Federal University of Technology, Owerri

Department of Mathematics, Harmattan Examinations 2018 / 2019 Session MTH 305. Functions of a Complex Variable and its applications I. Answer any five questions, Time 3 hours.

Q1a) Express $(1+i)^{1/4}$ in the form $a + ib$ (7 marks)

1b) If $\sin z = \alpha$ show that

$$z = -i\log\left[\sqrt{1-\alpha^2} + i\alpha\right]$$ (7 marks)

Q2a) Find all the roots of the equation $(1+z)^8 = (1-z)^8$ and show that all the roots lie on the imaginary axis (7 marks)

b) If z is complex show that the real part of $\log(z-1) = \frac{1}{2}\,\mathrm{Log}\,(1 - 2r\cos\theta + r^2)$ where $|z| = r$ and $\mathrm{Arg}\,z = \theta$

Q3a) Find the analytic function $w = u + iv$ given that $u = x^3y - xy^3 + k$, where k is a real constant, If $w(0) = 2 - 3i$ find the value of k. (6 marks)

b) Find the linear transformation which maps the line segment $\sqrt{2} \le x \le 2\sqrt{2}$; $\sqrt{2} \le y \le 2\sqrt{2}$ onto the line segment $u = 0$ $1 \le v \le 7$ such that $(\sqrt{2}, \sqrt{2}) \to (0, 1)$; $(2\sqrt{2}, 2\sqrt{2}) \to (0, 7)$ (8 marks)

Q4. Prove that under the inverse transformation, lines and circles map onto lines or circles. (6 marks) Hence show that the image of the two lines $x + y + 2 = 0$ and $x - y + 2 = 0$ are intersecting circles (8 marks)

Q5. Evaluate the following integrals along the given path, C.

$z(\sqrt{2} + i)$

a) $$\int_c \frac{dz}{z(z^2+1)} \quad ; \quad c = |z| = 4$$ (7 marks)

$z_3 = 0 + i1$

$z_3 = 1[\cos 90 + i \sin 90]$

$\cos \frac{10 + 2\pi}{2}$ $(2)^{9}$

b) $$\int_c z^2\,dz \quad ; \quad c = (0,0) \to (2,0) \to (3,4)$$ (7 marks)

$z_3 +$

Q6. Show that if $(z_1, z_2, z_3) \to (w_1, w_2, w_3)$ in order under the bilinear transformation then

$$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)} = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$ (6 marks)

Hence find the bilinear transformation which maps the triple $(i, 2, -i)$ onto the triple $(-i, 3, i)$ in order (8 marks)