FEDERAL UNIVERSITY OF TECHNOLOGY OWERRI

DEPARTMENT OF MATHEMATICS

TIME: 1HR MTH101 TEST 2015/2016 SESSION DATE: 13/04/2016

1.(a) Solve the following equations

I. $\log_3 x + \log_x 3 = \frac{10}{3}$ (II) $\log_{10}(x^2+9) - 2\log_{10} x = 1$

(b) Find n such that ${}^{n+2}P_3 = 6.\,{}^{n+2}P_1$

2.(a) Find the positive square root of $19 + 6\sqrt{2}$

(b). If $\mathcal{U} = \{x: 0 \le x \le 10, x \in \mathbb{Z}\}$ is a universal set with subsets $A = \{x: x \text{ is prime}\}$, $B = \{x: x \text{ is odd}\}$ $C = \{x: x \text{ is a factor of } 10\}$. Verify that (i) $(A \cup B)' = A' \cap B'$ (ii) $A - (B \cup C) = (A - B) \cap (A - C)$

3(a). How many four digit even numbers can be formed from the set $A = \{x: x \in \mathbb{N} \text{ and } x < 10\}$ and the numbers must be less than 5000.

(b) Given the equation $2x^2 - 3x + 5 = 0$. If α, β are roots of the equation, find $\alpha^2\beta + \alpha\beta^2 - \alpha\beta$.

4(a) Solve $\frac{x+6}{x+1} > 2$

(b) Write down and simply the first four terms in the expansion in ascending powers of x of $(1+3x)^{1/3}$. Hence evaluate $\sqrt[3]{1.03}$ to 5 d.p.

SOLUTION TO 2015/2016 MTH 101 TEST

1a (i) $\log_3 x + \log_x 3 = \frac{10}{3}$

$\log_3 x + \frac{1}{\log_3 x} = \frac{10}{3}$

let $\log_3 x = P$

$P + \frac{1}{P} = \frac{10}{3}$

Multiplying through by $3P$

$3P^2 + 3 = 10P \Rightarrow 3P^2 - 10P + 3 = 0$

$3P^2 - 9P - P + 3 \Rightarrow 3P(P-3) - 1(P-3) = 0$

$\Rightarrow (3P-1)(P-3) = 0$

$P = \frac{1}{3}$ or 3

But $\log_3 x = P$

$\Rightarrow \log_3 x = \frac{1}{3}$ or $\log_3 x = 3$

$x = 3^{1/3}$ or $x = 3^3$

$x = \sqrt[3]{3}$ or 27

1a (ii) $\log_{10}(x^2+9) - 2\log_{10} x = 1$

$\Rightarrow \log_{10}(x^2+9) - \log_{10} x^2 = \log_{10} 10$

$\log_{10} \frac{x^2+9}{x^2} = \log_{10} 10$

$\frac{x^2+9}{x^2} = 10$

$x^2 + 9 = 10x^2 \Rightarrow 10x^2 - x^2 - 9 = 0$

$9x^2 - 9 = 0 \Rightarrow 9(x^2-1) = 0$

$x^2 - 1 = 0 \Rightarrow (x-1)(x+1) = 0$

$x = 1$ or -1

1b ${}^{n+2}P_3 = 6 \cdot {}^{n+2}P_1$

$\frac{(n+2)!}{(n+2-3)!} = 6 \cdot \frac{(n+2)!}{(n+2-1)!}$

$\frac{(n+2)!}{(n-1)!} = 6\frac{(n+2)!}{(n+1)!}$

$\frac{1}{(n-1)!} = \frac{6}{(n+1)!} \Rightarrow \frac{1}{(n-1)!} = \frac{6}{(n+1)n(n-1)!}$

$\frac{1}{1} = \frac{6}{(n+1)n} \Rightarrow (n+1)n = 6$

$n^2 + n = 6 \Rightarrow n^2 + n - 6 = 0$

$n^2 + 3n - 2n - 6 = 0 \Rightarrow n(n+3) - 2(n+3) = 0$

$(n-2)(n+3) = 0 \Rightarrow n = 2$ or -3

Since n cannot be negative, $n = 2$

2a let $\sqrt{19 + 6\sqrt{2}} = \sqrt{a} + \sqrt{b}$

Squaring both sides

$19 + 6\sqrt{2} = a + 2\sqrt{ab} + b$

$19 + 6\sqrt{2} = a + b + 2\sqrt{ab}$

Comparing real and rational parts

$a + b = 19$ — *

$2\sqrt{ab} = 6\sqrt{2}$

$\sqrt{ab} = 3\sqrt{2} \Rightarrow ab = (3\sqrt{2})^2$

$ab = 9 \times 2 \Rightarrow ab = 18$ — **

From $a = 19 - b$ — ***

Put *** into **

$(19-b)b = 18 \Rightarrow 19b - b^2 = 18$

$\Rightarrow b^2 - 19b + 18 = 0 \Rightarrow b^2 - 18b - b + 18 = 0$

$b(b-18) - 1(b-18) = 0$

$(b-1)(b-18) = 0 \Rightarrow b = 1$ or 18

Substitute in ***

$a = 19 - 1$ or $19 - 18$

$a = 18$ or 1

Set $a = 1$ and $b = 18$

$\therefore \sqrt{19 + 6\sqrt{2}} = \sqrt{1} + \sqrt{18} = 1 + \sqrt{9 \times 2}$

$= 1 + 3\sqrt{2}$

(5b) $U = \{0,1,2,3,4,5,6,7,8,9,10\}$

$A = \{2,3,5,7\}$

$B = \{1,3,5,7,9\}$

$C = \{2,5,10\}$

i) $A \cup B = \{1,2,3,5,7,9\}$

$(A \cup B)' = \{0,4,6,8,10\}$

$A' = \{0,1,4,6,8,10\}$

$B' = \{0,2,4,6,8,10\}$

$A' \cap B' = \{0,4,6,8,10\}$

Thus $(A \cup B)' = A' \cap B'$

ii) $B \cup C = \{1,2,3,5,7,9,10\}$

$A-(B\cup C) = \{2,3,5,7\} - \{1,2,3,5,7,9,10\}$
$= \phi$

$A - B = \{2,3,5,7\} - \{1,3,5,7,9\} = \{2\}$

$A - C = \{2,3,5,7\} - \{2,5,10\} = \{3,7\}$

$(A-B) \cap (A-C) = \{2\} \cap \{3,7\} = \phi$

Thus, $A-(B\cup C) = (A-B)\cap(A-C)$

(3a) Even numbers in set A include $\{2,4,6,8\}$

To form a 4-digit even number less than 5000, we must either begin with 2 or 4.

Hence we are arranging 4 numbers taking 2 at a time.

$\Rightarrow {}^4P_2 = \frac{4!}{(4-2)!} = \frac{4\times3\times2!}{2!} = \underline{\underline{12}}$

(3b) Given $2x^2 - 3x + 5 = 0$

$\alpha+\beta = -\frac{b}{a} = -\frac{(-3)}{2} = \frac{3}{2}$

$\alpha\beta = \frac{c}{a} = \frac{5}{2}$

To find $\alpha^2\beta + \alpha\beta^2 - \alpha\beta$

$\alpha\beta(\alpha+\beta-1)$

$\frac{5}{2}\left[\frac{3}{2} - 1\right] = \frac{5}{2}\left(\frac{1}{2}\right) = \frac{5}{4}$

(4a) Given $\frac{x+6}{x+1} > 2$

Multiply both sides by $(x+1)^2$

$(x+6)(x+1) > 2(x+1)^2$

$(x+6)(x+1) - 2(x+1)^2 > 0$

$(x+1)[x+6-2(x+1)] > 0$

$(x+1)(x+6-2x-2) > 0$

$(x+1)(-x+4) > 0$

$(x+1)[-(x-4)] > 0$

$-(x+1)(x-4) > 0$

$(x+1)(x-4) < 0$

The turning values are $x = -1, 4$

Using the truth table

	$x<-1$	$-1<x<4$	$x>4$
$x+1$	−	+	+
$x-4$	−	−	+
$(x+1)(x-4)$	+	−	+

The solution set is $= \{x : -1 < x < 4\}$

(4b) $(1+3x)^{\frac{1}{3}} = 1 + \frac{1}{3}(3x) + \frac{\frac{1}{3}(\frac{1}{3}-1)(3x)^2}{2!} + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)(3x)^3}{3!} + \cdots$

$= 1 + x + \frac{\frac{1}{3}(-\frac{2}{3})}{2}9x^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{6}27x^3 + \cdots$

$= 1 + x - \frac{\frac{2}{9}(9x^2)}{2} + \frac{\frac{10}{27}(27x^3)}{6} + \cdots$

$= 1 + x - \frac{2x^2}{2} + \frac{10x^3}{6} + \cdots$

$= 1 + x - x^2 + \frac{5}{3}x^3 + \cdots$ *

$\therefore \sqrt[3]{1.03} = (1.03)^{\frac{1}{3}} = (1 + 0.03)^{\frac{1}{3}}$

$\Rightarrow x = 0.03$

Put $x = 0.03$ into *

$1 + 0.03 - (0.03)^2 + \frac{5}{3}(0.03)^3 + \cdots$

$= 1 + 0.03 - 0.0009 + \frac{5}{3}(0.000027)$

$= 1 + 0.03 - 0.0009 + 0.000045$

$= 1.029145$

$\simeq 1.02915$ to 5 d.p

FEDERAL UNIVERSITY OF TECHNOLOGY

SCHOOL OF PHYSICAL SCIENCES
DEPARTMENT OF MATHEMATICS
HARMATTAN SEMESTER EXAMINATIONS
ELEMENTARY MATHEMATICS I MTH 101
2015/2016 SESSION TIME: 2 HRS

Attempt all the questions. All questions carry equal marks. Choose correct options from A to E.

1. Using the linear expansion of $(1-2x)^{1/3}$, find, correct to 4 significant figures, the value of $(1.05)^{1/3}$. (A) -0.0250 (B) 0.0250 (C) 0.5 (D) 1.016 (E) 1639
2. What is the coefficient of x^5 in the expansion of $(x+y)^{10}$? (A) 210 (B) 120 (C) 252 (D) 160 (E) 45
3. Use the remainder theorem to determine the remainder when $(3x^3-2x^2+x-5)$ is divided by $(x+2)$. (A) -39 (B) -29 (C) 29 (D) 11 (E) 9
4. Solve the cubic equation $x^3-2x^2-5x+6=0$ by using the factor theorem. (A) $x=1,2,-1$ (B) $x=-1,2,-1$ (C) $x=1,-3,-2$ (D) $x=1,3,-2$ (E) $x=-1,-3,-2$.
5. Solve for n if $\log_3(2n+2)=\log_3(n-5)$ (A) -7 (B) 14 (C) 16 (D) 20 (E) -5
6. What is the value of $\frac{\log_{27}125}{\log_9 25}$? (A) 0.301 (B) 1 (C) 100 (D) 0.002 (E) 2.4
7. If $9^{2x+1}=\frac{81^{x-2}}{3^x}$, find x. (A) -3 (B) -2 (C) -10 (D) 11 (E) 12
8. Resolve $\frac{7x^2+5x+13}{(x^2+2)(x+1)}$ into partial fractions (A) $\frac{2x+3}{(x^2+7)}+\frac{1}{x-2}$ (B) $\frac{2x+3}{(x^2+7)}-\frac{3}{x-2}$ (C) $\frac{2x+3}{(x^2+2)}+\frac{5}{x+1}$ (D) $\frac{1}{x-4}+\frac{2-x}{(x^2+3)}$ (E) $\frac{2x-3}{(x^2+2)}-\frac{5}{x+1}$
9. In how many ways can 3 identical jobs be filled by selections from 12 different people? (A) 210 (B) 180 (C) 230 (D) 213 (E) 220
10. A box contains 8 good items and 2 defective items. In how many ways can we draw 3 good items? (A) 213 (B) 336 (C) 116,280 (D) 42 (E) 120
11. From a group of 10 democrats and 8 socialists, how many committees of 7 can be formed so as to include exactly 4 democrats. (A) 210 (B) 56 (C) 70 (D) 11760 (E) 8400
12. Solve the inequality $\frac{x-3}{x+4}\geq 2$ (A) $(-\infty,-11)$ (B) $(4,\infty)$ (C) $[-11,-4]$ (D) $[-11,4]$ (E) $(-\infty,-11)\cup(-11,\infty)$

$\log_3(2n+2)=\log_3(n-5)$

13. Solve $-x^2-9x-22\leq -4$. (A) $[-6,-3]$ (B) $[-3,\infty)$ (C) $(-\infty,-6)$ (D) $(-\infty,-6]\cup[-3,\infty)$ (E) $(-\infty,\infty)$
14. Solve $|x+5|-6\leq -1$ (A) $(-\infty,-10)$ (B) $(-10,\infty)$ (C) $[-10,0]$ (D) $(0,\infty)$ (E) $(-\infty,\infty)$
15. If $x=\frac{a}{(1+t)^2}$ and $y=\frac{a(1-t)}{1+t}$. Find $\frac{y+a}{2}$ in terms of a and x. (A) $(ax)^2$ (B) $2ax$ (C) $4ax$ (D) $\sqrt{ax}$ (E) a^2x
16. Find the sum of an infinite series given as $x+\frac{x}{1+x}+\frac{x}{(1+x)^2}+\frac{x}{(1+x)^3}+\cdots$ (A) x^2 (B) $\frac{x^2}{1+x}$ (C) $(1+x)^2$ (D) $\frac{1+x}{x^2}$ (E) $1+x$
17. Find all possible values of n when ${}^nP_5=18\,{}^{n-2}P_4$ (A) 10 (B) 9 (C) 10, 9 (D) 5, 4 (E) 5
18. If the coefficient of x^n in $(1+x)^{2n}$ is k, find the coefficient of x^n in $(1+x)^{2n-1}$ in terms of k. (A) k^2 (B) $2k$ (C) $\frac{k}{2}$ (D) $k-1$ (E) $k+1$
19. Express the following complex number in its Euler form, $z=\frac{1+i}{1-i}$. (A) $2e^{i\pi}$ (B) $e^{i\frac{\pi}{2}}$ (C) $e^{i\pi}$ (D) $\sqrt{2}e^{i\pi/2}$ (E) $e^{i2\pi}$
20. Given that α and β are the roots of the equation $x^2-5x+10=0$. Find the value of $\alpha^3+\beta^3$. (A) 25 (B) -25 (C) 35 (D) 5 (E) -5
21. Find the cube roots of $z=-i$ (A) $\frac{\sqrt{3}}{2}-\frac{i}{2},-i,-\frac{\sqrt{3}}{2}-\frac{i}{2}$ (B) $\frac{1}{2}+i,-i,0$ (C) $0,i,\frac{i\sqrt{3}}{2}$ (D) $-\frac{1}{2}+\frac{i\sqrt{3}}{2},i,-\frac{\sqrt{3}}{2}-\frac{i}{2}$ (E) $i,-i,1$
22. Find the different arrangements of the letters of the word MISSISSIPPI. (A) $\frac{11!}{4!4!2!}$ (B) $\frac{11!}{4!2!}$ (C) $\frac{11!}{8!4!}$ (D) $\frac{11!}{2!2!}$ (E) $\frac{10!}{4!4!2!}$
23. Find P_{k+1} such that $P_n=8^n+6$ is divisible by 7. (A) $8(9P_k+1)$ (B) $7(8P_k-6)$ (C) $7(10P_k+6)$ (D) $8(7P_k-6)$ (E) $7P_{k+1}+7$
24. Find the term independent of y in the expansion of $\left(\frac{x^4}{y}+\frac{y^2}{2x}\right)^{15}$ (A) $\frac{20}{7}x^{35}$ (B) $\frac{32}{3}x^{15}$ (C) $\frac{50}{3}x^{25}$ (D) $\frac{63}{8}x^{35}$ (E) $\frac{20}{7}x^{25}$
25. If the sum to infinity of a G.P is eight times its first term, find its common ratio. (A) $\frac{7}{8}$ (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{2}{5}$
26. Resolve $\frac{2+3x-x^2}{x(x^2-1)}$ into partial fractions (A) $\frac{2}{(x-1)}-\frac{2}{x}-\frac{1}{x+1}$ (B) $\frac{2}{(x-1)}+\frac{2}{x}-\frac{1}{x+1}$

91/99

(C) $\frac{2}{(x-1)}+\frac{2}{x}+\frac{1}{x+1}$ (D) $\frac{2}{(x-1)}-\frac{2}{x}+\frac{1}{x+1}$
(E) $\frac{2}{(x-2)}-\frac{2}{x}-\frac{1}{x+2}$

27. What is the coefficient of x^6 in the expansion of $\left(\frac{1}{x^2}-x\right)^{18}$ (A) 816 (B) -816 (C) -3060 (D) 3060 (E) 3006

28. If $6x^2-3kx+5=0$, then for what value of k will the sum of the roots equal product of roots? (A) 3 (B) 5 (C) $\frac{3}{5}$ (D) $\frac{5}{3}$ (E) 1

29. Find the value of x for which the equation $\sqrt{3x+4}-\sqrt{x+2}-\sqrt{x-3}=0$
(A) $-\frac{7}{3}$ (B) $\frac{7}{3}$ (C) 7 (D) -7 (E) 7, -7

30. If x, y, z are in GP, and their sum is 28 and their product is 512. Find x, y, z (A) 16, 8, 4 (B) 16, 10, 20 (C) 15, 10, 3 (D) 18, 8, 2 (E) 10, 5,6

31. The subsets P, Q, and R of the universal set $\mathcal{U}$ are defined as follows:
$\mathcal{U}=\{x: 10 \le x \le 30\}$,
$P=\{x: x \text{ is a prime number}\}$,
$Q=\{x: x \text{ is an odd number}\}$,
$R=\{x: x \text{ is a multiple of } 3\}$
What is the cardinality of the set $(P \cup Q)^C$
(A) 11 (B) 12 (C) 21 (D) 10 (E) 13

32. Given that one of the roots of the equation $9x^2+kx+3=0$ is the square of the reciprocal of the other, the roots are
(A) $\left(-3,-\frac{1}{9}\right)$ (B) $\left(3,\frac{1}{9}\right)$ (C) $\left(\frac{1}{3},9\right)$
(D) $(3,4)$ (E) $(3,9)$

33. Find the quotient when $x^4+3x^3-x^2+11x-4$ is divided by $x+4$. (A) x^3+x^2+3x-1
(B) x^3-x^2+3x-1 (C) x^3-x^2-3x+1
(D) x^3-x^2-3x-1 (E) x^3+3x-1

34. Evaluate $\frac{(1+i)(2+i)}{3-i}$ (A) -1 (B) 1 (C) $-i$ (D) i
(E) $2i$

35. The sum of 3 consecutive terms of an AP is 27 and their product is 693. Find the common difference of the AP. (A)27 (B) ± 2 (C) 4 (D) 125 (E) ± 3

36. In a group of 200 FUTO students, 133 are enrolled in MTH101, 117 are enrolled in CHM101 and 85 students are enrolled in both subjects. Find the number of students who are not enrolled in either of the courses. (A) 48 (B) 25 (C) 35 (D) 27 (E) 40

37. Simplify $\frac{(2^{p+1})^q}{(2^{q+1})^p} \times \frac{2^{2p}}{2^{2q}} \times 2^q$ (A) 2^p (B) 2^q
(C) 2^{pq} (D) 2^{p+1} (E) 2^{q+1}

38. The first two terms of an AP are -2 and 3. How many terms are needed for the sum to be 306? (A) 13 (B) 12 (C) 11 (D) 10 (E) 14

39. Find P_{k+1} such that 9^n-1 is divisible by 8.
(A) $8(10P_k-11)$ (B) $8(19P_k+1)$
(C) $8(9P_k+1)$ (D) $8(7P_k-8)$
(E) $8(10P_k+11)$

40. Simplify $\frac{3(2^{n+1})-4(2^{n-1})}{2^{n+1}-2^n}$ (A) 3 (B) 5 (C) 4
(D) 10 (E) 14

41. If $(x+y)+i(2x-y)=2+i$, find x and y.
(A) (1, 1) (B) (1, 2) (C) (1, -1) (D) (1, -2)
(E) (0, 1)

42. Find the positive square root of $6+\sqrt{32}$.
(A) $2+\sqrt{3}$ (B) $2+\sqrt{2}$ (C) $3+\sqrt{2}$
(D) $2+\sqrt{5}$ (E) $3-\sqrt{3}$

43. Find the values of p and q respectively if $x-2$ and $x+1$ are both factors of px^3+3x^2-9x+q. (A) 2, 10 (B) 2, -10
(C) $-2, 10$ (D) $-3, -10$ (D) 3, 10

44. Find the fifth term of $(x^3+2y)^8$
(A) $448x^{15}y^3$ (B) $1120x^{12}y^4$ (C) $448x^3y^{15}$
(D) $1120x^4y^{12}$ (E) $155x^{15}y^3$

45. Find the sum of an infinite series given by $1+\frac{2}{3}+\frac{4}{9}+\cdots$ (A) 1 (B) 0 (C) ∞ (D) 3

46. Determine the truth set of the inequality $(x-2)(2x+1)(x+4)(3-x)<0$
(A) $\{x: x<-4\}$ (B) $\{x: x<-4\} \cup \{x>3\}$
(C) $\{x: x<-4\} \cup \left\{x: -\frac{1}{2}<x<2\right\} \cup \{x: x>3\}$
(D) $\{x: x>3\} \cup \{x: x<1\}$ (E) $\{x: -5<x<-4\}$

47. Evaluate all the cube roots of unity.
(A) $1, -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$ (B) $-1, -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$
(C) $1, \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$ (D) $1, -\frac{1}{2}, \frac{1}{2}$ (E) $1, -1, 0$

48. Find a and n if the first three terms in the expansion of $(1+ax)^n$ are $1-10x+40x^2$.
(A) -2, 5 (B) 2,5 (C) -2, -5 (D) 2, -5 (E) 2,2

49. Solve for x and y if $3^{x+y}=243$; $2^{2x-5y}=8$.
(A) 4, 1 (B) 2,2 (C) 2, 1 (D) 1, 2 (E) 1,3

50. If the equation $(7p+1)x^2+(5p-1)x+p=1$ has equal roots, find p. (A) 1/3, 1 (B) 2, 1 (C) 5, 1/3 (D) -5, 1/3 (E) 5, -1/3

$\frac{5}{2} + \frac{\sqrt{15}}{2} i$

+5120

MTH101 EXAM SOLUTION 2015/2016

① $(1-2x)^{1/3} = 1 + \frac{1}{3}(-2x) + \frac{\frac{1}{3}(\frac{1}{3}-1)(-2x)^2}{2!} + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)(-2x)^3}{3!} + \cdots$

$= 1 - \frac{2}{3}x - \frac{\frac{1}{3}(-\frac{2}{3})4x^2}{2} + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})(-8x^3)}{6} + \cdots$

$= 1 - \frac{2}{3}x + \frac{2}{9\times 2}4x^2 - \frac{10}{27\times 6}8x^3 + \cdots$

$= 1 - \frac{2}{3}x + \frac{4}{9}x^2 - \frac{40}{81}x^3 + \cdots$

To find $(1.05)^{1/3}$

let $1-2x = 1.05 \Rightarrow 2x = -0.05$

$\Rightarrow x = -0.025$

$(1.05)^{1/3} = 1 - \frac{2}{3}(-0.025) + \frac{4}{9}(-0.025)^2 - \frac{40}{81}(-0.025)^3 + \cdots$

$= 1 + 0.0167 + 0.000278 + 0.00000772$

$= 1.0169857$ (D)

② $(x+y)^{10}$

${}^nC_r x^{n-r} y^r \Rightarrow {}^{10}C_r x^{10-r} y^r$ — *

Coefficient of x^5, will be gotten thus

$x^{10-r} = x^5$

$10 - r = 5 \Rightarrow r = 10-5 \Rightarrow r = 5$

Substitute in *

${}^{10}C_5 x^{10-5} y^5 = 252x^5y^5$

∴ Coefficient of x^5 is $252y^5$

closest option (C)

③ $x+2=0 \Rightarrow x=-2$

$P(x) = 3x^3 - 2x^2 + x - 5$

$P(-2) = 3(-2)^3 - 2(-2)^2 + (-2) - 5$

$= 3(-8) - 2(4) - 2 - 5$

$= -24 - 8 - 2 - 5 = -39$ (A)

④ By trial and error

$x=1$ gives $P(x)=0$

Thus $x-1$ is a factor of $P(x) = x^3 - 2x^2 - 5x + 6$

$$
\begin{array}{r|l}
 & x^2 - x - 6 \\
x-1 & x^3 - 2x^2 - 5x + 6 \\
 & -(x^3 - x^2) \\
 & \quad -x^2 - 5x \\
 & \quad -(-x^2 + x) \\
 & \qquad -6x + 6 \\
 & \qquad -(-6x+6) \\
 & \qquad\quad 0
\end{array}
$$

Then $x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$(x+2)(x-3) = 0$

$x = -2$ or 3

∴ $x = 1, 3, -2$ (D)

⑤ $\log_3(2n+2) = \log_3(n-5)$

$2n+2 = n-5$

$2n - n = -5 - 2$

$n = -7$ (A)

⑥ $\frac{\log_{27}125}{\log_9 25} = \frac{\log_{3^3}125}{\log_{3^2}25}$

$\frac{\frac{1}{3}\log_3 125}{\frac{1}{2}\log_3 25} = \frac{\frac{1}{3}\log_3 5^3}{\frac{1}{2}\log_3 5^2}$

$\frac{3\times\frac{1}{3}\log_3 5}{2\times\frac{1}{2}\log_3 5} = \frac{\log_3 5}{\log_3 5} = 1$ (B)

⑦ $9^{2x+1} = \frac{81^{x-2}}{3^x}$

$(3^2)^{2x+1} = \frac{(3^4)^{x-2}}{3^x}$

$3^{4x+2} = \frac{3^{4x-8}}{3^x} \Rightarrow 3^{4x+2} = 3^{4x-8-x}$

$3^{4x+2} = 3^{3x-8} \Rightarrow 4x - 3x = -8-2$

$x = -10$ (C)

⑧ $\frac{7x^2+5x+13}{(x^2+2)(x+1)} = \frac{Ax+B}{x^2+2} + \frac{C}{x+1}$

$7x^2+5x+13 = (Ax+B)(x+1) + C(x^2+2)$

$$7x^2+5x+13 = Ax^2+Ax+Bx+B+Cx^2+2C$$
$$7x^2+5x+13 = (A+C)x^2+(A+B)x+B+2C$$
Comparing like coefficients
$$A+C=7 \quad \text{—(i)}$$
$$A+B=5 \quad \text{—(ii)}$$
$$B+2C=13 \quad \text{—(iii)}$$
From equation (i) $A=7-C$

Substitute in (ii) $7-C+B=5$
$$B-C=-2 \quad \text{—(iv)}$$
Solving (iii) and (iv) simultaneously
$$B+2C=13 \quad \text{—(iii)}$$
$$B-C=-2 \quad \text{—(iv)}$$
$$3C=15 \Rightarrow C=\frac{15}{3} \Rightarrow C=5$$
Put $C=5$ into (i)
$$A+5=7 \Rightarrow A=7-5 \Rightarrow A=2$$
Put $A=2$ into (ii)
$$2+B=5 \Rightarrow B=5-2 \Rightarrow B=3$$
$$\therefore \frac{7x^2+5x+13}{(x^2+2)(x+1)} = \frac{2x+3}{x^2+2}+\frac{5}{x+1}$$

(9) Selection implies combination (C)
$$\Rightarrow {}^{12}C_3 = \frac{12!}{(12-3)!\,3!} = \frac{12\times11\times10\times9!}{9!\,3!}$$
$$\frac{12\times11\times10}{3\times2\times1} = 2\times11\times10 = 220 \quad (C)$$

(10) No of ways of selecting 3 good items out of 8 (not considering the defective)
$$= {}^8P_3 = \frac{8!}{(8-3)!} = \frac{8\times7\times6\times5!}{5!} = 336 \text{ ways} \quad (B)$$

(11) $${}^{10}C_4 \times {}^8C_3 = \frac{10!}{(10-4)!\,4!} \times \frac{8!}{(8-3)!\,3!}$$
$$\frac{10\times9\times8\times7\times6!}{6!\times4\times3\times2\times1} \times \frac{8\times7\times6\times5!}{5!\,3\times2\times1}$$
$$= 10\times3\times7\times8\times7 = 11760 \quad (D)$$

(12) $$\frac{x-3}{x+4} \geq 2$$
$$\frac{x-3}{x+4}\times(x+4)^2 \geq 2(x+4)^2$$
$$(x-3)(x+4) \geq 2(x+4)^2$$
$$(x-3)(x+4) - 2(x+4)^2 \geq 0$$
$$(x+4)[(x-3)-2(x+4)] \geq 0$$
$$(x+4)[x-3-2x-8] \geq 0$$
$$(x+4)(-x-11) \geq 0$$
$$-(x+4)(x+11) \geq 0$$
dividing both sides by -1
$$(x+4)(x+11) \leq 0$$
The turning values are $-4, -11$

Using the truth table

	$x\leq-11$	$-11\leq x\leq-4$	$x\geq-4$
$x+4$	$-$	$-$	$+$
$x+11$	$-$	$+$	$+$
$(x+4)(x+11)$	$+$	$-$	$+$

Thus the solution set is $\{x: -11\leq x\leq -4\}$ or $[-11,-4]$ (C)

(13) $$-x^2-9x-22 \leq -4$$
$$-x^2-9x-22+4 \leq 0$$
$$-x^2-9x-18 \leq 0$$
$$-(x^2+9x+18) \leq 0$$
$$x^2+9x+18 \geq 0$$
$$x^2+6x+3x+18 \geq 0$$
$$x(x+6)+3(x+6) \geq 0$$
$$(x+3)(x+6) \geq 0$$
The turning points are $x=-3,-6$.

	$x\leq-6$	$-6\leq x\leq-3$	$x\geq-3$
$x+3$	$-$	$-$	$+$
$x+6$	$-$	$+$	$+$
$(x+3)(x+6)$	$+$	$-$	$+$

The solution set is $\{x: -6\leq x \cup x\geq -3\}$ or $(-\infty,-6]\cup[-3,\infty)$ (D)

(14) $$|x+5|-6 \leq -1$$
$$|x+5| \leq -1+6$$
$$|x+5| \leq 5$$
Square both sides
$$(x+5)^2 \leq 5^2$$
$$x^2+10x+25 \leq 25$$
$$x^2+10x \leq 0$$
$$x(x+10) \leq 0$$
The turning values are $x=0,-10$

	$x\leq-10$	$-10\leq x\leq 0$	$x\geq 0$
x	$-$	$-$	$+$
$x+10$	$-$	$+$	$+$
$x(x+10)$	$+$	$-$	$+$

The solution is $[-10,0]$ (C)

(15) $x = \dfrac{a}{(1+t)^2}$ and $y = \dfrac{a(1-t)}{1+t}$

$$\frac{y+a}{2} = \frac{1}{2}[y+a] = \frac{1}{2}\left[\frac{a(1-t)}{1+t} + a\right]$$

$$= \frac{1}{2}\left[\frac{a(1-t)+a(1+t)}{1+t}\right] = \frac{1}{2}\left[\frac{a-at+a+at}{1+t}\right]$$

$$= \frac{1}{2}\left[\frac{2a}{1+t}\right] = \frac{a}{1+t} \; \ldots\ldots *$$

but $x = \dfrac{a}{(1+t)^2} \Rightarrow (1+t)^2 = \dfrac{a}{x}$

$$1+t = \frac{\sqrt{a}}{\sqrt{x}}$$

Substitute in * (D)

$$\frac{a}{\sqrt{a}/\sqrt{x}} = a \times \frac{\sqrt{x}}{\sqrt{a}} = \sqrt{a}\cdot\sqrt{x} = \sqrt{ax}$$

(16) $x + \dfrac{x}{1+x} + \dfrac{x}{(1+x)^2} + \dfrac{x}{(1+x)^3} + \cdots$

$a = x \quad r = \dfrac{x}{1+x} \times \dfrac{1}{x} = \dfrac{1}{1+x}$

$$S_\infty = \frac{a}{1-r} = \frac{x}{1-\frac{1}{1+x}} = \frac{x}{\frac{1+x-1}{1+x}}$$

$$\frac{x}{\frac{x}{1+x}} = x \times \frac{1+x}{x} = 1+x \quad \text{(E)}$$

(17) ${}^{n}P_5 = 18\;{}^{n-2}P_4$

$$\frac{n!}{(n-5)!} = 18\,\frac{(n-2)!}{(n-2-4)!}$$

$$\frac{n!}{(n-5)!} = \frac{18(n-2)!}{(n-6)!} \Rightarrow \frac{n(n-1)(n-2)!}{(n-5)(n-6)!} = \frac{18(n-2)!}{(n-6)!}$$

$$\frac{n(n-1)}{n-5} = \frac{18}{1} \Rightarrow n(n-1) = 18(n-5)$$

$n^2 - n = 18n - 90 \Rightarrow n^2 - n - 18n + 90 = 0$

$n^2 - 19n + 90 = 0$

$n^2 - 10n - 9n + 90 = 0$

$n(n-10) - 9(n-10) = 0$

$(n-9)(n-10) = 0$

$n = 9, 10$ (C)

(18) Given $(1+x)^{2n}$

$${}^{2n}C_n\, 1^{2n-n}\, x^n = {}^{2n}C_n\, x^n$$

Coefficient of $x^n = {}^{2n}C_n = K$

$$\frac{(2n)!}{(2n-n)!\,n!} = K \Rightarrow \frac{2n(2n-1)!}{n!\,n!} = K$$

$$\frac{2n(2n-1)!}{n!\,n(n-1)!} = K \Rightarrow 2\left[\frac{(2n-1)!}{n!(n-1)!}\right] = K$$

$$\Rightarrow \frac{(2n-1)!}{n!(n-1)!} = \frac{K}{2} \; \text{—} *$$

Next, Given $(1+x)^{2n-1}$

We have ${}^{2n-1}C_n\, 1^{(2n-1)-n}\, x^n = {}^{2n-1}C_n\, x^n$

Coefficient of $x^n = {}^{2n-1}C_n$

$$\Rightarrow \frac{(2n-1)!}{(2n-1-n)!\,n!} = \frac{(2n-1)!}{(n-1)!\,n!} = \frac{(2n-1)!}{n!(n-1)!} = \frac{K}{2}$$

From * (C)

(19) $z = \dfrac{1+i}{1-i}$

$$\Rightarrow \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+i+i+i^2}{1^2 - i^2}$$

$$\frac{1+2i-1}{1^2-(-1)} = \frac{2i}{2} = i \Rightarrow 0+i$$

$a = 0, \; b = 1$

$r = \sqrt{a^2+b^2} = \sqrt{0^2+1^2} = \sqrt{1^2} = 1$

$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{1}{0}\right) = 90 = \frac{\pi}{2}$

Euler's form $\Rightarrow re^{i\theta}$

$= 1e^{i\frac{\pi}{2}} = e^{i\frac{\pi}{2}}$ (B)

(20) Given $x^2 - 5x + 10 = 0$

$\alpha + \beta = -\dfrac{b}{a} = \dfrac{-(-5)}{1} = 5$

$\alpha\beta = \dfrac{c}{a} = \dfrac{10}{1} = 10$

But $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha+\beta)[\alpha^2 + \beta^2 - \alpha\beta]$

$= (\alpha+\beta)[(\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta]$

$= (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

$= 5[5^2 - 3(10)] = 5[25-30]$

$= 5(-5) = -25$ (B)

(21) $z^3 = -i = 0 - i$

or $z = (-i)^{1/3} = (\cos\theta - i\sin\theta)^{1/3}$

but $\theta = \tan^{-1}\left(-\frac{1}{0}\right) = 90 = \frac{\pi}{2}$

$r = \sqrt{0^2+(-1)^2} = \sqrt{0+1} = 1$

$z(-1)^{1/3} = \left[\cos\left(\frac{\pi}{2}+2\pi K\right) + i\sin\left(\frac{\pi}{2}+2\pi K\right)\right]^{1/3}$

$= \cos\frac{\pi/2+2\pi K}{3} + i\sin\frac{\pi/2+2\pi K}{3}$

for $K = 0, 1, 2$

For $K=0 \Rightarrow \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}+\frac{i}{2}$

for $K=1 \Rightarrow \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}+\frac{i}{2}$

for $K=2 \Rightarrow \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = 0 - i = -i$

(22) MISSISSIPPI $= \frac{11!}{4!4!2!}$ (A)

4S's, 4I's and 2P's

(23) Given $P_n = 8^n + 6$

For $n=1 \Rightarrow 8^1+6 = 8+6 = 14$
$= 7\times 2$

Assume true for $n=K$

$7P_K = 8^K + 6 \Rightarrow 8^K = 7P_K - 6$ — *

Show true for $n = K+1$

$P_{K+1} = 8^{K+1}+6 = 8^K \cdot 8 + 6$

$= 8(8^K)+6$ ——— **

Put * into **

$8[7P_K - 6] + 6 = 8(7P_K) - 48 + 6$

$8(7P_K) - 42 = 7(8P_K) - 42$

$= 7(8P_K - 6)$

(24) $\left(\frac{x^4}{y}+\frac{y^2}{2x}\right)^{15} = \left(x^4y^{-1} + 2^{-1}x^{-1}y^2\right)^{15}$

${}^{15}C_r\,(x^4y^{-1})^{15-r}(2^{-1}x^{-1}y^2)^r$

${}^{15}C_r\,x^{60-4r}\,y^{-15+r}\,2^{-r}x^{-r}y^{2r}$

${}^{15}C_r\,x^{60-5r}\,y^{-15+3r}\,2^{-r}$ — *

The term independent of y is gotten thus

$y^{-15+3r} = y^0$

$-15+3r = 0 \Rightarrow 3r = 15$

$r = 5$

Put $r=5$ into *

${}^{15}C_5\,x^{60-5(5)}\,y^{-15+3(5)}\,2^{-5}$

$3003\,x^{60-25}\,y^0\,\frac{1}{2^5}$

$\frac{3003}{32}x^{35}$

(25) $S_\infty = \frac{a}{1-r} = 8a$

$a = 8a(1-r)$

$1 = 8(1-r)$

$\frac{1}{8} = 1-r$

$r = 1-\frac{1}{8} = \frac{7}{8}$ (A)

(26) $\frac{2+3x-x^2}{x(x^2-1)} = \frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$

$2+3x-x^2 = A(x^2-1)+Bx(x+1)+Cx(x-1)$

$2+3x-x^2 = Ax^2-A+Bx^2+Bx+Cx^2-Cx$

$2+3x-x^2 = (A+B+C)x^2+(B-C)x - A$

Comparing Coefficients

$-A = 2 \Rightarrow A = -2$ — *

$B-C = 3$ — **

$A+B+C = -1$ — ***

put * into ***

$-2+B+C = -1 \Rightarrow B+C = 1$ — ****

Solving ** and **** Simultaneously

$+\begin{array}{l} B-C=3 \text{ — **} \\ B+C=1 \text{ — ****} \\ \hline 2B = 4 \Rightarrow B=2 \end{array}$

Put $B=2$ into ****

$2+C=1 \Rightarrow C=-1$

$\therefore \frac{2+3x-x^2}{x(x^2-1)} = \frac{-2}{x}+\frac{2}{x-1}-\frac{1}{x+1}$

$= \frac{2}{x-1}-\frac{2}{x}-\frac{1}{x+1}$ (A)

(27) Given $\left(\frac{1}{x^2}-x\right)^{18} = \left[x^{-2}+(-x)\right]^{18}$

${}^{18}C_r\,(x^{-2})^{18-r}(-x)^r = -{}^{18}C_r\,x^{-36+2r+r}$

$= -{}^{18}C_r\,x^{-36+3r}$ — *

Coefficient of x^6 is gotten thus

$x^{-36+3r} = x^6$

$-36+3r=6$

$3r = 6+36 \Rightarrow 3r = 42$

$r = 14$

Put $r=14$ into *

$-{}^{18}C_{14}\, x^{-36+3(14)} = -3060x^{-36+42}$

$= -3060x^{6}$

$\therefore$ Coefficient of $x^6 = -3060$ (C)

(28) Given $6x^2-3Kx+5=0$

sum of root $= \frac{-b}{a} = \frac{-(-3K)}{6} = \frac{3K}{6}$

product of root $= \frac{c}{a} = \frac{5}{6}$

$\frac{3K}{6} = \frac{5}{6} \Rightarrow 3K = 5 \Rightarrow K = \frac{5}{3}$ (D)

(29) $\sqrt{3x+4} - \sqrt{x+2} - \sqrt{x-3} = 0$ — *

$\sqrt{3x+4} - \sqrt{x+2} = \sqrt{x-3}$

$(\sqrt{3x+4} - \sqrt{x+2})^2 = (\sqrt{x-3})^2$

$3x+4-2(\sqrt{3x+4})(\sqrt{x+2}) + x+2 = x-3$

$3x+x-x+4+2+3 = 2(\sqrt{3x+4})(\sqrt{x+2})$

$3x+9 = 2(\sqrt{3x+4})(\sqrt{x+2})$

$(3x+9)^2 = [2(\sqrt{3x+4})(\sqrt{x+2})]^2$

$9x^2+27x+27x+81 = 4(3x+4)(x+2)$

$9x^2+54x+81 = 4(3x^2+6x+4x+8)$

$9x^2+54x+81 = 4(3x^2+10x+8)$

$9x^2+54x+81 = 12x^2+40x+32$

$12x^2-9x^2+40x-54x+32-81 = 0$

$3x^2-14x-49=0$

$3x^2-21x+7x-49=0$

$3x(x-7)+7(x-7)=0$

$(3x+7)(x-7)=0$

$x = -\frac{7}{3}$ or 7

But $x = -\frac{7}{3}$ does not satisfy the given equation *

$\therefore x = 7$ (C)

(30) let $x = \frac{a}{r}$, $y = a$, $z = ar$

Product, $\frac{a}{r} \times a \times ar = a^3 = 512$

$a = \sqrt[3]{512} \Rightarrow a = 8$

Sum, $\frac{a}{r} + a + ar = 28$

$\frac{8}{r} + 8 + 8r = 28$

$8 + 8r + 8r^2 = 28r$

$8r^2+8r-28r+8=0$

$8r^2-20r+8=0$

$2r^2-5r+2=0$

$2r^2-4r-r+2=0$

$2r(r-2)-1(r-2)=0$

$(2r-1)(r-2)=0$

$r = \frac{1}{2}$ or 2

$\therefore x = \frac{a}{r} = \frac{8}{1/2}$ or $\frac{8}{2} = 16$ or 4

$y = a = 8$

$z = ar = 8\times\frac{1}{2}$ or $8\times 2 = 4$ or 16

$16, 8, 4$ (A)

(31) $U = \{10, 11, 12, 13, 14, 15, \ldots, 30\}$

$P = \{11, 13, 15, 17, 19, 23, 29\}$

$Q = \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$R = \{12, 15, 18, 21, 24, 27, 30\}$

$P \cup Q = \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$

$(P\cup Q)^c = \{10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\}$

Cardinality $= 11$ (A)

(i.e there are 11 elements in the set

(32) Given $9x^2+Kx+3=0$

let the first root be $= \alpha$

The second root becomes $= \left(\frac{1}{\alpha}\right)^2 = \frac{1}{\alpha^2}$

Product of root $= \alpha \times \frac{1}{\alpha^2} = \frac{3}{9} = \frac{1}{3}$

i.e $\frac{1}{\alpha} = \frac{1}{3} \Rightarrow \alpha = 3$

$\frac{1}{\alpha^2} = \frac{1}{3^2} = \frac{1}{9}$

Thus, the roots are $(3, \frac{1}{9})$ (B)

(33)

$$\begin{array}{r|l} & x^3 - x^2 + 3x - 1 \\ \hline x+4 & x^4+3x^3-x^2+11x-4 \\ & -(x^4+4x^3) \\ \hline & -x^3 - x^2 \\ & -(-x^3-4x^2) \\ \hline & 3x^2+11x \\ & -(3x^2+12x) \\ \hline & -x-4 \\ & -x-4 \\ \hline & 0 \end{array}$$

$\therefore$ Quotient $= x^3-x^2+3x-1$ (B)

(34) $\frac{(1+i)(2+i)}{3-i} = \frac{2+i+2i+i^2}{3-i}$

$= \frac{2+3i+(-1)}{3-i} = \frac{1+3i}{3-i}$

$\frac{1+3i}{3-i} \times \frac{3+i}{3+i} = \frac{3+i+9i+3i^2}{3^2-i^2}$

$= \frac{3+10i+3i^2}{9-i^2} = \frac{3+10i+3(-1)}{9-(-1)}$

$= \frac{3+10i-3}{9+1} = \frac{10i}{10} = i$ (D)

(35) let the AP be $a-d$, a and $a+d$

sum $\Rightarrow a-d+a+a+d = 3a = 27$

$a = \frac{27}{3} \Rightarrow a = 9$

Product $\Rightarrow (a-d)a(a+d) = (9-d)9(9+d)$

$(9-d)(81+9d) = 729+81d-81d-9d^2$

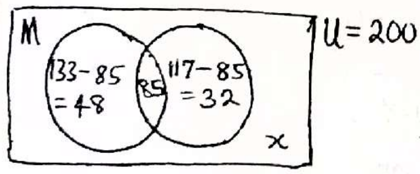
$729-9d^2 = 693$

$9d^2 = 729-693 \Rightarrow 9d^2 = 36$

$d^2 = \frac{36}{9} \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

(B)

(36)



$48+85+32+x = 200$

$165+x = 200$

$x = 200-165 \Rightarrow x = 35$ (C)

(37) $\frac{(2^{p+1})^q}{(2^{q+1})^p} \times \frac{2^{2p}}{2^{2q}} \times 2^q$

$= \frac{2^{pq+q}}{2^{pq+p}} \times \frac{2^{2p}}{2^{2q}} \times 2^q = \frac{2^{pq} \times 2^q}{2^{pq} \times 2^p} \times \frac{2^{2p}}{2^{2q}} \times 2^q$

$= \frac{2^{q+q} \times 2^{2p}}{2^p \times 2^{2q}} = \frac{2^{2q} \times 2^{2p}}{2^p \times 2^{2q}} = 2^p$ (A)

(38) $a = -2 \Rightarrow a+d = 3$

$-2+d = 3 \Rightarrow d = 3+2 \Rightarrow d = 5$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$306 = \frac{n}{2}[2(-2)+(n-1)5]$

$306 = \frac{n}{2}[-4+5n-5]$

$306 = \frac{n}{2}[5n-9] \Rightarrow 612 = n[5n-9]$

$612 = 5n^2 - 9n$

$5n^2 - 9n - 612 = 0$

$5n^2 - 60n + 51n - 612 = 0$

$5n(n-12) + 51(n-12) = 0$

$(5n+51)(n-12) = 0$

$n = -\frac{51}{5}$ or 12

Since number of terms n must be a positive integer, thus $n=12$ (B)

(39) Given $9^n - 1$

For $n=1$

$9^1 - 1 = 9-1 = 8 = 8 \times 1$
true

Assume true for $n=K$

$9^K - 1 = 8P_K$

$9^K = 8P_K + 1$ —— *

Show true for $n = K+1$

$9^{K+1} - 1 = 9^K \cdot 9 - 1$

$9(9^K) - 1$ —— **

Put * into **

$9(8P_K+1) - 1 = 9(8P_K) + 9 - 1$

$= 8(9P_K) + 8 = 8(9P_K + 1)$ (C)

(40) $\frac{3(2^{n+1}) - 4(2^{n-1})}{2^{n+1} - 2^n} = \frac{3(2^n \cdot 2) - 4\left(\frac{2^n}{2}\right)}{2^n \cdot 2 - 2^n}$

$\frac{6 \cdot 2^n - 2 \cdot 2^n}{2 \cdot 2^n - 2^n} = \frac{(6-2)2^n}{(2-1)2^n} = \frac{4 \cdot 2^n}{2^n}$

$= 4$ (C)

(41) $(x+y) + i(2x-y) = 2+i$

Comparing real and imaginary part.

$x+y = 2$ —— *

$+ \quad 2x-y = 1$ —— **

$3x = 3 \Rightarrow x = 1$

Put $x=1$ into *

$1+y = 2$

$y = 2-1 \Rightarrow y = 1$

$(x, y) = (1, 1)$ (A)

(42) let $\sqrt{6+\sqrt{32}} = \sqrt{a} + \sqrt{b}$

$6+\sqrt{32} = (\sqrt{a}+\sqrt{b})^2$

$6+\sqrt{32} = a + 2\sqrt{ab} + b$

$6+\sqrt{32} = a + b + 2\sqrt{ab}$

Comparing both sides

$a+b=6$ —— *

$2\sqrt{ab}=\sqrt{32}$

$2\sqrt{ab}=\sqrt{4\times8} \Rightarrow 2\sqrt{ab}=2\sqrt{8}$

$\sqrt{ab}=\sqrt{8} \Rightarrow ab=8$ — **

Solving * and ** simultaneously

$a+b=6$ — *

$ab=8$ —— **

From * $a=6-b$ — ***

substitute in **

$(6-b)b=8$

$6b-b^2=8 \Rightarrow b^2-6b+8=0$

$b^2-4b-2b+8=0$

$b(b-4)-2(b-4)=0$

$(b-2)(b-4)=0$

$b=2$ or 4

Substitute in ***

$a=6-2$ or $a=6-4$

$a=4$ or 2

take $a=4$ and $b=2$

$\sqrt{6+\sqrt{32}}=\sqrt{4}+\sqrt{2}=2+\sqrt{2}$ (B)

(43) $P(x)=Px^3+3x^2-9x+q$

$x-2=0 \Rightarrow x=2$

$P(2)=P(2)^3+3(2)^2-9(2)+q=0$

$8P+12-18+q=0$

$8P+q=6$ —— *

$x+1=0 \Rightarrow x=-1$

$P(-1)=P(-1)^3+3(-1)^2-9(-1)+q=0$

$-P+3+9+q=0$

$-P+q=-12$ —— **

Solving * and ** respectively

$8P+q=6$

$-\ \ -P+q=-12$

$9P=18 \Rightarrow P=\frac{18}{9} \Rightarrow P=2$

put $P=2$ into **

$-2+q=-12 \Rightarrow q=-12+2 \Rightarrow q=-10$

$P,q=2,-10$ (B)

(44) Given $(x^3+2y)^8$

Fifth term $=(4+1)$th term $={}^8C_4(x^3)^{8-4}(2y)^4$

$=70(x^3)^4\,2^4y^4=1120x^{12}y^4$ (B)

(45) $a=1$, $r=\frac{2}{3}$

$S_\infty=\frac{a}{1-r}=\frac{1}{1-\frac{2}{3}}=\frac{1}{\frac{1}{3}}=1\times\frac{3}{1}=3$ (D)

(46) $(x-2)(2x+1)(x+4)(3-x)<0$

The turning values are $x=-4,-\frac{1}{2},2,3$

using the truth table

	$x<-4$	$-4<x<-\frac{1}{2}$	$-\frac{1}{2}<x<2$	$2<x<3$	$x>3$
$x-2$	−	−	−	+	+
$2x+1$	−	−	+	+	+
$x+4$	−	+	+	+	+
$3-x$	+	+	+	+	−
Product	−	+	[illegible]	+	−

The solution set is $\{x: x<-4\}\cup\{x: -\frac{1}{2}<x<2\}\cup\{x: x>3\}$ (C)

(47) $z^3=1=1+0i$

$z=(1)^{1/3}$, $r=\sqrt{1^2+0^2}=1$

$\theta=\tan^{-1}\left(\frac{0}{1}\right)=0$

$z=\left[r\left(\cos(0+2k\pi)+i\sin(0+2k\pi)\right)\right]^{1/3}$

$z=1\left[\cos\frac{2k\pi}{3}+i\sin\frac{2k\pi}{3}\right]$

$k=0,1,2$

For $k=0 \Rightarrow z=\cos 0+i\sin 0=1$

for $k=1 \Rightarrow z=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$

for $k=2 \Rightarrow z=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$

$\therefore z=1, -\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$ (A)

(48) $1+nax+\frac{n(n-1)}{2}(ax)^2=1-10x+40x^2$

Comparing, $na=-10 \Rightarrow a=-\frac{10}{n}$ — * & $\frac{n(n-1)}{2}a^2=40$

$\frac{n(n-1)}{2}\left(\frac{-10}{n}\right)^2=40 \Rightarrow \frac{n(n-1)}{2}\cdot\frac{100}{n^2}=40$

$(n-1)50=40n \Rightarrow (n-1)5=4n \Rightarrow 5n-5=4n$

$5n-4n=5 \Rightarrow n=5$

put $n=5$ into * $a=-\frac{10}{5} \Rightarrow a=-2$

$a,n=-2,5$ (A)

(49) $3^{x+y}=243 \Rightarrow 3^{x+y}=3^5 \Rightarrow x+y=5$ ⋯ *

$2^{2x-5y}=8 \Rightarrow 2^{2x-5y}=2^3 \Rightarrow 2x-5y=3$ ⋯ **

From * $x=5-y$ ⋯ ***

Substitute in ** $2(5-y)-5y=3 \Rightarrow 10-2y-5y=3$

$-7y=-7 \Rightarrow y=1$

put $y=1$ into ***

$x=5-1 \Rightarrow x=4$

$x,y=4,1$ (A)

(50) Given $(7P+1)x^2+(5P-1)x+P=1$

$\Rightarrow (7P+1)x^2+(5P-1)x+P-1=0$

$a=7P+1$, $b=5P-1$, $c=P-1$

For equal roots, $b^2-4ac=0$

$(5P-1)^2-4(7P+1)(P-1)=0$

$25P^2-10P+1-4(7P^2-6P-1)=0$

$25P^2-10P+1-28P^2+24P+4=0$

$-3P^2+14P+5=0 \Rightarrow 3P^2-14P-5=0$

$\Rightarrow 3P^2-15P+P-5=0 \Rightarrow 3P(P-5)+1(P-5)=0$

$\Rightarrow (P-5)(3P+1)=0$

$P=5,-\frac{1}{3}$ (E)