FEDERAL UNIVERSITY OF TECHNOLOGY, OWERRI
SCHOOL OF PHYSICAL SCIENCES
DEPARTMENT OF MATHEMATICS
MATHEMATICAL METHODS I (MTH 201)
2017/2018 HARMATTAN SEMESTER EXAMINATION

DATE: 27 / 04 / 18 TIME: $2\frac{1}{2}$ HOURS

INSTRUCTION: ANSWER ANY SEVEN (7) QUESTIONS.

1a. Evaluate $\lim_{(x,y,z)\to(0,0,0)}\left(\frac{\sin(x+y+z)}{x+y+z}\right)$.

1b. Find the domain and range of the following functions:

(a) $f(x,y,z) = \ln\sqrt{x^2+y^2+z^2}$ (b) $g(x,y) = \cos^{-1}(x-y)$

2. Find all the first partial derivativesof the following functions at the indicated points:

(a) $f(x,y) = \ln(x^2+xy^2)$ at (-1,4) (b) $f(x,y,z) = (x+y+z)e^{xyz}$ at (0,1,-1).

3a. Find $\frac{\partial^3 f}{\partial y \partial x^2}$ for $f(x,y) = e^{xy}$.

3b. Eliminate the arbitrary function f from

(i) $z = x^n f\left(\frac{y}{x}\right)$ (ii) $z = y^2 + 2f\left(\frac{1}{x} + \ln y\right)$

4a. Find and classify the critical points of the following function: $f(x,y) = x^3 + y^3 - 6xy$

4b. Prove that $\lim_{(x,y)\to(1,2)} 3x^2 + y^2 = 7$

5a. Find the tangent plane and normal line to the following surface at the indicated point:

$x^2 - y^2 + z^2 + 1 = 0, at\ (1,3,\sqrt{7})$.

5b. Compute $\frac{dz}{dt}$ for the following function: $z(x,y) = xe^{xy}$, $x = t^2$, $y = t^{-1}$

6(a) Eliminate the arbitrary functions f and g if $u = f(3y+x) + xg(3y+x)$

6b. Find the directional derivative of the function, $f(x,y) = \ln(\sqrt{x^2+y^2})$ at (1, 1) in the direction of (2,1).

7a Find the critical points of the function, $f(x,y) = 4xy - x^2 - y^2 + 6x - 2$.

7b. Compute the total differentials for each of the following functions:

(i) $f(x,y) = e^{x^2+y^2}\tan(2x)$ (ii) $f(x,y,z) = \frac{x^3y^6}{z^2}$

8a. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for each of the following function: $x^3z^2 - 5xy^5z = x^2 + y^3$

8b. Evaluate f_x and f_y at $(1,2)$ from first principle if $f(x,y) = 25x^3y^2 + 72x^2y^3$.

9(a) Given the series, $\sum_{n=1}^{\infty}\frac{1}{3^{n-1}}$, find its sequence of partial sums and hence show that the series converges and find its sum.

9b. Find all the second order partial derivatives for $f(x,y) = \cos 2x - x^2e^{5y} + 3y^2$.

Moderated by Dr (Mrs) E.E. Omizha.
17/4/18

FEDERAL UNIVERSITY OT TECHNOLOGY, OWERRI

DEPARTMENT OF MATHEMATICS

HARMATTAN SEMESTER 2018 / 2019 EXAMINATIONS

MTH 201: MATHEMATICAL METHODS 1

TIME: THREE HOURS

INSTRUCTION: ANSWER QUESTION ONE (1) AND ANY OTHER FOUR (4) QUESTIONS

1a) Find the domain and range of the following functions:

(i) $f(x,y) = \frac{\sqrt{x^2+y^2-9}}{\text{[illegible]}}$ (ii) $g(x,y,z) = \frac{x}{\sqrt{16-x^2-y^2-z^2}}$ (iii) $f(x,y) = 3 + 2x^2 - 4y^2$

1bi) Find all the first partial derivatives of the function, $f(x,y,z) = x^2\sin(2y-5z) - y\cos(6zx) - 1$.

(ii) Evaluate the following limits:

(a) $\lim_{(x,y)\to(1,-1)} \frac{x^3+y^3}{x+y}$ (b) $\lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1}$

1c) If $w(x,y) = 3x^3 - 3x^2y + y^3x$, find $\frac{\partial^2 w}{\partial x \partial y}$.

2a) Given the function, $f(x,y)$ defined by

$$f(x,y) = \begin{cases} \dfrac{-3xy}{x^2+y^2} & ; \quad if(x,y) \neq (0,0) \\ 0 & ; \quad if(x,y) = (0,0) \end{cases}$$

(i) Find from first principle the value of $f_x(0,0)$ and $f_y(0,0)$

(ii) Discuss the continuity of the function $f(x,y)$ at the point (0,0)

2b) Prove that $\lim_{(x,y)\to(1,0)} 3x^2 + y^2 + xy = 3$.

3a) Differentiate implicitly to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ given that, $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$

3b) Given that $w(x,y) = x^2 - 2xy + y^2$, where $x = r + \theta$ and $y = r - \theta$; find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ using the appropriate chain rule.

4a) Find the directional derivative of the function, $f(x,y,z) = x^2 + 3xy - y^2 - 2yz + z^2$ at (1,1,2) in the direction of the vector $2i + 6j + 3k$.

4b) Show that the function, $u = x^2 - y^2$ is harmonic.

4c) Eliminate the arbitrary functions in $w(x,y) = f(y+2x) + xg(y+2x)$.

5a) Find and classify all the critical points of the function, $w(x,y) = 4 + x^3 + y^3 - 3xy$.

5b) Find (i) the equation of the tangent plane and (ii) the normal line to the surface $w = \ln(2x+y)$ at (-1, 3).

6a) Given that $w(x,y) = x^2 - xy$, obtain the total differential and show that the total differential is exact.

6b) Given that $\sin xy + e^{xy} + x^2 + y = 1$, find $\frac{dy}{dx}$.

6c) Find the gradient of the function $f(x,y,z) = \sin(xz) + \ln(xy) + \cos(xyz)$, at the point (1, 1, 1).