MTH125 TUTORIAL QUESTIONS

1. Determine the order and degree of the differential equation

   $\frac{d^4y}{dt^4}+4\frac{d^2y}{dt^2}+7y=9$

2. The degree and order of the differential equation

   $\left(\frac{dy}{dt}\right)^3+y^2=8$ are respectively:

3. What are the order and degree of the differential equation?

   $\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+\left(\frac{dy}{dt}\right)^{3/2}-3y=49$

4. Determine the order and degree of the differential equation

   $\left(\frac{d^5y}{dt^5}\right)^2-\left(\frac{dy}{dt}\right)^{5/2}=11$

5. Determine the differential equation whose general solution is:

   $y=Ae^{2x}+Be^{-x}$

6. Form a differential equation for the function

   $y=x+\frac{A}{x}$

7. Classify the following differential equations as linear or nonlinear and homogeneous or inhomogeneous

   a) $\frac{dy}{dt}+y\sin t=e^t$

   b) $\frac{d^3y}{dt^3}+7t\left(\frac{d^2y}{dt^2}\right)^{\frac{1}{3}}-y^{-1}=11$

   c) $\frac{d^5y}{dt^5}--e^y=0$

   d) $\frac{d^3y}{dt^3}=t^2y$

8. Determine which of the given equations are ordinary differential equations and partial differential equations. State the dependent and independent variables in each equation.

   a) $t\frac{dy}{dt}-16t^2y=\cos t$

   b) $t\frac{dy}{dt}-e^{2t}=0$

   c) $\frac{\partial^2u}{\partial t^2}=c^2\frac{\partial^2u}{\partial x^2}$

d) $\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial^2 x} = 5x$

9. Solve the following homogeneous differential equations

a) $\sin t \frac{dy}{dt} + y \cos t = 0$

b) $t^{-3} \frac{dy}{dt} + y = 0$

c) $\frac{dy}{dt} - 5t^2 y = 0$

d) $\frac{dy}{dx} = 3x^2 - 6x + 5$

10. Solve the following initial value problem

$\frac{dy}{dt} + y \sin t = 0, \ y(o) = 2$

11. Solve the following initial value problem

$t\frac{dy}{dt} + 5y = 0, \ y(1) = 1$

12. Find the particular solution of the equation

$e^{x} \frac{dy}{dx} = 4$ given that y=3 when x=0

13. Solve $\frac{dy}{dt} = t^2 (1 + y^2)$

14. Solve (sint) (siny) + cosy $\frac{dy}{dt} = 0$

15. $\frac{dy}{dt} = t^2 (t+1) e^{-2y}$

16. Solve $\frac{dy}{dx} = \frac{y^2 + xy^2}{x^2 y - x^2}$

17. Solve $\frac{dy}{dt} = \frac{t+y}{t-y}$

18. Solve $\frac{dy}{dt} = \frac{t^2 + y^2}{2ty}$

19. Solve $\frac{dy}{dx} = \frac{x+3y}{2x}$

20. Solve $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$

21. Solve $5\frac{dy}{dt} - y = 20t$

22. Solve $(1+t^2)\frac{dy}{dt} - 7ty = 6(1+t^2)$

23. Solve cost $\frac{dy}{dt} - \sin t \ y = 17$

24. Solve $t\frac{dy}{dt}-2y=-t^2,\ y(1)=1$

25. Solve $\frac{dy}{dx}+\frac{1}{x}y=x^2$

26. Solve $x\frac{dy}{dx}-5y=x^7$

27. Solve $(x+1)\frac{dy}{dx}+y=(x+1)^2$

28. Solve $\frac{dy}{dx}+\frac{1}{x}y=xy^2$

29. Solve $\frac{dy}{dt}+t^3y=y^4,\ y(1)=4$

30. Solve $\frac{dy}{dt}+y=y^2$

31. Solve $\frac{dy}{dt}+18y=y^{-5}e^{2t}$

32. Solve $3t\frac{dy}{dt}+2ty=y^{-1}t^5$

33. Solve $(1-t^2)\frac{dy}{dt}-2ty=y^3$

34. Compute the Wronskian w(t) of the differential equation $\frac{d^2y}{dt^2}-4y=0$ given $y_1(t)=e^{2t}$, and $y_2(t)=e^{-2t}$ are solutions.

35. Compute the Wronkian w(t) of a set of solution $y_1(t)$ and $y_2(t)$ of the differential equation $t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+7y=0$ given that $w(1)=1$.

36. Solve the general solution of $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=0 \quad y(0)=9 \quad \frac{dy(0)}{dt}=5$.

37. Given that $A\frac{d^2y}{dt^2}+B\frac{dy}{dt}+Cy=0$ is a second order linear homogeneous differential equation, if the discriminant is greater than zero. What is the possible general solution if the roots are m_1 and m_2?

38. Solve $\frac{d^2y}{dt^2}-2\frac{dy}{dt}+5y=0 \quad y(0)=5,\ \frac{dy(0)}{dt}=5$.

39. If a differential equation of the form $A\frac{d^2y}{dt^2}+B\frac{dy}{dx}+Cy=f(t)$ it is said to be constant coefficient inhomogeneous equation if $f(t)$ is

40. Solve the inhomogeneous differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=Q$ where $Q=9$.

41. Determine the general solution to the differential equation $\frac{d^2y}{dt^2}+9\frac{dy}{dt}+20y=f(t)$ where $f(t)=112e^{3t}$

42. Solve the inhomogeneous differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=f(t)$ where $f(t)=sin3t$ has a particular solution of

43. A particle moves in a straight line such that for a short time its velocity is defined by $v=(3t^2+4t)m/s$ where t is in seconds. Determine the position of the particle when $t=5$sec with initial value of $t=0$ and $s=0$.

44. A particle moves on a straight line from a position $s=5t^3+3t^2$ in metre at time t. Determine the acceleration of the particle at time $t=3sec$.

45. A car moves in a three dimensional space such that for a short time its velocity is given by $v=(3t+5t, 2t+3t^3, t+5t^2)ms^{-1}$ where t is the time in seconds. Determine its position in 25seconds

46. A particle moves along 3-dimensional plane whose parametric equations are $x = 3t^2$, $y = 8t$, $z = 2t^3$ in metres where t is the time in seconds. Determine the magnitude of the velocity at $t = 2s$.

47. Suppose a particle moves from rest with acceleration $\vec{a} = (2t, 4cos3t, 4sin3t)ms^{-2}$. Find the velocity in magnitude.

48. A particle moves in 3-dimensional plane with the following parametric equations $x = 3exp(-t)$, $y = 5sin2t$ and $z = 4cos2t$ in metres where t is the time. Determine the acceleration at the time $t = 0$.

49. A particle moves on a straight line from a position $s = 5t^3 + 3t^2$ in metres, at time(t). Determine the velocity of the particle at time $t = 3secs$.

50. Solve $\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 20y = 0$

51. Solve for the general solution of the differential equation given as $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0 \quad y(1) = e = y^1$

52. What is the solution to the equation $A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = 0$ where A, B, C are constant, the root are distinct real root m_1 and m_2 but $m_1 \neq m_2 \neq 0$?

53. Determine the differential equation of the Wronskian $w(t)$ for which value of t is $w(t)$ non zero if $y_1 = e^{-2t}$ and $y_2(t) = e^{5t}$.

54. Given $y_1(t) = \cosh mt$ and $y_2(t) = \sinh mt$. Compute the differential equation of the Wronskian $w(t)$ for which value of t is $w(t)$ non zero.

55. $A\frac{d^2y}{dx^2} + B\frac{dy}{dt} + C(t)y = f(t)$ where $A(t)$ $B(t)$ $C(t)$ and $f(t)$ are functions of variable t that are continuous on the open interval $a < t < b$ and at $A = 0$ if $f(t)$ in the differential equation is zero then it is said to be.

56. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 0 \quad y(\pi) = y^1(\pi) = e^{3\pi/2}$

57. A differential equation $A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = Q(x)$ where A, B and C are constant parameters is said to be homogeneous of first order if

58. Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} = 0$.

59. Solve $y'' + 0.2y' + 4.01y = 0 \quad y(0) = 0 \quad y'(0) = 2$

60. Solve $y'' + 2ay' + a^2y = 0$

61. Solve $8y'' - 2y' - y = 0 \quad y(0) = -0.2 \quad y'(0) = -0.325$

62. A particle moves on a straight line from a position $s = (7/_2 t^4 - 3t^3 + 5t^2 + 9t - 10)m$ at time t. Determine the magnitude of the acceleration of the particle at time $t = 5secs$.

63. $\frac{dy}{dx} + P(x)y = Q(x)y^n$ has a general solution of

64. A particle with an initial velocity $\vec{v}_0(e^3 \ e^{-2} \ 0)m/s$ where e^x stand for $exp(x)$ and $\vec{r}_0(0,0,0)m$ travelled with acceleration $\vec{a} = (3e^{-2t}, 4\sin 5t, 4\cos 5t)ms^{-2}$. Determine the magnitude of the acceleration at $t = 0sec$.

65. A particle of mass 30kg is pulled horizontally through the coordinate $x(t)=3t^2$ and $y(t)=2t^3+5t^2$ in meters. Find the Position vector at t = 2s.

66. A particle of mass 30kg is pulled horizontally through the coordinate $x(t)=3t^2$ and $y(t)=2t^3+5t^2$ in meters. Find the Velocity at t = 2s.

67. A particle of mass 30kg is pulled horizontally through the coordinate $x(t)=3t^2$ and $y(t)=2t^3+5t^2$ in meters. Find the Acceleration at t = 2s.

68. A particle of mass 30kg is pulled horizontally through the coordinate $x(t)=3t^2$ and $y(t)=2t^3+5t^2$ in meters. Find the Force at t = 2s.

69. A particle of mass 30kg is pulled horizontally through the coordinate $x(t)=3t^2$ and $y(t)=2t^3+5t^2$ in meters. Find the work done if the force is in the direction of motion at t = 2s.

70. A particle of mass 30kg is pulled horizontally through the coordinate $x(t)=3t^2$ *and* $y(t)=2t^3+5t^2$ in meters. Find the work done if the force is inclined at an angle of 60 degrees to the horizontal line t = 2s.

71. Work done by a particle in Joules is obtained by the...

72. Which of the following is **TRUE** about Force (**F**) in Newton?

73. Calculate the Work done when a horizontal force pulls a load of 80kg through the coordinate $x(t)=t^2$ *and* $y(t)=2t^2$ in meters, along a rough horizontal floor given that the coefficient of friction is 0.4 and time, t = 3 seconds ($Take\ \vec{g}=9.8\vec{e}m/s^2$).

74. Calculate the Velocity of a particle of mass 4kg in the coordinates $x(t)=3t^4-2t^3$ *and* $y(t)=4t^3+5t$ in 4 seconds.

75. Calculate the Acceleration of a particle of mass 4kg in the coordinates $x(t)=3t^4-2t^3$ *and* $y(t)=4t^3+5t$ in 4 seconds.

76. Calculate the Resistance acting on a particle of mass 4kg in the coordinates $x(t)=3t^4-2t^3$ *and* $y(t)=4t^3+5t$ in 4 seconds.

77. Calculate the power of a particle of mass 4kg in the coordinates $x(t)=3t^4-2t^3$ *and* $y(t)=4t^3+5t$ in 4 seconds.

78. Find the Work done by a particle of mass 25kg which ran up a stair case of vertical height in coordinate (x,y) = (2,3) in 3 seconds. ($Take\ \vec{g}=9.8\vec{e}m/s^2$).

79. Find the power of a particle of mass 25kg which ran up a stair case of vertical height in coordinate (x,y) = (2,3) in 3 seconds. ($Take\ \vec{g}=9.8\vec{e}m/s^2$).

80. Calculate the Kinetic Energy of a particle of mass 10kg whose position coordinate are $x(t)=5t$ *and* $y(t)=3t$ in 2 seconds.

81. Find the value of Z if the Force (Z,17)N is required to reduce the velocity of a particle of mass 10kg with position coordinate $x(t)=5t$ *and* $y(t)=3t$ in 2 seconds.

82. Which of the following represent the equation of motion for a body of mass, Mkg shot upward against gravity at a velocity, **U** and the air resistance is Mkv^2, where **k** is a constant?

83. Which of the following represent the equation of motion for a body of mass, Mkg falling under gravity at a velocity, **U** and the air resistance is Mkv^2, where **k** is a constant?

84. Which of the following represent the equation of motion for a body of mass, Mkg shot upward against gravity at a velocity, **U** in a medium whose air resistance is Mkv, where **k** is a constant?

85. Which of the following represent the equation of motion for a body of mass, Mkg falling under gravity at a velocity, **U** in a medium whose air resistance is Mkv, where **k** is a constant?

86. A particle of mass 20kg moves along a plane whose parametric equations are $x(t)=3t^2, y(t)=8t,$ *and* $z(t)=3t^3$ in meters, where t is the time in seconds. Find the Position vector at t = 2s.

87. A particle of mass 20kg moves along a plane whose parametric equations are $x(t)=3t^2, y(t)=8t,$ *and* $z(t)=3t^3$ in meters, where t is the time in seconds. Find the velocity at t = 2s.

88. A particle of mass 20kg moves along a plane whose parametric equations are $x(t)=3t^2, y(t)=8t,$ *and* $z(t)=3t^3$ in meters, where t is the time in seconds. Find the acceleration at t = 2s.

89. A particle of mass 20kg moves along a plane whose parametric equations are $x(t)=3t^2, y(t)=8t,$ *and* $z(t)=3t^3$ in meters, where t is the time in seconds. Find the force exerted by the particle at t = 2s.

90. A particle of mass 5kg which was initially at rest and is acted upon by a force $(7t+t^2)N$ at time t seconds. Find the velocity of the particle.

91. A particle of mass 5kg which was initially at rest and is acted upon by a force $(7t+t^2)N$ at time t seconds. Find the velocity of the particle after 3 seconds.

92. A particle of mass 5kg which was initially at rest and is acted upon by a force $(7t+t^2)N$ at time t seconds. Find the distance covered by the particle.

93. A particle of mass 5kg which was initially at rest and is acted upon by a force $(7t+t^2)N$ at time t seconds. Find the distance covered by the particle after 3 seconds.

94. A particle of mass 10kg moves along a curve whose parametric equations are $x(t)=e^{-t}$, $y(t)=2Cos5t, and\ z(t)=2\sin 5t$, all in meters, where t is the time in seconds. Find the position vector of the particle at time t = 0 seconds.

95. A particle of mass 10kg moves along a curve whose parametric equations are $x(t)=e^{-t}$, $y(t)=2Cos5t, and\ z(t)=2\sin 5t$, all in meters, where t is the time in seconds. Find the velocity of the particle at time t = 0 seconds.

96. A particle of mass 10kg moves along a curve whose parametric equations are $x(t)=e^{-t}$, $y(t)=2Cos5t, and\ z(t)=2\sin 5t$, all in meters, where t is the time in seconds. Find the acceleration of the particle at time t = 0 seconds.

97. A particle of mass 10kg moves along a curve whose parametric equations are $x(t)=e^{-t}$, $y(t)=2Cos5t, and\ z(t)=2\sin 5t$, all in meters, where t is the time in seconds. Find the Force of the particle at time t = 0 seconds.

ANSWERS TO ~~8~~ MTH 125

1. Order $=4$ Degree $=1$

2. Order $=1$ Degree $=3$

3. Order $=3$ Degree $=2$

4. Degree $=4$ Order $=5$

5. $y = Ae^{2x} + Be^{-x}$ --- * so we have

$y' = 2Ae^{2x} - Be^{-x}$ --- **

$y'' = 4Ae^{2x} + Be^{-x}$ --- ***

Add * and ** & ** and *** ie

$y + y' = 3Ae^{2x}$ ... (a) and $y' + y'' = 6Ae^{2x}$ ... (b)

By taking the ratio of a and b we have

$$\frac{y+y'}{y'+y''} = \frac{1}{2} \Rightarrow 2y + 2y' = y' + y''$$

So $y'' - y' - 2y = 0$

6. $y = x + \frac{A}{x}$ --- *

$y' = 1 + \left(-\frac{A}{x^2}\right)$ --- **

Make A subject in * and substitute into **

$$y' = 1 - \frac{xy - x^2}{x^2} \Rightarrow x^2 y' = x^2 - xy + x^2$$

$$\Rightarrow x^2 y' + xy - 2x^2 = 0$$: Stop here if you want

That is $y = \frac{2x^2}{x^2 - x}$

Clearly $y = \frac{2x}{x-1}$ as required

7. (a) linear, inhomogeneous

(b) Non-linear, Inhomogeneous

(c) linear, homogeneous

(d) linear, homogeneous

8. (a) O.D.E, $y = f(t)$ y is dependent and t is independent

(b) O.D.E, $y = f(t)$ y is " and t is "

(c) P.D.E, $u = f(x,t)$ u is " and x, t is "

(d) P.D.E, $u = f(x,y)$ u is " and x, y " "

9 (a) $\sin t \frac{dy}{dt} + y\cos t = 0$

using variable seperable

$$\Rightarrow \frac{1}{y}\frac{dy}{} = -\frac{\cos t}{\sin t}$$

Integrating both sides

$\ln y = -\ln \sin t + C$ $\qquad D = e^c$

$\Rightarrow y \sin t = D$ by logarithmic simplication

(b) $t^{-3} \frac{dy}{dt} + y = 0$

Using variable seperable

$\frac{1}{y} dy = -t^3 dt$ so $\ln y = -\frac{t^4}{4} + C$

In conclusion $y = Ae^{-t^4/4}$ where $A = e^c$

(c) $\frac{dy}{dt} - 5t^2 y = 0$; Variable seperable

$$\frac{1}{y} dy = 5t^2 dt$$

$\ln y = \frac{5t^3}{3} + c \Rightarrow$ if $e^c = A \Rightarrow y = Ae^{5t^3/3}$

(d) $\frac{dy}{dx} = 3x^2 - 6x + 5$ Variable seperable

$$dy = (3x^2 - 6x + 5)dx$$

$$y = x^3 - 3x^2 + 5x + C$$

10. $\frac{dy}{dt} + y\sin t = 0$ at $y(0) = 2$ Note that $y(0) = 2 \Rightarrow t = 0$ y

which is required to get a particular solution ie the value of

$\frac{1}{y} dy = -\sin t\, dt$

$\Rightarrow$ Integration occurred

$\ln y = \cos t + c$

Recall $y = 2$ and $t = 0$ ie $\ln 2 = \cos 0 + c \Rightarrow c = \ln 2 - 1$

Therefore $\ln y = \cos t + \ln 2 - 1$

By simplification $\ln\left(\frac{y}{2}\right) = \cos t - 1$

$$y = 2e^{\cos t - 1}$$

11. Solve $t\frac{dy}{dt} + 5y = 0$, $y(1) = 1$. Using variable seperable

3

$\frac{1}{y} dy = \frac{-5}{t} dt$ ⇒ Integrating both sides we have the following

$\ln y = -5 \ln t + c$

By logarithmic law $a \ln b = \ln b^a$

$\ln y = -\ln t^{5} + c$

Take $c = -\ln B$

$\ln y + \ln t^5 + \ln B = 0$

$\ln B y t^5 = 0$

Logarithmic simplification once more

$B y t^5 = e^0 = 1$

So $y = \frac{1}{Bt^5}$ Take $\frac{1}{B} = D$

$y = \frac{D}{t^5}$

Recall $t=1$ $y=1$ so $D=1$ hence $y = \frac{1}{t^5}$

12 $e^x \frac{dy}{dx} = 4$ given that $y=3$ when $x=0$

Using variable seperable

$dy = 4e^{-x} dx$

Integrating both sides

$y = -4e^{-x} + C$

Recall $y=3$ $x=0$ so $3 = -4 + c \Rightarrow c = 7$

$y = 7 - 4e^{-x}$

13 Solve $\frac{dy}{dt} = t^2(1+y^2)$ Variable Seperable

$\frac{1}{1+y^2} dy = t^2 dt$ → Integrating both sides

Note $\int \frac{1}{1+x^2} dx = \tan^{-1} x$ so

$\tan^{-1} y = \frac{t^3}{3} + C$ or $y = \tan\left(\frac{t^3}{3} + c\right)$

14 Solve $(\sin t)(\sin y) + (\cos y)\frac{dy}{dt} = 0$, using variable seperable

$\frac{\cos y}{\sin y} dy = -\sin t \, dt$

Integrating both sides

4.

$\ln \sin y = \cos t + c$

$\sin y = e^{\cos t + c}$

$\sin y = A e^{\cos t}$ where $A = e^c$

15 $\frac{dy}{dt} = t^2(t+1)e^{-2y}$, Variable Seperable

$e^{2y}dy = t^2(t+1)dt$

Integrate both sides

$\frac{e^{2y}}{2} = \frac{t^4}{4} + \frac{t^3}{3} + c$

By further simplication

$y = \frac{1}{2}\left[\ln\left(\frac{t^4}{2} + \frac{2t^3}{3} + D\right)\right]$ where $D = 2c$

16 $\frac{dy}{dx} = \frac{y^2 + xy^2}{x^2y - x^2}$ by factorization

$\frac{dy}{dx} = \frac{y^2(1+x)}{x^2(y-1)}$ using variable seperable

$\frac{y-1}{y^2}dy = \frac{1+x}{x^2}dx$

Seperating the above eq

$\left(\frac{y}{y^2} - \frac{1}{y^2}\right)dy = \left(\frac{1}{x^2} + \frac{x}{x^2}\right)dx$

$\int\left(\frac{1}{y} - \frac{1}{y^2}\right)dy = \int\left(\frac{1}{x^2} + \frac{1}{x}\right)dx$

Integrating both sides

$\ln y + \frac{1}{y} = -\frac{1}{x} + \ln x + c$

By simplication using logarithm

$\ln y - \ln x = -\frac{1}{x} - \frac{1}{y} + c$

$\ln\left(\frac{y}{x}\right) = -\frac{1}{x} - \frac{1}{y} + c$

Unless you want to go further

17 Solve $\frac{dy}{dt} = \frac{t+y}{t-y}$, Homogeneous so we use

$y = vt$ and $\frac{dy}{dt} = v + t\frac{dv}{dt}$

So $v + t\frac{dv}{dt} = \frac{t + vt}{t - vt}$

$v + t\frac{dv}{dt} = \frac{1+v}{1-v}$

$t\frac{dv}{dt} = \frac{1+v}{1-v} - v$ } by taking V to RHS

$t\frac{dv}{dt} = \frac{1+v^2}{1-v}$

Variable seperable

$\frac{1-v}{1+v^2}\,dv = \frac{1}{t}\,dt$

$\left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\right)dv = \frac{1}{t}\,dt$

Integrate both sides

$\tan^{-1}v - \frac{1}{2}\ln(1+v^2) = \ln t + c$

Recall $y = vt$ so $v = y/t$

$\tan^{-1}\left(\frac{y}{t}\right) - \ln\left(1 + \frac{y^2}{t^2}\right)^{1/2} = \ln t + c$

18 $\frac{dy}{dt} = \frac{t^2+y^2}{2ty}$, Homogeneous $y = Vt$ $\frac{dy}{dt} = v + t\frac{dv}{dt}$

$v + t\frac{dv}{dt} = \frac{t^2 + v^2t^2}{2t^2v} = \frac{1+v^2}{2v}$

so $t\frac{dv}{dt} = \frac{1+v^2}{2v} - v$ by moving V to RHS

$t\frac{dv}{dt} = \frac{1-v^2}{2v}$

Variable seperable

$\frac{2v}{1-v^2}\,dv = \frac{1}{t}\,dt$

$-\ln(1-v^2) = \ln t + c$

Let $c = \ln B$

$0 = \ln(1-v^2) + \ln t + \ln B$

$0 = \ln(Bt(1-v^2))$

ie $Bt(1-v^2) = 1$

Let $\frac{1}{B} = D$ So $t(1-v^2) = D$

Recall $y = vt$ ie $v = \frac{y}{t}$ Thus $t\left(1 - \frac{y^2}{t^2}\right) = D$

6.

19 Solve $\frac{dy}{dx} = \frac{x+3y}{2x}$ $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So we have the follow

$v + x\frac{dv}{dx} = \frac{1+3v}{2}$

$x\frac{dv}{dx} = \frac{1+v}{2}$ ie $\frac{2}{1+v}dv = \frac{1}{x}dx$

So $2\ln(1+v) = \ln x + c$

By further simplification

$\ln\left[\frac{B(1+v)^2}{x^2}\right] = 0$ Note $C = -\ln B$

$\frac{B(1+v)^2}{x^2} = 1$

$\left(\frac{1+v}{x^2}\right)^2 = D$ if $D = 1/B$

but $v = y/x$

$\left(\frac{x+y}{x^2}\right)^2 = D$

20 $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$ $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So $v + x\frac{dv}{dx} = \frac{1+v^2}{v}$ $\Rightarrow$ $x\frac{dv}{dx} = \frac{1}{v}$

Variable seperable

$v\,dv = \frac{1}{x}dx$

So $\frac{v^2}{2} = \ln x + c$

but $v = y/x$ so $\frac{y^2}{2x^2} = \ln x + c$

ie $y^2 = 2x^2\ln x + 2x^2C$

(ll ... x))

21 Solve $5\frac{dy}{dt} - y = 20t$ Now divide through by 5

$\frac{dy}{dt} - \frac{1}{5}y = 4t$

Now $P(t) = -1/5$ $\int P(t)dt = -\frac{1}{5}t$ so $I.F = e^{-1/5t}$

Clearly $\frac{d}{dt}\left(ye^{-1/5t}\right) = 4te^{-1/5t}$

Integrate both sides

$$ye^{-1/5t} = 4\int te^{-1/5t}\,dt$$

Using integration by part for RHS

$$\int te^{-1/5t}\,dt = t\left(\frac{e^{-1/5t}}{-1/5}\right) - \int (1)\frac{e^{(-1/5)t}}{-1/5}\,dt$$

$$= -5te^{-1/5t} - \int -5e^{-1/5t}\,dt$$

$$= -5te^{-1/5t} + 5\int e^{-1/5t}\,dt$$

$$= -5te^{-1/5t} + 5\frac{e^{-1/5t}}{-1/5} + c$$

$$= -5te^{-1/5t} - 25e^{-1/5t} + c$$

So $ye^{-1/5t} = -20te^{-1/5t} - 100e^{-1/5t} + D$ $\quad D = 4c$

$$y = -20t - 100 + De^{1/5t}$$

22 $(1+t^2)\frac{dy}{dt} - 7ty = 6(1+t^2) \Rightarrow$ linear

Dividing through by $1+t^2$

$$\frac{dy}{dt} - \frac{7t}{1+t^2}y = 6$$

$p(t) = -\frac{7t}{1+t^2}$ $\quad \int \frac{-7t}{1+t^2} = \frac{-7}{2}\ln(1+t^2)$ $\quad I.F = e^{-7/2\ln(1+t^2)}$

So $\frac{d}{dt}\left(ye^{-7/2\ln(1+t^2)}\right)' = 6e^{-7/2\ln(1+t^2)}$

$$ye^{-7/2\ln(1+t^2)} = 6\int e^{-7/2\ln(1+t^2)}\,dt$$

Recall that $e^{\ln a} = a$ so

$$y(1+t^2)^{-7/2} = 6\int (1+t^2)^{-7/2}\,dt$$

integrating $\int (1+t^2)^{-7/2}\,dt$ is quite lengthy but note that we have

$$\frac{y}{\sqrt{(1+t^2)^7}} = 6\sqrt{\frac{t^2}{t^2+1}} - 4\sqrt{\frac{t^3}{(t^2+1)^3}} + \frac{6}{5}\sqrt{\frac{t^5}{(t^2+1)^5}} + c$$

23 $\cos t\frac{dy}{dt} - \sin t\,y = 17$, linear first order

$$\frac{dy}{dt} - \frac{\sin t}{\cos t}y = \frac{17}{\cos t} \qquad p(t) = \frac{-\sin t}{\cos t}$$

$\int -\sin t/\cos t\,dt = \ln\cos t$ $\quad$ so $I.F = e^{\ln\cos t} = \cos t$

So $\frac{d}{dt}(y\cos t) = \frac{17}{\cos t} \times \cos t$

Integrating both sides

$y\cos t = 17t + c$

$y = 17t\sec t + c\sec t$ by dividing by $\cos t$

24 $t\frac{dy}{dt} - 2y = t^2$ $y(1) = 1$

Dividing through by t

$\frac{dy}{dt} - \frac{2}{t}y = -t$ $P(t) = -\frac{2}{t}$

$\int P(t) = -2\ln t$ $I.F = e^{-2\ln t} = t^{-2}$

So

$\frac{d}{dt}(yt^{-2}) = -t \cdot t^{-2}$

Integrating both sides

$yt^{-2} = -\ln t + c$

$y = -t^2\ln t + ct^2$

Recall $y(1) = 1$ so

$1 = -1^2\ln 1 + c(1)^2$

$c = 1$

So $y = -t^2\ln t + t^2$

25 Solve $\frac{dy}{dx} + \frac{1}{x}y = x^2$

$P(x) = \frac{1}{x}$ $\int P(x) = \ln x$

$I.F = e^{\ln x} = x$ so

$\frac{d}{dx}(yx) = x^2 \cdot x$

$yx = \int x^3 dx$

$yx = \frac{x^4}{4} + c$

$y = \frac{x^3}{4} + cx^{-1}$

26 $x\frac{dy}{dx} - 5y = x^7$

Dividing through by x

$$\frac{dy}{dx} - \frac{5y}{x} = x^6 \qquad P(x) = -\frac{5}{x} \quad so \quad \int P(x) = -5\ln x$$

$$I \cdot F = e^{-5\ln x} = +x^{-5}$$

$$\frac{d}{dx}(y x^{-5}) = x^6 \cdot x^{-5}$$

Integrating both sides

$$y x^{-5} = \frac{x^2}{2} + C$$

$$y = \frac{x^7}{2} + Cx^5$$

27 $(x+1)\frac{dy}{dx} + y = (x+1)^2$

Dividing through by $x+1$

$$\frac{dy}{dx} + \frac{1}{x+1}y = x+1 \qquad P(x) = \frac{1}{x+1} \qquad \int P(x) = \ln x+1$$

$$I \cdot F = e^{\ln x+1} = x+1$$

$$\frac{d}{dx}(y(x+1)) = (x+1)(x+1)$$

Integrating both sides

$$y(x+1) = \frac{(x+1)^3}{3} + C$$

$$y = \frac{(x+1)^2}{3} + C(x+1)^{-1}$$

28 $\frac{dy}{dx} + \frac{1}{x}y = xy^2$, Bernoulli equation

Divide through by y^2

$$y^{-2}\frac{dy}{dx} + \frac{1}{x}y^{-1} = x \qquad *$$

Let $v = y^{-1}$ so $\frac{dv}{dx} = -y^{-2}\frac{dy}{dx}$

$$-\frac{dv}{dx} = +y^{-2}\frac{dy}{dx}$$

By substitution in *

$$-\frac{dv}{dx} + \frac{v}{x} = x \implies \frac{dv}{dx} - \frac{v}{x} = -x$$ Multiply by -1

Do not write in this margin

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Question............................

Write on both sides of the paper

$P(x) = -1/x \quad \int P(x)\,dx = -\ln x \quad$ so $I.F = e^{-\ln x} = +x^{-1}$

so $\frac{d}{dx}(v\cancel{x}^{-1}) = -x \cdot (x^{-1}$

$\frac{d}{dx}(vx^{-1}) = \cancel{x} - 1$

Integrating both sides

$vx^{-1} = -x + c$

But ~~by~~ $y^{-1} = v$ so

$y^{-1}x^{-1} = -x + c$

$\frac{1}{xy} = -x + c$

$-x^2y + cxy = 1$

29 $\frac{dy}{dt} + t^3y = y^4 \qquad y(1) = 4 \qquad$ Bernoulli

Dividing through by y^{-4}

$y^{-4}\frac{dy}{dt} + ty^{-3} = 1$

$v = y^{-3}$ and $\frac{-1}{4}\frac{dv}{dt} = y^{-4}\frac{dy}{dt}$

$\frac{-1dv}{4dt} + vt = 1 \Rightarrow \frac{dv}{dt} - vt = -1$

$P(t) = -t \qquad \int P(t)\,dt = \frac{-4t^2}{2}$

so $\frac{d}{dt}\left(-v\frac{t^2}{2}\right) = -1\left(-\frac{t^2}{2}\right)$

Integrating both sides

$\frac{-vt^2}{2} = \frac{4t^3}{6} + c$

but $v = y^{-1}$

$\frac{-y^{-1}t^2}{2} = \frac{4t^3}{6} + c$

Further simplication

$\frac{4yt^3}{3} + 2cy + t^2 = 0$

Let $2c = D$

$\frac{4yt^3}{3} + Dy + t^2 = 0 \quad$ you can go further if you want

Recall $y(1) = 4$ Do that yourself.

11.

30. $\frac{dy}{dt} + y = y^2 \Rightarrow y^{-2}\frac{dy}{dt} + y^{-1} = 1$

$v = y^{-1} \quad -\frac{dv}{dt} = y^{-2}\frac{dy}{dt}$

$\frac{dv}{dt} - v = -1 \qquad P(t) = -1 \qquad \int P(t)\,dt = -t$

$\frac{d}{dt}\left(v(-t)\right) = -1 \cdot (-t)$

Integrating both sides

$-vt = \frac{t^2}{2} + c$

$v = y^{-1} \Rightarrow \frac{t}{y} + \frac{t^2}{2} + c$

So $\frac{yt^2}{2} - t + Cy = 0$

Go further if you want

31. $\frac{dy}{dt} + 18y = y^{-5}e^{2t} \Rightarrow y^5\frac{dy}{dt} + 18y^6 = e^{2t}$

$v = y^6 \Leftrightarrow \frac{1}{6}\frac{dv}{dt} = y^5\frac{dy}{dt}$ thus

$y^5\frac{dy}{dt} + 18y^6 = e^{2t}$ becomes $\frac{1}{6}\frac{dv}{dt} + 18v = e^{2t}$

$\Rightarrow \frac{dv}{dt} + 108v = 6e^{2t}$

$P(t) = 108 \qquad \int P(t)\,dt = 108t \qquad I.F = e^{108t}$

$\frac{d}{dt}\left(ve^{108t}\right) = 6e^{2t} \cdot e^{108t}$

Integrating both sides

$ve^{108t} = \int 6e^{110t}\,dt$

$ve^{108t} = \frac{3}{55}e^{110t} + c$

by $v = y^6$

$y^6 = \frac{3}{55}e^{2t} + Ce^{-108t}$

32. $3t\frac{dy}{dt} + 2ty = y^{-1}t^5 \Rightarrow y\frac{dy}{dt} + \frac{2}{3}y^2 = \frac{t^4}{3} \qquad P(t) = \frac{2}{3}$

$$\cancel{\int P(t)dt = \int 2/3\, dt = 2/3\, t \;/\; I.F = e^{2/3 t}}$$

$$\frac{d}{dt}\left(ve^{2/3t}\right) =$$

$$v = y^{+2},\quad \frac{1}{2}\frac{dv}{dt} = y\frac{dy}{dt}$$

$$\frac{1}{2}\frac{dv}{dt} + \frac{2}{3}y = \frac{t^4}{3}$$

$$\frac{dv}{dt} + \frac{4}{3}v = \frac{2t^4}{3} \qquad P(t) = \frac{4}{3} \qquad \int P(t)dt = \frac{4t}{3}$$

$$I.F = e^{4/3 t}$$

$$\frac{d}{dt}\left(ve^{4/3t}\right) = \frac{2}{3}t^4 e^{4/3t}$$

Integrating both sides

$$ve^{4/3t} = \frac{2}{3}\left[\frac{t^4 e^{4/3t}}{4/3} - 4t^3\left(\frac{e^{4/3t}}{4/3^2}\right) + 12t^2\left(\frac{e^{4/3t}}{(4/3)^3}\right) - 24t\left(\right.\right.$$

$$\left.+ 24\left(\frac{e^{4/3t}}{(4/3)^5}\right)\right]$$

$$ve^{4/3t} = \frac{2}{3}\left[\frac{3t^4 e^{4/3t}}{4} - \frac{9t^3 e^{4/3t}}{4} + \frac{81t^2 e^{4/3t}}{16} - \frac{243 t e}{32}\right.$$

$$\left. + \frac{729}{128}e^{4/3t}\right] + c$$

$$\cancel{v = t^4 e^{4}}$$

$$v = \frac{t^4}{2} - \frac{3}{2}t^3 + \frac{27}{8}t^2 - \frac{81}{16}t + \frac{243}{64} + c$$

but $v = y^2$

$$y^2 = \frac{t^4}{2} - \frac{3}{2}t^3 + \frac{27}{8}t^2 - \frac{81}{16}t + \frac{243}{16} + c$$

33 $(1-t^2)\frac{dy}{dt} - 2ty = y^3 \Rightarrow y^{-3}\frac{dy}{dt} - \frac{2t}{1-t^2}y^{-2} = \frac{1}{1-t^2}$

$$v = y^{-2} \Rightarrow \frac{-1}{2}\frac{dv}{dt} = y^{-3}\frac{dy}{dt}$$

$$-\frac{1}{2}\frac{dv}{dt} - \frac{2t}{1-t^2}y^{-2} = \frac{1}{1-t^2}$$

$$\frac{dv}{dt} + \frac{2t}{1-t^2}v = -\frac{1}{1-t^2}$$

Let $P(t) = \frac{4t}{1-t^2} \Rightarrow I.F\; e^{\int P(t)dt} = e^{-2\ln(1-t^2)} = (1-t^2)^{-2}$

$$\frac{d}{dt}\left(v\left(1-t^2\right)^{-2}\right) = \left(1-t^2\right)^{-3}$$

Integrating both side

$$v\left(1-t^2\right)^{-2} = \int \frac{1}{(1-t^2)^3}\, dt$$

Will stop here for this. Go further if you want

34 $W = \begin{vmatrix} y_1(t) & y_2(t) \\ dy_1(t)/dt & dy_2(t)/dt \end{vmatrix}$

$$W = \begin{vmatrix} e^{2t} & e^{-2t} \\ 2e^{2t} & -2e^{-2t} \end{vmatrix} = -2-2 = -4$$

35 $W(t) = W(t_0)e^{\int_{t_0}^{t} \frac{B(t)}{A(t)} dt}$ Note $W(1)=1$

Note $B(t) = t$ and $A(t) = t^2$ so

$$\int_{t_0}^{t} \frac{t}{t^2}\, dt = \int_{t_0}^{t} \frac{1}{t}\, dt = [\ln t]_{t_0}^{t} = \ln t - \ln t_0$$

Hence

$$W(t) = W(1)e^{-(\ln t - \ln t_0)} \quad t_0 = 1 \text{ given}$$

$$W(t) = W(1)e^{\ln t_0 - \ln t}$$

$$W(t) = 1 \times e^{\ln 1 - \ln t}$$

$$W(t) = e^{-\ln t}$$

$$W(t) = 1/t$$

36 $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 0$ (Second Order) where $y(0) = 9$ & $\frac{dy(0)}{dt} = 5$

Form a characteristics equation

$$k^2 + 2k - 3 = 0$$

By solving quadratically $k = -3$ or 1 so

$$y = Ae^{-3t} + Be^{t} \quad \ldots *$$

differentiate $*$

$$\frac{dy}{dt} = -3Ae^{-3t} + Be^{t} \quad \ldots **$$

Given if $t=0$ $y=9$ we have in $*$

$$9 = A + B \quad \ldots (a)$$

Also if $t=0$ $\frac{dy}{dt} = 5$ so we have in $**$

$$5 = -3A + B \quad \ldots (b)$$

By solving (a) and (b) simultaneously
$A = 1 \quad B = 8$ so our particular solution is
$y = e^{-3t} + 8e^{t}$

37 Note if Discriminant is greater than zero we have real distinct roots so
$$y = Ae^{m_1 x} + Be^{m_2 x}$$

38 $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 0 \quad y(0) = 5, \; \frac{dy}{dt}(0) = 5$

Characteristic equation is $k^2 - 2k + 5 = 0$

so $k = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2}$

$$k = \frac{2 \pm \sqrt{-16}}{2} \Rightarrow \frac{2 \pm 4i}{2}$$

so $k = 1 \pm 2i$

Hence
$$y = e^{t}(A\cos 2t + B\sin 2t) \quad \ldots *$$
Now $\frac{dy}{dt} = e^{t}(-2A\sin 2t + 2B\cos 2t) + e^{t}(A\cos 2t + B\sin 2t) \quad \ldots **$

From * when $t = 0$ $y = 5$
$5 = e^{0}(A\cos 2(0) + B\sin 2(0))$
i.e. $5 = A$

From ** when $t = 0$ $\frac{dy}{dt} = 5$ so

$$5 = e^{0}(-2A\sin 2(0) + 2B(\cos 2(0)) + e^{0}(A\cos 2(0) + B\sin 2(0))$$

$5 = 2B + A$ but $A = 5$ so $B = 0$

Therefore from *
$$y = 5e^{t}\cos 2t.$$

39. $f(t) \neq 0$ since equation is inhomogeneous

40 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = Q$ but $Q = 9$

Note $y = C.F + P.I$

First solve RHS using characteristics form
From Question 38 C.F $= Ae^{-3t} + Be^{t}$

To get P.E, we assume a solution of the form

$y_p = D$

Because Q is a constant polynomial

Now differentiate y_p twice

$\frac{dy_p}{dx} = 0$ and $\frac{d^2y_p}{dx^2} = 0$

Substitute this into equation gives

$0 + 2(0) - 3(D) = Q$ but $Q = 9$ so

$-3D = 9$

$D = -3$

So $P.I = y_p = -3$ Hence

$y = Ae^{-3t} + Be^{t} - 3.$

41 $\frac{d^2y}{dt^2} + \frac{9dy}{dt} + 20y = f(t)$ $f(t) = 112e^{3t}$

From LHS $k^2 + 9k + 20 = 0$ so $k = -5$ or -4

$C.F = Ae^{-5t} + Be^{-4t}$

For P.I $f(t) = 112e^{3t}$ so $y_p = Ce^{3t}$

Differentiate twice

$\frac{dy_p}{dt} = 3Ce^{3t}$ $\frac{d^2y_p^2}{dt} = 9Ce^{3t}$

Substitute.

$9Ce^{3t} + 27Ce^{3t} + 20Ce^{3t} = f(t)$ but $f(t) = 112e^{3t}$

$56Ce^{3t} = 112e^{3t}$

Divide both sides by $56e^{3t}$

$C = 2.$

$P.I = 2e^{3t}$

Hence $y = Ae^{-5t} + Be^{-4t} + 2e^{3t}$

42 $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = f(t)$ where $f(t) = \sin 3t$

$y = C.F + P.I$

We acquire C.F is

$k^2 + 2k - 3 = 0$ From question (36)

$C.F = Ae^{-3t} + Be^{t}$

P.I, since $f(t) = \sin 3t$ $y_p = A\cos 3t + B\sin 3t$

Differentiate twice

$\frac{dy_p}{dt} = -3A\sin 3t + 3B\cos 3t$ $\frac{d^2y_p}{dt^2} = -9A\cos 3t - 9B\sin 3t$

Substitute into equation given

$-9A\cos 3t - 9B\sin 3t + 2(-3A\sin 3t + 3B\cos 3t) - 3(A\cos 3t + B\sin 3t)$

Recall $f(t) = \sin 3t$

$-9A\cos 3t - 9B\sin 3t - 6A\sin 3t + 6B\cos 3t - 3A\cos 3t - 3B\sin 3t =$

Group sines and cosines

$\cos 3t(-9A + 6B - 3A) + \sin 3t(-9B - 6A - 3B) = \sin 3t$

Comparing both sides

$\cos 3t(-9A + 6B - 3A) = 0\cos 3t$ & $\sin 3t(-9B - 6A - 3B) = \sin$

$\Rightarrow -9A + 6B - 3A = 0$ and $-9B - 6A - 3B = 1$

$-12A + 6B = 0$ and $-12B - 6A = 1$

From first $B = 2A$ put into second $-12(2A) - 6A = 1$

Thus $A = -\frac{1}{30}$ and $B = -\frac{1}{15}$ Hence we conclude

$$y_p = -\frac{1}{30}\cos 3t - \frac{1}{15}\sin 3t$$

$$y = Ae^{-3t} + Be^{t} - \frac{1}{30}\cos 3t - \frac{1}{15}\sin 3t$$

43 velocity $= v = (3t^2 + 4t)$ m/s

Note that $v = \frac{ds}{dt}$ (rate of change of distance - displ-

So $v = \frac{ds}{dt} = (3t^2 + 4t)$ m/s

ie $\frac{ds}{dt} = (3t^2 + 4t)$ m/s

To get distance in terms of t integrate both sides

$$s = t^3 + 2t^2 + c$$

At initial point when $t = 0$ $s = 0$ so $c = 0$

ie $s = t^3 + 2t^2$

Therefore distance at $t = 5$ secs

$$s = 5^3 + 2(5^2)$$

$$s = 175\,m$$

44 Given $s = (5t^3 + 3t^2)$ m

Acceleration $= \frac{d^2s}{dt^2}$ (Note acceleration is rate of change of v and velocity is rate of change of displ

at $t = 3$ sec $a = 96\,m/s^2$

so $a = \frac{d^2s}{dt^2} = 30t + 6$

45. $V = (3t+5t,\ 2t+3t^3,\ t+5t^2)\ m/s^2$

Note this is velocity in 3-Dimensions we can write

$V = (3t+5t)\hat{i} + (2t+5t^3)\hat{j} + (t+5t^2)\hat{k}$

$V = (8t)\hat{i} + (2t+5t^3)\hat{j} + (t+5t^2)\hat{k}$ Recall $v = \frac{ds}{dt}$

From Integration of vectors

$s = (4t^2)\hat{i} + (t^2 + \frac{5}{4}t^4)\hat{j} + (\frac{t^2}{2} + \frac{5}{3}t^3)\hat{k}$

So position at $t = 25$

$s = 4(25)^2\hat{i} + (25^2 + \frac{5}{4}(\frac{25}{4})^4)\hat{j} + (\frac{25^2}{2} + \frac{5}{3}(25)^3)\hat{k}$

$s = 2500\hat{i} + \frac{1955625}{4}\hat{j} + \frac{158125}{6}\hat{k}$

i.e. $s = \left(2500, \frac{1955625}{4}, \frac{158125}{6}\right)$

46 Given $x = 3t^2$, $y = 8t$ and $z = 2t^3$ then

$s = 3t^2\hat{i} + 8t\hat{j} + 2t^3\hat{k}$

so $\frac{ds}{dt} = v = 6t\hat{i} + 8\hat{j} + 6t^2 k$

At $t = 2$

$v = 12\hat{i} + 8\hat{j} + 24k$

Magnitude v

i.e. $|v| = \sqrt{12^2 + 8^2 + 24^2}$

$|v| = \sqrt{144 + 64 + 576}$

$|v| = \sqrt{784}$

$|v| = 4\sqrt{177}\ m/s$

47 $\vec{a} = (2t, 4\cos 3t, 4\sin 3t)\ m/s$

Note $\vec{a} = \frac{d\vec{v}}{dt}$ so

$\frac{dv}{dt} = 2t\hat{i} + 4\cos 3t\hat{j} + 4\sin 3t\hat{k}$

so $v = t^2\hat{i} + \frac{4}{3}\sin 3t\hat{j} - \frac{4}{3}\cos 3t\hat{k}$

A: $|v| = \sqrt{(t^2)^2 + (\frac{4}{3})^2\sin^2 3t + (\frac{4}{3})^2\cos^2 3t}$

$|v| = \sqrt{t^4 + (\frac{4}{3})^2(\sin^2 3t + \cos^2 3t)}$ Note $\sin^2 3t + \cos^2 3t = 1$

$|v| = \sqrt{t^4 + (\frac{4}{3})^2(1)}$

$|v| = \sqrt{t^4 + (\frac{4}{3})^2}$ at $t = 0$ because it was at rest

$|v| = \frac{4}{3}\ m/s$

48. $x = 3e^{-t}$ $y = 5\sin 2t$ and $z = 4\cos 2t$

$s = 3e^{-t}\hat{i} + 5\sin 2t\hat{j} + 4\cos 2t\hat{k}$

$a = \frac{d^2s}{dt^2}$, we differentiate twice

$\frac{ds}{dt} = -3e^{-t}\hat{i} + 10\cos 2t\hat{j} - 8\sin 2t\hat{k}$

$\frac{d^2s}{dt^2} = 3e^{-t}\hat{i} - 20\sin 2t\hat{j} - 16\cos 2t\hat{k}$

at $t = 0$

$\vec{a} = \frac{d^2s}{dt^2} = 3\hat{i} - 16\hat{k}$

hence $|\vec{a}| = \sqrt{265}$ m/s²

49 $s = (5t^3 + 2t^2)$ m

$\vec{v} = \frac{ds}{dt}$ so $v = 15t^2 + 4t$ hence $-$ t = 3 ...

$\cancel{v} = \cancel{240 + 16} = \cancel{256\,m/s}$ $v = 135 + 12 = 147$ m/s

50 $\frac{d^2y}{dt^2} + \frac{9dy}{dt} + 20y = 0$

$k^2 + 9k + 20 = 0$ $\Rightarrow k = -5$ or -4 so

$y = Ae^{-5t} + Be^{-4t}$

51 $\frac{d^2y}{dt} - 2\frac{dy}{dt} + y = 0$ $y(1) = e = y'$

Note $y' = dy/dt$

$k^2 - 2k + k = 0$ $\Rightarrow k = 1$ or 1

$y = (A + Bt)e^t$ ... *

Differentiate y

$y' = (A + Bt)e^t + Be^t$ (Product rule) ... **

In * we have ($t = 1$ $y = e$)

$\cancel{e} = (A + B)\cancel{e}$ $1 = A + B$ ... (a)

In * we have $t = 1$ and $y' = e$

$\cancel{e} = (A + B)\cancel{e} + B\cancel{e}$ $\Rightarrow$ $1 = A + 2B$ ... (b)

From (a) and (b) $B = 0$ and $A = 1$

So $y = Ae^t$. from *.

52 $y = Ae^{m_1x} + Be^{m_2x}$

54 $W(t) = \begin{vmatrix} y_1 & y_2 \\ dy_1/dt & dy_2/dt \end{vmatrix}$ $y_1 = \cosh mt$ and $y_2 = \sinh mt$

So $\frac{dy_1}{dt} = m \sinh mt$ $\frac{dy_2}{dt} = m \cosh mt$

$$W(t) = \begin{vmatrix} \cosh mt & \sinh mt \\ m \sinh mt & m \cosh mt \end{vmatrix}.$$

$$= m \cosh^2 mt - m^2 \sinh^2 mt$$

$$= m(\cosh^2 mt - \sinh^2 mt)$$

Note $\cosh^2 mt - \sinh^2 mt = 1$ so

$$W(t) = m$$

55 Since $A(t) = 0$ equation becomes linear and also $f(t) = 0$ so we have a "homogeneous linear first order"

56 $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 5 = 0$ $y(\pi) = y'(\pi) = e^{3\pi/2}$

$$K^2 - 6K + 5 = 0 \Rightarrow K = 5 \text{ or } 1. \text{ so}$$

$$y = Ae^{5t} + Be^{t} \quad --- *$$

Differentiate $*$ we have

$$y' = 5Ae^{5t} + Be^{t} \quad --- **$$

At $t = \pi$ $y = e^{3\pi/2}$

$$e^{3\pi/2} = Ae^{5\pi} + Be^{\pi} \quad --- (a)$$

Also $t = \pi$ $y' = e^{3\pi/2}$

$$e^{3\pi/2} = 5Ae^{5\pi} + Be^{\pi} \quad --- (a)$$

Equating (a) and (b)

$$Ae^{5\pi} + Be^{\pi} = 5Ae^{5\pi} + Be^{\pi}$$

$$4Ae^{5\pi} = 0 \quad \text{so}$$

$$A = 0.$$

Put A into (a) $e^{3\pi/2} = Be^{\pi}$ so $B = e^{\pi/2}$

Hence

$$y = e^{t + \pi/2}$$

57 Homogeneous of first order if $A = 0$ and $Q(x) = 0$

58 $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} = 0$ $K^2 - 3K = 0$ $K = 0$ or 3

So $y = Ae^{3t} + Be^{0t}$

$$y = Ae^{3t} + B$$

59 $y'' + 0.2y' + 4.01y = 0 \quad y(0) = 0 \quad y'(0) = 2$

$$k^2 + 0.2k + 4.01 = 0$$

So

$$k = \frac{-0.2 \pm \sqrt{(0.2)^2 - 4(4.01)}}{2}$$

$$k = \frac{-0.2 \pm \sqrt{0.04 - 16.04}}{2}$$

$$k = \frac{-0.2 \pm \sqrt{-16}}{2}$$

$$k = \frac{-0.2 \pm 4i}{2}$$

$$k = -0.1 \pm 2i$$

Solution generally is $y = e^{-0.1x}(A\cos 2x + B\sin 2x)$ - - *

Now we differentiate *

$$y' = e^{-0.1x}(-2A\sin 2x + 2B\cos 2x) + -0.1e^{-0.1x}(A\cos 2x + B\sin 2x) \quad \text{- - **}$$

From * $\quad 0 = e^0(A) \Rightarrow A = 0$

From **

$$2 = e^0(2B) - 0.1e^0(A) \Rightarrow B = 1$$

Hence

$$y = e^{-0.1x}\sin 2x$$

60 $y'' + 2ay' + a^2y = 0$

$$k^2 + 2ak + a^2 = 0 \Rightarrow k = \frac{-2a \pm \sqrt{4a^2 - 4a^2}}{2}$$

$$k = \frac{-2a}{2} = -a \quad \text{ie}$$

$$y = (A + Bx)e^{-ax}$$

61 $8y'' - 2y' - y = 0 \qquad y(0) = -0.2 \quad y'(0) = -0.325$

$$8k^2 - 2k - 1 = 0 \Rightarrow k = \frac{2 \pm \sqrt{4 + 32}}{16}$$

$$k = \frac{2 \pm 6}{16} \Rightarrow k = \frac{1}{2} \text{ or } k = -\frac{1}{4}$$

So $y = Ae^{+\frac{1}{2}x} + Be^{-\frac{1}{4}x}$ - - *

Differentiating * we have

$$y' = \frac{1}{2}Ae^{\frac{1}{2}x} - \frac{1}{4}Be^{-\frac{1}{4}x} \quad \text{- - **}$$

If $x = 0$ $y = -0.2$ from *

$-0.2 = A + B$ - - (a)

If $x = 0$ $y' = -0.325$

$-0.325 = \frac{1}{2}A - \frac{1}{4}B$ - - (b)

From (a) and (b)

So $A = -0.5$ $B = 0.3$

So $y = -0.5e^{\frac{1}{2}x} + 0.3e^{-\frac{1}{4}x}$

mistake check Question 65

65 $m = 30\,Kg$ & $x(t) = (3t^2)m$, $y(t) = (2t^3 + 5t^2)m$

Note given only x and y implies 2-D

Recall Position is given

$$s = x\hat{i} + y\hat{j} \quad \text{so we have}$$

$$s = 3t^2\hat{i} + (2t^3 + 5t^2)\hat{j}$$

at $t = 2s$

$$s = 3(2)^2\hat{i} + (2(2)^3 + 5(2)^2)\hat{j}$$

$$s = (12\hat{i} + 36\hat{j})m$$

We are asked for position vector not magnitude

62 Given $s = \left(\frac{7}{2}t^4 - 3t^3 + 5t^2 + 9t - 10\right)m$ $\quad \vec{a} = \frac{d^2s}{dt^2}$ is

$$\frac{ds}{dt} = 14t^3 - 9t^2 + 10t + 9$$

$$\Rightarrow \frac{d^2s}{dt^2} = \vec{a} = 42t^2 - 18t + 10$$

So at $t = 5s$

$$\vec{a} = 42(5^2) - 18(5) + 10$$

$$= 1050 - 90 + 10$$

$$= 970\,m/s^2$$

63 Bernoulli equation. Work this in class.

64 We are only concerned with $\vec{a}$ as

$$\vec{a} = 3e^{-2t}\hat{i} + 4\sin 5t\,\hat{j} + 4\cos 5t\,\hat{k}$$

$$|\vec{a}|^2 = (3e^{-2t})^2 + 4^2\sin^2 5t + 4^2\cos^2 5t$$

$$|\vec{a}|^2 = 9e^{-6t} + 4^2(\sin^2 5t + \cos^2 5t)$$

$y = 1.3x - 2.12$

but $\cos^2\theta + \sin^2\theta = 1$, compare with equation

22 - $|\vec{a}|^2 = 9e^{-6t} + 16$

at $t = 0$

$|a|^2 = 9 + 16 \Rightarrow |a| = 5 m/s^2$

65 Already solved 65 on page 21

66 Mass not needed ; $x(t) = (3t^2)m$ $y(t) = (2t^3 + 5t^2)m$

So $\vec{s} = 3t^2\hat{i} + (2t^3 + 5t^2)\hat{j}$

$\vec{v} = \frac{d\vec{s}}{dt} = 6t\hat{i} + (6t^2 + 10t)\hat{j}$

at $t = 2s$

$\vec{v} = (12\hat{i} + 44\hat{j})\ m/s$

67 Mass not needed ; $x(t) = (3t^2)m$ $y(t) = (2t^3 + 5t^2)m$

So $\vec{s} = 3t^2\hat{i} + (2t^3 + 5t^2)\hat{j}$

$\vec{a} = \frac{d^2s}{dt^2}$ hence

$\vec{a} = 6\hat{i} + (12t + 10)\hat{j}$

at $t = 2s$

$\vec{a} = (6\hat{i} + 34\hat{j})\ m/s$

68 Mass $= 30kg$ and $x(t) = (3t^2)m$ and $y(t) = (2t^3 + 5t^2)m$

Recall $\vec{F} = m\vec{a}$

Now $\vec{a} = (6\hat{i} + 34\hat{j})$ as solved in question 67

so $\vec{F} = 30(6\hat{i} + 34\hat{j})$

$\vec{F} = (180\hat{i} + 1020\hat{j})\ N$

69 $M = 30kg$ and $x(t) = (3t^2)m$ and $y(t) = (2t^3 + 5t^2)m$

$W = |F| \times |\vec{s}|$

But $\vec{F} = (180\hat{i} + 1020\hat{j})N$ as gotten from question 68

Also $s = 3t^2\hat{i} + (2t^3 + 5t^2)\hat{j}$

at $t = 2s$ $s = 12\hat{i} + 34\hat{j}$

Now

$|\vec{F}| = \sqrt{180^2 + 1020^2} \approx 1036 N$

$|\vec{s}| = \sqrt{12^2 + 34^2} \approx 36 m$

so $W = 1036N \times 36m = 37\ 296\ kJ$

23

70 Mass = 30 Kg $|\vec{F}| = 1036N$ from (69) and $|\vec{S}| = 36m$

Given $\theta = 60°$

$W = Fs\cos\theta$

$W = 1036N \times 36m \times \cos 60°$

$W = 18.648\ KJ$

71 Workdone $= Fs\cos\theta$

72 Not optional

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Question.........................

Write on both sides of the paper

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(1) $\frac{d^4y}{dt^4} + 4\frac{d^2y}{dt^2} + 7y = 9$

order $=4$, deg $=1$

(2) $\left(\frac{dy}{dt}\right)^3 + y^2 = 8$ deg $=3$, ord $=1$

(3) $\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^{3/2} - 3y = 49$

$\therefore \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} - 3y - 49 = \left(\frac{dy}{dt}\right)^{3/2}$

$\therefore \left(\frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} - 3y - 49\right)^2 = \left(\frac{dy}{dt}\right)^3$

ord $=3$, deg $=2$.

(4) $\left(\frac{d^5y}{dt^5}\right)^2 - \left(\frac{dy}{dt}\right)^{5/2} = 11$

i.e. $\left(\left(\frac{d^5y}{dt^5}\right)^2 - 11\right)^2 = \left(\frac{dy}{dt}\right)^5$

ord $=5$, deg $=4$ (i.e. $2+2$)

which is $\left(\left(\frac{d^5y}{dt^5}\right)^2\right)^2 = \left(\frac{d^5y}{dt^5}\right)^4$

(5) $y = Ae^{2x} + Be^{-x}$

put $k=2$ and $k=-1$

$(k-2)(k+1)=0$

$\therefore k^2+k-2k-2=0$

i.e. $k^2-k-2=0$

$\therefore \boxed{y''-y'-2y=0}$ or

$\boxed{\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0}$

(6) $y = x + \frac{A}{x}$ $\therefore y' = 1 - \frac{A}{x^2}$

but $y - x = \frac{A}{x}$ $\therefore A = x(y-x)$

and so;

$y' = 1 - \frac{A}{x^2} = 1 - \frac{x(y-x)}{x^2}$

$\therefore y' = 1 - \frac{(y-x)}{x}$

$\therefore xy' = x - y + x$ $\therefore \boxed{xy' + y - 2x = 0}$

(7) (a) $\frac{dy}{dt} + y\sin t = e^t$ is linear but not homo.

(b) $\frac{d^3y}{dt^3} + 7t\left(\frac{d^2y}{dt^2}\right)^{1/3} - y^{-1} = 11$ not linear not homo.

(c) $\frac{d^5y}{dt^5} - e^y = 0$ not linear, and homo.

(d) $\frac{d^3y}{dt^3} = t^2y$. is linear and homo.

(8) (a) $t\frac{dy}{dt^2} - 16t^2y = \cos t$ Ord. dep $=y$, Ind $=t$

(b) $t\frac{dy}{dt} - e^{2t} = 0$, ord., dep $=y$, ind $=t$.

(c) $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, part., dep $=u$, ind $=x,t$

(d) $\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial^2 x} = 5x$, part., dep $=u$, ind $=x,y$

(9) (a) (V.S)

$\sin t\frac{dy}{dt} + y\cos t = 0$ $\therefore \sin t\frac{dy}{dt} = -y\cos t$

separating

$\therefore \frac{dy}{dt} = -y\frac{\cos t}{\sin t}$ $\therefore \frac{1}{y}dy = -\cot t\,dt$

$\therefore \int\frac{1}{y}dy = -\int\cot t\,dt$ $\therefore \ln y = -\ln(\sin t) + \ln A$

$\therefore \ln y = \ln A - \ln(\sin t)$ $\therefore \cancel{\ln}y = \cancel{\ln}\left(\frac{A}{\sin t}\right)$ $\therefore y = \frac{A}{\sin t}$

or $\boxed{y\sin t = A}$

(b) (V.S) $t^{-3}\frac{dy}{dt} + y = 0$ (V.S) $\therefore t^{-3}\frac{dy}{dt} = -y$ $\therefore \frac{1}{y}dy = \frac{-1}{t^{-3}}dt$

$\therefore \int\frac{1}{y}dy = -\int t^3dt$ $\therefore \ln y = -\frac{t^4}{4} + A$

$\ln y = A - \frac{t^4}{4}$

$\therefore y = e^{A - t^4/4}$ $\therefore y = e^A \cdot e^{-t^4/4}$ $\therefore \boxed{y = Be^{-\frac{t^4}{4}}}$

In general for homo linear first order:

$\frac{dy}{dt} + P(t)y = 0$ $\therefore \boxed{y = Be^{-\int P dt}}$

(2)

(V.S)

(c) (V.S) first order linear homo.

$\frac{dy}{dt} - 5t^2 y = 0$ ∴ $P = -5t^2$

∴ Soln ⇒ $y = B e^{-\int -5t^2 dt}$

∴ $\boxed{y = B e^{5t^3/3}}$

(d) $\frac{dy}{dx} = 3x^2 - 6x + 5$ (V.S)

∴ $\int dy = \int (3x^2 - 6x + 5) dx$

∴ $\boxed{y = x^3 - 3x^2 + 5x + A}$

(12) $e^x \frac{dy}{dx} = 4$ $y(0) = 3$

∴ $\int dy = 4 \int e^{-x} dx$ ∴ $y = 4 \frac{e^{-x}}{-1} + A$

i.e $y = A - 4e^{-x}$ ∴ at $x = 0, y = 3$ ∴ $A = 7$

∴ $\boxed{y = 7 - 4e^{-x}}$

(13) $\frac{dy}{dt} = t^2 (1 + y^2)$ (V.S)

$\int \frac{1}{1+y^2} dy = t^2 dt$ ∴ $\boxed{\tan^{-1} y = \frac{t^3}{3} + A}$

(e) (V.S) or first order linear homo.

$\frac{dy}{dt} + y \sin t = 0$, $y(0) = 2$

∴ $y = B e^{-\int P dt}$

∴ $y = B e^{-\int \sin t dt}$

∴ $y = B e^{\cos t}$ at $t = 0$, $y = 2$

∴ $2 = B e^1$ ∴ $B = 2/e$

∴ $y = \frac{2}{e} \cdot e^{\cos t}$

∴ $\boxed{y = 2 \cdot e^{(\cos t - 1)}}$

(14) (V.S) $\sin t \sin y + \cos y \frac{dy}{dt} = 0$ ∴ $\cos y \frac{dy}{dt} = -\sin t \sin y$

∴ $\frac{\cos y}{\sin y} dy = -\sin t dt$ ∴ $\int \cot y \, dy = -\int \sin t dt$

∴ $\ln(\sin y) = \cos t + A$

∴ $\sin y = e^{\cos t} \cdot e^A$ ∴ $\boxed{\sin y = B e^{\cos t}}$

(15) (V.S) $\frac{dy}{dt} = t^2 (t+1) e^{-2y}$ ∴ $\frac{1}{e^{-2y}} dy = (t^3 + t^2) dt$

∴ $\int e^{2y} dy = \int (t^3 + t^2) dt$ ∴ $\frac{e^{2y}}{2} = \frac{t^4}{4} + \frac{t^3}{3} + A$

∴ $\boxed{6e^{2y} = 3t^4 + 4t^3 + B}$, where $B = 12A$.

(f) (V.S) $t \frac{dy}{dt} + 5y = 0$, $y(1) = 1$

$t \frac{dy}{dt} = -5y$ ∴ $\frac{1}{y} dy = -\frac{5}{t} dt$

∴ $\ln y = -5 \ln t + \ln A$

∴ $\ln y = \ln A - 5 \ln t$

∴ $\ln y = \ln A - \ln t^5$

∴ $\cancel{\ln} y = \cancel{\ln} \left(\frac{A}{t^5}\right)$

∴ $y = \frac{A}{t^5}$ ∴ $A = y t^5$

at $t = 1, y = 1$ ∴ $A = 1$

∴ $\boxed{1 = t^5 y}$

(16) (V.S) $\frac{dy}{dx} = \frac{y^2 + xy^2}{x^2 y - x^2}$ ∴ $\frac{dy}{dx} = \frac{y^2(1+x)}{x^2(y-1)}$

∴ $\frac{dy}{dx} = \left(\frac{y^2}{y-1}\right)\left(\frac{1+x}{x^2}\right)$ ∴ $\left(\frac{y-1}{y^2}\right) dy = \left(\frac{1+x}{x^2}\right) dx$

∴ ⇒ $\left(\frac{1}{y} - \frac{1}{y^2}\right) dy = \left(\frac{1}{x^2} + \frac{1}{x}\right) dx$ ∴ $\int \left(\frac{1}{y} - y^{-2}\right) dy = \int \left(x^{-2} + \frac{1}{x}\right) dx$

$\ln y - \frac{y^{-1}}{-1} = \frac{x^{-1}}{-1} + \ln x + A$ ∴ $\boxed{\ln y + \frac{1}{y} = \ln x - \frac{1}{x} + A}$

(17) (HM) $\frac{dy}{dt} = \frac{t+y}{t-y}$ put $y = vt$ ∴ $\frac{dy}{dt} = v + t\frac{dv}{dt}$

∴ $v + t\frac{dv}{dt} = \frac{\cancel{t} + v\cancel{t}}{\cancel{t} - v\cancel{t}}$ ∴ $v + t\frac{dv}{dt} = \frac{1+v}{1-v}$

∴ $t\frac{dv}{dt} = \frac{1+v}{1-v} - v = \frac{1 + v - v + v^2}{1-v} = \frac{1+v^2}{1-v}$

$$t\frac{dv}{dt} = \frac{1+v^2}{1-v} \quad \therefore \frac{1-v}{1+v^2}\,dv = \frac{1}{t}\,dt$$

$$\int\left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\right)dv = \int\frac{1}{t}\,dt$$

$$\int\left(\frac{1}{1+v^2} - \frac{1}{2}\cdot\frac{2v}{1+v^2}\right)dv = \int\frac{1}{t}\,dt$$

$$\therefore \tan^{-1}v - \frac{1}{2}\ln(1+v^2) = \ln t + A$$

$$\therefore \tan^{-1}(y/t) - \frac{1}{2}\ln\left(1+\frac{y^2}{t^2}\right) = \ln t + A$$

$$\tan^{-1}(y/t) - \frac{1}{2}\ln\left(\frac{t^2+y^2}{t^2}\right) = \ln t + A$$

$$\tan^{-1}(y/t) = \ln t + \ln\left(\frac{t^2+y^2}{t^2}\right)^{1/2} + A$$

$$\tan^{-1}(y/t) = \ln\left(t\cdot\frac{\sqrt{t^2+y^2}}{t}\right) + A$$

$$\therefore \boxed{\tan^{-1}(y/t) = \ln\left(\sqrt{y^2+t^2}\right) + A}$$

18 (H.m) $\frac{dy}{dt} = \frac{t^2+y^2}{2ty}$. put $y = vt$

$$v + t\frac{dv}{dt} = \frac{t^2+v^2t^2}{2t\cdot vt} \quad \therefore = \frac{1+v^2}{2v}$$

$$\therefore t\frac{dv}{dt} = \frac{1+v^2}{2v} - v = \frac{1+v^2-2v^2}{2v}$$

$$\therefore t\frac{dv}{dt} = \frac{1-v^2}{2v} \quad \therefore \int\frac{2v}{1-v^2}\,dv = \int\frac{1}{t}\,dt$$

$$\therefore -\ln(1-v^2) = \ln t + \ln A$$

$$\therefore \cancel{\ln}(1-v^2)^{-1} = \cancel{\ln}(At)$$

$$\therefore \frac{1}{1-v^2} = At \quad \therefore 1 = At\left(1 - \frac{y^2}{t^2}\right)$$

$$\therefore 1 = At\left(\frac{t^2-y^2}{t^2}\right) \quad \therefore 1 = \frac{A(t^2-y^2)}{t}$$

$$\therefore \boxed{A(t^2-y^2) = t}$$

19 (H.m) $\frac{dy}{dx} = \frac{x+3y}{2x}$, put $y = vx$

$$\therefore v + x\frac{dv}{dx} = \frac{x+3vx}{2x} = \frac{1+3v}{2}$$

$$x\frac{dv}{dx} = \frac{1}{2} + \frac{3}{2}v - v = \frac{1}{2} + \frac{1}{2}v$$

i.e $x\frac{dv}{dx} = \frac{1}{2}(1+v)$

$$\therefore \int\frac{1}{1+v}\,dv = \int\frac{1}{2x}\,dx \quad \therefore \ln(1+v) = \frac{1}{2}\ln x + \ln A$$

$$\therefore \cancel{\ln}(1+v) = \cancel{\ln}(Ax^{1/2}) \quad \therefore (1+v) = A(\sqrt{x})$$

$$\therefore (1+v)^2 = A^2x \quad \therefore \left(1+\frac{y}{x}\right)^2 = Bx \quad . B = A^2$$

$$\therefore \left(\frac{x+y}{x}\right)^2 = Bx \quad \therefore \boxed{(x+y)^2 = Bx^2} \quad \text{or}$$

$$x^2+2xy+y^2 = Bx^2 \quad \therefore y^2+2xy = Bx^2 - x^2$$

$$\therefore y^2+2xy = x^2(B-1) \quad \boxed{\therefore y^2+2xy = Cx^2}$$

$$\boxed{C = B-1}$$

20 $\frac{dy}{dx} = \frac{x^2+y^2}{xy}$, put (H.m) $y = vx$.

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{vx^2} = \frac{1+v^2}{v}$$

$$\therefore x\frac{dv}{dx} = \frac{1+v^2}{v} - v = \frac{1+v^2-v^2}{v} = \frac{1}{v}$$

$$\therefore v\,dv = \frac{1}{x}\,dx \quad \therefore \frac{v^2}{2} = \ln x + A$$

$$\therefore \frac{y^2}{2x^2} = \ln x + A \quad \therefore y^2 = 2x^2(\ln x + A) \quad \text{or}$$

$$y^2 = x^2(\ln x^2 + B) \quad \text{where } B = 2A.$$

21 L $\quad 5\frac{dy}{dt} - y = 20t \quad \therefore \frac{dy}{dt} - \frac{1}{5}y = 20t$

$$I.f = e^{\int P} = e^{-\frac{1}{5}\int dt} = e^{-\frac{1}{5}t} \qquad P = -1/5 ,\ Q = 20t.$$

$$\therefore (I.f)\,y = \int (I.f)\,Q\,dt$$

$$y\,e^{-1/5t} = \int 20t\,e^{-1/5t}\,dt \qquad \text{using Int. by Part i.e (D.I)}$$

$$= 20\int t.e^{-1/5t}\,dt$$

	D	I
+	t	$e^{-1/5t}$
−	1	$e^{-1/5t}/-\frac{1}{5}$
	0	$e^{-1/5t}/\frac{1}{25}$

$$y\,e^{-1/5t} = 20\left[-5t\,e^{-1/5t} - 25\,e^{-1/5t}\right] + A$$

$$y = A\,e^{1/5t} - 100t - 500$$

22) (L) $(1+t^2)\frac{dy}{dt} - 7ty = 6(1+t^2)$

$\therefore \frac{dy}{dt} - \frac{7t}{1+t^2}\cdot y = 6$

$\therefore P = \frac{-7t}{1+t^2} \quad \therefore Q = 6.$

$I.f = e^{-\frac{7}{2}\int\frac{2t}{1+t^2}dt} = e^{-\frac{7}{2}\ln(1+t^2)}$

$= \cancel{e}^{\cancel{\ln}(1+t^2)^{-\frac{7}{2}}} = (1+t^2)^{-\frac{7}{2}}$

$\therefore (I.f)y = \int (I.f)Q\,dt$

$y(1+t^2)^{-\frac{7}{2}} = \int (1+t^2)^{-\frac{7}{2}}.6\,dt$

$$\boxed{\frac{y}{(1+t^2)^{\frac{7}{2}}} = 6\int\frac{1}{(1+t^2)^{\frac{7}{2}}}dt.} \ (*)$$

Consider (*) $\int\frac{1}{(1+t^2)^{\frac{7}{2}}}dt$, put $t = \tan\theta$, $dt = \sec^2\theta\,d\theta$

$\Rightarrow \int\frac{1}{(1+\tan^2\theta)^{\frac{7}{2}}}\cdot\sec^2\theta\,d\theta = \int\frac{1}{(\sec^2\theta)^{\frac{7}{2}}}\cdot\sec^2\theta\,d\theta$

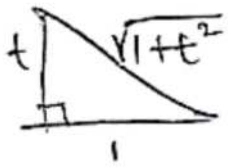
$= \int\frac{1}{\sec^7\theta}\sec^2\theta\,d\theta = \int\frac{1}{\sec^5\theta}d\theta = \int\cos^5\theta\,d\theta$

$= \int\cos^4\theta.\cos\theta\,d\theta = \int(1-\sin^2\theta)^2\cos\theta\,d\theta$

$= \int(1-2\sin^2\theta+\sin^4\theta)\cos\theta\,d\theta$

$= \int(\cos\theta - 2\sin^2\theta\cos\theta + \sin^4\theta\cos\theta)d\theta = \sin\theta - \frac{2\sin^3\theta}{3} + \frac{\sin^5\theta}{5}$

where $\frac{t}{1} = \tan\theta$



$\therefore \sin\theta = \frac{t}{\sqrt{1+t^2}} \quad \therefore \int = \frac{t}{\sqrt{1+t^2}} - \frac{2}{3}\frac{t^3}{(\sqrt{1+t^2})^3} + \frac{1}{5}\frac{t^5}{(\sqrt{1+t^2})^5}$

i.e $\int = \frac{t}{(1+t^2)^{1/2}} - \frac{2}{3}\frac{t^3}{(1+t^2)^{3/2}} + \frac{1}{5}\frac{t^5}{(1+t^2)^{5/2}}$

$\therefore \frac{y}{(1+t^2)^{\frac{7}{2}}} = 6\left[\frac{t}{(1+t^2)^{1/2}} - \frac{2}{3}\frac{t^3}{(1+t^2)^{3/2}} + \frac{1}{5}\frac{t^5}{(1+t^2)^{5/2}}\right] + A$

$\therefore y = 6t(1+t^2)^3 - 4t^3(1+t^2)^2 + \frac{6}{5}t^5(1+t^2) + A(1+t^2)^{\frac{7}{2}}$

23) (L), $\cos t\frac{dy}{dt} - \sin t\, y = 17$

$\therefore \frac{dy}{dt} - \frac{\sin t}{\cos t}y = \frac{17}{\cos t} \quad \therefore \frac{dy}{dt} - \tan t\, y = \frac{17}{\cos t}$

$P = -\tan t, \quad Q = 17/\cos t.$

$\therefore I.f = e^{-\int\tan t\,dt} = \cancel{e}^{(\cancel{\ln}(\cos t))}$

$(I.f) = \cos t.$

$\therefore (I.f)y = \int(I.f)Q\,dt$

$\cos t\, y = \int \cancel{\cos t}.\frac{17}{\cancel{\cos t}}dt = 17t + A$

$\therefore \cos t\, y = 17t + A \quad \therefore \boxed{y = \sec t(17t+A)}$

4) (L) $t\frac{dy}{dt} - 2y = -t^2$, $y(1) = 1$

$\frac{dy}{dt} - \frac{2}{t}y = -t$, $P = -2/t$, $Q = -t$

$I.f = e^{-2\int\frac{1}{t}dt} = e^{-2\ln t} = \cancel{e}^{\cancel{\ln}t^{-2}}$

$= t^{-2}, \quad \therefore (I.f)y = \int(I.f)Q\,dt$

$\therefore t^{-2}.y = \int t^{-2}.(-t)\,dt = -\int\frac{t}{t^2}dt = -\int\frac{1}{t}dt$

$\therefore \frac{y}{t^2} = -\ln t + A \quad \boxed{\therefore y = t^2(A - \ln t)}$

25) (L) $\frac{dy}{dx} + \frac{1}{x}y = x^2$, $P = 1/x$, $Q = x^2$

$\therefore I.f = e^{\int\frac{1}{x}dx} = e^{\ln x} = x.$

$\therefore (I.f)y = \int(I.f)Q\,dx \Rightarrow$

$xy = \int x.x^2dx = \int x^3dx = \frac{x^4}{4} + A$

$\therefore xy = \frac{x^4}{4} + A \quad \therefore \boxed{4xy = x^4 + B} \quad \boxed{B = 4A}$

(L.) $x\frac{dy}{dx} - 5y = x^7$

$\therefore \frac{dy}{dx} - \frac{5}{x}y = x^6$ $\quad P = \frac{-5}{x}, Q = x^6$

$I.f = e^{-5\int \frac{1}{x}dx} = e^{-5\ln x} = \cancel{e^{\ln x^{-5}}}$

$= x^{-5}$

$\therefore y(I.f) = \int (I.f)Q\,dx$

$y \cdot x^{-5} = \int (x^{-5}).x^6.dx$

$y/x^5 = \int x\,dx = x^2/2 + A$

$\therefore y = x^5(x^2/2 + A)$ or

$\boxed{2y = x^5(x^2+B)}$, $B = 2A$

27 (L.) $(x+1)\frac{dy}{dx} + y = (x+1)^2$

$\therefore \frac{dy}{dx} + \frac{1}{x+1}.y = (x+1)$

$P = \frac{1}{x+1}, Q = x+1$

$I.f = e^{\int \frac{1}{1+x}dx} = \cancel{e^{\ln(x+1)}}$

$= (x+1)$

$\therefore y(I.f) = \int (I.f)Q\,dx$

$y(x+1) = \int (x+1)^2 dx = \frac{(x+1)^3}{3} + A$

$y(x+1) = \frac{(x+1)^3}{3} + A$

$(x+1)y = \left(\frac{(x+1)^3}{3} + A\right)$

28 (B.) $\frac{dy}{dx} + \frac{1}{x}y = xy^2$

$y^{-2}\frac{dy}{dx} + \frac{1}{x}y^{-1} = x$, put $v = y^{-1}$

$\therefore -\frac{dv}{dx} = y^{-2}\frac{dy}{dx}$

$\therefore -\frac{dv}{dx} + \frac{1}{x}v = x$ $\therefore \frac{dv}{dx} - \frac{1}{x}v = -x$

$\therefore P = -1/x, Q = -x$

$I.f = e^{-\int \frac{1}{x}dx} = e^{-\ln x} = \cancel{e^{\ln x^{-1}}}$

$\therefore I.f = 1/x$ $\therefore (I.f)y = \int (I.f)Q\,dx$

$\therefore v.1/x = \int -x.1/x\,dx = -\int dx = -x + A$

i.e $\frac{v}{x} = -x + A$ $\therefore \frac{y^{-1}}{x} = A - x$ i.e $\boxed{1 = xy(A-x)}$

29 (B.) $\frac{dy}{dt} + t^3y = y^4$, $y(1) = 4$

$\therefore y^{-4}\frac{dy}{dt} + t^3.y^{-3} = 1$, put $v = y^{-3}$

$\therefore -\frac{1}{3}\frac{dv}{dt} = y^{-4}\frac{dy}{dt}$

$-\frac{1}{3}\frac{dv}{dt} + t^3.v = 1$ $\Rightarrow \frac{dv}{dt} - 3t^3.v = -3$

$P = -3t^3$, $Q = -3$ $\quad I.f = e^{\int -3t^3dt} = e^{-3t^4/4}$

$\therefore v(I.f) = \int -3.e^{-3t^4/4}dt$

30 $\frac{dy}{dt} + y = y^2$ (B.) or use (V.S).

$\therefore y^{-2}\frac{dy}{dt} + 1.y^{-1} = 1$ $\quad v = y^{-1}$ $\therefore -\frac{dv}{dt} = y^{-2}\frac{dy}{dt}$

$\therefore -\frac{dv}{dt} + 1.v = 1$ $\therefore \frac{dv}{dt} - v = -1$ $\therefore P = -1, Q = -1$

$\therefore I.f = e^{\int -dt} = e^{-t}$

$v(I.f) = \int (I.f)Q\,dt$ $\therefore ve^{-t} = \int -.e^{-t}dt$

$\therefore ve^{-t} = -\frac{e^{-t}}{-1} + A$ $\therefore ve^{-t} = e^{-t} + A$

$\therefore v = 1 + Ae^t$ $\therefore y^{-1} = 1 + Ae^t$

i.e $1 = y(1 + Ae^t)$

31 (B.) $\frac{dy}{dt} + 18y = y^{-5}e^{2t}$.

$y^5\frac{dy}{dt} + 18y^6 = e^{2t}$ $\therefore$ put $v = y^6$ $\therefore \frac{1}{6}\frac{dv}{dt} = y^5\frac{dy}{dt}$

$\therefore \Rightarrow \frac{1}{6}\frac{dv}{dt} + 18v = e^{2t}$ $\therefore \frac{dv}{dt} + 108v = 6e^{2t}$

$\therefore P = 108$, $Q = 6e^{2t}$

$\therefore I.f = e^{\int 108dt} = e^{108t}$

$\therefore y(I.F) = \int (I.F) Q \, dt$

$y e^{105t} = \int e^{105t} \cdot 6 e^{2t} dt$

$y^6 e^{108t} = 6 \int e^{110t} dt$

$y^6 e^{108t} = 6 \dfrac{e^{110t}}{110} + A$

$$\underline{y^6 = \frac{3}{55} e^{2t} + A e^{-108t}}$$

(32) $3t \dfrac{dy}{dt} + 2ty = y^{-1} t^5$

$\therefore \dfrac{dy}{dt} + \dfrac{2}{3} y = y^{-1} \dfrac{t^4}{3}$

$\therefore y \dfrac{dy}{dt} + \dfrac{2}{3} y^2 = \dfrac{t^4}{3}$

Put $v = y^2 \therefore \dfrac{1}{2} \dfrac{dv}{dt} = y \dfrac{dy}{dt}$

$\therefore \dfrac{1}{2} \dfrac{dv}{dt} + \dfrac{2}{3} v = \dfrac{t^4}{3}$

$\dfrac{dv}{dt} + \dfrac{4}{3} v = \dfrac{2}{3} t^4$, $P = \dfrac{4}{3}$

$Q = \dfrac{2}{3} t^4$

$(I.F) = e^{\int \frac{4}{3} dt} = e^{\frac{4}{3} t}$

$\therefore y(I.F) = \int (I.F) Q \, dt$

$y e^{4/3 t} = \int e^{4/3 t} \cdot \frac{2}{3} t^4 dt$

$= \frac{2}{3} \int t^4 e^{4/3 t} dt$

Using Int. by Part (D.I)

D	I	
t^4	$e^{4/3 t}$	
$4t^3$	$\frac{3}{4} e^{4/3 t}$	$+$
$12t^2$	$\frac{9}{16} e^{4/3 t}$	$-$
$24t$	$\frac{27}{64} e^{4/3 t}$	$+$
24	$\frac{81}{256} e^{4/3 t}$	$-$
0	$\frac{243}{1024} e^{4/3 t}$	$+$

$= \frac{3}{4} t^4 e^{4/3 t} - \frac{9}{4} t^3 e^{4/3 t} + \frac{81}{16} t^2 e^{4/3 t} - \frac{243}{32} t e^{4/3 t}$

$+ \frac{729}{128} e^{4/3 t}$

i.e

$y e^{4/3 t} = \frac{2}{3} \left[\frac{3}{4} t^4 e^{4/3 t} - \frac{9}{4} t^3 e^{4/3 t} + \frac{81}{16} t^2 e^{4/3 t} - \frac{243}{32} t e^{4/3 t} + \frac{729}{128} e^{4/3 t} \right] + A$

$$\therefore \underline{y = \frac{1}{2} t^4 - \frac{3}{2} t^3 + \frac{27}{8} t^2 - \frac{81}{16} t + \frac{243}{64} + A e^{-4/3 t}}$$

(33) $(1-t^2) \dfrac{dy}{dt} - 2ty = y^3 \quad \therefore \dfrac{dy}{dt} - \dfrac{2t}{1-t^2} \cdot y = y^3 \cdot \dfrac{1}{1-t^2}$

$\therefore y^{-3} \dfrac{dy}{dt} - \dfrac{2t}{1-t^2} y^{-2} = \dfrac{1}{1-t^2}$, put $v = y^{-2}$

$\therefore -\frac{1}{2} \dfrac{dv}{dt} = y^{-3} \dfrac{dy}{dt}$

$\therefore -\dfrac{1}{2} \dfrac{dv}{dt} - \dfrac{2t}{1-t^2} v = \dfrac{1}{1-t^2}$ i.e $\boxed{\dfrac{dv}{dt} + \dfrac{4t}{1-t^2} v = \dfrac{-2}{1-t^2}}$

i.e $P = \dfrac{4t}{1-t^2}$ and $Q = \dfrac{-2}{1-t^2}$

$\therefore I.F = e^{\int P} = e^{\int \frac{4t}{1-t^2} dt} = e^{-2 \int \frac{-2t}{1-t^2} dt} = e^{-2 \ln(1-t^2)}$

$= e^{\ln (1-t^2)^{-2}} = \dfrac{1}{(1-t^2)^2}$

Now; $y(I.F) = \int (I.F) Q \, dt$

$\therefore v \dfrac{1}{(1-t^2)^2} = \int \dfrac{1}{(1-t^2)^2} \cdot \dfrac{-2}{1-t^2} dt$

$\dfrac{v}{(1-t^2)^2} = -2 \int \dfrac{1}{(1-t^2)^3} dt$

Now; let $1-t^2 = 1 - \sin^2\theta$ i.e $t = \sin\theta \therefore dt = \cos\theta d\theta$

$= -2 \int \dfrac{1}{(1-\sin^2\theta)^3} \cos\theta d\theta = -2 \int \dfrac{1}{\cos^6\theta} \cos\theta d\theta$

$= -2 \int \dfrac{1}{\cos^5\theta} d\theta = -2 \int \sec^5\theta d\theta$

Using the Reduction formula: $\boxed{\int \sec^n x = I_n = \frac{1}{n-1} \tan x \sec^{n-2} x - \frac{(n-2)}{(n-1)} I_{n-2}}$

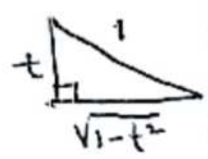
we get;

$\boxed{\int \sec^5\theta d\theta = \frac{1}{4} \tan\theta \sec^3\theta - \frac{3}{8} \tan\theta \sec\theta + \frac{3}{8} \ln(\sec\theta + \tan\theta)}$

where $\frac{t}{1} = \sin\theta$

$\tan\theta = \frac{t}{\sqrt{1-t^2}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-t^2}}$



$\therefore \dfrac{v}{(1-t^2)^2} = -2 \left[\dfrac{1}{4} \dfrac{t}{(1-t^2)^2} - \dfrac{3}{8} \dfrac{t}{(1-t^2)} + \dfrac{3}{8} \ln\left(\dfrac{1+t}{\sqrt{1-t^2}}\right) \right] + A$

where $v = y^{-2}$

$\therefore y^{-2} = -2t + \frac{3}{4} t(1-t^2) - \frac{3}{4}(1-t^2)^2 \ln\left(\frac{1+t}{\sqrt{1-t^2}}\right) + A(1-t^2)^2$

or

$1 = y^2 \left(A(1-t^2)^2 - 2t + \frac{3}{4} t(1-t^2) - \frac{3}{4} (1-t^2) \ln\left(\frac{1+t}{\sqrt{1-t^2}}\right) \right)$

$y_1 = e^{2t}$, $y_2 = e^{-2t}$ i.e $y_1' = 2e^{2t}$, $y_2' = -2e^{-2t}$

$\therefore$ Wronskian $= W(t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$= \begin{vmatrix} e^{2t} & e^{-2t} \\ 2e^{2t} & -2e^{-2t} \end{vmatrix} = (e^{2t})(-2e^{-2t}) - (e^{-2t})(2e^{2t})$

$= -2e^0 - 2e^0 = -2-2 = \underline{\underline{-4}}$.

(35) Note for: $A\frac{d^2y}{dx^2} + B\frac{dy}{dx} + Cy = 0$

i.e $\frac{d^2y}{dx^2} + \frac{B}{A}\frac{dy}{dx} + \frac{C}{A}y = 0$

Wronskian $W(t) = W_0 e^{-\int_{t_0}^{t} \frac{B}{A} dt}$

where at t_0, $W(t_0) = W_0$.

$\therefore$ for $t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 7y = 0$ i.e

$\frac{d^2y}{dt^2} + \frac{1}{t}\frac{dy}{dt} + \frac{7}{t^2}y = 0$, $\frac{B}{A} = \frac{1}{t}$.

$\therefore W(t) = W_0 e^{-\int_{t_0}^{t} \frac{1}{t} dt} = W_0 e^{-\ln t|_{t_0}^{t}}$

where $t_0 = 1$, $W(t_0) = W(1) = 1$

$\therefore W(t) = 1 . e^{-\ln t|_1^t} = e^{-(\ln t - \ln 1)} = e^{-\ln t - 0}$

$= e^{-\ln t} = e^{\ln t^{-1}} = t^{-1} = \underline{\underline{\frac{1}{t}}}$

(36) $y'' + 2y' - 3y = 0$, $y(0) = 9$, $y'(0) = 5$

The Auxiliary (characteristics) equ is;

$k^2 + 2k - 3 = 0 \therefore k_1 = -3$ and $k_2 = 1$

i.e it result in two real unequal roots

$\therefore$ solution: $\boxed{y = Ae^{k_1 t} + Be^{k_2 t}}$ G. solution

$\therefore \boxed{y = Ae^{t} + Be^{-3t}}$, using the initial condition

$y(0) = 9 \quad \therefore 9 = Ae^0 + Be^0 \therefore A + B = 9$ (I)

also $y' = Ae^t - 3Be^{-3t}$ and $y'(0) = 5$

$\therefore 5 = A - 3B$ i.e $A - 3B = 5$ —— (II)

Solving gives $A = 8$, $B = 1$ $\therefore \boxed{y = 8e^t + e^{-3t}}$

which is the particular solution.

(37) $Ay'' + By' + Cy = 0$, with the Aux. equ as $Ak^2 + Bk + C = 0$ if the equation has a discriminant $D = b^2 - 4ac > 0$ then it has two real unequal roots.

i.e

$D = b^2 - 4ac > 0$ two real unequal roots

$= 0$ two real equal roots

< 0 two complex roots

$\therefore$ Solution should look like

$\boxed{y = Ae^{m_1 t} + Be^{m_2 t}}$

(38) $y'' - 2y' + 5y = 0$ $y(0) = 5$, $y'(0) = 5$

$\therefore k^2 - 2k + 5 = 0 \quad \therefore k = 1 \pm 2i = u \pm iv$

i.e $y = e^{ut}[A\cos vt + B\sin vt]$, $u = 1$, $v = 2$

$\therefore \boxed{y = e^{t}[A\cos 2t + B\sin 2t]}$ → General soln.

using $y(0) = 5$ and $y'(0) = 5$

$5 = e^0[A\cos 0 + \cancel{B\sin 0}] \quad \therefore 5 = A(1) \therefore A = 1$.

and $y' = e^t[-2A\sin 2t + 2B\cos 2t]$

$+ e^t[A\cos 2t + B\sin 2t]$

$\therefore 5 = 1[0 + 2B] + 1[A + 0]$

$\therefore 5 - A = 2B \quad \therefore B = 2$.

$\boxed{y = e^t[\cos 2t + 2\sin 2t]}$ —— P.S.

(39) $Ay'' + By + Cy = f(t)$ is inhomogeneous when $f(t) \neq 0$ and homo. when $f(t) = 0$.

(40) $y'' + 2y' - 3y = Q$, $Q = 9$

Solution is of the form $y = y_H + y_p$

where $y_H \Rightarrow$ Soln of the homo. part

$y_p \Rightarrow$ Soln to the inhomo part

$y_H \Rightarrow$ is called Complementary function

$y_p \Rightarrow$ is called Particular Integral.

Now; $k^2 + 2k - 3 = 0 \quad \therefore k_1 = -3$ and $k_2 = 1$

$\therefore y = Ae^{-3t} + Be^{t}$

∴ $y(I.F) = \int (I.F)\,Q\,dt$

$y e^{108t} = \int e^{108t}\cdot 6e^{2t}\,dt$

$y^6 e^{108t} = 6\int e^{110t}\,dt$

$y^6 e^{108t} = 6\frac{e^{110t}}{110} + A$

$y^6 = \frac{3}{55}e^{2t} + Ae^{-108t}$

(32) $3t\frac{dy}{dt} + 2ty = y^{-1}t^5$

∴ $\frac{dy}{dt} + \frac{2}{3}y = y^{-1}\frac{t^4}{3}$

∴ $y\frac{dy}{dt} + \frac{2}{3}y^2 = \frac{t^4}{3}$

put $v = y^2$ ∴ $\frac{1}{2}\frac{dv}{dt} = y\frac{dy}{dt}$

∴ $\frac{1}{2}\frac{dv}{dt} + \frac{2}{3}v = \frac{t^4}{3}$

$\frac{dv}{dt} + \frac{4}{3}v = \frac{2}{3}t^4$, $P = \frac{4}{3}$, $Q = \frac{2}{3}t^4$

$(I.F) = e^{\int \frac{4}{3}dt} = e^{\frac{4}{3}t}$

∴ $y(I.F) = \int (I.F)\,Q\,dt$

$ye^{4/3t} = \int e^{4/3t}\cdot\frac{2}{3}t^4\,dt$

$= \frac{2}{3}\int t^4 e^{4/3t}\,dt$

Using Int. by part (D.I)

D	I	
t^4	$e^{4/3t}$	
$4t^3$	$\frac{3}{4}e^{4/3t}$	+
$12t^2$	$\frac{9}{16}e^{4/3t}$	−
$24t$	$\frac{27}{64}e^{4/3t}$	+
24	$\frac{81}{256}e^{4/3t}$	−
0	$\frac{243}{1024}e^{4/3t}$	+

$= \frac{3}{4}t^4e^{4/3t} - \frac{9}{4}t^3e^{4/3t} + \frac{81}{16}t^2e^{4/3t} - \frac{243}{32}te^{4/3t} + \frac{729}{128}e^{4/3t}$

i.e

$ye^{4/3t} = \frac{2}{3}\left[\frac{3}{4}t^4e^{4/3t} - \frac{9}{4}t^3e^{4/3t} + \frac{81}{16}t^2e^{4/3t} - \frac{243}{32}te^{4/3t} + \frac{729}{128}e^{4/3t}\right]$

∴ $y = \frac{1}{2}t^4 - \frac{3}{2}t^3 + \frac{27}{8}t^2 - \frac{81}{16}t + \frac{243}{64} + Ae^{-4/3t}$

(33) $(1-t^2)\frac{dy}{dt} - 2ty = y^3$ ∴ $\frac{dy}{dt} - \frac{2t}{1-t^2}\cdot y = y^3\cdot\frac{1}{1-t^2}$

∴ $y^{-3}\frac{dy}{dt} - \frac{2t}{1-t^2}y^{-2} = \frac{1}{1-t^2}$, put $v = y^{-2}$

∴ $-\frac{1}{2}\frac{dv}{dt} = y^{-3}\frac{dy}{dt}$

∴ $-\frac{1}{2}\frac{dv}{dt} - \frac{2t}{1-t^2}v = \frac{1}{1-t^2}$ i.e $\boxed{\frac{dv}{dt} + \frac{4t}{1-t^2}v = \frac{-2}{1-t^2}}$

i.e $P = \frac{4t}{1-t^2}$ and $Q = \frac{-2}{1-t^2}$

∴ $I.F = e^{\int P} = e^{\int\frac{4t}{1-t^2}dt} = e^{-2\int\frac{-2t}{1-t^2}dt} = e^{-2\ln(1-t^2)}$

$= e^{\ln(1-t^2)^{-2}} = \frac{1}{(1-t^2)^2}$

Now; $y(I.F) = \int (I.F)\,Q\,dt$

∴ $v\frac{1}{(1-t^2)^2} = \int\frac{1}{(1-t^2)^2}\cdot\frac{-2}{1-t^2}\,dt$

$\frac{v}{(1-t^2)^2} = -2\int\frac{1}{(1-t^2)^3}\,dt$

Now; let $1-t^2 = 1-\sin^2\theta$ i.e $t = \sin\theta$ ∴ $dt = \cos\theta\,d\theta$

$= -2\int\frac{1}{(1-\sin^2\theta)^3}\cos\theta\,d\theta = -2\int\frac{1}{\cos^6\theta}\cos\theta\,d\theta$

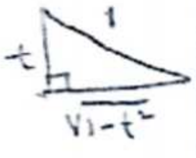
$= -2\int\frac{1}{\cos^5\theta}\,d\theta = -2\int\sec^5\theta\,d\theta$

Using the Reduction formula: $\boxed{\int\sec^n x = I_n = \frac{1}{n-1}\tan x\sec^{n-2}x - \frac{(n-2)}{(n-1)}I_{n-2}}$

we get;

$\int\sec^5\theta\,d\theta = \frac{1}{4}\tan\theta\sec^3\theta - \frac{3}{8}\tan\theta\sec\theta + \frac{3}{8}\ln(\sec\theta + \tan\theta)$

where $\frac{t}{1} = \sin\theta$



$\tan\theta = \frac{t}{\sqrt{1-t^2}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-t^2}}$

where $v = y^{-2}$

∴ $\frac{v}{(1-t^2)^2} = -2\left[\frac{1}{4}\frac{t}{(1-t^2)^2} - \frac{3}{8}\frac{t}{(1-t^2)} + \frac{3}{8}\ln\left(\frac{1+t}{\sqrt{1-t^2}}\right)\right] + A$

∴ $y^{-2} = -2t + \frac{3}{4}t(1-t^2) - \frac{3}{4}(1-t^2)\ln\left(\frac{1+t}{\sqrt{1-t^2}}\right) + A(1-t^2)^2$

or

$1 = y^2\left(A(1-t^2)^2 - 2t + \frac{3}{4}t(1-t^2) - \frac{3}{4}(1-t^2)\ln\left(\frac{1+t}{\sqrt{1-t^2}}\right)\right)$

$y_1 = e^{2t}$, $y_2 = e^{-2t}$ i.e $y_1' = 2e^{2t}$, $y_2' = -2e^{-2t}$

Wronskian $= W(t) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$= \begin{vmatrix} e^{2t} & e^{-2t} \\ 2e^{2t} & -2e^{-2t} \end{vmatrix} = (e^{2t})(-2e^{-2t}) - (e^{-2t})(2e^{2t})$

$= -2e^0 - 2e^0 = -2-2 = \underline{-4}$.

(35) Note for: $A\frac{d^2y}{dx^2} + B\frac{dy}{dx} + Cy = 0$

i.e $\frac{d^2y}{dx^2} + \frac{B}{A}\frac{dy}{dx} + \frac{C}{A}y = 0$

Wronskian $W(t) = W_0\, e^{-\int_{t_0}^{t}\frac{B}{A}dt}$

where at t_0, $W(t_0) = W_0$.

$\therefore$ for $t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} + 7y = 0$ i.e

$\frac{d^2y}{dt^2} + \frac{1}{t}\frac{dy}{dt} + \frac{7}{t^2}y = 0$, $\frac{B}{A} = \frac{1}{t}$.

$\therefore W(t) = W_0\, e^{-\int_{t_0}^{t}\frac{1}{t}dt} = W_0\, e^{-\ln t|_{t_0}^{t}}$

where $t_0 = 1$, $W(t_0) = W(1) = 1$

$\therefore W(t) = 1 . e^{-\ln t|_1^t} = e^{-(\ln t - \ln 1)} = e^{-\ln t - 0}$

$= e^{-\ln t} = e^{\ln t^{-1}} = t^{-1} = \underline{\frac{1}{t}}$

(36) $y'' + 2y' - 3y = 0$, $y(0) = 9$, $y'(0) = 5$

The Auxiliary (characteristics) equ is;

$k^2 + 2k - 3 = 0$ $\therefore$ $k_1 = -3$ and $k_2 = 1$

i.e it result in two real unequal roots

$\therefore$ Solution: $\boxed{y = Ae^{k_1 t} + Be^{k_2 t}}$

$\therefore$ $\boxed{y = Ae^{t} + Be^{-3t}}$ G. solution, using the initial condition

$y(0) = 9$ $\therefore 9 = Ae^0 + Be^0$ $\therefore A + B = 9$ (I)

also $y' = Ae^t - 3Be^{-3t}$ and $y'(0) = 5$

$\therefore 5 = A - 3B$ i.e $A - 3B = 5$ —— (II)

Solving gives $A = 8$, $B = 1$ $\therefore y = 8e^t + e^{-3t}$

which is the particular solution.

(37) $Ay'' + By' + Cy = 0$, with the Aux. equ as $Ak^2 + Bk + C = 0$ if the equation has a discriminant $D = b^2 - 4ac > 0$ then it has two real unequal roots.

i.e

$D = b^2 - 4ac > 0$ two real unequal roots
$= 0$ two real equal roots
< 0 two complex roots

$\therefore$ Solution should look like

$\boxed{y = Ae^{m_1 t} + Be^{m_2 t}}$

(38) $y'' - 2y' + 5y = 0$ $y(0) = 5$, $y'(0) = 5$

$\therefore k^2 - 2k + 5 = 0$ $\therefore$ $k = 1 \pm 2i = u \pm iv$

i.e $y = e^{ut}[A\cos vt + B\sin vt]$, $u = 1$, $v = 2$

$\boxed{y = e^t[A\cos 2t + B\sin 2t]}$ → General soln.

Using $y(0) = 5$ and $y'(0) = 5$

$5 = e^0[A\cos 0 + \cancel{B\sin 0}]$ $\therefore 5 = A(1)$ $\therefore A = 1$.

and $y' = e^t[-2A\sin 2t + 2B\cos 2t]$
$+ e^t[A\cos 2t + B\sin 2t]$.

$\therefore 5 = 1[0 + 2B] + 1[A + 0]$

$\therefore 5 - A = 2B$ $\therefore B = 2$.

$\boxed{y = e^t[\cos 2t + 2\sin 2t]}$ —— P.S.

(39) $Ay'' + By + Cy = f(t)$ is inhomogeneous when $f(t) \neq 0$ and homo. when $f(t) = 0$.

(40) $y'' + 2y' - 3y = Q$, $Q = 9$

Solution is of the form $y = y_H + y_p$

where $y_H \Rightarrow$ Soln ~~of~~ the homo. part
$y_p \Rightarrow$ Soln to the inhomo part

$y_H \Rightarrow$ is called Complementary function

$y_p \Rightarrow$ is called Particular Integral.

Now; $k^2 + 2k - 3 = 0$ $\therefore k_1 = -3$ and $k_2 = 1$

$\therefore y = Ae^{-3t} + Be^{t}$

Now: $y_p = C$ since the inhomo. part $= 9$

i.e $Q = 9$ a Constant $\therefore$

$y_p = y = C$ so that $y' = 0$ and $y'' = 0$

$\therefore$ Substituting into $y'' + 2y' - 3y = p = 9$

$\Rightarrow 0 + 2(0) - 3(c) = 9 \quad \therefore -3C = 9$

$\therefore C = -3.$

hence $y_p = C = -3.$

$\therefore y = y_H + y_p$

$$\boxed{y = Ae^{-3t} + Be^{t} - 3}$$

(41) $y'' + 9y' + 20y = f(t)$, $f(t) = 112e^{3t}$

$k^2 + 9k + 20 \quad \therefore k_1 = -4$ and $k_2 = -5$

real unequal roots $\therefore$

$$\boxed{y_H = Ae^{-4t} + Be^{-5t}}$$

$\therefore y_p = Ce^{3t} = y$ so that;

$y' = 3Ce^{3t}$ and $y'' = 9Ce^{3t}$

Substituting into $y'' + 9y' + 20y = 112e^{3t}$

$\therefore 9C\cancel{e^{3t}} + 9(3C\cancel{e^{3t}}) + 20(C\cancel{e^{3t}}) = 112\cancel{e^{3t}}$

$\therefore 9C + 27C + 20C = 112$

$\therefore 56C = 112$

$\therefore C = 2.$

$\therefore y_p = Ce^{3t} = 2e^{3t}.$

$\therefore y = y_H + y_p \Rightarrow \boxed{y = Ae^{-4t} + Be^{-5t} + 2e^{3t}}$

(42) $y'' + 2y' - 3y = f(t)$, $f(t) = \sin 3t$.

$k^2 + 2k - 3 = 0 \quad \therefore k_1 = -3$ and $k_2 = 1$

$\therefore y_H = Ae^{-3t} + Be^{t}$ and y_p

$y_p = C\cos 3t + D\sin 3t$, $\boxed{\text{even if } f(t) = \cos 3t}$

$\therefore y = y_p = C\cos 3t + D\sin 3t \quad \therefore$

$y' = -3C\sin 3t + 3D\cos 3t$

$y'' = -9C\cos 3t - 9D\sin 3t.$

Substituting into $y'' + 2y' - 3y = \sin 3t$

$\therefore -9C\cos 3t - 9D\sin 3t$
$+ 2(-3C\sin 3t + 3D\cos 3t) = 1\sin 3t$
$-3(C\cos 3t + D\sin 3t) \quad + 0\cos 3t$

i.e

$-9C\cos 3t - 9D\sin 3t$
$-6C\sin 3t + 6D\cos 3t = 1\sin 3t$
$-3C\cos 3t - 3D\sin 3t \quad + 0\cos 3t$

Comparing coefficients of sine and cosine

$(-9C + 6D - 3C) = 0$ for cosine

$(-9D - 6C - 3D) = 1$ for sine.

$\therefore 6D - 12C = 0$ i.e $D = -\frac{1}{15}$

$-12D - 6C = 1 \qquad C = -\frac{1}{30}$

i.e $y_p = -\frac{1}{15}\sin 3t - \frac{1}{30}\cos 3t$

$= -\frac{1}{30}\left[2\sin 3t + \cos 3t\right]$

$\therefore y = y_H + y_p$

$y = Ae^{-3t} + Be^{t} - \frac{1}{30}\left[\cos 3t + 2\sin 3t\right]$

(43) $v = (3t^2 + 4t)$ m/s Note $v = \frac{ds}{dt}$

where s = position = displacement.

$\therefore \frac{ds}{dt} = v$ i.e $ds = v\,dt$

$\therefore \int_{s_0}^{s} ds = \int_{t_0}^{t} v\,dt = \int_{t_0}^{t} (3t^2 + 4t)\,dt$

$\therefore s\Big|_{s_0}^{s} = t^3 + 2t^2\Big|_{t_0}^{t}$ where at $t_0 = 0$, $s_0 = 0$

$\therefore s\Big|_0^s = t^3 + 2t^2\Big|_0^t \quad \therefore \boxed{s = t^3 + 2t^2}$

after $t = 5$ secs $\therefore s = 5^3 + 2(5)^2 = \underline{175\text{m}}$

(44) $\vec{s} = (5t^3 + 3t^2)$ m $\therefore \vec{v} = \frac{d\vec{s}}{dt}$, $\vec{a} = \frac{d\vec{v}}{dt}$

$\therefore \vec{a} = \frac{d}{dt}(\vec{v}) = \frac{d}{dt}\left(\frac{d\vec{s}}{dt}\right) = \frac{d^2\vec{s}}{dt^2}.$

$\vec{V} = \frac{d\vec{s}}{dt} = \frac{d}{dt}\left(5t^3+3t^2\right) = 15t^2+6t$

$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}\left(15t^2+6t\right) = 30t+6$

∴ at $t = 3$ secs ∴ $\vec{a} = 30(3)+6$
$= 90+6 = \underline{96\,m/s^2}$

(45) $\vec{V} = (3t^2+5t,\ 2t+3t^3,\ t+5t^2)\,m/s$. Note the velocity is given in terms of the 3-Dimensional coordinate vector $v = (x,y,z) = x\hat{i}+y\hat{j}+z\hat{k}$.

∴ position = displacement = s ⇒
$\vec{V} = \frac{d\vec{s}}{dt}$ ∴ $d\vec{s} = \vec{v}\,dt$

∴ $\int_{s_0}^{s} d\vec{s} = \int_{t_0}^{t} \vec{v}\,dt$ i.e $t_0 = 0$ (Initial), $s_0 = 0$.

$\vec{s}\Big|_0^s = \int_0^t (3t^2+5t,\ 2t+3t^3,\ t+5t^2)\,dt$

$\vec{s} = \left(t^3+\frac{5t^2}{2},\ t^2+\frac{3t^4}{4},\ \frac{t^2}{2}+\frac{5t^3}{3}\right)\Big|_0^t$

∴ $\vec{s} = \left(t^3+\frac{5t^2}{2},\ t^2+\frac{3t^4}{4},\ \frac{t^2}{2}+\frac{5t^3}{3}\right)$ at $t=$

∴ $\vec{s} = \left(8+\frac{5.4}{2},\ 4+\frac{3.16}{4},\ \frac{4}{2}+\frac{5.8}{3}\right)$

$\vec{s} = \left(8+10,\ 4+12,\ 2+\frac{40}{3}\right) = \begin{pmatrix}18\\16\\46/3\end{pmatrix} m.$

(46) $x = 3t^2,\ y = 8t,\ z = 2t^3$.

∴ position $= \vec{r} = \vec{s} = x\hat{i}+y\hat{j}+z\hat{k} = (x,y,z)$
i.e $\vec{r} = (3t^2, 8t, 2t^3)$

∴ velocity $= \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t^2, 8t, 2t^3)$
$= (6t, 8, 6t^2)$

∴ Magnitude of velocity $= |\vec{v}| = \sqrt{(6t)^2+8^2+(6t^2)^2}$

$|\vec{v}| = \sqrt{36t^2+64+36t^4}$

∴ at $t = 2s$ ∴ $|\vec{v}| = \sqrt{(36)(4)+64+36(16)}$
$= \sqrt{36(4+2+16)}$
$= \sqrt{36(22)} = \underline{\sqrt{784}\,m/s}$

(47) $\vec{a} = (2t,\ 4\cos 3t,\ 4\sin 3t)\,m/s^2$.

$\vec{a} = \frac{d\vec{v}}{dt}$ ∴ $\int_{v_0}^{v} d\vec{v} = \int_{t_0}^{t} \vec{a}\,dt$ where $t_0 = 0$, $v_0 = 0$

∴ $v\Big|_0^v = \int_0^t (2t,\ 4\cos 3t,\ 4\sin 3t)\,dt$

$\vec{V} = \left[t^2,\ \frac{4\sin 3t}{3},\ -\frac{4\cos 3t}{3}\right]_0^t$

$\vec{V} = \left(t^2,\ \frac{4}{3}\sin 3t,\ -\frac{4}{3}\cos 3t\right) - \left(0, 0, -\frac{4}{3}\right)$

∴ $\vec{V} = \left(t^2,\ \frac{4}{3}\sin 3t,\ \frac{4}{3}-\frac{4}{3}\cos 3t\right)$ i.e

$\underline{\vec{V} = \left(t^2,\ \frac{4}{3}\sin 3t,\ \frac{4}{3}(1-\cos 3t)\right)\,m/s}$

(48) $x = 3e^{-t},\ y = 5\sin 2t,\ z = 4\cos 2t$

∴ position $\vec{r} = \vec{s} = (3e^{-t},\ 5\sin 2t,\ 4\cos 2t)$

$\vec{a} = \frac{d^2\vec{s}}{dt^2} = \frac{d}{dt}\left(-3e^{-t},\ 10\cos 2t,\ -8\sin 2t\right)$

$\vec{a} = \left(3e^{-t},\ -20\sin 2t,\ -16\cos 2t\right)$

at $t = 0$

$\underline{\vec{a} = (3, 0, -16)\,m/s^2}$ $e^0 = 1$, $\sin 0 = 0$, $\cos 0 = 1$

(49) $s = 5t^3+3t^2$ ∴ $\vec{v} = \frac{ds}{dt} = 15t^2+6t$

at $t = 3s$ ∴ $\vec{v} = 15(9)+6(3)$
$= 135+18 = \underline{153}$

(50) $y''+9y'+20 = 0$ ∴ $k^2+9k+20 = 0$

$k_1 = -4$ and $k_2 = -5$ ∴ $\underline{y = Ae^{-4t}+Be^{-5t}}$

(51) $y''-2y'+y = 0$, $y(1) = e = y'(1)$

∴ $k^2-2k+1 = 0$ ∴ $k_1 = 1$ and $k_2 = 1 = k$

real equal roots ∴ $y = (A+Bt)e^{kt}$

∴ $y = (A+Bt)e^t$ ∴ at $t = 1$, $y = e$

$e = (A+B)e$ ∴ $\boxed{1 = A+B}$ — (1)

and $y' = (A+Bt)e^t + Be^t$ (product rule)

$\therefore Y' = (A+Bt)e^t + Be^t$ at $t=1, Y'=e$

$\therefore \cancel{e} = (A+B)\cancel{e} + B\cancel{e}$

$\therefore 1 = A+B+B \quad \therefore \boxed{A+2B=1}$

$\therefore A+B=1$ and
$A+2B=1$ Solving $A=1$, $B=0$

$\therefore y = (A+Bt)e^t \quad \therefore y = (1+0)e^t$

$\boxed{y=e^t}$ — G.S.

(52) If $m_1 \neq m_2$ (i.e real unequal roots) then solution $\therefore y = Ae^{m_1 t} + Be^{m_2 t}$

(53) Since W_t = Wronskian $\neq 0 \Rightarrow$ the ODE has a unique solution $\therefore$

$y_1 = e^{-2t}$ and $y_2 = e^{5t}$ i.e $y = Ay_1 + By_2$

$\therefore y = Ae^{-2t} + Be^{5t}$ is the Soln.

$\therefore k_1 = -2$ and $k_2 = 5 \quad \therefore (k+2)(k-5) = 0$

$\therefore k^2 - 3k - 10 = 0$

$\boxed{y'' - 3y' - 10y = 0}$ — is the ODE.

(54) $Y = AY_1 + BY_2$ "

$y = A\cosh mt + B\sinh mt$

i.e real equal roots but different in sign. $\therefore k_1 = m$ and $k_2 = -m$

$\therefore (k+m)(k-m) = 0$

$\therefore k^2 - m^2 = 0$

$\boxed{y'' - m^2 = 0}$

(55) $Ay'' + By' + cy = f(t)$, if $f(t) = 0$ then it is said to be a homogeneous second order ODE, but at $A=0$, if $f(t) = 0$ then it is a homogeneous first order ODE.

(56) $y'' - 6y' + 5y = 0$, $y(\pi) = y'(\pi) = e^{3\pi/2}$

$k^2 - 6k + 5 = 0$, $k_1 = 5$, $k_2 = 1$

$\boxed{y = Ae^{5t} + Be^t}$ $\quad e^{3\pi/2} = Ae^{5\pi} + Be^{\pi}$

and

$y' = 5Ae^{5t} + Be^t$ and $t=\pi$, $y' = e^{3\pi/2}$

$e^{3\pi/2} = 5Ae^{5\pi} + Be^{\pi}$ — (ii)

Now $\quad Ae^{5\pi} + \cancel{Be^{\pi}} = \cancel{e^{3\pi/2}}$

$(-)\quad 5Ae^{5\pi} + \cancel{Be^{\pi}} = \cancel{e^{3\pi/2}}$

$4Ae^{5\pi} = 0 \quad \therefore A = 0$

Sub into (i)

$e^{3\pi/2} = 0 + Be^{\pi}$

$\therefore B = e^{\frac{3\pi}{2} - \pi} = e^{\pi/2}$

$\therefore y = 0 + e^{\pi/2}.e^t$

$\boxed{\therefore y = e^{t + \pi/2}}$

(57) Homo of first order if $A=0$ and $\phi(t) = 0$.

(58) $y'' - 3y' = 0$ i.e $k^2 - 3k = 0$

$\therefore k(k-3) = 0 \quad \therefore k_1 = 0$ and $k_2 = 3$

or use calculator on $k^2 - 3k + 0 = 0$

$\therefore y = Ae^{0t} + Be^{3t}$

$\therefore y = A + Be^{3t}$

(59) $y'' + 0.2y' + 4.01y = 0$, $y(0) = 0$, $y'(0) = 2$.

$k^2 + 0.2k + 4.01 = 0$

$\therefore k_1 = ? \quad k_2 = ?$ i.e $k = -\frac{1}{10} \pm 2i$

i.e $u = -1/10$, $v = 2$.

$\therefore y = e^{ut}[A\cos vt + B\sin vt]$

$y = e^{-\frac{1}{10}t}[A\cos 2t + B\sin 2t]$

Using the initial condition:

"e" at $t=0$, $y=0$

$0 = e^0[A(1) + B(\cancel{0})]$ $\therefore A = 0$, also

$y' = e^{-1/10 t}[-2A\sin 2t + 2B\cos 2t]$

$-\frac{1}{10}e^{-1/10 t}[A\cos 2t + B\sin 2t]$

at $t=0$, $y'(0) = 2$

$\therefore 2 = \cancel{e^0}[0 + 2B] - \frac{1}{10}\cancel{e^0}[A(1) + 0]$

$2 = B - \frac{A}{10}$ $\therefore 2 = B$ and $B = 1$, $A = 0$

Hence;

$y = e^{-\frac{1}{10}t}[0 + 1\sin 2t]$

$\therefore \boxed{y = e^{-\frac{1}{10}t}\sin 2t}$

(60) $y'' + 2ay' + a^2y = 0$ $\therefore k^2 + 2ak + a^2 = 0$

$\Rightarrow (k+a)(k-a) = 0$ i.e $k = -a$ twice

i.e two real equal roots.

$\therefore \boxed{y = (A + Bt)e^{-at}}$

(61) $8y'' - 2y' - y = 0$, $y(0) = -0.2$

$y'(0) = -0.325$.

$8k^2 - 2k - 1 = 0$ $\therefore k_1 = \frac{1}{2} = 0.5$, $k_2 = -\frac{1}{4} = -0.25$

$\therefore \boxed{y = Ae^{0.5t} + Be^{-0.25t}}$

now;

$-0.2 = Ae^0 + Be^0$ $\therefore \boxed{A + B = -0.2}$ (1)

also

$y' = 0.5Ae^{0.5t} - 0.25Be^{-0.25t}$

$\therefore -0.325 = 0.5A\cancel{e^0} - 0.25B\cancel{e^0}$

$\therefore \boxed{0.5A - 0.25B = -0.325}$ (11)

Solving $\Rightarrow A = -\frac{1}{2} = -0.5$

$B = 3/10 = 0.3$

$\therefore \boxed{y = 0.3e^{-0.25t} - 0.5e^{0.5t}}$

(62) $s = (\frac{7}{2}t^4 - 3t^3 + 5t^2 + 9t - 10)$ m.

$\therefore ds/dt = 14t^3 - 9t^2 + 10t + 9$

$\therefore a = \frac{d^2s}{dt^2} = 42t^2 - 18t + 10$, at $t = 5s$

$\therefore a = 42(25) - 18(5) + 10 = \underline{\underline{970}}\ m/s^2$

(63) $y' + Py = Qy^n$, has a general soln of the Bernoulli first order ODE, reducible to a linear first order ODE.

(64) $\vec{a} = (3e^{-2t}, 4\sin 5t, 4\cos 5t)\ m/s^2$

$\therefore$ Magnitude $\Rightarrow |\vec{a}| =$

$= \sqrt{(3e^{-2t})^2 + (4\sin 5t)^2 + (4\cos 5t)^2}$

$= \sqrt{9e^{-4t} + 16\sin^2 5t + 16\cos^2 5t}$

at $t = 0$ sec

$|\vec{a}| = \sqrt{9(1) + \cancel{16(0)} + 16(1)}$

$= \sqrt{9 + 16} = \sqrt{25} = \underline{\underline{5}}$

(65) $x = 3t^2$ and $y = 2t^3 + 5t^2$

$\therefore$ position $\vec{r} = \vec{s} = x\hat{i} + y\hat{j} = (x, y)$

$\vec{s} = (3t^2, 2t^3 + 5t^2)$ at $t = 2$

$\therefore \vec{s} = (3(2)^2, 2(2)^3 + 5(2)^2) = \underline{\underline{(12, 36)\ m}}$

(66) $\therefore \vec{v} = \frac{d\vec{s}}{dt} = (6t, 6t^2 + 10t)$

$\therefore$ at $t = 2$ $\therefore \vec{v} = \underline{\underline{(12, 44)\ m/s}}$

(67) $\vec{a} = \frac{d\vec{v}}{dt} = (6, 12t + 10)$, at $t = 2$

$\therefore \vec{a} = (6, 34)\ m/s^2$

(68) $\vec{F} = m\vec{a} = 30(6, 34)$

$= \underline{\underline{(180, 1020)\ N}}$

(69) Work done $= \vec{F}\cdot\vec{d}$ = force x disp.

$= \vec{F}\cdot\vec{s}$

$W = (180, 1020) \cdot (12, 36)$

$= 2160 + 36720 = \underline{38{,}880 \, J}$

i.e $= (180)(12) + (1020)(36)$. dot product of two vectors

(70) Work done by force inclined at an angle θ is

$W = (\vec{F}\cos\theta) \cdot \vec{d} = \vec{F}\cos\theta \cdot \vec{s}$

$\therefore W = (180, 1020)(\cos 60) \cdot (12, 36)$

$= \begin{pmatrix} 180 \\ 1020 \end{pmatrix} \cos 60 \cdot \begin{pmatrix} 12 \\ 36 \end{pmatrix}$

$= \begin{pmatrix} 180\cos 60 \\ 1020\cos 60 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 36 \end{pmatrix}$ — $\cos 60 = 0.5$

$= \begin{pmatrix} 90 \\ 510 \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 36 \end{pmatrix} = \left((90)(12) + (510)(36) \right)$

dot product $= 1080 + 18360$

$= \underline{19440 \, J}$

(71) Work done is obtained by the product of force ($\vec{F}$) and distance ($\vec{d}$) covered by the particle at time t.

i.e $W = \vec{F} \cdot \vec{d}$ in Joules.

(72) Question incomplete.

(73) Work $= \vec{F} \cdot \vec{d}$ but since it's on a rough surface $W = \vec{F} \cdot \vec{d}$ where $\vec{F} = \mu m \vec{g}$

where μ = Coefficient of friction.

$\therefore W = \mu m \vec{g} \cdot \vec{r}$ where $\vec{r} = (t^2, 2t^2)$

$\therefore$ at $t = 3$ secs $\therefore \vec{r} = (9, 18)$.

and $g = 9.8$ $\therefore \vec{g} = (1,1) \cdot 9.8 = (9.8, 9.8)$

i.e change g to a vector quantity $\vec{g}$

$\therefore W = (0.4)(80)(9.8, 9.8) \cdot (9, 18)$

$= (32)(9.8, 9.8) \cdot (9, 18)$

$= (313.6, 313.6) \cdot (9, 18)$

$= (313.6)(9) + (313.6)(18)$

$= 2822.4 + 5644.8 = \underline{8467.2 \, J}$

(74) $\vec{r} = \vec{s} = (3t^4 - 2t^3, 4t^3 + 5t)$

$\therefore \vec{v} = \dfrac{d\vec{r}}{dt} = (12t^3 - 6t^2, 12t^2 + 5)$ at $t = 4$

$= (768 - 96, 192 + 5) = \underline{\begin{pmatrix} 672 \\ 197 \end{pmatrix} m/s}$

(75) $\vec{a} = \dfrac{d\vec{v}}{dt} = (36t^2 - 12t, 24t)$

$\therefore$ at $t = 4$

$\vec{a} = (36(4)^2 - 12(4), 24(4))$

$= \underline{\begin{pmatrix} 528 \\ 96 \end{pmatrix} m/s^2}$

(76) Resistance = Resulting force $= \vec{F}$

$\vec{F} = m\vec{a} = (4)\begin{pmatrix} 528 \\ 96 \end{pmatrix} = \underline{\begin{pmatrix} 2112 \\ 384 \end{pmatrix} N}$

(77) Power $= \dfrac{\text{Work}}{\text{time}}$ i.e $P = \dfrac{W}{t} = \dfrac{\vec{F} \cdot \vec{d}}{t}$

i.e $P = \vec{F} \cdot \dfrac{\vec{d}}{t} = \vec{F} \cdot \vec{v}$, where $\vec{v} = \dfrac{\vec{d}}{t}$

where $\vec{d}$ = change in distance i.e $d\vec{r}$

$\therefore P = \vec{F} \cdot \vec{v} = \begin{pmatrix} 2112 \\ 384 \end{pmatrix} \cdot \begin{pmatrix} 672 \\ 197 \end{pmatrix}$ dot product

$= (2112)(672) + (384)(197)$

$= 1419264 + 75648 = \underline{1494912 \, W}$

where the unit of Power is Watt (W)

(78) Work $= \vec{F} \cdot \vec{d}$ $\quad \vec{s} = (2, 3)$ $\quad h = 5$

$= m\vec{g} \cdot \vec{h}$

$\therefore W = (25)(9.8, 9.8) \cdot (2, 3)$

$= (245, 245) \cdot (2, 3) = 490 + 735 = \underline{1225 \, J}$

79) $P = \frac{W}{t} \therefore = \frac{1225}{3} = \underline{408.33W}$

$= \underline{408.33W}$

(80) $K.E = \frac{1}{2}m\vec{v}^2$, $\vec{s} = \vec{r} = (5t, 3t)$

$\therefore \frac{d\vec{r}}{dt} = \vec{v} = (5,3)$

$\therefore K.E = \frac{1}{2}(m)\vec{v}\cdot\vec{v} = \frac{1}{2}(10)(5,3)\cdot(5,3)$

$= \frac{10}{2}[(5)(5) + (3)(3)] = 5(25+9)$

$= 5(34) = \underline{170J}$

(81) $\vec{s} = \vec{r} = (5t, 3t)$ at $t=2$, $\vec{s} = (10,6)$

$\therefore \frac{d\vec{r}}{dt} = (5,3) = \vec{v}$, since velocity was reduced from rest i.e at $t=0$, $V = V_0 = (0,0)$ and after time $t=2$, $V = V_2 = (5,3)$

$\therefore K.E = \frac{1}{2}m\vec{v}_2^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\vec{v}^2 = \Delta K.E$

$\therefore K.E = \frac{1}{2}(10)(5,3)\cdot(5,3) = 170J.$

but $\vec{F}\cdot\vec{d} = \text{Work} = K.E$

i.e $\vec{F}\cdot\vec{d} = \Delta KE \therefore$

$(z, 17)\cdot(10,6) = 170$

$10z + 102 = 170 \therefore 10z = 170 - 102$

$\therefore z = \underline{6.8}$

(82) $m\vec{a} = F - R$

$m\vec{a} = -m\vec{g} - mkv^2$ or

$\boxed{m\frac{d\vec{v}}{dt} = -m\vec{g} - mk\vec{v}^2}$

upward gravity $= -g$

(83) $m\vec{a} = mg - mkv^2$,

$\therefore \boxed{m\frac{d\vec{v}}{dt} = m\vec{g} - mkv^2}$

downward gravity $= +g$

(84) $m\vec{a} = -mg - mkv$

$\boxed{m\frac{d\vec{v}}{dt} = -mg - mkv}$

(85) $m\vec{a} = mg - mkv$

$\boxed{m\frac{d\vec{v}}{dt} = +mg - mkv}$

(86) $\vec{r} = \vec{s} = (3t^2, 8t, 3t^3)$, at $t = 2s$

$\vec{s} = \underline{(12, 16, 24)\,m}$

(87) $\vec{v} = \frac{d\vec{s}}{dt} = (6t, 8, 9t^2)$ at $t=2$

$\vec{v}(2) = \underline{(12, 8, 36)}\,m/s.$

(88) $\vec{a} = \frac{d\vec{v}}{dt} = (6, 0, 18t)$ at $t=2$

$\vec{a}(2) = \underline{(6, 0, 36)\,m/s^2}$

(89) $\vec{F} = m\vec{a} = 20(6, 0, 36) = \begin{pmatrix}120\\0\\720\end{pmatrix}N$

(90) Applying Newtons law of motion

$\vec{F} = m\vec{a}$ i.e $\vec{F} = m\frac{d\vec{v}}{dt}$ $\frac{\vec{F}}{m}dt = d\vec{v}$

$\therefore \int_{v_0}^{v} dv = \int_{t_0}^{t} \frac{\vec{F}}{m}dt = \int_{t_0}^{t}\left(\frac{7t+t^2}{5}\right)dt$

$\therefore v\Big|_{v_0}^{v} = \frac{1}{5}\left(\frac{7t^2}{2} + \frac{t^3}{3}\right)\Big|_0^t$, $t_0 = 0$, $V_0 = 0$

$\vec{v} = \underline{\left(\frac{7}{10}t^2 + \frac{1}{15}t^3\right)m/s}$

(91) at $t=3$. $V(3) = \frac{7\cdot9}{10} + \frac{27}{15}$

$V(3) = \frac{63}{10} + \frac{9}{5} = \frac{81}{10} = \underline{8.1\,m/s}$

(92) $\frac{ds}{dt} = v \therefore ds = v\,dt$

$\therefore \int_{s_0}^{s} ds = \int_{t_0}^{t} v\,dt = \int_0^t\left(\frac{7t^2}{10} + \frac{1}{15}t^3\right)dt$

$\therefore s\Big|_0^s = \frac{7t^3}{30} + \frac{t^4}{60} \therefore s = \underline{\frac{7t^3}{30} + \frac{t^4}{60}}$

(93) $s(3) = \frac{7(3)^3}{30} + \frac{3^4}{60} = \frac{189}{30} + \frac{81}{60}$

$= 153/20 \Rightarrow s = 153/20\,m$ or $\underline{7.65m}$

(14)

94) $\vec{s} = (e^{-t}, 2\cos 5t, 2\sin 5t)$ m at $t=0$

$\vec{s}(0) = (e^{0}, 2\cos 0, 2\cancel{\sin 0}) = \underline{(1, 2, 0)}$ m

95) $\vec{v} = \frac{d\vec{s}}{dt} = (-e^{-t}, -10\sin 5t, 10\cos 5t)$

$\therefore \vec{v}(0) = (-e^{0}, -10\cancel{\sin 0}, 10\cos 0)$

$= \underline{(-1, 0, 10)}$ m/s

96) $\vec{a} = \frac{d\vec{v}}{dt} = (e^{-t}, -50\cos 5t, -50\sin 5t)$

$\therefore \vec{a}(0) = (e^{0}, -50\cos 0, -50\cancel{\sin 0})$

$= \underline{(1, -50, 0)}$ m/s^2

97) $\vec{F} = m\vec{a} = 10(1, -50, 0)$

$= \underline{(10, -500, 0)}$ N.