Catalog

DEPT OF MATHS FUNAAB 2021 MTS 105 TUTORIAL QUESTIONS1

1. (a) By searching through the internet and reading through the Chinese, Babylonian, Greece and Arabian history of Mathematics, write briefly and concisely on the evolution of the following number systems:
   i. $\mathbb{N}$ of natural numbers
   ii. $\mathbb{Z}$ of integers
   iii. $\mathbb{Q}$ of rational numbers
   iv. $\mathbb{R}$ of real numbers
   v. $\mathbb{C}$ of complex numbers

   (b) Search the internet for the name of the cardinality of the set $\mathbb{N}$?

   (c) Search the internet for the types of sets we have in the literature and their real life applications.

2. Determine with proof which of the following statements are true and which are false:
   (a) $\mathbb{Z}^+ \subseteq \mathbb{Q}^+$.
   (b) $\mathbb{Z}^+ \subseteq \mathbb{Q}$.
   (c) $\mathbb{Q}^+ \subseteq \mathbb{R}$.
   (d) $\mathbb{R}^+ \subseteq \mathbb{Q}$.
   (e) $\mathbb{Q}^+ \cap \mathbb{R}^+ = \mathbb{Q}^+$.
   (f) $\mathbb{Z}^+ \cup \mathbb{R}^+ = \mathbb{R}^+$.
   (g) $\mathbb{R}^+ \cap \mathbb{C} = \mathbb{R}^+$.
   (h) $\mathbb{C} \cup \mathbb{R} = \mathbb{R}$.
   (i) $\mathbb{Q}^* \cap \mathbb{Z} = \mathbb{Z}$.
   (j) $\mathbb{Z} \cup \mathbb{Q} = \mathbb{Z}$.

3. Let A, B and C be nonempty subsets of the reference set X. Use Venn diagrams only to investigate the truth or falsity of each of the following:
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
   (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
   (c) $(A - B) - C = (A - C) - (B - C)$.
   (d) $A - (B \cup C) = (A - B) \cap (A - C)$.
   (e) $A \triangle (B \cap C) = (A \triangle B) \cap (A\ trC)$.
   (f) $A \cap (B \triangle C) = (A \cap B) \triangle (A \triangle C)$.
   (g) $A \triangle (B \cup C) = (A \triangle B) \cup (A \triangle C)$.
   (h) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$.

   Note that
   $$A \triangle B \equiv (A - B) \cup (B - A)$$
   $\triangle$ is called the symmetric difference.

4. For $A = \{1, 2, 3, 4, 5, 6, 7\}$, compute the number of:
   (a) Subsets of A.
   (b) Nonempty subsets of A
   (c) Proper subsets of A
   (d) Nonempty proper subsets of A.
   (e) Subsets of A containing three elements.
   (f) Subsets of A containing 1,2.
   (g) Proper subsets of A containing 1,2.
   (h) Subsets of A with an even number of elements.
   (i) Subsets of A with an odd number of elements.
   (j) Subsets of A with an odd number of elements, including the element 3.

5. (a) If a set A has 63 proper subsets, what is $\mid A \mid$?
   (b) If a set B has 64 subsets of odd cardinality, what is $\mid B \mid$?
   (c) Let $X = \{1, 2, 3, \ldots, 29, 30\}$. How many subsets A of X satisfy :
      i. $\mid A \mid = 5$?
      ii. $\mid A \mid = 5$ and the smallest element in A is 5 ?
      iii. $\mid A \mid = 5$ and the smallest element in A is less than 5 ?

6. Let A, B, C, D, E be subsets of $\mathbb{Z}$ defined as follows:

$$A = \{2n : n \in \mathbb{Z}\}, B = \{3n : n \in \mathbb{Z}\}, C = \{4n : n \in \mathbb{Z}\}, D = \{6n : n \in \mathbb{Z}\}, E = \{8n : n \in \mathbb{Z}\}.$$

   (a) Which of the following are true and which are false ?
      i. $E \subseteq C \subseteq A$
      ii. $A \subseteq C \subseteq E$
      iii. $B \subseteq D$
      iv. $D \subseteq B$
      v. $D \subseteq A$
      vi. $D^c \subseteq A^c$
   (b) Determine each of the following sets:
      i. $C \cap E$.
      ii. $B \cup D$.
      iii. $A \cap B$.
      iv. $B \cap D$.
      v. A^c.
      vi. $A \cap E$.
      vii. $B^c \cap E^c$.
      viii. $C^c \cup D^c$.

      Note that A^c is the compliment of A.

DEPT OF MATHS FUNAAB 2021 MTS 105 TUTORIAL QUESTIONS2

1. Let A, B, C, D be nonempty subsets of a reference set X. Show that:

   (a) $(X - A) \cup (X - B) = X - (A \cap B)$.

   (b) $(X - A) \cap (X - B) = X - (A \cup B)$.

   (c) $(((A \cup B) \cap C)^c \cup B^c))^c = B \cap C$.

   (d) $(A \triangle B)^c = A \triangle B^c = A^c \triangle B$.

   (e) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$.

   (f) $(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap D^c))] = B \cap (A \cup C)$.

   (g) $[(A \cap B) \cup (A^c \cap C)]^c = (A \cap B^c) \cup (A^c \cap C^c)$.

   (h) $A \times (B - C) = (A \times B) - (A \times C)$.

   (i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

2. (a) Let $I = \mathbb{Z}^+$ be an index set and for each $n \in I$, consider the set $A_n = \{1, 2, 3, \ldots, n - 1, n\}$. Determine the following:

      i. $\bigcup_{n=1}^{7} A_n$.

      ii. $\bigcap_{n=1}^{11} A_n$.

   (b) Let $A = \{1, 2, 4, 8, 16\}$ and let $B = \{1, 2, 3, 4, 5, 6, 7\}$. Determine $A \times B$. If $(2 - x, 5), (4, y - 2) \in A \times B$, find the values of x and y for $(2 - x, 5) = (4, y - 2)$.

   (c) Let A, B, C, D be nonempty sets. Show that:

      i. $A \times B = B \times A$ if and only if $A = B$.

      ii. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

   (d) Let $X = \{1, 2, 3, 4, 5, 6, 7\}$ be a reference set and let $A = \{1, 3, 5, 7\}$, $B = \{2, 4, 6\}$, $C = \{3, 6\}$ be subsets of X. Find the following:

      i. $A \times B$.

      ii. $B \times C$.

      iii. $A \times B \times C$.

      iv. $A \times A \times C$.

      v. $C \times C \times C$.

3. In a survey of 120 passengers, an airline found that 48 enjoyed wine their meals, 78 enjoyed mixed drinks, and 66 iced tea. In addition, 36 enjoyed any given pair of these beverages and 24 passengers enjoyed them all. If two passengers are selected at random from the survey sample of 120, what is the probability that:

   (a) they both want only iced tea with their meals?

   (b) they both enjoy exactly two of the three beverage offerings?

4. A professor has two dozen introductory textbooks on mathematics and is concerned about their coverage of the topics (A) algebra, (B) analysis, and (C) calculus. The following data are the numbers of books that contain materials on these topics:

$$| A |= 8, | B |= 13, | C |= 13, | A \cap B |= 5, | A \cap C |= 3, | B \cap C |= 6, | A \cap B \cap C |= 2.$$

   (a) How many of the textbooks include material on exactly one of these topics ?

   (b) How many have no material on algebra ?

5. Professor Agboola gave his MTS 708 class a test consisting of three questions. There are 21 students in his class, and every student answered at least one question. Five students did not answer the first question, seven failed to answer the second question, and six did not answer the third question. If nine students answered all three questions, how many answered exactly one question ?m

6. At an undergraduate science competition, 34 students received awards for scientific projects. 14 awards were given for projects in biology, 13 in chemistry, and 21 in physics. If 3 students received awards in all three subject areas, how many received awards for exactly:

   (a) one subject area ?

   (b) two subject areas ?

7. (a) Let $A = \{n : n \in \mathbb{Z}^+, 1 \leq n \leq 100\}$. if $B \subseteq A$, where no element in B is three times another element in B, what is the maximum value possible for $| B |$?

   (b) Let X be a given reference set with $A, B \subseteq X, A \cap B = \emptyset, | A |= 12$ and $| B |= 10$. If seven elements are selected from $A \cup B$, what is the probability the selection contains four elements from A and three from B ?

DEPT OF MATHS FUNAAB 2021 MTS 105 TUTORIAL QUESTIONS0

1. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 4 & -5 \\ 5 & 6 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 3 & -2 \\ -3 & 4 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 & -2 \\ -1 & 3 & -5 \\ 8 & -7 & -8 \end{bmatrix}$ be given matrices. Use the matrices to show that:

(a) $A(BC) = (AB)C$.

(b) $A(B + C) = AB + AC$.

(c) $B(A - C) = BA - BC$.

(d) $k(ABC) = A(kB)C$, where $k \in \mathbb{R}$.

2. Given that $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & 1 \end{bmatrix}$, find the values of k if $k^3A + k^2B + kC = D$.

3. (a)If $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 & 4 & 7 \\ -6 & 1 & -3 & 0 \end{bmatrix}$, find the product AB. Is the product BA possible ? Explain.

(b) Let $A = \begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 & 16 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 & 2 \\ -1 & 3 & 4 \\ -1 & 2 & 3 \end{bmatrix}$ and $D = \frac{1}{4}\begin{bmatrix} 2 & 5 & 4 \\ 2 & 3 & -4 \\ 1 & 2 & -3 \end{bmatrix}$ be given matrices.

(i) Compute the determinants of A, B, C and D.

(ii) Compute the adjoints of A, B, C and D.

(iii) Compute the inverses of A, B, C and D.

(iv) Show that: $(AB)^{-1} = B^{-1}A^{-1}$ and $(BCD)^{-1} = D^{-1}C^{-1}B^{-1}$.

4. (a) Let $A = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$. Show that $A^2 - 2A + I = O$ where I is a 2×2 unit matrix. Hence find the inverse of A and solve the system of equations: $x + 2y = -1, 2x + 3y = 2$.

(b) Find the values of k given that $\begin{vmatrix} k & -1 & -8 \\ 1 & 1 & -1 \\ 3 & k & -5 \end{vmatrix} = 0$.

(c) Solve for x given that

$$\begin{vmatrix} \frac{1}{3} - x & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} - x & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} - x \end{vmatrix} = 0$$

5. Let A be a matrix given by

$$\begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}.$$

(a) Show that

$$A^3 - A^2 + A - 83I = O,$$

where I and O are 3×3 unit and zero matrices respectively.

(b) Use your result in (a) to show that

$$A^{-1} = \frac{1}{83}\left[A^2 - A + I\right].$$

(c) Using your result in (b) only, compute the inverse of A and hence solve the system of linear equations

$$\begin{aligned} 2x - 2y + 7 &= 5 \\ 3x - 4y - 2z &= 5 \\ 5x + 7y + 3z &= 1. \end{aligned}$$

(d) Use Crammer's Rule to confirm your solution to the linear system in (c).

6. (a) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ be a given matrix. Compute the inverse of A and hence solve the system of linear equations

$$\begin{aligned} x + 2y - z &= -2 \\ 2x + y + 3z &= 16 \\ 3x + y + 2z &= 14. \end{aligned}$$

(b) Solve for x given that $\begin{vmatrix} -1 & 3 & x \\ 2x-3 & 1-x & 3x+1 \\ 2 & x & -2 \end{vmatrix} = 9x - 28.$

(c) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, show that $A^2 - 4A - I = O$ and then compute A^{-1}. Hence solve the system of linear equations $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} \\ \frac{3}{4} \end{bmatrix}$.

7. (a) Given that $\begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ x^2 \end{bmatrix} = \begin{bmatrix} 3 \\ k \end{bmatrix}$, find the value(s) of k.

(b) Let A be any 2×2 square matrix. Show that AA^T is symmetric where A^T is the transpose of A. Is $A^T A$ also symmetric ?

(c) Find the values of k given that $\begin{vmatrix} k & 3+k & -10 \\ 1-k & 2-k & 5 \\ 2 & 4+k & -k \end{vmatrix} = 48.$

8. Let $A = \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$ be a given matrix.

(a) Show that $A^3 - 5A^2 - A + I = O$, where I is the 3×3 unit matrix.

(b) By pre/post multiply the equation $A^3 - 5A^2 - A + I = O$ by A^{-1}, the inverse of A, show that $A^{-1} = I + 5A - A^2$ and hence compute the inverse of A.

(c) Using the inverse of A obtained in (b), solve the system of linear equations given by

$$\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}.$$

Name :- ADEAGBO - MICHAEL - OLUMIDE.

COPLANT

Dept :- PLANT · BREEDING · AND · SEED TECHNOLOGY · [PBST].

Matric No :- [20193716]

SOLUTIONS TO THE DEPT OF MATHS FUNAAB

2021 MTS 105 TUTORIAL QUESTIONS 1.

Lecturer :- PROFESSOR · ADESINA AGBOOLA.

1ai N → This refers to a set of natural numbers they are presumed to have started before recorded history when humans began to count things. The Babylonians developed a place-value system based on the numerals for 1 (one) and 10 (ten). The ancient Egyptians added to this system to include all the powers of 10 up to one million. Natural numbers were first studied seriously by such greek philosophers and mathematicians as Pythogeras (582-500BC) and Archimedes (287-212BC).
They include; number from 1 to positive infinity $(+\infty)$ $\{1, 2, 3 - - -, \infty\}$

aii Z of Integers.

This refers to set of integer. These are numbers that can be represented on the number line i.e $\{-\infty - - -, -2, -1, 0, 1, 2, 3, - - - +\infty\}$.

aiii Q of Rational numbers.

These are set of rational numbers. These refers to set of fractions.
$Q = \{x : x = a/b,\ a, b \in Z,\ b \neq 0\}$.

aiv R of Real numbers.

These are set of real numbers from negative to positive. It is combination of rational and irrational numbers.

$$R = \{x \mid -\infty \leq x \leq \infty\}.$$

av C of Complex numbers.

These are set of complex numbers and are written in the form $a+bi$ where $i = \sqrt{-1}$. therefore $C = \{x : a+ib,\ a, b \in Z\}$.

1b The Cardinality of the set N [Natural number] is referred to as INFINITE QUANTITY $\langle\infty\rangle$.

1c Types of sets in literature and their real life applications.

i. Universe.

There are millions of galaxies present in our world which are seperated from each other by some distance. Here the universe act as a set.

ii. Playlist

A playlist has different kinds of songs present in our smartphones or computers. Rock songs are often separated from classical or any other genre. Hence, playlists also form the example of sets.

iii Rules.

Every school or company have different sets of rules which have to be followed by every student and employee. there are disciplinary rules, rules for leave, timing rules, hostel rules, and many others. Hence, all different types of rules are separated from others.

iv Representative House.

Representative houses are examples of sets. Here the people belonging to various departments have to sit seperately from other departments. For example, the legal department and finance department don't sit intermixed with each other. It has the lower house and upper house called senate, where only senior members sit whereas the juniors sit in the lower house.

v Shopping Malls.

When we go shopping in a mall, we all have noticed that there are separate portions for each kind of things. For instance, clothing shops are on another floor whereas the food court is at another part of the mall.

Question 2

(a) $Z^+ \subseteq Q^+$.

$Z^+ = \{$All set of positive integers$\}$

$Q^+ = \{$Set of all positive fractions$\}$

Therefore $Z^+ \subseteq Q^+$ because all positive integers are positive fractions.

(b) $Z^+ \subseteq Q$.

$Z^+ = \{$All set of positive Integers$\}$, $Q = \{$All set of fractions$\}$

Therefore $Z^+ \subseteq Q$ because all positive integers are subset of fractions.

c. $Q^+ \subseteq R$.

Q = {All set of rational number i.e fractions}

R = {Real numbers}.

$Q \not\subseteq R$ because not all element in R are in Q.

d. $R^+ \subseteq Q$.

R = {Set of real numbers that are positive}.

Q = {Set of rational number}.

$R^+ \not\subseteq Q$

e. $Q^+ \cap R^+ = Q^+$

Q = {All set of positive fractions},

R = {Set of positive fractions}.

Therefore $Q^+ \cap R^+ = Q^+$ because all positive fractions are positive real numbers.

f. $Z^+ \cup R^+ = R^+$

Z^+ = {All positive integers}.

R^+ = {All positive real numbers}.

Therefore $Z^+ \cup R^+ = R^+$ because all positive integers are contained in Positive.

g. $R^+ \cap C = R^+$

R^+ = {Positive real numbers}.

C = {Complex numbers}.

Therefore, $R^+ \cap C = R^+$ because C is a superset of R^+.

h. $C \cup R = R$.

C = {set of Complex numbers}

R = {All real numbers from negative and positive}.

Therefore $C \cup R \neq R$

i $Q^+ \cap Z = Z$

Q^+ = {set of positive fractions}

Z = {set of positive and negative Integers}

Therefore $Q^+ \cap Z \neq Z$

j) $Z \cup Q = Z$

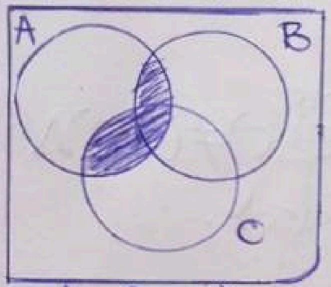
$Z = \{$Set of positive and negative integers$\}$

$Q = \{$Set of fractions; $Q = \frac{a}{b} : a, b \in Z\}$

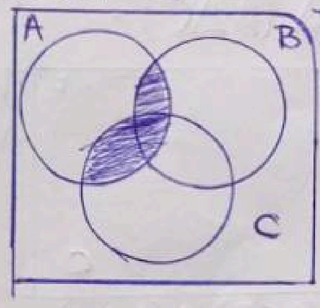
Therefore $Z \cup Q = Z$ because Z is a super set of Q.

Question 3

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.



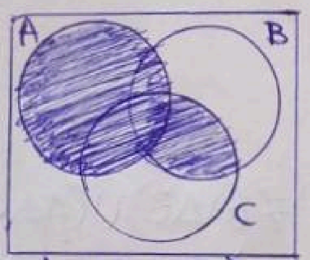
$A \cap (B \cup C)$



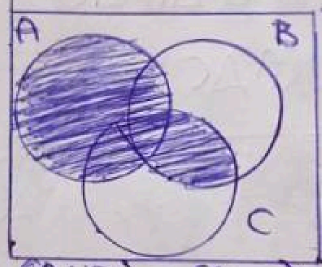
Therefore,

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b)



$A \cup (B \cap C)$

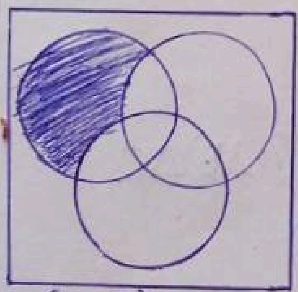


$(A \cup B) \cap (A \cup C)$

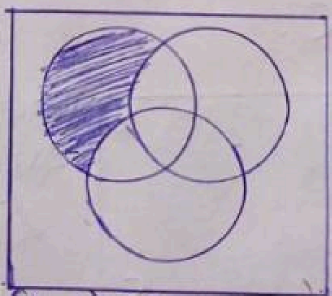
Therefore,

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$

(c) $(A-B)-C = (A-C)-(B-C)$.



$(A-B)-C$



$(A-C) \div (B-C) = (A-B)$

Therefore,

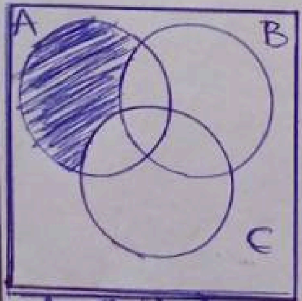
$\therefore (A-B)-C$

$= (A-C) \div (B-C)$

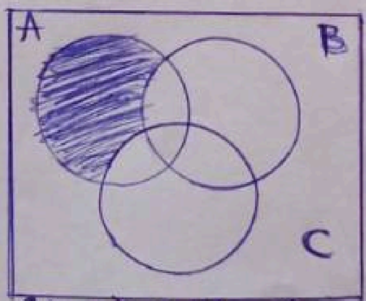
<True.>.

(d) $A-(B \cup C) = (A-B) \cap (A-C)$.

$= A-B$



$A-(B \cup C)$



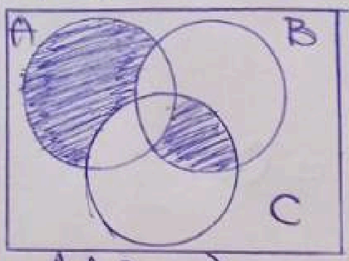
$(A-B) \cap (A-C)$

Therefore

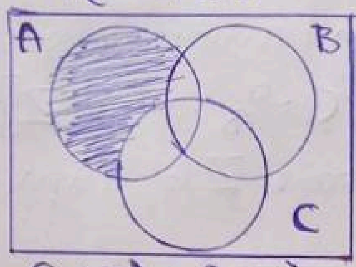
$\therefore A-(B \cup C) = (A-B) \cap (A-C)$

<True>.

(e) $A\triangle(B\cap C)=(A\triangle B)\cap(A\triangle C)$.



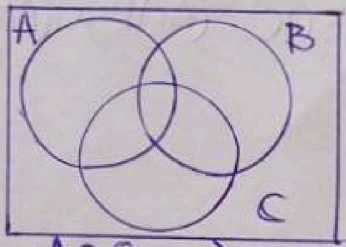
$A\triangle(B\cap C)$



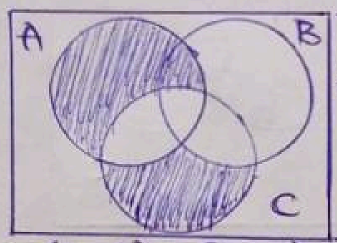
$(A\triangle B)\cap(A\triangle C)$

Therefore,
$A\triangle(B\cap C)\neq(A\triangle B)\cap(A\triangle C)$
$\langle$False$\rangle$.

(f) $A\cap(B\triangle C)=(A\cap B)\triangle(A\triangle C)$.



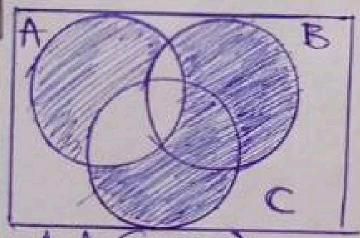
$A\cap(B\triangle C)$



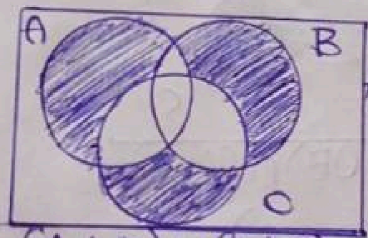
$(A\cap B)\triangle(A\triangle C)$

Therefore;
$A\cap(B\triangle C)\neq(A\cap B)\triangle(A\triangle C)$
$\langle$False$\rangle$.

(g) $A\triangle(B\cup C)=(A\triangle B)\cup(A\triangle C)$.



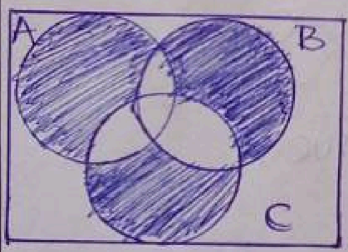
$A\triangle(B\cup C)$

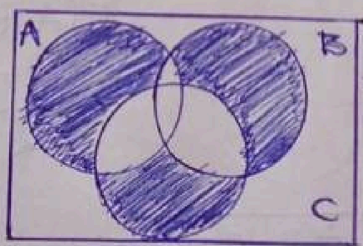


$(A\triangle B)\cup(A\triangle C)$

Therefore;
$A\triangle(B\cup C)\neq(A\triangle B)\cup(A\triangle C)$
$\langle$False$\rangle$.

h $A\triangle(B\triangle C)=(A\triangle B)\triangle C$.





Therefore
$A\triangle(B\triangle C)=(A\triangle B)\triangle C$
$\langle$True$\rangle$.

Question 4.

4 $A=\{1,2,3,4,5,6,7\}$.

4a Subset of $A = 2^n$
$= 2^7$
$= \underline{128}$

4b Non-empty subsets of A $= 2^n - 1$
$= 2^7 - 1$
$= 128 - 1$
$= \underline{127}.$

4c Proper subsets of A $= 2^n - 1$
$= 128 - 1$
$= \underline{127}.$

4d Non-empty proper subsets of A. $= 2^n - 2$
$= 2^7 - 2$
$= 128 - 2$
$= \underline{126}.$

4e Subsets of A containing three elements. $= {}^7C_3$
$= \frac{7!}{(7-3)!3!}$
$= \underline{35}$

4f Subsets of A containing $\{1, 2\}$
$n = 2$
$\therefore 2^n = 4$
$= \underline{4}.$

4g Proper subsets of A containing $\{1, 2\}$ $= 2^n - 2$
$= 2^2 - 2 = 4 - 2$
$= \underline{2}.$

4h Subsets of A with an even number of elements.
$= {}^7C_2 + {}^7C_4 + {}^7C_6$
$= 21 + 35 + 7$
$= \underline{63}$

4i Subsets of A with an odd number of elements.
$= {}^7C_1 + {}^7C_3 + {}^7C_5 + {}^7C_7$
$= 7 + 35 + 21 + 1$
$= \underline{64}$

4j Subsets of A with an odd number of elements, including the element 3.

5 Question 5

5a Proper subset of A = 63

$2^n - 1 = 63$

$2^n = 63 + 1$

$2^n = 64$

$2^n = 2^6$

$n = \underline{6}.$

5b $2^n = 64$

$2^n = 2^6$

$n = 6$

$n = 6 + 1$

$n = \underline{7}$

5c $X = \{1, 2, 3, \ldots\ldots, 29, 30\}$

(i) $\{A\} = 5?$

$\{A\}$ $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

$= 32$

(ii) $|A|$ is less than 5

$\{1, 2, 3, 4\}$

$\{4, 3, 2, 1, 0\}$

$|A| = \{4, 3, 2, 1\}$

(iii) Subset are less than 5

$= 2^4, 2^3, 2^2, 2^1$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$2^3 = 2 \times 2 \times 2 = 8$

$2^2 = 2 \times 2 = 4$

$2^1 = 2$

$\therefore = \{2, 4, 8, 16\}.$

Question 6

$A = \{2n: 2, 4, 6, 8, 10, \ldots\}$ $B = \{3n: 3, 6, 9, 12, 15, \ldots\}$,

$C = \{4n: 4, 8, 16, 20, \ldots\}$. $D = \{6n: 6, 12, 18, 24, 30, \ldots\}$.

$E = \{8n: 8, 16, 24, 32, 40, \ldots\}$.

Note: $\subseteq$ - Subset i.e the two or three sets are equal.

2ci $A\times B = B\times A$ if and only if $A=B$.

i $A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$

$B\times A=\{(3,1),(3,2),(4,1),(4,2)\}$

$\therefore A\times B = B\times A \Leftrightarrow A=B$.

ii $(A\times B)\cap(C\times D)=\{(1,3),(1,4),(2,3),(2,4)\}\cap\{(5,7),(5,8),(6,7),(6,8)\}$
$\Rightarrow \emptyset$

$(A\cap C)\times(B\cap D)=\{\emptyset,\emptyset\}\Rightarrow\emptyset$

Since L.H.S = R.H.S, ∴ It is proved.

di $A\times B=\{(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6),(7,2),(7,4),(7,6)\}$

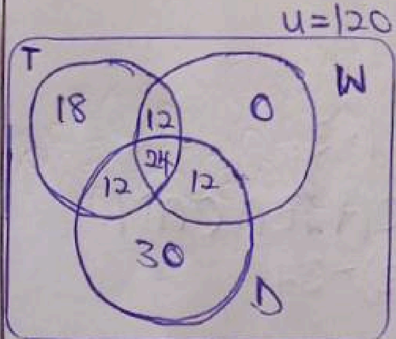
ii $B\times C=\{(2,3),(2,6),(4,3),(4,6),(6,3),(6,6)\}$

iii $A\times B\times C=\{(1,2,3),(1,2,6),(1,4,3),(1,4,6),(1,6,3),(1,6,6),(3,2,3),(3,2,6),(3,4,3),(3,4,6),(3,6,3),(3,6,6),(5,2,3),(5,2,6),(5,4,3),(5,4,6),(5,6,3),(5,6,6),(7,2,3),(7,2,6),(7,4,3),(7,4,6),(7,6,3),(7,6,6)\}$.

iv $A\times A\times C=\{(1,1,3),(1,1,6),(1,3,3),(1,3,6),(1,5,3),(1,5,6),(1,7,3),(1,7,6),(3,1,3),(3,1,6),(3,3,3),(3,3,6),(3,5,3),(3,5,6),(3,7,6),(5,1,3),(5,3,6),(5,5,3),(5,5,6),(5,7,3),(5,7,6),(7,1,3),(7,1,6),(7,3,3),(7,3,6),(7,5,3),(7,5,6),(7,7,3),(7,7,6)\}$.

v $C\times C\times C=\{(3,3,3),(3,3,6),(3,6,3),(6,3,3),(6,3,6),(6,6,3),(6,6,6)\}$.

3a



$n(W)=48, n(D)=78, n(T)=66, n(W\cap D\cap T)=24,$
$n(W\cap D)=36, n(W\cap D)=36, n(W\cap T)=36, n(D\cap T)=36$
$n(W\cap D\cap T')=36-24\Rightarrow 12$
$n(W\cap T\cap W')=36-24\Rightarrow 12$
$n(W\cap D'\cap T')=48-(24+12+12)$
$=0$.

$n(D\cap W'\cap T')=78-(12+24+12)$
$=78-48$
$=30$

$n(T\cap D'\cap W')=66-(12+12+24)$
$=66-48$

b $Pr(\text{only Tea})=18$
$=\frac{18}{120}\times\frac{17}{119}=3/140$

c $Pr(\text{Exactly any two of the item})=12/120+12/120+12/120\Rightarrow 3/10$

6ai $E \subseteq C \subseteq A$

Since $E = \{\text{Multiples of } 8\}$.

$C = \{\text{Multiples of } 4\}$

$A = \{\text{Multiples of } 2\}$.

$E \nsubseteq C \nsubseteq A$

6aii $A \subseteq C \subseteq E$

from 6ai, $A \nsubseteq C \nsubseteq E$

6aiii $B \subseteq D$

$B = \{\text{Multiples of } 3\}$

$D = \{\text{Multiples of } 6\}$

$B \nsubseteq D$

6aiv $D \subseteq B$

from 6aiii $D \nsubseteq B$

6v $D \subseteq A$

$D = \{\text{Multiples Of } 6\}$

$A = \{\text{Multiples of } 2\}$

$D \nsubseteq A$

6vi $D^c \subseteq A^c$

$D^c = \{\text{Multiples of } 2, 3, 4, \text{ and } 8\}$.

$A^c = \{\text{Multiples of } 3, 4, 6 \text{ and } 8\}$.

$D^c \nsubseteq A^c$

6b(i) $C \cap E = \{8, 16, 24, 32, 40, \ldots\} \equiv \{E\}$.

(ii) $B \cup D = \{3, 6, 9, 12, 15, 18, 24, \ldots\} \equiv \{\text{Multiples of } 3 \text{ and } 6\}$

(iii) $A \cap B = \{6, 12, 18, 24, 30, \ldots\} \equiv \{\text{Multiples of } 6\}$.

(iv) $B \cap D = \{6, 12, 18, 24, 30, \ldots\} \equiv \{\text{Mutiples of } 6\}$.

(v) $A^c = \{3, 5, 7, 9, 11, 13, \ldots\} \equiv \{\text{Set of odd numbers from } 3\}$.

(vi) $A \cap E = \{8, 16, 24, 32, 40, \ldots\} \equiv \{\text{Multiples of } 8\}$

(vii) $B^c \cap E^c \Rightarrow B^c = \{2, 4, 5, 7, 8, 10, 11, \ldots\}$

$E^c = \{9, 10, 11, 12, 13, \ldots\}$.

$B^c \cap E^c = \{10, 11, \ldots\}$.

viii $C^c \cup D^c \Rightarrow C^c = \{\text{Multiples of } 3\} \equiv \{2,3,5,6,7,\ldots\}$

$D^c = \{2,4,6,7,8,9,\ldots\}$

$\therefore C^c \cup D^c = \{2,3,4,5,6,7,8,\ldots\}$.

Name: Adeagbo ~ Michael ~ Olumide.

College: COPLANT

Department: Plant Breeding And Seed Technology {PBST}

Matric NO: [20193716].

Solutions To The Dept Of Maths Funaab 2021 MTS105 Tutorial Questions2.

Lecturer: Professor Adesina Agboola.

Part 2

Question 1

1a $(X-A) \cup (X-B) = X-(A \cap B)$.

Let $x \in (x-A) \cup (x-B)$

$\Leftrightarrow x \in (X-A)$ or $x \in (x-B)$.

$\Leftrightarrow x \in X$ and $x \notin A$ or $x \in X$ and $x \notin B$

$\Leftrightarrow x \in X$ and $x \notin A$ and $x \notin B$

$\Leftrightarrow x \in X$ and $x \notin (A \cap B)$

$\Leftrightarrow x \in X-(A \cap B)$.

(b) $(x-A) \cap (X-B) = X-(A \cup B)$

Let $x \in (x-A) \cap (x-B)$

$\Leftrightarrow x \in (X-A)$ and $x \in (x-B)$

$\Leftrightarrow x \in X$ and $x \notin A$ and $x \notin X$ and $x \notin B$

$\Leftrightarrow x \in X$ and $x \notin A$ or $x \notin B$

$\Leftrightarrow x \in X$ and $x \notin (A \cup B)$

$\Leftrightarrow x \in X-(A \cup B)$.

1c

1f) $(A\cap B)\cup[B\cap((C\cap D)\cup(C\cap D^c))]$
$= B\cap(A\cup C)$
$\Rightarrow (A\cap B)\cup[B\cap(C\cap(D\cup D^c))]$
$\Rightarrow (A\cap B)\cup[B\cap(C\cap U)]$
$\Rightarrow (A\cap B)\cup(B\cap C)$
$\Rightarrow \underline{B\cap(A\cup C)}$

1d $(A\Delta B)^c = A\Delta B^c = A^c\Delta B$.

$x\in(A\Delta B)^c$
$x\notin(A\Delta B)$
$x\notin(A-B)\cup(B-A)$
$x\notin A$ and $x\in B$ or $x\notin B$ and $x\in A$
$x\in A$ and $x\in B$ or $x\notin A$ and $x\notin B$
$x\in A$ and $x\in B$ or $x\in A^c$ and $x\in B^c$
$x\in(A-B^c)$ or $x\in(B^c-A)$
$\underline{x\in(A\Delta B^c)}$

1e $A\Delta(B\Delta C) = (A\Delta B)\Delta C$.

1g $[(A\cap B)\cup(A^c\cap C)]^c = (A\cap B^c)\cup(A^c\cap C^c)$.

$\Rightarrow (A\cap B)^c \cap (A^c\cap C)^c$

$\Rightarrow (A^c\cup B^c)\cap(A\cup C^c)$

$\Rightarrow (A^c\cap C^c)\cup((B^c\cap A)$

$\Rightarrow \underline{(A\cap B^c)\cup(A^c\cap C^c)}$

1h $A\times(B-C) = (A\times B)-(A\times C)$.

from the left hand side

$\Rightarrow A\times(B\cap C^c)$

$\Rightarrow (A\times B)\cap(A\times C^c)$

$\Rightarrow \underline{(A\times B)-(A\times C^c)}$

1i

* Question 2

2ai $\bigcup_{n=1}^{7} A_n = A_1\cup A_2 A_3\cup A_4\cup A_5\cup A_6\cup A_7 \Rightarrow \bigcup_{n=1}^{7} = \underline{A_7}$

(ii) $\bigcap_{n=1}^{11} A_n = A_1\cap A_2\cap A_3\cap A_4\cap A_5\cap A_6\cap A_7\cap A_8\cap A_9\cap A_{10}\cap A_{11}$

$\Rightarrow \bigcap_{n=1}^{11} A_n = \underline{A_1}$

b $A=\{1,2,4,8,16\}$, $B=\{1,2,3,4,5,6,7\}$

$A\times B = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(2,1),(2,2),(2,3),(2,4),$
$(2,5),(2,6),(2,7),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(4,7),(8,1),$
$(8,2),(8,3),(8,4),(8,5),(8,6),(8,7),(16,1),(16,2),(16,3),(16,4),$
$(16,5),(16,6),(16,7)\}$.

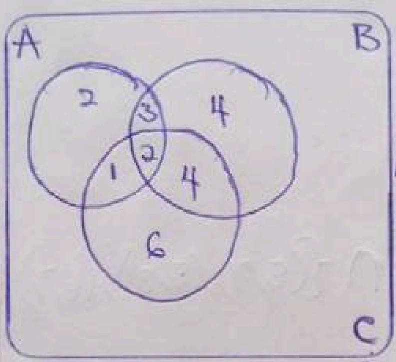
ii $(2-x,5) = (4,y-2)$

$2-x=4$, $y-2=5$

$\therefore x=2-4$ $\quad y=5+2$

$x=-2$ $\quad y=7$

4)



$|A|=8, |B|=13, |A\cap B|=5, |A\cap C|=3$

$|B\cap C|=6, |A\cap B\cap C|=2$

$|A\cap B\cap C'|=5-2 \Rightarrow 3$

$|A\cap B'\cap C|=3-2 \Rightarrow 1$

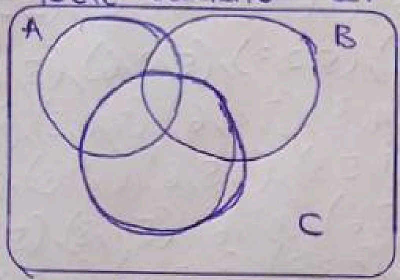
$|A\cap B'\cap C'|=8-(3+2+1) \Rightarrow 2$

$|A'\cap B\cap C'|=13-(3+2+4) \Rightarrow 4$

(a) Exactly one of this topics $= 2+4+6$
$= \underline{12}$

(b) No material on algebra $= 4+6+4$
$= \underline{14}$

5) Total student = 21



$|Q_1, A| = 21-5$, $|Q_2, A| = 21-7$; $|Q_3, A| = 21-6$
$= \underline{16}$ $= \underline{14}$ $= \underline{15}$

$|Q_1, A| = 5$, $|Q_2, F| = 7$, $|Q_3, F| = 6$

$|Q_1 A \cap Q_2 A \cap Q_3 A| = 9$

$|\text{Exactly One}| = [|Q_1, A| - 9] + [|Q_2, A| - 9] + [|Q_3, A| - 9]$
$= (16-9)+(14-9)+(15-9)$
$= 7+5+6$
$= \underline{18}$

6a) $U = 34$, $|B| = 14$, $|C| = 13$, $|P| = 21$, $|B\cap C\cap P| = 3$

$U = |B|+|C|+|P| - |C\cap P| - |P\cap B| - |B\cap C| + |B\cap C\cap P|$.

$|P\cap B| + |C\cap P| + |B\cap C| = (14+13+21+3)-34$
$= 51-34$
$= 17$.

b. Exactly One subject area $= |B| + |C| + |P| - [|P\cap B| + |C\cap P| + |B\cap C|]$
$= (14+13+21)-17$
$= 48-17$
$= \underline{31}$

(c) Exactly two subject area $= |C\cap P| + |B\cap C| + |B\cap P| - |B\cap C\cap P|$
$= 17-3$
$= \underline{14}$.

7a $A = \{1 \le n \le 100\}$

$A = \{1, 2, 3, 4, 5, \cdots, 100\}$

$B = \{$Most multiples of 3$\}$

Hence $B^c = \{$multiples of 3 between 1 and 100$\}$

$|B| = |A| - |B^c|$

$|A| = 100$

$|B^c| = 30$

$|B| = 100 - 30$

$= \underline{70}$

7b $A, B \subseteq X$

Since $A \cap B = \varnothing$

$|A| = 12, |B| = \cancel{10}\ 10$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 12 + 10 - 0$

$= 22$

Selection of 7 from $|A \cup B|$

$= {}^{22}C_n = 170544$ $P_r(3,4) = {}^7C_3 \times {}^7C_4$

$P_r(3,4) = \dfrac{1225}{170544}$.

Name :- Adeagbo - Michael - Olumide.

Dept :- Plant · Breeding · And · Seed · Technology · [PBST]

Matric No :- [20193716]

Solutions To The Dept Of Maths Funaab 2021 Mts 105 Tutorial Questions O [MATRICES].

Lecturer :- Professor · Adesina Agboola.

MATRICES

1. $A=\begin{pmatrix}1 & 2 & -3\\ 3 & 4 & -5\\ 5 & 6 & -7\end{pmatrix}$ $B=\begin{pmatrix}1 & -1 & -2\\ 2 & 3 & -2\\ -3 & 4 & -5\end{pmatrix}$ $C=\begin{pmatrix}1 & 4 & -2\\ -1 & 3 & -5\\ 8 & -7 & -8\end{pmatrix}$

a. Show $A(BC)=(AB)C$.

b. $A(B+C)=AB+AC$.

c. $B(A-C)=BA-BC$.

d. $K(ABC)=A(KB)C$, Where $K \in \mathbb{R}$.

Solution

(a) $A(BC)=(AB)C$ from LHS,

$$BC=\begin{pmatrix}1 & -1 & -2\\ 2 & 3 & -2\\ -3 & 4 & -5\end{pmatrix}\begin{pmatrix}1 & 4 & -2\\ -1 & 3 & -5\\ 8 & 7 & -8\end{pmatrix}$$

$$BC=\begin{pmatrix}1(1)-1(-1)-2(8) & 1(4)-1(3)-2(-7) & 1(-2)-1(-5)-2(-8)\\ 2(1)+3(-1)-2(8) & 2(4)+3(3)-2(-7) & 2(-2)+3(-5)-2(-8)\\ -3(1)+4(-1)-5(8) & -3(4)+4(3)-5(-7) & -3(-2)+4(-5)-5(-8)\end{pmatrix}$$

$$BC=\begin{pmatrix}1+1-16 & 4-3+14 & -2+5+16\\ 2-3-16 & 8+9+14 & -4-15+16\\ -3-4-40 & -12+12+35 & 6-20+40\end{pmatrix}=\begin{pmatrix}-14 & 15 & 19\\ -17 & 31 & -3\\ -47 & 35 & 26\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}1 & 2 & -3\\ 3 & 4 & -5\\ 5 & 6 & -7\end{pmatrix}\begin{pmatrix}-14 & 15 & 19\\ -17 & 31 & -3\\ -47 & 35 & 26\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}1(-14)+2(-17)-3(-47) & 1(15)+2(31)-3(35) & 1(19)+2(-3)-3(26)\\ 3(-14)+4(-17)-5(-47) & 3(15)+4(31)-5(35) & 3(19)+4(-3)-5(26)\\ 5(-14)+6(-17)-7(-47) & 5(15)+6(31)-7(35) & 5(19)+6(-3)-7(26)\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}-14-34+141 & 15+62-105 & 19-6-78\\ -42-68+235 & 45+124-175 & 57-12-130\\ -70-102+329 & 75+186-245 & 95-18-182\end{pmatrix}$$

$$A(BC)=\begin{bmatrix}93 & -28 & -65\\ 123 & -6 & -85\\ 157 & 16 & -105\end{bmatrix} \rightarrow (1)$$ and $(AB)C$

$$AB=\begin{bmatrix}1 & 2 & -3\\ 3 & 4 & -5\\ 5 & 6 & -7\end{bmatrix}\begin{bmatrix}1 & -1 & -2\\ 2 & 3 & -2\\ -3 & 4 & -5\end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & -1+6-12 & -2-4+15 \\ 3+8+15 & -3+12-20 & -6-8+25 \\ 5+12+21 & -5+18-28 & -10-12+35 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -7 & 9 \\ 26 & -11 & 11 \\ 38 & -15 & 13 \end{bmatrix}$$

$$AB(C) = \begin{bmatrix} 14 & -7 & 9 \\ 26 & -11 & 11 \\ 38 & -15 & 13 \end{bmatrix} \begin{bmatrix} 1 & 4 & -2 \\ -1 & 3 & -5 \\ 8 & -7 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 14+7+72 & 56-21-63 & -28+35-72 \\ +26+11+88 & 104-33-77 & -52+55-88 \\ 38+15+104 & 152-45-91 & -76+75-104 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 93 & -28 & -65 \\ 125 & -6 & -85 \\ 157 & 16 & -105 \end{bmatrix} \longrightarrow (2)$$

Hence; $A(BC) = (AB)C$

1b $A(B+C) = AB + AC$

$$B+C = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 3 & -2 \\ 3 & 4 & -5 \end{bmatrix} + \begin{bmatrix} 1 & 4 & -2 \\ -1 & 3 & -5 \\ 8 & -7 & -8 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -4 \\ 1 & 6 & -7 \\ 5 & -3 & -13 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 4 & -5 \\ 5 & 6 & -7 \end{bmatrix} \begin{bmatrix} 2 & 3 & -4 \\ 1 & 6 & -7 \\ 5 & -3 & -13 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2-15 & 3+12+9 & -4-14+39 \\ 6+4-25 & 9+24+15 & -12-28+65 \\ 10+6-35 & 15+36+21 & -20-42+91 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -11 & 24 & 21 \\ -15 & 48 & 25 \\ -19 & 72 & 29 \end{bmatrix} \longrightarrow (1)$$

$$AB = \begin{bmatrix} 14 & -7 & 9 \\ 26 & -11 & 11 \\ 38 & -15 & 13 \end{bmatrix} \langle \text{from } 1a \rangle$$

$$AC = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 4 & -5 \\ 5 & 6 & -7 \end{bmatrix} \begin{bmatrix} 1 & 4 & -2 \\ -1 & 3 & -5 \\ 8 & -7 & -8 \end{bmatrix}$$

$$AC = \begin{bmatrix} -25 & 31 & 12 \\ -41 & 59 & 14 \\ -57 & 87 & 16 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 14 & -7 & 9 \\ 26 & -11 & 11 \\ 38 & -15 & 13 \end{bmatrix} + \begin{bmatrix} -25 & 31 & 12 \\ -41 & 59 & 14 \\ -57 & 87 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 24 & 21 \\ -15 & 48 & 25 \\ -19 & 72 & 29 \end{bmatrix} \longrightarrow (2)$$

Comparing (1) and (2)

$\therefore A(B+C) = AB + AC$

1c. $B(A-C) = BA - BC$

$$A - C = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 4 & -5 \\ 5 & 6 & -7 \end{bmatrix} - \begin{bmatrix} 1 & 4 & -2 \\ -1 & 3 & -5 \\ 8 & -7 & -8 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -1 \\ 4 & 1 & 0 \\ -3 & 13 & 1 \end{bmatrix}$$

$$B(A-C) = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 3 & -2 \\ -3 & 4 & -5 \end{bmatrix} \begin{bmatrix} 0 & -2 & -1 \\ 4 & 1 & 0 \\ -3 & 13 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -29 & -3 \\ 18 & -27 & -4 \\ 31 & -55 & -2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 3 & -2 \\ -3 & 4 & -5 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 3 & 4 & -5 \\ 5 & 6 & -7 \end{bmatrix} = \begin{bmatrix} -12 & -14 & 16 \\ 1 & 4 & -7 \\ -16 & -20 & 24 \end{bmatrix}$$

$$BC=\begin{bmatrix}1&-1&-2\\2&3&-2\\-3&4&-5\end{bmatrix}\begin{bmatrix}1&4&-2\\-1&3&-5\\8&-7&-8\end{bmatrix}=\begin{bmatrix}-14&15&19\\-17&31&-3\\-47&35&26\end{bmatrix}$$

$$BA-BC=\begin{bmatrix}-12&-14&16\\1&4&-7\\-16&-20&24\end{bmatrix}-\begin{bmatrix}-14&15&19\\-17&31&-3\\-47&35&26\end{bmatrix}$$

$$=\begin{bmatrix}2&-29&-3\\18&-27&-4\\31&-55&-2\end{bmatrix}\longrightarrow (2)$$

$\therefore B(A-C)=BA-BC.$

1d $K(ABC)=A(KB)C$, Where $K\in R$

Let $K=2$

$$ABC=\begin{bmatrix}93&-28&-65\\125&-6&-85\\157&16&-105\end{bmatrix} \text{(From 1a)}$$

$$K(ABC)=2\begin{bmatrix}93&-28&-65\\125&-6&-85\\157&16&-105\end{bmatrix}$$

$$K(ABC)=\begin{bmatrix}186&-56&-130\\250&-12&-170\\314&32&-210\end{bmatrix}\longrightarrow (1)$$

$$KB=2\begin{bmatrix}1&-1&-2\\2&3&-2\\-3&4&-5\end{bmatrix}=\begin{bmatrix}2&-2&-4\\4&6&-4\\6&8&-10\end{bmatrix}$$

$$A(KB)=\begin{bmatrix}1&2&-3\\3&4&-5\\5&6&-7\end{bmatrix}\times\begin{bmatrix}2&-2&-4\\4&6&-4\\-6&8&-10\end{bmatrix}=\begin{bmatrix}28&-14&18\\52&-22&22\\76&-30&26\end{bmatrix}$$

$$A(KB)C=\begin{bmatrix}28&-14&18\\52&-22&22\\76&-30&26\end{bmatrix}\begin{bmatrix}1&4&-2\\-1&3&-5\\8&-7&-8\end{bmatrix}$$

$$=\begin{bmatrix}186&-56&-130\\250&-12&-170\\314&32&-210\end{bmatrix}\quad \therefore K(ABC)=A(KB)C.$$

2 $A=\begin{bmatrix}2 & 4 & 2\\ 1 & 3 & 1\end{bmatrix}$, $B=\begin{bmatrix}-1 & 1 & 1\\ 1 & -2 & -1\end{bmatrix}$, $C=\begin{bmatrix}2 & 4 & 2\\ 1 & 3 & 1\end{bmatrix}$, $D=\begin{bmatrix}1 & -1 & -1\\ -1 & 2 & 1\end{bmatrix}$

$K^3A + K^2B + KC = D$

$\Rightarrow \begin{bmatrix}2K^3 & 4K^3 & 2K^3\\ K^3 & 3K^3 & K^3\end{bmatrix} + \begin{bmatrix}-K^2 & K^2 & K^2\\ K^2 & -2K^2 & -K^2\end{bmatrix} + \begin{bmatrix}2K & 4K & 2K\\ K & 3K & K\end{bmatrix} = \begin{bmatrix}1 & -1 & -1\\ -1 & 2 & 1\end{bmatrix}$

We can obtain the following from the matrices above;

$4K^3 + K^2 + 4K = -1 \longrightarrow (1)$

$2K^3 + K^2 + 24 = -1 \longrightarrow (2)$

Equate Equation (1) and (2)

$4K^3 + K^2 + 4K = 2K^3 + K^2 + 2K$

$4K^3 - 2K^3 + K^2 - K^2 + 4K - 2K = 0$

$2K^3 + 2K = 0$

$2K(K^2+1) = 0$

$2K = 0$ or $K^2 + 1 = 0$

$K = 0/2$ or $K = \sqrt{-1}$

$\therefore K = \sqrt{-1}$ since D is not a zero or null matrix.

3a $A=\begin{bmatrix}2 & 3\\ 1 & -1\\ 0 & 4\end{bmatrix}$, $B=\begin{bmatrix}5 & -2 & 4 & 7\\ -6 & 1 & -3 & 0\end{bmatrix}$

$AB=\begin{bmatrix}2 & 3\\ 1 & -1\\ 0 & 4\end{bmatrix}\begin{bmatrix}5 & -2 & 4 & 7\\ -6 & 1 & -3 & 0\end{bmatrix}$

$AB \Rightarrow \begin{bmatrix}10-18 & -4+3 & 8-9 & 14+0\\ 5+6 & -2-1 & 4+3 & 7-0\\ 0-24 & 0+4 & 0-12 & 0+0\end{bmatrix}$

$AB=\begin{bmatrix}-8 & -1 & -1 & 14\\ 11 & -3 & 7 & 7\\ -24 & 4 & -12 & 0\end{bmatrix}$

The product of BA is impossible because the number of rows in B(4) is not equal to the number of Columns In A(3).

3bi Compute the determinants of A, B, C and D.

$$[A] = \begin{bmatrix} -5 & 7 & 1 \\ 1 & -5 & 7 \\ 7 & 1 & -5 \end{bmatrix}$$

$$= -5(25-7) - 7(-5-49) + 1(1+35)$$

$$= -90 + 378 + 36$$

$$= 324.$$

$$[B] = \begin{bmatrix} 1 & 4 & 16 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 1(1-0) - 4(0-0) + 16(0-0)$$

$$= 1 - 0 + 0 = \underline{1}$$

$$[C] = \begin{bmatrix} 4 & 1 & 2 \\ -1 & 3 & 4 \\ -1 & 2 & 3 \end{bmatrix} \Rightarrow 4(9-8) - 1(-3+4) + 2(-2+3)$$

$$4 - 1 + 2 = \underline{5}$$

$$[D] = \tfrac{1}{4}\begin{bmatrix} 2 & 5 & 4 \\ 2 & 3 & -4 \\ 1 & 2 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/2 & 5/4 & 1 \\ 1/2 & 3/4 & -1 \\ 1/4 & 1/2 & -3/4 \end{bmatrix}$$

$$1/2\left(-9/16 + 1/2\right) - 5/4\left(-3/8 + 1/4\right) 1\left(14 - 3/16\right)$$

$$\Rightarrow 1/32 - 25/32 + 1/6 = \frac{23}{32}$$

3ii Compute the adjoints of A, B, C and D.

Co-factor of A

$$\begin{bmatrix} +\begin{bmatrix} -5 & 7 \\ 1 & -5 \end{bmatrix} & -\begin{bmatrix} 1 & 7 \\ 7 & -5 \end{bmatrix} & +\begin{bmatrix} 1 & -3 \\ 7 & 1 \end{bmatrix} \\ -\begin{bmatrix} 7 & 1 \\ 1 & 5 \end{bmatrix} & +\begin{bmatrix} -5 & 1 \\ 7 & -5 \end{bmatrix} & -\begin{bmatrix} -5 & 7 \\ 7 & 1 \end{bmatrix} \\ +\begin{bmatrix} 7 & 1 \\ -5 & 7 \end{bmatrix} & -\begin{bmatrix} -5 & 1 \\ 1 & 7 \end{bmatrix} & +\begin{bmatrix} -5 & 7 \\ 1 & -5 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 18 & 54 & 36 \\ 36 & 18 & 54 \\ 54 & 36 & 18 \end{bmatrix}$$

Adjoint A = Transpose of the Co-factor

$$\text{Adjoint } A = \begin{bmatrix} 18 & 36 & 54 \\ 58 & 18 & 36 \\ 36 & 54 & 18 \end{bmatrix}$$

Inverse Of A^{-1}

$$\frac{1}{324}\begin{bmatrix}18 & 36 & 54\\ 54 & 18 & 36\\ 36 & 54 & 18\end{bmatrix}$$

$$=\begin{bmatrix}1/18 & 1/9 & 1/6\\ 1/6 & 1/18 & 1/9\\ 1/9 & 1/6 & 1/18\end{bmatrix}$$

Co-factor of B =
$$\begin{bmatrix}+\begin{vmatrix}1 & 4\\ 0 & 1\end{vmatrix} & -\begin{vmatrix}0 & 4\\ 0 & 1\end{vmatrix} & +\begin{vmatrix}0 & 1\\ 0 & 0\end{vmatrix}\\ -\begin{vmatrix}4 & 16\\ 0 & 1\end{vmatrix} & +\begin{vmatrix}1 & 16\\ 0 & 1\end{vmatrix} & -\begin{vmatrix}1 & 4\\ 0 & 0\end{vmatrix}\\ +\begin{vmatrix}4 & 16\\ 1 & 4\end{vmatrix} & -\begin{vmatrix}1 & 16\\ 0 & 4\end{vmatrix} & -\begin{vmatrix}1 & 4\\ 0 & 1\end{vmatrix}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ -4 & 1 & 0\\ 0 & -4 & 1\end{bmatrix}$$

Adjoint of B = Transpose of the Cofactor.

$$\text{Adjoint of } B = \begin{bmatrix}1 & -4 & 0\\ 0 & 1 & -4\\ 0 & 0 & 1\end{bmatrix}$$

Inverse Of B^{-1}

$$\frac{1}{1}\begin{bmatrix}1 & -4 & 0\\ 0 & 1 & -4\\ 0 & 0 & -1\end{bmatrix} \Rightarrow \underline{\underline{\begin{bmatrix}1 & -4 & 0\\ 0 & 1 & -4\\ 0 & 0 & 1\end{bmatrix}}}$$

Co-factor of C

$$\begin{bmatrix}1 & 1 & 1\\ -1 & 14 & 9\\ -2 & -18 & 13\end{bmatrix}$$

Adjoint of C = Transpose Of Co-factor

$$\text{Adjoint of } C = \begin{bmatrix}1 & -1 & -2\\ 1 & 14 & -18\\ 1 & 9 & 13\end{bmatrix}$$

Inverse Of C^{-1}

$$\frac{1}{5}\begin{bmatrix}1 & -1 & -2\\ 1 & 14 & -18\\ 1 & 9 & 13\end{bmatrix} \Rightarrow \begin{bmatrix}1/5 & -1/5 & -2/5\\ 1/5 & 14/5 & -18/5\\ 1/5 & 9/5 & 13/5\end{bmatrix}$$

Co-factor of D

$$\begin{bmatrix} -1 & -2 & 1 \\ 23 & -10 & 1 \\ -32 & 16 & 4 \end{bmatrix}$$

Adjoint Of D = Transpose Of Co-factor.

$$\text{Adjoint of } D = \begin{bmatrix} -1 & 25 & -32 \\ -2 & -10 & 16 \\ 1 & 1 & 4 \end{bmatrix}$$

Inverse Of D^{-1}

$$\frac{1}{3}\begin{bmatrix} -1 & 25 & -32 \\ -2 & -10 & 16 \\ 1 & 1 & 4 \end{bmatrix} = \begin{bmatrix} -1/3 & 25/3 & -32/3 \\ -2/3 & -10/3 & 16/3 \\ 1/3 & 1/3 & 4/3 \end{bmatrix}$$

3bh) $$B^{-1} = \begin{bmatrix} 1 & -4 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/18 & 1/9 & 1/6 \\ 1/6 & 1/18 & 1/9 \\ 1/9 & 1/6 & 1/18 \end{bmatrix}$$

$$\therefore B^{-1}A^{-1} = \begin{bmatrix} 1 & -4 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/18 & 1/9 & 1/6 \\ 1/6 & 1/18 & 1/9 \\ 1/9 & 1/6 & 1/18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -11/18 & -1/9 & -5/18 \\ -5/18 & -11/18 & -1/9 \\ 1/9 & 1/6 & 1/18 \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}.$$

4a $A = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$, $A^2 = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1-4 & 2-6 \\ -2+6 & -4+9 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 4 & 5 \end{bmatrix}$$

$$2A = 2\begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ 4 & 6 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 - 2A + I = 0$$

$$\Rightarrow \begin{bmatrix} -3 & -4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} -2 & -4 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow (1)$$

$$A^{-1} = \frac{1}{|A|} (\text{Adjoint of } A)$$

$$|A| = (3 \times -1) - (2 \times -2)$$

$$= -3 - (-4) = 1$$

Co-factor of $A = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$

Adjoint of A = Transpose of the co-factor

Adjoint of $A = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$

$$A^{-1} = 1/1 \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$$

4b $\begin{vmatrix} K & -1 & -8 \\ 1 & 1 & -1 \\ 3 & K & -5 \end{vmatrix} = 0$

$$K(-5+K) + 1(-5+3) - 8(K-3) = 0$$

$$-5K + K^2 - 5 + 3 - 8K + 24 = 0$$

$$K^2 - 13K + 22 = 0, \; K^2 - 11K - 2K + 22 = 0$$

$$K(K-11) - 2(K-11) = 0$$

$$(K-2)(K-11) = 0$$

$$(K-2)(K-11) = 0$$

$\therefore K=2$ or $K=11$

4C $\begin{bmatrix} \frac{1}{3}-x & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3}-x & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3}-x \end{bmatrix} = 0$

Multiply through by 3

$\begin{bmatrix} 1-3x & -2 & -2 \\ -2 & 1-3x & -2 \\ -2 & -2 & 1-3x \end{bmatrix} = 0$

$1-3x\left[(1-3x)(1-3x)-4\right]+2\left[-2(1-3x)-4\right]-2\left[4+2(1-3x)\right]=0$

$1-3x\left[1-6x+9x^2-4\right]+2\left[-2+6x-4\right]-2\left[4+2-6x\right]=0$

$1-3x\left[9x^2-6x-3\right]+2\left[-6+6x\right]-2\left[6-6x\right]=0$

$9x^2-6x-3-27x^3+18x^2+9x-24+24x=0$

$-27x^3+18x^2+9x^2-6x+9x+24x-3-24=0$

$-27x^3+27x^2-27x+27=0$

Divide through by 27

$x^3+x^2-x+1=0$

$\therefore x = -1,1,1$

Question 5

5a $A=\begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}$; $A^3=\begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}^3$

$A^3=\begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}\begin{bmatrix} 2 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}\begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 3 & 11 & 9 \\ -16 & -4 & 5 \\ 46 & -17 & 0 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 34 & 13 & 8 \\ -19 & 83 & 7 \\ 41 & -24 & 80 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 11 & 9 \\ -16 & -4 & 5 \\ 46 & -17 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 - A^2 + A - 83I = 0$$

$$\Rightarrow \begin{bmatrix} 84 & 13 & 8 \\ -19 & 83 & 7 \\ 41 & -24 & 80 \end{bmatrix} - \begin{bmatrix} 3 & 11 & 9 \\ -16 & 4 & 5 \\ 46 & -17 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix} - \begin{bmatrix} 83 & 0 & 0 \\ 0 & 83 & 0 \\ 0 & 0 & 83 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5b $A^{-1} = \frac{1}{83}\left[A^2 - A + I\right]$.

$$A^{-1} = \frac{1}{[A]}\left[\text{Adjoint of } A\right].$$

$$[A] = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix} = 2(-12+14) + 2(9+10) + 1(21+20) = 83$$

$$\text{Co-factor of } A = \begin{bmatrix} +\begin{vmatrix} -4 & -2 \\ 7 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & -2 \\ 5 & 3 \end{vmatrix} & +\begin{vmatrix} 3 & -4 \\ 5 & 7 \end{vmatrix} \\ -\begin{vmatrix} -2 & 1 \\ 7 & 3 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & -2 \\ 5 & 7 \end{vmatrix} \\ +\begin{vmatrix} -2 & 1 \\ -4 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} & +\begin{vmatrix} 2 & -2 \\ 3 & -4 \end{vmatrix} \end{bmatrix}$$

$$\text{Co-factor of } A = \begin{bmatrix} 2 & -19 & 41 \\ 13 & 1 & -24 \\ 8 & 7 & -2 \end{bmatrix}$$

$$\text{Transpose of } A = \begin{bmatrix} 2 & 13 & 8 \\ -19 & 1 & 7 \\ 41 & -24 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{83}\begin{bmatrix} 2 & 13 & 8 \\ -19 & 1 & 7 \\ 41 & -24 & -2 \end{bmatrix} \longrightarrow (1)$$

$$A^2 - A + I = \begin{bmatrix} 3 & 11 & 9 \\ -16 & -4 & 5 \\ 46 & -17 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 13 & 8 \\ -19 & 1 & 7 \\ 41 & -24 & -2 \end{bmatrix}$$

$$\frac{1}{83}\left[A^2 - A + I\right] = \frac{1}{83}\begin{bmatrix} 2 & 13 & 8 \\ -19 & 1 & 7 \\ 41 & -24 & -2 \end{bmatrix} \longrightarrow (2)$$

Compare (1) and (2), therefore

$$A^{-1} = \frac{1}{83}\left(A^2 - A + I\right)$$

5c **Solution To Questions 5 ~~b~~ ~~4~~ C**

$$2x - 2y + 1 = 5$$
$$3x - 4y - 2z = 5$$
$$5x + 7y + 3z = 1$$

Using $Ax = B$, We derive

$$\begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$$

Recall that $A^{-1} = \frac{1}{83}\begin{bmatrix} 2 & 13 & 8 \\ -19 & 1 & 7 \\ 41 & -24 & -2 \end{bmatrix}$

Using the formula $A^{-1}B = x$, We derive

$$\frac{1}{83}\begin{bmatrix} 2 & 13 & 8 \\ -19 & 1 & 7 \\ 41 & -24 & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\frac{1}{83}\begin{bmatrix} 10 + 65 + 8 \\ -95 + 5 + 7 \\ 205 + 120 - 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\frac{1}{83}\begin{bmatrix} 83 \\ -83 \\ 83 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\therefore x = 1,\ y = -1,\ z = 1$

5d Using Crammer's Rule

$$\begin{bmatrix} 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$$

Let $D = 2(-12+14) + 2(9+10) + 1(21+20)$

$D = 83$

$D_1 \begin{bmatrix} 5 & -2 & 1 \\ 5 & -4 & -2 \\ 1 & 7 & 3 \end{bmatrix}$

$D_1 = 5(-12+14) + 2(15+2) + 1(35+4)$

$= 83$

$D_2 = \begin{bmatrix} 2 & 5 & 1 \\ 3 & 5 & -2 \\ 5 & 1 & 3 \end{bmatrix} = 2(15+2) - 5(9+10) + 1(3-25)$

$= -83$

$D_3 = \begin{bmatrix} 2 & -2 & 5 \\ 3 & -4 & 5 \\ 5 & 7 & 1 \end{bmatrix} = 2(-4-35) + 2(3-25) + 5(21+20)$

$= 83.$

Now $x = \frac{D_1}{D} = \frac{83}{83} = 1$

$y = \frac{D_2}{D} = {-83}/{83} = -1$

$z = \frac{D_3}{D} = {83}/{83} = 1$

$\therefore x = 1, y = -1,$ and $z = 1$

Question 6

6) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix}$; $A^{-1} \frac{1}{|A|}$ (Adjoint of A).

$|A| = 1(2-3) - 2(4-9) + 1(2-3)$
$= \underline{10}$

Co-factor of A = $\begin{pmatrix} +\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \end{pmatrix}$

$= \begin{bmatrix} -1 & 5 & -1 \\ -5 & 5 & 5 \\ 7 & -5 & -3 \end{bmatrix}$

Adjoint of A = Transpose of Co-factor

Adjoint of A = $\begin{bmatrix} -1 & -5 & 7 \\ 5 & 5 & -5 \\ -1 & 5 & -3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \langle \text{Adjoint of } A \rangle = \frac{1}{10} \begin{bmatrix} -1 & -5 & 7 \\ 5 & 5 & -5 \\ -1 & 5 & -3 \end{bmatrix}$

Using $Ax = B$

$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 16 \\ 4 \end{bmatrix}$

$\therefore A^{-1}B = x$

$\frac{1}{10} \begin{bmatrix} -1 & -5 & 7 \\ 5 & 5 & -5 \\ -1 & 5 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ 16 \\ 14 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\frac{1}{10}\begin{bmatrix} 2 & -80 & +98 \\ -10 & +80 & -70 \\ 2 & +80 & -42 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\frac{1}{10}\begin{bmatrix} 20 \\ 0 \\ 40 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\therefore x = 2,\ y = 0,\ z = 4.$

6b Solve for x given that

$$\begin{bmatrix} -1 & 3 & x \\ 2x-3 & 1-x & 3x+1 \\ 2 & x & -2 \end{bmatrix} = 9x - 28.$$

$$-1\left[(1-x)(-2) - (3x+1)(x)\right] - 3\cancel{(x-3)}\left([2x-3][-2] - [3x+1][2]\right)$$
$$+x\left([2x-3][x] - [1-x]2\right) = 9x - 28.$$

$$-1[-2+2x] - [3x^2+x] - 3[-4x+6] - [6x+2] + x[2x^2-3x] -$$
$$[2-2x] = 9x - 28.$$

$$-1[-2+2x-3x^2-x] - 3[-4x+6-6x-2] + x[2x^2-3x] - 2 + 2x]$$
$$= 9x - 28$$

$$2 - 2x + \cancel{3x^2} + x + 12x - 18 + 18x + 6$$
$$+2x^3 - \cancel{3x^2} - 2x + 2x^2 = 9x - 28.$$

Collect like terms.

$$2x^3 + 2x^2 + 26x + x = 9x - 28 + 18 - 6 - 2$$
$$2x^3 + 2x^2 + 26x - 8x + 18 = 0$$

Divide through by 2.

$$x^3 + x^2 + 9x + 9 = 0$$
$$x^2(x+1) + 9(x+1) = 0$$
$$x^2(x+1) + 9(x+1) = 0$$

x^2+9 or $x+1=0$

$x^2=-9$ or $x=-1$

$x=\sqrt{-9}$ or $x=-1$

$x=\sqrt{-9}$ or -1

6c $A=\begin{bmatrix}1&2\\2&3\end{bmatrix}$, Show that $A^2-4A-I=0$

$A^2=\begin{bmatrix}1&2\\2&3\end{bmatrix}^2=\begin{bmatrix}1&2\\2&3\end{bmatrix}\begin{bmatrix}1&2\\2&3\end{bmatrix}=\begin{bmatrix}1+4&2+6\\2+6&4+9\end{bmatrix}\Rightarrow\begin{bmatrix}5&8\\8&13\end{bmatrix}$

$4A=4\begin{bmatrix}1&2\\2&3\end{bmatrix}=\begin{bmatrix}4&8\\8&12\end{bmatrix}$

$I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$O=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$A^2-4A-I=0$

$\begin{bmatrix}5&8\\8&13\end{bmatrix}-\begin{bmatrix}4&8\\8&12\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}\longrightarrow (1)$

$A^{-1}=\frac{1}{|A|}\{\text{Adjoint of } A\}.$

$|A|=\langle 3-4\rangle=-1$

Co-factor of $A=\begin{bmatrix}3&-2\\2&-1\end{bmatrix}$

Adjoint of $A=\begin{bmatrix}3&2\\-2&1\end{bmatrix}$

$A^{-1}=\cancel{\frac{1}{[-1]}}\ {}^{1}|-1\begin{bmatrix}3&2\\-2&1\end{bmatrix}=\begin{bmatrix}-3&-2\\2&-1\end{bmatrix}$

Using $Ax=B$

$\begin{bmatrix}1&2\\2&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2/3\\3/4\end{bmatrix}$

$\therefore A^{-1}B=x$

$\begin{bmatrix}-3&-2\\2&-1\end{bmatrix}\begin{bmatrix}-2/3\\3/4\end{bmatrix}=\begin{bmatrix}x\\y\end{bmatrix}$

$$\begin{bmatrix} 2 & -3/2 \\ -4/3 & -3/4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 7/2 \\ -25/12 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$x = 7/2$, $y = -25/12$

Question 7

7a Given that $\begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ x^2 \end{bmatrix} = \begin{bmatrix} 3 \\ k \end{bmatrix}$

$5x + 2x^2 = 3$

$2x + 4x^2 = k$

$2x^2 + 5x = 3 \longrightarrow$ (i)

$4x^2 + 2x = k \longrightarrow$ (ii)

$2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$

$2x(x-3) - 1(x+3) = 0$

$(2x-1)$ or $(x+3) = 0$

$x = 1/2$ or $x = -3$

Substitute x in equation (2)

When $x = 1/2$

$4(x^2 + 2x) = k$

$4(1/2)^2 + 2(1/2) = k$

$2 = k$

$\therefore k = 2$

When $x = -3$

$4(x^2 + 2x) = k$

$4(-3)^2 + 2(-3) = k$

$30 = k$

$k = 30$

$\underline{x = 1/2,\ k = 2}$ or $\underline{x = -3,\ k = 30}$.

7b & 7c Let $A = \begin{bmatrix}2&3\\3&2\end{bmatrix}$; $A^T = \begin{bmatrix}2&3\\3&2\end{bmatrix}$

$AA^T = \begin{bmatrix}2&3\\3&2\end{bmatrix}\begin{bmatrix}2&3\\3&2\end{bmatrix} = \begin{bmatrix}4+9&6+6\\6+6&9+4\end{bmatrix}$

$AA^T = \begin{bmatrix}13&12\\12&13\end{bmatrix}$

AA^T is Symmetric

Yes A^TA is also symmetric since the transpose $A^T = \underline{A}$

8) $A = \begin{bmatrix}8&-1&-3\\-5&1&2\\10&-1&-4\end{bmatrix}$; $A^3 = \begin{bmatrix}8&-1&-3\\-5&1&2\\10&-1&-4\end{bmatrix}^3$

$= \begin{bmatrix}202&-31&-73\\-130&20&47\\235&-36&-85\end{bmatrix}$

$A^2 = \begin{bmatrix}39&-6&-14\\-25&4&9\\45&-7&-16\end{bmatrix}$; $5A^2 = 5\begin{bmatrix}39&-6&-14\\-25&4&9\\45&-7&-16\end{bmatrix} = \begin{bmatrix}195&-30&-70\\-125&20&45\\225&-35&-80\end{bmatrix}$

$A^3 - 5A^2 - A + I = 0$

$\Rightarrow \begin{bmatrix}202&-31&-73\\-130&20&47\\235&-36&-85\end{bmatrix} - \begin{bmatrix}195&-30&-70\\-125&20&45\\225&-35&80\end{bmatrix} - \begin{bmatrix}8&-1&-3\\-5&1&2\\10&-1&-4\end{bmatrix} +$

$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}0&0&0\\0&0&0\end{bmatrix}$

8b $A^{-1} = I + 5A - A^2$

$A^{-1} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} + \begin{bmatrix}40&-5&-15\\-25&5&10\\50&-5&-20\end{bmatrix} - \begin{bmatrix}39&-6&14\\-25&4&9\\45&-7&-16\end{bmatrix}$

$A^{-1} = \begin{bmatrix}2&1&-1\\0&1&1\\5&2&-3\end{bmatrix}$

8c $Ax = B$

i.e $\begin{bmatrix}8&-1&-3\\-5&1&2\\10&-1&-4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-3\\2\\-4\end{bmatrix}$

Recall that $A^{-1}B = x$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} -6 & +2 & +4 \\ 0 & +4 & -4 \\ -15 & +4 & +12 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x = 0, y = 0, z = 1$

Ilerioluwa