PHY 112 LECTURE- Part 2

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MODULE 1- WHEATSTONE BRIDGE CIRCUIT AND ITS APPLICATIONS

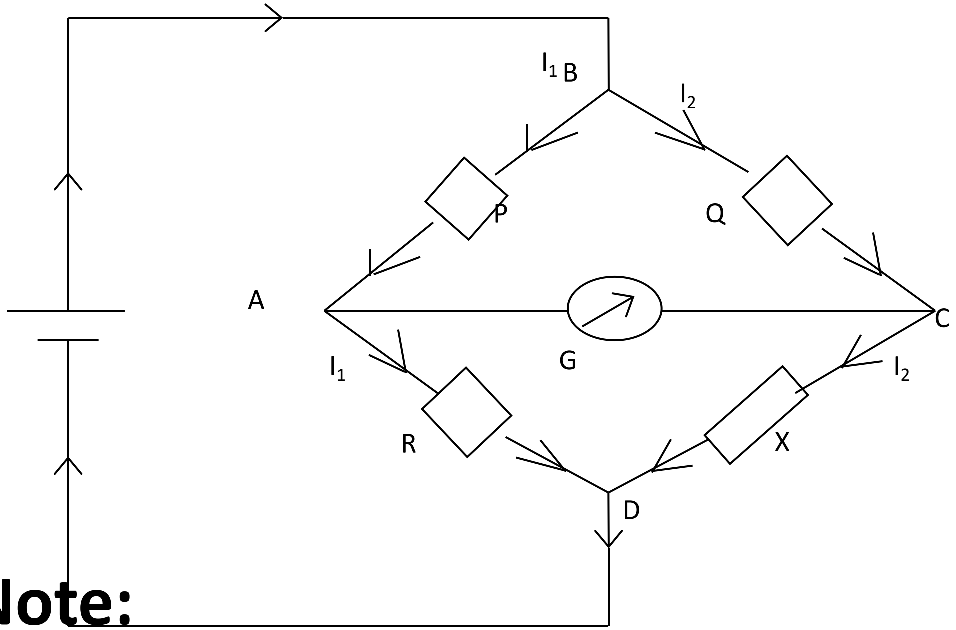
- The Wheatstone bridge is one of the electrical measuring devices operating on the principle of null measurement. The circuit in its original form, as shown below, was designed by Charles Wheatstone in 1943.
- The Wheatstone bridge is an electrical circuit designed to give an accurate method for measuring resistance. Generally, Wheatstone bridge is used to determine unknown resistances. This measurement will allow us to measure how the resistance of the wire varies with length and perhaps the resistivity of the material. The relationship between resistance (R), resistivity (ρ), length (L) and cross sectional area (A) is given as:

$$R = \frac{\rho L}{A} \Rightarrow \rho = \frac{RA}{L}$$

Every simple well behaved material has its own resistivity and at least in principle, we can identify the material by measuring its resistivity. It is possible to measure resistances without resorting to an ohm meter or to plotting the V vs I curve (i.e. V-I plot).

WHEATSTONE BRIDGE CIRCUIT(CONTD.)

We can set up a circuit called a Wheatstone bridge as shown below:



- **Note:**
- I_1, I_2 are currents; P, Q, R are known resistance
- X is unknown resistance; G is the Galvanometer
- **Balance Condition:** $V_A = V_C$ or $V_A - V_C = 0$

WHEATSTONE BRIDGE CIRCUIT(CONTD.)

- If the potential between points A and C (i.e. V_A and V_C) is zero i.e. balance condition, then no current will flow through the galvanometer. In this case, we can write:

$$V = I_1 (P + R) = I_2 (Q + X)$$

We can also write for this special case of no current through the galvanometer that:

$$I_1 P = I_2 Q \quad \text{and} \quad I_1 R = I_2 X$$

and the ratio of these equations gives: $$\frac{P}{R} = \frac{Q}{X}$$

The Wheatstone bridge takes advantage of the property that resistance is directly proportio to length for a wire. If we construct the resistances P and R by dividing a known length of wir with uniform cross sectional area A into two segments with lengths L_P and L_R, we can write:

$$P = \frac{\rho L_P}{A} \quad \text{and} \quad R = \frac{\rho L_R}{A}$$

WHEATSTONE BRIDGE CIRCUIT(CONTD.)

We can write the ratio of these resistances as: $\frac{P}{R} = \frac{L_p}{L_R}$

The unknown resistance can now be solved in terms of the lengths of wire that form two of the resistors i.e. P and R and a known resistor Q i.e. $\frac{P}{R} = \frac{Q}{X}$ and $\frac{L_P}{L_R} = \frac{Q}{X} \Rightarrow X = \frac{Q\,L_R}{L_P}$

Thus, our measurement does not depend on an ohm meter. We only need to measure two lengths and know the resistance of one resistor.

Applications of Wheatstone Bridge

A number of resistance measuring devices have been devised on the principle of Wheatstone bridge. For example meter bridge, post office box, potentiometer, Carey foster's bridge, Callendar bridge, Griffiths bridge.

Potential Divider Circuits

- Two resistances in series can be arranged so as to provide a fraction of a given potential difference. The arrangement is known as a potential divider . A potential divider circuit is formed whenever two resistance are connected in series across a source of e.m.f The e.m.f produces a voltage drop across each component in the ratio of their resistances, and the potential is divided between the resistances. A potential divider is like a potentiometer.

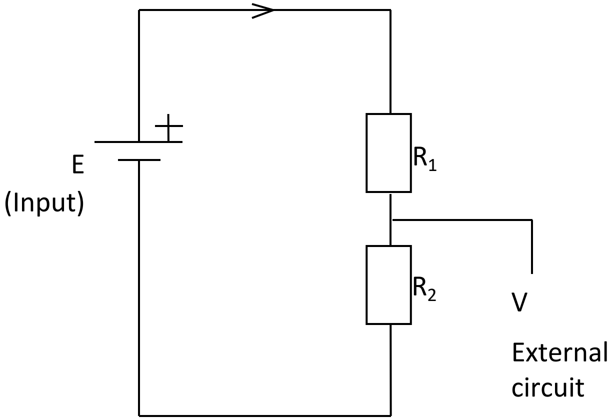


Figure 1

Pd across external circuit, V_O $= \dfrac{R_2}{R_1 + R_2} \times E$

This expression was obtained from: $\dfrac{V_O}{R_2} = \dfrac{E}{R_1 + R_2}$

Moving Coil/Pointer Instruments

Pointer instruments are electrical equipments with pointers as indicators in measuring electrical parameters including resistance, voltage, capacitance, current etc but are not as accurate as potentiometers because of intrinsic errors involved in their full scale deflection (FSD). Examples include galvanometer, voltmeter, ammeter, multimeter (i.e. Avometer) etc.

The Usage of Meters

Ammeters are connected in series and so need to have very low resistance, while Voltmeters are connected in parallel and must have a very high resistance.

Moving Coil/Pointer Instruments (CONTD.)

Shunts and Multipliers

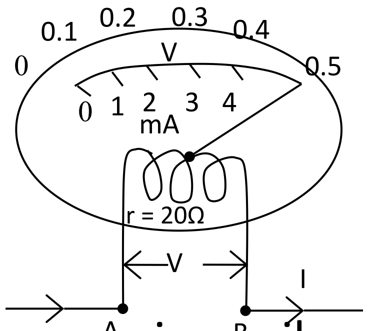
Shunt resistors (i.e. shunts) are used to divert current round ammeters, allowing them to read current greater than full scale deflection (FSD), while Multiplier resistances (i.e. multipliers) drop excess voltage, allowing voltmeters to read a potential difference that is greater than that which would produce the FSD current in the meter.

Multi-Range Meter

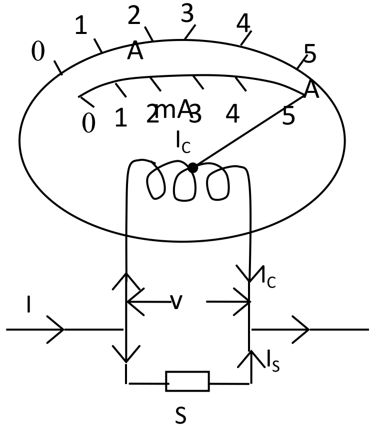
Multi-range meter or multimeters widely used in the radio and electrical industries are moving-coil meters which can read potential differences or currents on the same scale, by switching to series or shunt resistors at the back of the meter.

Converting a Milliammeter to a Voltmeter and an Ammeter

A milliammeter is an instrument used in measuring current. It can also be used as a voltmeter, which measures potential difference.



pd across moving coil ammeter



- If a milliameter is to be converted to an ammeter, a low resistance material called shunt is connected in parallel across the terminals of a moving coil meter. The shunt diverts most of the current to the measured I, away from the coil.

The Metre (Slide Wire) Bridge

- This is used for measuring voltages below 1.5 volts. In this circuit the unknown voltage is connected across a section of resistance wire typically 1 m in length, the ends of which are connected to a standard electrochemical cell E_0 that provides a constant current through the wire. The unknown emf E_1, in series with a galvanometer, is then connected across a variable-length section of the resistance wire AB, using a sliding contact. The sliding contact is moved until no current flows into or out of the standard cell, as indicated by a galvanometer in series with the unknown emf.
- The metre bridge is a practical form of the Wheatstone Bridge. As shown in the figure below, when the balance point, say C, is obtained, then

$$\frac{P}{Q} = \frac{R_{AC}}{R_{CB}} \tag{4}$$

where R_{AC} and R_{CB} are the resistances of the wire segments AC and CB respectively.

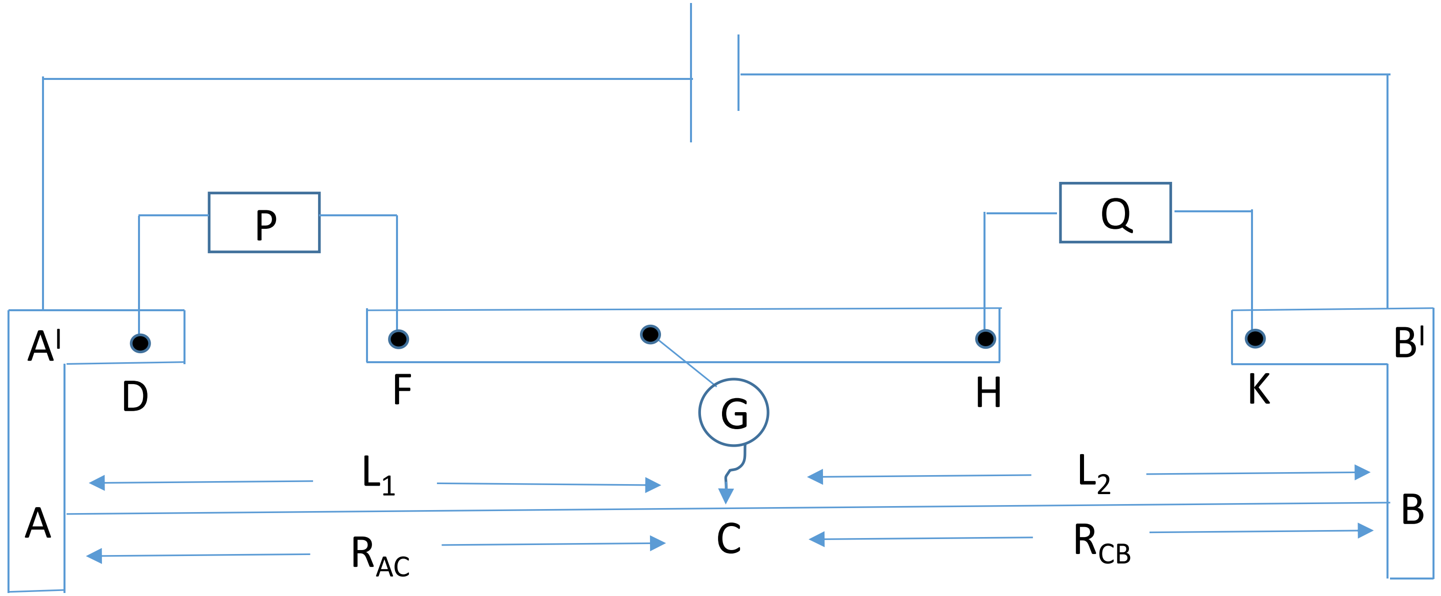


Fig. 2.0: The Metre Bridge

- Since the wire is uniform, the resistances R_{AC} and R_{CB} are proportional to the lengths l_1 and l_2 respectively. Hence,

$$\frac{P}{Q} = \frac{l_1}{l_2} \qquad (5)$$

- In using the metre bridge, it is assumed that the resistances of the end connections AA^I and BB^I are negligible. For this reason, P and Q are chosen so that the balance point C is as close to the middle of the bridge wire as possible. The galvanometer is usually protected by connecting a resistor in series with it. The lowest resistance the bridge can measure accurately is 1Ω.

- The voltage across the selected section of wire is then equal to the unknown voltage. The unknown voltage E_1 can be calculated from the current and the fraction of the length of the resistance wire that was connected to the unknown emf. The galvanometer does not need to be calibrated, as its only function is to read zero. When the galvanometer reads zero, no current is drawn from the unknown electromotive force and so the reading is independent of the source's internal resistance.

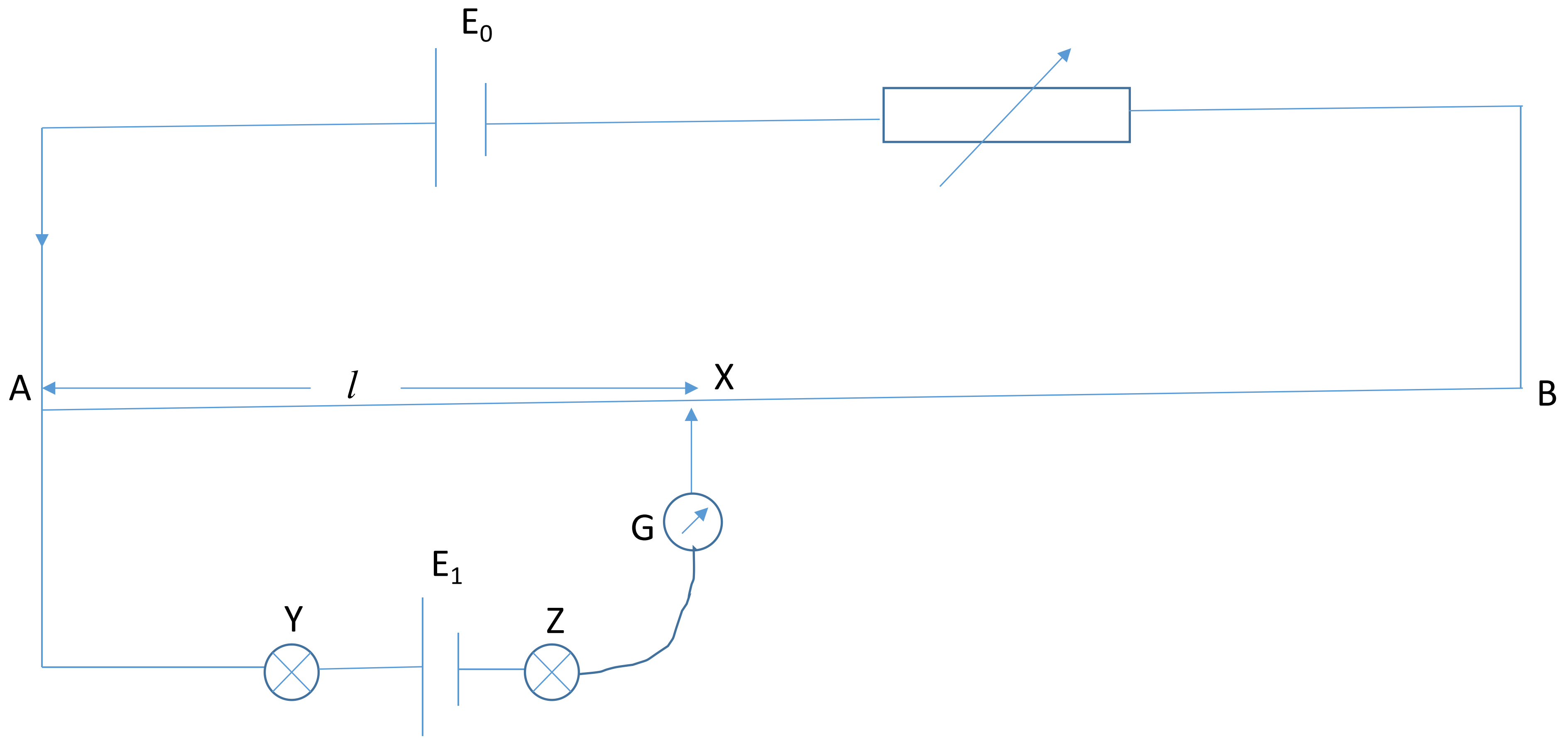


Fig 3.0: Basic Arrangement for the Measurement of an e.m.f.

E_0 = Source e.m.f.

E_1= Unknown e.m.f.

l = distance AX

YZ = terminals

AB = Uniform resistance wire

G = galvanometer

AB is exactly 1.0 m and *l*, the distance between *A* and *X,* is the sliding contact. At point *X*, the galvanometer indicates no current; *l* is therefore a measure of the e.m.f. *E*.

Example: If a balance point AX (=50.3cm) is obtained in an experiment to determine an unknown value of the e.m.f of a cell, and after a replacement with another cell AX is obtained to be 72.3 cm. Then

$$\frac{E1}{E2} = \frac{50.3}{72.3} = 0.696$$

- **Practical Uses**

The metre bridge can be used in the following ways:

(1) Measurement of an unknown resistance (as described above).

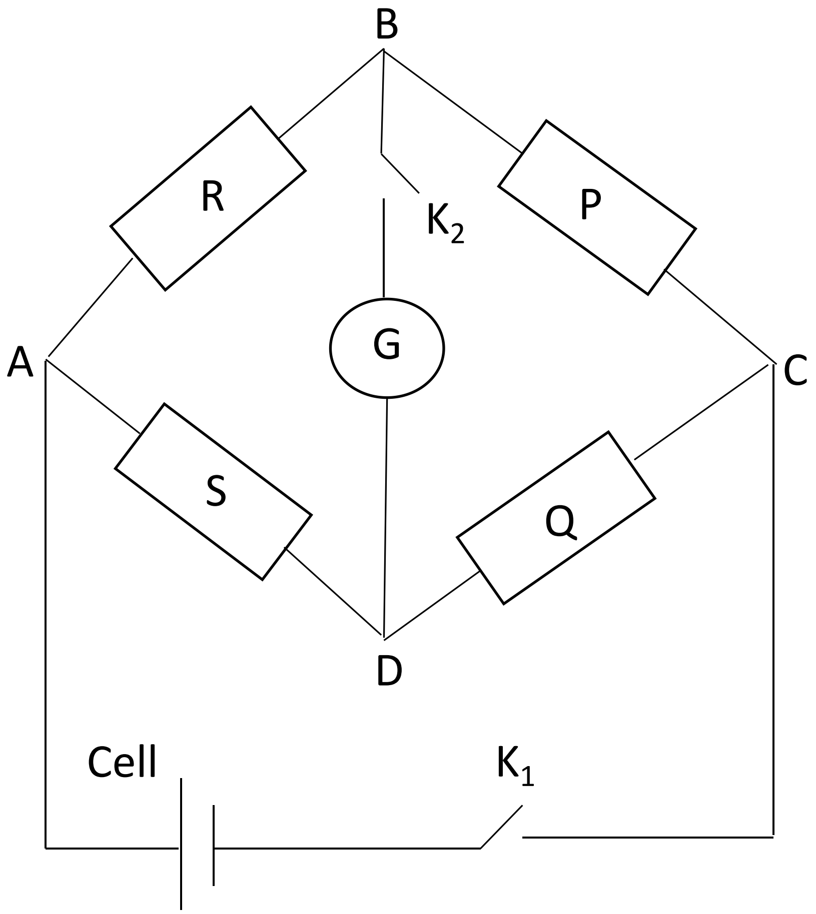
(2) Measurement of the resistivity of the material of a wire.

(3) Temperature coefficient of Resistance.

(4) The Bolometer.

(5) To test whether a material obeys Ohm’s law.

Example



The Wheatstone Bridge

(1) A typical Wheatstone bridge is balanced for the following values P = 20.0Ω. Q = 50.0Ω and S = 85.0Ω. Calculate the value of R.

Solution

Using Fig. 1.0 and equation $\frac{R}{P} = \frac{S}{Q}$ above, we have

R = 20/50 x 85

= 34.0 Ω

- **The Potential Divider**

Worked Example

Suppose in Fig. below, R_1 = 1 Ω and R_2 = 3 Ω. .

Since the same current I flows through (series connection) the resistors, then

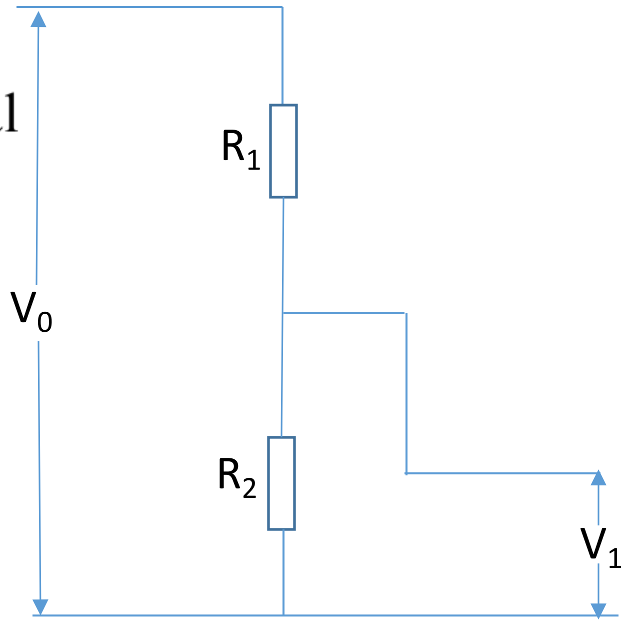
$$V = IR$$

and V α R

It means that the potential difference across each resistor is in proportion to its resistance value.

The ratio of the p.d. across R_1 and R_2 = 1:3.

The p.d. across both resistors is V_0



So, p.d. across R_1 is $\frac{1}{1 + 3} V_0 = \frac{1}{4V_0}$

p.d. across R_2 is $\frac{3}{1+3} V_0 = \frac{3}{4V_0}$

MODULE 2 - ELECTRODYNAMICS OF CHARGED PARTICLES

- Electrodynamics is the study of charges in motion and their interactions at boundary interfaces
- When both electrical and magnetic effects are present, the interaction is referred to as electromagnetism. Therefore Electrodynamics or electromagnetism studies the consequences of electromagnetic forces between electric charges and currents..

Charged particles are found to interact with both electric and magnetic fields. A charged particle placed in a magnetic field experiences a force when the following conditions are met:

(1) The charge must be moving, for no magnetic force acts on a stationary charge.

(2) The velocity of the moving charge must have a component that is perpendicular to the magnetic field.

Basics -Definitions

- Electromagnetism describes the relationship between electricity and magnetism.
- Most of us are acquainted with bar magnets or those thin magnets that usually end up on refrigerators. These magnets are known as permanent magnets.
- Although permanent magnets receive a lot of exposure, we use and depend on electromagnets much more in our everyday lives.
- Electromagnetism is essentially the foundation for all of electrical engineering.
- We use electromagnets to generate electricity, store memory on our computers, generate pictures on a television screen, diagnose illnesses, and etc
- Electromagnetism works on the principle that an electric current through a wire generates a magnetic field.
- This magnetic field is the same force that makes metal objects stick to permanent magnets. In a bar magnet, the magnetic field runs from the north to the south pole. In a wire, the magnetic field forms around the wire.
- If we wrap that wire around a metal object, we can often magnetize that object. In this way, we can create an electromagnet.

Basics-Making of Magnets

- Magnets are made up of poles, there is no monopole even at atomic level

- Magnets can be made using:
  - Electrical methods
  - Single touch
  - Divided touch
  - Hammering
- De-magnetisation is possible by:
  - Electrical methods and must be placed in a E – W direction
  - Heating E – W direction

- **Defn:** Magnetism is the effect of moving charges
- **Effect:** A compass needle fluctuates or deflects during a thunderstorm or if placed below a wire carrying current
- A magnetic flux is always induced in the space around a current carrying wire
- The direction of deflection of a compass needle depends on the direction of the electric current
- Hans Christian Oersted (1820), a Danish physicist discovered that electric currents can produce magnetic fields

Basics-Magnetic Properties of Matter

- Materials may be classified by their response to externally applied magnetic fields as diamagnetic, paramagnetic, or ferromagnetic.
- These magnetic responses differ greatly in strength for a given type of material.

- Diamagnetism is a property of all materials and opposes applied magnetic fields, but is very weak.

- Paramagnetism, when present, is stronger than diamagnetism and produces magnetization in the direction of the applied field, and proportional to the applied field.

- Ferromagnetic effects are very large, producing magnetizations sometimes orders of magnitude greater than the applied field and as such are much larger than either diamagnetic or paramagnetic effects

Basics-Magnetic Properties of Matter

- Examples are:
  - Ferromagnetic – Fe, Ni, Gd and Dy
  - Paramagnetic – Al, Cr, K, My and Na
  - Diamagnetic – Co, Bi, C, Ag, Pb, and Zn
  - The magnetization of a material is expressed in terms of density of net magnetic dipole moments μ in the material.
- Magnetic dipole moment and spin angular momentum are used to distinguish them/magnetic domains

Maxwell's Equations

The theory of electromagnetism was developed over the course of the 19th century, most prominently by Maxwell. In the 1860s, Maxwell developed a mathematical description of faraday's field lines

Maxwell's equations describe (1) how the field depends on the distribution and movement of charges in space and (2) how the field connects to itself.

From these equations, Maxwell derived an equation for electromagnetic waves in a vacuum and showed that these would travel at a velocity equal to the speed of light.

Maxwell's Laws or Equations

It is essential to mention the Maxwell's laws as they are crucial to electrodynamics.

The Maxwell's equations are;

(i) $\nabla.\vec{B} = 0$

(ii) $\nabla \mathrm{x}\vec{\mathrm{E}} = -\frac{\partial \vec{B}}{\partial t}$

(iii) $\nabla \mathrm{x}\vec{\mathrm{B}} = \mu_0(j + \epsilon_0 \frac{\partial D}{\partial t})$

where $\vec{j}=\sigma E$ and $\vec{D} = \epsilon_0 \vec{E}$ represent the current density and electric displacement respectively.

(iv) $\nabla.\vec{\mathrm{E}} = {}^{\rho}/_{\epsilon_0}$

Where ρ represents charge distribution density.

Motion of a Charged Particle in an Electromagnetic Field

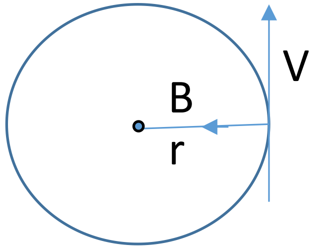
The Lorentz's force on a charged particle in a magnetic field is;

$$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B}) \qquad (1)$$

From the Lorentz force expression, the force on a charged particle (q) traversing a magnetic field $\vec{B}$ with velocity $\vec{V}$ can be computed.

Examples of practical applications are cyclotron, and mass spectrometer where electrons are accelerated to a high speed.

Considering a uniform circular magnetic field, $\vec{B}$ where a charged particle, q orbits,



The particle cuts the field at a right angle, thus from classical electrodynamics, the mechanical force is:

$$F = \frac{mv^2}{r} \tag{2}$$

From the Lorentz's force on a charged particle moving in a uniform magnetic field $\vec{B}$ at a right angle;

$$q\vec{V} \times \vec{B} = \frac{mv^2}{r} \text{ (centripetal force)} \tag{3}$$

$$qvB = \frac{mv^2}{r} \tag{4}$$

$$\omega = \frac{v}{r}$$

$$qB = m\omega$$

where $\omega = 2\pi f$

$$qB = m \times 2\pi f \qquad (5)$$

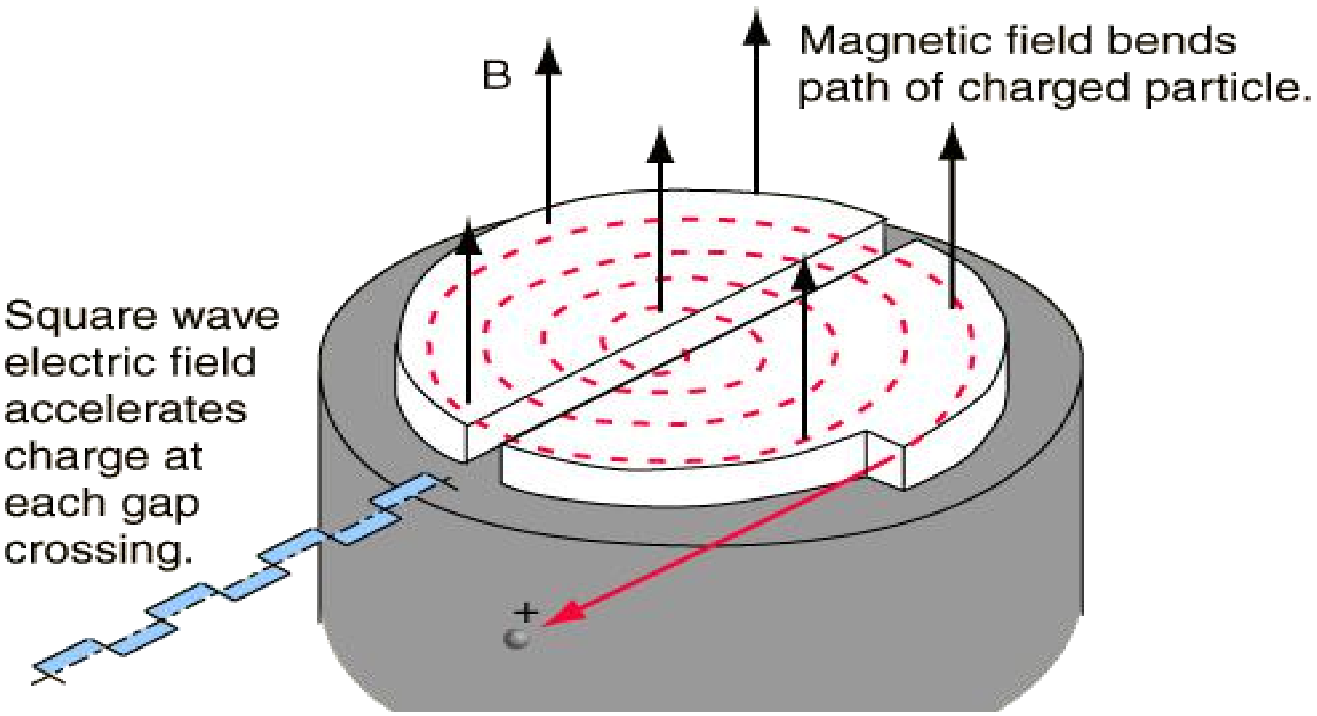
$f = \frac{Bq}{2\pi m}$ is the frequency of the particle in its orbit.

Charge particles in a Magnetic field- A Reminder

- Particles will move in a circular path at constant velocity v, such that:
- Centripetal force equals magnetic force, that is: $m v^2 / r = q v B$
- The period and frequency are independent of the speed of the particle, i.e. $f = 1/T = qB/2\prod m$
- Particles with the same e/m ratio have the same period (cyclotron acceleration)

Cyclotron

The cyclotron was one of the earliest types of particle accelerators, and is still used as the first stage of some large multi-stage particle accelerators. It makes use of the magnetic force on a moving charge to bend moving charges into a semicircular path between accelerations by an applied electric field. The applied electric field accelerates electrons between the "dees" of the magnetic field region. The field is reversed at the cyclotron frequency to accelerate the electrons back across the gap.



A cyclotron

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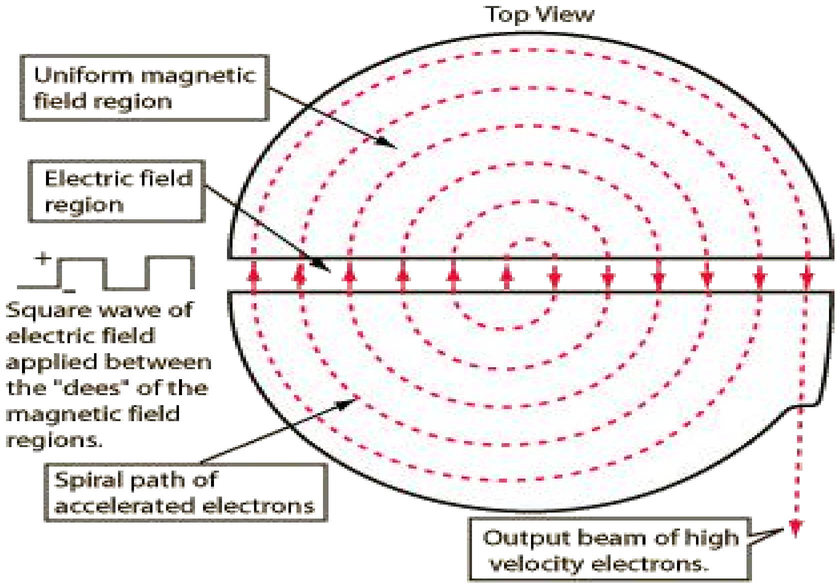
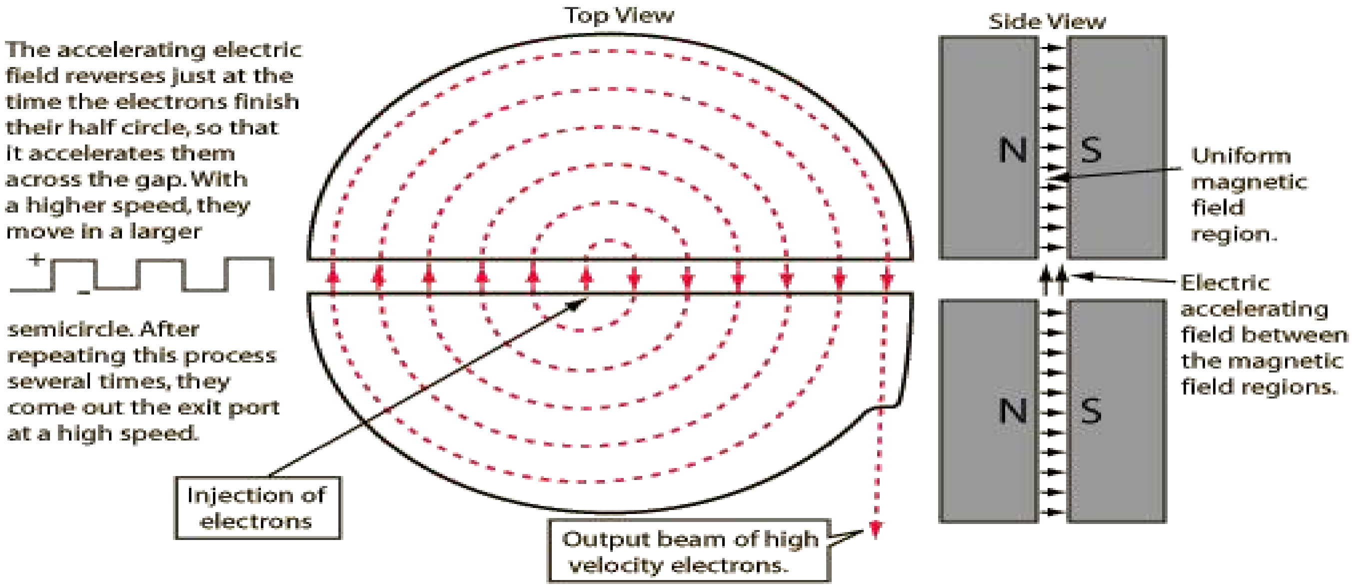




A moving charge in a cyclotron will move in a circular path under the influence of a constant magnetic field. If the time to complete one orbit is calculated:

$$T = \frac{2\pi r}{v} = \frac{2\pi m v}{qBv} = \frac{2\pi m}{qB}$$

$$\omega_{cyclotron} = \frac{qB}{m}$$

Cyclotron

- Radionuclide may be produced in cyclotrons where protons or deuterons area added to stable nuclides
- Cyclotron produced radionuclides generally decay by either beta plus process or by electron capture.
- In nuclear medicine, generators are often used to produce radionuclides for clinical use.
- Radionuclides produced by generators decay by beta minus process.

Torque on a Current Carrying Coil

From the Lorentz force expression, the force acting on a charged particle in a magnetic field is $F = BqV\sin\theta$. Also, the force on a moving current can be obtained. The force on a current carrying coil or conductor is given by $F = BIL\sin\theta$ and maximum when $\theta = 90^0$. It vanishes when $\theta = 0^0$ or 180^0 i.e when current is parallel or anti-parallel to the magnetic field. This is the concept applied in the operation of hi-fi speakers.

However, a current carrying conductor can experience a force that makes it rotate when placed in a magnetic field. If a loop or wire is suspended in a magnetic field, the magnetic force produces a torque, Γ that tends to rotate the loop. This torque is responsible for the operation of a number of devices including galvanometers and electric motors (We shall see this later in this course).

For $\Gamma = NAIB\sin\theta$ where θ is the angle between the normal to the plane of the coil and the magnetic field, N is the number of turns in the coil or loop of area, A carrying current, I under the action of a magnetic field, B, which is measured in Tesla (T). The expression $m = NIA$ is the magnetic moment, and is measured in $A.m^2$. The S.I. unit of the torque is Newton.metre (Nm).

Worked Examples

(1) An electron has a velocity perpendicular to a uniform magnetic field (0,0,B). Show that it executes circles with an angular frequency, $\omega = {}^{eB}/_{m}$. Give one other application and calculate the cyclotron frequency for B = 1.0 T.

(me = 9.1x10^-31 kg, e = 1.6x10^-19 C) — i.e. $(m_e = 9.1 \times 10^{-31}\ kg,\ e = 1.6 \times 10^{-19}\ C)$

Solution

F = qv x B is the Lorentz force

Thus, for the magnetic part of the field, *B = (0, 0, B)* and velocity *v = (0, 0, V)*.

$F = evB\sin\theta,\ \theta = 90^0$

$\Rightarrow F = evB,$ thus the electron executes circles with

centripetal force, $F_{cep} = \dfrac{mv^2}{r}$

$\therefore\ evB = \dfrac{mv^2}{r}$

$\Rightarrow \dfrac{v^2}{r} = \dfrac{evB}{m}$

With $\dfrac{v}{r} = \omega$, giving the angular frequency,

$\Rightarrow \omega = \dfrac{eB}{}$

Apart from the cyclotron, another application of this principle is the mass spectrometer.

For $B = 1.0\ T$, $m = 9.1\text{x}10^{-31}\ kg$ and $e = 1.6\text{x}10^{-19}\ C$, the cyclotron frequency, ω is:

$$\omega = \frac{1.6\text{x}10^{-19}\times 1.0}{9.1\text{x}10^{-31}}\ \text{rad/s}$$

(2) The electron and proton in a hydrogen atom are separated by $0.53x10^{-10}$m. Calculate the gravitational force between them.

$$F = \frac{Gm_1m_2}{r^2}$$

where $G = 6.07\text{x}10^{-11} Nm.m/kg^2$

$m_1 = 9.1\text{x}10^{-31}\ kg$ and $m_p = 1.67\text{x}10^{-27}\ kg$

Thus, the gravitational force between them is:

$$F = \frac{6.07\times10^{-11}\times9.1\times10^{-31}\times1.67\times10^{-27}}{(0.53\times10^{-10})^2}$$

$$= 1.74\text{x}10^{-19}\ \text{N}.$$

MAGNETIC FIELDS AND MAGNETIC FORCES OF AND ON CURRENT-CARRING CONDUCTORS

Magnetic Fields of Current-Carrying Conductors

- A current flowing through a wire produces a magnetic field with a circular geometry, wrapping itself around the wire.
- A current-carrying conductor can produce a magnetic field like that of a bar magnet if the wire is shaped in a certain way.
- An electric current consists of moving charges, so a bunch of moving charged electrons constitutes an electric current.
- An electric current produces a magnetic field. Figure 4.0 shows magnetic field lines from a wire carrying an electric current. The magnetic field lines of a straight current-carrying wire form concentric circles around the wire.
- The direction of the magnetic field (arrows) is given by the right-hand rule: when the thumb of the right hand points in the direction of the conventional current, the fingers curl around the wire in the direction of the magnetic field.

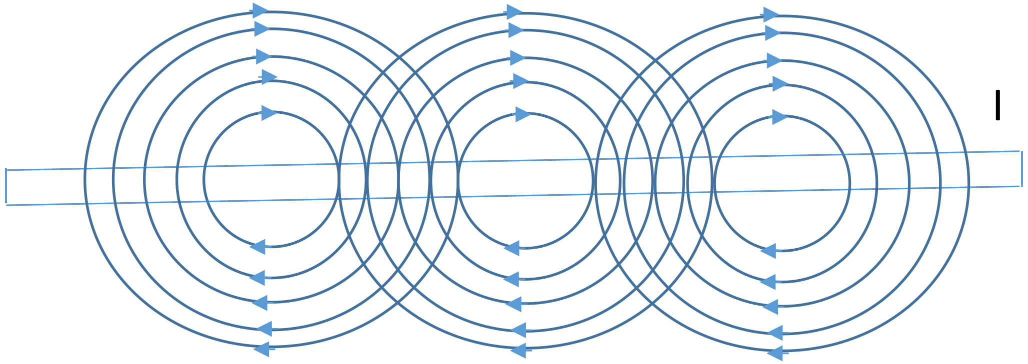


Fig. 4.0: Magnetic field lines from a wire carrying an electric current.

- Moving electrons generate a magnetic field that we cannot see but other moving electrons will experience a resulting force in the presence of the magnetic field.
- When two current-carrying conductors are next to each other, the current in one conductor will create a magnetic field around that conductor.
- The electric current in the other conductor therefore experiences a force exerted by the magnetic field.

- This also works in reverse. The electric current in each conductor reacts to the magnetic field produced by the other conductor.
- There is a force exerted on each of the conductors as shown in Figure 5.0. Thus, a magnetic field can exert a force on a moving charged particle.
- The magnitude of this force is found to be proportional to the particle's charge and its speed.

When the particle's velocity ($\bar{v}$) is perpendicular to the magnetic field ($\bar{B}$), the magnitude of the field is defined as

$$B = \frac{F}{qv} \qquad (6)$$

Equation (6) is valid only when $\bar{v}$ is perpendicular to $\bar{B}$. The SI unit of magnetic field is Newton per ampere-meter [*N/(A.m),* or *tesla (T)*].

Physically, *B* means the magnetic force exerted on a charged particle per unit charge (coulomb) and per unit speed (m/s). Thus, the unit of B is *N/(C.m/s)* or *N/(A.m),* because $1A = 1C/s$. This combination of units is named the tesla (T) after Nikola Tesla (1856 – 1943), an early researcher in magnetism, and $1T = 1N/(A.m)$.

A non-SI unit commonly used by geologists and geophysicists, called the gauss (G), is defined as one ten-thousandth of a tesla ($1G = 10^{-4}\ T = 0.1\ mT$). For example, the Earth's magnetic field is on the order of several tenths of a gauss or several hundredths of a millitesla.

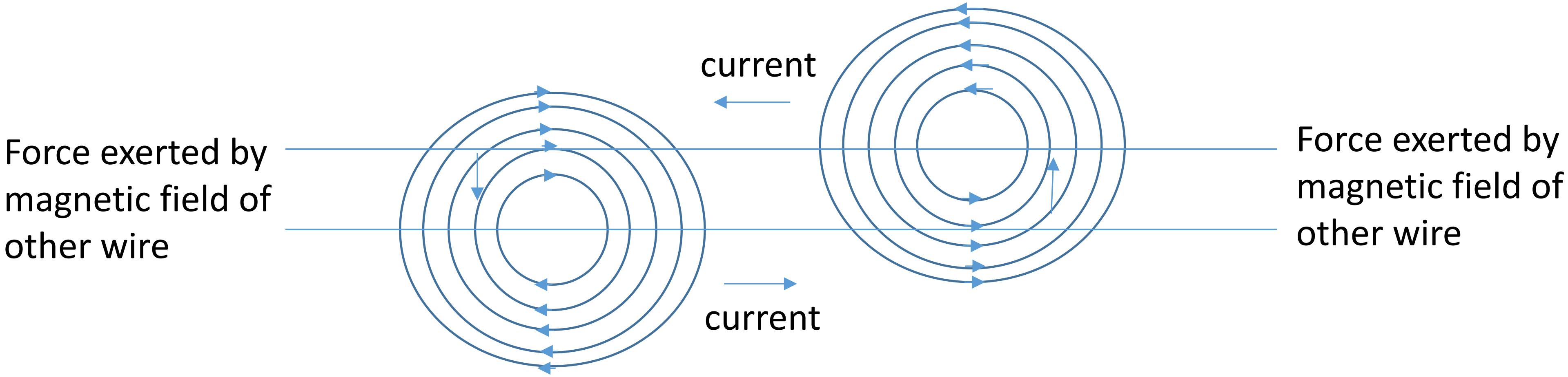


Fig. 5.0: Magnetic field lines for two conductors carrying current in opposite directions.

The magnetic field of an infinitely long straight wire can be obtained by applying Ampere's law. Ampere's law takes the form:

$$\sum B_{II}\Delta l = \mu_0 I \qquad (7)$$

and for a circular path centred on the wire, the magnetic field is everywhere parallel to the path. The summation then becomes:

$$\sum B_{II}\Delta l = B2\pi r \qquad (8)$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad (9)$$

This is the expression for the magnetic field. The constant μ_0 is the permeability of free space and is given as: $\mu_0 = 4\pi \times 10^{-7}\ T.m/A$.

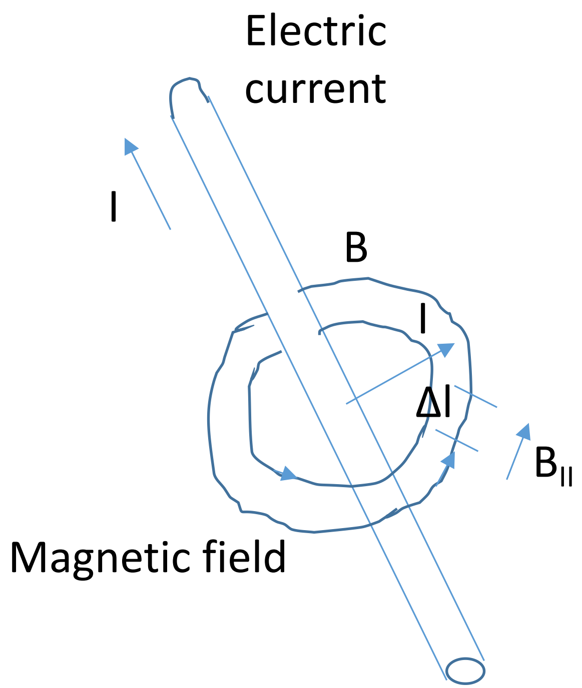


Fig. 6.0: Magnetic field of a current-carrying conductor

Magnetic Forces on Current-Carrying Conductors

The magnetic force on a charged particle depends on the relative orientation of the particle's velocity and the magnetic field.

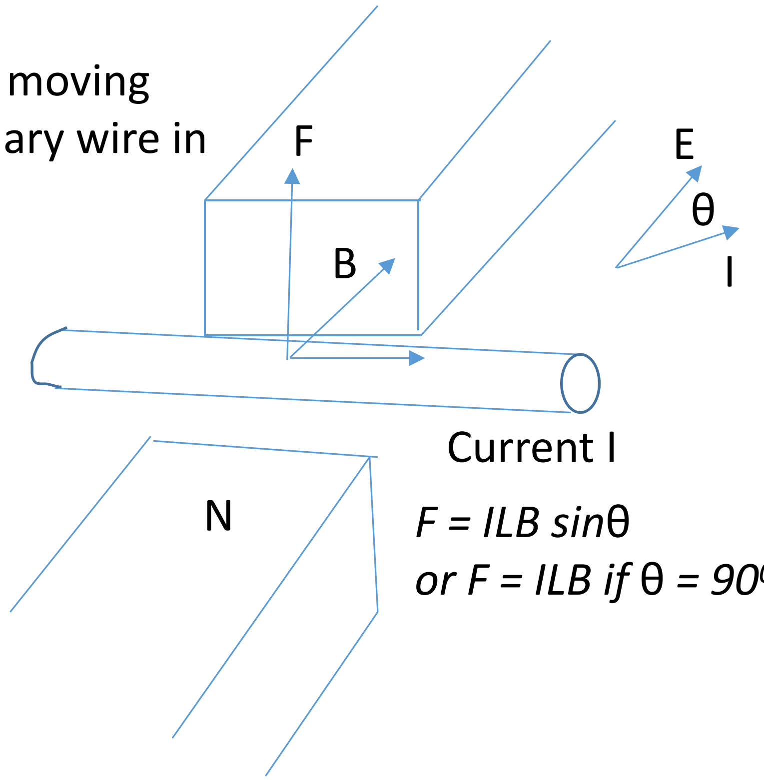
A magnetic force cannot change the speed of a charged particle, only its direction. When a charged particle enters a uniform magnetic field in a direction perpendicular to that field, its motion is continuously changed by the magnetic force; it ends up moving in a circle, with radius,

$$r = \frac{m \times v}{q \times B} \quad (10)$$

where, *r* is radius, *m, v, q* and *B* are mass, velocity, charge and magnetic field respectively.

When a conductor carrying no current and fixed in place at both ends, extends through the gap between the vertical pole faces of a magnet, the magnetic field between the faces is directed outward from the page (Fig.7.0). If a current is sent through the conductor, the magnetic force on the current-carrying conductor is perpendicular to both the conductor and the magnetic field with direction given by the right hand rule. This is shown in Figure 8.0.

Positive charge moving through stationary wire in magnetic field



*F = ILB sin*θ

or F = ILB if θ *= 90*^{0}

This relationship arises from the basic magnetic force:

$$F = qvB \sin \theta$$

which for a charge q travelling length L in a wire can be written

$$F = q\frac{L}{t}B\sin\theta$$

$$F = \frac{q}{t}LB\sin\theta$$

$$F = ILB\sin\theta$$

Fig. 7.0: A conductor between the vertical pole faces of a magnet.

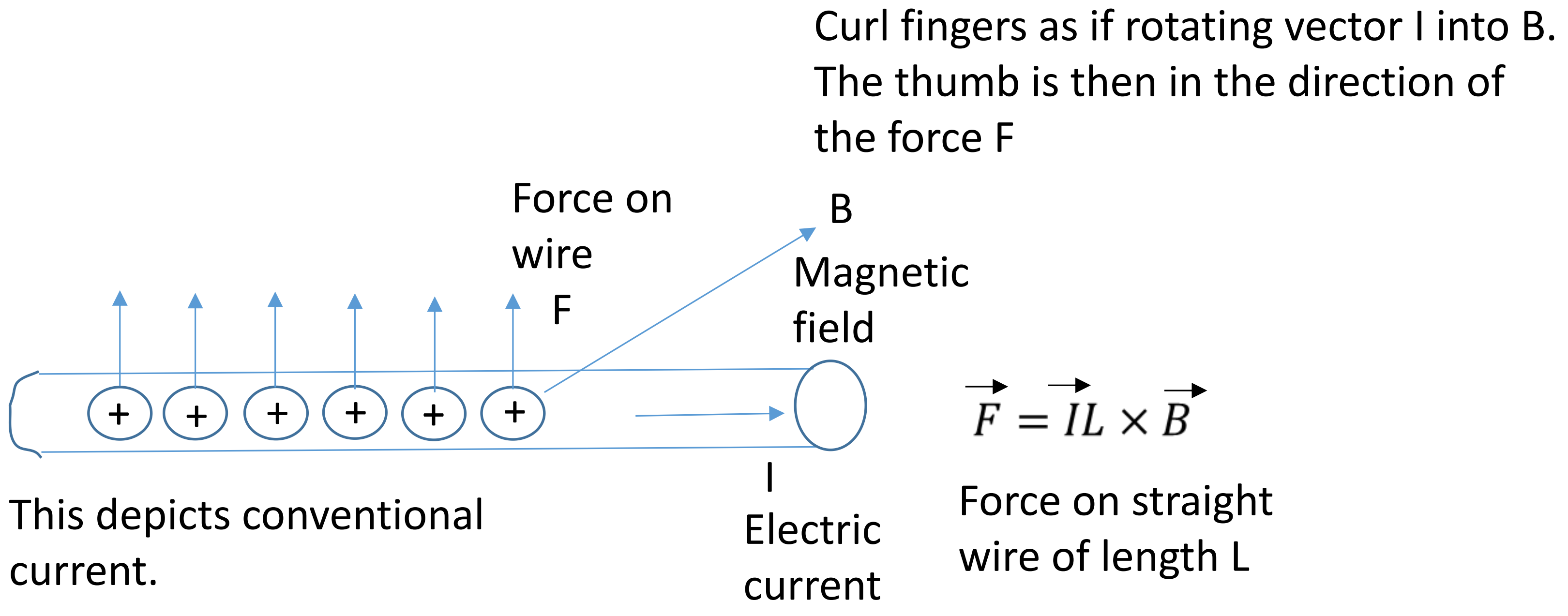


Fig. 8.0: Magnetic field with direction given by the right hand rule.

- A current consists of many small charged particles running through a conductor. If immersed in a magnetic field, the particles will experience a force; they can transmit this force to the wire through which they travel. The force on a section of wire of length *L* carrying a current *I* through a magnetic field *B* is given by:

$$F = I\,(L \times B) \quad (11)$$

$$= I\,L\,B\,\sin(\theta) \quad (12)$$

where, θ is the angle between the directions of *L* and *B*. Equation (11) implies that *F* is always perpendicular to the plane defined by vectors *L* and *B*, as indicated in Figure 8.

- If a conductor is not straight or the field is not uniform, we can imagine the conductor broken up into small straight segments and apply equation (11) to each segment.
- The force on the conductor as a whole is then the vector sum of all the forces on the segments that make it up.

- Mathematically, we can write,

$$d\bar{F}_B = I d\bar{L} \times \bar{B} \tag{13}$$

and this expression can be used to find the resultant force on any given arrangement of currents by integrating equation (13) over that arrangement.

Thus, $d\bar{F} = I\int d\bar{L} \times \bar{B}$.

Summary

Magnetic field $\bar{B}$. A magnetic field $\bar{B}$ is defined in terms of the force $\bar{F}_B$ acting on a test particle with charge q moving through the field with velocity $\bar{v}$:

$$\bar{F}_B = q\bar{v} \text{ x } \bar{B}$$

The SI unit for $\bar{B}$ is the tesla (T):

$$1T = 1N/(A.m) = 10^4\ gauss.$$

Magnetic force on a current-carrying conductor. A straight conductor carrying a current I in a uniform magnetic field experiences a sideways force:

$$\bar{F}_B = I\bar{L} \times \bar{B}$$

The force acting on a current element $Id\bar{L}$ in a magnetic field is,

$$d\bar{F}_B = Id\bar{L} \times \bar{B}$$

The direction of the length $\bar{L}$ or $d\bar{L}$ is that of the current I.

Worked Examples

(1) A straight horizontal length of copper wire has a current $I = 46A$ flowing through it. What are the magnitude and direction of the minimum magnetic field $\overline{B}$ needed to suspend the wire, that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 45.2g/m.

Solution

To balance the downward gravitational force $\overline{F}_g$ on the wire, we want $\overline{F}_B$ to be directed upward as in Figure 9.0. $\overline{B}$ must be horizontal and rightward to give the required upward $\overline{F}_B$.

The magnitude of $\overline{F}_B$ is given as:

$$F_B = ILBsin\theta \qquad (14)$$

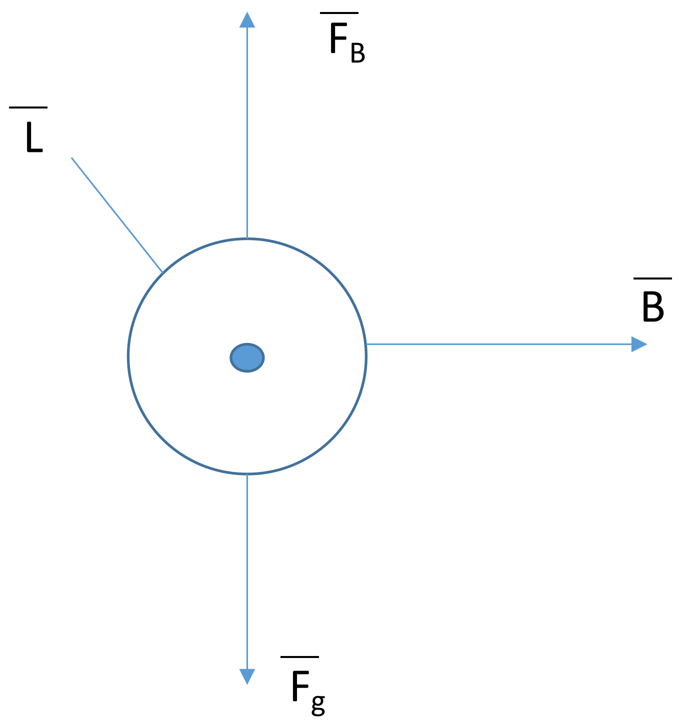


Fig. 9.0: A Cu wire carrying current out of the paper.

Since we want $\overline{F}_B$ to balance $\overline{F}_g$, we want

$$ILB sin\theta = mg = \overline{F}_g \quad (15)$$

where mg is the magnitude of $\overline{F}_g$ and m is mass of the wire.
For the minimal field magnitude B for $\overline{F}_B$ to balance $\overline{F}_g$, we set $\theta = 90^0$ (making $\overline{B}$ perpendicular to the wire). Therefore, $\sin \theta = \sin 90^0 = 1$, so equation (15) gives:

$$B = \frac{mg}{IL sin\theta} = \frac{(m/L)g}{I} \quad (16)$$

Where (m/L) is the linear density of the wire.

So,

$$B = \frac{\left(45.2\times10^{-3}\,{kg}/{m}\right)\left(9.8\,{m}/{s^2}\right)}{46A}$$

$= 9.6\text{x}10^{-3}$T.

(2) A He^{2+} ion travels at right angles to a magnetic field of 0.45 T with a velocity of 10^6 m/s. Find the magnitude of the magnetic force on the ion.

Solution

$F = qvB\sin\theta \qquad \theta = 90^0 \qquad \sin\theta = 1$

$F = 2(1.60\text{x}10^{-19})10^6\,(0.45) = 1.44\text{x}10^{-13}$ N.

(3) A conductor bearing a current of 15A lies perpendicular to a uniform magnetic field. A force of 0.5N is found to exist on a section of the wire 60cm long. Determine the magnetic induction B.

Solution

For a straight segment of the conductor of length L,

$F = B\, IL \sin 90^0$

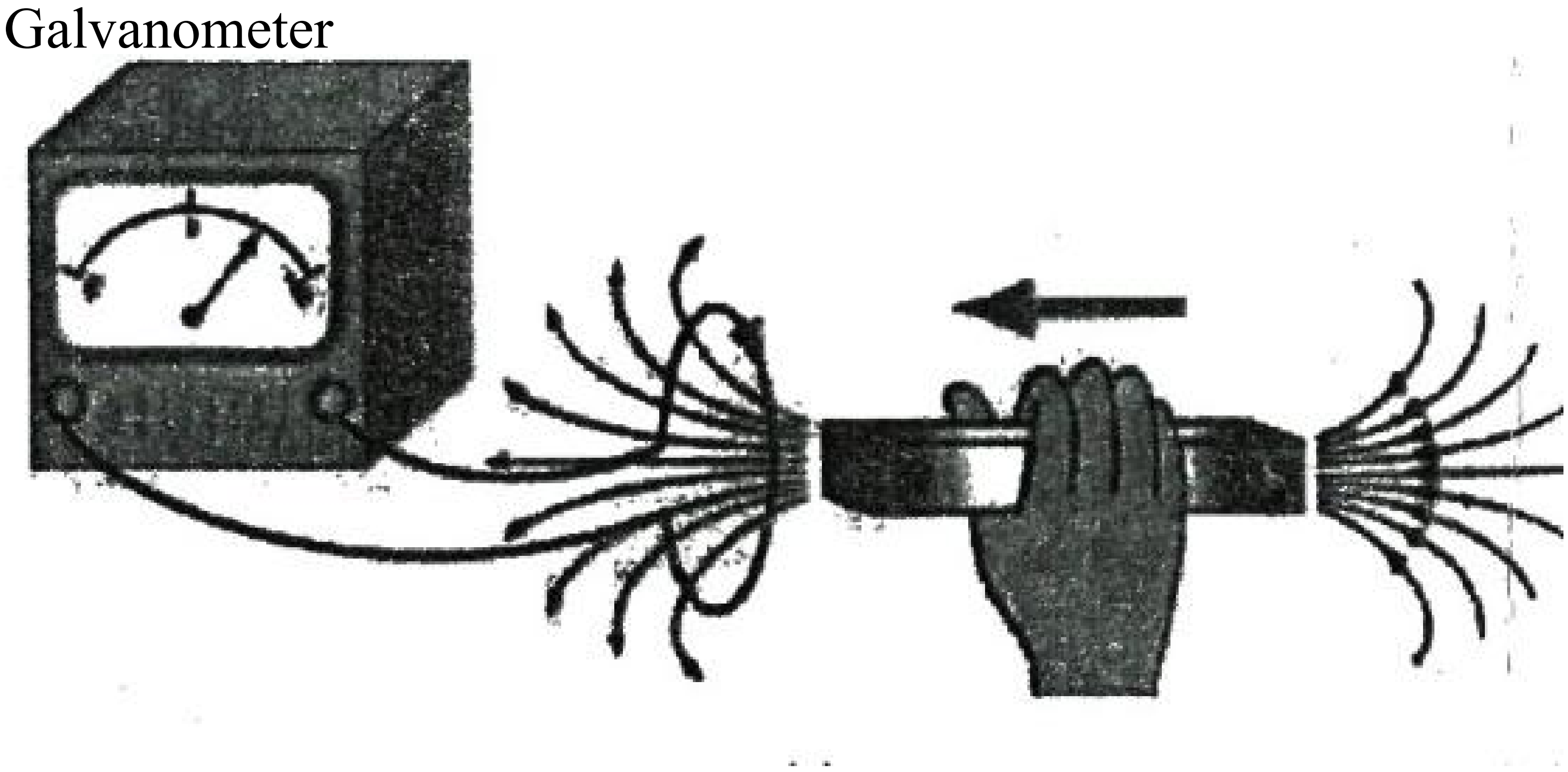
$0.5 = 15(0.60)B$

$B = 0.056T$

ELECTROMAGNETIC INDUCTION

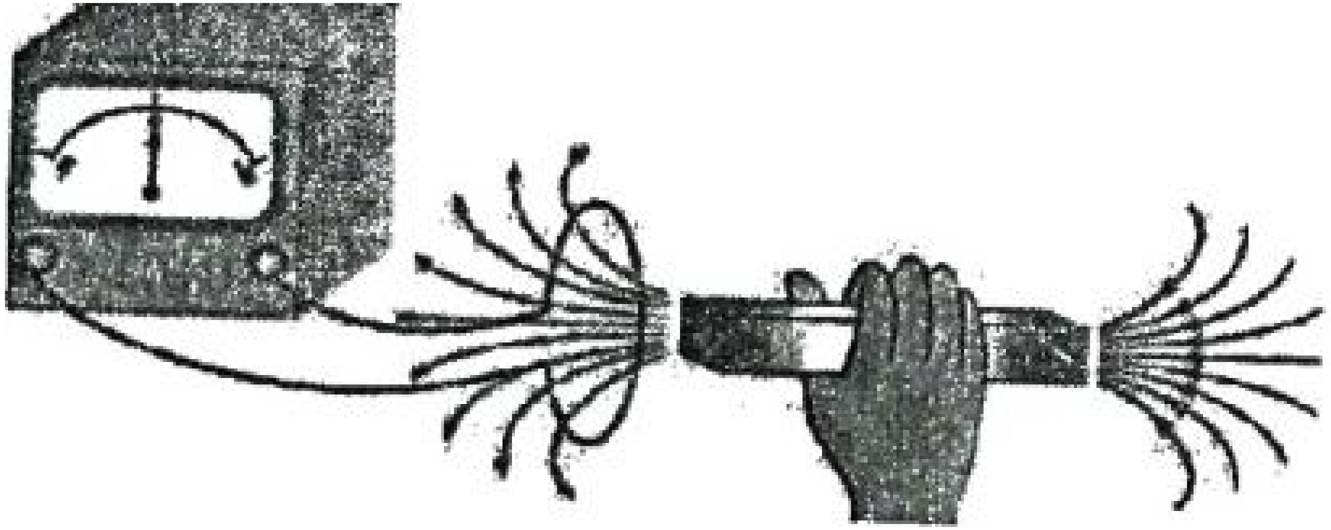
Electromagnetic induction is the phenomenon whereby a current is produced in a conductor whenever the current cut across magnetic field lines.

A charged object produced an electric field (E) radially around itself, so also a bar magnet creates a magnetic field (B) around itself based on the orientation of its north (N) and south (S) poles. The magnetic field lines start from the north pole and ends on the south pole. It has been shown and as earlier stated that an electric current produces a magnetic field and versa depending on the position and orientation of the conductor and the magnet..



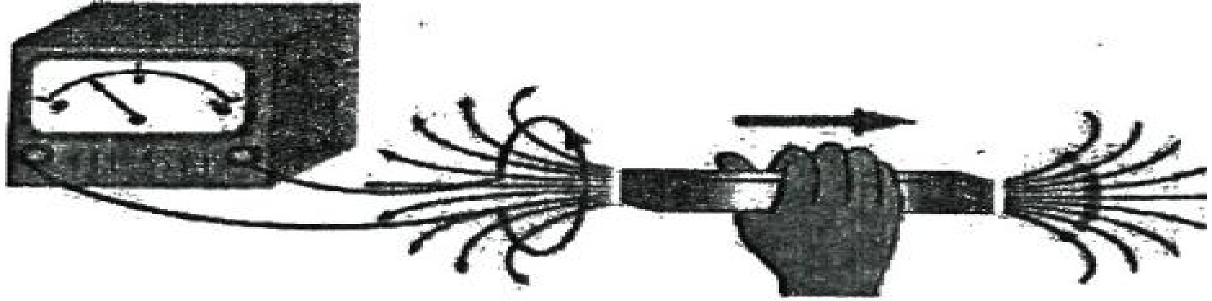
(a)

Galvanometer



(b)

Galvanometer



(C)

(c)

Fig. 10.0: Electromagnetic induction

LORENTZ's FORCE

The electric field ($\vec{E}$) can be defined as the force ($\vec{F}_E$) per unit charge (q) and it is measured in Newton per coulomb.

$$\vec{E} = \frac{\vec{F}_E}{q} \text{ (N/C)} \qquad (17)$$

Electromotive force (e.m.f) ε is generally defined as the work done by a non electrostatic agent in carrying a unit charge round a closed loop:

$$\varepsilon = \frac{W_{nonelectrostatic}}{q} \oint \frac{\vec{F} \cdot dl}{q} \qquad (18)$$

When a charges q moves through a magnetic field of strength B with velocity *v,* a force,

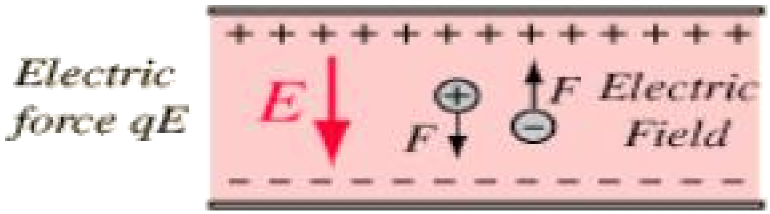
$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad (19)$$

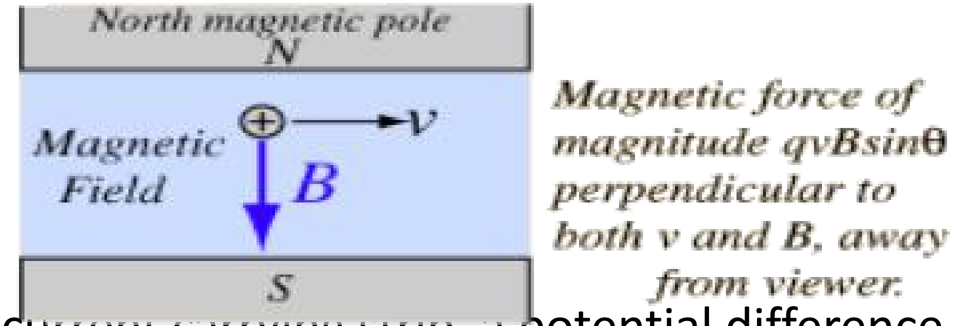
acts on it. Equation (19) shows that the magnitude and direction of F_B depends on v and B and when the sign of the charge of the particle changes, the direction of the magnetic force also changes.

Lorentz Force

- When a particle is subject to both E and B in the same region, total force is called Lorentz force

$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric force}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic force}}$$





- Hall Effect: when B is applied perpendicular to a current carrying strip, a potential difference develops across the width, $V_H = IB/nqt$

The SI unit of magnetic field is tesla (T)

$$1\text{T} = \frac{Newton}{Coulomb.(^{meter}/_{second})} = N/A.m$$

It is also possible for the charges to be relatively stationary while the whole conductor moves through the field as shown in figure 11.0

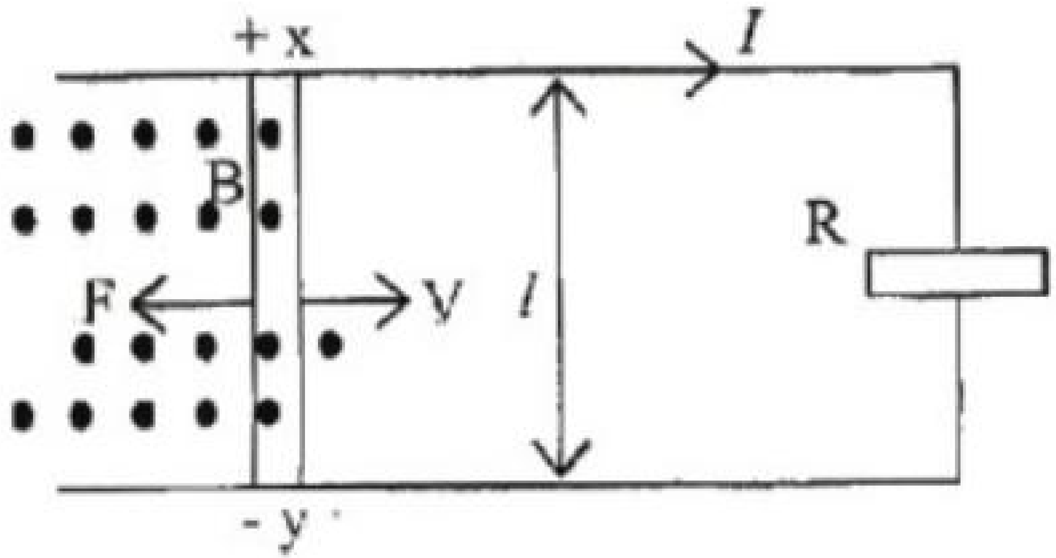


Fig. 11.0: Motion of a conductor in a magnetic field strength B, directed out of the page.

- Then the force on a charge is called the Lorentz's force F given by:

$$F = q(E + v \times B) \qquad (20)$$

From equation (20) observe that:

$$\varepsilon = \oint (E + v \times B).dl \qquad (21)$$

- The terms $\varepsilon = \oint E.dl$ and $\varepsilon = \oint (v \times B).dl$ are both contained in Faraday's law.
- The first term represents induced electric field associated with time varying magnetic field while the second term represents motional e.m.f. which occurs when a conductor placed in a magnetic field moves relative to the field.

Magnetic Flux

The magnetic flux can be defined as the number of field lines passing through an area, A, where the area is a vector perpendicular to the plane of the magnetic field lines (Fig. 12.0)

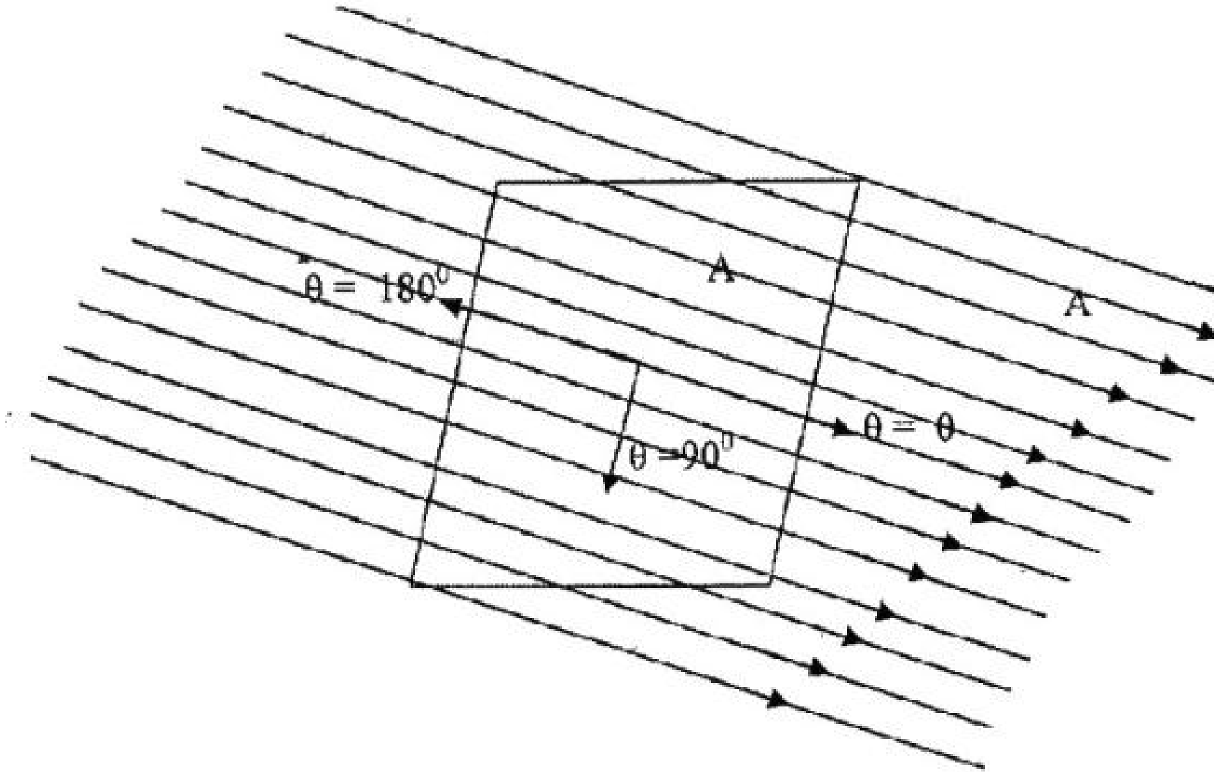


Fig. 12.0: Magnetic flux through a conducting loop.

If the plane of rotation conducting loop is perpendicular to the magnetic field such that

$$\theta = 0, \text{ then } \phi_B = \phi_{B\,max} \quad (22)$$

when $\theta = 180^0$, the magnitude of the magnetic flux is the same but in the opposite direction.

$$\phi_B = -\phi_{B\,max} \quad (23)$$

when $\theta = 90^0$

$$\phi_B = 0 \quad (24)$$

As the loop is rotated, the area through which the line of flux cuts across the conducting loop changes, and the magnetic flux depends on the angle between the area and the magnetic field lines following,

$$\phi_B = BA\cos\theta \tag{25}$$

The above can be summarized as follows: the magnetic flux ϕ_B through a plane area A in a uniform magnetic field is given by:

$$\phi_B = B.A \tag{26}$$

For a non-uniform field and a rough surface the flux is given by:

$$\phi_B = \int B \, . \, dA \tag{27}$$

The SI unit of magnetic flux is weber (Wb).

FARADAY'S LAW

Electric fields are made by stationary charges, and magnetic fields are made by moving charges. The idea of faraday describing electrical and magnetic inductions as fields revolutionized physics and is still central to its development.

The work done in circuits, where other forces apart from electrostatic forces, like magnetic or chemical forces are acting cannot only be defined using potential difference alone, rather e.m.f. is used instead. Using the system described in figure 11.0 above.

$$\varepsilon = \oint (v \times B).dl \qquad (28)$$

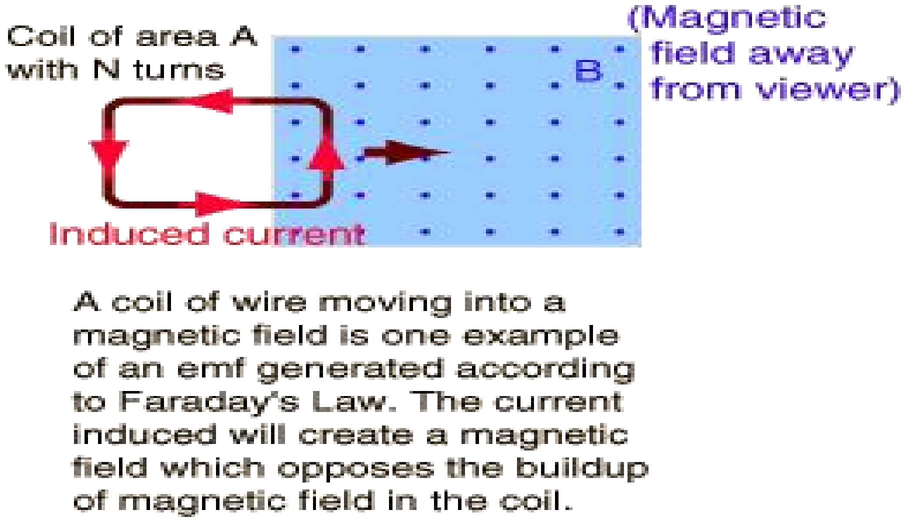
$\varepsilon = lvB$ (*l, v and B* are all at right angles), *lvdt* is the area cut out by the conductor in time *dt, lv*Bdt is the decrease in the flux for the interval. Therefore we have:

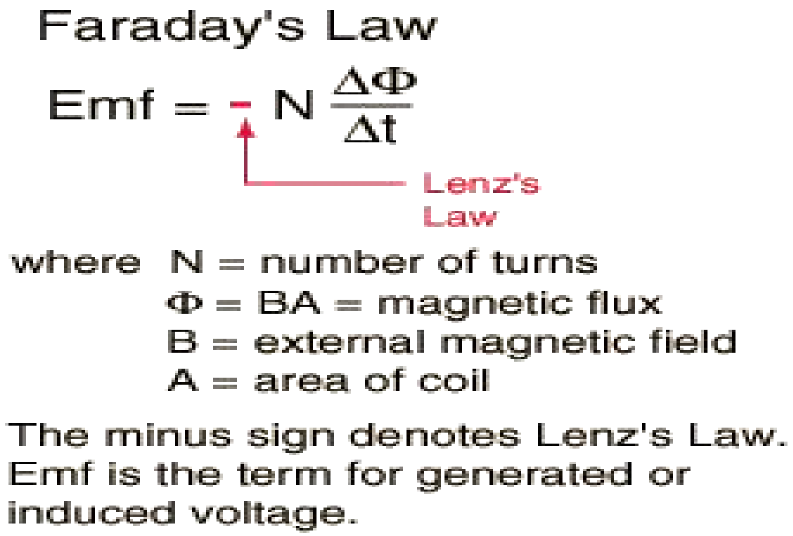
$$- d\phi_B = lvBdt \qquad (29)$$

and $\varepsilon = -\frac{d\phi_B}{dt}$ (Faraday's Law) (30)

Faraday’s Law of Electromagnetic Induction

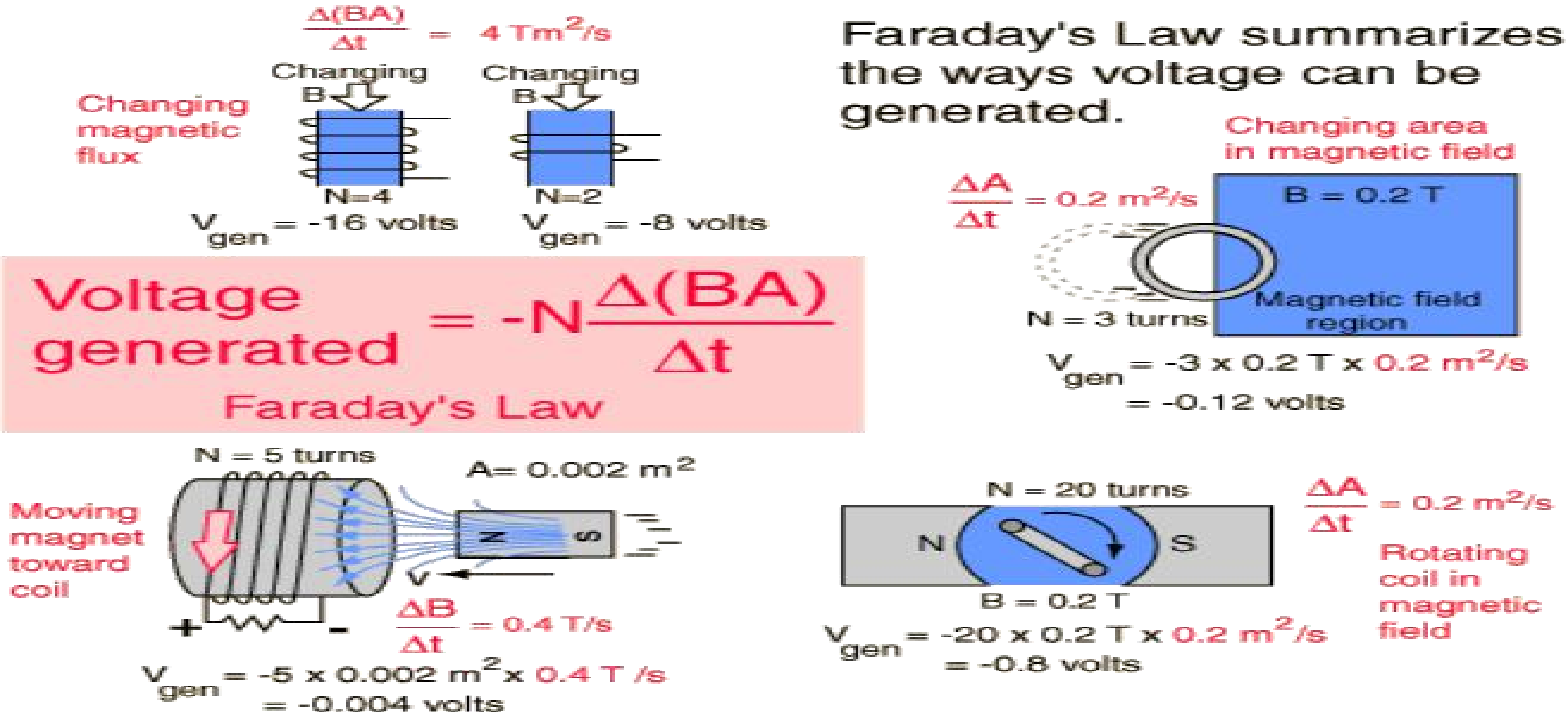
- Faraday's law is a fundamental relationship which comes from Maxwell's equations. It serves as a succinct summary of the ways a voltage (or emf) may be generated by a changing magnetic environment. The induced emf in a coil is equal to the negative of the rate of change of magnetic flux times the number of turns in the coil. It involves the interaction of charge with magnetic field.





Faraday's Law of Electromagnetic Induction

- .



Equation (30) is the e.m.f. induced in the circuit by the motion of the conductor through the magnetic field and this phenomenon is called electromagnetic induction.

Faraday's law states that, the induced e.m.f along any closed path is proportional to the rate of variation of the magnetic flux through the area bounded by the path.

$$\varepsilon \ \alpha = \frac{d\phi_B}{dt} \qquad (31)$$

The derivative of ϕ_B in equation (66) with respect to time is,

$$\frac{d\phi_B}{dt} = \frac{dB}{dt} A\cos\theta + B\frac{dA}{dt}\cos\theta - BA\sin\theta\frac{d\theta}{dt}$$

Since $\phi_B = B.A = \mathrm{BA}\cos\theta$.

- Faraday discovered that a voltage would be generated across a length of wire if that wire was exposed to a perpendicular magnetic field flux of changing intensity.
- A magnetic field of charging intensity can easily be created by moving a permanent magnet to or from a wire or coil of wire.
- The magnetic field must increase or decrease in intensity perpendicular to the wire before any voltage can be induced (equation 30).
- The change in magnetic flux can be due to changes in the
- (i) field strength B,
- (ii) area of the conductor placed in a uniform magnetic field, and
- (iii) change in orientation of the conductor relative to the magnetic field.
-

Induced Electric Field

Self induction

Following Faraday's law, the magnetic field produced by the current-carrying wire is always perpendicular to the wire, and the flux intensity of that magnetic field varies with the amount of current through it so it is possible for a wire to induce voltage along its own length if there is a change in the current passing through it. This effect is known as self-induction where changes in the magnetic field produced by changes in the current through a wire, induces voltage along the length of the wire.

This effect is applied in inductors where the self induction is intensified by bending the wire into the shape of a coil and sometimes wrapping the coil around a material of high magnetic permeability (μ_o).

Mutual Induction

If two stationary conducting loops are brought close to each other such that a link is formed between their magnetic fields, if a steady current flows through either if the loops, magnetic field will be induced in the second loop but no voltage will be induced because there is no change in the magnetic field

(Faraday's law requires a change in the magnetic field before voltage can be induced). On the other hand, if the current changes in the first loop either by switching on or off the current source to it, then the magnetic field in the other loop will change and voltage is induced in the loop. Mutual induction has application in transformers which operates on the principle that voltage is generated to oppose any change in the magnetic field.

For induced electric field with stationary conductor, the motional part of equation (21) becomes zero and Faraday's law reduces to:

$$\varepsilon = - \frac{d\phi_B}{dt} = E.dl = - A \frac{dB}{dt} \qquad (32)$$

Motional Emf

Equations (28-30) describe the rate of change of flux of magnetic field over a moving conductor and so for motional e.m.f. with no induced electric field, Faraday's law reduces to:

$$\varepsilon = \oint (\, v \times B).dl = \frac{d\phi_B}{dt} \qquad (33)$$

Condition For Induced Emf

When the number of field lines passing through the plane of conducting loop changes, an emf is induced. This can be achieved by the motion of either the magnet or the conducting loop. If there is no motion, no emf is induced.

The magnitude of the induced emf depends directly on:

(i) The number of turns in the coil;

(ii) The speed of rotation of the coil.

(iii) The strength of the magnetic field.

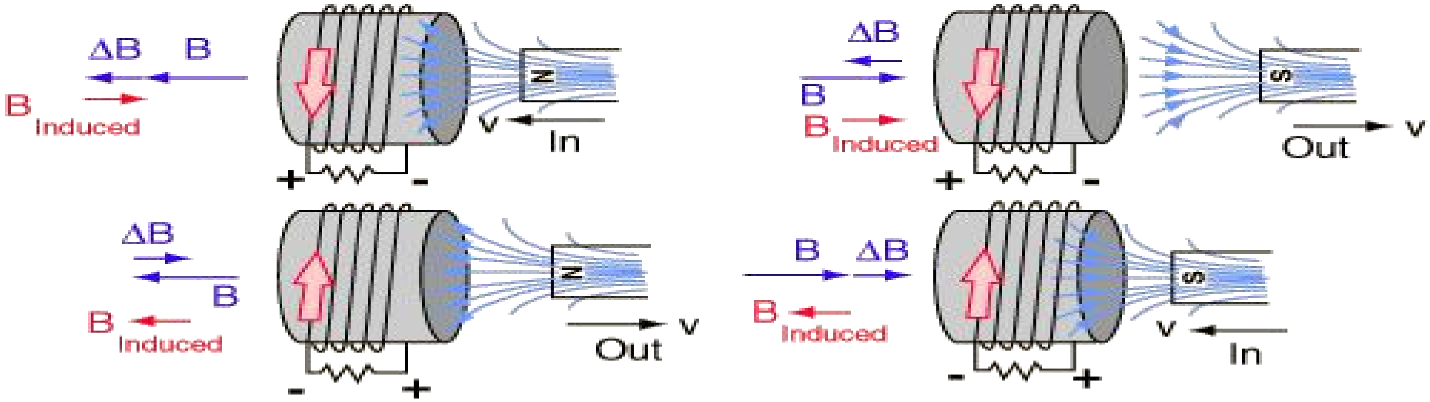
LENZ'S LAW

H.F Lenz improved upon Faraday's statement on the direction of the induced current. Considering the system in figure 18.0, the current which flows in the circuit is in such a direction as to exert a force opposing the motion of the conductor *xy* and the change brought about by it. This is required for conservation of energy since power is dissipated in the circuit. ***Lenz's law states that, the direction of the induced e.m.f is such that any current caused by it tends to appose the change of flux which produced it.***

Lenz's law upholds the law of conservation of energy and ensures that induced currents get their energy from the effect creating them. It indicates the direction of the induced current which is determined by **the *right hand rule which states that if the fingers of a right hand move in the same direction as the current, then the thumb indicates the direction of the magnetic field as it passes through the center of the loop*.**

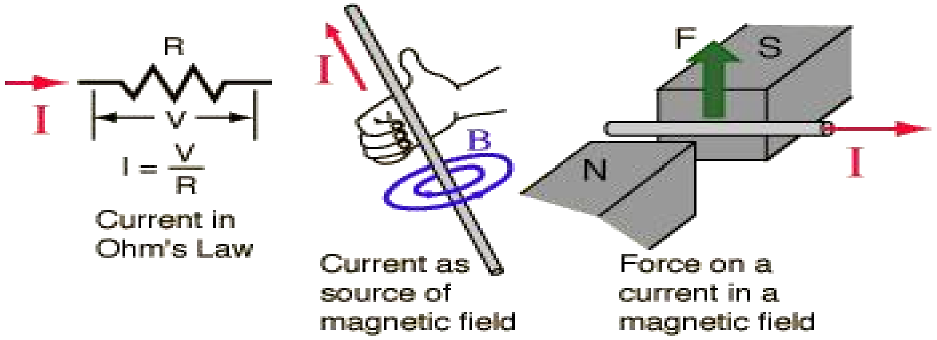
Lenz Law

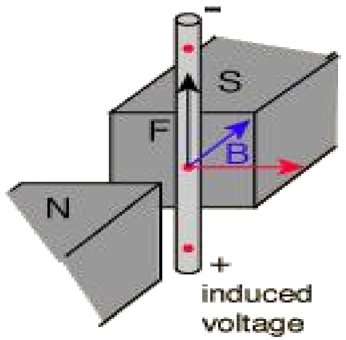
- When an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it.
- The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. In the examples here, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.



Right hand rule

- The direction of the induced magnetic flux can be determined by the right rule





APPLICATIONS OF ELECTROMAGNETIC INDUCTION

- Electromagnetic induction has found application in the construction of electrical generators, which use mechanical power to move a magnetic field pas coils of wire to generate voltage .
- Faraday's discovery of electromagnetic inductions was the beginning of electrical engineering.
- Nearly all the commercial electric current used today is generated by induction, in machines which contain coils moving continuously in a magnetic field.
- In electric generator, a rotating armature coil in a magnetic field generates an electric current. when the device is used as motor, a current is passed through the coil. The interaction of the magnetic field with the current causes the coil to spin. It is also used to produce eddy currents for the breaks of some trains. An eddy current is a swirling current set up in a conductor in response to a changing magnetic field.
- Generators of alternating current are often called alternators. The magnetic field of an alternator is provided by an electromagnetic called a field magnet.
- The rotating core called the armature, is wound on an iron core, which is shaped so that it can turn within the pole-pieces of the field magnet.

TRANSFORMERS

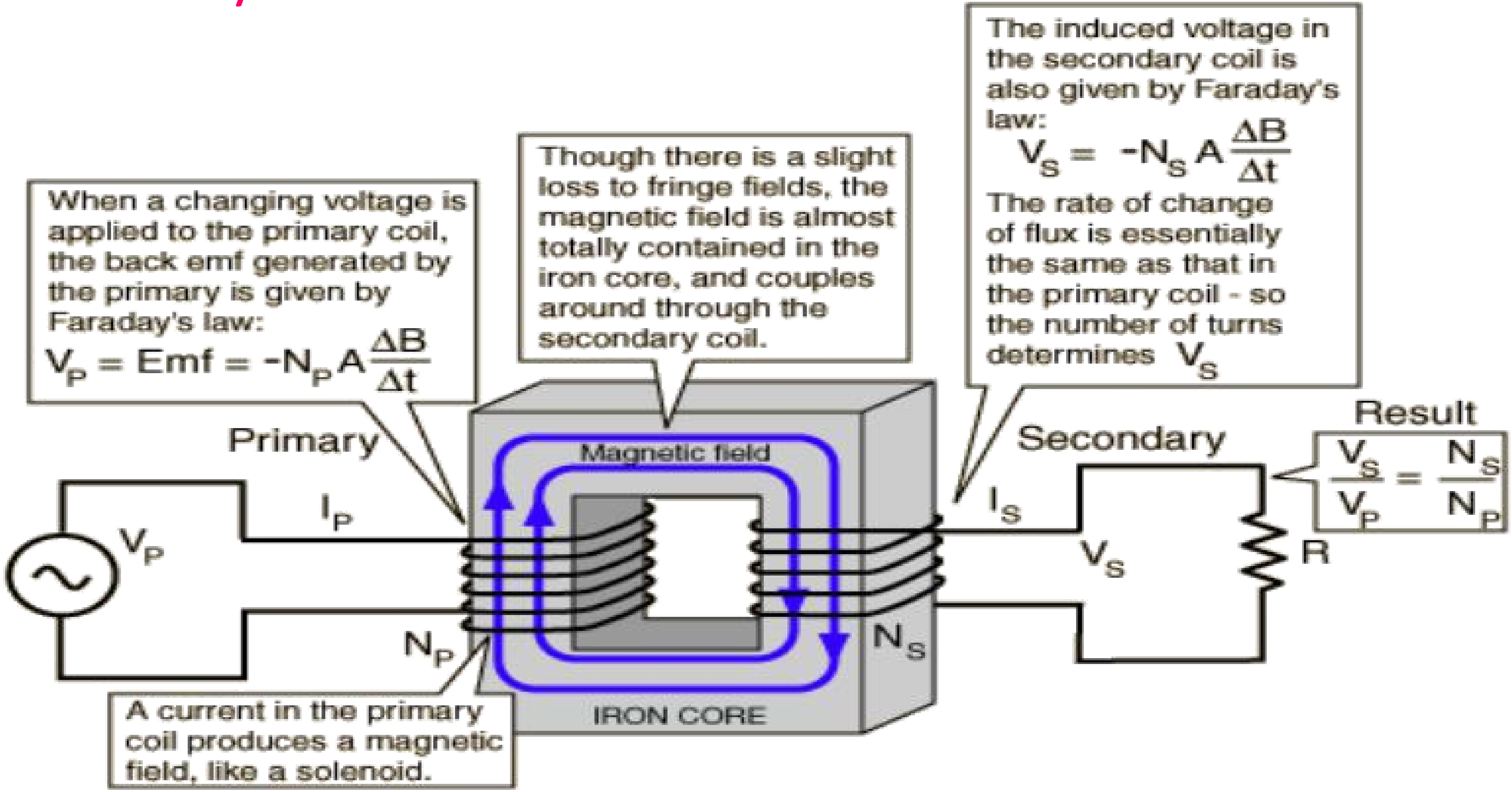
- Transformers employ the principles of electromagnetic induction. When an A.C is passed through a coil, alternating emf is induced. The magnitude of emf depends on the relative number of turns.
- Electricity is often generated a long way from where it is used, and is transmitted long distances through power lines. Although the resistance of a short length of power line is relatively low over a long distance the resistance can become substantial.
- A power line of resistance r causes a power loss of I^2R; this is wasted as heat. By reducing the current (I), therefore, the I^2R losses can be minimized. At the generating station, the power (P) generated is given by P= VI. To reduce the current while keeping the power constant,

the voltage V can be increased. Using AC power and Faraday's law of induction, there is a very simple way to increase voltage and decrease current (or vice versa), and that is to use a transformer. A transformer is made up of two coils, each with a different number of loops, linked by an iron core, so the magnetic flux from one passes through the other. When the flux generated by one coil changes (as it does continually if the coil is connected to an AC power source), the flux passing through the other will change, inducing a voltage in the second coil. With AC power, the voltage induced in the second coil will also be AC.

A transformer is designed so that little energy is lost. To ensure this

- Low resistance copper coils are used
- Cores are laminated to reduce eddy current losses
- Cores be made of soft magnetic materials
- Core design linearity

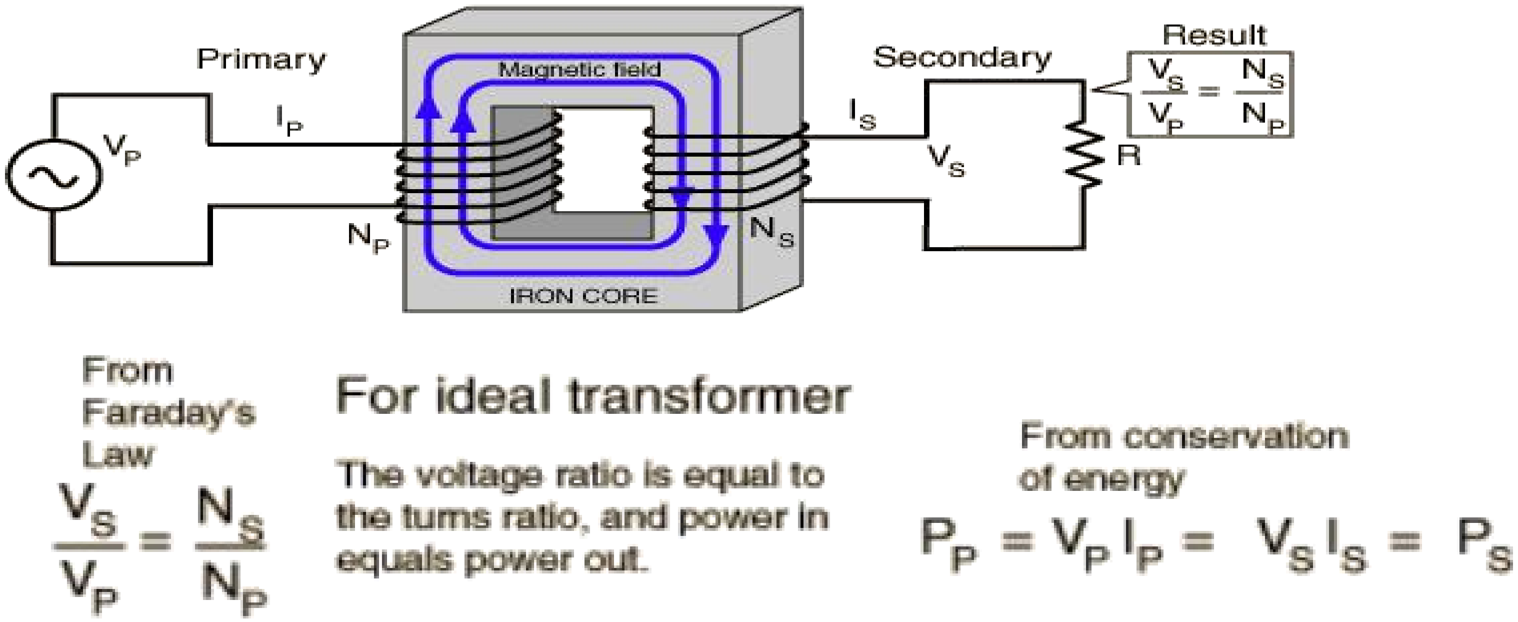
Faraday’s and Transformers



Faraday’s and Transformers

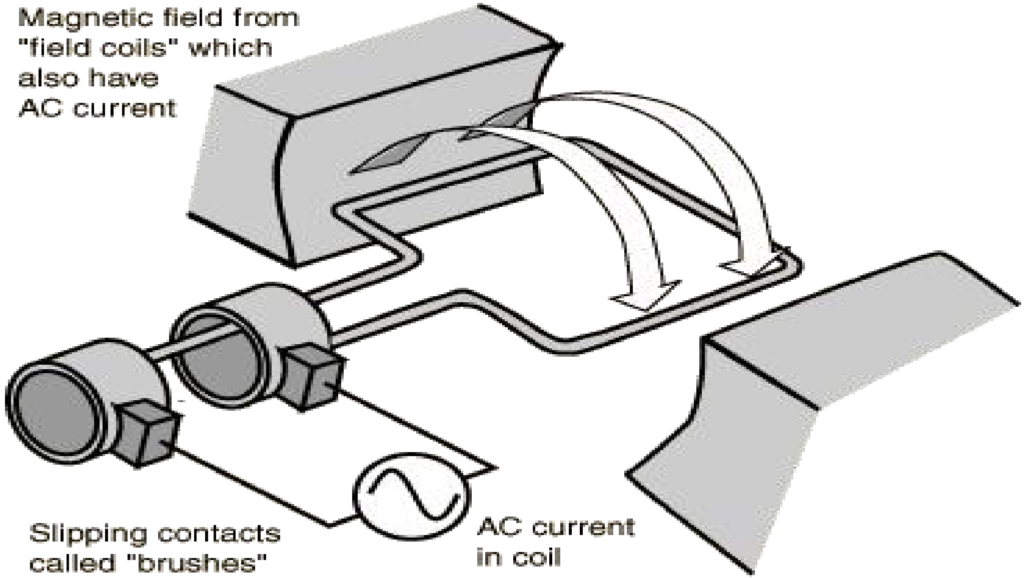
- A transformer makes use of Faraday's law and the ferromagnetic properties of an iron core to efficiently raise or lower AC voltages. It of course cannot increase power so that if the voltage is raised, the current is proportionally lowered and vice versa.

The ideal transformer neglects losses to resistive heating in the primary coil and assumes ideal coupling to the secondary (i.e., no magnetic losses



AC Motor

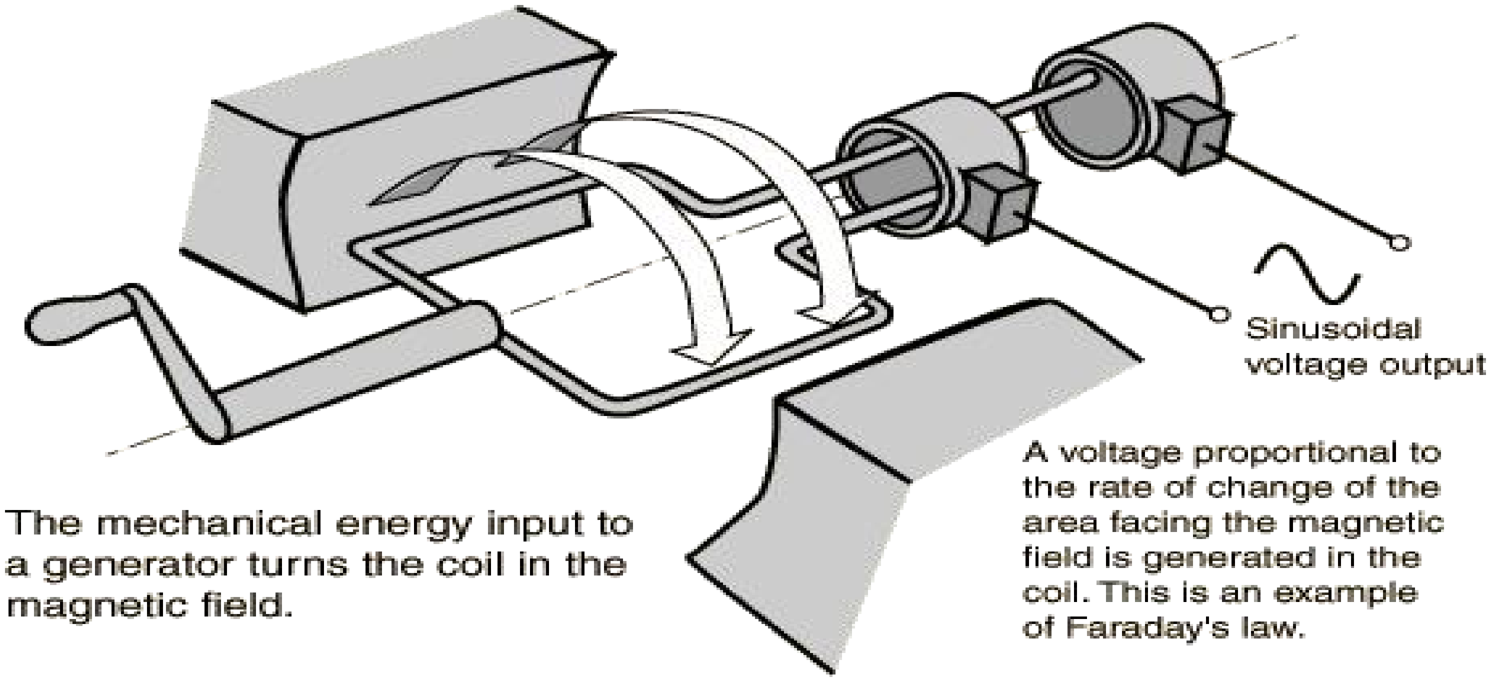
As in the DC motor case, a current is passed through the coil, generating a torque on the coil. Since the current is alternating, the motor will run smoothly only at the frequency of the sine wave. It is called a synchronous motor. More common is the induction motor, where electric current is induced in the rotating coils rather than supplied to them directly.



One of the drawbacks of this kind of AC motor is the high current which must flow through the rotating contacts. Sparking and heating at those contacts can waste energy and shorten the lifetime of the motor. In common AC motors the magnetic field is produced by an electromagnet powered by the same AC voltage as the motor coil. The coils which produce the magnetic field are sometimes referred to as the "stator", while the coils and the solid core which rotates is called the "armature". In an AC motor the magnetic field is sinusoidally varying, just as the current in the coil varies

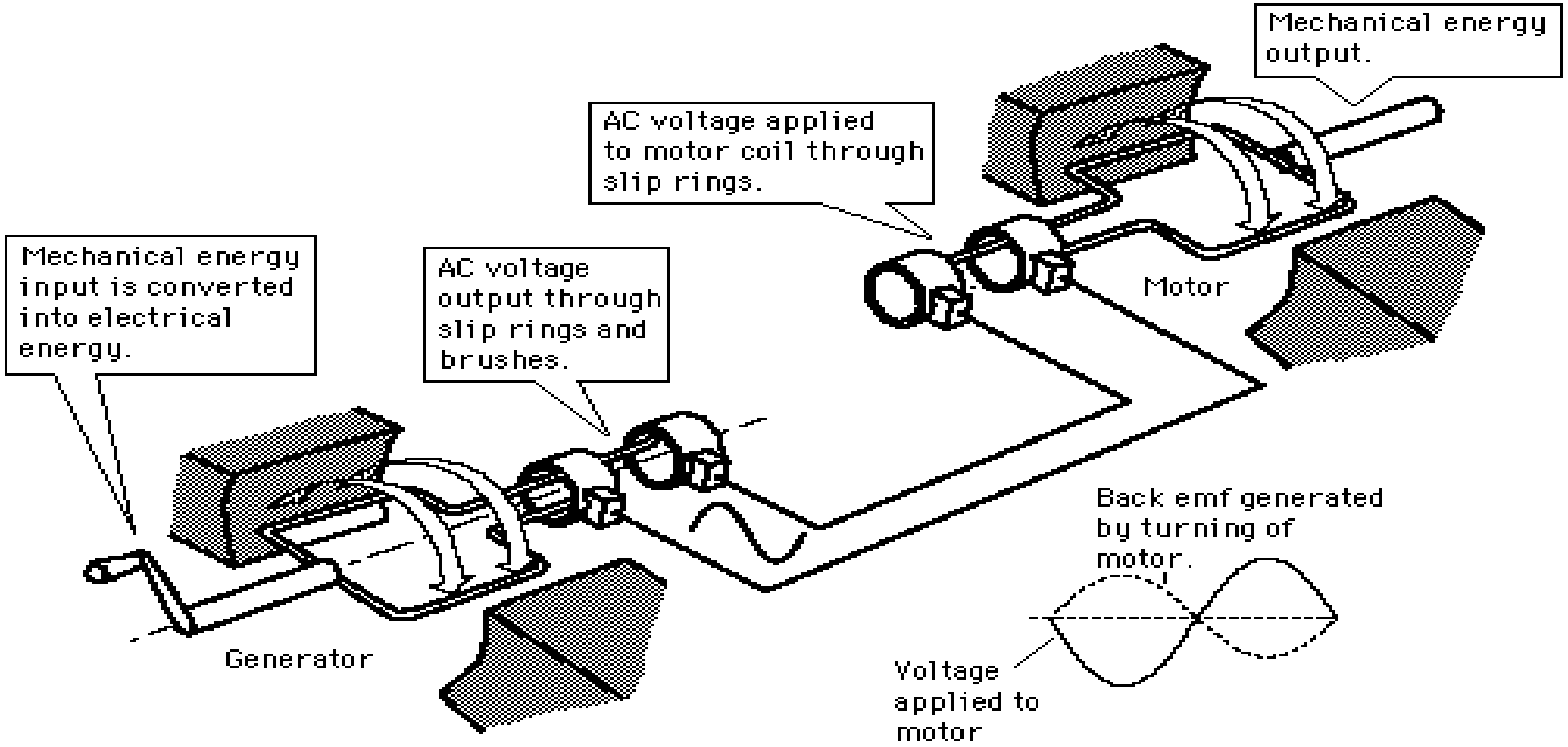
AC Generator

The turning of a coil in a magnetic field produces motional emfs in both sides of the coil which add. Since the component of the velocity perpendicular to the magnetic field changes sinusoidally with the rotation, the generated voltage is sinusoidal or AC. This process can be described in terms of Faraday's law when you see that the rotation of the coil continually changes the magnetic flux through the coil and therefore generates a voltage



Generator and Motor

A hand-cranked generator can be used to generate voltage to turn a motor. This is an example of energy conversion from mechanical to electrical energy and then back to mechanical energy



DC Generators- A Dynamo

The D.C generator or dynamo is different from an alternator in that the armature winding is connected to a commutator instead of slip rings. A commutator consists of two half rings of copper insulated from one another and turning with the coil. The commutator is arranged so that it reverses the connections from the coil to the circuit at the instant when the e.m.f reverses in the coil.

Examples:

(1) A square coil of side 0.18m is pivoted about the y axis. It is oriented at an angle 35^0 to *x* axis. If the orientation changes to 55^0 and the external field, B=0.6iT. Find the changes in flux.

Solution

$\Delta\phi = B.A = BAcos\alpha$ (definition)

A and B are constants (given)

$\Delta\phi = BAcos\alpha\ (\beta_2 - \beta_1)$

The coil is oriented at angle 35^0 to the x-axis, it makes angle β_1 with the z-axis; when the orientation changes, it makes angle 55^0 with the x-axis and β_2 with the z-axis.

$\beta_1 = (90^0 – 35^0) = 55^0$

$\beta_2 = (90^0 – 55^0) = 35^0$

$\Delta\phi = 0.6 \text{ x } (0.18)^2 (\cos 35^0 - \cos 55^0)$

Using the identity,

$\cos a – \cos b = [-2\sin ½ (a-b)\sin ½ (a+b)]$

$a= 35^0 , b=55^0$

$\Delta\phi = 4.8 \text{ x } 10^{-3} Wb$

(2) A straight wire length 50m is travelling at velocity 640kmh^{-1} in a direction perpendicular to the earth's magnetic field of strength, B = 0.6 x 10^{-4}iT. Find the induced e.m.f. between the ends of the wire.

Solution

Induced emf, $\varepsilon = \frac{d\phi}{dt} = -\mathbf{B}\frac{dA}{dt}$ (**B** is not changing)

$$\varepsilon = -Blv$$

$$= -0.6 \times 10^{-4} \times 50 \times (640 \times 10^{3}) / 3600$$

$$= 0.5333V$$

Worked Example

(3) The normal to a coil of radius 7cm makes an angle of 60^0 with the magnetic field during rotation through 100 turns which changes from 5 to 25 Tesla in 3 seconds. If the coil has a resistance of 20Ω, what is the magnitude of the induced current? What is the direction of the induced current?

Solution

$$E = \frac{-d\phi}{dt} = -\ d/dt\ (NAB\cos\theta)$$

where $N = 100$, $A = \pi r^2 = \frac{22}{7} \text{ x } (7)^2$

$$= \quad 154cm^2$$

$$E = \frac{-100x154x10^{-4}x\ (25 - 5)x\ \cos 600}{3} \text{volt}$$

The current flows clockwise.

(4)An emf of 0.35V is generated between the ends of a bar moving through a magnetic field of 0.11T equivalent to the moving conductor shown in figure 21.0. What field strength would be required to produce an emf of 1.5V between the ends of the bar, assuming all other factors remain the same.

Solution

$E = Blv$ then $0.35 = 0.11 \times k$ where $k = lv$,

$k = 0.35/0.11 = 3.18$

Hence, the field strength required to produce an emf of 1.5V is given by:

$B = 1.5/k \quad = 1.5/3.18 = 0.47T$

Worked Examples

(5) A conducting rod is moving at a speed of 5.0ms^{-1} in a direction perpendicular to a 0.80T magnetic field. The rod has a length of 1.6m with a negligible electrical resistance.

(a) What is the e.m.f produced by the rod if the bulb has a resistance of 96Ω?

(b) Find the induced current in the circuit, and

(c) calculate the electrical power delivered to the bulb,

and

(d) the energy consumed by the bulb in 60.0seconds.

Solution

(a) The motioned e.m.f is given as

$\varepsilon = VBL = (5.)(0.80 \text{ X } 1.6) = 6.4V$

(b) Induced current

$$I = {}^{\varepsilon}/_{R} = {}^{6.4}/_{96\Omega} = 0.067A$$

(c) The electrical power

$$P = I\varepsilon = 0.067 \times 6.4 = 0.43W$$

(d) The energy consumed in 60 seconds becomes

$$E = Pt = 0.43 \times 60.0 = 26J$$

(6) Determine the work done in 60.0seconds by the external agent supplying a 0.086N force that keeps a conducting rod moving at a constant speed of 5.0ms^{-1}.

Solution

The work done, W, for the distance x, travelled by the rod in time t becomes:

$$W = Fx = F(vt)$$

$$= (0.086N)\ (5.0ms^{-1})\ (60.0secs) = 26J$$

Worked Examples

(7) A transformer for home use of a portable radio reduces 120V ac to 9.0V ac. The secondary contains 30 turns and the radio draws 400mA. Calculate

(i) the number of turns in the primary

(ii) the current in the primary; and

(iii) the power transformed.

Solution

(i) This is a step-down transformer, therefore, we have

$$N_p = N_s \frac{V_P}{N_p} = \frac{30 \text{ x } 120}{9.0} = 400 \text{ turns}$$

(ii) $$I_p = I_s \frac{N_s}{N_p} = (0.40A) \text{ x } \left(\frac{30}{400}\right) = 0.030A$$

(iii) The power transformed is

$$P = I_s V_s = 0.40 \text{ x } 9.0 = 3.6W$$

(8) An average of 120 kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.40Ω. Calculate the power loss if the power is transmitted at (i)240V, and (ii) 24,000V.

Solution

We determine the current I in the lines for each case and then find the power loss from $p = I^2R$.

(i) If 120kW is sent at 240V, then the total current will be

$$I = \frac{P}{V} = \frac{1.2\ X\ 105W}{240V} = 500A$$

Therefore, the power loss in the lines is

$$P_L = I^2R = (500\text{A})^2 \text{ x } (0.40\Omega) = 100\text{kW}$$

Over 80% of all the power would be lost or wasted in the power lines.

(ii) When V = 24,000V

$$I = \frac{P}{V} = \frac{1.2\ X\ 10^5 W}{2.4\ X\ 10^4 V} = 5.0A$$

Therefore, power loss is

$$P_L = I^2R = (5.0\text{A})^2\ (0.40\Omega) = 10\text{W}$$

(9) The north pole of a magnet is partially withdrawn in 0.40s from a flat coil of 50 turns, so that the flux in each coil changes from $8.2\text{x}10^{-2}$ Wb to $0.2\text{x}10^{-2}$Wb. Find the magnitude and direction of the average e.m.f. induced in the coil.

Solution

The induced e.m.f. by Faraday's law of electromagnetic induction is given by:

$$E = -N\frac{d\Phi}{dt}$$

Hence; $E = \frac{-(change\ in\ flux)\times N}{time\ taken}$

$$= \frac{-(0.2 \times 10^{-2} - 8.2 \times 10^{-2}) \times 50}{0.4}$$

$$= \frac{(-8.0 \times 10^{-2})Wb \times 50}{0.4s}$$

$= 50 \text{ x } 2.0\text{x}10^{-1}$ *Volt*

$= 10.0\ V$

The current flows clockwise due to a decrease observed in the magnetic fields.

MODULE 3 - AC CIRCUITS: ALTERNATING CURRENT AND VOLTAGES APPLIED TO INDUCTORS, CAPACITORS AND RESISTORS

- Electromagnetic induction is of immense practical importance since it is the means whereby nearly all the world's electric power is produced.
- Most of our electricity comes from generators in power stations which use electromagnetic induction.
- Most generators give out alternating currents (AC). Alternating currents are involved in all aspects of modern communication. Alternating current behaves in a circuit in a different way from direct currents (DC) earlier discussed in part- 1 of this course

- Direct currents are less easy to generate than alternating current, and alternating em.fs are more convenient to step up and to step down, and to distribute over a wide area.

- The national grid system, which supplies electricity to the whole country, is therefore fed with alternating current.

- Small motors, of the size used in vacuum cleaners and common machine tools, run satisfactorily on alternating current, but large ones, as a general rule do not.

- Direct current is therefore used on most electric railway systems.

Alternating Current

- Direct current (DC) circuits involve current flowing in one direction.
- In alternating current (AC) circuits, instead of a constant voltage supplied by a battery, the voltage oscillates in a sine wave pattern, varying with time as:

$$V = V_0 \sin \omega t \qquad (34)$$

Where V_0 represents the maximum potential difference or the voltage amplitude, V in the instantaneous potential difference, and ω the angular frequency, f, by:

$$\omega = 2\pi f \qquad (35)$$

Root-mean-square Currents and Voltages

- The average value of a pure A.C current is zero, and the effective value of the current cannot be measured with an ordinary A.C. meter. A.C meters are normally designed to respond to the heating effect in a pure resistor; the effective value of the A.C being that which produces the same heating effect as a direct current. The effective or root mean square (r.m.s) value of a sinusoidal current bears a simple relation to the peak value.

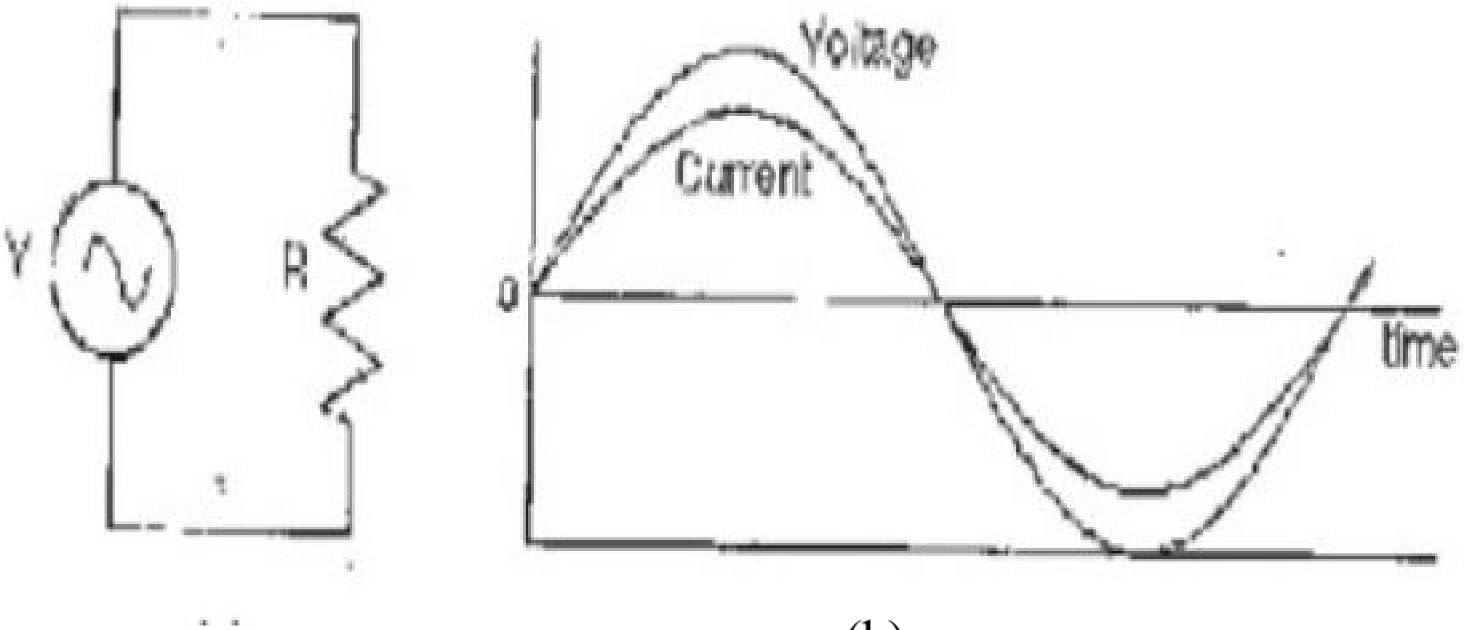
- In commercial practice, alternating current are always measured and expressed in terms of their root-mean-square (r.m.s) value.

- One example is the heat dissipated by a resistor. The instantaneous rate at which a resistance produces overall energy is I^2R where I is the size of the current at that instant. The direction of the current in the resistor is unimportant. Only the variation in size of the current affects the power.

Resistance in an AC Circuit

Figure 13.0 shows a resistor of resistance R connected to the terminals of an AC source. The instantaneous terminal potential difference is $V = V_0$ sinωt, and the instantaneous current in the resistor is:

$$I = \frac{V}{R} = \frac{V_0}{R} sin\ \omega t \qquad (98)$$



(a) (b)

Fig. 13.0: Resistance in an ac circuit.

In AC circuits we'll talk a lot about the phase of the current relative to the voltage. In a circuit which only involves resistors, the current and voltage are in phase with each other, which means that the peak voltage is reached at the same instant as peak current. In circuits which have capacitors and inductors (coils) the phase relationships will be quite different.

The r.m.s. current and potential difference are related as

$$I_{rms} = \frac{V_{rms}}{R} \qquad (36)$$

and the average power dissipated as heat in R is

$$P_{av} = I_{rms} V_{rms}$$

$$= I^2_{rms} R \qquad (37)$$

Capacitance in an AC Circuit

If we now have a circuit which has only a capacitor of capacitance C and an AC power source (such as a wall outlet) (Fig. 14.0).

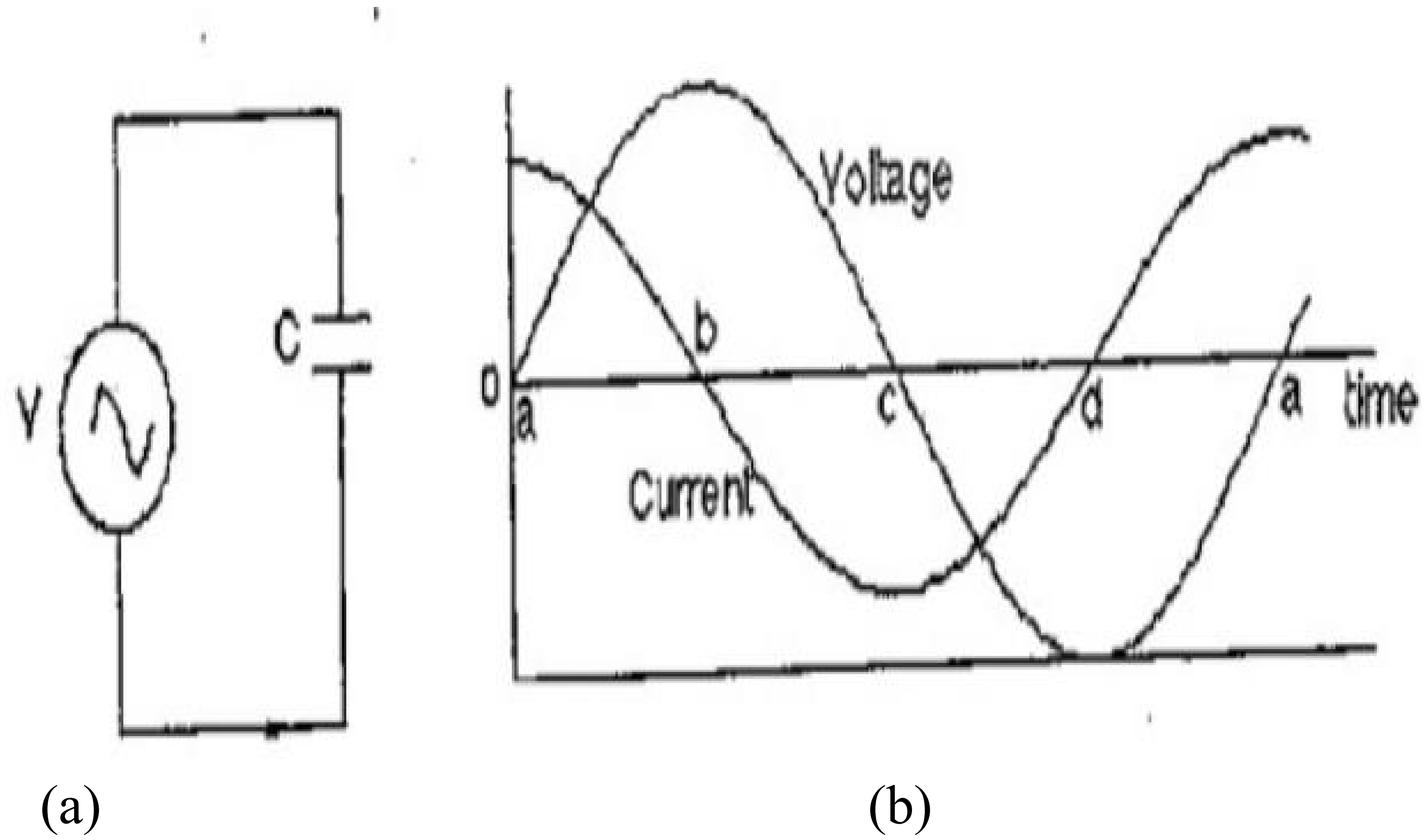


Fig. 14.0: Capacitance in a ac circuit.

The instantaneous charge Q on the capacitor is

$$Q = CV = CV_0 \sin\omega t \quad (37)$$

And the current *I* is

$$I = \frac{dQ}{dt} = \omega CV_0 \cos\omega t \quad (38)$$

The maximum current *Io* is:

$$I_0 = \omega CV \quad (39)$$

And we can write

$$I = I_0 \cos\omega t \quad (40)$$

- It turns out that there is a 90^0 phase difference between the current and voltage, with the current reaching its peak, 90^0 (1/4 cycle) before the voltage reaches its peak. Put another way, the current leads the voltage by 90^0 in a purely capacitive circuit.

- A capacitor in an A.C. circuit exhibits a kind of resistance called capacitive reactance in ohms. This depends on the frequency of the A.C. voltage, and is given by:

$$\text{capacitive reactance: } X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad (41)$$

We can use this like a resistance (because, really, it is a resistance) in an equation of the form $V=IR$ to get the voltage across the capacitor:

$$V_0 = I_0 X_c \tag{42}$$

Note that V and I are generally the rms values of the voltage and current respectively.

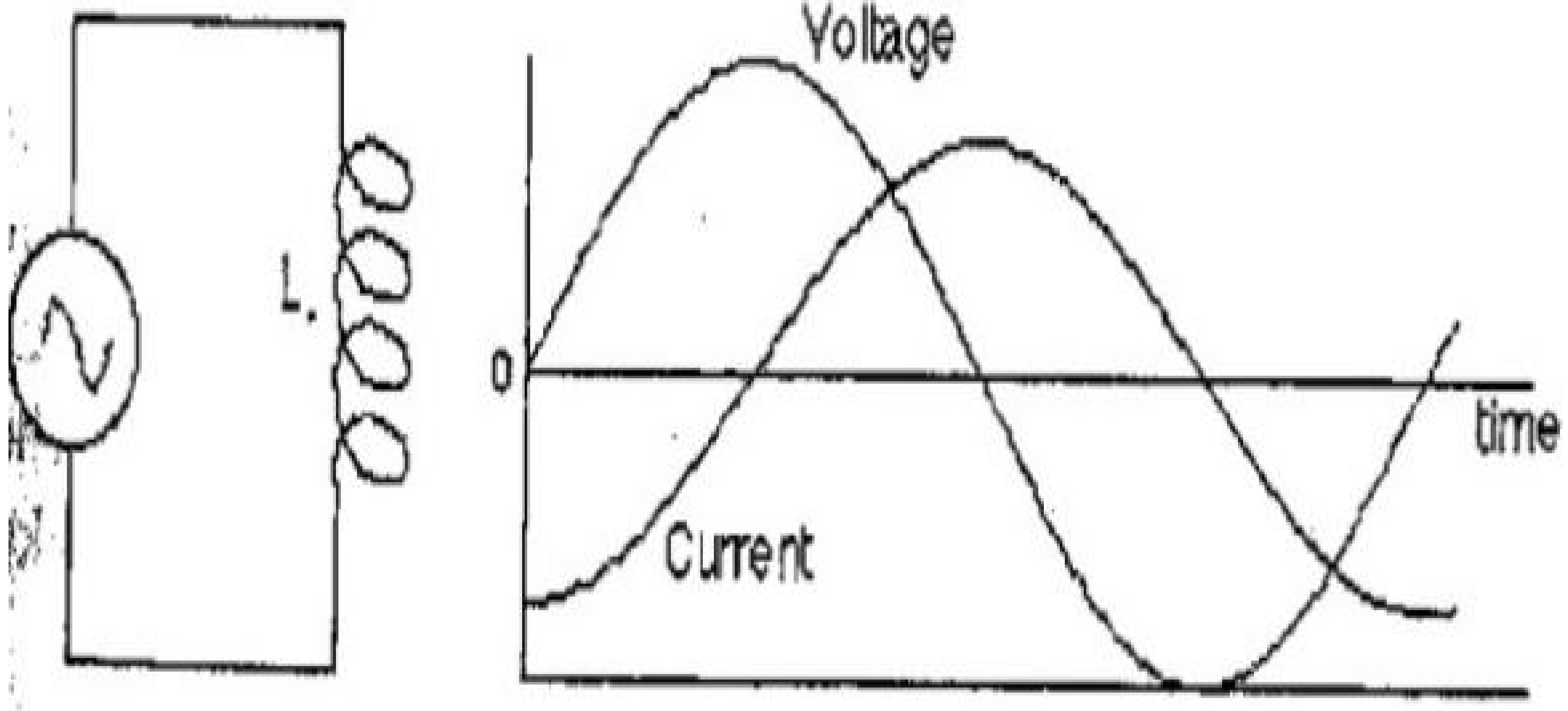
Inductance in an AC Circuit

An inductor is simply a coil of wire (often wrapped around a piece of ferromagnet).

If we now look at a circuit composed only of an inductor and an AC power source (Fig. 33.0), we will again find that there is a 90^0 phase difference between the voltage and the current in the indicator. This time, however, the current lags behind the voltage by 90^0, so it reaches its peak ¼ cycle after the voltage peaks.

The reason for this has to do with the law of induction:

$$\varepsilon = N\frac{d\phi}{dt} \text{ or } \varepsilon = -L\frac{dI}{dt} \qquad (43)$$



(a) (b)

Fig. 15.0: Inductance in an ac circuit.

Applying Kirchoff's loop rule to the circuit above gives:

$$V - L\frac{dI}{dt} = 0 \quad so \quad V = L\frac{dI}{dt} = V_0 \sin \omega t$$

and

$$dI = \frac{V_0}{L} \sin \omega t dt$$

integrating of both sides gives

$$I = -\frac{V_0}{\omega L} \cos \omega t \quad (44)$$

The maximum current is

$$I_0 = \frac{V_0}{\omega L} \quad (45)$$

and

$$I = -I_0 \cos \omega t \quad (46)$$

- From fig 15.0 as the voltage from the power source increases from zero, the voltage on the inductor matches it.
- With the capacitor, the voltage came from the charge stored on the capacitor plates (or, equivalently, from the electric field between the plates).
- With the inductor, the voltage comes from changing the flux through the coil, or, equivalently, changing the current through the coil, which changes the magnetic field in the coil.
- To produce a large positive voltage, a large increase in current is required. When the voltage passes through zero, the current should stop changing just for an instant.
- When the voltage is large and negative, the current should be decreasing quickly.
- These conditions can all be satisfied by having the current vary like a negative cosine wave, when the voltage follows a sine wave.

- How does the current through the inductor depend on the frequency and the inductance? If the frequency is raised, there is less time to change the voltage. If the time interval is reduced, the change in current is also reduced, so the current is lowered. The current is also reduced if the inductance is increased.
- As with the capacitor, this is usually put in terms of the effective resistance of the inductor. This effective resistance is known as the inductive reactance and is given by:

$$X_L = \omega L = 2\pi f L \qquad (47)$$

Where *L* is the inductance of the coil. The unit of inductance is the Henry (H).

As with capacitive reactance, the voltage across the inductor is given by:

$$V_0 = I_0 X_L \qquad (48)$$

The reactance of an indicator is directly proportional both to its inductance *L* and the angular frequency ω: the greater the inductance, and the higher the frequency, the larger is the reactance.

The R-L-C Series Circuits

In many instances, AC circuits include resistance, inductive reactance, and capacitive reactance. In this case, resistors, capacitors and inductors are combined in one circuit. If all three component are present, the circuit is known as an RLC circuit. A simple series circuit is shown in figure 16.0.

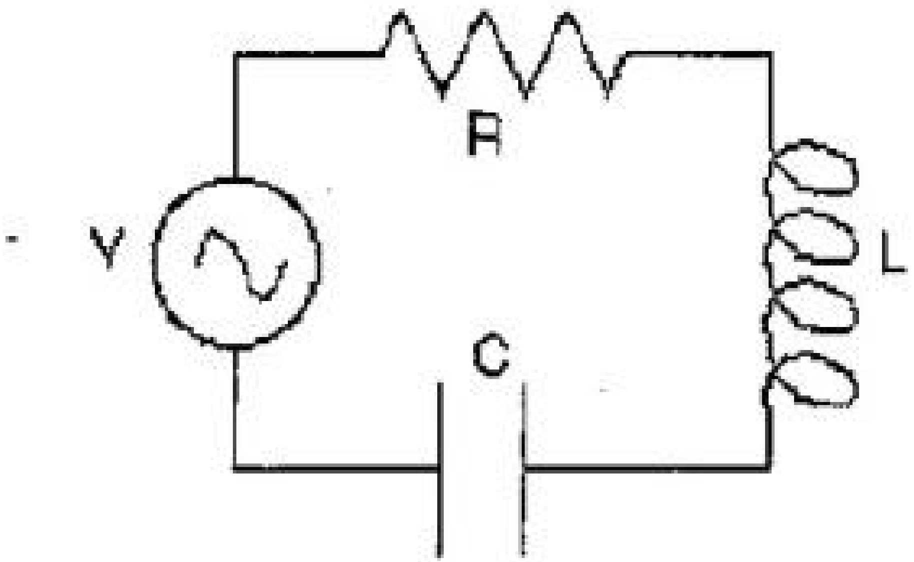
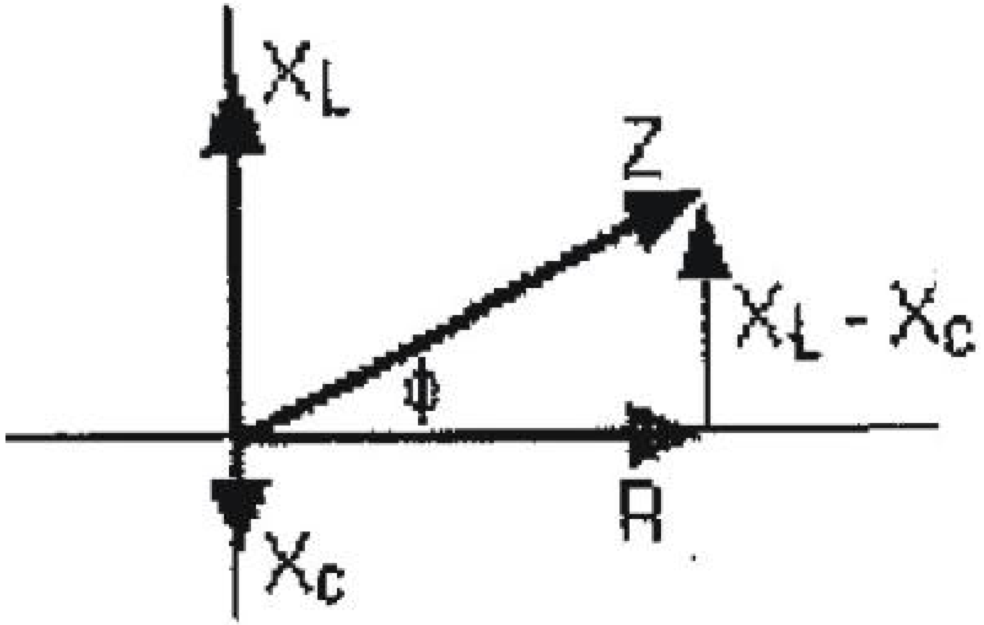


Fig. 16.0: A series RLC Circuit.

- The overall resistance to the flow of current in an RLC circuit is known as the impedance, symbolized by Z.
- The impedance is found by combining the resistance, the capacitive reactance and the inductive reactance.
- Unlike a simple series circuit with resistors, however, where the resistance and reactance are added, in an RLC circuit the resistance and reactance are added as vector. This is because of the phase relationships.
- In a circuit with just a resistor, voltage and current are in phase. With only a capacitor, current is 90^0 ahead of the voltage and with just an inductor the reverse is true, the voltage leads the current by 90^0. when all three components are combined into one circuit, there has to be some compromise.

- To figure out the overall effective resistance, as well as to determine the phase between the voltage and current, the impedance is calculated thus:
- The resistance R is drawn along the +x-axis of an x-y coordinate system.
- The inductive reactance is at 90^0 to this, and is drawn along the +y-axis.
- The capacitive reactance is also at 90^0 to the resistance, and is 180^0 different from the inductive reactance, so it's drawn along the y-axis.
- The impedance, Z, is the sum of these vectors and is given by:

$$Z = \sqrt{R^2 + (X_L - XC)^2} = \sqrt{R^2 + (\omega L - {}^1/_{\omega C})^2} \qquad (49)$$



The current and voltage in an RLC circuit are related by *V=IZ*. The phase relationship between the current and voltage can be found from the vector diagram: it is the angle between the impedance, Z, and the resistance, R. the angle can be found from

$$\tan \varphi = \frac{X_L - X_C}{R} \qquad (50)$$

- If the angle is positive, the voltage leads the current by that angle. If the angle is negative, the voltage lags behind the current.

The power dissipated in an RLC circuit is given by:

$$P = VI \cos \varphi \qquad (51)$$

Cos φ is known as the power factor in the circuit.

- Note that all of this power is lost in the resistor, the capacitor and inductor alternately store energy in electric and magnetic fields and then give the energy back to the circuitry system.

Summary

The ***phase relationship*** between the instantaneous voltage and instantaneous current in AC-circuit components are:

- The voltage across a pure resistor is in phase with the current;
- The voltage across a pure inductor leads the current by ¼ cycle;
- The voltage across a pure capacitor lags behind the current by ¼ cycle.

The ***inductive reactance*** X_L of an inductor is a measure of its effect on an alternating current.

The ***capacitive inductance*** X_C of the capacitor is a measure of its effect on an alternating current. Both X_L and X_C vary with the frequency of the current.

The ***impedance Z*** of an AC-circuit is analogous to the resistance of a DC-circuit.

Worked Examples

(1) At what frequency will the reactance of a 5μF capacitor be (a) 0.1Ω, (b) 10Ω and (c) 50Ω?

Solution

(a) $X_C = \frac{1}{\omega C}$

$$\omega = \frac{1}{X_C C} = \frac{1}{0.1\Omega \ X \ 5 \ X 10^{-6} F}$$

$$= 2.0 \text{ x } 10^6 \ rad \ s^{-1}$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{2.0 \ X \ 10^6 \ rad \ s^{-1}}{2\pi} = 31.8 \ \times 10^4 \ Hz$$

(b) $\omega = \frac{1}{10\Omega \times 5 \times 10^{-6} \ F} = 2.0 \times 10^4 \ rad \ s^{-1}$

$$f = \frac{2.0 \times 10^4 \ rad \ s^{-1}}{2\pi} = 3.18 \ kHz$$

(c) $\omega = \frac{1}{50\Omega \times 5\times10^{-6}\ F} = 4.0 \text{ x } 10^3\ rad\ s^{-1}$

$$f = \frac{4.0\times10^3\ rad\ s^{-1}}{2\pi} = 636.6\ Hz$$

(2) An A.C. voltage source has an amplitude of 45V and an angular frequency of 1000 rad s^{-1}. Find the current amplitude if it is connected across a capacitor of capacitance (a) 0.02μF (b) 4.0μF and (c) 100μF.

Solution

(a) $X_c = \frac{1}{\omega C} = \frac{1}{1000\ rad\ s^{-1}\times 0.02\times10^{-6}F}$

$= 5.0 \text{ x } 10^4\ \Omega$

$$I_0 = \frac{V_0}{X_c} = \frac{45V}{5.0 \times 10^5 \Omega} = 0.9\ mA$$

(b) $$X_c = \frac{1}{1000\ rad\ s^{-1} \times 4.0 \times 10^{-6} F}$$

$$= 2.5 \text{ x } 10^2\ \Omega$$

$$I_0 = \frac{V_0}{X_c} = \frac{45V}{2.5 \times 10^2 \Omega} = 0.18\ A$$

(c) $$X_c = \frac{1}{1000\ rad\ s^{-1} \times 100 \times 10^{-6} F}$$

$$= 10\Omega$$

$$I_0 = \frac{45V}{10\Omega} = 4.5A$$

Module 3 in Summarised form- A.C. Circuits

An alternating current has a sinusoidal wave form (i.e. having the form of a sine wave). Since an alternating current varies sinusoidally, it could be represented by the equation: $I = I_m \sin wt$

Note: $w = 2\pi f$ and I_m represents the peak (maximum) value of the current.

Root mean square current and power

In commercial practice, alternating current are always measured and expressed in terms of their root-mean-square (r.m.s) value.

One example is the heat dissipated by a resistor. The instantaneous rate at which a resistance produces overall energy is I^2R where I is the size of the current at that instant. The direction of the current in the resistor is unimportant. Only the variation in size of the current affects the power.

A.C. Circuits (contd.)

The average power produced in the resistor over one cycle is: =

$$\bar{p} = \frac{1}{2} I_m^2 R \qquad \left(\frac{I_m}{\sqrt{2}}\right) \times \left(\frac{I_m}{\sqrt{2}}\right) \times R \qquad \left(\frac{I_m}{\sqrt{2}}\right)^2 \times R$$

This average power is the same as the constant power produced by a d.c. current of. This dc current is known as the r.m.s equivalent current to the a.c.

The r.m.s equivalent current is defined as that d.c. that will provide the same power in the resistor as the a.c. does on average i.e.

$$I_{rms}^2 R = \frac{1}{2} I_m^2 R \quad \therefore I_{rms} = \sqrt{\frac{1}{2} I_m^2} = \frac{I_m}{\sqrt{2}}$$

Using r.m.s values of current, voltage drop, and emf, a.c. can be treated as d.c. provided the circuit only contains components with resistance and not capacitance or inductance. The ordinary dc rules apply:

$$I_{rms} = \frac{V_{rms}}{R}$$

$$\bar{p} \quad I_{rms} \; V_{rms}$$

$$I_{rms}^2 R = \frac{V_{rms}^2}{R} = \frac{1}{2} I_m V_m$$

A.C. with Capacitor

DC current cannot flow through a capacitor because of the insulating medium between its plates. However, a.c. current can flow because the capacitor plates are being continually charged, discharged and charged the other way round by the alternating voltage of the mains.

Note: $V = V_m Sin\ 2\pi ft$ and recall : $Q = CV$ $\therefore Q = CV_m Sin\ 2\pi ft$

Also: $I = \frac{dQ}{dt} \therefore I = \frac{d}{dt}(CV_m Sin\ 2\pi ft) = 2\pi f CV_m Cos\ 2\pi ft$ Hence, $I = I_m Cos\ 2\pi ft$

i.e. $I_m = 2\pi f CV_m$ So I_m is proportional to f, C and V_m

Thus, the greater the voltage, or the capacitance, the greater the charge on the plates, and therefore the greater the current required to charge or discharge the capacitor. The higher the frequency, the more rapidly is the capacitor charged and discharged and therefore again the greater is the current.

Reactance and Inductance of a capacitor

The **reactance of a capacitor** X_C is its opposition in Ohms to the passage of alternating current.

Note: Reactance is applicable to a.c. while resistance is applicable to D.C.

Recall $I_m = 2\pi f CV_m$ ∴

Hence, $X_c = \frac{V_m}{I_m}$ or $X_c = \frac{V_m}{2\pi f CV_m}$

$$X_c = \frac{1}{2\lambda f C} \qquad X_c = \frac{1}{WC}$$

Inductance of a capacitor

Self-inductance opposes changes of current and the changing current sets up a back e.m.f in the coil of magnitude: $E = -L\frac{dI}{dx}$

Note: Self-induced emf is often termed back emf because of its tendency to oppose the external emf.

Inductance of a capacitor (contd.)

To maintain the current, the applied supply voltage must be equal to the back emf. The voltage applied to the coil must therefore be given by:

$$V = -L\frac{dI}{dt}$$

Recall: $I = I_m \cos 2\pi ft$

$$\therefore V = -L\frac{d}{dt}(I_m \cos 2\pi ft) = -L2\pi fI_m(-\sin 2\pi ft) = 2\pi fLI_m \sin 2\pi ft$$

i.e. $V = V_m \sin 2\pi ft$

$\Rightarrow V_m = 2\pi fLI_m$

$$\therefore \frac{V_m}{I_m} = 2\pi fL = X_L$$