FEDERAL UNIVFERSITY OF TECHNOLOGY, OWERRI
DEPARTMENT OF MATHEMATICS
2017/2018 HARMATTAN SEMESTER EXAMINATION
MTH 203: ODE ANSWER 5 QUESTIONS TIME ; 3HRS

1. a) Obtain the differential equation corresponding to the family of curves
$$y = C_1 \sin x + C_2 \cos x$$
and classify them as linear or nonlinear. State the order and degree.
b) Solve the differential equation
$$\left(y^2 e^{xy^2} + 6x\right)dx + \left(2xye^{xy^2} - 4y\right)dy = 0$$

2. a) Define the concept of i) Complete Fundamental Set ii) the Wronskian of n-solution function of an nth order differential equation.
b) To solve $y'' + 4y' + 8y = \sin x$ by variation of parameters method, Henry obtained $v_1(x) = -\frac{1}{2}\int \frac{\sin 2x \sin x}{e^{-2x}} dx$ and $v_2(x) = \frac{1}{2}\int \frac{\cos 2x \sin x}{e^{-2x}} dx$. Trace your working to these expressions.

3. a) Given the equation of the form $y' + p(x)y = q(x)y^n$; $n \neq 0$. Obtain its linear differential equation equivalent.
b) Solve the initial value problem (IVP) $\begin{cases} xdx + e^{-x^2}(y^5 - 1)dy = 0 \\ y(0) \end{cases}$

4. a) Given that $M(x, y)dx + N(x, y)dy = 0$ is not Exact, obtain the integrating factor $\mu(x)$ that will make it Exact.
b) Show that $y = 2e^{3x} - 5e^{4x}$ is a solution of the d.e. $y'' - 7y' + 12y = 0$

5. a) Solve by the method of undetermined coefficients, the differential equation
$$y'' + 4y = 3\sin 2x$$
b) Solve the IVP $\frac{dy}{dx} = \frac{x+y+1}{x+y+3}$; $y(0) = -1$

6. a) Solve the differential equation ; $y' + \frac{2}{x}y = x^6 y^3$
b) Solve the differential equation : $y' = \frac{x^3 + y^3}{3xy^2}$

SCHOOL OF PHYSICAL SCIENCES
DEPARTMENT OF MATHEMATICS
2018/2019 HARMATTAN SEMESTER EXAMINATIONS
MTH 203: Ordinary Differential Equations I
Instruction: Answer any 5 (five) questions. **TIME: 3 Hours**

1(a) Given that the Laplace transform of a piecewise continuous function f is defined for $t \geq 0$ as

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt,$$

obtain the Laplace transform of the function

$$f(t) = \begin{cases} \sin t, & 0 \leq t < \pi \\ 0, & t \geq \pi \end{cases}$$

1(b) Solve $y'' - 3y' + 2y = e^{-4t}$, $y(0) = 1, y'(0) = 5$

2(a) Solve $\frac{dy}{dx} + y\cot x = 5e^{\cos x}$;

$y'' - 3y' + 2y = e^{-4t}$

2(b) Solve $(\cos^2 x - y\cos x)dx - (1 + \sin x - 2y - 3)dy = 0$

3(a) Solve the equation $\frac{dx}{dt} + \frac{x}{t} = tx^2$

3(b) Find the solution of the initial value problem

$$y'' - 2y' - 8y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

4(a) Find the solution of the equation

$$\frac{dy}{dx} = \frac{2x - 5y + 3}{2x + 4y - 6}$$

4(b) Obtain a differential equation from $y = e^{x+A} = -Be^x$, where A and B are constants.

5(a) Determine whether the following equation is exact, and then solve it

$$(e^x \sin y + 3y)dx - (3x - e^x \sin y)dy = 0$$

5(b) Prove that if $y_1(x)$ and $y_2(x)$ are solutions of

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$$

then $c_1y_1(x) + c_2y_2(x)$ is also a solution of this equation where c_1 and c_2 are arbitrary constants

6(a) Find the general solution of $y'' - 2y' + y = \frac{e^x}{2}$ using variation of parameters.

6(b) Find a second linearly independent solution for the differential equation $(1 - x^2)y'' - 2xy' + 2y = 0$, given that $y_1(x) = x$ is a solution.

Moderated by Dr. C. A. Nse