255 + j\,0.00996$ S

$Y_A = 0.0274\,\angle 21.3^\circ$ S In polar form

Also,

$X_L = 2\pi fL$

$X_L = 2\pi \times 60 \times 34.5 \times 10^{-3}$

$X_L = 13.0\,\Omega$

$\therefore Z_B = 42 + j\,13\,\Omega$

$Y_B = \frac{1}{Z_B}$

Where Y_B = admittance of branch B

Z_B = impedance of branch B

$$Y_B = \frac{1}{42 + j\,13}$$

$$Y_B = \frac{1}{42 + j\,13} \times \frac{42 - j\,13}{42 - j\,13}$$

$$Y_B = \frac{42 - j\,13}{42^2 + 13^2} = \frac{42 - j\,13}{1933}$$

$Y_B = 0.0217 - j\,0.00673$ S

$Y_B = 0.0227 \angle -17.2°$S

Total admittance, Y is

$Y = Y_A + Y_B$

$Y = 0.0255 + j\,0.00996 + 0.0217 - j\,0.00673$

$Y = 0.0472 + j\,0.00323$ S

$Y = 0.04731 \angle 3.9°$ S

(a) Circuit impedance, Z

$$Z = \frac{1}{Y} \quad \text{-----------------} \quad 4.4.4$$

where: Z = Circuit impedance

Y = Circuit admittance

$$Z = \frac{1}{0.04731 \angle 3.9°} = \frac{1}{0.04731} \angle -3.9°$$

$Z = 21.14 \angle -3.9°\,\Omega$

$\therefore$ the circuit impedance is $21.14\,\Omega$ □

(b) Supply current, I

$$I = \frac{V}{Z}$$

$$I = \frac{250 \angle 0°}{21.14 \angle -3.9°}$$

$I = 11.83 \angle 3.9°$ A □

- The supply current is 11.83A and leads the supply voltage by 3.9°

(c) Circuit resistance, R

$R = Z \cos \Phi$

$R = 21.14 \cos(-3.9°)$

$R = 21.09\,\Omega$ □

(d) Circuit reactance, X

$X = Z \sin \Phi$

$X = 21.14 \sin(-3.9°)$

$X = -1.44\,\Omega$ □

Since X is negative, the reactance must be capacitive. Thus the circuit is equivalent to a $21.35\,\Omega$ resistor in series with a $1.44\,\Omega$ capacitive reactance.

Question 3

Use Circuit Figure 4.24 to Calculate:

(a) the circuit current and its phase difference from the supply voltage

(b) branch currents and their phase difference from the supply voltage

(c) Voltage across the inductor.

(d) Draw the complete vector diagram not to scale.

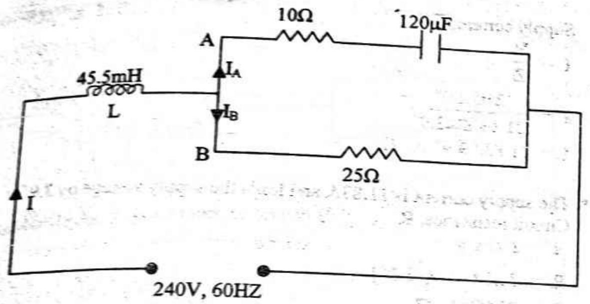


Fig. 4.24

Answer 4

$$Z_A = 10 - \frac{j}{120 \times 10^{-6} \times 2\pi \times 60} = 10 - j\,22.1\,\Omega$$

$$Y_A = \frac{1}{10 - j\,22.1} = \frac{10 + j\,22.1}{588.41}$$

$Y_A = 0.017 + j\,0.0376$ S

$Z_B = 25 + j\,0\,\Omega$

$Y_B = 0.04 + j\,0$ S

Let Y_{AB} be the combined admittance of branches A and B

$\therefore Y_{AB} = Y_A + Y_B$

$Y_{AB} = 0.017 + j\,0.0376 + 0.04 + j\,0$ S

$Y_{AB} = 0.057 + j\,0.0376$ S

Let Z_{AB} be the effective impedance of branches A and B

$$Z_{AB} = \frac{1}{0.057 + j\,0.0376} = \frac{1}{0.0683\,\angle 33.4^\circ}$$

$Z_{AB} = 14.64\angle -33.4^\circ$

$Z_{AB} = 12.22 - j\,8.06\,\Omega$

$X_L = j\,2\pi fL$

$X_L = j\,2\pi \times 60 \times 45.5 \times 10^{-3}$

$X_L = j\,17.15\,\Omega$

Let Z be the total impedance of the circuit,

$Z = Z_{AB} + Z_L$

$Z = 12.22 - j\,8.06 + j\,17.15$

$Z = 12.22 + j\,9.09\,\Omega$

$Z = 15.23\,\angle 36.6^\circ\Omega$

(a) Circuit current, I

$I = \frac{V}{Z}$ where: V = supply voltage

$$I = \frac{240\angle 0^\circ}{15.23\angle 36.6^\circ}$$

$I = 15.76\,\angle -36.6^\circ$A □

∴ The circuit current is 15.76A lagging the supply voltage by 36.6° □

(b) To find the branch currents, I_A and I_B

$V_{AB} = I\,Z_{AB}$

$V_{AB} = 15.76\angle -36.6^\circ \times 14.64\,\angle -33.4^\circ$

$V_{AB} = 230.73\angle -70^\circ$V

(i) $$I_A = \frac{V_{AB}}{Z_A} = \frac{230.73\angle -70}{10 - j22.1}$$

$$I_A = \frac{230.73\angle -70^\circ}{24.26\angle -65.7^\circ}$$

$I_A = 9.51\,\angle -4.3^\circ$ A □

∴ The current through branch A is 9.5 1A lagging the supply voltage by 4.3°

(ii) $$I_B = \frac{V_{AB}}{Z_B} = \frac{230.73\angle -70^\circ}{25\angle 0^\circ}$$

$I_B = 9.23\angle -70^\circ$A □

∴ The current through branch B is 9.23A lagging the supply voltage by 70°.

(c) Let V_L be the voltage across the inductor

$V_L = IX_L$

Where $X_L = j17.15\Omega$

$V_L = 15.76\angle -36.6^\circ \times 17.15\angle 90^\circ$

$V_L = 270.28\angle 53.4^\circ$ V □

(d) Figure 4.25 shows the vector diagram.

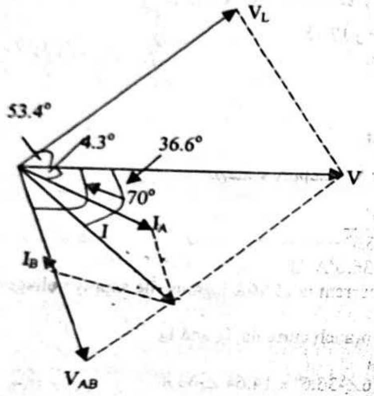


Fig. 4.25

Question 5

Two circuits A and B are connected in series across a 180V a.c supply. Circuit A consists of 5Ω resistor connected in parallel with a coil of inductive reactance 10Ω. Circuit B consists of a capacitive reactance of 10Ω in parallel with a pure resistive load of 10Ω resistance. Calculate;

(a) the total circuit impedance
(b) the total circuit current
(c) the voltage across each parallel circuit.
(d) the branch currents in the two circuits
(e) Draw the vector diagram not to scale

Answer 5

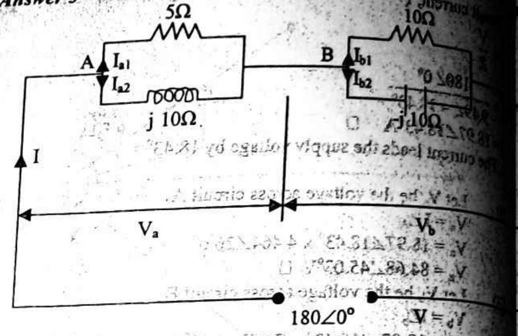


Fig. 4.26

Figure shows the circuit arrangement.

$Y_A = \frac{1}{5} + \frac{1}{j10} = 0.2 - j0.1\ S$

$Y_A = 0.224\ \angle{-26.6^\circ}\ S$

$Y_B = \frac{1}{10} + \frac{1}{-j10} = 0.1 + j0.1\ S$

$Y_B = 0.1414\angle 45^\circ\ S$

$Z_A = \frac{1}{Y_A} = \frac{1}{0.224\angle -26.6^\circ} = 4.464\angle 26.6^\circ\Omega$

$Z_A = 4.0 + j2.0\Omega$

$Z_B = \frac{1}{Y_B} = \frac{1}{0.1414\angle 45^\circ} = 7.072\angle{-45^\circ}\Omega$

$Z_B = 5 - j5\Omega$

(a) Total circuit impedance, Z

$Z = Z_A + Z_B$

$Z = 4 + j2 + 5 - j5 = 9 - j3\Omega$

$Z = \sqrt{9^2 + 3^2}\ \angle \tan^{-1}\left(\frac{-3}{9}\right)$

$Z = 9.49\ \angle{-18.43^\circ}\Omega$ □

(b) Circuit current, I

$I = \frac{V}{Z}$

$I = \frac{180\angle 0^\circ}{9.49\angle -18.43^\circ}$

$I = 18.97\angle 18.43^\circ A$ □

∴ The current leads the supply voltage by 18.43°

(c) (i) Let V_a be the voltage across circuit A

$V_a = I Z_A$

$V_a = 18.97\angle 18.43^\circ \times 4.464\angle 26.6^\circ$

$V_a = 84.68\angle 45.03^\circ V$ □

(ii) Let V_b be the voltage across circuit B

$V_b = IZ_b$

$V_b = 18.97\angle 18.43^\circ \times 7.072\angle -45^\circ$

$V_b = 134.16\angle -26.6^\circ V$ □

(d) Branch currents

(i) $I_{a1} = \frac{V_a}{R_5} = \frac{84.68\angle 45.03^\circ}{5}$

$I_{a1} = 16.94\angle 45.03^\circ A$ □

(ii) $I_{a2} = \frac{V_a}{X_L}$

$I_{a2} = \frac{84.68\angle 4530^\circ}{j10} = \frac{84.68\angle 45.03^\circ}{10\angle 90^\circ}$

$I_{a2} = 8.468\angle -44.97^\circ A$ □

(iii) $I_{b1} = \frac{V_b}{R_{10}}$

$I_{b1} = \frac{134.16\angle -26.6^\circ}{10}$

$I_{b1} = 13.416\angle -26.6^\circ A$ □

(iv) $I_{b2} = \frac{V_b}{X_C}$

$I_{b2} = \frac{134.16\angle -26.6^\circ}{-j10} = \frac{134.16\angle -26.6^\circ}{10\angle -90^\circ}$

$I_{b2} = 13.416\angle 63.4^\circ A$ □

(e) the vector diagram is shown in figure 4.27

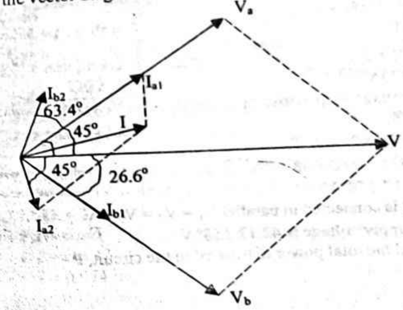


Fig. 4.27

Question 6

Three impedance $Z_1 = (5 + j8)\Omega$, $Z_2 = (4 - j6)\Omega$ and $Z_3 = (7 + j10)\Omega$ are connected in parallel to an a.c. supply. If the power dissipated in Z_1 is 100W, find:

a. The supply voltage
b. The total power dissipated in the combination and
c. The power factor

Answer 6

The circuit arrangement is shown in figure 4.28.

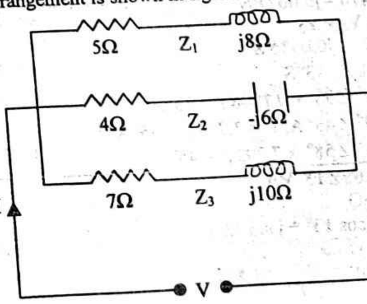


Fig. 4.28

Let P_1 be the power dissipated in impedance one

$P_1 = 100W$ (given)

$P_1 = I_1^2 R$

$100 = I_1^2 \times 5$

$I_1 = 4.472A$

Let V_1 be the voltage drop across Z_1

$V_1 = I_1 Z_1$

$Z_1 = 5 + j8 = 9.43\angle 58°$

$V_1 = 4.472 \times 9.43\angle 58°$

$V_1 = 42.17\angle 58° V$

Since the circuit is connected in parallel $V_1 = V_2 = V_3 = V$

a. the supply voltage is $42.17\angle 58° V$

b. to find the total power dissipated in the circuit, P

$S = VI^*$

But $I = \frac{V}{Z} = VY$ ---------------- 4.4.5

Where Y = total admittance of the circuit.

$Y_1 = \frac{1}{5 + j8} = \frac{5 - j8}{89}$

$Y_1 = 0.0562 - j0.0899 \ S$

$Y_2 = \frac{1}{4 - j6} = \frac{4 + j6}{52}$

$Y_2 = 0.0769 + j0.1154 \ S$

$Y_3 = \frac{1}{7 + j10} = \frac{7 - j10}{149}$

$Y_3 = 0.0470 - j0.0671 \ S$

$Y = Y_1 + Y_2 + Y_3$

$Y = 0.1801 - j0.0416 \ S$

$Y = 0.1848\angle -13° S$

$\therefore I = 42.17\angle 58° \times 0.1848\angle -13° A$

$I = 7.793\angle 45° A, I^* = 7.793\angle -45° A$

$\therefore S = 42.17\angle 58° \times 7.793\angle -45°$

$S = 328.63\angle 13° VA$

$S = P + jQ$

$S = 328.63(\cos 13° + j \sin 13°)$

$S = 320.2 + j73.9$

$\therefore P = 320.2W$ □

(c) the power factor, P.F

$P.F = \cos \emptyset = \frac{P}{S}$ ------- 4.4.6

Where: P = real power, S = apparent power

$P.F = \frac{320.2}{328.63}$

P.F = 0.974 Lag □

Alternatively, the phase difference between the supply voltage and the supply current is $58° - 45°$

$\therefore \emptyset = 13°$

$P.f = \cos\emptyset$

$P.f = \cos 13°$

P.f = 0.974 lag □

Question 7

The circuit of figure 4.29 shows a series – parallel network. The voltage across the parallel branch is $100\angle 0° V$.

a. Calculate:

i. the branch currents

ii. the circuit total current

iii. the voltage across the series part

iv. the supply voltage, $V\angle\emptyset$

(b) Draw the phasor diagram showing all the currents and voltages.

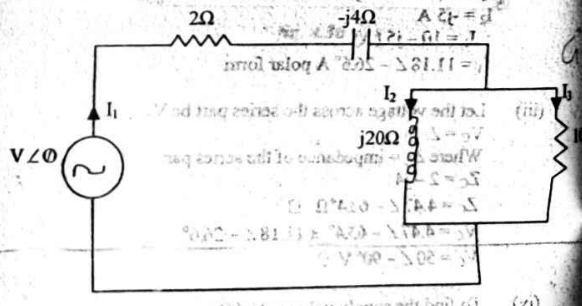


Fig. 4.29

Answer 7

(a) (i) **The branch currents are I_2 and I_3.**

$I_2 = \frac{V_P}{X_L}$

Where,
V_P = voltage across the parallel branch.
X_L = reactance of the inductor

$I_2 = \frac{100\angle 0^\circ}{20\angle 90^\circ}$

$I_2 = 5\angle -90^\circ$ A □

Similarly,

$I_3 = \frac{V_P}{R}$

Where, R = resistance of the resistor

$I_3 = \frac{100\angle 0^\circ}{10}$

$I_3 = 10\angle 0^\circ$ A □

The branch currents are $5\angle -90^\circ$ A and $10\angle 0^\circ$ A □

(ii) To find the circuit current, I_1

$I_1 = I_2 + I_3$ from KCL

We now convert the current, I_2 to rectangular coordinate for easy manipulation.

$I_2 = 5(\cos 90^\circ - j\sin 90^\circ)$

$I_2 = -j5$ A

$\therefore I_1 = 10 - j5$ (A)

$I_1 = 11.18\angle -26.6^\circ$ A polar form

(iii) Let the voltage across the series part be V_c

$V_C = Z_C I_1$

Where Z_C = impedance of the series part

$Z_C = 2 - j4$

$Z_C = 4.47\angle -63.4^\circ\Omega$ □

$V_C = 4.47\angle -63.4^\circ \times 11.18\angle -26.6^\circ$

$V_C = 50\angle -90^\circ$ V □

(iv) To find the supply voltage, $V\angle\Phi$

$V\angle\Phi = I_1 Z_T$

where : Z_T = Total impedance of the circuit
Z_P = Total impedance of the parallel branch.

$Z_P = \frac{j20 \times 10}{10 + j20} = \frac{200\angle 90^\circ}{22.36\angle 63.4^\circ}$

$Z_P = 8.95\angle 26.6^\circ\Omega = (8 + j4)\Omega$

$Z_T = 8 + j4 + 2 - j4$

$Z_T = 10\Omega$

$\therefore V\angle\Phi = 11.18\angle -26.6^\circ \times 10$

$V\angle\Phi = 111.8\angle -26.6^\circ$ V □

(b) The phasor diagram is now drawn as shown in figure 4.30

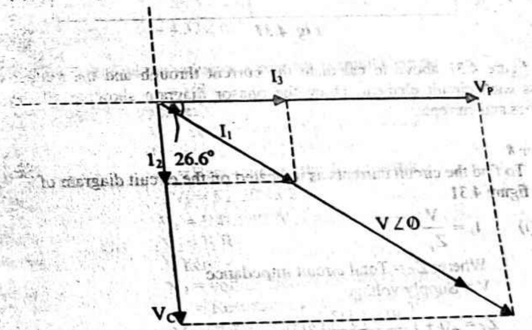


fig. 4.30

Question 8

Fig. 4.31

Use figure 4.31 above to calculate the current through and the voltage across each circuit element. Draw the phasor diagram showing all the voltages and currents.

Answer 8

(a) To find the circuit currents as indicated on the circuit diagram of figure 4.31

(i) $I_1 = \frac{V}{Z_T}$

Where: Z_T = Total circuit impedance

V = Supply voltage

$$Z_T = 20 + \left[\frac{40 \times (-j23.1)}{40 - j23.1}\right]$$

$$Z_T = 20 + \frac{924\angle -90^\circ}{46.2\angle -30^\circ}$$

$$Z_T = 20 + 20\angle -60^\circ$$

$$Z_T = 20 + 20(\cos 60^\circ - j\sin 60^\circ)$$

$$Z_T = 20 + 10 - j17.32$$

$$Z_T = (30 - j17.32)\Omega$$

$$Z_T = 34.64\angle -30^\circ\,\Omega$$

$$\therefore I_1 = \frac{173\angle 0^\circ}{34.64\angle -30^\circ}$$

$I_1 = 5\angle 30^\circ$ A □

(ii) To find I_2 and I_3 using current – divider method

$$I_2 = \frac{-j23.1}{40 - j23.1} \times \frac{5\angle 30^\circ}{1}$$

$$I_2 = \frac{23.1\angle -90^\circ \times 5\angle 30^\circ}{46.2\angle -30^\circ} = \frac{115.5\angle -60^\circ + 30^\circ}{46.2}$$

$I_2 = 2.5\angle -30^\circ$ A □

(iii) Also, $I_3 = \frac{40}{40 - j23.1} \times \frac{5\angle 30^\circ}{1}$

$$I_3 = \frac{200\angle 30^\circ}{46.2\angle -30^\circ} = \frac{200\angle 30^\circ + 30^\circ}{46.2}$$

$I_3 = 4.33\angle 60^\circ$ A □

(b) To find the voltages across each circuit element.

(i) $V_1 = I_1 R_1$

where:

V_1 = voltage across the 20Ω resistor

I_1 = current in the 20Ω resistor

R_1 = Resistance of the 20Ω resistor.

$\therefore V_1 = 5\angle 30^\circ \times 20$

$V_1 = 100\angle 30^\circ$ V □

(ii) $V_2 = I_2 R_2$

Where:

V_2 = voltage across the 40Ω resistor

R_2 = Resistance of the 40Ω resistor

$V_2 = 2.5\angle -30^\circ \times 40$

$V_2 = 100\angle -30^\circ$ V □

(iii) $V_3 = I_3 X_C$

Where, V_3 = voltage across the capacitor

X_C = capacitive reactance

$V_3 = 4.33\angle 60^\circ \times 23.1\angle -90^\circ$

$V_3 = 100\angle -30^\circ$ V □

This is expected since the capacitor and the 40Ω res[istor] are connected in parallel.

(b) The phasor diagram is shown in figure 4.32.

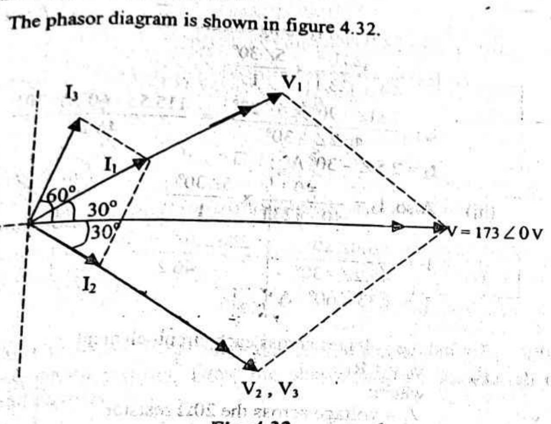


Fig. 4.32

4.5 Resonance In Series And Parallel R – L – C Circuits.

Question 1

A series R-L-C circuit is shown in Figure 4.33. Show that at resonance the resonant frequency is given by

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Where:

L = inductance of the coil

C = capacitance of the capacitor

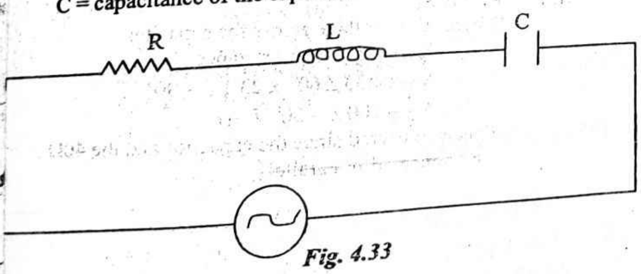


Fig. 4.33

Answer 1

Required to show that

$f_o = \frac{1}{2\pi\sqrt{LC}}$

$X_L = 2\pi fL$ = inductive reactance

$X_C = \frac{1}{2\pi fC}$ = capacitive reactance

$Z = \left(R^2 + (X_L - X_C)^2\right)^{\frac{1}{2}}$ = impedance of the circuit.

For resonance,

$V_L = V_C$ ---------- (1)

$X_L = X_C$ ---------- (2)

Thus Z = R

∴ From the impedance equation,

$Z^2 = Z^2 + (X_L - X_C)^2$

$0 = (X_L - X_C)^2$

$X_L = X_C$

$2\pi f_o L = \frac{1}{2\pi f_o C}$

$4\pi^2 f_o^2 LC = 1$

$f_o = \left(\frac{1}{4\pi^2 LC}\right)^{\frac{1}{2}}$

$f_o = \frac{1}{2\pi\sqrt{LC}}$ □ ---------------------- 4.5.1

Question 2

A coil of resistance 8 – Ω and inductance 2.4mH is connected in series with an 1 μ F capacitor. The circuit is connected to a 15V, variable – frequency supply. Calculate

(i) the frequency at which resonance occurs

(ii) the voltages across the coil and the capacitor at this frequency

(iii) the Q – Factor of the circuit.

Answer 2

(i) $f_o = \frac{1}{2\pi\sqrt{LC}}$

where f_o = resonant frequency

L = 2.4×10^{-3}H, C = 1×10^{-6}F

$$f_o = \frac{1}{2\pi(2.4 \times 10^{-3} \times 1 \times 10^{-6})^{\frac{1}{2}}}$$

$$f_o = \frac{1}{2\pi \times 4.90 \times 10^{-5}} = 3248HZ$$

$$f_o = 3.25KHZ \quad \square$$

(ii) Let V_{Lo} be the voltage across the coil at resonance

$$V_{Lo} = I_o X_L \text{ ---------- } 4.5.2$$

I_o = resonant current

$$X_L = 2\pi f_o L$$

$$= 2\pi \times 3.25 \times 10^3 \times 2.4 \times 10^{-3}$$

$$X_L = 49.0\Omega$$

$$I_o = \frac{V}{Z} = \frac{V}{R}$$

$$I_o = \frac{15}{8} = 1.875 \text{ A}$$

$$\therefore V_{Lo} = 49 \times 1.875$$

$$V_{Lo} = 91.875V \quad \square$$

Let V_{Co} be the voltage across the capacitor at resonance.

$$V_{Co} = I_o X_C$$

$$X_C = \frac{1}{2\pi f_o C} = \frac{1}{2\pi \times 3.25 \times 10^3 \times 1 \times 10^{-6}}$$

$$X_C = 49.0\Omega$$

$$V_{Co} = 1.875 \times 49$$

$$V_{Co} = 91.875V \quad \square$$

(iii) Q – factor,

$$Q - factor = \frac{1}{R}\sqrt{\frac{L}{C}} \text{ ---------------------- } 4.5.3$$

$$Q - factor = \frac{1}{8}\left(\frac{2.4 \times 10^{-3}}{1 \times 10^{-6}}\right)^{\frac{1}{2}}$$

$$Q\text{-factor} = \frac{49}{8}$$

$$Q\text{-factor} = 6.125 \quad \square$$

Question 3

A coil of resistance 12Ω and inductance 250μH is in parallel with a variable capacitor. This combination is in series with a resistor of 16.2KΩ. The voltage of the supply is 180V at a frequency of 650KHZ.

Calculate:

(a) the value of C to give resonance

(b) the Dynamic impedance of the circuit

(c) the current in each branch of the circuit at resonance.

Answer 3

Figure 4.34 shows the circuit diagram.

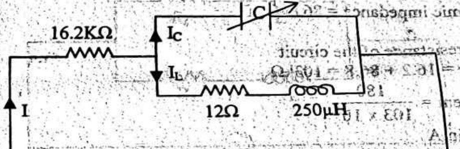


Fig. 4.34

$$X_L = 2\pi f L$$

$$X_L = 2\pi \times 650 \times 10^3 \times 250 \times 10^{-6}$$

$$X_L = 1021\Omega$$

This implies that the resistance of the coil is very small compared to reactance.

∴ Neglecting the resistance, the resonant frequency becomes

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

(a) To calculate the value of C

$$650 \times 10^3 = \frac{1}{2\pi(250 \times 10^{-6} \times C)^{\frac{1}{2}}}$$

$$(650 \times 10^3)^2 \times (2\pi)^2 \times (250 \times 10^{-6} C) = 1$$

$$C = \frac{1}{4\pi^2(650 \times 10^3)^2 (250 \times 10^{-6})}$$

$$C = 2.40 \times 10^{-10} F$$

$$C = 240PF \quad \square$$

(b) $Q - \text{factor} = \dfrac{2\pi fL}{R}$ ---------------------- 4.5.4

$Q - \text{factor} = \dfrac{2\pi \times 650 \times 10^3 \times 250 \times 10^{-6}}{12}$

$Q - \text{factor} = 85.1$ □

(c) Dynamic impedance $= \dfrac{L}{CR}$ -- 4.5.5

$= \dfrac{250 \times 10^{-6}}{240 \times 10^{-12} \times 12}$

Dynamic impedance = 86805.56Ω

Dynamic impedance = 86.8kΩ □

(d) Total resistance of the circuit

$= 16.2 + 86.8 = 103k\Omega$

$\therefore$ current $= \dfrac{180}{103 \times 10^3}$

$I = 1.75mA$

p.d across the parallel branch = current x dynamic impedance

$= 1.75 \times 10^{-3} \times 86.8 \times 10^3$

$= 151.90V$

$\therefore$ current through inductive branch, I_L

$I_L = \dfrac{151.90}{\sqrt{12^2 + 1021^2}} = \dfrac{151.90}{1021}$

$I_L = 0.1488$ A

$I_L = 148.8mA$ □

Current through C, I_C

$I_C = \dfrac{V_P}{X_C}$

V_P = voltage across the parallel branch

$X_C = \dfrac{1}{2\pi fC}$

$I_C = 2\pi fC\, V_P$

$I_C = 2\pi \times 650 \times 10^3 \times 240 \times 10^{-12} \times 151.90$

$I_C = 0.1489A$

$I_C = 148.9mA$ □

Question 4

Figure 4.35 shows a resistor, R_L and an inductor, L connected in parallel with a capacitor C and a resistor, R_C. Show that at resonance, the resonant frequency is given by

$$f_o = \frac{1}{2\pi\sqrt{LC}}\left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)^{\frac{1}{2}}$$

(ii) Find the expression for f_o when $R_L = R_C = 0\Omega$

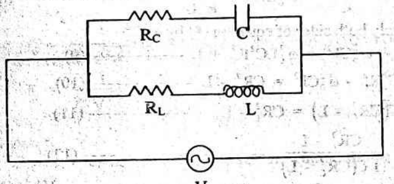


Fig. 4.35

Answer 4

Let Y_L be the admittance of inductive branch

Let Y_C be the admittance of capacitive branch

Let Y be the total admittance of the circuit.

$Y = Y_L + Y_C$ ---------------------------------- (1)

$Y = \dfrac{1}{R_L + jX_L} + \dfrac{1}{R_C - jX_C}$ ---------------------------------- (2)

$Y = \dfrac{R_L - jX_L}{R_L^2 + X_L^2} + \dfrac{R_C + jX_C}{R_C^2 + X_C^2}$ ---------------------------------- (3)

$Y = \left(\dfrac{R_L}{R_L^2 + X_L^2} + \dfrac{R_C}{R_C^2 + X_C^2}\right) + j\left(\dfrac{X_C}{R_C^2 + X_C^2} - \dfrac{X_L}{R_L^2 + X_L^2}\right)$ -(4)

The circuit is at resonance when the complex admittance is a real number or when the susceptance of the circuit is equal to zero.

$\therefore \left(\dfrac{X_C}{R_C^2 + X_C^2} - \dfrac{X_L}{R_L^2 + X_L^2}\right) = 0$ ---- (5)

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2} \text{ --------- } (6)$$

But $X_C = \frac{1}{\omega_o C}$, $X_L = \omega_o L$

Where: ωo = angular velocity at resonance

$$\therefore \frac{1}{\omega_o C}\left(R_L^2 + \omega_0^2 L^2\right) = \omega_o L\left(R_C^2 + \frac{1}{\omega_o^2 C^2}\right) \text{ --------- } (7)$$

$$\frac{R_L^2}{\omega_o C} + \frac{\omega_o L^2}{C} = \omega_o L R_C^2 + \frac{L}{\omega_o C^2} \text{ --------- } (8)$$

Multiply both sides of equation (8) by $\omega_o C^2$

$$CR_L^2 + \omega_0^2 CL^2 = \omega_0^2 LC^2 R_C^2 + L \text{ --------- } (9)$$

$$\omega_0^2 LC^2 R_C^2 - \omega_0^2 CL^2 = CR_L^2 - L \text{ --------- } (10)$$

$$\omega_0^2 LC\left(CR_C^2 - L\right) = CR_L^2 - L \text{ --------- } (11)$$

$$\omega_0^2 = \frac{CR_L^2 - L}{LC\left(CR_C^2 - L\right)} \text{ --------- } (12)$$

$$\omega_0^2 = \frac{R_L^2 - L/C}{LC\left(R_C^2 - L/C\right)} \text{ --------- } (13)$$

But $\omega_0 = 2\pi f_o$ --------- (14)

Putting equation (14) into equation (13) we have,

$$(2\pi f_o)^2 = \frac{R_L^2 - \frac{L}{C}}{LC(R_C^2 - L/C)} \text{ --------- } (15)$$

$$f_o^2 = \frac{\left(R_L^2 - \frac{L}{C}\right)}{4\pi^2 LC\left(R_C^2 - \frac{L}{C}\right)} \text{ --------- } (16)$$

$$f_o = \left[\frac{\left(R_L^2 - \frac{L}{C}\right)}{4\pi^2 LC\left(R_C^2 - \frac{L}{C}\right)}\right]^{\frac{1}{2}} \text{ --------- } (17)$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}\left(\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}\right)^{\frac{1}{2}} \text{ --------- } 4.5.6$$

(ii) when $R_L = R_C = 0$

Equation (4.5.6) becomes,

$$f_o = \frac{1}{2\pi\sqrt{LC}}\left(\frac{0 - \frac{L}{C}}{0 - \frac{L}{C}}\right)^{\frac{1}{2}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}(1)^{\frac{1}{2}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Question 5

Find C which results in resonance for the circuit shown in fig. 5 if 1591.55 HZ

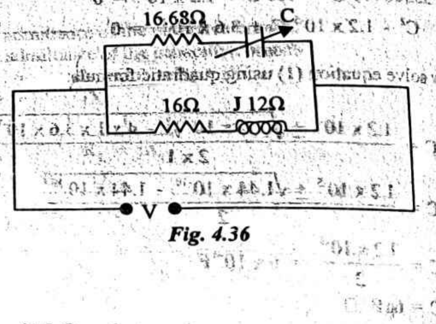


Fig. 4.36

Answer 5

To find C.

From equation (4.5.6) we have,

$$f_o = \frac{1}{2\pi\sqrt{LC}}\left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)^{\frac{1}{2}}$$

$$f_o^2 \times 4\pi^2 LC = \left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)$$

$$1591.55^2 \times 4 \times \pi^2 LC = \left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)$$

$$X_L = 2\pi fL = 12$$

$$L = \frac{12}{2\pi \times 1591.55} = 1.2 \times 10^{-3} H$$

$$1591.55^2 \times 4\pi^2 \times 1.2 \times 10^{-3} C = 16^2 - \frac{\frac{1.2 \times 10^{-3}}{C}}{16.68^2 - \frac{1.2 \times 10^{-3}}{C}}$$

$$120000\,C = 256 - \frac{\frac{1.2 \times 10^{-3}}{C}}{278.22 - \frac{1.2 \times 10^{-3}}{C}}$$

$$33386400C - 144 = 256 - \frac{1.2 \times 10^{-3}}{C}$$

$$33386400\,C^2 - 400\,C + 1.2 \times 10^{-3} = 0$$

$$C^2 - 1.2 \times 10^{-5} C + 3.6 \times 10^{-11} = 0 \quad \text{----------} (1)$$

We now solve equation (1) using quadratic formula.

$$C = \frac{1.2 \times 10^{-5} \pm \sqrt{(1.2 \times 10^{-5})^2 - 4 \times 1 \times 3.6 \times 10^{-11}}}{2 \times 1}$$

$$C = \frac{1.2 \times 10^{-5} \pm \sqrt{1.44 \times 10^{-10} - 1.44 \times 10^{-10}}}{2}$$

$$C = \frac{1.2 \times 10^{-5}}{2} = 6 \times 10^{-6} F$$

$C = 6\mu F$ □

Question 6

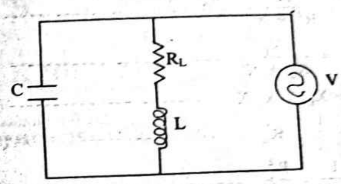


Fig. 4.37

Figure 4.37 shows a capacitor, C connected in parallel with a coil of resistance, R_L and inductance L. Show that the resonant frequency is given by

$$f_o = \frac{1}{2\pi}\left(\frac{1}{LC} - \frac{R_L^2}{L^2}\right)^{\frac{1}{2}}$$

Answer 6

Let Y_L be the admittance of the inductive branch.
Let Y_C be the admittance of the capacitive branch

$$Z_L = R_L + j\omega L = R_L + jX_C$$

$$Z_C = -jX_C$$

$$Y_L = \frac{1}{R_L + jX_C}$$

$$Y_L = \frac{R_L - jX_C}{R_L^2 + X_C^2} \quad \text{----------------} (1)$$

$$Y_C = \frac{1}{-jX_C} = \frac{jX_C}{X_C^2} = \frac{j}{X_C} \quad \text{--------} (2)$$

$$Y = Y_C + Y_L$$

$$Y = \frac{j}{X_C} + \frac{R_L - jX_C}{R_L^2 + X_C^2} \quad \text{----------------} (3)$$

$$Y = \frac{R_L}{R_L^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R_L^2 + X_L^2}\right) \quad \text{-------} (4)$$

The circuit is at resonance when the complex part (susceptance) of equation (4) is equal to zero

$$\therefore \frac{1}{X_C} - \frac{X_L}{R_L^2 + X_L^2} = 0 \quad \text{------------ (5)}$$

$$\frac{1}{X_C} = \frac{X_L}{R_L^2 + X_L^2} \quad \text{------------ (6)}$$

$$R_L^2 + X_L^2 = X_C X_L \quad \text{------------ (7)}$$

$$\omega_0^2 L^2 = \frac{L}{C} - R_L^2 \quad \text{------------ (8)}$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R_L^2}{L^2} \quad \text{------------ (9)}$$

$$\omega_o = 2\pi f_o \quad \text{------------ (10)}$$

$$4\pi^2 f_0^2 = \frac{1}{LC} - \frac{R_L^2}{L^2} \quad \text{------------ (11)}$$

$$f_0^2 = \frac{1}{4\pi^2}\left[\frac{1}{LC} - \frac{R_L^2}{L^2}\right] \quad \text{------------ (12)}$$

$$f_0 = \frac{1}{2\pi}\left[\frac{1}{LC} - \frac{R_L^2}{L^2}\right]^{\frac{1}{2}} \quad \text{------------ (13)}$$

Question 7

Use figure 4.38 to calculate:

a. **the value of C for resonance to occur in the circuit**
b. **the effective impedance of the network**
c. **the supply current**

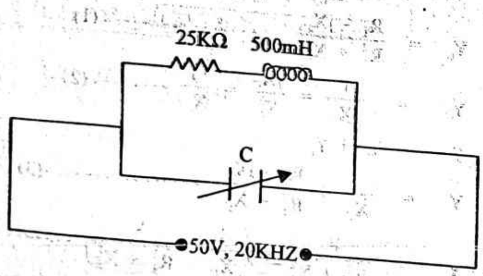


Fig. 4.38

Answer 7

a. To find C at resonance. At resonance,

$$f_0 = \frac{1}{2\pi}\left(\frac{1}{LC} - \frac{R_L^2}{L^2}\right)^{\frac{1}{2}}$$

Given:

L = 500mH = 0.5H

R_L = 25kΩ = 25 x 10^3Ω, f_0 = 20KHZ

$$f_0 = \frac{1}{2\pi}\left(\frac{1}{0.5C} - \left(\frac{25 \times 10^3}{0.5}\right)^2\right)^{\frac{1}{2}}$$

$$f_0 = \frac{1}{2\pi}\left(\frac{2}{C} - 2.5 \times 10^9\right)^{\frac{1}{2}}$$

$$\left(20 \times 10^3\right)^2 \times 4\pi^2 = \frac{2}{C} - 2.5 \times 10^9$$

$$1.579 \times 10^{10} = \frac{2}{C} - 2.5 \times 10^9$$

$$1.579 \times 10^{10}C + 2.5 \times 10^9 C = 2$$

$$1.829 \times 10^{10}C = 2$$

$$C = \frac{2}{1.829 \times 10^{10}} = 1.09 \times 10^{-10}F$$

C = 109PF

(b) Effective impedance, Z

$$Z = \frac{L}{CR}$$

$$Z = \frac{0.5}{109 \times 10^{-12} \times 25 \times 10^3} = 183486.24\Omega$$

Z = 183.5KΩ

(c) Supply current, I

$$I = \frac{V}{Z}$$

$$I = \frac{50}{183.5 \times 10^3} = 2.725 \times 10^{-4}A$$

$$I = 272.5\,\mu A$$

Question 8

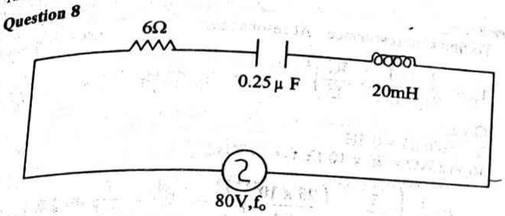


Fig. 4.39

Use figure 4.39 to find:

a. the resonant frequency, f_0
b. Q-factor at resonance
c. The bandwith
d. The current in the circuit at resonance
e. The voltage across each element in the circuit at resonance.

Answer 8

(a) At resonance,

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi(20 \times 10^{-3} \times 0.25 \times 10^{-6})^{\frac{1}{2}}}$$

$$f_o = 2250.79\text{HZ}$$

$$f_o = 2.25\text{KHZ} \ \square$$

(b) Q – factor at resonance, Q_o

$$Q_o = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q_o = \frac{1}{6}\left(\frac{20 \times 10^{-3}}{0.25 \times 10^{-6}}\right)^{\frac{1}{2}} = \frac{282.84}{6}$$

$$Q_o = 47.14$$

(c) Bandwidth, B.W

$$\text{B.W} = \frac{f_o}{Q_o} \quad \text{---------- 4.5.7}$$

$$\text{B.W} = \frac{2.25 \times 10^3}{47.14}$$

$$\text{B.W} = 47.73 \text{ HZ} \quad \square$$

(d)

$$I_o = \frac{V}{R}$$

$$I_o = \frac{80}{6}$$

$$I_o = 13.3\text{A} \ \square$$

(e) Voltage across each circuit element

(i)

$$V_R = I_oR$$

$$V_R = 13.3 \times 6$$

$$V_R = 79.8\text{V} \quad \square$$

(ii)

$$V_C = I_o X_C$$

$$X_C = \frac{1}{2\pi f_o C} = \frac{1}{2\pi \times 2.25 \times 10^3 \times 0.25 \times 10^{-6}}$$

$$X_C = 282.94\Omega$$

$$V_C = 13.3 \times 282.94 = 3763\text{V}$$

$$V_C = 3.76\text{KV} \quad \square$$

(iii)

$$V_L = I_oX_L$$

$$X_L = 2\pi f_oL$$

$$X_L = 2\pi \times 2.25 \times 10^3 \times 20 \times 10^{-3}$$

$$X_L = 282.74\Omega$$

$$V_L = 13.3 \times 282.74$$

$$V_L = 3.76\text{KV} \quad \square$$

4.6 Summary of Equations Used

1. Alternating Quantities

$$I_{av} = \frac{i_1 + i_2 + \dots + i_n}{n} \quad \text{---------- 4.1}$$

$$V_{av} = \frac{V_1 + V_2 + \dots + V_n}{n} \quad \text{---------- 4.2}$$

$I_{r.m.s} = \sqrt{\frac{(i_1^2 + i_2^2 + ... + i_n^2)}{n}}$ ------------- 4.3

$V_{r.m.s} = \sqrt{\frac{V_1^2 + V_2^2 + ... + V_n^2}{n}}$ ------------- 4.4

- $I_{av} = 0.637I_m$ (sinusoidal wave) ----4.5
- $V_{av} = 0.637V_m$ (sinusoidal wave) ----4.6
- $I_{r.m.s} = 0.707\, I_m$ (sinusoidal wave) ----4.7
- $V_{r.m.s.} = 0.707V_m$ (sinusoidal wave) --4.8
- Form factor = $\frac{\text{r.m.s. value}}{\text{Average value}}$ ------4.9
- Peak factor = $\frac{\text{Maximum value}}{\text{r.m.s. value}}$ ------4.10

$I_m = \sqrt{2}\, I_{r.m.s.}$ ----------------4.11

$V_m = \sqrt{2}\, V_{r.m.s}$ ----------------4.12

$\omega = 2\pi f$ ----------------4.13

$T = \frac{1}{f}$ ----------------4.14

2. R - L CIRCUIT

$X_L = 2\pi fL$ ----------------4.15

$Z = (R^2 + X_L^2)^{\frac{1}{2}}$ ----------------4.16

$I = \frac{V}{Z}$ ----------------4.17

$\emptyset = \tan^{-1}\left[\frac{X_L}{R}\right]$ ----------------4.18

- P.f = $\cos\emptyset$ ----------------4.19

$S = IV$ or $I^*V\angle\emptyset$ ----------------4.20

$P = IV\cos\emptyset$ ---------------- 4.21

$Q = IV\sin\emptyset$ ---------------- 4.22

3. R - C CIRCUIT

$X_C = \frac{1}{2\pi fC}$ ----------------4.23

$Z = \left(R^2 + X_C^2\right)^{\frac{1}{2}}$ ----------------4.24

$\emptyset = -\tan^{-1}\left[\frac{X_C}{R}\right]$ ----------------4.25

$V_C = \sqrt{V^2 - V_R^2}$ ----------------4.26

$V_C = IX_C$ ----------------4.27

4. R - L - C CIRCUIT

$Z = \left[R^2 + (X_L - X_C)^2\right]^{\frac{1}{2}}$ ----------------4.28

$\emptyset = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$ ----------------4.29

$Y = \frac{1}{Z}$ ----------------4.30

$P.f = \frac{P}{S}$ ----------------4.31

5. **RESONANCE**

$f_0 = \frac{1}{2\pi\sqrt{LC}}$ (series R - L - C circuit) ----------4.32

Q factor = $\frac{1}{R}\sqrt{\frac{L}{C}}$ ----------------4.33

Dynamic Impedance = $\frac{L}{CR}$ ----------------4.34

$f_0 = \frac{1}{2\pi\sqrt{LC}}\left(\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}\right)^{\frac{1}{2}}$ (parallel circuit) ---- 4.35

$B.W = \frac{f_0}{Q_0}$ ----------------4.36

4.7 Tutorial Problems Four

1. The equation relating the current in a circuit with time is $i = 141.4 \sin 377t$ where the current is measured in amperes and the time is measured in seconds. Find the values of:
   a. the r.m.s. Current
   b. the frequency
   c. the instantaneous value of the current when t is 3ms.

2. Determine the value of the supply voltage and the power factor in the circuit shown in figure 2

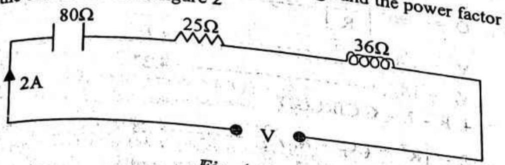


Fig. 4.40

3. An alternating voltage wave is represented by $V = 353.5 \sin\omega t$. What are the maximum and r.m.s. values of the voltage?

4. Find the resultant of the following four voltages:
$e_1 = 50 \sin 377t$ V
$e_2 = 25 \sin (377t + 60^{0})$V
$e_3 = 40 \cos 377t$ V
$e_4 = 30 \sin (377t - 45^{0})$ V

5. Three e.m.f.s, $e_A = 50\sin\omega t$, $e_B = 50 \sin (\omega t - \frac{\pi}{6})$ and $e_C = 60 \cos \omega t$ volts, are induced in three coils connected in series so as to give the phasor sum of the three e.m.f.s. Calculate the maximum value of the resultant e.m.f. and its phase relative to e.m.f. e_A.

If the connections to coil B were reversed what will be the maximum value of the resultant e.m.f. and its phase relative to e_A?

6. A triangular voltage wave has the following value over one-half cycle, both half-cycles being symmetrical about the zero axis:

Time (ms)	0	10	20	30	40	50	60	70	80	90	100
Voltage (V)	0	2	4	6	8	10	8	6	4	2	0

Plot half cycle of the waveform and hence determine:
a. the average value;
b. the r.m.s. value
c. the form factor

7. A voltage has the following steady values in volts for equal intervals of time, changing instantaneously from one value to the next: 0, 5, 10, 20, 50, 60, 50, 20, 10, 5, 0, -5, -10, etc.

Calculate:
i. The r.m.s. value of the voltage
ii. Its form factor
iii. The peak factor of the voltage

8. A steel – cored coil connected to a 100V, 50HZ supply is found to take current of 5A and to dissipate a power of 200W. Find:
a. the impedance
b. the effective resistance
c. the inductance
d. the circuit power factor

9. A 200Ω resistor and a 0.8H inductor are connected in series to a supply of 240V, 50HZ alternating current. Calculate:
a. the circuit current
b. the power in the resistor

10. Find the average and effective values for the wave form in figure 4.41

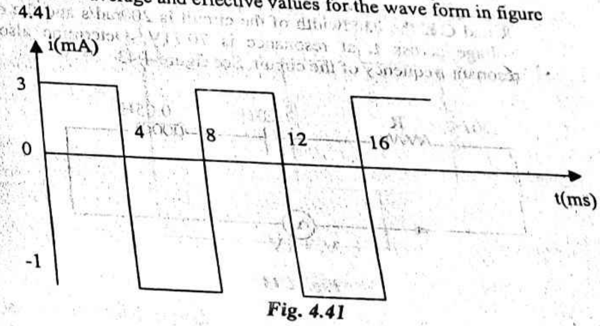


Fig. 4.41

11. In the circuit shown in figure 4.42, calculate
a. the peak value of the applied voltage
b. the peak voltage across the L – C combination
c. the phase difference between the alternating electric current and the alternating voltage
d. real and reactive power.

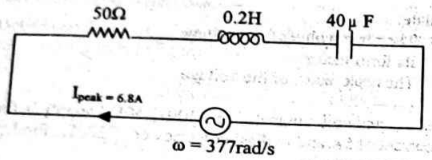


Fig. 4.42

12. A series circuit consisting of R = 5Ω, L = 0.02H and C = 80 μ F has a variable frequency sinusoidal voltage applied. Find the values of ω for which the current
    a. will lead the voltage by 45°
    b. be in phase
    c. lag by 45°

13. If the power consumed at resonance is 0.1W, calculate the value of R and C if the bandwidth of the circuit is 200rad/s and the r.m.s. voltage across L at resonance is 70.71V. Determine also the resonant frequency of the circuit. See figure 4.43

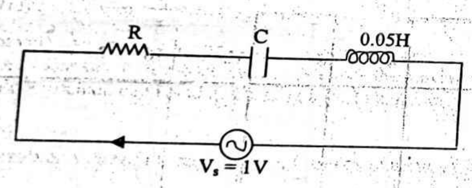


Fig. 4.43

14. In the three branch parallel circuit, determine the impedance Z_1 in fig. 4.44

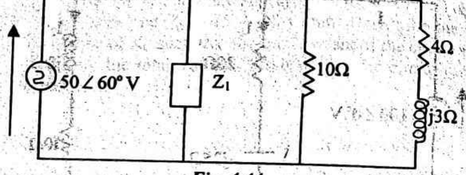


Fig. 4.44

15. Use the circuit in figure 4.45 to calculate:

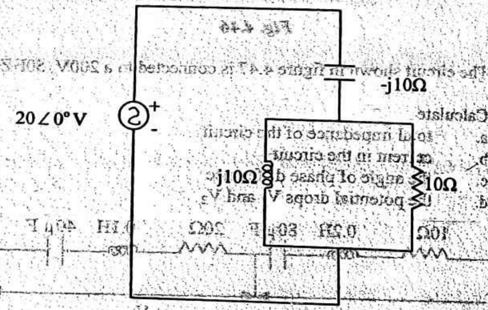


Fig. 4.45

a. the total impedance
b. the power dissipated in the 10Ω resistor.

16. In a three-branch parallel circuit, $I_1 = 10\angle 45°$A, $I_2 = 10\angle -60°$A, and the total current drawn from the supply is $25\angle -10°$A. Calculate the value of I_3.

17. Calculate a polar expression for the impedance of each branch of the circuit, the impedance and admittance of the complete circuit, the current in each branch, and the total current. See figure 4.46

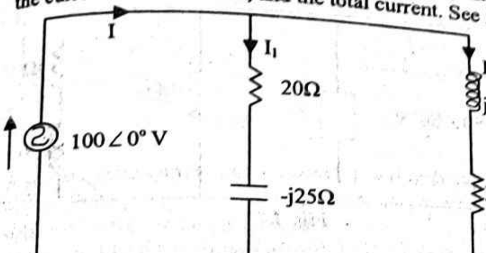
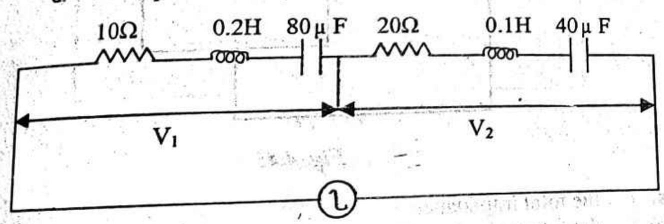


Fig. 4.46

18. The circuit shown in figure 4.47 is connected to a 200V, 50HZ supply.

Calculate

a. total impedance of the circuit
b. current in the circuit
c. the angle of phase difference
d. the potential drops V_1 and V_2



200V, 50HZ

Fig. 4.47

19. An inductive circuit, in parallel with a non-inductive circuit of 20Ω is connected across a 50HZ supply. The inductive current is 4.3A and the non-inductive current is 2.7A. The total current is 5.8A

Find

a. the power absorbed by the inductive resistance
b. its inductance

c. the P.F of the combined circuit.

20. Impedances Z_2 and Z_3 in parallel are in series with an impedance Z_1 across a 100V, 50HZ a.c supply. $Z_1 = (6.25 + j1.25)$ ohm, $Z_2 = (5 + j0)$ ohm and $Z_3 = (5 - jX_C)$ ohm. Determine the value of capacitance of X_C such that the total current of the circuit will be in phase with the total voltage. What is then the circuit current and power?

Chapter Five

5.0 TRANSFORMERS AND ELECTRICAL MACHINES

5.1 Transformers

Question 1

(a) **The volts per turn of a certain single phase transformer is 2.5. The transformer has a step-down ratio of 5000V to 350V. Calculate:**

(i) **the respective number of turns in each winding**
(ii) **the secondary current if the primary current is 20 A.**

(b) **The input to a power transformer with 250 turns in the primary is to be 100V at 50HZ. The transformer has two secondary coils, one secondary provides 150mA at 600V and another provides 5A at 12V. Estimate**

(i) **the number of turns required in the secondaries**
(ii) **the full-load primary current.**

Answer 1

(i) To calculate the number of turns.

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} = \text{Volts per turn} \quad \text{------------ 5.1.1}$$

Where:

V_1 = Primary voltage
V_2 = Secondary voltage
N_1 = number of turns in the primary
N_2 = number of turns in the secondary

$$\therefore \frac{V_1}{N_1} = 2.5$$

$$\frac{5000}{N_1} = 2.5$$

$$N_1 = \frac{5000}{2.5}$$

N_1 = 2000 Turns □

Also,

$$\frac{V_2}{N_2} = 2.5$$

$$\frac{350}{N_2} = 2.5$$

$$N_2 = \frac{350}{2.5}$$

N_2 = 140 Turns □

(ii) Secondary current, I_2

From the transformer equation.

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} \quad \text{------------ 5.1.2}$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1}$$

$$\frac{2000}{140} = \frac{I_2}{20}$$

$$I_2 = \frac{2000 \times 20}{140}$$

I_2 = 285.7A □

(b) Fig. 5.1 below shows the transformer connection

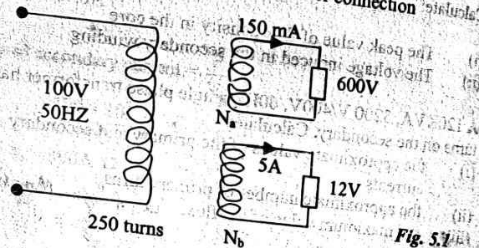


Fig. 5.1

(i) $\frac{N_a}{N_1} = \frac{V_a}{V_1}$ From transformer equation

$$\frac{N_a}{250} = \frac{600}{100}$$

$$N_a = \frac{600 \times 250}{100} = 1500 \text{ turns}$$

Also, $\frac{N_b}{250} = \frac{12}{100}$

$N_b = \frac{12 \times 250}{100} = 30$ turns

∴ Total number of turns in the secondaries
= 1500 + 30
= 1530.0 turns □

(ii) For a transformer (ideal), the secondary ampere turns must be equal to the primary ampere turns.

$\therefore N_1I_1 = I_aN_a + I_bN_b$

$250I_1 = 150 \times 10^{-3} \times 1500 + 5 \times 30.0$

$250I_1 = 225 + 150$

$I_1 = \frac{375.0}{250}$

$I_1 = 1.50A$ □

Question 2

(a) A single – phase transformer has 500 primary and 1200 secondary turns. The net cross-sectional area of the core is 80cm². If the primary winding be connected to a 60HZ supply at 750V. Calculate:

(i) The peak value of flux density in the core

(ii) The voltage induced in the secondary winding

(b) A 120KVA, 3200 V/400V, 60HZ single phase transformer has 60 turns on the secondary. Calculate:

(i) the approximate values of the primary and secondary currents

(ii) the approximate number of primary turns

(iii) the maximum value of the flux. i.e flux Maximum

Answer 2

(a) (i) From the e.m.f equation of a transformer,

$E_1 = 4.44 fN_1 B_mA$ ---------------------- 5.1.3

Where:

E_1 = e.m.f induced in the primary coil

f = Supply frequency

N_1 = Number of turns in the primary

B_m = maximum flux density

A = Cross-sectional area.

∴ $750 = 4.44 \times 60 \times 500 \times B_m \times 80 \times 10^{-4}$

$750 = 1065.6 B_m$

$B_m = \frac{750}{1065.6}$

$B_m = 0.704$ T □

(ii) From the transformer equation,

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$

$\frac{500}{1200} = \frac{750}{V_2}$

$V_2 = \frac{1200 \times 750}{500}$

$V_2 = 1800V$

$V_2 = 1.8kV$ □

(b) Power = V_1I_1

$120 \times 10^3 = 3200 I_1$

$I_1 = \frac{120 \times 10^3}{3200}$

$I_1 = 37.5A$ □

Full-load secondary current = $\frac{P}{V_2}$

$I_2 = \frac{120 \times 10^3}{400}$

$I_2 = 300A$ □

(ii) $\frac{N_1}{N_2} = \frac{V_1}{V_2}$

$\frac{N_1}{60} = \frac{3200}{400}$

$N_1 = \frac{3200 \times 60}{400}$

$N_1 = 480$ turns □

(iii) $E_2 = 4.44 fN_2 Ø_m$ ——————— 5.1.4

Where:

E_2 = E.m.f. induced in the secondary winding

f = supply frequency

N_2 = Number of turns in the secondary

$Ø_m$ = maximum flux

$400 = 4.44 \times 60 \times 60 \times Ø_m$

$Ø_m = 0.025$ Wb

$Ø_m = 25$mWb □

Question 3

A 4000/240V, 60HZ, one phase transformer has a core of effective cross sectional area 150cm^2 and a low-voltage winding of 120 turns. Determine:

- (i) the number of turns of the high – voltage winding
- (ii) the maximum flux density in the core
- (iii) if the transformer supplied a load of 40KW at 0.75 power factor lagging when connected to a 4KV supply, calculate the approximate values of secondary and primary currents.

Answer 3

(i) $\frac{N_1}{N_2} = \frac{V_1}{V_2}$

$\frac{N_1}{120} = \frac{4000}{240}$

$N_1 = \frac{4000 \times 120}{240}$

$N_1 = 2000$ Turns □

(ii) the maximum flux density, B_m

$Ø_m = \frac{E_1}{4.44fN_1}$

$Ø_m = \frac{4000}{4.44 \times 60 \times 2000}$

$Ø_m = 7.51 \times 10^{-3}$ Wb

But $Ø_m = B_m A$

$B_m = \frac{Ø_m}{A}$

$B_m = \frac{7.51 \times 10^{-3}}{150 \times 10^{-4}}$

$B_m = 0.5$T □

(iii) $P = IV \cos Ø$

$\cos Ø$ = power factor = 0.75

$40 \times 10^3 = I_1 \times 4 \times 10^3 \times 0.75$

$I_1 = \frac{40 \times 10^3}{4 \times 10^3 \times 0.75}$

$I_1 = 13.3$A □

Also, $40 \times 10^3 = I_2 \times 240 \times 0.75$

$I_2 = \frac{40 \times 10^3}{240 \times 0.75}$

$I_2 = 222.2$A □

Question 4

A 1ϕ– Ph, 440/4400 V transformer has a resistance of 3Ω connected in series with its primary winding and a 1KΩ resistor connected across the secondary winding. Calculate the primary current when the circuit is supplied at 440V.

Answer 4

The transformer connection is shown in fig. 5.2

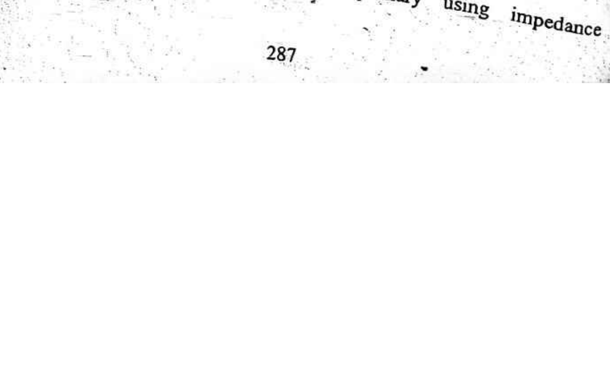


Fig. 5.2

Referring the resistance, R_2 to the primary using impedance transformation, we have

- $R_2^1 = \left(\frac{N_1}{N_2}\right)^2 R_2$ ---------------------- 5.1.5

- or $R_2^1 = \left(\frac{V_1}{V_2}\right)^2 R_2$ ---------------------- 5.1.6

- R_2^1 = equivalent secondary resistance referred to the primary.

$\therefore\ R_2^1 = \left(\frac{440}{4400}\right)^2 \times 1 \times 10^3$

$R_2^1 = 10\Omega$

Figure 5.2 now becomes

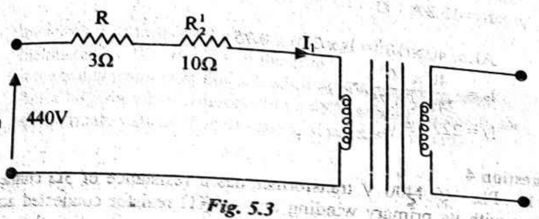


Fig. 5.3

$\therefore\ R_1 = R + R_2^1 = 3 + 10$

$R_1 = 13\Omega$

The primary current, I_1

$I_1 = \frac{V_1}{R_1}$

$I_1 = \frac{440}{13}$

$I_1 = 33.85$ A □

Question 5

The primary and secondary windings of a 120KVA, 4800/480V transformer with reactances taken out of the windings are shown in figure 5.4

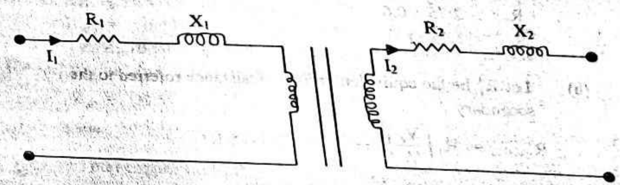


Fig. 5.4

Circuit Parameters:

$R_1 = 2.75\Omega$ $R_2 = 6m\Omega$

$X_1 = 4.5\Omega$ $X_2 = 10m\Omega$

Calculate for the transformer:

(a) equivalent resistance as referred to the primary

(b) equivalent resistance as referred to secondary

(c) equivalent reactance as referred to both primary and secondary.

(d) equivalent impedance as referred to both primary and secondary

(e) total copper loss.

Answer 5

Full – load primary current, $= I_1 = \frac{120 \times 10^3}{4800}$

$I_1 = 25$A

Full – load secondary current, $I_2 = \frac{120 \times 10^3}{480}$

$I_2 = 250$ A

(a) Let R_2^1 be the equivalent secondary resistance referred to the primary

$$R_2^1 = R_2\left(\frac{N_1}{N_2}\right)^2 = R_2\left(\frac{V_1}{V_2}\right)^2$$

$$R_2^1 = 6 \times 10^{-3}\left(\frac{4800}{480}\right)^2 = 0.6\Omega$$

Let R_{01} be the equivalent resistance as referred to primary

$R_{01} = R_1 + R_2^1$

$R_{01} = 2.75 + 0.6$

$R_{01} = 3.35\ \Omega$ □

(b) Let R_1^1 be the equivalent primary resistance referred to the secondary.

$$R_1^1 = R_1\left(\frac{V_2}{V_1}\right)^2 \text{ -------------- 5.1.7}$$

$$R_1^1 = 2.75\left(\frac{480}{4800}\right)^2 = 0.0275\Omega = 27.5\,m\Omega$$

Let R_{02} be the equivalent resistance as referred to secondary

$R_{02} = R_2 + R_1^1$

$R_{02} = 6m\ \Omega + 27.5m\Omega$

$R_{02} = 33.5m\Omega$ □

(c) $X_{01} = X_1 + X_2^1$

$$X_2^1 = X_2\left(\frac{V_1}{V_2}\right)^2 \text{ ---------- 5.1.8}$$

$$X_2^1 = 10\times10^{-3}\left(\frac{4800}{480}\right)^2 = 1\Omega$$

$\therefore X_{01} = 4.5 + 1.0$

$X_{01} = 5.5\Omega$

$X_{02} = X_2 + X_1^1$

$$X_1^1 = X_1\left(\frac{V_2}{V_1}\right)^2 \text{ ----------- 5.1.9}$$

$$X_1^1 = 4.5\left(\frac{480}{4800}\right)^2 = 45\,m\ \Omega$$

$X_{02} = 10 + 45$

$X_{02} = 55\,m\Omega$

(d) Let Z_{01} be the equivalent impedance as referred to primary

$$Z_{01} = \left(R_{01}^2 + X_{01}^2\right)^{1/2} \text{ -------------- 5.1.10}$$

$Z_{01} = \left(3.35^2 + 5.5^2\right)^{1/2}$

$Z_{01} = 6.44\ \Omega$ □

Let Z_{02} be the equivalent impedance as referred to secondary

$$Z_{02} = \left(R_{02}^2 + X_{02}^2\right)^{1/2} \text{ -------------- 5.1.11}$$

$Z_{02} = \left(33.5^2 + 55^2\right)^{1/2}$

$Z_{02} = 64.4\,m\Omega$ □

(e) Total copper Loss, P_C

$$P_C = I_1^2 R_1 + I_2^2 R_2 \text{ ------------- 5.1.12}$$

$P_C = 25^2 \times 2.75 + 250^2 \times 6\times10^{-3}$

$= 1718.75 + 375$

$P_C = 2093.75\ W$

$P_C = 2.09\ KW$ □

Alternatively,

$$P_C = I_1^2 R_{01} = I_2^2 R_{02} \text{ ------------- 5.1.13}$$

$P_C = 25^2 \times 3.35$

$P_C = 2.09\ KW$ □

Question 6

A 240 KVA, 2400 / 480 V real transformer has the below parameters:

$R_1 = 10.25\,\Omega,\ X_1 = 18\,\Omega,\ R_C = 80\,\Omega$

$X_m = 400\Omega, R_2 = 0.6\Omega, X_2 = 1.8\Omega$

(a) Draw the equivalent circuit of the real transformer showing all the parameters

(b) Refer all the resistances and reactances to the secondary.

(c) Refer the primary current and the primary terminal voltage to the secondary.

(d) Draw the equivalent circuit of the transformer referred to the secondary.

Answer 6

(a)

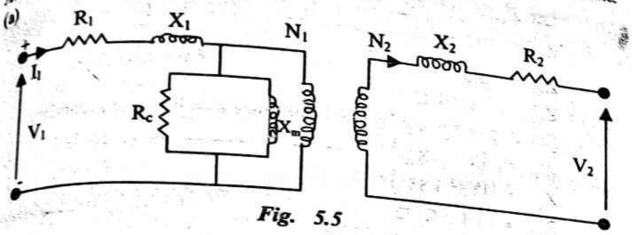


Fig. 5.5

Fig. 5.5 shows the equivalent circuit of the real transformer.

(b) $R_1^I = R_1\left(\frac{N_2}{N_1}\right)^2 = R_1\left(\frac{V_2}{V_1}\right)^2$

$R_1^I = 10.25\left(\frac{480}{2400}\right)^2$

$R_1^I = 0.41\Omega$ □

$X_1^I = X_1\left(\frac{N_2}{N_1}\right)^2 = X_1\left(\frac{V_2}{V_1}\right)^2$

$X_1^I = 18\left(\frac{480}{2400}\right)^2$

$X_1^I = 0.72\Omega$ □

$R_C^I = R_C\left(\frac{V_2}{V_1}\right)^2$ ------------------ 5.1.14

$R_C^I = 80\left(\frac{480}{2400}\right)^2$

$R_C^I = 3.2\Omega$ □

$X_m^I = X_m\left(\frac{V_2}{V_1}\right)^2$ ------------------ 5.1.15

$X_m^I = 400\left(\frac{480}{2400}\right)^2$

$X_m^I = 16\Omega$ □

(c) $I_1^I = I_1\left(\frac{N_1}{N_2}\right)$ ---------------- 5.1.16

$I_1 = \frac{240\times10^3}{2400} = 100\text{ A}$

$I_1^I = 100\left(\frac{2400}{480}\right)$

$I_1^I = 500\text{ A}$ □

$V_1^I = V_1\left(\frac{N_2}{N_1}\right) = V_1\left(\frac{V_2}{V_1}\right)$ -------------- 5.1.16a

$V_1^I = 2400\left(\frac{480}{2400}\right)$

$V_1^I = 480\text{ V}$ □

(d) the equivalent circuit of the transformer referred to the secondary is shown in figure 5.6.

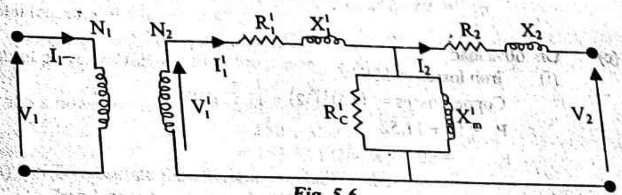


Fig. 5.6

Question 7

The full – load copper and Iron losses for a large power transformer ar 32KW and 18KW respectively. If the full-load output of the transformer i 1000KW, Calculate the losses and efficiency of the transformer

(a) on full load

(b) on half load

(c) on 60% load

Answer 7

(a) (i) Copper losses = 32 KW

Iron losses = 18 KW

∴ Total losses at full – load, P_L

$P_L = 32 + 18$

$P_L = 50\text{ KW}$ □

(ii) Efficiency =

$$\frac{\text{Output power}}{\text{Output power + losses}} \times \frac{100}{1}\% \text{ ---------- } 5.1.17$$

$$\eta_T = \frac{1000}{1000+50} \times \frac{100}{1}\%$$

η_T = 95. 24 % □

(b) on half load

(i) Iron losses = 18KW = Constant

Copper losses = $(½)^2 \times 32$ = 8 KW

$\therefore P_L = 18 + 8$

P_L = 26KW □

(ii) $$\eta_T = \frac{1000(½)}{1000(½)+26} \times \frac{100}{1}\%$$

$$\eta_T = \frac{500}{526} \times \frac{100}{1}\%$$

η_T = 95.06 % □

(c) On 60% load

(i) Iron losses = 18KW

Copper losses = $(0.6)^2 (32)$ = 11.52 KW

P_L = 18 + 11.52

P_L = 29. 52 KW □

(ii) $$\eta_T = \frac{1000(0.6)}{1000(0.6)+29.52} \times \frac{100}{1}\%$$

$$\eta_T = \frac{600}{600+29.52} \times \frac{100}{1}\%$$

η_T = 95.31% □

Question 8

The primary and secondary windings of a 200KVA transformer have resistances of 0.25Ω and 0.82mΩ respectively. The primary and secondary voltages are 2500V and 200 V respectively and the iron loss is 900W. Calculate the efficiency on (a) half load (b) 80% load, assuming the power factor of the load to be 0.75.

Answer 8

Full – load primary current, $I_1 = \frac{200 \times 1000}{2500}$

I_1 = 80 A

Full – load secondary current, $I_2 = \frac{200 \times 1000}{200}$

I_2 = 1000 A

∴ Primary copper loss = $I_1^2 R_1$ (on full – load)

= $80^2 \times 0.25$

= 1600 W

Primary copper loss = 1. 6 KW

∴ Secondary copper loss = $I_2^2 R_2$ (on full – load)

= $1000^2 \times 0.82 \times 10^{-3}$

= 820W

Secondary copper loss = 0.82 KW

∴ total copper loss on full – load = 1.6 + 0.82

= 2.42KW

Total loss on full – load = 2.42 + 0.90

= 3.32 KW

output power on full – load = 200 x 0.75

= 150 KW

∴ input power on full- load = output power + losses

= 150 + 3.32

= 153.32 KW

(a) To calculate the efficiency on half load

Iron loss = constant = 0.9KW

Copper loss varies as the squre of the current

∴ Copper loss = $(½)^2 \times 2.42$ = 0.605 KW

total loss on half – load = 0. 9 + 0.605

= 1.505 KW

input power on half – load = $\frac{150}{2} + 1.505$ = 76. 505KW

$$\eta_T = \left(1 - \frac{\text{Losses}}{\text{input power}}\right) \times \frac{100}{1}\% \quad 5.1.18$$

$$\eta_T = \left(1 - \frac{1.505}{76.505}\right) \times \frac{100}{1}\%$$

η_T = 98.03% □

(b) At 80 % load

Iron loss = 0.9 KW

Copper loss = $(0.8)^2 \times 2.42$ = 1.55KW

Total loss on 80 % load = 0.9 + 1.55

= 2. 45 KW

Input power on 80 % load = 150 (0.8) + 2.45

= 122. 45 KW

$$\eta_T = \left(1 - \frac{2.45}{122.45}\right) \times \frac{100}{1} \%$$

η_T = 98.0% □

Question 9

The following figures were obtained from tests on a 30 KVA, 3000 /110 V transformer

0. C test : 3000V 0.5 A 350 W

S. C. test: 150V 10 A 500 W

Calculate the efficiency of the transformer at

(a) full – load, 0.8 p.f

(b) half full – load, unity p.f

(c) Voltage regulation for power factor 0.8 lagging.

(d) Calculate the KVA output at which the efficiency is maximum,

Answer 9

(a) Full – load and 0.8 p.f

Iron loss = 350W

Copper loss = 500W

∴ total loss = 350 + 500 = 0.850KW

output power = 30 x0.8 = 24KW

$$\eta_T = \frac{24}{24 + 0.85} \times \frac{100}{1} \%$$

η_T = 96..58 % □

(b) half full –load and unity p.f

Iron loss = 0.35KW

$$\text{Copper loss} = \left(\frac{1}{2}\right)^2 \times 0.5 = 0.125 \text{ KW}$$

Total loss = 0.35 + 0.125 = 0.475 KW

$$\text{Output power} = \frac{30}{2} \times 1 = 15 \text{ KW}$$

$$\therefore \eta_T = \frac{15}{15 + 0.475} \times \frac{100}{1} \% = \frac{1500}{15.475} \%$$

η_T = 96.93 % □

(c)

$$V.R = \frac{V_{sc} \cos(\Phi_e - \phi_2)}{V_1} \times \frac{100}{1} \% \qquad \text{5.1.19}$$

Where:

V. R = Voltage regulation

V_{sc} = short – circuit Voltage

Cos ϕ_2 = the given p.f. in which the regulation is to be found

Cos ϕ_e = power factor on short – circuit test.

V_1 = Primary Voltage on open – circuit test.

$$\cos \phi_e = \frac{P_{sc}}{I_1 V_{sc}} \qquad \text{5. 1. 20}$$

where :

P_{sc} = Short – circuit power

I_1 = Short – circuit primary current

$$\therefore \cos \Phi_e = \frac{500}{10 \times 150} = 0.3333$$

$\Phi_e = \cos^{-1} [0.3333]$

$\Phi_e = 70.5°$

$\cos \Phi_2 = 0.8$

$\Phi_2 = \cos^{-1} [0.8] = 36.9°$

$$\therefore V.R = \frac{150 \cos(70.5° - 36.9°)}{3000} \times \frac{100}{1} \%$$

V.R = 4.2 % □

(d) To Calculate the KVA output at which the efficiency is maximum.

Full – load output = 30KVA

Copper loss = 0.5KW

Let y be the fraction of full – load KVA at which the efficiency is a maximum.

But at maximum efficiency,

Copper loss = iron loss

$\therefore y^2(0.5) = 0.35$

$y = \left(\frac{0.35}{0.5}\right)^{½}$

$y = 0.837$

∴ the KVA output at maximum efficiency
= 30 y
= 30 x 0.837
= 25. 11 KVA □

Question 10

Find the all-day efficiency of a 200KVA distributing transformer whose copper loss and iron loss at full – load are 1000W and 800W respectively. During a day of 24 hours, it is loaded as under:

NO. of hours	Load in KW	Power factor
8	150	0.75
10	100	0.80
4	72	0.72
2	0	

Answer 10

Energy output over 24 hrs
= 150 x 8 + 100 x 10 + 72 x 4
= 1200 + 1000 + 288
= 2488 KWh

Iron loss for 24 hrs = 0.80 x 24
= 19.2 KWh

To find the copper loss, we first of all find the KVA equivalent of the load using

$S = \frac{P}{\text{Cos}\,\Phi} = \frac{P}{p.f}$

For p = 150KW, p.f = 0.75

$S = \frac{150}{0.75} = 200\text{ KVA etc}$

Copper loss is

$= 1\left(\frac{200}{200}\right)^2 \times 8 + 1\left(\frac{125}{200}\right)^2 \times 10 + 1\left(\frac{100}{200}\right)^2 \times 4$

Copper loss = 8 + 3.91 + 1.0

Copper loss = 12.91 KWh

Total energy loss = 19.2 + 12.91

Total energy loss = 32.11 KWh

$$\eta_{AD} = \frac{\text{Output energy in KWh in 24 hours}}{\text{input energy in KWh in 24 hours}} \text{ ------------ } 5.1.21$$

$$\eta_{AD} = \frac{2488}{2488 + 32.11} \times \frac{100}{1}\%$$

$\eta_{AD} = 98.73\,\%$ □

Note that the iron loss occurs throughout the day whereas the copper loss occurs only when the transformer is loaded.

5.2 A.C. Machines

Question 1

(a) An 8 – pole cage induction motor runs at 5% slip. Calculate the motor speed if the supply frequency is 60 H Z.

(b) A 6 – pole induction motor runs at 19.2 revs/second and is supplied from a 60 HZ supply. Calculate the percentage slip.

Answer 1

(a) $S\,(\%) = \frac{N_s - N_r}{N_s} \times \frac{100}{1}$ ------------ 5.2.1

Where :
S = Slip
N_s = Synchronous speed
N_r = rotor speed

Also ,

$N_s = \frac{f}{p}$ ------------ 5.2.2

where :
f = the supply frequency in Hertz
P = the number of pairs of poles

$\therefore N_s = \frac{60}{4} = 15$ revs/second

$\therefore 0.05 = \frac{15 - N_r}{15}$

$0.05 \times 15 = 15 - N_r$

$N_r = 15 - 0.75$

$N_r = 14.25$ r.p.s □

(b) $N_s = \frac{f}{p}$

$N_s = \frac{60}{3} = 20$ r. p. s.

$\therefore S(\%) = \frac{20 - 19.2}{20} \times \frac{100}{1}$

$S(\%) = \frac{0.8}{20} \times \frac{100}{1}$

$S(\%) = 4\%$ □

Question 2.

(a) The supply frequency of an eight- pole induction motor is 50 H Z and the shaft speed is 735 r.p.m. Calculate:

(i) Synchronous speed

(ii) Percentage slip

(iii) Speed of slip.

(b) A 6 – pole, 50 HZ squirrel – cage induction motor runs on load at a shaft speed of 970 r. p. m. Calculate:

(i) the percentage slip

(ii) the frequency of induced current in the rotor.

Answer 2

(a) (i) $N_s = \frac{60f}{p}$

N_s = Synchronous speed in r. p. m.

$N_s = \frac{60 \times 50}{4}$

$N_s = 750$ r. p. m. □

(ii) $S = \frac{N_s - N_r}{N_s} \times \frac{100}{1}\%$

$S = \frac{750 - 735}{750} \times \frac{100}{1}\%$

$S = 2\%$ □

(iii) • Slip speed $= N_s - N_r$

$= 750 - 735$

Slip speed = 15 r. p. m. □

(b) (i) $S = \frac{N_s - N_r}{N_s} \times \frac{100}{1}\%$

$N_s = \frac{60f}{p} = \frac{60 \times 50}{3} = 1000$ r.p.m

$S = \frac{1000 - 970}{1000} \times \frac{100}{1}\%$

$S = 3\%$ □

(ii) $f_r = Sf$ ———————— 5.2.3

Where:

f_r = frequency of induced current in the rotor

S = Slip,

f = supply frequency

$\therefore f_r = \frac{3}{100} \times 50$

$f_r = 1.5$ H Z □

Question 3

The rotor e m f of a 3 – phase, 8 – pole, 440V, 50 H Z induction motor alternates at 2 H Z. Calculate the speed and percentage slip of the motor. Find the motor copper loss per phase if the full input to the motor is 105 KW.

Answer 3

(i) $S = \frac{f_r}{f}$ ———————— 5.2.3

$S = \frac{2}{50} = 0.04$p.u.

$S = 4\%$ □

(ii) $N = (1 - S) N_s$ ———————— 5. 2. 4

N = actual speed

N_s = synchronous speed

$N_s = \frac{60f}{p}$

$N_s = \frac{60 \times 50}{4} = 750$ r. p. m

$N = (1 - 0.04) \times 750$

N = 720 r. p. m □

(iii) Rotor copper loss = S x rotor input ———————— 5.2.5

rotor input = 105 KW

Rotor copper loss = 0.04 x 105 x 10^3

= 4200 W

Rotor copper loss per phase = $\frac{4200}{3}$ = 1400 W

Rotor copper loss per phase = 1.40 KW □

Question 4

A 4 – pole, 400V, 3 – phase, 50 H Z induction motor runs at 1440 r.p.m. at 0.8 p.f. lag and delivers an output of 10.8 KW. The stator loss is 1060 W and friction and windage losses are 390 W. Calculate

(i) Slip
(ii) rotor copper loss
(iii) rotor frequency
(iv) line current
(v) efficiency

Answer 4

(i) $S = \frac{N_s - N_r}{N_s} \times \frac{100}{1}\%$

$N_s = \frac{60f}{p} = \frac{60 \times 50}{2} = 1500\,r.p.m$

$\therefore S = \frac{1500 - 1440}{1500} \times \frac{100}{1} = \frac{60 \times 100}{1500}$

S = 4 % □

(ii) Let P_c be the rotor copper loss

$P_c = \frac{SP_m}{1-S}$ ---------------------- 5.2.6

Where:

S = Slip, P_m = rotor gross output

P_m = 10.8 + 0.390

P_m = 11.19 KW

$P_c = \frac{0.04 \times 11.19 \times 10^3}{1 - 0.04} = \frac{447.6}{0.96}$

P_c = 466.25 W □

(iii) rotor frequency, $f_r = Sf$

f_r = 0.04 x 50

f_r = 2 HZ □

(iv) power = $\sqrt{3}V_L I_L \cos Ø$ ---------------- 5.27

Where:

V_L = line Voltage

I_L = Line Current

Cos Ø = power factor

Rotor input power = $P_m + P_c = P_r$ ---------------- 5.2.8

P_r = 11.19 + 0.466

P_r = 11.656 KW

$\therefore$ motor input power, $P_i = P_r$ + stator losses

P_i = 11.656 + 1.060

P_i = 12.716 KW

$\therefore 12.716 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.8$

$I_L = \frac{12716}{\sqrt{3} \times 400 \times 0.8} = \frac{12716}{554.26}$

I_L = 22.94 A □

(v) Efficiency, $\eta = \frac{\text{output power}}{\text{input power}} \times \frac{100}{1}\%$ ---------- 5.2.9

$\eta = \frac{10.8}{12.716} \times \frac{100}{1}\%$

η = 84.93 % □

Question 5

The power input to the rotor of a 500V, 50Hz, 3 – phase, 6 – pole induction motor is 40 KW. It is observed that the induction motor is running at 4 % slip.

Calculate

(a) rotor speed
(b) rotor copper loss per phase
(c) the mechanical power developed
(d) the rotor resistance per phase if rotor current is 80 A.

Answer 5

(a) $S = \frac{N_s - N_r}{N_s}$

$N_s = \frac{60 \times 50}{3} = 1000$ r.p.m

$\therefore 0.04 = \frac{1000 - N_r}{1000}$

$40 = 1000 - N_r$

$N_r = 1000 - 40$

$N_r = 960$ r.p.m □

(b) Rotor copper loss = 0.04 x 40 from eq (5.2.5)

= 1.60 KW □

(c) Mechanical power developed, Pm

$P_m = P_r\,(1 - S)$

$P_m = 40\,(1 - 0.04)$ ⇒ $S = \frac{P_r - P_m}{P_r} \times 100\,\%$

$P_m = 38.4$ KW □

(d) rotor resistance per phase, R_2

$P_r = 3I_2^2 R_2$ ------------------------ 5.2.10

Where :

P_r = rotor copper loss

I_2 = rotor current

R_2 = rotor resistance per phase

$\therefore 1.60 \times 10^3 = 3 \times 80^2 \times R_2$

$R_2 = \frac{1.60 \times 10^3}{3 \times 80^2} = \frac{1600}{19200}$

$R_2 = 0.0833\Omega$ □

Question 6

Determine approximately the starting torque of an induction motor in terms of full – load torque when started by means of

(a) a star – delta switch

(b) an auto – transformer with 60 % tapping.

Ignore magnetizing current. The short – circuit current of the motor at normal voltage is 6 times the full – load current and the full – load slip is 3 percent.

Answer 6

(a) Start – delta starting.

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 S_f \text{ ----------------------------- } 5.2.11$$

Where:

T_{st} = starting torque

T_f = full – load torque

S_f = full – load slip

I_f = full – load current

I_{st} = starting current

Here, $I_{st} = \frac{1}{\sqrt{3}} I_{sc}$

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{\sqrt{3}I_f}\right)^2 S_f$$

$$\frac{T_{st}}{T_{st}} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 S_f$$

$$= \frac{1}{3}(6)^2 \times 0.03$$

$$\frac{T_{st}}{T_f} = 0.36$$

$T_{st} = 0.36\,T_f$

$\therefore T_{st} = 36\%$ of T_f □

(b) Auto – transformer starting.

$$\frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right) S_f \text{ ------------------------- } 5.2.12$$

where:

K = Transformation ratio

K = 60% = 0.6

$$\frac{T_{st}}{T_f} = (0.6)^2\,(6)^2 \times 0.03$$

$$\frac{T_{st}}{T_f} = 0.389$$

$T_{st} = 0.389\,I_f$

$\therefore T_{st} = 38.9\%$ of I_f □

Question 7

The power supplied to a three – phase induction motor is 60KW and the corresponding stator losses are 10.5KW. Calculate:

(a) the total mechanical power developed and the rotor copper loss when the slip is 0.05 per unit:

(b) the output power of the motor if the friction and windage losses are 8.2KW and (c) the efficiency of the motor. Ignore the motor iron loss.

Answer 7

(a) Input power to rotor = 60 – 10.5

= 49.5 KW

$$S = \frac{\text{rotor copper loss}}{\text{input power to rotor}} \quad \text{------------------- 5.2.13}$$

$$\therefore 0.05 = \frac{\text{rotor copper loss}}{49.5}$$

∴ rotor copper loss = 0.05 x 49.5

rotor copper loss = 2.475 KW □

Mechanical power developed, P_m

P_m = 49.5 – 2.475

P_m = 49.025 KW □

(b) Output power of motor = 47.025 – 8.2

= 38.825 KW □

(c) $\eta = \frac{38.825}{60} \times \frac{100}{1} \%$

$\eta = \frac{3882.5}{60} \%$

∴ η = 64.7 % □

Question 8

A 1200KVA, 15KV, three – phase, star-connected alternator has a resistance per phase of 1.2 Ω, a core loss of 35KW, a field copper loss of 11.4KW and a combined friction and windage loss of 13.6KW. Determine the full-load efficiency at power factor 0.8 lag.

Answer 8

Full load current, $I = \frac{1200}{\sqrt{3} \times 15}$

I = 46.19 A

Stator copper loss = $3 I^2 R$

= $3 \times (46.19)^2 \times 1.2$

= 7.68 KW

Output power = 1200 x 0.8 = 960 KW

Stator core loss = 35 KW

Stator copper loss = 7.68 KW

Field copper loss = 11.4 KW

Friction and windage loss = 13.6KW

Total input power = 960 + 35 + 7.68 + 11.4 +13.6

Total input power = 1027.68 KW

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times \frac{100}{1} \%$$

$$\eta = \frac{960}{1027.68} \times \frac{100}{1} \%$$

η = 93.41 % □

5.3 D.C. Machines

Question 1

(a) A d. c. shunt generator delivers 400A at 240V and the resistances of the shunt field and armature are 40 Ω and 0.05 Ω respectively. Calculate the generated e.m.f.

(b) A d. c. shunt generator has an induced e.m.f. of 280 V when the armature current is 60 A and the terminal voltage is 247V. Assuming a brush Volt drop of 3V, Calculate:

(i) the resistance of the armature,

(ii) the terminal voltage for an armature current of 40 A

(iii) the generated e.m.f. if the field current is increased to give a terminal voltage of 320V when delivering 80 A.

Answer 1

(a) The generator circuit is shown in figure 5.7 below

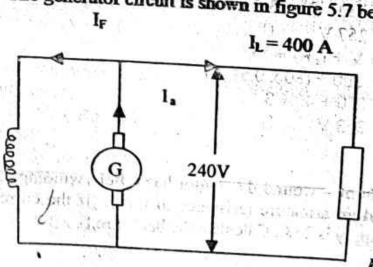


Fig . 5.7

From figure 5.7, $I_a = I_f + I_L$

Where:

I_a = armature current, I_f = current in the shunt field

I_L = load current

$I_f = \frac{240}{40} = 6\,A$

$I_a = 6 + 400 = 406\,A$

From generator e.m.f. equation,

$E = V + I_a R_a$ ———————— 5.3.1

Where :

E = e.m.f. generated in the armature

V = terminal Voltage

I_a = Armature current

R_a = Armature resistance

$E = 240 + 406 \times 0.05 = 240 + 20.3$

$E = 260.3\,V$ □

(b) (i) $E = V + I_a R_a + V_b$ ———————— 5.3.2

where:

V_b = brush voltage drop

$280 = 247 + 60 R_a + 3$

$280 = 250 + 60 R_a$

$R_a = \frac{280 - 250}{60}$

$R_a = 0.50\,\Omega$ □

(ii) $V = E - I_a r_a - V_b$

$V = 280 - (40 \times 0.5) - 3$

$V = 280 - 20 - 3$

$V = 257\,V$ □

(iii) $E = V + I_a R_a + V_b$

$E = 320 + (80 \times 0.5) + 3$

$E = 320 + 40 + 3$

$E = 363\,V$ □

Question 2.

(a) A 250 V Shunt – wound d.c. motor has a field winding resistance of 50Ω and an armature resistance of 0.2Ω. If the current taken from the supply is 25A, Calculate the **back e.m.f.**

(b) A d.c. motor with an armature resistance of 0.42Ω and a constant brush volt drop of 1.8V has a back e.m.f. of 220V when the armature current is 10A. Calculate the terminal Voltage.

Answer 2

(a) The motor circuit is shown in figure 5.8 below.

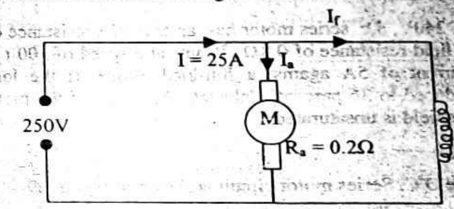


Fig. 5.8

$I = I_a + I_f$

$I_f = \frac{250}{50} = 5\,A$

$I_a = 25 - 5 = 20\,A$

The motor e.m.f. equation is given by

$E = V - I_a R_a$ ———————— 5.3.3

Where:

E = back e.m.f. of the motor

V = terminal Voltage

I_a = Armature current

R_a = Armature resistance

$E = 250 - (20 \times 0.2)$

$E = 250 - 4$

$E = 246\,V$ □

(b) When the brush volt drop is not negligible, then equation (5.3.3) modifies to

$E = V - I_a Ra - V_b$ ———————— 5.3.4

Where:

V_b = brush volt drop.

$220 = V - (10 \times 0.42) - 1.8$

$220 = V - 4.2 - 1.8$

$V = 220 + 6$

$V = 226\,V$ □

Question 3

(a) A 220 V series motor has a field winding resistance of 0.25 Ω and an armature resistance of 0.8 Ω. If the current taken at 360 r.p.m. is 40 A, Calculate:

(i) the back e.m.f of the motor

(ii) the torque on the armature

(b) A 240V, d.c. series motor has an armature resistance of 0.2 Ω and a field resistance of 0.8 Ω. It runs at a speed of 500 r.p.m taking a current of 5A against a full-load torque. If the load torque is reduced to 36 percent, Calculate the speed of the motor assuming the field is unsaturated.

Answer 3

(a) The D.C. Series motor circuit is shown in figure 5.9

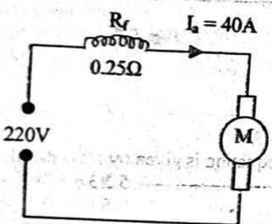


Fig. 5.9

(i) $E = V - I_a R_a$

$E = 220 - (40) R_a$

But the total armature circuit resistance is $(R_a + R_f)$ since the two resistances are in series.

$E = 220 - (40)(0.25 + 0.8)$

$E = 220 - 42$

$E = 178V$

(ii) $T = \frac{EI_a}{2\pi \times n}$ ---------------------- 5.3.5

Where:

T = torque, E = back e.m.f.

I_a = armature current

n = motor speed in rev per second

$n = 360$ r.p.m

$n = \frac{360}{60}$ r.p.s $= 6$ r.p.s

$T = \frac{178 \times 40}{2\pi \times 6}$

$T = 188.86$ Nm

(b) $\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$

Where:

N_1 = speed in the first case

N_2 = speed in the second case

E_{b1} = Back e.m.f. in the first case

E_{b2} = Back e.m.f. in the second case

I_{a1} and I_{a2} = Armature current in the 1st and 2nd case

Φ_1 = Flux per pole in the first case

Φ_2 = Flux per pole in the second case.

Also,

$T \alpha \Phi I_a$ ---------------- 5.3.7

But $\Phi = I_a$

$\therefore T \alpha I_a^2$ ---------------------- 5.3.8

$\therefore T_1 = K(5)^2$ Where K = constant

$T_2 = K\ I_{a2}^2$

$\frac{T_2}{T_1} = \left(\frac{K I_{a2}}{K \times 5}\right)^2 = \frac{I_{a2}^2}{25}$

$\therefore 0.36 = \frac{I_{a2}^2}{25}$

$I_{a_2}^2 = 0.36 \times 25 = 9$

$I_{a_2}^2 = 3.0$ A

$E_{b_1} = 240 - 5(0.2 + 0.8) = 235$ V

$E_{b_2} = 240 - 3(0.2 + 0.8) = 237$ V

$\therefore \frac{N_2}{500} = \frac{237}{235} \times \frac{5}{3}$

$\frac{N_2}{500} = \frac{1185}{705}$

$N_2 = \frac{500 \times 1185}{705}$

$N_2 = 840.43$ r.p.m □

Question 4

A Series motor runs at 600 r.p.m when taking 110A from a 200V supply. The resistance of the armature circuit is 0.18 ohm and that of the series winding is 0.02 ohm. Calculate the speed when the current has fallen to 60 A, assuming the useful flux per pole for 110A to be 0.025 Wb and that for 60 A to be 0.0158 Wb.

Answer 4

Total resistance of armature and series windings = 0.18 + 0.02 = 0.2Ω

Back e.m.f when I_a = 110A

$E = 200 - 110 \times 0.20$

$E = 178V$

$E = K \Phi N$ ——————— 5.3.9

Where:

K = Contant, Φ = flux per pole

N = speed

$178 = K \times 600 \times 0.025$

$K = \frac{178}{15} = 11.87$

When $I_a = 60$ A,

$E = 200 - 60 \times 0.20$

$E = 188$ V

$E_{new} = K\ \Phi_{new}\ N_{new}$

$188 = 11.87 \times 0.0158 \times N_{new}$

$N_{new} = \frac{188}{0.1875} = 1002.67$ r.p.m.

speed at 60 A = 1003 r.p.m. □

Question 5

(a) Calculate the flux in a 4 – pole d.c generator with 722 armature conductors generating 500V when running at 1000 r.p.m. when the armature is

(i) Lap connected

(ii) Wave connected.

(b) A Six – pole armature is wound with 420 conductors. The magnetic flux and the speed are such that the average e. m.f. generated in each conductor is 1. 5 V, and each conductor is capable of carrying a full-load current of 80 A. Calculate the terminal voltage on no load, the output current on full load and the total power generated on full load when the armature is

(i) Lap – connected and

(ii) Wave – connected.

Answer 5

(a) $E = \frac{\Phi ZPN}{60C}$ ——————— 5.3.10

where:

E = e.m.f. induced in any parallel path in armature

N = armature speed in r.p.m

P = Number of poles

Z = total number of armature conductors

Φ = flux / pole in weber

C = Number of parallel paths in armature

(i) Lap – connected

Here, C = 4

$\Phi = \frac{60 \times 500 \times 4}{722 \times 1000 \times 4} = 0.04155$ Wb

$\Phi = 41.55$ mWb □

(ii) Wave – connected. Here, C = 2

$Ø = \frac{60 \times 500 \times 2}{722 \times 1000 \times 4}$

$Ø = 0.02078$Wb

$Ø = 20.78$mWb □

(b) (i) Lap – connected.

Number of parallel paths = 6

Number of conductors per path = $\frac{420}{6}$ = 70

Terminal Voltage on no load = (e.m.f. per conductor).

x (no. of conductors per path)

= 1.5 x 70

= 105 V

Output current on full load = (full-load current conductors

x (no. of parallel paths)
= 80 x 6
= 480 A
∴ Total power generated on full load
= Output current x generated e.m.f.
= 480 x 105 = 50400W
= 50. 4 KW □

(ii) Wave - connected
Number of parallel paths = 2

No. of conductors / path = $\frac{420}{2}$ = 210

Terminal voltage on no load = 1.5 x 210
= 315 V

Output current on full load = 80 x 2
= 160 A

Total power generated on full load
= 160 x 315 = 50400W
= 50.4 KW □

Question 6

(a) A 20 KW, 1500 r.p.m. generator operating at rated load has a terminal Voltage of 240V. Calculate its voltage regulation if the no-load Voltage of the generator is 250 V.

(b) A 100KW, 250V Shunt motor takes a full-load line current of 42 A. The armature and field resistances are 0.15Ω and 200Ω respectively. The total brush-contact drop is 2.5 V and the core and friction losses are 320W. Calculate the efficiency of the motor assuming that stray – load loss is 0.05 percent of rated output.

Answer 6

(a) $V.R = \frac{V_{nL} - V_{rated}}{V_{rated}}$ ---------------------------- 5.3.11

Where:
V. R. = Voltage regulation
V_{nL} = Voltage at no-load
V_{rated} = rated Voltage

$V.R = \frac{250 - 240}{240} \times \frac{100}{1} \%$

V.R = 4.2 % □

(b) Input power = 250 x 42 = 10500 W

Field resistance loss = $\frac{250^2}{200}$ = 321.5 W

$I_f = \frac{250}{200}$ = 1.25A

I_a = 42 – 1.25 = 40.75 A

Armature resistance loss = 40.75^2 x 0.15
= 249 W

Core loss and friction loss = 320 W

Brush – contact loss = 40.75 x 2.5 = 101.88 W

Stray – load loss = $\frac{0.05}{100} \times \frac{100 \times 10^3}{1}$ = 50 W

Total losses = 312.5 + 2.49 + 320 + 101.88 +50
Total losses = 1033.38 W
Power output = 10500 – 1033.38 = 9466.62 W

$\eta = \frac{9466.62}{10500} \times \frac{100}{1} \%$

η = 90.16 % □

Question 7

A shunt – connected d.c motor draws a total current of 40 A from a 500 Supply when delivering its rated output at its rated speed of 950 r.p.m. The resistances of the field circuit and armature are 210Ω and 0.48 respectively, and the mechanical losses are 850 W. Find the useful power output, torque and efficiency.

Answer 7

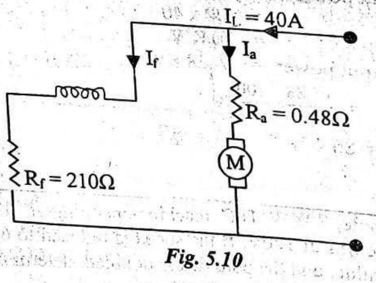


Fig. 5.10

Figure 5.10 shows the shunt motor arrangement.

$I_f = \frac{V_a}{R_f} = \frac{500}{210} = 2.38\ A$

$I_a = I_L - I_f$

$I_a = 40 - 2.38 = 37.62\ A$

Back e.m.f. at rated output and rated speed,

$E = V_a - I_a R_a$

$E = 500 - 37.62 \times 0.48$

$E = 481.9V$

Gross power output $= E I_a$

$= 481.9 \times 37.62$

$= 18.13\ KW$

Net power output = Gross power – mechanical losses

$= 18.13 - 0.85$

$= 17.28\ KW$

(i) the useful power output is 17.28 KW

(ii) Speed, N = 950 r.p.m

$\omega = \frac{2\pi \times 950}{60} = 99.48\ rad/s$

$$\text{Torque, } T = \frac{P}{\omega} \qquad \text{5.3.12}$$

P = Useful power output

ω = angular speed

$T = \frac{17.28 \times 10^3}{99.48}$

$T = 173.7\ Nm$ □

(iii) Input power $= V_a I_L$

$= 500 \times 40$

$= 20\ KW$

Output power = 17.28 KW

$\therefore \eta = \frac{17.28}{20} \times \frac{100}{1}\ \%$

$\eta = 86.4\ \%$ □

Question 8

An eight – pole, 30KW, D.C. machine operating at 1120 r.p.m has a generated e.m.f of 120V. If the speed is reduced to 60 percent of its original value, and the pole flux is doubled, determine

(i) induced e.m.f,

(ii) frequency of the rectangular voltage wave in the armature winding.

(b) A 230 V Shunt motor has an armature resistance of 0.32Ω and a field resistance of 108 Ω. At no – load the motor takes a line current of 4.5A while running at 1180 r.p.m. If the line current at full – load is 48.5A, Calculate the full – load speed.

Answer 8

(a) (i) $\frac{E_1}{E_2} = \frac{N_1\Phi_1}{N_2\Phi_2}$ from eq (5.3.6)

$E_2 = E_1 \times \frac{N_2\Phi_2}{N_1\Phi_1}$

$E_2 = \frac{120 \times 0.60 \times N \times 2\Phi}{N \times \Phi} = 144V$

$E_2 = 144V$ □

(ii) $f = \frac{NP}{120}$

$f = \frac{1120 \times 8 \times 0.60}{120}$

$f = 44.8\ HZ$ □

(b) The motor circuit is shown in figure 5.11

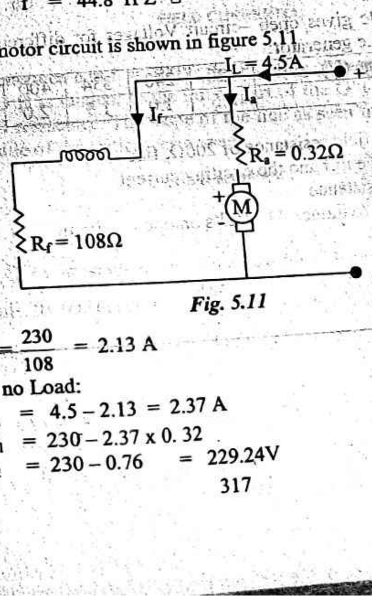


Fig. 5.11

$I_f = \frac{230}{108} = 2.13\ A$

At no Load:

$I_a = 4.5 - 2.13 = 2.37\ A$

$E_1 = 230 - 2.37 \times 0.32$

$E_2 = 230 - 0.76 = 229.24V$

$N_1 = 1180$ r.p.m

At full – load

Armature current, $I_a = 48.5 - 2.37$

$I_a = 46.13$ A

$E_2 = 230 - 46.13 \times 0.32$

$E_2 = 230 - 14.76$

$E_2 = 215.24$ V

$$\frac{N_1}{N_2} = \frac{E_1}{E_2} \qquad \text{------------------ 5.3.13}$$

Where:

N_1 = Speed at no – load

N_2 = Speed at full – load

E_1 = Back e.m.f. at no – load

E_2 = Back e.m.f. at full – load

$$\therefore \frac{1180}{N_2} = \frac{229.24}{215.24}$$

$$N_2 = \frac{1180 \times 215.24}{229.24}$$

$N_2 = 1108$ r.p.m ▯

Question 9

The following table gives open –circuit Voltages for different values of field current for a d.c generator:

Open-circuit voltage (V)	120	240	334	400	444	470
Field current (A)	0.5	1.0	1.5	2.0	2.5	3.0

If the machine has a field resistance of 200Ω, find

(a) the generated e.m.f and the exciting current

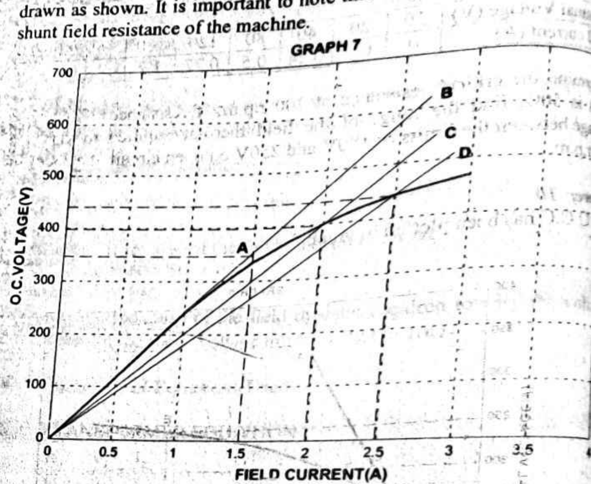
(b) the critical resistance

(c) the resistance to induce 444 Volts on open – circuit.

Answer 9

The O.C.C. has been plotted in Graph 7. The shunt resistance line O drawn as shown. It is important to note that the slope of the line give shunt field resistance of the machine.

GRAPH 7



(a) The generated e.m.f = 400V; the Exciting current = 2 A ▯

(b) Line O B is tangent to the initial part of the O. C. C. It represen critical resistance. The slope of the line as seen from point A give the value of the critical resistance:

$$\text{Critical resistance} = \frac{360}{1.5}$$

$$= 240\Omega \ ▯$$

(c) Line OD represents shunt resistance for inducing 444V on open - Circuit. Its resistance is = $\frac{444}{2.5}$

$= 177.6\Omega$ ▯

Question 10

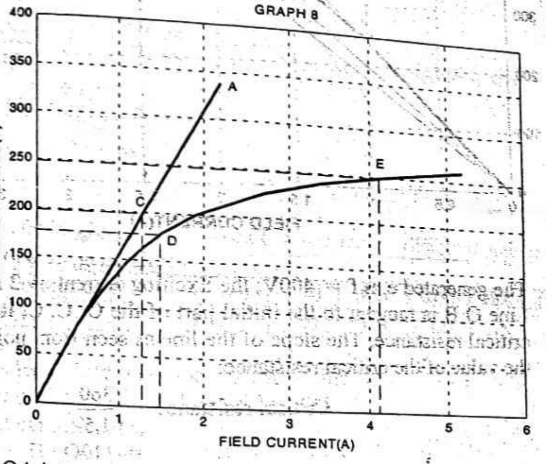
The following figures give the O. C. C of d.c. Shunt generator driven at a constant speed of 700 r.p.m.

Terminal Voltage (V)	10	20	40	80	120	160	200	240	260
Field current (A)	0	0.1	0.24	0.5	0.77	1.2	1.92	3.43	5.2

Determine the critical resistance at 700 r.p.m. If resistance of the field coils is 50Ω, find the range of the field rheostat required to vary the voltage between the limits of 180V and 250V on open circuit at a speed of 700 r.p.m.

Answer 10

The O.C.C has been plotted in graph 8



Line OA is tangent to the initial part of the O. C. C. It represents critical resistance. The slope of the line as seen from point C gives the value of the critical resistance.

Critical resistance $= \dfrac{200}{1.25}$

$= 160\ \Omega$ □

Point D on the graph gives a Voltage of 180 V. The corresponding value of the exciting current is 1.5 A

Field circuit resistance $= \dfrac{180}{1.5} = 120\Omega$ □

∴ The value of the field rheostat required

$= 120\ \Omega - 50\Omega$

$= 70\Omega$ □

Point E on the graph gives a voltage of 250V. The corresponding value of the exciting current is 4.15A

Field circuit resistance $= \dfrac{250}{4.15}$

$= 60\Omega$

∴ the value of the field rheostat required

$= 60 - 50$

$= 10\Omega$ □

∴ the range of the field rheostat required to vary the voltage between the above voltage limits is 70Ω to 10Ω □

5.4 Summary of Equations Used

1. TRANSFORMER EQUATION

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} \quad \text{---------- 5.1}$$

2. E.M.F. Equation of Transformer

$$E = 4.44\ fN\text{Ø} \quad \text{---------- 5.2}$$

3. Impedance Transformation in Transformer

$$R_2^1 = \left(\frac{N_1}{N_2}\right)^2 R_2 \quad \text{----------5.3}$$

$$R_2^1 = \left(\frac{V_1}{V_2}\right)^2 R_2 \quad \text{----------5.4}$$

$$R_1^1 = R_1\left(\frac{V_2}{V_1}\right)^2 \quad \text{----------5.5}$$

$$X_2^1 = X_2\left(\frac{V_1}{V_2}\right)^2 \quad \text{----------5.6}$$

$$X_i^1 = X_i\left(\frac{V_2}{V_1}\right)^2 \text{------------} 5.7$$

$$R_C^1 = R_C\left(\frac{V_2}{V_1}\right)^2 \text{------------} 5.8$$

$$X_m^1 = X_m\left(\frac{V_2}{V_1}\right)^2 \text{------------} 5.9$$

4. **Current and Voltage Transformation in Transformer**

$$I_1^1 = I_1\left(\frac{N_1}{N_2}\right) \text{------------} 5.10$$

$$V_i^1 = V_i\left(\frac{N_2}{N_1}\right) \text{------------} 5.11$$

5. Transformer Efficiency

$$\eta_T = \frac{\text{Output Power}}{\text{Output power + losses}} x \frac{100}{1}\% \text{------------} 5.12$$

$$\eta_T = \left(1 - \frac{\text{Losses}}{\text{input power}}\right) x \frac{100}{1}\% \text{------------} 5.13$$

6. Voltage Regulation for Transformer

$$V.R = \frac{V_{sc}\,Cos\,(\phi_c - \phi_2)}{V_1} x \frac{100}{1}\% \text{------------} 5.14$$

$$Cos\,\O_c = \frac{P_{sc}}{I_1 V_{sc}} \text{------------} 5.15$$

7. Transformer All-Day Efficiency

$$\eta_{AD} = \frac{\text{Output energy in KWh in 24 hours}}{\text{input energy in KWh in 24 hours}} \text{------------} 5.16$$

8. Percentage Slip

$$S(\%) = \frac{N_S - N_r}{N_S} x \frac{100}{1} \text{------------} 5.17$$

9. Synchronous Speed

$$N_s = \frac{f}{P} \text{------------}$$

10. Slip Speed

Slip Speed = $N_S - N_r$ ------------

11. Rotor Frequency

$f_r = Sf$ ------------

12. Actual Speed

$N = (1 - S)\,N_S$ ------------

13. Rotor Copper Loss

$$P_C = \frac{SP_m}{1-S} \text{------------} 5.$$

14. Power

$P = \sqrt{3}\,V_L I_L\,Cos\,\O$ ------------ 5.2

$P_m = P_r\,(I - S)$ ------------ 5.24

$P_r = 3I_2^2 R_2$ ------------ 5.25

15. Motor Efficiency

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} x \frac{100}{1}\% \text{------------} 5.26$$

16. Torque

$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 S_f \text{------------} 5.27$$

$$\frac{T_{st}}{T_f} = k^2\left(\frac{I_{sc}}{I_f}\right)^2 S_f \text{------------} 5.28$$

$T = EI_a/2\pi N$ ------------ 5.29

17. E.M.F. Equation of Generator

$E = V + I_a R_a$ ------------ 5.30

$E = V + I_a R_a + V_b$ ------------ 5.31

18. Speed – Voltage – Current Relation

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \quad \text{--- 5.32}$$

Speed – Voltage-Flux Relation

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\emptyset_1}{\emptyset_2} \quad \text{--- 5.3.3}$$

$$E = K \emptyset N \quad \text{--- 5.34}$$

$$E = \frac{\emptyset ZPN}{60C} \quad \text{--- 5.35}$$

20. Voltage Regulation of Machine

$$V.R = \frac{V_{nL} - V_{rated}}{V_{rated}} \quad \text{--- 5.36}$$

21. E.M.F Equation of Motor

$$E = V - I_a R_a \quad \text{--- 5.37}$$

$$E = V - I_a R_a - V_b \quad \text{--- 5.38}$$

5.5 Tutorial Problems Five

1. A 50KVA single-phase transformer has a turns ratio of 300/20. The primary winding is connected to a 2200V, 50Hz supply. Calculate:
   (e) the secondary voltage on no load
   (f) the approximate values of the primary and secondary currents on full load
   (g) the maximum value of the flux

2. A 200KVA, 3300/240 V, 50Hz, single-phase transformer has 80 turns on the secondary winding. Assuming an ideal transformer, find
   a. the primary and secondary currents on full load
   b. the maximum value of the flux
   c. the number of primary turns

3. A step-down single-phase transformer has a ratio of 15:1 and a primary voltage of 3300. Calculate the secondary voltage and the primary and secondary currents when a load of 30KVA at unity power factor is supplied. Ignore transformer losses.

4. A 2000KVA transformer is known to have copper losses of 35KW and iron losses of 25KW, when supplying full load at unity power factor. The transformer is contained in a tank holding 3600 litres of insulating oil. Calculate
   a. the efficiency at full load, unity power factor
   b. the efficiency at half-full load, unity power factor

5. A step up transformer in a small radio set has 200 turns in the primary coil and 600 turns in the secondary coil. When the primary coil is connected across a 240 Va.c. supply, the electric current through it is 45mA. Find the e.m.f. and the electric current in the secondary coil.

6. A transformer steps down voltage from 2400V to 240V
   a. If the primary coil has 1000 turns, find the number of turns in the secondary coil
   b. Find the primary and secondary electric currents when 4KW of electrical power is transferred from the primary circuit to the secondary circuit at 85% efficiency.

7. A 220/110V, 10KVA non ideal transformer has a primary winding resistance of 0.25Ω and a secondary winding resistance of 0.06Ω. Determine
   a. the primary and secondary currents at rated load; and the total resistance
   b. referred to the primary and
   c. referred to the secondary.

8. A 1Ø, 100KVA, 2000/200V two-winding transformer is connected as an auto transformer as shown in figure 5.12 below such that more than 2000V is obtained at the secondary. The portion ab is the 200V winding. Compute the KVA rating as an auto transformer.

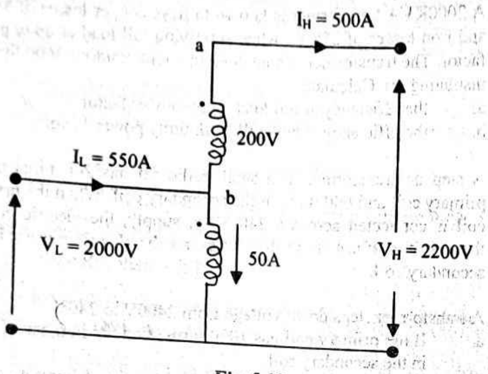


Fig. 5.12

9. A 50KVA, 2400/240V transformer has a core loss P_c = 200W at rated voltage and a copper loss P_{cu} = 500W at full load. It has the following load cycle

% load	0.0%	50%	75%	100%	110%
Power factor	-	1	0.8lag	0.9 lag	1.0
Hours	6	6	6	3	3

Determine the all-day efficiency of the transformer.

10. The following results were obtained on a 50KVA transformer:
O.C. test – primary voltage, 3300V; secondary voltage, 400V; primary power, 430W.
S.C. test – primary voltage, 124V; primary current, 15.3A; primary power, 525W; secondary current, full-load value. Calculate
    a. the efficiencies at full load and at half load for 0.7 power factor;
    b. the voltage regulations for power factors 0.7
        i. Lagging,
        ii. leading and
    c. the secondary terminal voltages corresponding to (i) and (ii).

11. Determine the voltage induced in the armature of a d.c. machine running at 1750 rpm and having four poles. The flux per pole is

25mWb, and the armature is lap-wound with 728 conductors. Calculate also the electromagnetic torque if the armature current is 120A.

12. A d.c motor with an armature resistance of 0.25Ω and a constant brush volt drop of 1.5V has a back e.m.f. of 216V when the armature current is 10A. Calculate the terminal voltage.

13. A 25KW, 415V, 50Hz three-phase squirrel-cage induction motor is 87% efficient and has a power factor of 0.92 lagging. Calculate the line current of the motor.

14. A 200V series motor has a field winding resistance of 0.1Ω and an armature resistance of 0.3Ω. If the current taken at 5 revs/second is 30A, calculate the torque on the armature. Calculate also the back e.m.f. of the motor.

15. A d.c. shunt generator has an induced e.m.f of 300V when the armature current is 80A and the terminal voltage is 274V. Assuming a brush volt drop of 2V, calculate
    (a) the resistance of the armature
    (b) the terminal voltage for an armature current of 60A, and
    (c) the generated e.m.f. if the field current is increased to give a terminal voltage of 300V when delivering 100A.

16. The power supplied to a three-phase induction motor is 40KW and the corresponding stator losses are 1.5KW. Calculate
    (a) the total mechanical power developed and the rotor copper loss when the slip is 4%;
    (b) the output power of the motor if the friction and windage losses are 0.8KW, and
    (c) the efficiency of the motor.

17. The power input to a 400V, 60Hz, 6 – pole, 3 – phase induction motor running at 1140 r.p.m. is 40KW at 0.8 p.f. lag. Stator losses are 1KW and the friction and windiage losses are 2KW. Calculate
    (i) the slip
    (ii) the rotor copper loss
    (iii) the brake h.p.
    (iv) efficiency and
    (v) input current.

18. A 18.65KW, 4-pole, 50Hz, 3 – phase induction motor has friction and windage losses of 2.5 percent of the output. The full-load slip is 4%. Compute for full-load:
   (a) the rotor copper loss
   (b) the rotor input
   (c) the output torque
   (d) the gross electromagnetic torque

19. The open-circuit characteristic of a separately- excited d.c. generator driven at 1000 r.p.m. is as follows:

Field current (A)	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
E.M.F. (Volts):	30.0	55.0	75.0	90.0	100.0	110.0	115.0	120.0

If the machine is connected as a shunt generator and driven at 1000 r.p.m. and has a field resistance of 100Ω, find
   (a) open – circuit voltage and exciting current
   (b) the critical resistance and
   (c) resistance to induce 115 volts on open-circuit

20. The open-circuit characteristic of a shunt generator when separately-excited and running at 1000 r.p.m. is given by:

Open-circuit voltage	56	112	150	180	200	216	230
Field amperes	0.5	1.0	1.5	2.0	2.5	3.0	3.5

If the generator is shunt-connected and runs at 1100 r.p.m. with a total field resistance of 80Ω, determine
   a. no-load e.m.f
   b. the output when the terminal voltage is 200V if the armature resistance is 0.1Ω.
   c. the terminal voltage of the generator when giving the maximum output current.

Neglect the effect of armature reaction and of brush contact drop.

Chapter Six

6.0 THREE PHASE A.C. SYSTEMS

6.1 Star-Delta Transformation

Question 1

(a) Three resistors having resistances 2Ω, 3Ω and 5Ω are star-connected to terminals A, B and C respectively. Calculate the resistances of equivalent delta-connected resistors.

(b) Three resistors having resistances 10Ω, 5Ω and 25Ω are delta-connected between terminals AB, BC and CA respectively. Calculate the resistances of equivalent star-connected resistors.

Answer 1

a. The star-connected resistors are shown in figure 6.1 and the equivalent delta in Figure 6.2.

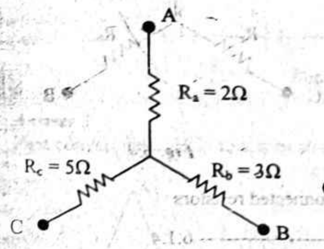


Fig. 6.1

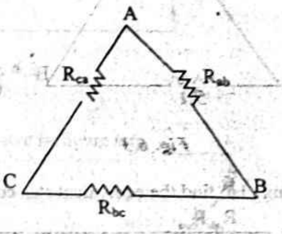


Fig. 6.2

To find the equivalent delta-connected resistors

$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c} \quad \text{------------ 6.1.1}$$

Where:

R_a, R_b and R_c are respective star resistances. R_{ab} = Equivalent delta resistance connected between A and B

$$R_{ab} = 2 + 3 + \frac{2 \times 3}{5} = 5 + \frac{6}{5}$$

$R_{ab} = 6.2\Omega$ □

$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a} \quad \text{------------ 6.1.2}$$