

UNIVERSITY OF ILORIN, ILORIN FACULTY OF PHYSICAL SCIENCES DEPARTMENT OF MATHEMATICS

2017/2018 B.Sc. DEGREE RAIN SEMESTER EXAMINATION IN MATHEMATICS, JUL/AUG 2018

Course Title: COMPLEX ANALYSIS II. Level: 300

Course Code: MAT 326. No. of Credits: 3. Time Allowed: 2hrs

Instructions: Answer ANY FOUR questions.

1 (a) If f(z) is analytic inside and on the boundary of a ring-shaped region R bounded by two concentric circles C_1 and C_2 with centre at a and radii r_1 , r_2 respectively with $r_1 > r_2$. Prove that

$$\lim_{n \to \infty} \frac{1}{2\pi i} \oint_{C_1} \left(\frac{z-a}{w-a} \right)^n \frac{f(w)}{w-z} dw, \ w \in C_1 = 0.$$

(b) Prove also that

$$\lim_{n \to \infty} \frac{1}{2\pi i} \oint_{C_2} \left(\frac{w - a}{z - a} \right)^n \frac{f(w)}{z - w} dw, \ w \in C_2 = 0.$$

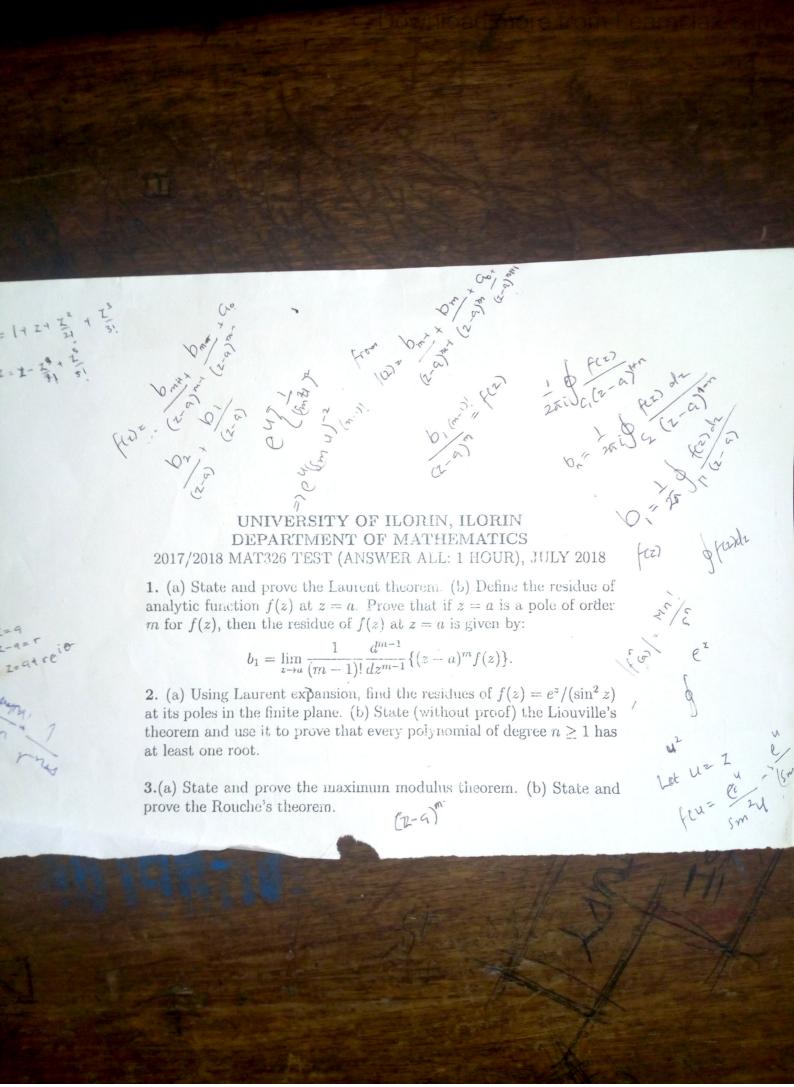
- 2. (a) Using Laurent expansion, find the residue of $f(z) = 1/[z^2(z-3)^2]$ at the origin. (b) State and prove the Liouville's theorem and use it to prove that every polynomial of degree $n \ge 1$ has at least one root.
- 3.(a) Prove that if z = a is a pole of order m for f(z), then the residue of f(z) at z = a is given by:

$$b_1 = \lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{ (z-a)^m f(z) \}.$$

- (b) Find the residue of $f(z) = (\cot z \coth z)/z^3$ at its singular point in the finite plane.
- (4)(a) State and prove the Residue theorem. (b) Evaluate the integral

$$\frac{2}{\pi} \int_{-\infty}^{\infty} \frac{x \cos \pi x dx}{x^2 + 2x + 5}$$

- (a) State and prove the Cauchy's inequality. (b) State and prove the Rouche's theorem.
 - **6.**(a) Discuss briefly the concept of analytic continuation. (b) Obtain an analytic continuation of $f(z) = \sum_{n=0}^{\infty} z^n/2^{n+1}$ whose centre is z = 2i and determine its region of convergence.



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