FEDERAL UNIVERSITY OF TECHNOLOGY, OWERRI
DEPARTMENT OF MATHEMATICS
RAIN SEMESTER EXAMINATIONS, 2017/2018 SESSION
MTH 332, VECTOR AND TENSOR ANALYSIS

Answer any Five Questions

1(a) If $\bar{f}(t)$ and $\bar{g}(t)$ are differentiable vector functions of t, simplify the following:

(i) $\frac{d}{dt}\left(\frac{d^2\bar{f}}{dt^2} \times \frac{d^3\bar{f}}{dt^3} \cdot \frac{d^4\bar{f}}{dt^4}\right)$, (ii) $\frac{d^2}{dt^2}\left(\frac{d^3\bar{f}}{dt^3} \cdot \frac{d\bar{f}}{dt} \times \frac{d^2\bar{f}}{dt^2}\right)$

(b) If $\bar{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\quad \bar{b} = -\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ and
$\bar{c} = 4\hat{\imath} - \hat{\jmath} - 3\hat{k}$, are vectors,
find constants α, μ *and* γ such that
$3\hat{\imath} + 2\hat{\jmath} + \hat{k} = \alpha\bar{a} + \gamma\bar{b} + \mu\bar{c}$

2(a) Using tensor indicial notations, evaluate
$(\bar{a} \times \bar{b}) \times \bar{c}$.

(b) Show that
$(\bar{a} \times \bar{b}) \times \bar{c} + (\bar{b} \times \bar{c}) \times \bar{a} + (\bar{c} \times \bar{a}) \times \bar{b} = \bar{o}$

(c) Parametrize the following curve with respect to arc length s. .

$x = sin3t - 3t\,cos3t + 5$

$y = cos3t + 3tsin3t + 7, \qquad z = 9t^2 + 1$, $0 \leq t \leq 1$

3. Find the line integral of the tangential component of
$\bar{f} = (x + 2y + z^2)\hat{\imath} + (y^2 + 2xy + z^2)\hat{\jmath} + (x^2 + y^2)\hat{k}$
along the curve C given by
$x = t,\ \ y = 2t^2,\ \ z = t^3,\ \ 0 \leq t \leq 1$

4(a) Find a potential $\emptyset\,(x, y, z)$ for the vector field

$\bar{f} = (9x^2ye^z - Zsinx + 6ze^x + 5)\hat{\imath}$

$+ (3x^3e^z + Zcosy - y^3z^4 + 7)\hat{\jmath}$

$+ (3x^3ye^z + cosx + siny - y^4z^3 + 6e^x + 9)\hat{k}$

Cosx Cix = $\frac{\sin 2x}{2}$

(b) Parametrize the curve below with respect to arc length

$x = t,\ y = \sin 3\pi t,\ z = \cos 3\pi t,\ 0 \le t \le 1.$

5. Find the line integral of

$$\bar{f} = (x^2 + y^2 + 2xy + 1)\hat{\imath} + (2x + y^2 + 2)\hat{\jmath}$$

from the point $(-1,0)$ to $(1,0)$ along the following curves

(a) along the x -axis, (b) along the upper part of the semi-circle $y = \sqrt{1 - x^2}$.

6(a) A charged particle of mass 1kg moves along a curve given by

$$r = \frac{1}{1+3\cos\theta},\ \frac{d\theta}{dt} = e^{-\alpha t}\cos\beta t,\ \alpha > 0, \beta > 0.$$

By differentiating $\bar{r} = r\hat{U}_r$ (where all terms have their usual meanings), find (i) $\frac{d\bar{r}}{dt}$, (ii) $\frac{d^2\bar{r}}{dt^2}$.

(b) Find the coriollis acceleration.