MECHANICS

PHS 105/ Department of Physics

AKINBORO F. G

LEARNING MANAGEMENT SYSTEM
LMS

DISPLACEMENT

Newton's Laws of motion

1. Newton's law of Motion state that every body continues on its state of rest or uniform acceleration unless an impulse force act on its.

2. Newton second law of motion state that:
Action and Reaction are equal and opposite

3. Newton third law of motion state that:
That forces equal to the product of mass and acceleration

$$F = Ma$$

- **Speed**: Distance moved also known as displacement of object from one position to the other an objects displacement is 10m in a certain direction of its speed

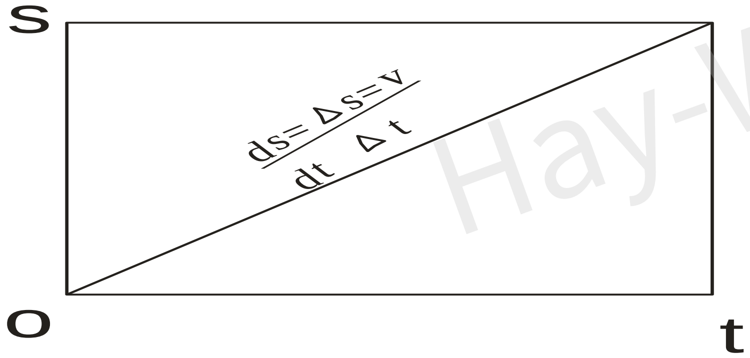
- **Velocity:** in the rate of change of displacement. it can also be expressed as: displacement time or distance covered per time

Velocity = Distance covered (ds) in sec.

Time taken (dt) in sec..

- **Vector Quantity:** **quantity with both size and direction of Displacement. eg velocity**

- **Scalar Quantity:** **quantity with size both no direction. example speed or mass**



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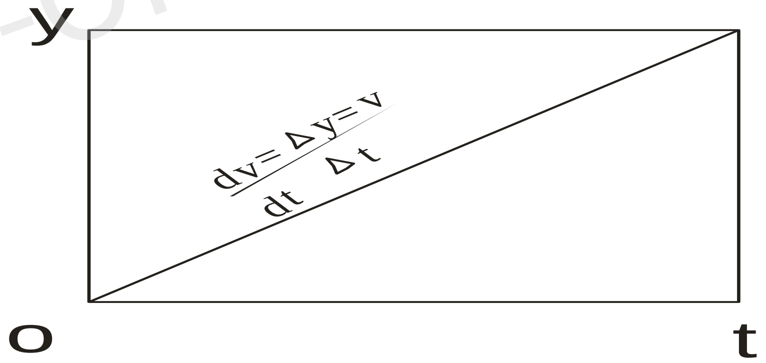
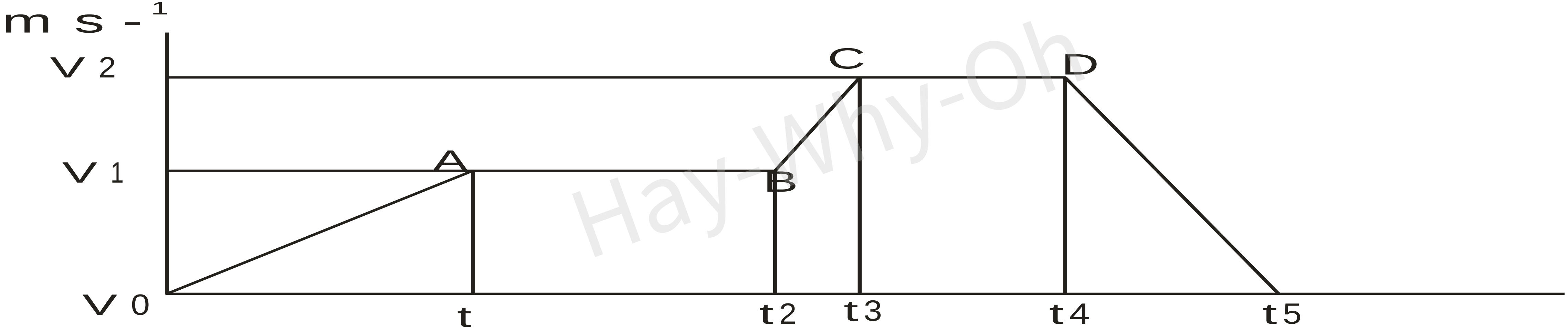


$\frac{ds}{dt} = \frac{s-o}{t-o} = V$ = velocity

$\frac{dv}{dt} = \frac{v-o}{t-o} = a$ = acceleration

VELOCITY - TIME GRAPH = ACCELERATION

Velocity time graph gives us more information about acceleration of object. Consider the motion of



Velocity-Time Graph

An object described by the graph above. Here the object work off from initial velocity o and attain end velocity V, in t second at A, it now maintain this velocity until B in reached at V_1 taking time T_1-T_2. The object now maintain the new velocity until t_3 at C, again become constant until D is reached and finally come down to 0, Vo at time t_5

It should be noted that there can only be velocity charge if there is an accelerator

From the above, we can deduce that the object accelerated two times during the course of its journey or flight it now decelerate once

Accelerate 1 from t_o to t_1 $\frac{V_1 - V_2}{T_1 - T_2}$

Accelerate 2 from t_2 to t_3 , $\frac{V_2 - V_1}{T_3 - T_2}$

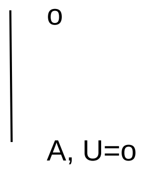
Deceleration from t_4 to $t_{5,}$` $\frac{V_2 - V_0}{T_5 - T_0}$

MOTION UNDER GRAVITY - FREE FALL

Object falling freely from a certain distance above the ground falls with a uniform acceleration of approximately 9.8 meters / second square ($9.8 m/s^2$). The distance traveled under acceleration due to gravity is independent of inertia irrespective of the type of object.

VERTICAL AND HORIZONTAL MOVEMENT

A footballer kick a ball 2.5ms vertically upwards from



Acceleration due to gravity is -g, because the ball moves from A, its initial position at A to the final position B against the gravitational pull which is downwards

$$= ut + \frac{1}{2} at^2$$

If the initial velocity of kick in 23m/s the acceleration due to gravity will act on the ball and start to retard the movement of the ball until it is momentarily at rest, then the final velocity will be zero.

The

$$v = u + at$$

$$023 + (-98)t$$

$$9.8t = 25$$

$$t= \frac{25}{9.8}$$

The distance between A and O

$$= S = ut + ½ at^2$$

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LEARNING MANAGEMENT SYSTEM
LMS

CIRCULAR MOTION

IN THE LAST CLASS
WE TREATED THE TOPICS- Displacement, Velocity And Acceleration of Object In Straight line
Today are going to treat-
Circular motion

LEARNING MANAGEMENT SYSTEM LMS

The following serves as reminder to last week's lecture:-

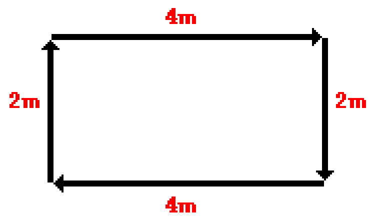
DISTANCE AND DISPLACEMENT

Distance and displacement are two quantities that may seem to mean the same thing yet have distinctly different definitions and meanings.

•Distance is a scalar quantity that refers to "how much ground an object has covered" during its motion.

•Displacement is a vector quantity that refers to "how far out of place an object is"; it is the object's overall change in position.

To test your understanding of this distinction, consider the motion depicted in the diagram below. A physics teacher walks 4 meters East, 2 meters South, 4 meters West, and finally 2 meters North.



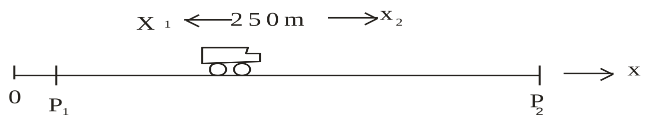
Even though the physics teacher has walked a total distance of 12 meters, her displacement is 0 meters. During the course of her motion, she has "covered 12 meters of ground" (distance = 12 m). Yet when she is finished walking, she is not "out of place" - i.e., there is no displacement for her motion (displacement = 0 m). Displacement, being a vector quantity, must give attention to direction. The 4 meters east *cancels* the 4 meters west; and the 2 meters south *cancels* the 2 meters north. Vector quantities such as displacement are *direction aware*. Scalar quantities such as distance are ignorant of direction. In deter

DISPLACEMENT TIME & AVERAGE VELOCITY

The velocity of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. Velocity is equivalent to a specification of an object's speed and direction of motion.

For **example**, someone who takes 40 minutes to drive 20 miles north and then 20 miles south (to end up at the same place), has an **average speed** of 40 miles divided by 40 minutes, or 1 mile per minute (60 mph). **Average velocity**, however, involves total displacement, instead of distance.

DISPLACEMENT TIME & AVERAGE VELOCITY



t1 *Δx=250m* *t2*

$$\text{Objects Average Velocity} = \frac{dx}{dt}$$

$$Dt = t_2 - t_1$$

$$slope = velocity = \frac{dx}{dt}$$

$$Dx = x_2 - x_1$$

$$\frac{dx}{dt} = instantaneous\ velocity$$

AVERAGE VELOCOTY

Someone who takes 40 minutes to drive 20 miles north and then 20 miles south (to end up at the same place), has an **average speed** of 40 miles divided by 40 minutes, or 1 mile per minute (60 mph). **Average velocity**, however, involves total displacement, instead of distance

Velocity – Time Graph = Acceleration

Velocity time graph gives us more information about acceleration of object.

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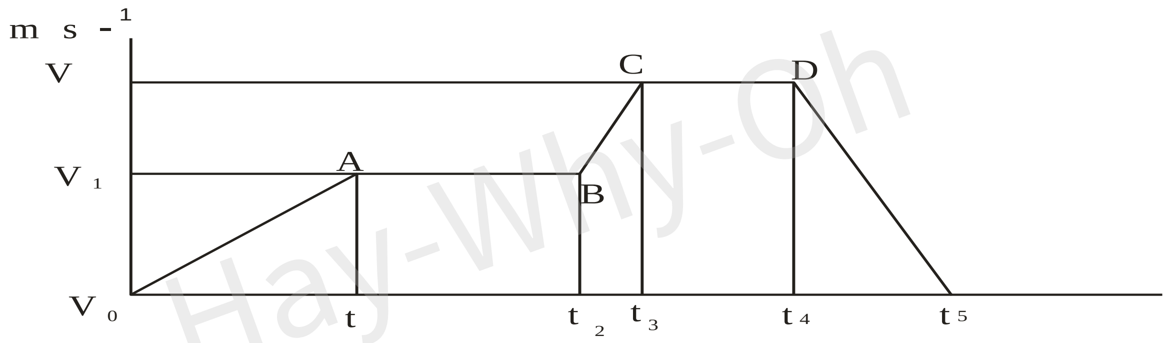


Velocity – Time graph $^t/_s$

Velocity – Time graph of an object described by the graph above

. Here the object took off from initial velocity 0 and attain and velocity $v_1\ in\ t_1\ second$

it now maintain this velocity until *t2 when* it reached it now attain a new velocity *v2* from initial velocity *v1* taking time*t3- t2*. The object now maintain the new velocity until *t4* is reached and finally come down back to *v0* at time *t5*

Note that there can only be velocity change if there is an acceleration

From the above we can deduce that the object accelerated two times during the course of its journey or flight and decelerated once

Accelerate 1 $\frac{v_1 - v_2}{t_1 - t_0}$

Accelerate 2 $\frac{V_2 - V_1}{t_3 - t_2}$

Decelerate 1 $\frac{V_2 - V_0}{t_4 - t_5}$

AVERAGE ACCLERATION

$$\frac{V_2x}{t_2} = \frac{v_x}{t_1} = \frac{\Delta v}{\Delta t} = Average\ acceleration$$

$$\frac{Ds}{dt} = Velocity ---- -(1)$$

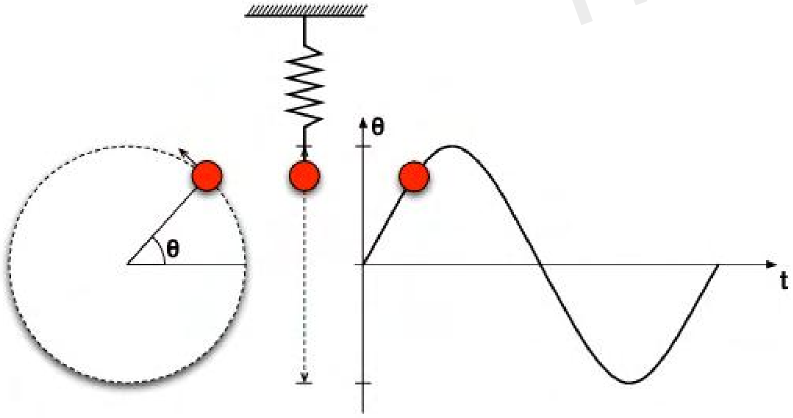
(s) $\frac{Displacement}{t} = velocity$

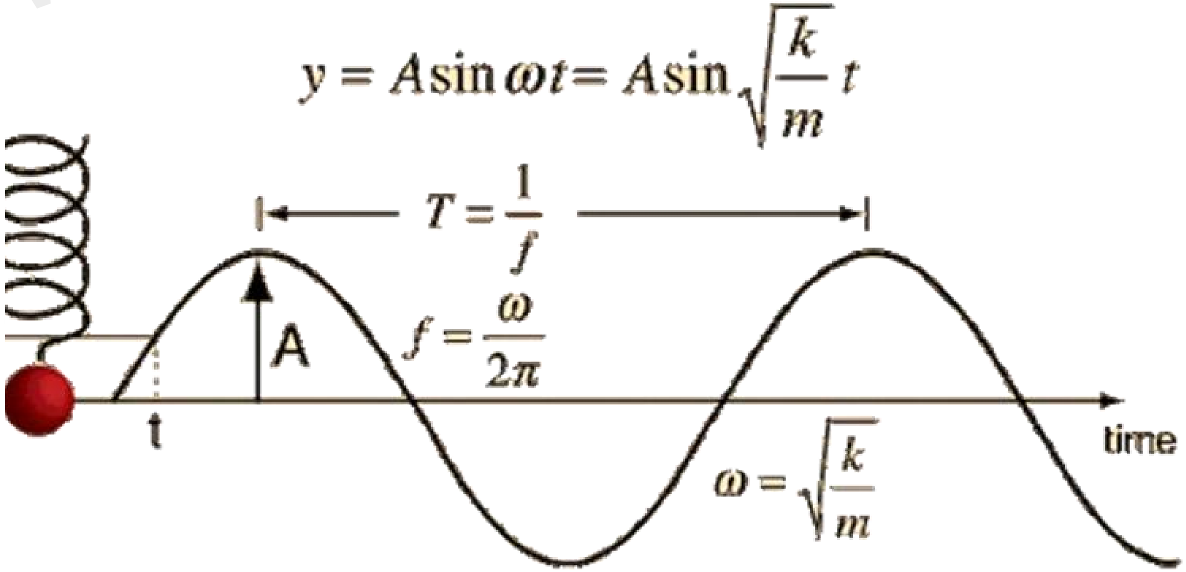
$$Average\ velocity = \frac{V+u}{2} --------- -(2)$$

$$Accleration = \frac{v-u}{t} -------- -(3)$$

Simple Harmonic Motion

Simple harmonic motion, in physics, repetitive movement back and forth through an equilibrium, or central, position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. The time interval of each complete vibration is the same





CIRCULAR MOTION

There are many cases of objects moving in a curve or circular path about some point, such as bicycles or cars turning round corners or racing cars going round circular tracks.

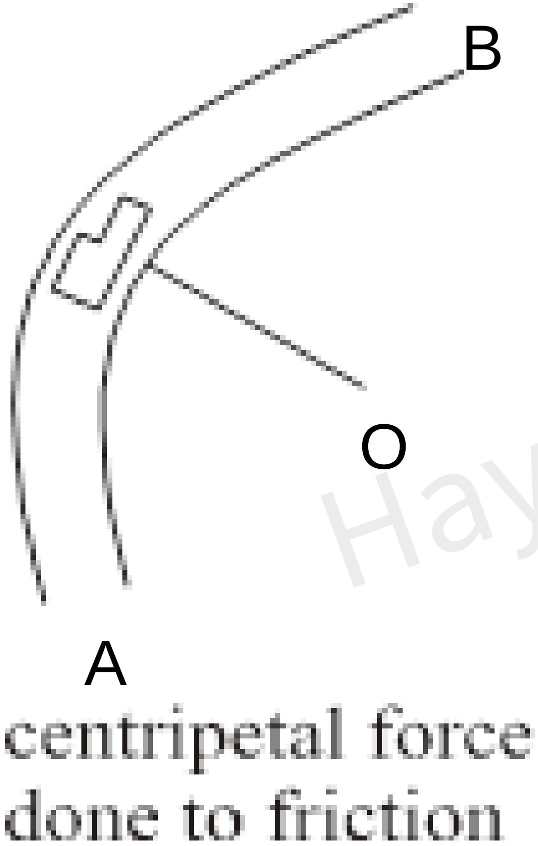
The earth and other planets move round the sun in roughly circular paths.

The speed is in linear motion, we then have to use ‘angular speed’.

This helps to find the ‘period’ or time to go once round the circle.

We shall also find tout how an objects moving at constant speed round a circle have acceleration towards the Centre of the circle.

Consider an object moving in a circle with a uniform speed round a fixed point O as centre, Figure below



If the object moves from A to B so that radius OA moves through an angleθ, its angular speed, ω, about O is defined as the change of the angle per second. So if t is the time taken by the object to move from A to B,

$$\omega = -\frac{\theta}{1} \quad .$$

The angle θ is measured in radians. (2π radians = 360°.) So angular speed is usually expressed in ‘radius per second’ (rad $s{-}^{1}$). From (1),

$$\theta = \omega t$$

This is similar to the formula ‘distance = uniform velocity x time’ for motions in a straight line, it will be noted that the time T to describe the circle once. Known as the period of the motion, is given by

$$T = \frac{2\pi}{\omega}$$

Since 2π radians is the angle in 1 revolution (360°)

If is the length of the arc AB, then $\frac{s}{r} = \theta$, by definition of an angle in radians.

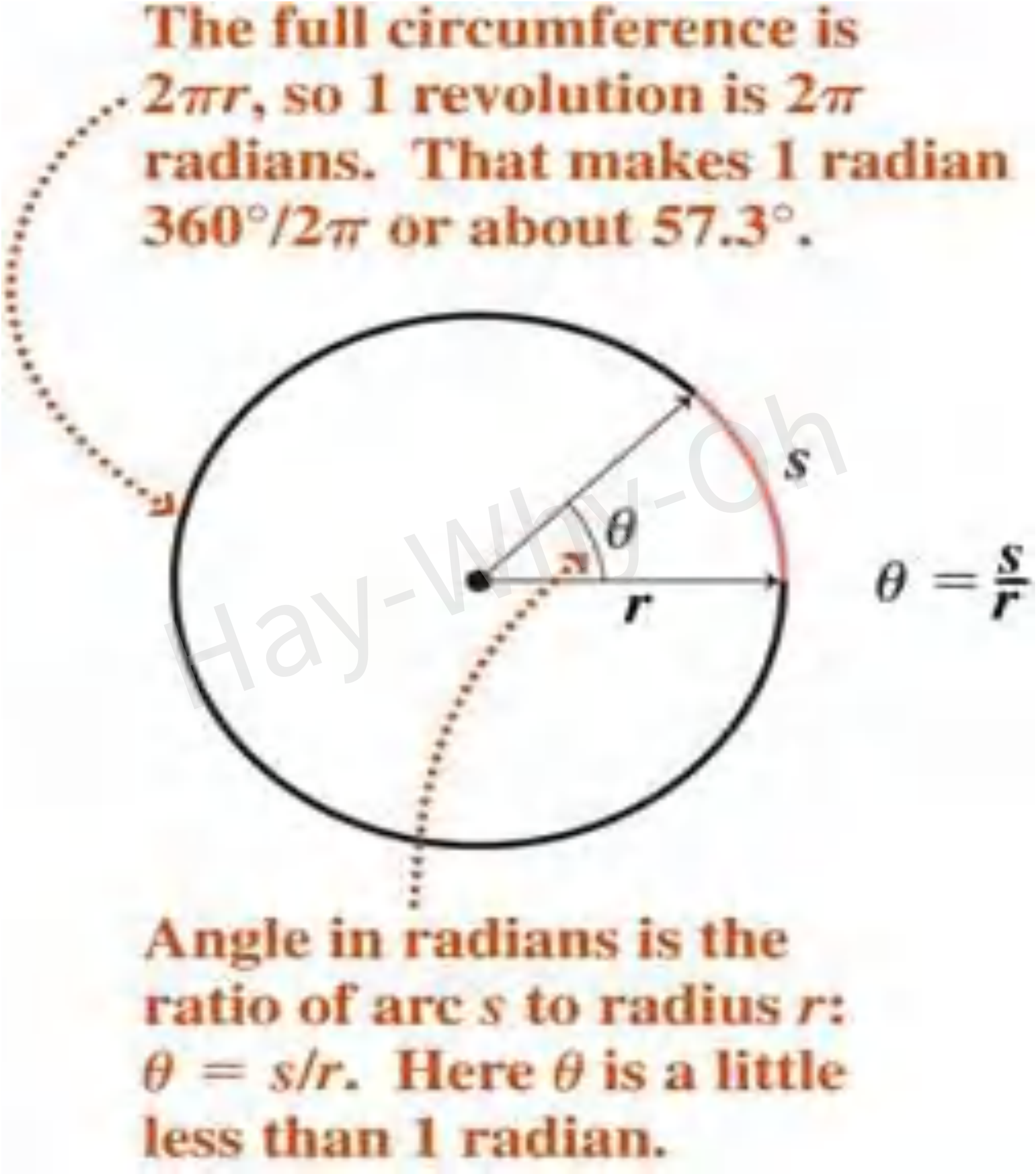
$$\therefore s = r\theta$$

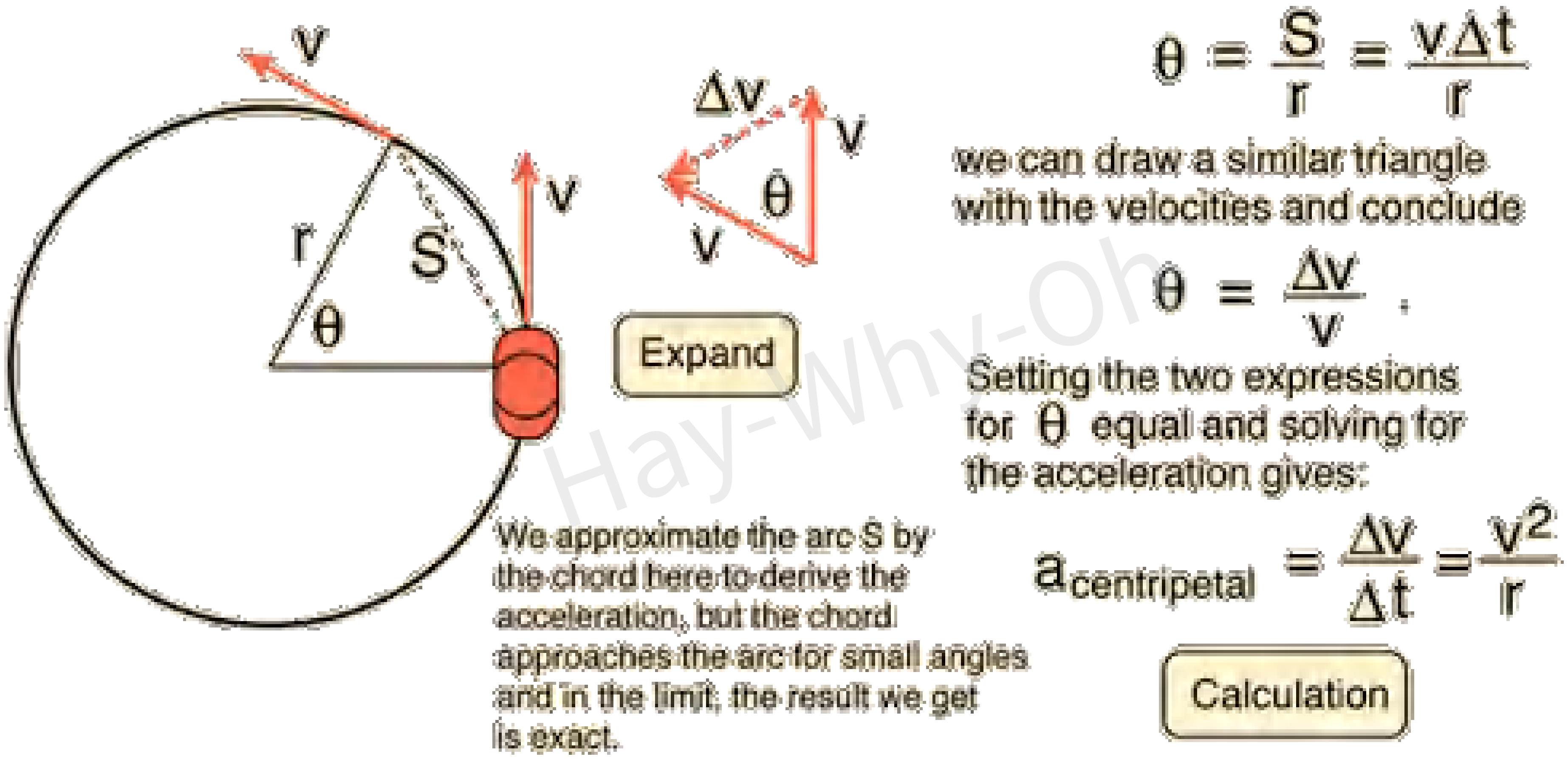
Dividing by t, the time taken to move from A to B,

$$\therefore \frac{s}{t} = \mathrm{r}\,\frac{\theta}{t}$$

But s/t = the speed, v, of the rotating object, and θ/t is the angular velocity.

$$\therefore v = \mathrm{r}\omega \quad . \; . \quad . \quad . \quad . \quad . \quad .(4)$$


The full circumference is $2\pi r$, so 1 revolution is 2π radians. That makes 1 radian $360°/2\pi$ or about 57.3°.
s
θ
r
$\theta = \frac{s}{r}$
Angle in radians is the ratio of arc s to radius r: $\theta = s/r$. Here θ is a little less than 1 radian.


v
r
S
θ
v
Δv
v
θ
v
Expand
$\theta = \frac{S}{r} = \frac{v\Delta t}{r}$
we can draw a similar triangle
with the velocities and conclude
$\theta = \frac{\Delta v}{v}$.
Setting the two expressions
for θ equal and solving for
the acceleration gives:
$a_{centripetal} = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$
Calculation
We approximate the arc S by
the chord here to derive the
acceleration, but the chord
approaches the arc for small angles
and in the limit, the result we get
is exact.

Example on Circular Motion

A model car moves round a circular track of radius 0.3m at 2 revolutions per second

What is

- A) the angular speed ω the angular speed ω
- b) the period T,
- C) the speed v of the car?
- D) Find also the angular speed of the car if it moves
- with a uniform speed of 2 m s-1

a) For 1 revolution, angle turned $\theta = 2\pi$ rad (360°). So

b) $\omega = 2 \times 2\pi = 4\pi$ rad s^{-1}

c) Period T= time for 1 rev = $\frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = 0.5$ s. (Or, T = 1 s/2 = 0.5 s.)

d) Speed $v = r\omega = 0.3 \times 4\pi = 1.2\pi = 3.8$ m s^{-1}

From $v = r\omega$

$$\omega = \frac{v}{r} = \frac{2\text{m s}^{-1}}{0.4m} = 5\text{rad s}^{-1}$$

MECHANICS

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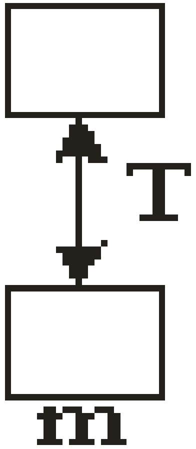
$$\frac{AB}{T} = Cos\ 60$$

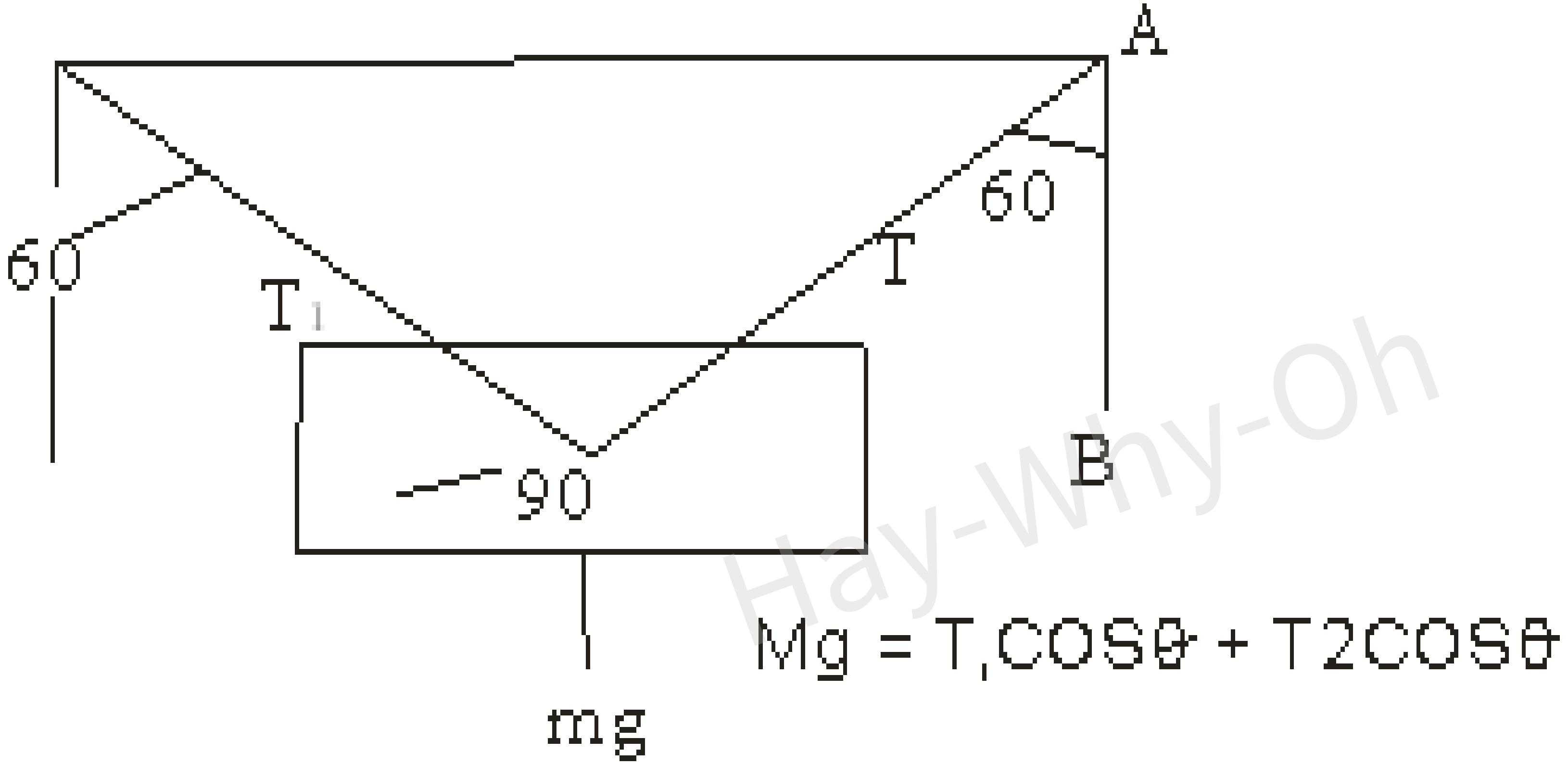
$$AB = TCos\ 60^0$$

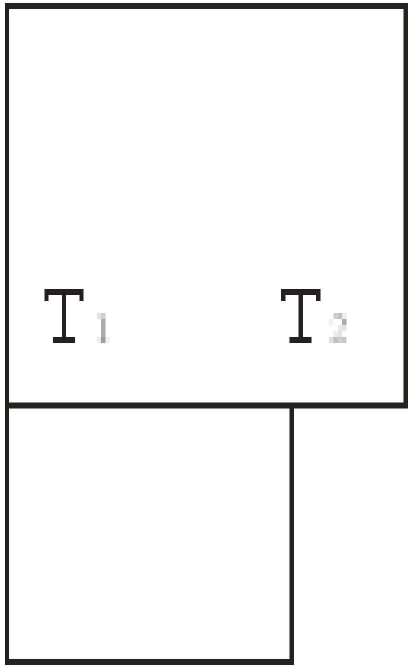
$$mg = T_1 + T_2$$



$T - mg = m$

FORCES, MASS & ACCELERATION





Consider the diagram above: $\frac{AB}{AO} = Cos\ 60$

LINEAR MOMENTUM

LINEAR MOMENTUM

Momentum of a moving object is the product of the mass and the velocity = mu

A mass m of body 50kg moving with velocity u of 1ms^{-1} $= The\ momentum = 50kgms^{-1}$

Newton second law of motion: force is directly proportional to rate of change of momentum change.

Suppose a boy of 50kg on a bicycle has a velocity of 1ms^{-1} and just realizes that he is late for school just peddles faster to speed of 5ms^{-1} for 3seconds.

$\therefore$ *Thus* the change in momentum is change per seconds.

Average time $= \frac{Change\ in\ momentum}{Time\ of\ the\ school}$

$$\frac{50 \times 3 - 50 \times 1}{5}$$

Here the velocity increases from 1ms^{-1} $to\ 3ms^{-1} = kg5^2 = Newton = Force$

F = momentum change / seconds

= mass x velocity change per seconds

= ma

State the unit of momentum in SI unit

A sand falls at the rate of 90g/sec into a horizontal belt moving at the rate of 40cm/s. Find the force of the belt in Newton inserted by the camel.

F = ma

$= \frac{90}{100}kgs \times \frac{40}{100}ms^{-1} = 0.0g \times 0.4ms^{-1} = 0.036N$

ACTION AND REACTION

Newton 3rd law of motion state that, action and reactionare opposite.
E.g. leaning on table. one exerts an action on the table while the table produces an equal and opposite reaction.

MOMENTUM CHANGES DUE TO ACTION AND REACTION

The effect of Acton and reaction can be study by two trolleys A & B.

A sand fall constantly @ the rate of 50g/s onto a belt moving horizontally at 40cm/s.

Find the force in the belt in newton exerted by the sand.

Force = momentum change per second = mass of the sand per second × velocity change.

$$\therefore F = 0.05(1g/sec \times (\frac{0.4m}{s})$$

= 0.02N

At takeoff, A jet releases gas at the rate of 90kg/s, if the force produce is 100n.

What is the velocity at which the gas is expelled?

Force = rate of change with v

100 = mv = mass x velocity…

VELOCITY OF A BULLET

Momentum change before collision = momentum change after collision
Suppose a bullet is fired from a rifle and the riffle falls backward.
Supposed the bullet ha a mass of 15g and leave the rifle of mass 3000J with a velocity of 100km/h.
Find the velocity at which the rifle jacks backward.

Mass of bullet = 15g
Mass of rice = 3000j
V of bullet = 100km/h
Velocity of which the angle jack

$$m_1 \, v_1 = \, m_2 \, v_2$$

$$0.015\times \, \frac{100k}{36 \times 36} \times 1000 = 3 \, \times V$$

FORCES & MOMENTS

Moment – Forces that keep machine working levels or keep objects in equilibrium

Examples

State force

Photo frame

Latides

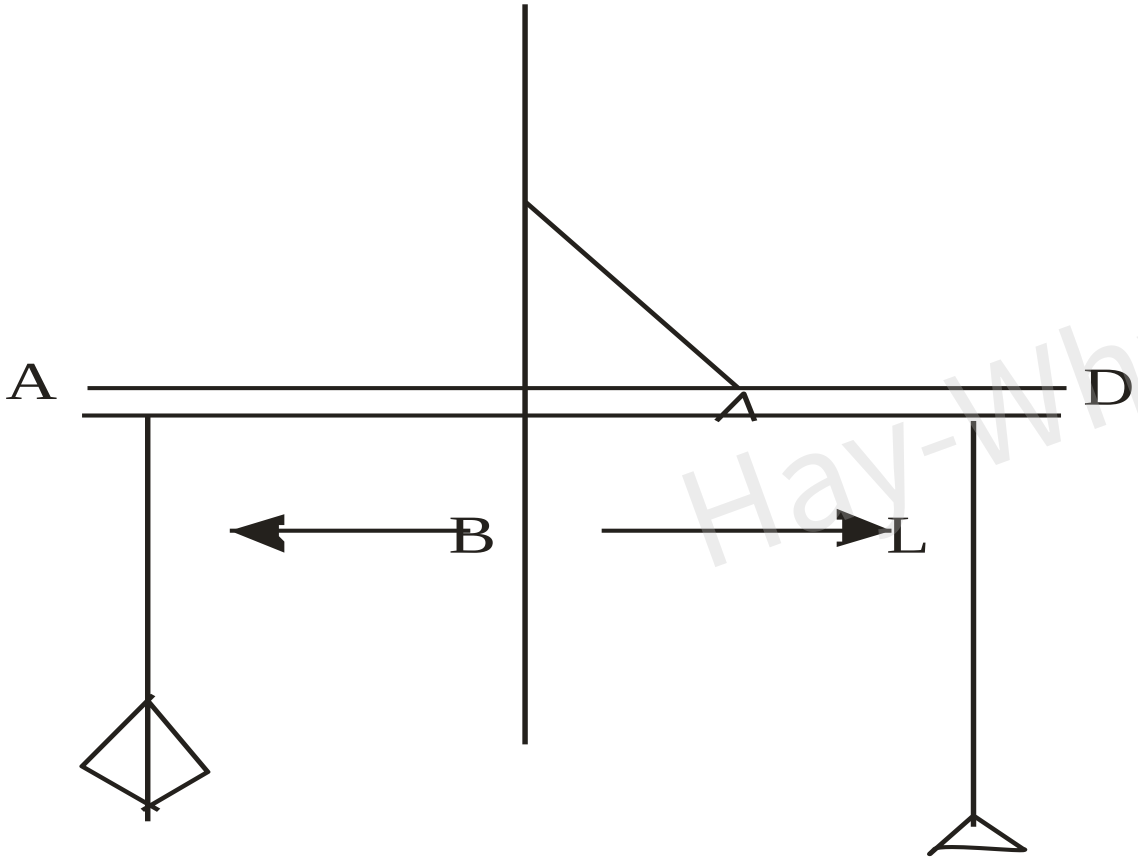
Bridges

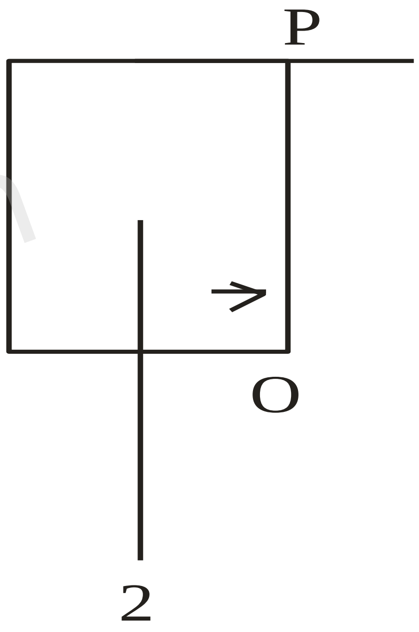
Turning Effect

Opening of door

See – saw

TURNING EFFECTS OF FORCES IS CALLED MOMENTS





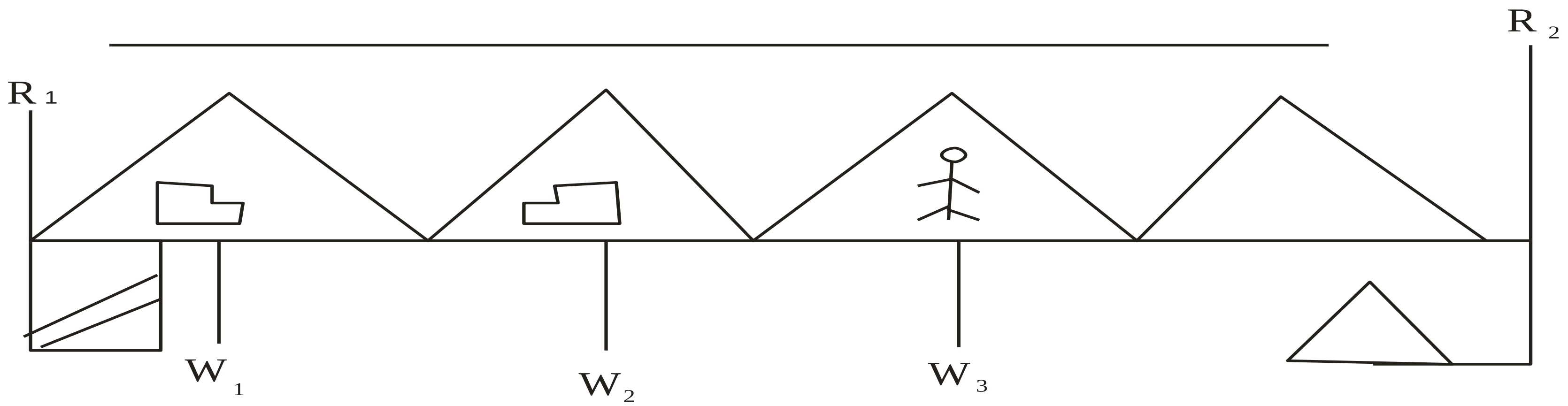
From the above diagram:

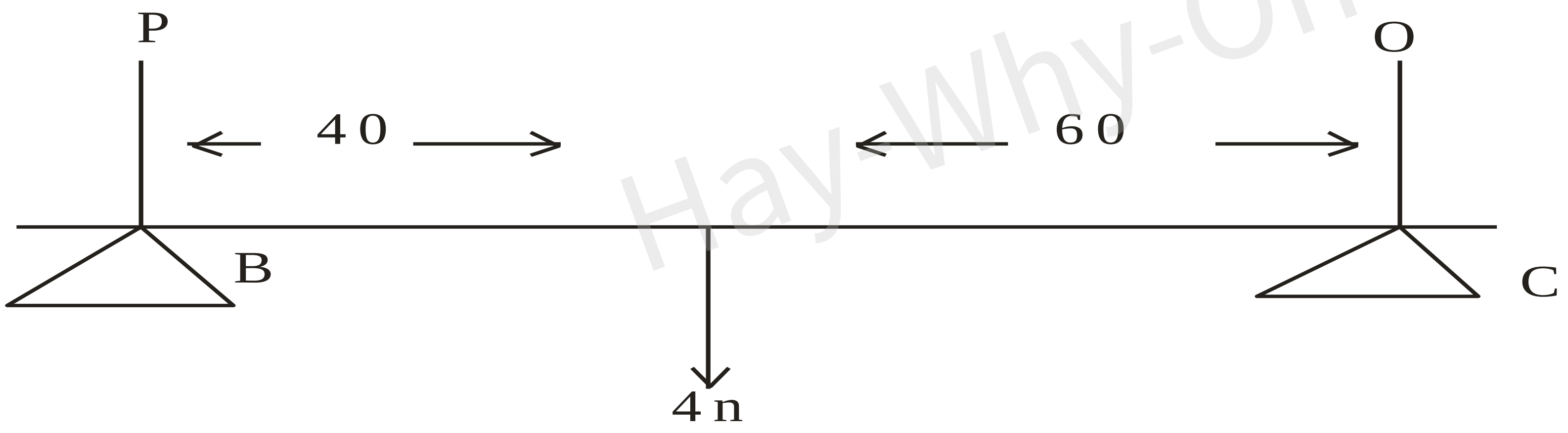
Total force in one direction = Total time in opposite direction

Moment of force about g point or axis 0 in force X perpendicular distance from the point of action.

Anticlockwise moment = @ any point clockwise moment @ the same per.

Effect of perpendicular about on moment





$$P + Q = 4n$$

Total Clockwise moment = Total anticlockwise moment

Anticipate

TEAM SYNERGY

Led By:- Hay-Why-Oh

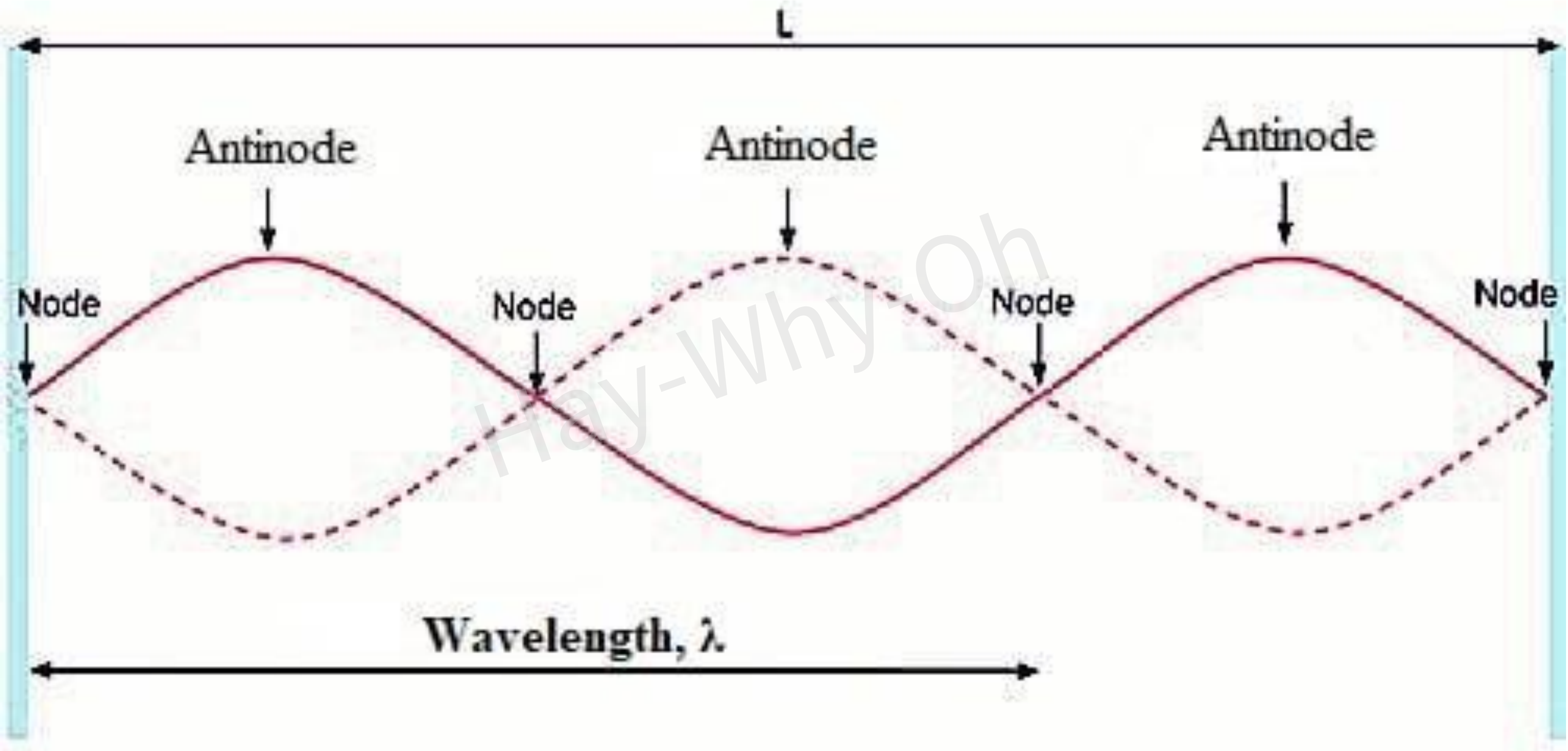
WAVE AND WAVES

PHS 231 Department of Physics

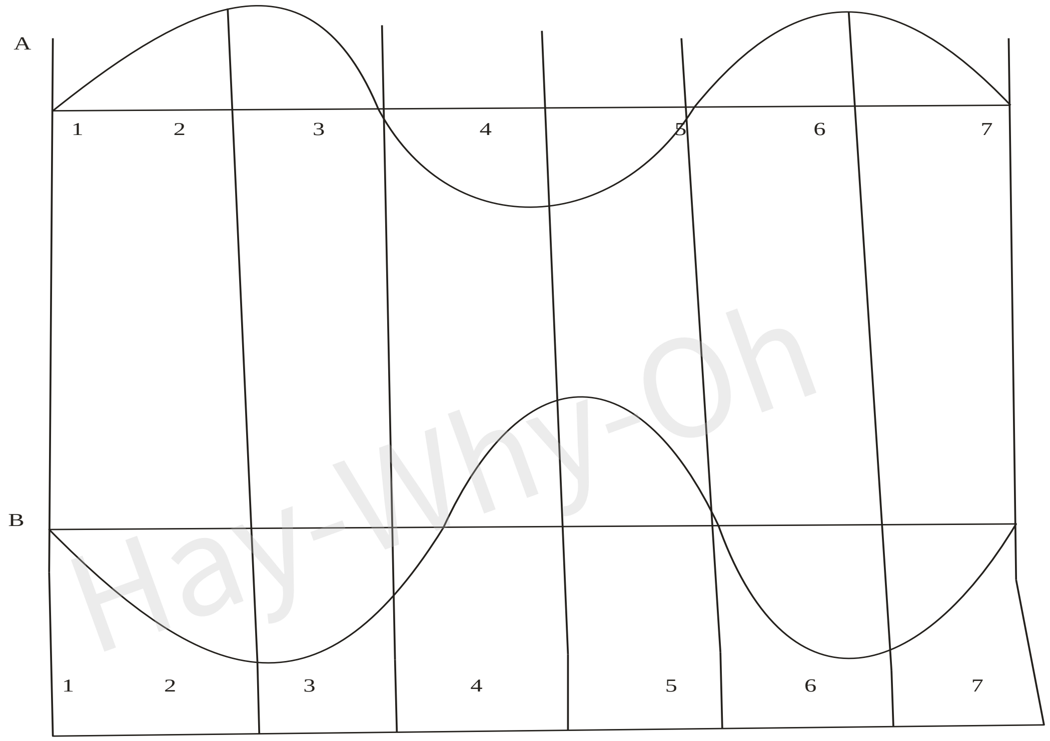
AKINBORO F. G

WAVE OPTICS-
TRANSVERSE STATIONARY WAVES

TRANSVERSE STATIONARY WAVES



TRANSVERSE STATIONARY WAVES



(1)

When two waves with the same amplitude, frequency and time period travel in opposite direction in a straight line, the result wave obtained is called stationary wave

1 2 3 4 5 6 7

Two waves with the same amplitude, frequency and time period travel in opposite direction. At the distance of t = 0 the waves are as shown above, the resultant displacement curve in a straight line.

All the particles is the medium in their mean position at time t = $\frac{T}{4}$the wave A will have advanced through a distance $\frac{\pi}{4}$towards right and B will have advance $\frac{T}{4}$towards the length. The resultant displacement in as

PROPERTY OF STATIONARY WAVES

Stationary waves are form due to superposition of two simple harmonic longitudinal progressive wave of the same amplitude and periodic time and traveling in opposite direction.

The important properties of those waves are

(1) In those -wave's nodes and antinodes are formed alternately. Nodes are the position where the particles are at their mean position having maximum strain. Antinodes are the position where particles vibrate with maximum amplitude and minimum strain.

(2) All the particles except the nodes vibrate harmonically with time period equal to each of the component waves.

(3) The amplitude of the vibration gradually increases from zero to maximum from node to antinodes.

(4) The medium is split into segment and all particle of each segment vibrate in phase.

(5) Condensation and verification do not travel toward as in the case of progressive wave but they appear and disappear alternately at the same place.

(6) Condensation and verification do not travel forward; therefore there are no transfers of energy.

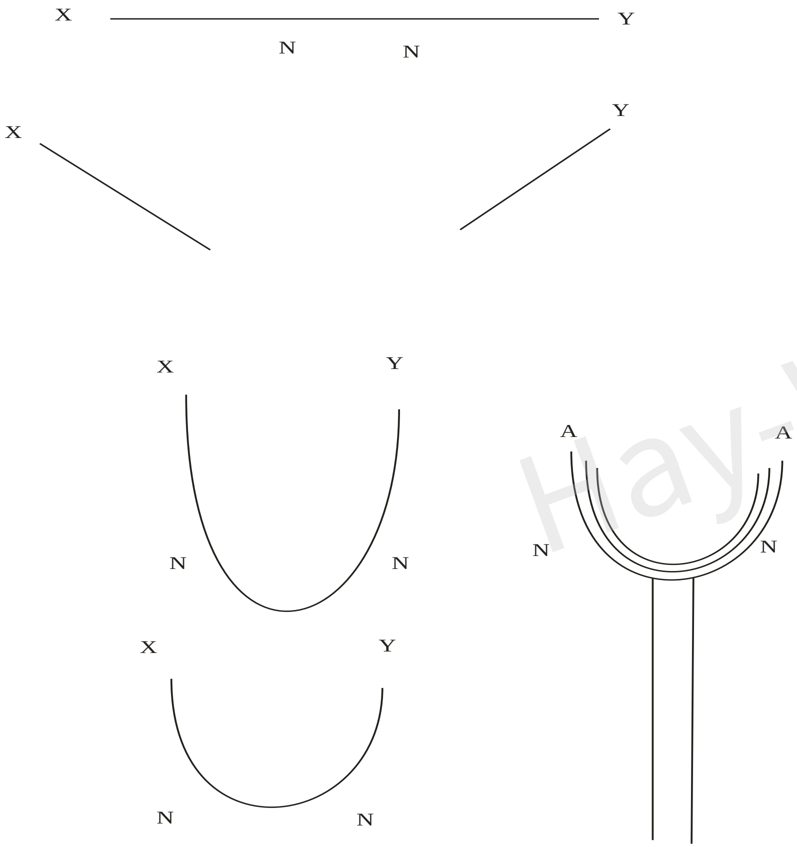
(7) The distance between from adjacent node and antinode is T/4

(8) The appearance of the wave can be represented by a sin e wave but reduces to straight line in each period.

(9) The velocity and acceleration are separated by a distance at any given instance.

(10) In the same segment at the same instance all particles will be in phase and the velocities and acceleration will be maximum or minimum of the same instance

THE TURNING FORK



A tuning fork is a two-pronged metal fork that can be used as an acoustic resonator. Traditionally, this tool has been used to tune musical instruments. Tuning forks work by releasing a perfect wave pattern to match a musician's instrument.

It is a fork-shaped acoustic resonator used in many applications to produce a fixed tone. The main reason for using the fork shape is that, unlike many other types of resonators, it produces a very pure tone, with most of the vibrational energy at the fundamental frequency

ENERGY OF A PROGRESSIVE WAVE

(1) The case of progressive wave, new waves are continuously formed at the end of the wave which means that there is continuous transfer of energy in the direction of propagation of waves.

The energy for the propagation is supplied from source.

(2) The energy transfer per second is equal to the energy processed by the particles in a length U which is equal to the velocity of waves.

(3) The energy of the partly in part of kinetic KE and partly potential PE.

(4) The kinetic energy is due to the velocity of vibrating particle.

(5) For a Particle executing simple harmonic motion the velocity in maximum sit the mean position and minimum at the extreme position and consequently the KE is maximum at the mean position and minimum at the extreme and

(6) The particle vibration with simple harmonic motion has their potential energy due to displacement and maximum at their extreme position and minimum of their mean position.

In longitudinal there is compression and verification, therefore the energy distribution is not limit at the point of no velocity there is no compression and there is no energy and at the point of maximum velocity there is compression hence maximum energy.

Note: In the case of longitudinal wave, there is not transfer of the medium in the direction of propagation but there is always transfer of energy along the direction of propagation

ENERGY OF A PLANE PROGRESSION WAVES

Total Energy = KE + PE

PE = mgh

∴ PE per uni tvolume = egh

Since – Density $= \frac{\text{mass}}{\text{Volume}}$

M = Density x Volume

Volume = 1 unit

∴ If the Density function density/ unit

M = e x 1

G = acceleration = F

Charge on h = dy

$\therefore$ PE = ρfdy ------ (1)

$$k2 = \frac{1}{2} mv2$$

$$= \frac{1}{2} \rho v2 \quad \text{--- (2)}$$

Energy per unit volume

$$\rho f dy + \frac{1}{2} \rho v2 \quad \text{---- (3)}$$

$$y = a\, Sin\, \frac{2\pi}{人}(vt - x)$$

$$u = \frac{dy2}{dt} = \frac{2\pi v}{人}\, a\, Cos\, \frac{2\pi}{人}(vt - x)$$

$$F = \frac{dy2}{dt2} = \frac{4\pi 2}{\pi 2} a\,Sin\ \frac{2\pi}{\text{人}}(vt - x)$$

From 1

PE = per unit volume

$$= \rho \frac{4\pi 2}{\pi 2}\ a\,Sin\ \frac{2\pi}{\text{人}}\ (vt - x)$$

$$e\ \frac{4\pi 2v}{\text{人}}\ ydy$$

Work done destroy the displacement y

$$\int_o^y \frac{4\pi 2}{\pi 2} \rho ay\ dy$$

$$\frac{4\pi 2}{\pi} \rho a\ \int_o^y ydy$$

$$\frac{a4\pi 2}{\pi 2}\ \frac{\rho}{2}\ y^2$$

$$\frac{a2\pi 2}{\pi 2} \rho y^2\ v^2 = PE\ per\ unit\ volume$$

$$\frac{2\pi 2}{\pi 2} \rho v^2 a^2 Sin\ \frac{2\pi}{\text{人}}(vt - x)$$

KE / unit volume

$$\frac{1}{2}mv^2$$

$$= \frac{1}{2}\rho\frac{4\pi}{\lambda}\ a^2v^2Cos^2\frac{eg^2}{2}\ (vt - x)$$

$$\frac{2\pi^2}{\pi^2}\ a^2v^2Cos\ \frac{2\pi}{\pi}\ (vt - x)$$

Energy/ unit volume $= \dfrac{2\pi^2}{\pi}\rho v^2a^2 - 1$

$$V = n\ \lambda$$

$$V = n\pi$$

$$E = \frac{2\pi^2}{\pi^2}\rho s a^2\ v^2$$

$$= \frac{2\pi^2}{\pi^2}\ e\ a^2\ (n\pi^2)$$

$$= \frac{2\pi^2}{\cancel{\pi^2}}\ e\ a^2\ n^2\ \cancel{\pi^2}$$

$$= 2\pi^2\ e\ ^2a^2\ n^2$$

Hence the average kinetic energy per unit volume and average potential energy per unit volume are equal and equal to ½ . The total energy Hence

$$\text{PE} = \pi^2\ e^2\ a^2\ n^2$$

$$\text{KE} = \pi^2\ e^2\ a^2\ n^2$$

$$Sin\ \frac{\pi x}{\pi} = 1 \quad Sin\ \frac{2vt}{\lambda} = 1$$

$$P = P_o v^2 e\ \frac{4\pi a}{\lambda}$$

$$P = P_o\ Sin\ \frac{2\pi x}{\lambda}\ Sin\ \frac{2\pi vt}{\lambda}$$

Take $P_o \, Sin \; {}^{2\pi}/_{\pi} = \; P_o$

$$P = P_x \, Sin \, \frac{2\pi vt}{\pi}$$

Velocity of the particle

$$U = \frac{dy}{dt} = \; \frac{4\pi av}{\pi} \, Cos \, \frac{2\pi x}{\pi} \, Cos \, \frac{2\pi vt}{\pi}$$

Taken

$$U = U_x \, Cos \, \frac{2\pi vt}{\pi}$$

Work done or energy transfer per unit area in a small interval of time dt = Pudt

Total energy transfer in time

$$= \int_o^T pudt$$

Rate of energy transfer

$$\int_o^T \frac{eudt}{T}$$

$$\frac{1}{2}\int_o^T \left(px\ Sin\ \frac{2\pi vt}{\lambda}\right) x\ (U\ Cos\ \frac{2\pi t}{\pi})$$

$$P_x\ Ux\ \int_o^T Sin\ \frac{4\pi vt}{\lambda}\ dt$$

$$\int_o^T Sin\ \frac{4\pi vt}{\lambda}\ dt = 0$$

Rate of energy transfer

$= Ux$

Thus is then of stationary wave no energy is transferred

Anticipate

TEAM SYNERGY

Led By:- Hay-Why-Oh

PHS 105

Course Outline (Mechanics Synopsis)

Linear Motion: (Motion in a straight)

* Measurement, Standard, Unit and Errors

* Displacement, Average Velocity

* Instantaneous Velocity

* Acceleration

* Acceleration of falling Bodies and Gravity

Motion in a Circle: (Circular Motion)

* Centripetal Acceleration

* Centripetal Force

* Inertia Force in Rotation (Moment of Inertia)

* Centrifugal Force

Simple Harmonic Motion

* Periodic Motion (Periodic time, Frequency and Amplitude)

* Dynamics of Simple Harmonic Motion

* Resonance

* Damped and Force Oscillations

Gravitation

* Newton' Law of Universal Gravitation

* Satellites and Weightlessness

* Kepler's Laws

Statics and Hydrostatics

Statics

* Mass, Force and Weight

* Forces in Equilibrium

* Resolution of Forces
* Moment of Forces
* Principles of Moment
* Couple
* General Conditions of Equilibrium

Hydrostatics (Fluid at Rest)

* Fluid, Pressure
* Transmission of Fluid Pressure
* Density, Relative Density, Specific Weight and Specific Gravity
* Pressure in a liquid due to its own weight and pressure measurements

Elasticity

* Stress, Strain
* Young’s Modules, Bulk and Sheer Moduli

Friction, Viscosity and Surface Tension

* Sliding and Static Friction
* Viscosity (Laminar and Turbulent flows)
* Surface Tension and Capillarity

Text Books

* Applied Mechanics ---- Hannah & Hillier
* Physics (principles with applications) ------ Douglas C. Giancoli
* General Physics ----- Sternhein and Kana
* Any other physics text books covering these synopses

Mechanics: Study of Motions of objects and of the forces that affect their motions.

Linear Motion

Physics like many other sciences is largely based on quantitative measurements. A quantitative discussion of motion requires measurements of time and distance, so that we can consider the standards, units and errors involved in physical measurement.

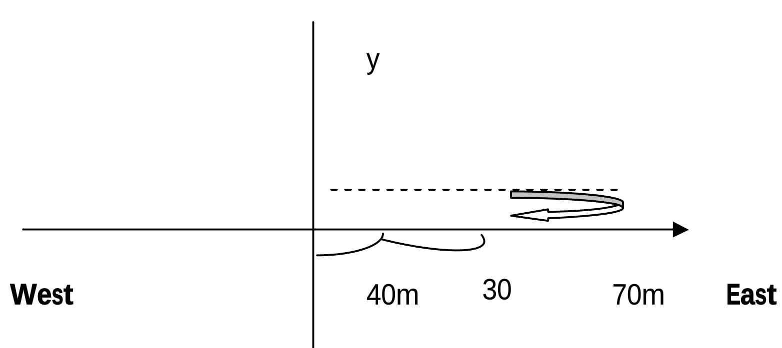
Measurements: Quantitative physical measurements must be expressed by numerical comparison of some agreed set of standards. All measuring devices are calibrated directly in terms of primary standards of L**ength, Time and Mass** as established by international scientific community. All physical quantities can be expressed in terms of some combinations of these three fundamental dimensions, which we denote as **L, T and M** respectively.

Internationally accepted set of metrics units called the **"Systeme Internationale** (S.I) units are: **Metre, Kilogram and Second**, termed as basic units i.e., **"M. K.S" System;** older units are **C.G.S. units.**

Errors: Measurements and predictions are both subject to errors.
Measurement errors are of two types: Random and systematic.

Both errors are present in all experiments and can be reduced by taking the average of many measurements (Random)

Displacement: Change in position of an object or distance between two points in a specified direction. To distinguish between distance and displacement, take for instance, a person walking 70m east and then turn around to walk 30m west.



The total distance walked is 100m but displacement is 40.0m I.e. 40m from the starting point

Velocity and Speed

Velocity: Signifies both magnitude (numerical value) and direction, which makes velocity a vector quantity. Speed: Signifies only magnitude.

Note: Average velocity is defined in terms of displacement rather than total distance travelled.

Hence, average velocity = $\frac{\text{Displacement}}{\text{Time Elapsed}}$

Unit = m/s.

Let $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$

Δx = Displacement and Δt = change in time or elapsed time

$\therefore$ Average velocity $\tilde{V} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

Instantaneous Velocity: Velocity at any instance of time, defined as average velocity over an indefinitely short time interval.

$V_{nst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$, unit is m/s.

Acceleration = $\frac{\text{Change of velocity}}{\text{Time elapsed}}$; the unit is m/s^2

Hence, average acceleration, $\bar{a} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$

Similarly, Instantaneous acceleration is:

$a_{inst} = \frac{\lim}{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

unit: m/s^2

Uniformly Accelerated Motion

This occurs when acceleration remains constant over time. But when acceleration changes, the change is sufficiently small such that we assume it to be constant; thus situation is treated as ***uniformly accelerated motion.*** In this case, instantaneous and average accelerations are equal.

Suppose the initial time $t_1 = 0$, then

$T = t_2$ (time elapsed)

Let the initial position x_1 and initial velocity v_1 be represented as x_0 and v_0

Hence, $\tilde{V}$ (average velocity) = $\frac{x-x0}{t} = \frac{x-x0}{t}$

So, also,

$$a = \frac{v-v0}{t}$$

$$At = v - v_0$$

$$V = V_0 + at \quad \ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

Recall $\tilde{V} = \frac{x2-x1}{t2-t1}$ and that $x_1 = x_0, x_2 = x$

When $t_1 = 0$

$$\tilde{V} = \frac{x-x0}{t}$$

$$\therefore \ x = \tilde{V}t + x_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (2)$$

Since velocity increases at a uniform rate, the average velocity $\tilde{V}$, will be midway between the initial and final velocities, hence

$\tilde{V} = \frac{v0+v}{2}$, now substituting this in equation (2),

We obtain $x = x_0 + (\frac{v0+v}{2})\,t \quad = \quad x_0 + (\frac{v0+v+at}{2})\,t$

$$X = x_0 + v_0t + \frac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

To obtain the velocity v at time t in term of v_0, a, x, and x_0, we begins as

$$x = x_0 + \tilde{V}t = x_0 + (\frac{v+v0}{2})\,t$$

Recall from equation (1) that

$$t = \frac{v + v0}{a}.$$

$$\therefore \varkappa = \varkappa_0 + \left(\frac{v+v0}{2}\right)\left(\frac{v-v0}{a}\right)$$

$$= \varkappa_0 + \frac{v^2 - v_0^2}{2a}$$

$$V^2 = v_0^2 + 2a(\varkappa - \varkappa_0) \quad \ldots\ldots\ldots\ldots (4)$$

Hence, kinematic equations for constant acceleration are:

$V = v_0 + at$	$V = v_0 + a\Delta t$
$\varkappa = \varkappa_0 v_0 + \frac{1}{2}at^2$	$\Delta\varkappa = v_0\Delta t + \frac{1}{2}a(\Delta t)^2$
$V^2 = v_0^2 + 2a(\varkappa - \varkappa_0)$	$V^2 = V_0 + 2a\Delta\varkappa$
$\Delta = \frac{v+v0}{2}$	$\tilde{V} = \frac{1}{2}(v_0 + v)$
	$\Delta\varkappa = \frac{1}{2}(v_0 + v)\Delta t$

Note that these equations are not valid unless (a) is a constant. In many cases $\varkappa_0 = 0$

Illustrations:

1. A car accelerates from rest to 30m/s. What is its average acceleration?

Solution: -

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v+v0}{t-t0} = \frac{30-0}{10} = 3 \text{ m/s}.$$

⟹ an increase in velocity of 3m/s in each second of the 10 seconds time interval.

(2) An object moves according to the formula

$\varkappa = b + ct^{3}$. What is the instantaneous acceleration at time t?

<u>Solution:</u>

1st find v

$$V = \frac{dx}{dt} = \frac{d}{dt}(b + ct^3) = 3ct^2$$

Then, $a_{hnat} = \lim \longrightarrow 0 \frac{dv}{dt} = \frac{d}{dt}(3tc^2) = 6ct$

(3). A car initially at rest at a traffic light accelerates at $2m/s^2$ when the light turns green. After 4 secs, what are its velocity and position?

Solution:

$a = 2m/s^2$, $\Delta t = 4secs$ and v_0

$\therefore$ (i) $v = v_0 + a\Delta t = 0 + 2(4) = 8mls$

(ii) $\Delta x = v_0\Delta t + \frac{1}{2}a(\Delta t)^2 = 0 + \frac{1}{2}(2)(4)^2 = 16m$

(4). A car accelerates from rest with a constant acceleration of $2m/s^2$ onto a highway where traffic is moving at a steady rate of 24m/s.

(a). How long will it take for the car to reach a velocity of 24m/s?

(b). How far will it travel in that time?

(c). The driver does not want the vehicle behind to come closer than 20m nor force it to slow down. How large a break in traffic must the driver wait for

Solution

(a) i. e time need to reach the velocity v= 24m/s

$$V = V_0 + a\Delta t$$

$$= \Delta t = \frac{V + V_0}{a} = \frac{24}{2} = 12sec$$

(b) $\Delta x = V_0\Delta t + \frac{1}{2}a(\Delta t)^2 = 0 + \frac{1}{2}(2)(12)^2 = 144m$

(c) The vehicle behind is moving at a constant velocity $V_0 = 24m/s$, so $a = 0$

$\therefore$ In 12sec., it moves a distance $\Delta x = V_0\Delta t + \frac{1}{2}a(\Delta t)^2$

$$= 24 \times 12 + \frac{1}{2}\, 12 \times 0 = 288m$$

Since the entering car travels 144m in this time, the oncoming vehicles gains (288 - 144) m or 144m, If it is to come no closer than 20m, the break in traffic must be at least (144 + 20) m or 164m.

Assignment 1

A car reaches a velocity of 20m/s with an acceleration of $2m/s^2$. How far will it travel while it is accelerating if it (a) Initially at rest? (b). initially moving at 10m/s.

A train accelerates uniformly from rest to reach 54km/h in 200sec after which the speed remains constant for 300 sec. At the end of this time the train decelerates to rest in 150 sec. Find the total distance travelled.

A baseball pitcher throws a fastball with a speed of 44m/s. It is observed that in throwing the baseball, the pitcher accelerates the bell through a displacement of about 3.5m from behind, estimate the average acceleration of the ball during the throwing motion.

Falling Bodies and Gravity (By Galileo Galilei 1564- 1642)

At a given location on the Earth and in the absence of air resistance, all objects fall with the same uniform acceleration due to gravity , denoted by $g = 9.8m/s^2$, downward. "g" varies slightly due as a result of changes in latitude, elevation and density of local geological features.

When dealing with freely falling objects, we make use of the same equation as described in kinematic by replacing a with g and since the motion is vertical, we put y in place of x, y_0 in place x_0.

Note: It is arbitrary whether we choose y to be position in the upward direction or in the downward direction, but we must be consistent about it throughout a problem's solution.

The equations for falling bodies will be

$$V = v_0 + gt$$

$$Y = y_0 + v_0t + \frac{1}{2}gt^2$$

$$V^2 = V_0^2 + 2g\,(y - y_0)$$ and taken $a = +g$ (as downward).

Example: Suppose that a ball is dropped from a tower 70.0m high, how far it will have fallen after 2.0 sec.

$a = g = +9.8m/s^2$ since we have chosen downward as +ve

$\therefore V_0 = 0, y_0 = 0$

$y = 0 + 0 + \frac{1}{2} gt^2 = \frac{1}{2}(9.8) \times 2^2 = 19.6m$

Now suppose the ball in the above is thrown downward with a speed of 3.0mls instead of being dropped, what they would be its position and speed after 2.0 sec?

Solution:

$V_0 = 3.0m/s$ and $t = 2.0Sec$, $y_0 = 0$

$y = V_0t + \frac{1}{2} gt^2 = 3 \times 2 + \frac{1}{2} \times 9.8 \times 4$

$= 6 + 9.8 \times 2 = 25.6m$

Its speed after 2.0 Sec,

$V = V_0 + gt = 3.0 + 9.8 + 2 = 22.6m$

Example: A ball is thrown upward into the air with an initial velocity of 15.0m/s Calculate (a) How high it goes (b) How long the ball is in the air before it comes back to his hand.

Solution

Let y be +ve in upward direction and –ve in the downward direction.

Note the difference in convention) i.e. $a = -9.8mls^2$

So, to determine d highest height, $V = 0$, and $V_0 = 15.0m/s$

(1) $V^2 = v_0^2 + 2gy$

$$Y = \frac{v^2 - v02}{2g} = \frac{0 - (15)^2}{2(-9.8)} = 11.5m$$

(2) $y = y_0 + V_0t + \frac{1}{2} gt^2$, $y_0 = 0$

$y = 15t + \frac{1}{2}(-9.8) t^2$

$y - y_0 = V_0t + \frac{1}{2} gt^2$

Displacement not total distance travelled

$0 = 15.0 t + \frac{1}{2}(-9.8) t^2$

$= 15t - 4.9t^2 = 0$

$= t(15 - 4.9t) = 0$

$t = 0$ or $t = \frac{15}{4.9} = 3.06 Sec$

t = 0 corresponds to initial point (A) and y = 0, while t = 3.06sec corresponds to C when the ball has returned to y = 0 i.e ball is in the air.

Assignment 2

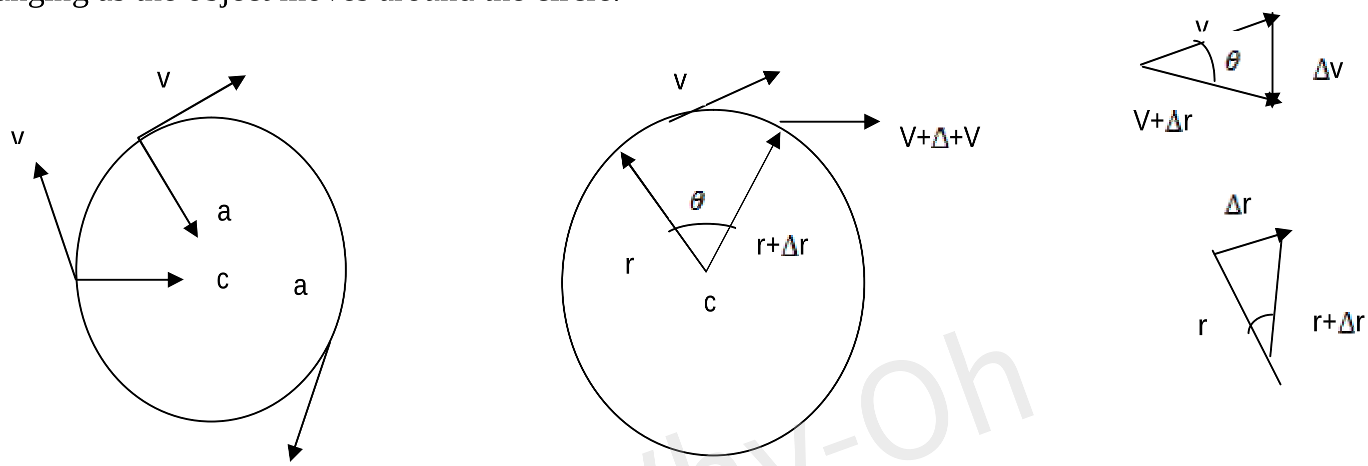
1. A ball thrown upward with velocity of 15.0mls, (a) how much time it takes for the ball to reach the maximum height. (b) The velocity of the ball when it returns to the thrower's hand. (c) at what time t the ball passes a point 8.0m above the ground.

2. A ball is dropped from a window 84m above the ground, (a) when does the ball strike the ground? (b) What is the velocity of the ball when it strikes the ground?

(3) A ball is thrown upward at 19.6mls from a window 58.8m above the ground (a) How high does it go? (b) When does it reach its highest point? (c) When does it strike the ground?

2.0 Circular Motion

An object moves in a straight line if the net force on it acts in the direction of motion or is zero. If the net force acts at an angle to the direction of motion or is zero. If the net force acts at an angle to the direction of motion at any moment, the object moves in curved paths. An object that moves in a circle at constant speed V is said to experience ***Uniform circular Motion.*** The magnitude of the velocity remains constant but the direction is continuously changing as the object moves around the circle.



2.1 Centripetal Acceleration:

"Centre-Seeking" acceleration or radial acceleration (it's directed along the radius, towards the centre of the circle, denoted by a_r.

Consider the above diagrams, since they are similar (isosceles Δ) then their sides are proportional i.e.

$$\frac{1}{v}\left|\Delta v\right| = \frac{1}{r}\left|1r\right|,\quad \text{now divide by } \Delta t,$$

$$\frac{1}{v}\left|\frac{\Delta v}{\Delta t}\right| = \frac{1}{r}\left|\frac{\Delta r}{\Delta t}\right|,\quad \text{taking the limit } \Delta t \longrightarrow 0,$$

(Instantaneous acceleration and velocity), hence $\frac{1}{v} a_r = \frac{1}{r} v$

$$\frac{a_r}{V} = \frac{v}{r},\qquad a_r = \frac{v^2}{r}$$

, called the centripetal acceleration

Comments: Acceleration varies inversely with the radius, the smaller the circle the greater the acceleration. It also varies as V^2, i.e. it increases rapidly with the speed.

Example:

(1). A 150g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.6m. The ball makes exactly 2.0 revolutions in a second; what is its centripetal acceleration?

$$V = \frac{2\pi r}{t}$$

$$= \frac{2(3.14)(0.6)}{0.5} = 7.54\text{m/s}$$

$$\therefore \quad a_r = \frac{V^2}{r} = \frac{(7.54)^2}{0.6} = 94.8\text{m/s}^2$$

Note: The period T of an object revolving in a circle is defined as the time required for one complete revolution.

$$V = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}$$

(2). The moon is nearly circular orbit about the Earth has a radius of about 384,000km and a period of 27.3days. Determine the acceleration of the moon towards the earth.

Solution: To orbit the earth, the moon travels a distance $2\pi r$,

$$r = 3.84 \times 10^8\text{m}$$

$$\therefore \quad v = \frac{2\pi r}{T} = \frac{6.28 \times 3.84 \times 10^8}{27.3 \times 24 \times 60 \times 60}$$

$$= 1.02 \times 10^3\text{mls.}$$

$$\text{Hence } a_r = \frac{v^2}{r} = \frac{(1.02 \times 10^3)^2}{3.84 \times 10^8} = 2.72 \times 10^{-3}\text{mls}^2$$

2.2: <u>Centripetal Force</u>: From Newton's 2nd, law, F= Ma, an object that is accelerating must have a net force acting on it. Therefore, for a ball on the end of a string, moving in a circle must have a force applied to keep it moving in that circle.

Since, $a_r = \frac{v^2}{}$ $\therefore$ Centripetal force is $F = \frac{Mv^2}{}$

r r

Since, a_r is directed towards the centre at any moment, the net force too must be directed toward the centre of the circle.

(In vector form, F= - M v^2 ř, ř = ⟹ is a unit vector in that direction

r r

Example: A car travels on flat circular track of radius 200m at 30m/s and has a centripetal acceleration a_r = 4.5m/s^2.

(a) If the mass of the car is 1000kg, what frictional force is required to provide the acceleration?

(b) If the co-efficient of static friction $\aleph_s$ is 0.8, what is the maximum speed at which the car can circle the track?

Solution

(a) Mass = 1000kg, a = 4.5m/s^2

F = Ma_r = 1000 x 4.5 = 4500N

(b) W = mg (i.e normal force N)

∴ Frictional force possible is N_s N= $N_s$mg

∴ $\frac{mv^2}{r}$ = N mg

= $\sqrt{Nsrg}$

= (0.8x200x9.8) = 39.6m/s

$\frac{F}{N}$ =N

F = u N

F= U_SN =$N_s$mg

Digression

Comment: If the driver attempt to exceed 39.6m/s, the car will not be able to continue on the circular course and it will skid off.

2.3 Moment of Inertial

Moment: Turning effect of a force about an axis. And Torque is just equal and opposite forces. Despite the fact that T = I ∝ is similar in form to F = Ma, it is important to realize that both the torque **T** and moment of inertial **I** depend on the position of the axis of rotation. **"I"** also depends on the shape and mass of the rotating object.

To calculate the moment of inertial of a complex object, we must initially separate the object into N small pieces of mass m_1, $m_{2,}$ m_3 , ………$m_{N,}$ with each piece having distance $r_{1,}$ r_2, r_3, ………., r_N from the axis of rotation.

∴ Moment of inertial I for first piece is

$$I = m_1r^2 + m_2\,r_2^2 + \ldots\ldots\ldots\ldots + M_Nr_N^2$$

$$\sum_{i=1}^{N} m1r2 \ ,$$

Note that I is large when the force are far from the axis of rotation.

When the masses are arbitrarily small, the sum becomes an integral, given by

$I = \int r2dm$, so, for several shape and sizes, we have different moments of inertial. For instance, a uniform disk or cylinder of radius R rotating about the axis,

$I = \frac{1}{2} mR^2$ and for a rod of length **l** rotating about the centre,

$I = \frac{1}{12} ml^2$

<u>Example:</u> Two equal point masses M_0 are at the ends of a mass less than bar of length **l**. Find the moment of Inertial for an axis perpendicular to the bar through (a) the centre (b) an end

Solution (a): For an axis through an end, the mass at that end has r = 0 while the other mass is at a distance **l**, so

$$I = M_0\left(\frac{l}{2}\right)^2 + M_0\left(\frac{l}{2}\right)^2 = \frac{M_0l^2}{2}$$

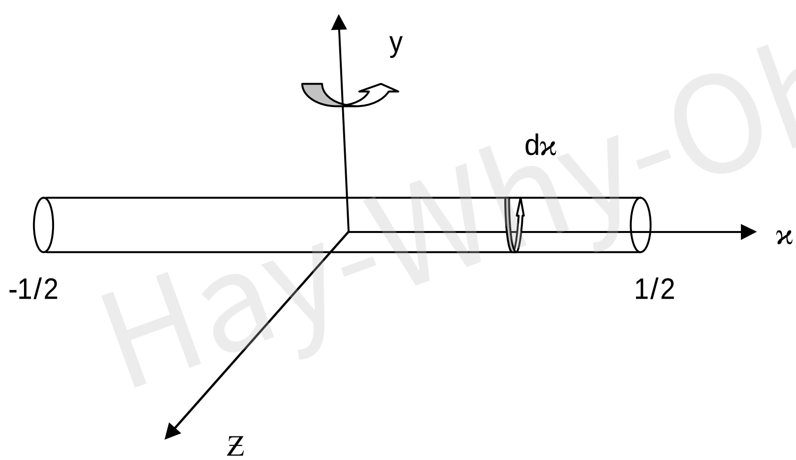
(b) For an axis through an end, the mass at that end has r = 0 while the other mass is at a distance l, so, $I = 0 + M_0\,l^2 = M_0l^2$

Comment: This shows that moment of Inertial depends on the position of the rotation axis.

2.4 <u>Centrifugal force:</u> This is equal and opposite to the centripetal force and therefore acts radially outwards. It is seen to be due to the tension in a cord, required to provide the motion in a circle.

Assignment 3

1. A 1000kg car rounds a curve on a flat road of radius 50m at a speed of 50km/h (14m/s). Will the car make the turn, or will it skid, if (a) the pavement is dry and the coefficient of static friction is $N_s = 0.25$?
2. A racing car starts from rest in the pit area and accelerates at a uniform rate to a speed of 35m/s in 11Sec; moving on a circular track of radius 500m. Assuming constant tangential acceleration, find (a) the tangential acceleration and (b) the centripetal acceleration when the speed is 30m/s.
3. Find the moment of Inertial of a thin rod of length **l** and mass m about an axis through its centre.



Simple Harmonic Motion

When an object moves back and forth repeatedly over the same path, it is said to be oscillating or vibrating. Examples are a Sheldon or swing, pendulum clock, violin string etc. S.H.M is characterized by several quantities like (1) Amplitude (maximum displacement of the oscillating object from equilibrium). Cycle (complete oscillation back and forth), Period T (time required for one complete oscillation). Frequency F (the number of cycles in a unit time).

In general, the period T and frequency F are related by $F = \frac{1}{T}$ in H_z

Now consider an object at the end of a coil spring, when displaced from its equilibrium position and released, the resulting oscillating motion is referred to as <u>simple harmonic motion</u>. The position, velocity and acceleration are related in a specific way which we now determine.

When a coil spring is stretched by application of force, the logarithm x and the applied force F are proportional.

$F = k\varkappa$, k is called the spring constant.

The spring exerts a restoring force that is opposite in direction

$F_r = -k\varkappa$

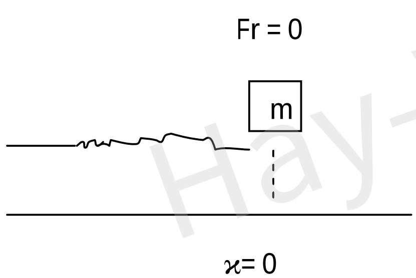
-ve sign indicates that the restoring force is always opposite to displacement

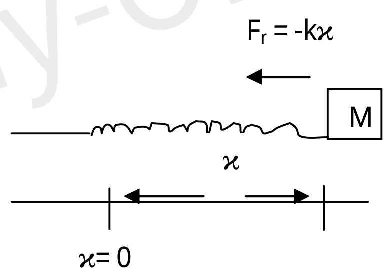
Take for instance, a mass resting on a frictionless table attached to a spring. Suppose the mass is pulled from its equilibrium point and it is released, then it moves under the influence of restoring force.

$\therefore$ F= ma

Definition of

$-kx = ma$, $a = -\frac{kx}{m}$ (simple harmonic motion)





Now recall that, $v = \frac{dx}{dt}$ and $a = \frac{dv}{dt} = \frac{d2x}{dt2}$

Hence $a = -\frac{kx}{m}$ $\Rightarrow$ 2^{nd} derivative of $\varkappa$ is proportional to $-\varkappa$

i.e $\frac{d2x}{dt2} = -\frac{k}{m}\varkappa$ by comparism

$\therefore$ $\varkappa = -\varkappa$ Two functions that have this properly are sines and cosines

For instance,

$\varkappa$ = A co scot, where A and W are constants to be determine shortly.

$\frac{d}{dt}$(cos ωt) = - ωsm ωt, V = $\frac{dx}{dt}$ = - A $\omega \sin \omega t$

Similarly, $\frac{d}{dt}$ (smωt) = $\omega \cos \omega t$

$\therefore$ a = $\frac{dv}{dt}$ = A ω^2 cos ωt

Recall S.H.M., equation, a = $\frac{-k}{m}$x, so by compassion

$\frac{-k}{m}$x = - A ω^2cos ωt = - ω^2 $\frac{A \cos \omega t}{x}$

$\therefore$ $\frac{-kx}{m} = -\omega^2$x

$\omega^2 = \frac{k}{m}$ or $\omega = \sqrt{\frac{k}{m}}$

A and ω A are the amplitude, maximum displacement in either direction from the equilibrium position.

Since $\omega = 2\lambda f$ and $f = \frac{1}{T}$

$\therefore$ $\omega = \frac{2\lambda}{T} = 2\lambda f$

Or $f = \frac{1}{T} = \frac{\omega}{2\lambda} = \frac{1}{2\lambda}\sqrt{\frac{k}{m}}$

Question: An object has a mass of 0.1kg and is on a flatless table. If a 5N force is applied, the spring in stretched 0.2m,

(a) What is the spring constant? (b) Find the characteristic frequency and period of oscillation that the mass is set in motion.

Solution:

a) F = kx = k = $\frac{f}{x} = \frac{5}{0.2} = 2.5\ N/m$

b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25}{0.1}} = 2.52.Hz$, T = $\frac{1}{f} = \frac{1}{2.52} = 0.397s$

$= 0.397 sec$

3.1 Energy in Simple Hamonia Motion

In S.H.M., like pendum, there is a continual interchange of potential and kinetic energy, i.e. when the pendulum is at its highest point, the velocity is zero and the energy is entirely potential.

Simply, when a mass oscillates on a spring, the total energy is constant and there is also a continual interchange of potential and kinetic energy. It is convenient to define potential energy to be zero at the equilibrium point. As the mass passes through **X** = 0, its energy is entirely kinetic.

The potential energy at a displacement **X** is equal to the work that must be done against the restoring force to stretch the spring to that extent.

Hence a displaced object, work done by a force **F** is $\int frds$ and the required force to stretch a spring is F = kx. Hence, work done in stretching the spring from 0 to R is

$$\omega = \int_0^x f dx = \int_0^x k_x d_x = \frac{1}{2} kx^2 =$$

∴ Potential Energy $u = \frac{1}{2} kx^2$

Total energy = $P.E + K.E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$

Example: A mass of 2kg on a spring is extended 0.3m from the equilibrium position and released from rest. The spring constant is 65N/m

(a) What is the initial potential energy of the spring?

(b) What is the maximum speed of the mass after it is released?

(c) Find the speed when the displacement is 0.2m

Solution (a) initially the displacement is 0.3m, so $u_0 = \frac{1}{2} kx^2$

$$u_0 = \frac{1}{2} kx^2 = \frac{1}{2} \times 65 \times (0.3)^2 = 2.92J$$

(b) The energy is totally kinetic when the spring and the mass passes through the unstretched position x = 0. So the K.Σ $\frac{1}{2} mv^2$

$$\frac{1}{2} mv^2 = u_0$$

$$v = \sqrt{\frac{2u_0}{m}} = \sqrt{\frac{2\,(2.92)}{2}} = 1.71\ m/s$$

(c) When x = 0.2m, potential and kinetic energies are non–zero since total energy is conserved:

$$\therefore \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = u_0$$

$$v = \sqrt{\frac{2}{m}}\left(u_0 - \frac{1}{2}kx^2\right)$$

$$= \sqrt{\frac{2}{2}}\left(2.92 - \frac{1}{2}(65)(0.2)^2\right. = 1.27\,m/s$$

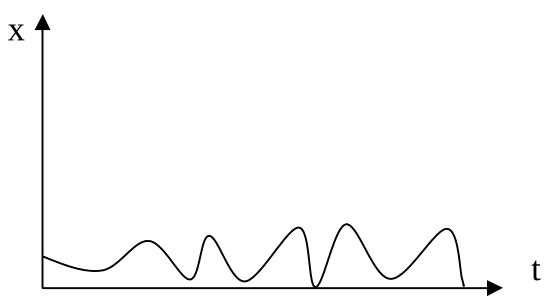
3.2. Damped Oscillation

Most real situation cannot be described precisely by the equations of S.H.M. because of the presence of dissipative forces such as friction or air resistance. For instance, a pendulum clock will gradually come to rest unless energy is supplied to replace the losses.

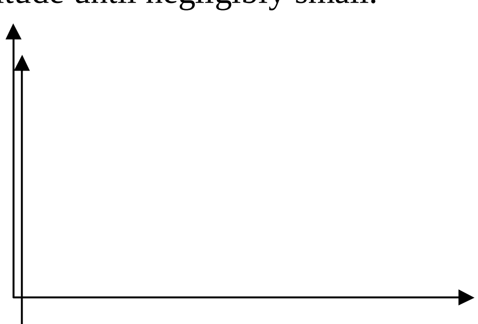
Damping is caused by dissipative forces, typically dependent on the velocity. Dissipative force $\mathbf{F_d}$ is linearly proportional to V i.e. F_d = rv, r = damping constant, while the minus sign indicates that the damping force opposes the motion.

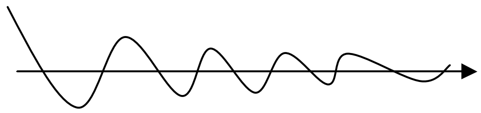
Now consider the effect of damping force in the equation of motion for a weight on a spring:

When r = 0, the oscillation continues with same amplitude indefinitely.

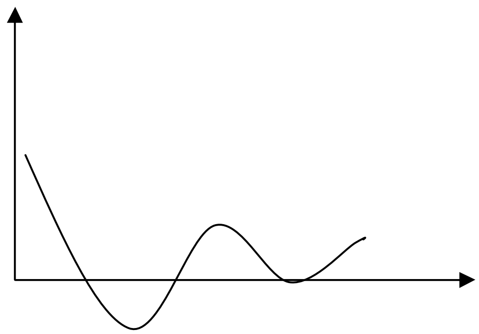


When a small amount of damping is present, oscillation is steedily decreases in amplitude until negligibly small.





If r is larger, then the oscillation is faster



But when very large, oscillation cannot occur and the body/weight returns to its equilibrium position without oscillation.

3.3 Forced Oscillation and Resonance

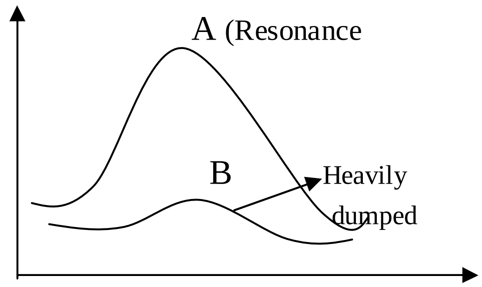
When a vibrating system is set in motion, it vibrates at its natural frequency. However, a system is often not left to merely oscillate on its own but may have an external force applied to it, which itself oscillates at a particular frequency.

For instance a mass on a spring when pulled, vibrates back and forth at a frequency f, the mass then vibrates at the frequency f of the external force, even if this frequency is different from the natural frequency of the spring, which we denote as f_0, where f_0 is:

$f_0 = \frac{1}{2x}\sqrt{\frac{k}{m}}$, this is an example of forced oscillation.

The amplitude of vibration and hence, the energy transferred into the vibrating system is found to depend on the difference between f and f_0, its maximum when the frequency of the external force is equal to the national frequency of the system.

i.e., f=f_0,
the amplitude can become large when the driving brief.
For lightly damped F, near the natural frequently ,
$f \approx f_0$. When the damping is small, the increase in amplitude near f = f_0 is very large. This effect is Known as **Resonance.** The natural vibrating



frequency f_0 of a system is called its resonant frequency, f_0, frequency.

Assignment 4

1. A spring stretches 0.150m when a 0.300kg mass is hung from it. The spring is then stretched an additional 0.100m from its equilibrium point and released.

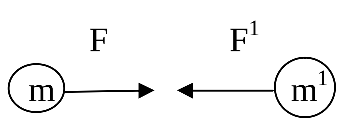
   Determine (a) the spring constant k (b) the amplitude of the oscillation A (c) the maximum velocity V_0 (d) the velocity v when the mass is 0.050m from equilibrium and (e) the maximum acceleration of the mass.

2. For a S.H.O. determine, (a) the total energy (E), the kinetic and potential energies of half amplitude: $\left(x = \pm \frac{A}{2}\right)$.

4.0 Gravitation

Newton's study on planetary motion has led to inferring a formular for gravitational force between two masses. This formula is termed the law of universal gravitation (law of nature). It states that for two uniform spheres of two objects of any shape that are so small compared with their separation, that, they may be considered as point particles, the law has a simple form. If two spheres or particles have gravitational masses m &m1 and their centres are separated by distance r, then the forces between the two spheres have a magnitude.

$F = \frac{Gmm^1}{r^2}$



G = Gravitational constant = 6.67 x 10^{-11} NM ^{2}kg $^{-2}$

Since the magnitude of the gravitational force varies as $\frac{1}{r^2}$,

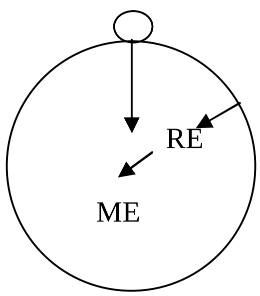
Then the law is called **Inverse Square Law**

Every particle in the universe attracts every other particle with a force that is proportional to the square of the distance between them. This force acts along the path of the two particles.

4.1 Weight

Weight of an object is the gravitational force it experiences. For an object on the surface of the Earth, its force is mainly due to the earth's attraction.

Consider an object with gravitational mass m at the surface of the earth, subjected to a gravitational force, **F,** by law of universal gravitation.



RE = Radius of the earth = 6400km

$$\therefore F = \frac{G_m M_E}{R^2{}_E}$$

Since F = m_a (Newton's gud law)

$$m_a = \frac{G_m M_E}{R^2{}_E}$$

a = g

$$m_g = \frac{G_m M_E}{r^2{}_E}$$

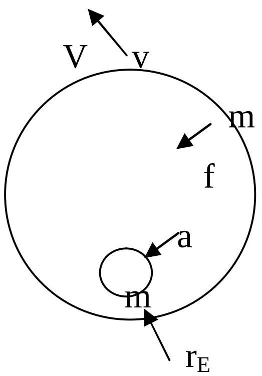
$g = \frac{G\ M_E}{R^2{}_E}$ i.e, gravitational acceleration is the same for all objects.

<u>Satellites & Weightlessness</u>

A satellite is put in an orbit by accelerating it to a sufficiently high tangential speed with the use of rocket. For instance in circular motion, satellites are usually put into circular orbits because they require at least take-off speed. Therefore, for satellites that move in orbit, it acceleration is, $\frac{v^2}{r}$, Hence recall that F = $\frac{G_m M_E}{r^2{}_E}$ and since

$a = \frac{v^2}{r}$ and F = $m_a = \frac{mv^2}{r}$

$\therefore \frac{G_m M_E}{r^2{}_E} = \frac{mv^2}{r}$



m = mass of satellite

r = sum of the Earth's radius

r_E plus the satellite's height h

above the Earth: r = r_E th

V_v = velocity of the orbit

V_{orb} $\sqrt{\frac{GM_E}{r}}$

Note: The mass of the satellite does not appear and the orbital speed decreases as the radius of the orbit increases.

Since velocity = $\frac{displacement}{Time}$ and displacement = 2πr, T = $\frac{2\pi r}{V_{orb}}$

Hence the period of the orbit is:

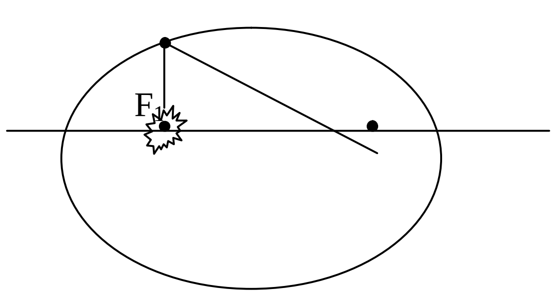
T = $\frac{2\pi r}{V_{orb}}$ = $\frac{2\pi r}{\sqrt{GM}}$ x $\sqrt{r^3}$

Square both sides

$T^2 = \frac{4\pi^2}{GM}$ x $r^3 = kr^3$, This is called the **Kepler's third law,** which states that the square of the period of the orbit is proportional to the cube of the radius of the orbit.

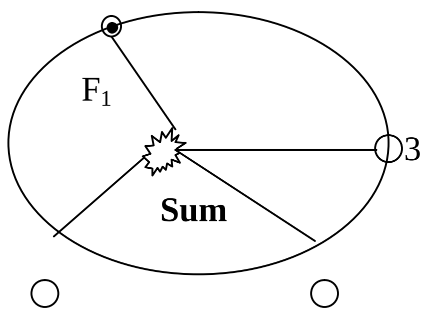
Summary of Keepler's Laws (Laws of Planetary Motion)

First Law: The path of each planet about the sum is an ellipse with the sun at one focus.



Sum F_2

Second Law: Each planet moves so that an imaginary line drawn from the sin to the planet sweeps out equal areas in equal periods of time



5.0 Statics and Hydrostatics

Statics: Study of forces in equilibrium

5.1 Mass, Forces and Weight

Just give definitions and then read-up:

5.2 Forces in Equilibrium

A single force cannot exist alone and is unbalanced. For equilibrium, it must be balanced by an equal and opposite force acting along the same straight line. Forces may be said to exist in pairs, however, a single force may also be balanced by any number of other forces.

Conditions for Equilibrium:

(1) Find a body to be at rest, the sum of the forces acting on it must add up to zero. Hence, if the forces on the object act in a plane, a condition for equilibrium is that $\sum F_k = 0$, $\sum F_y = 0$, and if it acts in 3-dimension, $\sum F_z = 0$,

(2) The sum of the torgues acting on a body must be zero

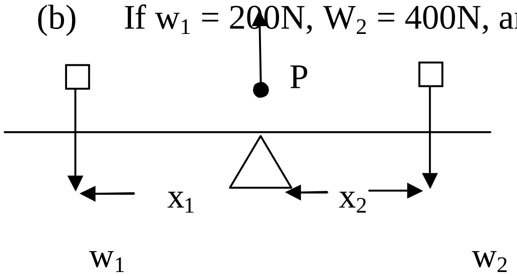
$$\sum \tau = 0$$

These two conditions ensure that a rigid body will be in both translationl and rotational equilibrium.

Example: Two weights w_1 and w_2 are balanced on a board pivoted about its centre: (

(a) What is the ratio of their distance $\frac{x_2}{x_1}$ from the pivot?

(b) If w_1 = 200N, W_2 = 400N, and x = 1m



X_1 = 1m, what is x_2?

(assume the board to be weightless)

Solution: First law of equilibrium condition, N force exerted by the support must balance their weights such that the net force is zero.

i.e. N = $w_1 + w_2$ ($N - w_1 - w_2 = 0$)

so, tongue about each weight: $\tau_1 = x_1\, w_1$

$$\tau_2 = x_2\, w_1$$

Hence $\tau = \tau_1 + \tau_2 = 0$

$x_1\, w_1 - x_2\, w_2 = 0$

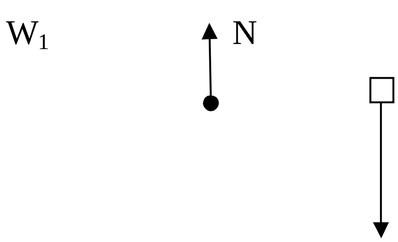
$x_1\, w_1 - x_2\, w_1$

$\frac{x_2}{x_1} = \frac{w_1}{w_2}$

(b) If w_1 = 200N, w_2 = 400N, $^0x_1$ = 5.0m

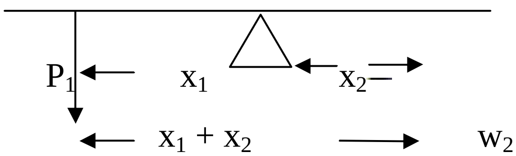
$$x_2 = x_1 \frac{w_1}{w_2} = \frac{200}{400} = 0.5\text{m}$$

Example 2: Again, find $\frac{w_1}{w_2}$ when the pivot is at the centre, i.e, torgue about point P, where w_1 is placed.



Torgue about (P_1), (N and w_2)

$x,\ N - (x_1 + x_2)\, w_2$



Smile, sum of torgues must be zero

$$(x_1 + x_2)\, w_2 + x_1 \mathrm{N} = 0$$

Again, the sum of forces must be added up to zero

$N - w_1 - w_2 = 0$

$N = w_1 + w_2$

∴ Putting N,

- $(x_1 + x_2)\, w_2 + (w_1 + w_2)x_1 = 0$

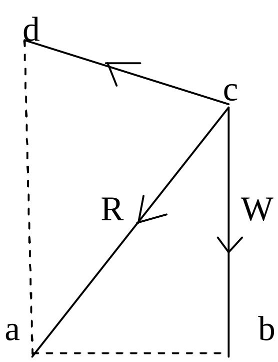
- $(x_1 + x_2)\, w_2 + (w_1 + w_2)x_1 = 0$

- $w_1 \; x_1 = x_2 \; w_2$

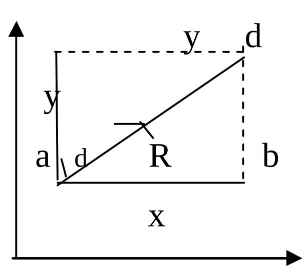
- $\frac{x_2}{x_1} = \frac{w_1}{w_2}$

Comment: No matter whereever P is, same answer is obtained.

5.3 Resolution of forces



Forces ab and ad can be replaced completely by a single force Ac i.e. to replace a single force by two other forces in any convenient direction. These two forces are known as the Components of the single force.



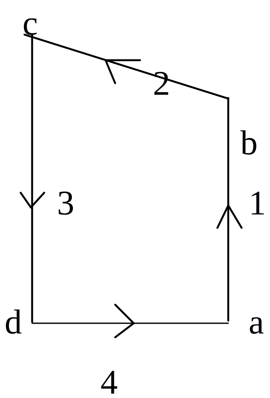
c

ab = ac cosθ i.e. x = R cos θ and d = ac sinθ i.e. Y = RSin θ

Polygon of Forces

If more than three forces act at the same point and are in equilibrium, they may be represented in magnitude, sense and direction by the sides of a polygon “taken in order”

1,2,3,4 are represented by ab, bc, cd and da



Hyrostatics (Fluid at Rest)

Preamble: Matter consists of 3 states: Solid, Liquid and Gases.

Fluid: Liquid and gases (has definite volume, but no define shape while gas has neither, definite, shape or volume.

Pressure: Study of fluid mechanics involves density of a substance (defined as mass per unit volume).

If **F** is the magnitude of the normal force on the paston and **A** is the surface area of the piston, then the pressure, **P** of the fluid at the level to which the device has been submerged, is defined as the ratio of force to area.

Pressure = $\frac{Force}{Area}$

Suppose the normal force exerted by the fluid is **F** over a surface element of area δA, the pressure at that point is:

$P = \lim_{\delta A \to 0} \frac{F}{\delta A} = \frac{dF}{dA}$. Unit is N/m^2 (Pascal P_a)

Transmission of Fluid Pressure

Pressure increases linearly with depth. Consider a liquid of density **P** at rest and open to the atmosphere as in the figure below. A sample of liquid in a cylinder of cross-sectional area, A, extending from the surface of the liquid to a depth **h**, pressure exerted by the fluid on the bottom face is **P** and on the tip is $\mathbf{P_o}$, hence, upward force is PA and downward force exerted is $\mathbf{P_oA}$.

Mass of liquid in the cylinder is m = Pv = PAh.

Weight w of the liquid in the cylinder is W = mg = Pvg = for the cylinder to be in equilibrium, upward force must be greater than the downward force.

$PA – P_oA = PghA – A (P – P2) = PghA$

$P = P_o + Pgh$

P_o = l atm pressure ≈

$1.01 \times 10^5 P_a$

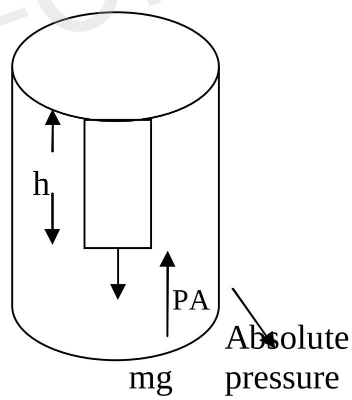


fig.

Pascal’s Law

A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the liquid and to the wall of the container i.e. pressure at every point in a liquid is the same.

Fluid Dynamics (Fluids in Motion)

This is the study of properties of a fluid as a function of time. Fluid in motion in characterized in two main types: Steady or Laminar and Non-Steady or Turbulent.

Steady or Laminar: If each particle of the fluid follows a smooth path such that different particle never cross each other. In this case, the velocity of the fluid at any point remains constant in lime.

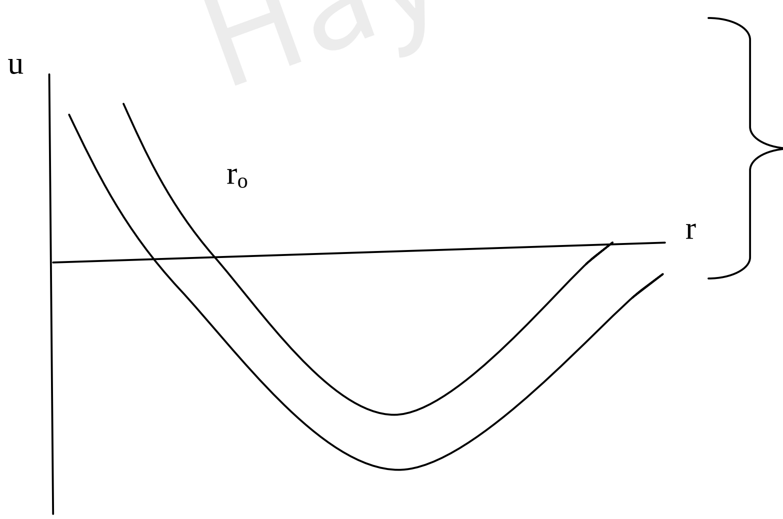
Non Steady or Turbulent: This is an irregular flow characterized by whirl pool–like region i.e. at certain critical speed, the fluid flow becomes non-steady.

Viscosity: Degree of internal friction in the fluid, viscos force is associated with the resistance of two adjacent layers of the fluid to move relative to each other.

Elasticity

<u>Elasticity Properties of Solids</u>

<u>Pre-ambles</u>: Kinetic theory of gasses has shown that matter consists of molecules, which behaves like free particles in gases. For solids, the molecules have small distance and so, exert significant forces on one another. The relationship between potential energy $U_{(r)}$ and $F_{(r)}$ is illustrated in the graph below:



Energy dependence of intermolecular distance.

Fig (2.0)

Note: That the molecules are normally at freed position (distance r_o from one another) and their forces in any molecule is zero; and potential energy (p.e) is minimum.

$$\frac{du_{(r)}}{dr} = 0$$

The response of a material to a given type of deforming force as explained in the above graph describes the principle of elasticity- the ability of a material to return back to its original shape and size.

<u>Stress</u>: This is the external force per unit cross sectional area acting on an object

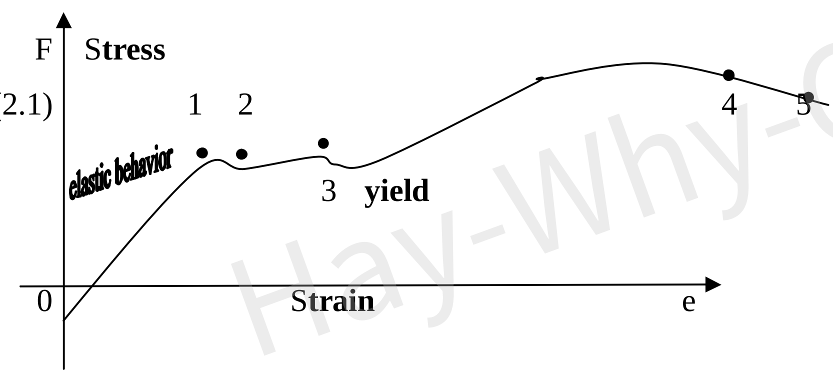
$$\text{Stress} = \frac{\text{Force}}{\text{Area}}, \text{ Unit is } Nm^{-2}$$

<u>Strain</u>: **-** This is the ratio of the change in original size or shape to the original size or shape.

$$\text{Strain} = \frac{\Delta R}{h}$$

<u>Illustration:</u> - A piece of wire stretched by an increasing force obeys Hook's law i.e. extension is proportional to force strain is proportional to stress.

Fig (2.1)



0 – 2, wire return to its original length after the load had been removed. **2,** is known as elastic limit. After this print, it is brittle. **3,** is the yield print beyond this print, extension is rapid, and this known as plasticity. At **4**, material is under maximum force or stress that can be sustained. At **5**, it is the breaking point.

Since strain is proportional to stress, the constant of protionality is called **<u>elastic modulus</u>**

Anticipate

TEAM SYNERGY

Led By:- Hay-Why-Oh

PHYSICS FOR BIOLOGICAL SCIENCES AND AGRICULTURAL STUDENTS

PHS 105 / PHYSICS DEPARTMENT

Name of Presenter: OLURIN OLUWASEUN T.

THE EQUATION OF STATE OF AN IDEAL GAS MAINTAINED AT LOW PRESSURE (OR LOW DENSITY)

The macroscopic state of a gas in thermodynamic equilibrium is determined by its temperature, pressure, and volume.

A **gas** is a substance that expands to fill the container in which it is placed. Thus, the volume of a gas is the volume of its container.
A gas is a substance that, when placed in a container, expands to fill the container.

The physical properties of a gas are pressure, volume, temperature, and number of molecules.

Several simple relationships exist among the four properties of a gas—pressure, volume, temperature, and number of molecules. The relationships are Boyle, Charles, Gay-Lussac, Avogadro's.

Emperical Gas Law

Boyle's Law states that the product of a gas's pressure, *p,* and its volume, *V*, at constant temperature is a constant.

$$PV = Constant$$

$$P_1V_1 = P_2V_2$$

Charles's Law, which states that for a gas kept at constant pressure, the volume of the gas, *V*, divided by its temperature, *T*, is constant

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Gay-Lussac's Law, which states that ratio of the pressure, *p*, of a gas to its temperature, *T*, at the same volume is constant

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Avogadro's Law states that the ratio of the volume of a gas, *V*, to the number of gas molecules, *N*, in that volume is constant,

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

One mole of any gas will have Avogadro's number of molecules

1 **mole** of a gas is defined to have $6.022 \times 10^{23}\, Molecules$

$$N = nNa$$

The combinations of the Empirical gas laws leads to Ideal Gas Law, thus

$$\frac{PV}{NT} = Constant$$

$$PV = KNT$$

$PV = NKT$, recall that $N = nNa$

$$PV = nN_a KT$$

This equation becomes,

$$PV = nRT$$

Where, $R = N_a K$ which is known as Universal gas constant. $R = 8.314 J / K.mol$

P is Pressure $(1atm = 101.3.KPa = 1.013\times10^{5}\,Pa = 1.013\times10^{5}\,Nm^{-2})$,

V is Volume $(1L = 10^{-3}\,m^{3})$

T is temperature (K)

N is number of gas molecules

n is number of moles

K is Boltzmann's constant $K = 1.38\times10^{-23}\,J/k$

Avogardo's constant $(N_a = 6.022\times10^{23}\,Molecules)$

Boyle's Law is concerned with the relationship of pressure and volume using a fixed amount of gas (a fixed number of mols of gas)

P*V = constant at constant temperature

Avogadro's Law is concerned with the relationship between the number of molecules or mols (n) and the volume of a gas under conditions of constant pressure and temperature

$V \propto n$ at constant pressure and temperature

Charles' Law is concerned with the relationship of temperature and volume when dealing with a constant amount of gas (mols)

$V \propto T$ when T is expressed in K. The K temperature scale is derived from the behavior of gases

if $V \propto T$ then $V = kT$ where k is a constant at constant pressure

Ideal gas law: $PV = nRT$ where R is a constant

$R = 0.0821$ L·atm/K·mol

Note that at constant n and T, PV = constant   Boyle's Law

Note that at constant P and T V/n = constant   Avogadro's Law

Note that at constant P and n, V/T = constant   Charles's Law

Standard conditions of pressure and temperature

T = 0 °C (273 K)

Pressure: 1 atm

What volume does a mol of any ideal gas occupy at STP?

$PV = nRT$

V = 1mol(0.0821 L*atm/K*mol)(273 K)/(1 atm)

V = 22.4 L

This means that equal volumes of gases under identical conditions of temperature and pressure contain equal number of molecules

Example: Gas having 150 cm^3 volume has pressure 120 cmHg. If we increase volume of container to 300 cm^3, find final pressure of the gas.

Since $P_1.V_1$ is constant from boyle's law;

$P_1.V_1=P_2.V_2$

$120.150=P_2.300$

$P_2=60$ cm H

Example: Gas at 127 ^0C has volume 240 ml. If we increase temperature of gas from 127 ^0C to 227 ^0C, find final volume of the gas.

Solution:

We first convert unit of temperature.

T_1=127+273=400 K

T_2=227+273=500 K

V_1=240 ml

V_2=?

We use Charles' law to solve this problem.

$V_1/T_1=V_2/T_2$

$240/400=V_2/500$

V_2=300 ml

Example: If we want to decrease pressure of gas, placed in a container having constant volume, from 4P to P how much we should change the temperature of it. Its current temperature is 127 ^{0}C.

P1=4P

P2=P

T_1=127 ^{0}C=127+273=400 K

$P_1/T_1=P_2/T_2$

$4P/400=P/T_2$

T_2=100 K=t+273

t=-173 ^{0}C

PHYSICS FOR BIOLOGICAL SCIENCES AND AGRICULTURAL STUDENTS

PHS 105 / PHYSICS DEPARTMENT

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HEAT TRANSFER MECHANISM

Heat transfer describes the exchange of thermal energy, between physical systems depending on the temperature and pressure, by dissipating heat.

Heat transfer is energy transfer due to a temperature difference in a medium or between two or more media

Mode of Heat transfer

The fundamental modes of heat transfer are conduction or diffusion, convection and radiation.

Conduction is the transfer of thermal energy by molecular action, without any motion of the medium.

Conduction heat transfer is due to a temperature gradient in a stationary medium or media

Convection is the transfer of thermal energy by the actual motion of the medium itself. The medium in motion is usually a gas or a liquid. Convection is the most important heat transfer process for liquids and gases.

Convection heat transfer occurs between a surface and a moving fluid at different temperatures

Radiation is a transfer of thermal energy by electromagnetic waves.

Radiation heat transfer occurs due to emission of energy in the form of electromagnetic waves by all bodies above absolute zero temperature

Net **radiation** heat transfer occurs when there exists a temperature difference between two or more surfaces emitting radiation energy

Heat Transfer by Conduction.

a) **Conduction** is the process by which heat is transferred via collisions of internal particles that make up the object $\Rightarrow$ *individual* (mass) particle transport.

i) Heat causes the molecules and atoms to move faster in an object.

ii) The hotter molecules (those moving faster) collide with cooler molecules (those moving slower), which in turn, speeds up the cooler molecules making them warm.

iii) This continues on down the line until the object reaches equilibrium.

The amount of heat transferred ΔQ from one location to another over a time interval Δt is

$$\boxed{\Delta Q = \mathcal{P}\,\Delta t} \quad .$$

ii) $\mathcal{P}$ is measured in **watts** when Q is measured in Joules and Δt in seconds.

iii) As such, $\mathcal{P}$ is the same thing as power since they are both measured in the same units.

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$$\mathcal{P}_{\text{cond}} = \frac{\Delta Q}{\Delta t} = KA\left(\frac{T_{\text{h}} - T_{\text{c}}}{L}\right).$$

Conduction

Conduction can take place in solids, liquids, or gases. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons.

The rate of heat conduction through a medium depends on the geometry of the medium, its thickness, and the material of the medium, as well as the temperature difference across the medium.

$$Q \alpha A(T_h - T_C)t \qquad 1$$

The thermal energy transmitted is also found to be inversely proportional to the thickness of the slab, that is,

$$Q \alpha \frac{1}{L} \qquad 2$$

These two proportionalities can be combined into one as,

$$Q \alpha \frac{A(T_h - T_c)t}{L}$$

$$Q = \frac{KA(T_h - T_c)t}{L} \qquad 3$$

Equation 3 gives the amount of thermal energy transferred by conduction.
Where K is Coefficient of thermal conductivity

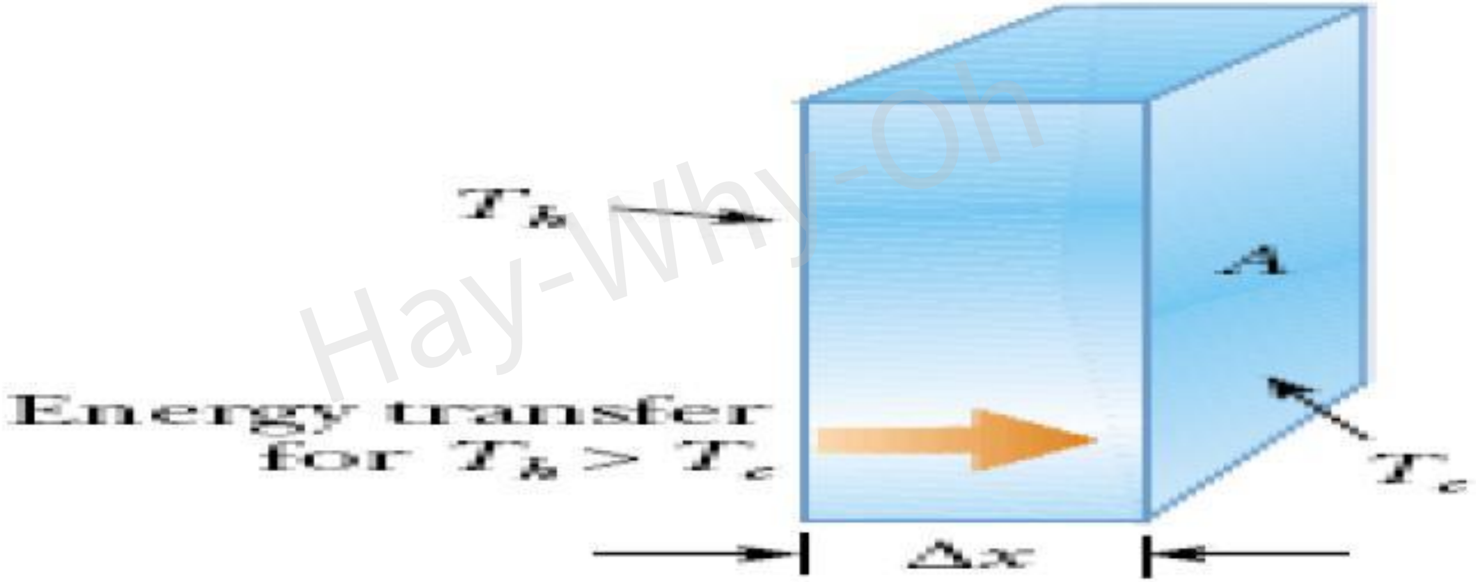


Figure 6: Heat conduction through slab

Convection

Convection

Convection is the transfer of heat from one place to another by the movement of fluids, a process that is essentially the transfer of heat via mass transfer.

Energy transferred by the movement of a warm substance is said to have been transferred by convection. When the movement result from differences in density, as with air around a fire, it is refers to as natural convection. Air flows at a beach is an example of natural convection. When the heated substances is forced to move by a fan or pump, as in same hot-air and hot-water heating system, the process is called forced convection. If it were not for convection current, it would be very difficult to boil water. As water is heated in a teakettle, the lower layers are warmed first. The heated water expands and rises to the top because its density is lowered. At the same time, the denser, cool water at the surface sinks to the bottom of the kettle and is heated.

We can distinguish two types of convection:

(a) Forced convection: This is a process in which a material is forced to move by a blower or pump leading to transfer of heat.

(b) Natural or free convection:- this is a process in which a material flows due to differences in density.

Heat Transfer by Convection.

a) When an ensemble of hot particles move in bulk to cooler regions of a gas or liquid, the heat is said to flow via **convection**.

b) Boiling water and cumulus clouds are 2 examples of convection.

The rate of thermal energy transfer by convection is given by Equation 4

$$P = \frac{Q}{T} = hA\Delta T \qquad 4$$

Where P is the rate of heat transferred,

h is the convective heat transfer coefficient

A is the area and ΔT is the change in temperature

Radiation

Radiation is the transfer of thermal energy by electromagnetic waves. Of all the heat energy transport mechanisms, only radiation does not require a medium =) it can travel through a vacuum. the Stefan–Boltzmann law states that the total energy radiated per unit surface area of a black body across all wavelengths per unit time (also known as the black-body radiant exitance or emissive power), is directly proportional to the fourth power of the black body's thermodynamic temperature T.

The rate at which an object emits radiant energy is given by the Stefan-Boltzmann Law:

$$P = \delta AeT^4 \qquad 5$$

Where P is the Power radiated (emitted) S.I. unit is Watts (W),

A is cross-sectional area (m^2),

e is emmisivity (e=1 for a perfect absorber or emitter),

T is temperature (K).

$\delta = 5.6696 \times 10^{-8} Wm^{-2}K^{-4}$ is Stefan-Boltzmann's constant $Wm^{-2}K^{-4}$

If an object is at temperature T and its surroundings are at a temperature T_0, then the net energy gained or lost each second by the object as a result of radiation is;

$$P_{net} = \sigma Ae(T^4 - T_0^4) \qquad 6$$

RADIATION is the mode of transport of radiant electromagnetic energy through vacuum and the empty space between atoms. Radiant energy is distinct from heat, though both correspond to energy in transit. Heat is heat; electromagnetic radiation is electromagnetic radiation – don't confuse the two.

A *blackbody* is a body that absorbs all the radiant energy falling on it. At thermal equilibrium, a body emits as much energy as it absorbs. Hence, a good absorber of radiation is also a good emitter of radiation.

In **convection,** a warm substance transfers energy from one location to another.

All objects emit **radiation** in the form of electromagnetic waves at the rate

$$\mathcal{P} = \sigma A e T^4$$

Example 1

A thin square steel plate, 10cm on a side, is heated in a blacksmith forge to a temperature of 800^0C. If the emissivity is 0.60. what is the total rate of radiation of energy?

Solution: Total surface area, including both sides is $2\,(0.10\text{m})^2 = 0.020\text{m}^2$

We must convert the temperature to the Kelvin scale

$$800^0\text{c} = 1073\text{K}$$

$$P = Ae\sigma T^4$$

$$= (0.020\text{m}^2) \times 0.60 \times 5.67 \times 10^{-8}\ \text{w/m}^2\text{k}^4)\ (10\ 73\text{k})^4$$

$$\mathbf{= 900W}$$

Example 2

If the total surface area of the human body is $1.2m^2$ and the surface temperature is $30^0C = 303K$. If the surroundings are at temperature of 20^0C, find the total rate of radiation of energy from the body and the net rate of heat loss from the body by radiation? The emissivity of the body is approximately equal to unity.

Solution:

Taking e = 1. The body radiates at a rate H = AeσT^4

= (1.20m2) x 1 x 5.67 x 10-8 W/m2k4) x (303K)^4

= 574W

This loss of heat is partly offset by absorption of radiation, which depends on the temperature of the surroundings.

∴ The net rate of radiative energy transfer is given by

$$H_{net} = Ae\sigma(T^4 - T^5)$$

$= (1.20m^2) \times 1 \times 5.67 \times 10^{-8}\ w/m^2.k^4 \times (303k)^4 - (293k)^4$

= 72W

Note: The value of H net is positive because the body is losing heat to its colder surroundings.

The temperature of a silver bar rises by 10.0°C when it absorbs 1.23 kJ of energy by heat. The mass of the bar is 525 g. Determine the specific heat of silver.

$$\Delta Q = mc_{\text{silver}}\Delta T$$

$$1.23\ \text{kJ} = (0.525\ \text{kg})c_{\text{silver}}(10.0^\circ\text{C})$$

$$c_{\text{silver}} = \boxed{0.234\ \text{kJ/kg}\cdot{}^\circ\text{C}}$$

A 50.0-g sample of copper is at 25.0°C. If 1 200 J of energy is added to it by heat, what is the final temperature of the copper?

From $Q = mc\Delta T$

we find $\Delta T = \frac{Q}{mc} = \frac{1\,200 \text{ J}}{0.050\,0 \text{ kg}(387 \text{ J/kg}\cdot°\text{C})} = 62.0°\text{C}$

Thus, the final temperature is $\boxed{87.0°\text{C}}$.

Course Title

Course Code: PHS 105 / Department: PHYSICS

Name of Presenter: PROF. V. MAKINDE

i) Density

$Density = \frac{mass\ of\ substance}{volume\ of\ substance}$; that is, $\rho = \frac{m}{V}$; Unit: gcm^{-3} or kgm^{-3}

Note: i) Density of water = 1 gcm^{-3} or 1000 kgm^{-3}

ii) 1 litre (10^{-3} m^3) of water has a mass of 1 kg; implying that 1 cm^3 of water has a mass of 1g

iii) Density decreases with increasing temperature

j) Relative Density, RD

The relative density, RD of a substance is defined as

$RD = \frac{density\ of\ substance}{density\ of\ water}$; that is, $RD = \frac{\rho_{substance}}{\rho_{water}}$ Unit: None

Again, $RD = \frac{mass\ (or\ weight)of\ a\ volume\ of\ substance}{mass\ (or\ weight)of\ an\ equal\ volume\ of\ water}$

Note: Density of a substance = RD of the substance × Density of water

k) Archimedes Principle

When a body is totally or partially immersed in afluid, it experiences an upthrust which is equal to the weight of the fluid displaced

l) Upthrust, U of a fluid on a body

$$Upthrust = Weight\ of\ body\ in\ air - Weight\ of\ body\ in\ fluid$$

$$= True\ weight - Apparent\ weight$$

$$= Apparent\ loss\ in\ weight\ of\ body\ in\ fluid$$

Again, $Upthrust = Weight\ of\ fluid\ displaced$ that is, $U = W_{fd}$

Hence $U = mass\ of\ fluid\ displaced \times acceleration\ due\ to\ gravity$;

or $U = m_{fd}g$ or $U = \rho_f V_{fd} g$

where ρ_f – density of fluid (in gcm^{-3} or kgm^{-3});

V_f – volume of fluid displaced (in cm^3 or m^3)

g – acceleration due to gravity (in ms^{-2})

m) Principle of Flotation

Weight of body = Weight of fluid displaced

that is, $m_b g = m_{fd} g$ or, $\rho_b V_b = \rho_f V_{fd}$

n) Methods of Measurement of RD (and hence Density) of solids and/or liquids

(i) Using Relative Density Bottle

Mass of dry, clean, and empty relative density bottle = m_1

Mass of relative density bottle + water = m_2 => mass of water = $m_2 - m_1$

Mass of relative density bottle + substance = m_3 => mass of substance = $m_3 - m_1$

Using $RD = \frac{mass\ (or\ weight)\ of\ a\ volume\ of\ substance}{mass\ (or\ weight)\ of\ an\ equal\ volume\ of\ water}$ => $RD = \frac{m_3 - m_1}{m_2 - m_1}$

(ii) Based on Archimedes Principle

- **Using a spring balance**

  Mass of solid in air, water, and liquid, m_1, m_2, m_3

  Weight of solid in air, water, and liquid, W_1, W_2, and W_3

- **Using a helical spring**

  Extension of solid in air, water, and liquid, e_1, e_2, e_3

- **Using a the principle of moments**

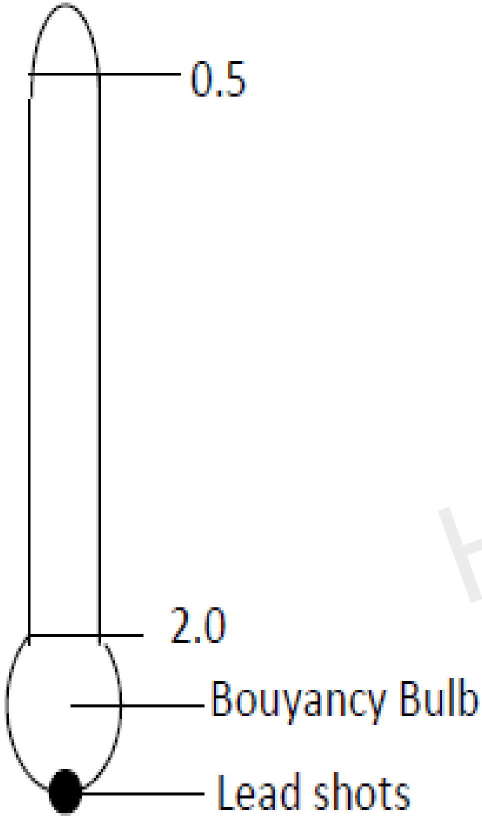
  Distance of balancing mass m from CG when sample was in air, water, and liquid; d_1, d_2, d_3

In general therefore,

$$RD\ of\ solid = \frac{m_1}{m_1 - m_2} = \frac{W_1}{W_1 - W_2} = \frac{e_1}{e_1 - e_2} = \frac{d_1}{d_1 - d_2}$$

$$RD\ of\ liquid = \frac{m_1 - m_3}{m_1 - m_2} = \frac{W_1 - W_3}{W_1 - W_2} = \frac{e_1 - e_3}{e_1 - e_2} = \frac{d_1 - d_3}{d_1 - d_2} = \frac{Upthrust\ in\ liquid}{Upthrust\ in\ water}$$

o) The Hydrometer



Weight of hydrometer ($m_h g$) = Weight of liquid displaced ($\rho_f V_{fd} g$)

that is, $m_h = \rho_f V_{fd}$

$m_h = \rho_f A_h l_h$

where

m_h – mass of hydrometer (in kg)

A_h – cross-sectional area of the stem of the hydrometer (in m^2)

ρ_f – density of the fluid inside which the hydrometer was floating (in kgm^{-3})

l_h – length of hydrometer stem immersed in the fluid (in m)

PRACTICE EXERCISE

1. Determine the weight of a body whose mass is 5kg?

   Solution: Data: $m = 5kg$, $W = ?$ given $g = 10\ ms^{-2}$

   Applying $W = mg$ => $W = 5 \times 10$ that is, $W = 50\ N$

2. What is the density of a solid cube whose mass is 2 kg with sides 5 cm

   Solution: Data: $m = 2kg$, $l = 5\ cm$ => $l = 5 \times 10^{-2}\ m$

   Applying $V = l^3$ => $V = (5 \times 10^{-2})^3\ m^3$ or $V = 125 \times 10^{-6}\ m^3$

   Using $\rho = \frac{m}{V}$, => $\rho = \frac{2}{125 \times 10^{-6}}$; that is, $\rho = 16 \times 10^3\ kgm^{-3}$

3. An empty density bottle has a mass of 20 g. When it is completely filled with kerosene, its mass is 68 g. If the bottle has a mass of 80 g when completely filled with water, calculate the relative density and density of kerosene

Solution:

Mass of empty bottle, m_1 = 20 g = 0.020 kg

Mass of bottle full of water, m_2 = 80 g

Mass of bottle full of kerosene, m_3 = 68 g

Applying $RD = \frac{m_3 - m_1}{m_2 - m_1}$, => $RD = \frac{0.068 - 0.020}{0.080 - 0.020} = \frac{0.048}{0.060} = 0.80$

Hence, the density of the kerosene is obtained using $\rho_{kerosene} = RD_{kerosene} \times$ density of water

That is, $\rho_{kerosene} = 0.80\ gcm^{-3}$ or $\rho_{kerosene} = 0.80 \times 10^3\ kgm^{-3}$ => $\rho_{kerosene} = 800\ kgm^{-3}$

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4. ¼ th of the volume of a cylindrical bucket of radius 0.2 m and height 0.5 m was immersed in a fluid of RD 0.75. What is the upthrust experienced by the bucket in the fluid?

Solution: r = 0.2 m, h = 0.5 m, $RD_{fluid} = 0.75$, U = ?

$V_{bucket} = \pi r^2 h$ => $V_{bucket} = \pi \times 0.2^2 \times 0.5$ hence $V_{bucket} = 0.0628\ m^3$

$V_{fluid\ displaced} = V_{bucket\ immesed} = \frac{1}{4} V_{bucket} = 0.01571\ m^3$

$\rho_{fluid} = RD_{fluid} \times$ density of water => $\rho_{fluid} = 0.75 \times 10^3\ kgm^{-3}$ or $\rho_{fluid} = 750\ kgm^{-3}$

Hence $U = \rho_{fluid} V_{fluid\ displaced}\ g$ that is, $U = 750 \times 0.01571 \times 10$ N => $U = 117.81$ N

Momentum

Momentum = mass × velocity   Unit: $kgms^{-1}$

p = mv   [Note that $p = \frac{d(k.e)}{dt}$]

Change in momentum = mass × change in velocity

$\Delta p = m(v - u)$ or $\Delta p = m\Delta v$

Equivalence of Impulse and Change in Momentum

If a force F is applied to a mass m for a duration t such that motion is effected, then

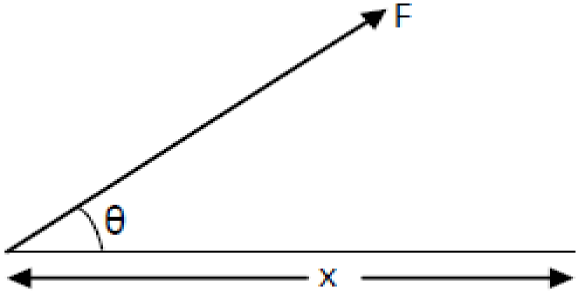
Impulse = Ft	Unit: Ns	Dimension: MLT^{-1}
Change in momentum = m(v - u)	Unit: kgms-1	Dimension: MLT^{-1}

But $F = ma = m\frac{(v-u)}{t}$;

=>   Ft = m(v –u)   that is,   Impulse = Change in momentum

Work, Energy and Power

a) Work



If a force F is applied to a body at an angle θ to the horizontal as shown such that motion is effected and the body moved through a distance x, then the work done is given by $W = Fx\cos\theta$ Unit: Nm

b) Energy

- **Mechanical Energy**

Kinetic energy = ½ mv^2 that is, k.e. = $\int p\,dv$; where $p = mv$ Unit: J

Potential energy = mgh that is, p.e. = Wh Unit: J

where h is the height of body above the reference level

- **Work – Energy theorem**

  $\frac{1}{2} m(v^2 - u^2)$ = Work done

  that is, change in mechanical energy = Work done

- **Elastic Potential Energy**

  Elastic potential energy = $\frac{1}{2} ke^2$ or Elastic potential energy = $\frac{1}{2}$ Fe

  where k – elastic constant of material (in Nm-1)

  and F – applied force (in N)

The area under the force – extension graph is equivalent to the elastic potential energy or work done.

- **Total Energy**

  E = K.E. + P.E. => E is constant

At equilibrium, K.E. = P.E.

For a body falling to the ground from a height h, the velocity just before it hits the ground is given by

$$v = \sqrt{2gh}$$

- **Power**

$$Power = \frac{dW}{dt} = \frac{dE}{dt};$$

$$that\ is, \quad Power = \frac{Work\ done}{Time} = \frac{Energy\ expended}{Time}$$

Therefore, W = Pt or E = Pt

For a body moving with uniform velocity, P = Fv where F – tractive force (in N)

PRACTICE EXERCISE

1. A 5 kg mass was dropped from a height of 10 m. Given that acceleration due to gravity is 9.8 ms^{-2}, determine i) the potential energy of the body ii) the velocity with which the body hits the ground iii) the kinetic energy of the body on hitting the ground.

Solution: Data: m = 5 kg, h = 10 m, g = 9.8 ms^{-2}, P.E. = ?, v = ?, K.E. = ?

i) To determine the potential energy of the body, we apply p.e. = mgh

that is, p.e. = 5 × 9.8 × 10 => p.e. = 490 J

ii) To find the velocity with which the body hits the ground, we equate p.e. to k.e.;

that is, $\frac{1}{2}mv^2 = mgh$ giving $v = \sqrt{2gh}$;

Hence $v = \sqrt{2 \times 9.8 \times 10}$ => v = 14 ms^{-1}

iii) To determine the kinetic energy of the body on hitting the ground we apply k.e. = $\frac{1}{2}mv^2$

that is, k.e. = ½ × 5 × 14^2 => k.e. = 490 J

2. A force of 5 N was applied to an elastic rubber band such that the rubber band was extended by 4 cm. Determine i) the elastic constant of the rubber band ii) the elastic potential energy stored in the stretched rubber band

Solution: $F = 5$ N, $e = 4$ cm $= 0.04$ m, $k = ?$, p.e. $= ?$

i) To determine the elastic constant of the rubber band,

we apply $F = ke$; that is, $5 = 0.04\,k$ => $k = 125\ Nm^{-1}$

ii) To determine the elastic potential energy stored in the stretched rubber band,

we apply p.e. $= \frac{1}{2} Fe$ or $\frac{1}{2} ke^2$ Using p.e. $= \frac{1}{2} Fe$ gives p.e. $= \frac{1}{2} \times 5 \times 0.04$

=> Stored elastic potential energy = 0.1 J