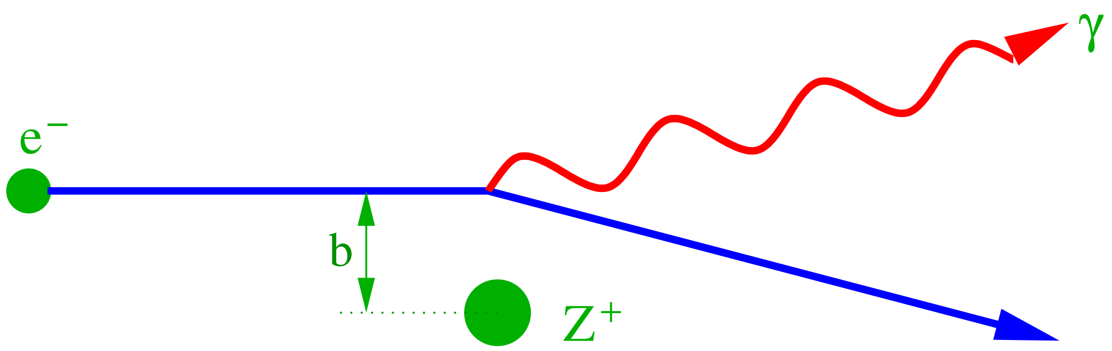
Free-Free or Bremsstrahlung Radiation

- Electrons in a plasma are accelerated by encounters with massive ions.
- This is the dominant continuum emission mechanism in thermal plasmas.
- An important *coolant* for plasmas at high temperature

Examples :

- Radio emission from HII regions
- Radio emission from ionised winds and jets
- X-ray emission from clusters of galaxies



Calculation of Bremsstrahlung Spectrum

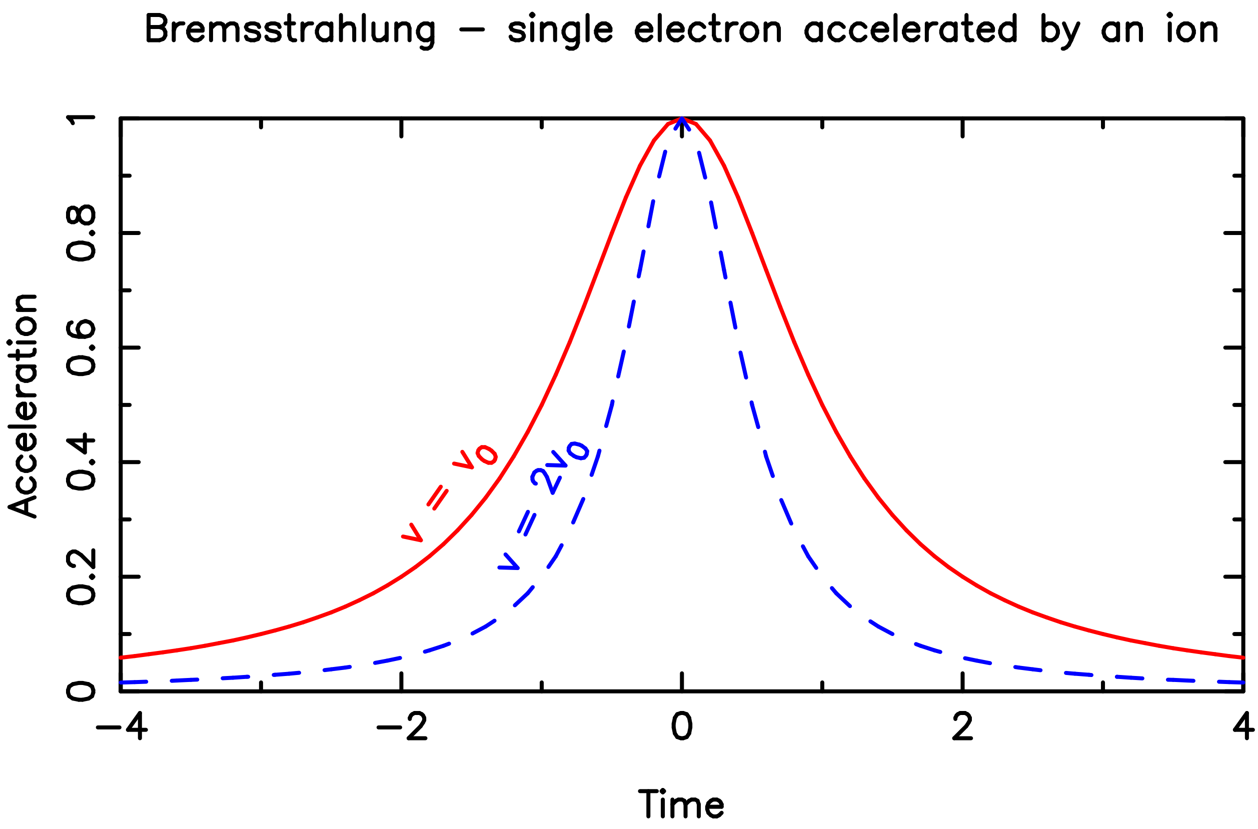
Important ingredients:

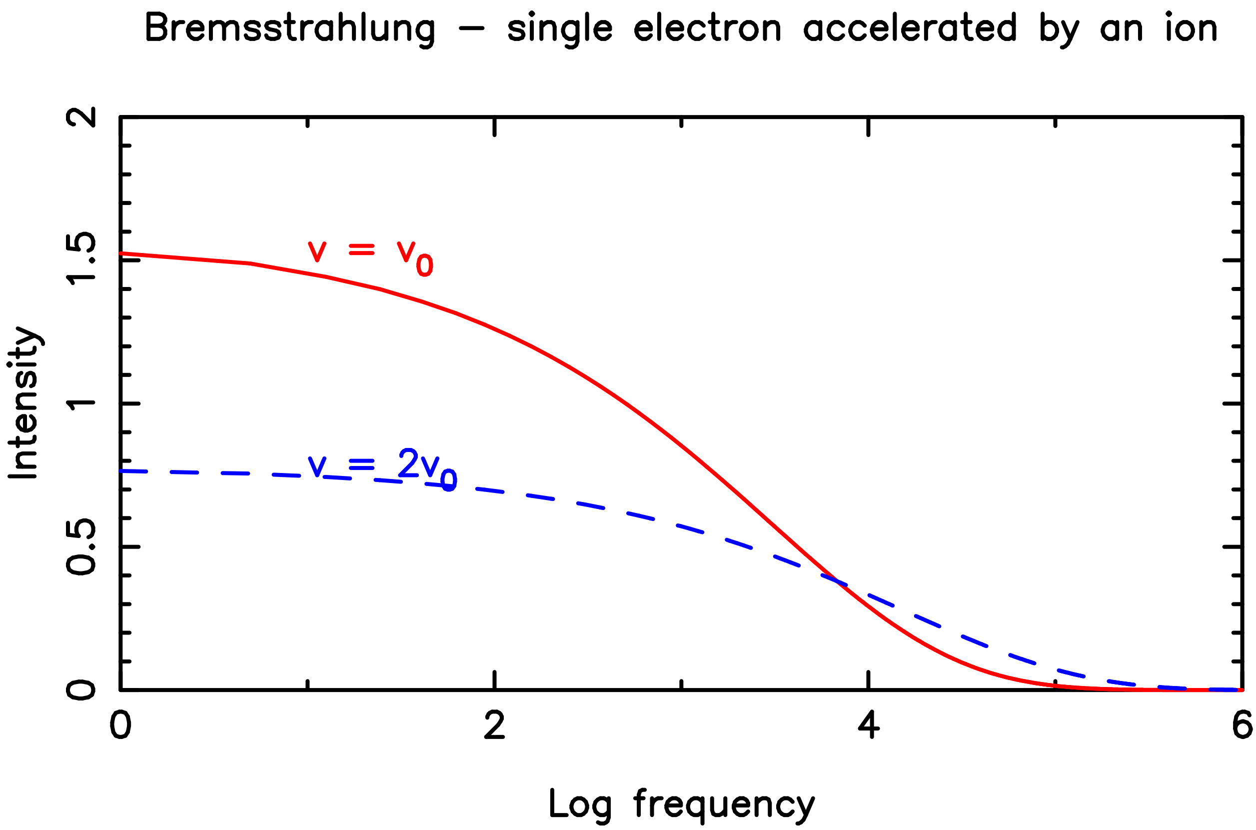
- Consider one particle at a specific b and v.
- When a charged particle accelerates it emits radiation (Larmor's formula). Acceleration is a function of b, v and Z.
- Acceleration as a function of time $\longrightarrow$ intensity spectrum via the Fourier Transform (Parseval's theorem).
- Integrate over b (exact details tricky – gives rise to the Gaunt Factor, $\bar{g}_{ff}$ which is a function of ν, T, Z).
- Include term for collision rate (depends on number densities n_e and n_i of electrons and ions respectively).
- Integrate over v. Assume plasma in thermal equilibrium $\longrightarrow$ Maxwellian distribution of v.

$$\Rightarrow \epsilon_\nu^{ff} = 6.8\times10^{-52}\, T^{-1/2}\, Z^2\, n_e\, n_i\, \exp[-h\nu/(k_B T)]\, \bar{g}_{ff}(\nu)$$

with the result having the units $\mathrm{W\,m^{-3}\,Hz^{-1}}$.

ϵ_ν^{ff} is the emissivity, the emitted *power* per unit *volume* per unit *frequency*. This is related to the spontaneous emission coefficient (the emitted power per unit volume per unit frequency per unit *solid angle*) by $\epsilon_\nu^{ff} = 4\pi j_\nu$.


Bremsstrahlung – single electron accelerated by an ion
Acceleration
0
0.2
0.4
0.6
0.8
1
−4
−2
0
2
4
Time
v = v0
v = 2v0


Bremsstrahlung – single electron accelerated by an ion
Intensity
0
0.5
1
1.5
2
0
2
4
6
Log frequency
v = v0
v = 2v0

Simple Example: Hydrogen Plasma

A common case is that of an optically thin hydrogen plasma, so $n_e = n_i$ and $Z = 1$.

Because the plasma is optically thin, the total emitted specific intensity is proportional to the emissivity integrated along the line of sight.

$$I_\nu \propto \int n_e^2 \, T^{-1/2} \, dl$$

This is proportional to n^2 as we would expect for a collisonal process.

The integral $\int n_e^2 \, dl$ is called the *emission measure*, and is often written in units of $\mathrm{cm^{-6}\,pc}$.

Total Emissivity

Integrate over frequency to get the *total emissivity*:

$$\epsilon^{ff} = 1.4 \times 10^{-28} T^{1/2} \, Z^2 \, n_e \, n_i \; \bar{g}_B$$

This has units of $\mathrm{W\,m^{-3}}$.

If we set $\bar{g}_B = 1.2$ we will probably be within 20% of the correct result.

Free-Free Absorption

- Have calculated how much radiation *emitted*.
- Now wish to find how much an observer *receives*. These two are not equal because free-free absorbtion occurs.
- Find how much absorbed as a function of frequency i.e. α_ν (= fraction of intensity lost per unit distance)
- Kirchoff's Law : $j_\nu = \alpha_\nu B_\nu(T) = \epsilon_\nu/4\pi$

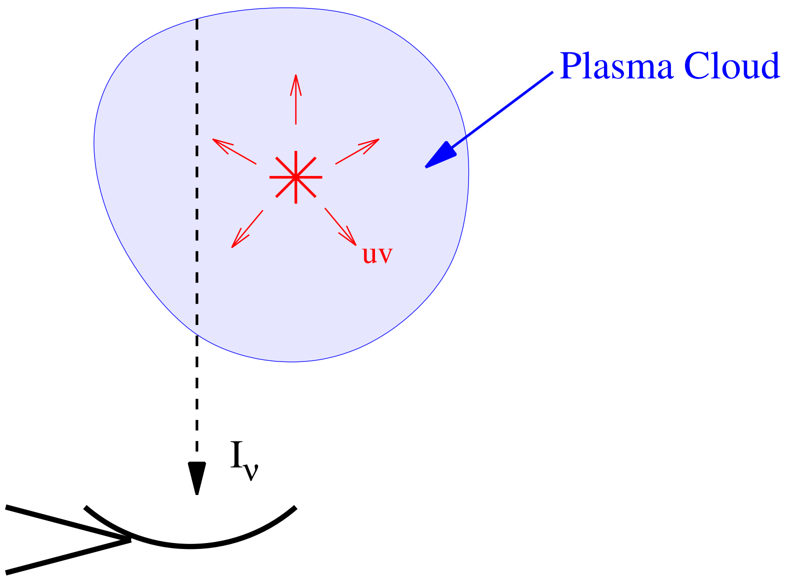
$$\Rightarrow \alpha_\nu^{ff} = \frac{\epsilon_\nu}{4\pi B_\nu(T)}$$
$$= 3.7 \times 10^{-2} \frac{Z^2 n_e n_i (1 - \exp[-h\nu/(k_B T)]) \bar{g}_{ff}(\nu)}{\nu^3 T^{1/2}}$$
$$= \frac{1.8 \times 10^{-12} Z^2 n_e n_i \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}}$$ [1]

in units of m^{-1}.

- Can now find optical depth $\tau_\nu = \int \alpha_\nu ds$
- If optically thin, spectrum is as calculated before (I_ν appoximately flat until turnover).
- If optically thick, spectrum is effectively blackbody.

[1] In the Rayleigh-Jeans region

Example: HII regions around OB stars



- The uv-photons from OB stars photoionises the gas surrounding them. The resulting plasma has a temperature of around 10^4 K.
- The optical depth in the R-J limit is given by

$$\tau \propto \int \frac{n^2 \, \bar{g}_{ff}(\nu)}{\nu^2 T^{3/2}} dl$$

- In this regime $\bar{g}_{ff}(\nu) \propto \nu^{-0.1} T^{0.15}$.
- $I_\nu = (1 - e^{-\tau_\nu}) B_\nu(T_e)$
- At low ν, $\tau_\nu >> 1 \;:\; I_\nu \propto B_\nu(T_e) \propto \nu^2$ – Blackbody like spectrum.
- At high ν, $\tau_\nu << 1 \;:\; I_\nu \propto \tau B_\nu(T_e) \propto \nu^{-0.1}$ – “Flat” spectrum,
- Turnover when $\tau_\nu \approx 1$. e.g. $\nu \approx 1$ GHz for Orion.

Example: X-ray emission from clusters of galaxies

- Gas in clusters of galaxies at temperatures of $T_e \approx 10^8$ K ($\equiv$ 8.6 keV). Therefore Bremsstrahlung emission extends into X-rays.
- Very low gas density, $n_e \approx 10^4\ \mathrm{m}^{-3}$, so emission optically thin. Cluster core radius $r_c \approx 200$ kpc.
- Estimate T_e from location of "knee" in spectrum.
- X-ray flux density $F_X \propto \int n_e^2 T_e^{-1/2} dl$.
- Bolometric (total) X-ray luminosity $L_X \propto \int n_e^2 T_e^{1/2} dl$.
- Cluster gas also gives rise to the Sunyaev–Zel'dovich effect $F_{SZ} \propto \int n_e T_e dl$.
- Can combine SZ and X-ray data to get n_e and the line of sight depth. If assume that line of sight depth is equal to distance across cluster, can then calculate Hubble's constant.

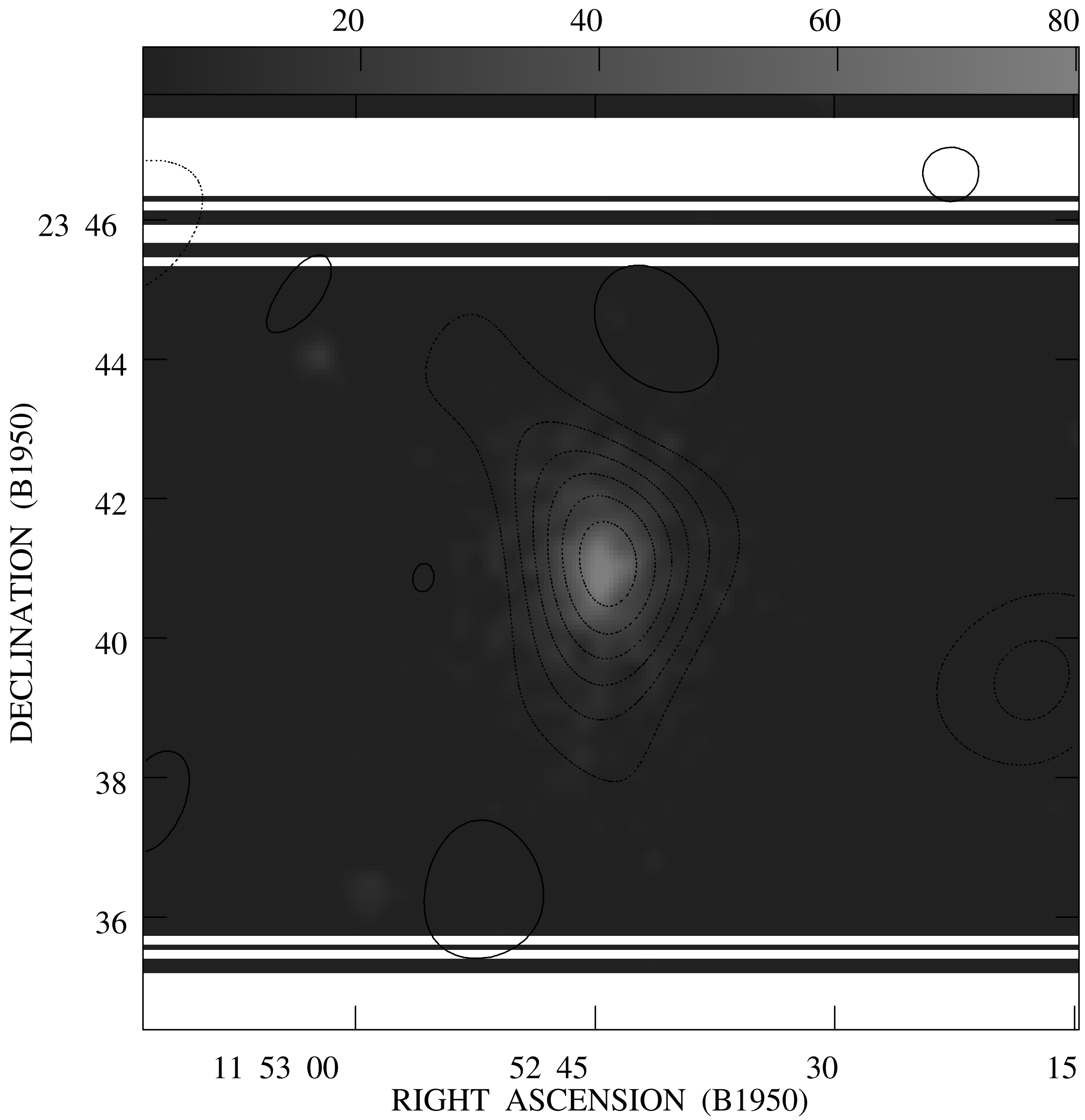


Figure 1: The cluster of galaxies A1413. Greyscale is X-rays from ROSAT PSPC. Contours are the S–Z effect from Ryle Telescope

Example: Ionised winds from stars

- If wind speed constant $\Rightarrow\ n \propto r^{-2}$

$$\bullet\ \tau_\nu = \int \alpha_\nu^{ff} ds$$
$$\propto \int n^2 \nu^{-2} ds$$

assuming T in wind is constant, in R-J region and $\bar{g}_{ff}(\nu) \approx 1$.

- In this case the optical depth τ_ν is a function of distance from the star x. Need to integrate along line of sight y where $r^2 = x^2 + y^2$.

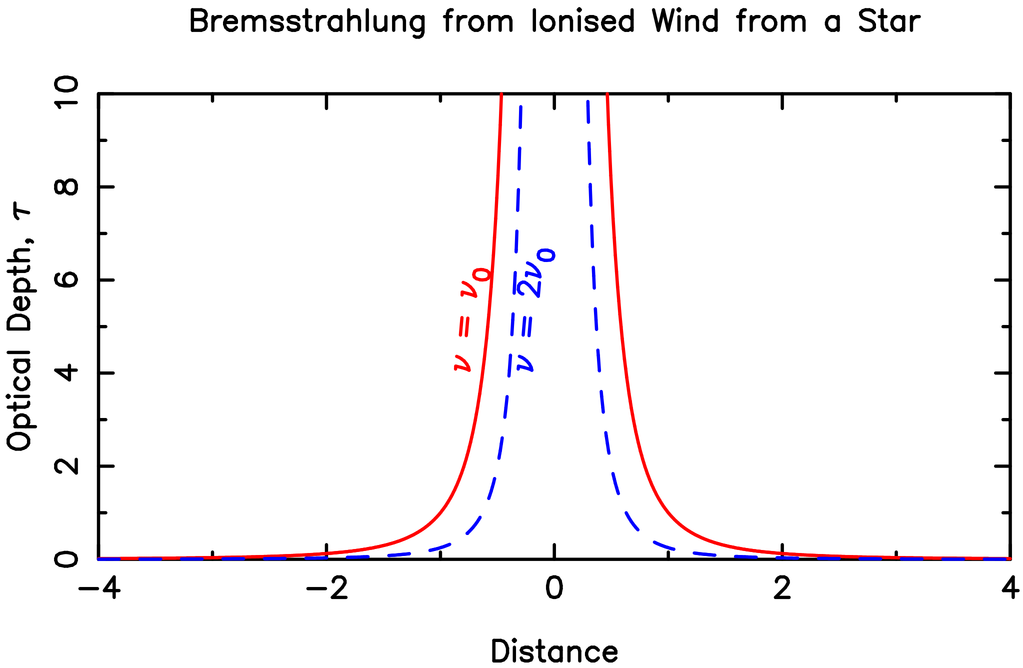
$\Rightarrow \tau_\nu(x) \propto \nu^{-2} x^{-3}$

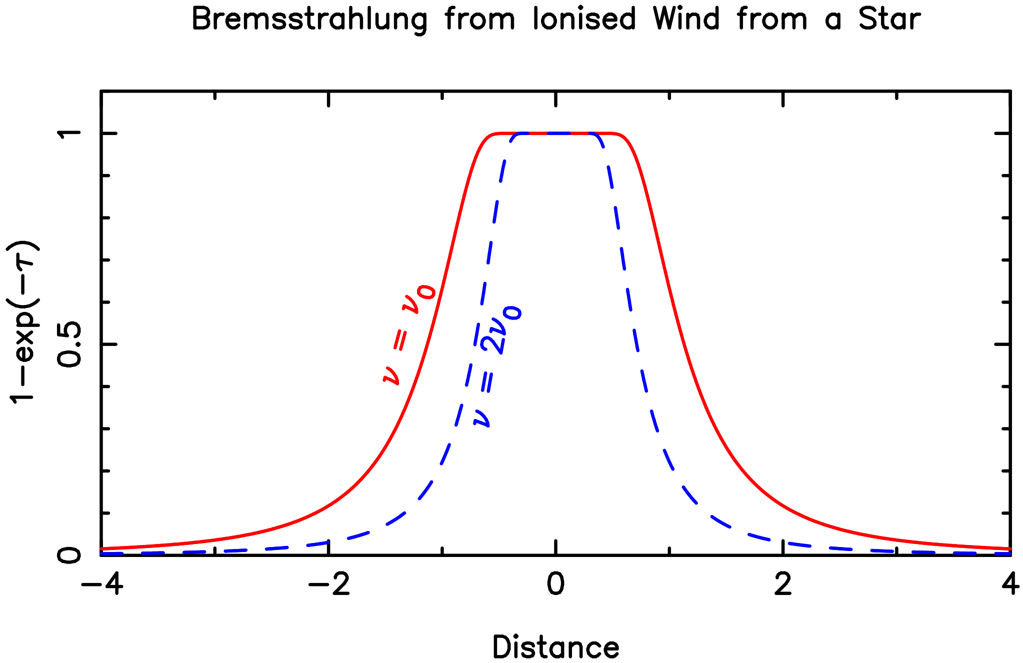
- The flux from the wind $F_\nu \propto I_\nu d\Omega$; $d\Omega = 2\pi x dx$
- $I_\nu = (1 - \exp(-\tau_\nu(x)))B_\nu \approx (1 - \exp(-\tau_\nu(x)))2kT/\lambda^2$.

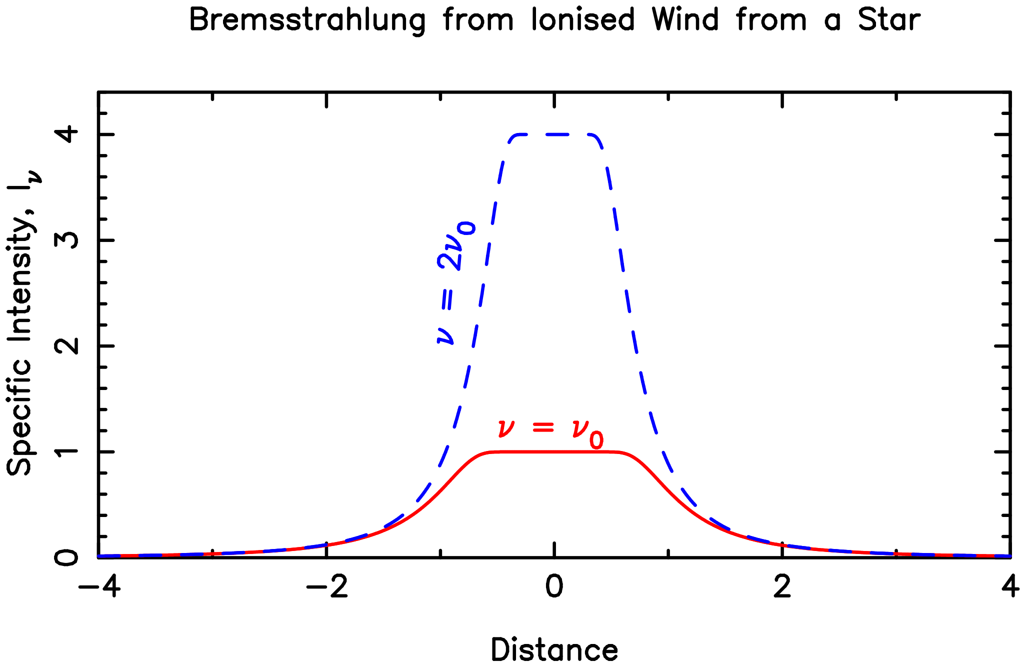
$$\Rightarrow F_\nu \propto \int 2\pi x dx (1 - e^{-\tau_\nu(x)}) 2kT \, \frac{\nu^2}{c^2}$$
$$\propto \nu^{2/3} \int \left(1 - e^{-1/w^3}\right) dw$$

using substitution $w = x\nu^{(2/3)}$

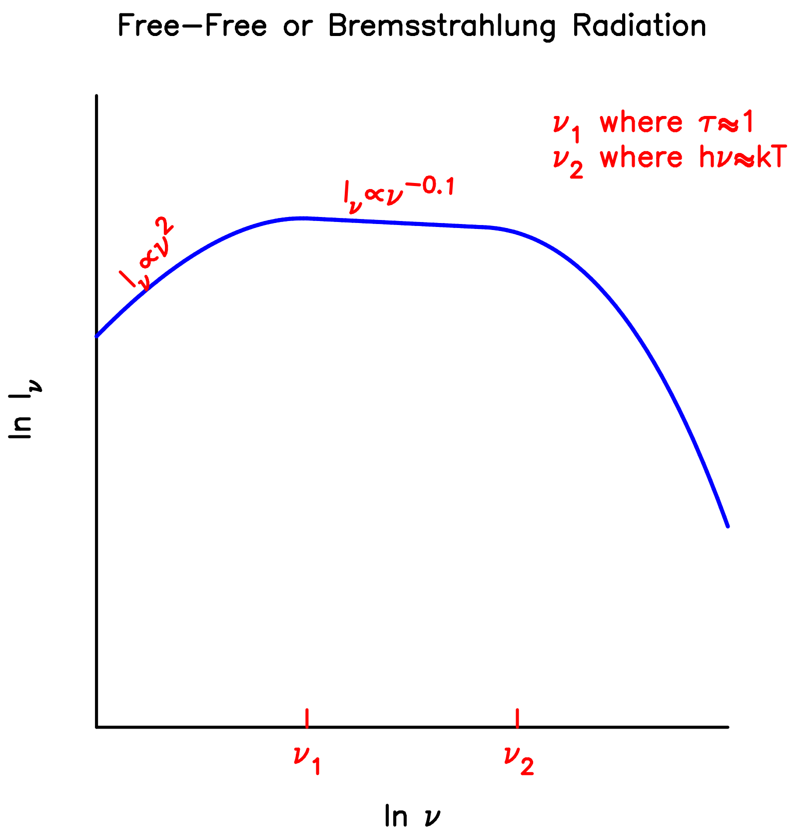
- $\boldsymbol{F_\nu \propto \nu^{2/3}}$
- A full analysis will allow calculation of the mass loss rate.

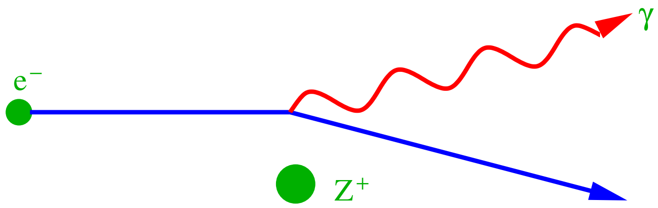

Bremsstrahlung from Ionised Wind from a Star
Optical Depth, τ
ν = ν0
ν = 2ν0
Distance


Bremsstrahlung from Ionised Wind from a Star
1−exp(−τ)
ν = ν0
ν = 2ν0
Distance


Bremsstrahlung from Ionised Wind from a Star
Specific Intensity, Iν
ν = 2ν0
ν = ν0
Distance

Free-Free or Bremsstrahlung Radiation





- Emission as result of collisions between charged particles, usually electrons and ions.
- Emitters in thermal equilibrium
- Unpolarised
- e.g. — HII regions
  — X-ray emission from clusters of galaxies
  — Ionised winds from stars

Appendix: Derivation of Bremsstrahlung Spectrum

Radiation from single accelerating electron

- Larmor's formula gives the power from an electron as a function of acceleration

$$\frac{dW}{dt\,d\Omega} = \frac{e^2\,|\ddot{r}(t)|^2}{16\pi^2\,\varepsilon_0\,c^3}\sin^2\theta$$

Integrating over solid angle gives

$$\frac{dW}{dt} = \frac{e^2\,|\ddot{r}(t)|^2}{6\pi\,\varepsilon_0\,c^3}$$

- If we introduce the fourier transform of $\ddot{r}(t)$

$$\ddot{r}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(i\omega t)\,\ddot{r}(t)\,dt$$

we can write the total energy emitted as $P = \int_0^\infty I_\omega\,d\omega$ where the spectral density is

$$I_\omega = \frac{e^2}{3\pi\varepsilon_0 c^3}|\ddot{r}(\omega)|^2$$

This follows from Parseval's theorem (that $\int|\ddot{r}(t)|^2 dt = \int|\ddot{r}(\omega)|^2 d\omega$) and from the symmetry property that $\ddot{r}(-\omega) = \ddot{r}^*(\omega)$.

- An electron in a harmonic field $E_0\exp(i\omega t)$ undergoes an acceleration

$$\ddot{r}(t) = \frac{-e}{m}E_0\exp(i\omega t)$$

$$\text{Averaging over time} \Rightarrow \frac{\mathrm{dW}}{\mathrm{dt}} = \frac{\mathrm{e}^2}{6\pi\,\varepsilon_0\,\mathrm{c}^3}\frac{\mathrm{e}^2\,\mathrm{E}_0^2}{2\mathrm{m}^2}$$

Now the incident flux in the wave is just $\varepsilon_0\, E_0^2\, c/2$, so writing the radiated power as a cross section so that the power radiated is $\sigma_T\times$ flux, we find

$$\sigma_T = \frac{1}{6\pi}\left(\frac{e^2}{\varepsilon_0 m c^2}\right)^2$$

Collision of single electron at one speed

- Consider an electron travelling at speed $\dot{r}$ colliding with an ion with impact parameter b. Assuming the deviation from a straight line is small:

$$\Delta\dot{r} = \frac{Ze^2}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{b\,dt}{(b^2+\dot{r}^2t^2)^{1.5}} = \frac{2Ze^2}{4\pi\varepsilon_0 m\dot{r}b}$$

- The frequency spectrum is given by

$$\ddot{r}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(i\omega t)\,\ddot{r}(t)\,dt$$

- We define a characteristic interaction time $\tau = b/\dot{r}$, where b is the impact parameter.

$$\omega\tau \ll 1 \Rightarrow \ddot{r}(\omega) \approx (2\pi)^{-0.5}\Delta\dot{r}$$
$$\omega\tau \gg 1 \Rightarrow \ddot{r}(\omega) \approx 0$$

• Thus for frequencies up to some value $\sim \dot{r}/b$, we have a flat frequency spectrum

$$I_\omega = \frac{Z^2\, e^6}{24\,\pi^4\, \varepsilon_0^3\, c^3 m^2 \dot{r}^2 b^2}$$

and most of the energy is emitted at a frequency $\sim \dot{r}/b$.

Collision of many electrons at one speed

• Suppose we have number densities n_e and n_i for the electrons and ions. The collison rate per unit volume between impact parameters b and $b + db$ is then

$$n_e\, n_i\, 2\pi\, b\, db\, \dot{r}$$

and the total emitted power per unit volume per unit (angular) frequency is found by integration to be

$$\epsilon_\omega^{ff} = \frac{n_e\, n_i\, Z^2\, e^6}{12\pi^3 \varepsilon_0^3\, c^3 m^2\, \dot{r}} \log\left(\frac{b_{\max}}{\mathrm{b}_{\min}}\right)$$

• The upper limit is set by the condition that we expect no emission beyond a frequency $\dot{r}/b$, so that $b_{\max} \approx \dot{r}/\omega$.

• There are two limits on the lower limit to b. The straight line assumption breaks down when the particle kinetic energy is smaller than the potential energy for a given b: this gives

$$b_{\min}^C = \frac{Z\, e^2}{2\,\pi\,\varepsilon_0\, m\, \dot{r}^2}$$

The second is the quantum limit set by uncertainty:

$$b_{\min}^{QM} = \frac{h}{m\,\dot{r}}$$

• We define the Gaunt Factor to encode all the uncertainties in the above analysis:

$$g_{ff}(\dot{r},\omega) = \frac{\sqrt{3}}{\pi}\,\log\left(\frac{b_{\max}}{\mathrm{b}_{\min}}\right)$$

$$\Rightarrow \epsilon_{\omega}^{ff} = \frac{n_e\,n_i\,Z^2\,e^6}{12\sqrt{3}\,\pi^3\varepsilon_0^3\,c^3 m^2\,\dot{r}}\,g_{ff}(\dot{r},\omega)$$

Thermal Free-Free emission

• If the particles obey a Maxwellian distribution, we have a probability density $p(\dot{r}) \propto \exp(-m\dot{r}^2/(2kT))$, and we can perform the necessary average to obtain the total specific emissivity:

$$\epsilon_{\nu}^{ff} = A\,T^{-1/2}\,Z^2\,n_e\,n_i\,\exp[-h\nu/(kT)]\,\bar{g}_{ff}(\nu)$$

where

$$A = \frac{32\,\pi\,e^6}{3\,m\,c^3}\left(\frac{2\,\pi}{3\,k\,m}\right)^{1/2}\left(\frac{1}{4\,\pi\,\varepsilon_0}\right)^3$$

• The Gaunt Factor $\bar{g}_{ff}(\nu)$ has now been averaged over the Maxwellian distribution.