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UNIVERSITY OF ILORIN, ILORIN
FACULTY OF PHYSICAL SCIENCES
DEPARTMENT OF MATHEMATICS

2017/2018 B.Sc. DEGREE RAIN SEMESTER EXAMINATION IN MATHEMATICS, JUL/AUG 2018

Course Title: COMPLEX ANALYSIS II. Level: 300
Course Code: MAT 326. No. of Credits: 3. Time Allowed: 2hrs
Instructions: Answer ANY FOUR questions.

1.(a) If $f(z)$ is analytic inside and on the boundary of a ring-shaped region R bounded by two concentric circles C_1 and C_2 with centre at a and radii r_1, r_2 respectively with $r_1 > r_2$. Prove that

$$\lim_{n\to\infty} \frac{1}{2\pi i}\oint_{C_1}\left(\frac{z-a}{w-a}\right)^n \frac{f(w)}{w-z}dw,\ w\in C_1 = 0.$$

(b) Prove also that

$$\lim_{n\to\infty} \frac{1}{2\pi i}\oint_{C_2}\left(\frac{w-a}{z-a}\right)^n \frac{f(w)}{z-w}dw,\ w\in C_2 = 0.$$

2. (a) Using Laurent expansion, find the residue of $f(z) = 1/[z^2(z-3)^2]$ at the origin. (b) State and prove the Liouville's theorem and use it to prove that every polynomial of degree $n \geq 1$ has at least one root.

3.(a) Prove that if $z = a$ is a pole of order m for $f(z)$, then the residue of $f(z)$ at $z = a$ is given by:

$$b_1 = \lim_{z\to a} \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\{(z-a)^m f(z)\}.$$

(b) Find the residue of $f(z) = (\cot z \coth z)/z^3$ at its singular point in the finite plane.

4.(a) State and prove the Residue theorem. (b) Evaluate the integral

$$\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{x\cos \pi x dx}{x^2+2x+5}.$$

5.(a) State and prove the Cauchy's inequality. (b) State and prove the Rouche's theorem.

6.(a) Discuss briefly the concept of analytic continuation. (b) Obtain an analytic continuation of $f(z) = \sum_{n=0}^{\infty} z^n/2^{n+1}$ whose centre is $z = 2i$ and determine its region of convergence.

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2017/2018 MAT326 TEST (ANSWER ALL: 1 HOUR), JULY 2018

1. (a) State and prove the Laurent theorem. (b) Define the residue of analytic function $f(z)$ at $z = a$. Prove that if $z = a$ is a pole of order m for $f(z)$, then the residue of $f(z)$ at $z = a$ is given by:

$$b_1 = \lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\}.$$

2. (a) Using Laurent expansion, find the residues of $f(z) = e^z/(\sin^2 z)$ at its poles in the finite plane. (b) State (without proof) the Liouville's theorem and use it to prove that every polynomial of degree $n \geq 1$ has at least one root.

3.(a) State and prove the maximum modulus theorem. (b) State and prove the Rouche's theorem.