EXERCISES FOR THE *Feynman* LECTURES ON PHYSICS

The NEW MILLENNIUM *Edition*

Richard Feynman, Robert Leighton, Matthew Sands, et al.,

edited by Michael A. Gottlieb and Rudolf Pfeiffer

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Preface

This complete set of exercises for *The Feynman Lectures on Physics* was compiled from three sources: Leighton and Vogt's *Exercises in Introductory Physics* (Addison-Wesley, 1969), and Volumes II and III of Caltech's *Exercises for The Feynman Lectures on Physics* (Addison-Wesley 1964–65). The original exercises have been given a facelift: they are reset in LaTeX with neatly redrawn figures, updated with modern units, clarified and corrected. The exercises for Volumes II and III have been supplemented with several new problems, and answers/solutions, which were completely lacking in previous editions, are now provided. This is the first time exercises for all three volumes of *The Feynman Lectures on Physics* have appeared together in a single volume, and the first time such exercises have been published with a (nearly) complete set of answers.

All of these exercises were at one time assigned to students as homework or test problems in Caltech's mandatory two year introductory physics course, either when Richard Feynman was teaching it (1961–64), or during the nearly two decades that followed when *The Feynman Lectures on Physics* was used as the textbook. Many people contributed to the creation of these exercises, whom you will find acknowledged in the *Introductions* to the exercises for each volume. In addition, we wish to acknowledge:

> The Division of Physics, Math and Astronomy of the California Institute of Technology, for giving us permission to make this book and including it in *The Feynman Lecture on Physics New Millennium Edition*,
>
> Rochus Vogt, for contributing his notebooks, compiled during many years of teaching introductory physics at Caltech,
>
> Eugene Cowan, for contributing his copies of the solutions to the Vols. II & III exercises,
>
> Aaron Zimmerman, for vetting the newly added material for Caltech,
>
> Adam Cochran, for skillfully negotiating the publishing agreement with Basic Books.

Michael A. Gottlieb & Rudolf Pfeiffer
Editors, *The Feynman Lectures on Physics New Millennnium Edition*
December 2013

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Introduction

These exercises were compiled by the authors for use with Volume I of *The Feynman Lectures in Physics* in the first year introductory physics course at the California Institute of Technology, and they are accordingly arranged in the same order as to subject as *The Feynman Lectures*. Within each topic or chapter, the exercises are subdivided into categories according to degree of generality or difficulty. In the order in which these appear in a given chapter, these are: proofs or generalizations, easy exercises, intermediate exercises, and more sophisticated or elaborate exercises. Usually, the proofs and generalizations supplement the discussion given in *The Feynman Lectures*, and the results are intended to be understood and used by the student. The average student should have little trouble solving the easy exercises, and should be able to solve most of the intermediate exercises within a reasonable time—perhaps ten to twenty minutes each. The more sophisticated exercises generally require a deeper physical insight or more extensive thought, and will be of interest principally to the better student.

The individual exercises were contributed and critically evaluated by many people. A substantial number were originated by R. B. Leighton in connection with the original Feynman Lecture series; some are reproduced by permission from an extensive set compiled by Foster Strong; many were adapted by R. E. Vogt from examination problems used in the introductory course. To the many contributors, known or anonymous, the authors express their sincere thanks.

The authors regard the present work as far from complete. It is hoped that in the course of time they, or others at Caltech, will refine the present material and add new exercises and explanations so that a self-study workbook may eventually result, whose utility might extend beyond the present limited range.

Robert B. Leighton & Rochus E. Vogt

1

Atoms in Motion

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 1 to 3. Use the ideas outlined in these chapters, together with your own experience and imagination, in analyzing the following exercises. Precise numerical results in most cases are *not* expected.

1.1 If heat is merely molecular motion, what is the difference between a hot, stationary baseball and a cool, rapidly moving one?

1.2 If the atoms of all objects are perpetually in motion, how can there be any permanent objects, such as fossil imprints?

1.3 Explain qualitatively why and how friction in a moving machine produces heat. Explain also, if you can, why heat cannot produce useful motion by the reverse process.

1.4 Chemists have found that the molecules of rubber consist of long criss-crossed chains of atoms. Explain why a rubber band becomes warm when it is stretched.

1.5 What should happen to a rubber band which is supporting a given weight, if it is heated? (To find out, try it.)

1.6 Can you explain why there are no crystals that have the shape of a regular pentagon? (Triangles, squares, and hexagons are common in crystal forms.)

1.7 You are given a large number of steel balls of equal diameter d and a container of known volume V. Every dimension of the container is much greater than the diameter of a ball. What is the greatest number of balls, N, that can be placed in the container?

1.8 How should the pressure P of a gas vary with n, the number of atoms per unit volume, and $\langle v\rangle$, the average speed of an atom? (Should P be proportional to n and/or to $\langle v\rangle$, or should it vary more, or less, rapidly than linearly?)

1.9 Ordinary air has a density of about $0.001\,\mathrm{g\,cm^{-3}}$, while liquid air has a density of about $1.0\,\mathrm{g\,cm^{-3}}$.

(a) Estimate the number of air molecules per cm^3 in ordinary air, n_G, and in liquid air, n_L.

(b) Estimate the mass m of an air molecule.

(c) Estimate the average distance l an air molecule should travel between collisions at normal temperature and pressure (NTP, 20 °C at 1 atm). This distance is called the *mean free path*.

(d) Estimate at what pressure P, in normal atmospheres, a vacuum system should be operated in order that the mean free path be about one meter.

1.10 The intensity of a collimated, parallel beam of potassium atoms is reduced 3.0 % by a layer of argon gas 1.0 mm thick at a pressure of 6.0×10^{-4} mmHg. Calculate the effective target area A per argon atom.

1.11 X-ray diffraction studies show that NaCl crystals have a cubic lattice, with a spacing of 2.820 Å between nearest neighbors. Look up the density and molecular weight of NaCl and calculate Avogadro's number N_A. (This is one of the most precise experimental methods for determining N_A.)

1.12 Boltwood and Rutherford found that radium in equilibrium with its disintegration products produced 13.6×10^{10} helium atoms per second per gram of radium. They also measured that the disintegration of 192 mg of radium produced $0.0824\,\mathrm{mm}^3$ of helium per day at standard temperature and pressure (STP, $0\,^\circ\mathrm{C}$ at 1 atm). Use these data to calculate:

(a) The number of helium atoms N_H per cm^3 of gas at STP.

(b) Avogadro's number N_A.

Reference: Boltwood and Rutherford, *Phil. Mag.* **22**, 586, (1911).

1.13 Rayleigh found that 0.81 mg of olive oil on a water surface produced a mono-molecular layer 84 cm in diameter. What value of Avogadro's number N_A results, assuming the approximate composition $H(CH_2)_{18}COOH$, in a linear chain, with density $0.8\,\mathrm{g\,cm^{-3}}$?

Reference: Rayleigh, *Proc. Roy. Soc.* **47**, 364 (1890).

1.14 About 1860, Maxwell showed that the viscosity of a gas is given by

$$\eta = \frac{1}{3}\,\rho v\, l,$$

where ρ is the density, v the mean molecular speed, and l, the mean free path. The latter quantity he had earlier shown to be $l = 1/(\sqrt{2}\,\pi N_g \sigma^2)$, where σ is the diameter of the molecule. Loschmidt (1865) used the measured value of η, ρ(gas), and ρ(solid) together with Joule's calculated v to determine N_g, the number of molecules per cm^3 in a gas at STP. He assumed the molecules to be hard spheres, tightly packed in a solid. Given $\eta = 2.0 \times 10^{-4}\,\mathrm{g\,cm^{-1}\,s^{-1}}$ for air at STP, $\rho(\text{liquid}) \approx 1.0\,\mathrm{g\,cm^{-3}}$, $\rho(\text{gas}) \approx 1.0 \times 10^{-3}\,\mathrm{g\,cm^{-3}}$, and $v \approx 500\,\mathrm{m\,s^{-1}}$, calculate N_g.

1.15 A glass full of water is left standing on an average outdoor windowsill in California.

(a) How much time T do you think it would take to evaporate completely?

(b) How many molecules J per cm^2 and per s would be leaving the water glass at this rate?

(c) Briefly discuss the connection, if any, between your answer to part (a) above and the average rainfall over the earth.

1.16 A raindrop of an afternoon thundershower fell upon a Paleozoic mud flat and left an imprint which is later dug up as a fossil by a hot, thirsty geology student. As he drains his canteen, the student idly wonders how many molecules of water, N, of that ancient raindrop he has just drunk. Estimate N using only data which you already know. (Make reasonable assumptions regarding necessary information which you do not know.)

2

Conservation of Energy, Statics

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 4.

2.1 Use the principle of virtual work to establish the formula for an unequal-arm balance, as shown in Fig. 2-1, $W_1 l_1 = W_2 l_2$. (Neglect the weight of the cross-beam.)

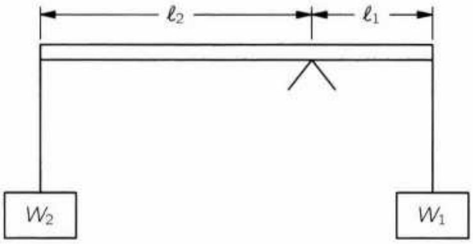


Figure 2-1

2.2 Extend the formula obtained in Ex. 2.1 to include a number of weights hung at various distances from the pivot point,

$$\sum_i W_i l_i = 0.$$

(Distances on one side of the fulcrum are considered positive and on the other side, negative.)

2.3 A body is acted upon by n forces and is in static equilibrium. Use the principle of virtual work to prove that:

(a) If $n = 1$, the magnitude of the force must be zero. (A trivial case.)

(b) If $n = 2$, the two forces must be equal in magnitude, opposite in direction, and collinear.

(c) If $n = 3$, the forces must be coplanar and their lines of action must pass through a single point.

(d) For any n, the sum of the products of the magnitude of a force F_i times the cosine of the angle Δ_i between the force and any fixed line, is zero:

$$\sum_{i=1}^{n} F_i \cos \Delta_i = 0.$$

2.4 Problems involving static equilibrium in the absence of friction may be reduced, using the *Principle of Virtual Work*, to problems of mere geometry: Where does one point move when another moves a given small distance? In many cases this question is easily answered if the following properties of a triangle are used (referring to Fig. 2-2):

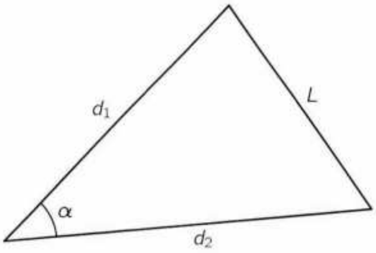


Figure 2-2

(a) If the sides d_1 and d_2 remain fixed in length, but the angle α changes by a small amount $\Delta\alpha$, the opposite side L changes by an amount

$$\Delta L = \frac{d_1 d_2}{L} \sin \alpha \, \Delta\alpha.$$

(b) If the three sides a, b, c of a right triangle change in length by small amounts Δa, Δb, and Δc, then

$$a\,\Delta a + b\,\Delta b = c\,\Delta c \qquad \text{(where } c \text{ is the hypotenuse).}$$

Prove these formulas.

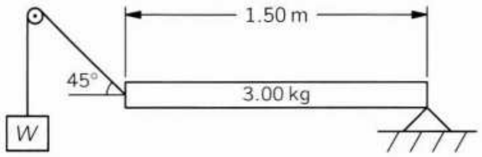


Figure 2-3

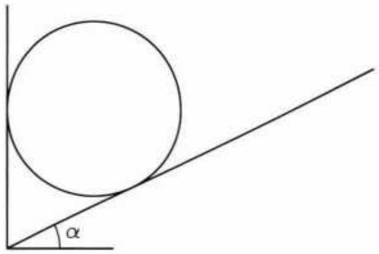


Figure 2-4

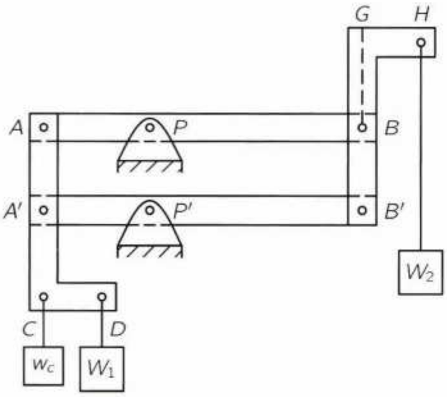


Figure 2-5

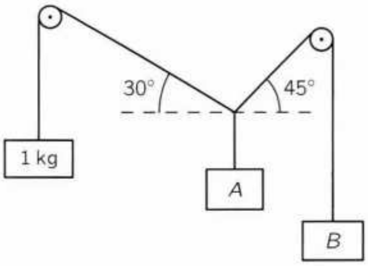


Figure 2-6

2.5 A uniform plank 1.5 m long and weighing 3.00 kg is pivoted at one end. The plank is held in equilibrium in a horizontal position by a weight and pulley arrangement, as shown in Fig. 2-3. Find the weight W needed to balance the plank. Neglect friction.

2.6 A ball of radius 3.0 cm and weight 1.00 kg rests on a plane tilted at an angle α with the horizontal and also touches a vertical wall, as shown in Fig. 2-4. Both surfaces have negligible friction. Find the force with which the ball presses on the wall F_W and on the plane F_P.

2.7 The jointed parallelogram frame $AA'BB'$ is pivoted (in a vertical plane) on the pivots P and P', as shown in Fig. 2-5. There is negligible friction in the pins at A, A', B, B', P, and P'. The members $AA'CD$ and $B'BGH$ are rigid and identical in size. $AP = A'P' = \frac{1}{2}PB = \frac{1}{2}P'B'$. Because of the counterweight w_c, the frame is in balance without the loads W_1 and W_2. If a 0.50 kg weight W_1 is hung from D, what weight W_2, hung from H, is needed to produce equilibrium?

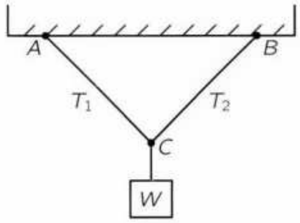


Figure 2-7

2.8 The system shown in Fig. 2-6 is in static equilibrium. Use the principle of virtual work to find the weights A and B. Neglect the weight of the strings and the friction in the pulleys.

2.9 A weight $W = 50$ lb is suspended from the midpoint of a wire ACB as shown in Fig. 2-7. $AC = CB = 5$ ft. $AB = 5\sqrt{2}$ ft. Find the tension T_1 and T_2 in the wire.

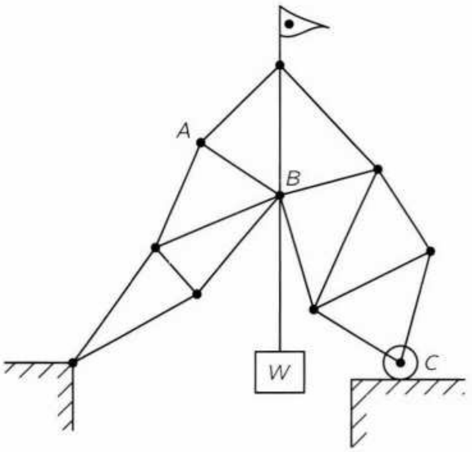


Figure 2-8

2.10 The truss shown in Fig. 2-8 is made of light aluminum struts pivoted at each end. At C is a roller which rolls on a smooth plate. When a workman heats up member AB with a welding torch, it is observed to increase in length by an amount x, and the load W is thereby moved vertically an amount y.

(a) Is the motion of W upward or downward?

(b) What is the force F in the member AB (including the sense, i.e., tension or compression)?

2.11 What horizontal force F (applied at the axle) is required to push a wheel of weight W and radius R over a block of height h, as shown in Fig. 2-9?

2.12 A horizontal turntable of diameter D is mounted on bearings with negligible friction. Two horizontal forces in the plane of the turntable of equal magnitude F, parallel to each other but pointing in opposite directions, act on the rim of the turntable on opposite ends of the diameter, as shown in Fig. 2-10.

(a) What force F_B acts on the bearing?

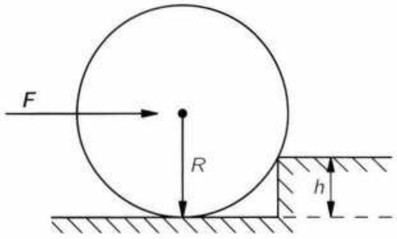


Figure 2-9

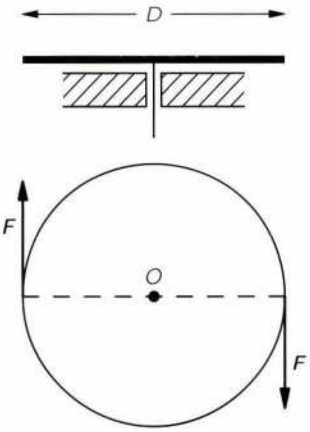


Figure 2-10

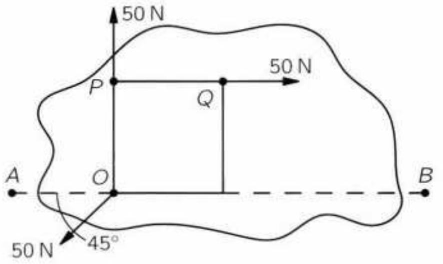


Figure 2-11

(b) What is the torque (= moment of this force couple) τ_O about a vertical axis through the center O?

(c) What would be the moment τ_P about a vertical axis through an arbitrary point P in the same plane?

(d) Is the following statement correct or false? Explain. "Any two forces acting on a body can be combined into a single resultant force that would have the same effect." In framing your answer, consider the case where the two forces are opposite in direction but not quite equal in magnitude.

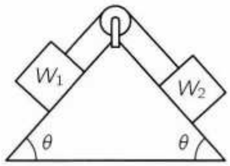


Figure 2-12

2.13 A flat steel plate floating on mercury is acted upon by three forces at three corners of a square of side 0.100 m, as shown in Fig. 2-11. Find a *single* fourth force $\boldsymbol{F}$ which will hold the plate in equilibrium. Give the magnitude, direction, and point of application of $\boldsymbol{F}$ along the line AB.

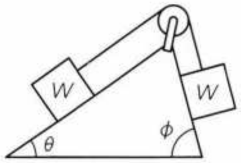


Figure 2-13

2.14 In the absence of friction, at what speed v will the weights W_1 and W_2 in Fig. 2-12 be moving when they have traveled a distance D, starting from rest? $(W_1 > W_2)$

2.15 In Fig. 2-13, the weights are equal, and there is negligible friction. If the system is released from rest, at what speed v will the weights be moving when they have traveled a distance D?

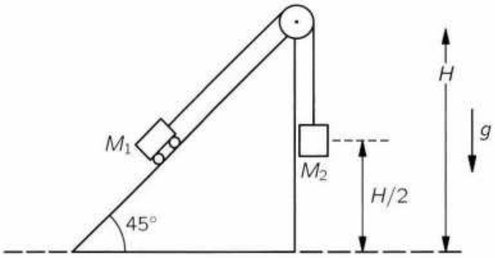


Figure 2-14

2.16 A mass M_1 slides on a 45° inclined plane of height H as shown in Fig. 2-14. It is connected by a flexible cord of negligible mass over a small pulley (neglect its mass) to an equal mass M_2 hanging vertically as shown. The length of the cord is such that the masses can be held at rest both at height $H/2$. The dimensions of the masses and the pulley are negligible compared to H. At time $t = 0$ the two masses are released.

(a) For $t > 0$ calculate the vertical acceleration a of M_2.

(b) Which mass will move downward?

(c) At what time t_1 will the mass identified in part b) strike the ground?

(d) If the mass identified in part b) stops when it hits the ground, but the other mass keeps moving, will it strike the pulley?

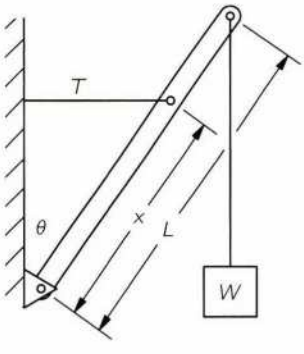
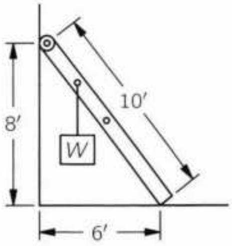


Figure 2-15

2.17 A derrick is made of a uniform boom of length L and weight w, pivoted at its lower end, as shown in Fig. 2-15. It is supported at an angle θ with the vertical by a horizontal cable attached at a point a distance x from the pivot, and a weight W is slung from its upper end. Find the tension T in the horizontal cable.



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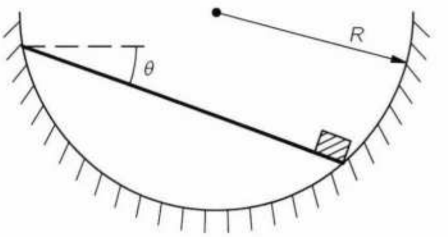
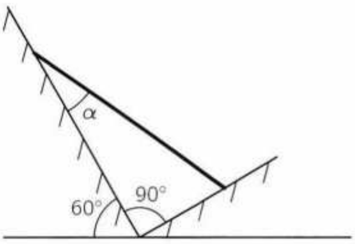


Figure 2-17



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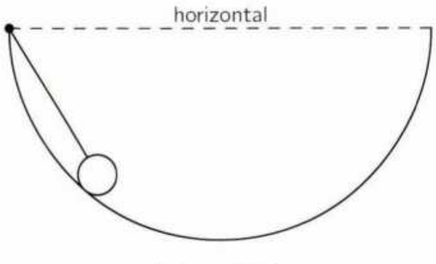

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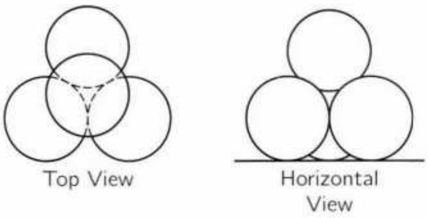

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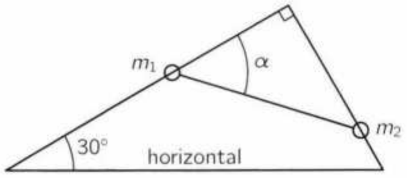


Figure 2-21

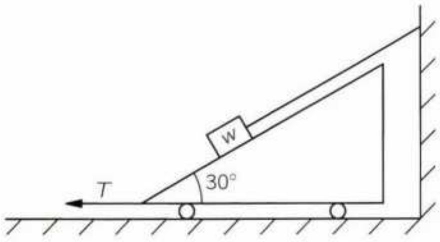


Figure 2-22

2.18 A uniform ladder 10 ft long with rollers at the top end leans against a smooth vertical wall, as shown in Fig. 2-16. The ladder weighs 30 lb. A weight $W = 60\,\text{lb}$ is hung from a rung 2.5 ft from the top end. Find

(a) the force F_R with which the rollers push on the wall.

(b) the horizontal and vertical forces F_h and F_v with which the ladder pushes on the ground.

2.19 A plank of weight W and length $\sqrt{3}R$ lies in a smooth circular trough of radius R, as shown in Fig. 2-17. At one end of the plank is a weight $W/2$. Calculate the angle θ at which the plank lies when it is in equilibrium.

2.20 A uniform bar of length l and weight W is supported at its ends by two inclined planes as shown in Fig. 2-18. From the principle of virtual work find the angle α at which the bar is in equilibrium. (Neglect friction.)

2.21 A small solid sphere of radius 4.5 cm and weight W, is to be suspended by a string from the ends of a smooth hemispherical bowl of radius 49 cm, as shown in Fig. 2-19. It is found that if the string is any shorter than 40 cm, it breaks. Use the principle of virtual work to find the breaking strength F of the string.

2.22 An ornament for a courtyard at a World's Fair is to be made up of four identical, frictionless metal spheres, each weighing $2\sqrt{6}$ ton-wt. The spheres are to be arranged as shown in Fig. 2-20, with three resting on a horizontal surface and touching each other; the fourth is to rest freely on the other three. The bottom three are kept from separating by spot welds at the points of contact with each other. Allowing for a factor of safety of 3, how much tension T must the spot welds withstand?

2.23 A rigid wire frame is formed in a right triangle, and set in a vertical plane as shown in Fig. 2-21. Two beads of masses $m_1 = 100\,\text{g}$, $m_2 = 300\,\text{g}$ slide without friction on the wires, and are connected by a cord. When the system is in static equilibrium, what is the tension T in the cord, and what angle α does it make with the first wire?

2.24 Find the tension T needed to hold the cart shown in Fig. 2-22 in equilibrium, if there is no friction.

(a) Using the principle of virtual work.

(b) Using force components.

2.25 A bobbin of mass $M = 3\,\text{kg}$ consists of a central cylinder of radius $r = 5\,\text{cm}$ and two end plates of radius $R = 6\,\text{cm}$. It is placed on a slotted incline on which it will roll but not slip, and a mass $m = 4.5\,\text{kg}$ is suspended from a cord wound around the bobbin, as shown in Fig. 2-23. It is observed that the system is in static equilibrium. What is the angle of tilt θ of the incline?

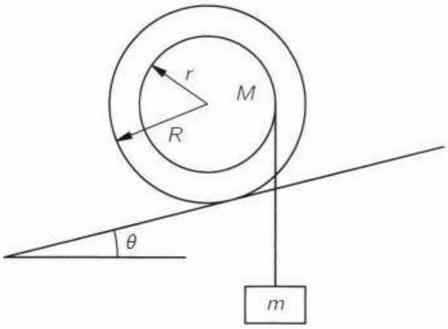


Figure 2-23

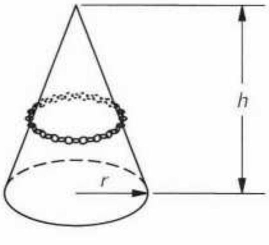


Figure 2-24

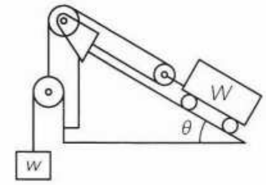


Figure 2-25

2.26 A loop of flexible chain, of total weight W, rests on a smooth right circular cone of base radius r and height h, as shown in Fig. 2-24. The chain rests in a horizontal circle on the cone, whose axis is vertical. Find the tension T in the chain. Neglect friction.

2.27 A cart on an inclined plane is balanced by the weight w as shown in Fig. 2-25. All parts have negligible friction. Find the weight W of the cart.

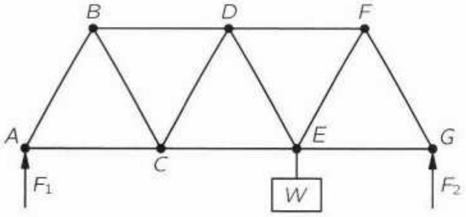


Figure 2-26

2.28 A bridge truss is constructed as shown in Fig. 2-26. All joints may be considered frictionless pivots and all members rigid, weightless, and of equal length. Find the reaction forces F_1 and F_2 and the force F_{DF} in the member DF.

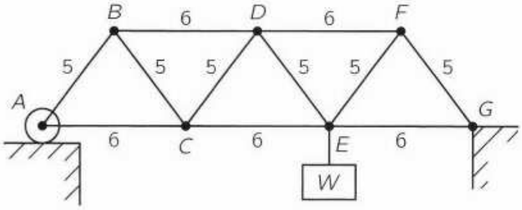


Figure 2-27

2.29 In the truss shown in Fig. 2-27, all diagonal struts are of length 5 units and all horizontal ones are of length 6 units. All joints are freely hinged, and the weight of the truss is negligible.

(a) Which of the members could be replaced with flexible cables, for the load position shown?

(b) Find the forces in struts BD and DE.

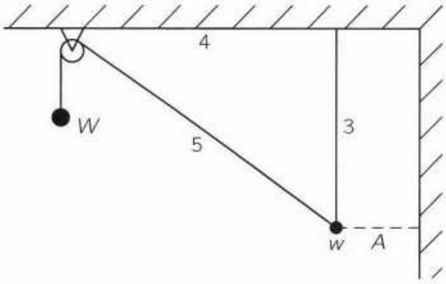


Figure 2-28

2.30 In the system shown in Fig. 2-28, a pendulum bob of weight w is initially held in the vertical position by a thread A. When this thread is burned, releasing the pendulum, it swings to the left and barely reaches the ceiling at its maximum swing. Find the weight W. (Neglect friction, the radius of the pulley, and the finite sizes of the weights.)

2.31 Two equal masses m are attached to a third mass $2m$ by equal lengths of fine thread and the thread is passed over two small pulleys with negligible friction situated 100 cm apart, as shown in Fig. 2-29. The mass $2m$ is initially held level with the pulleys midway between them, and is then released from rest. When it has descended a distance of 50 cm it strikes a table top. What is it's speed v when it reaches the table top?

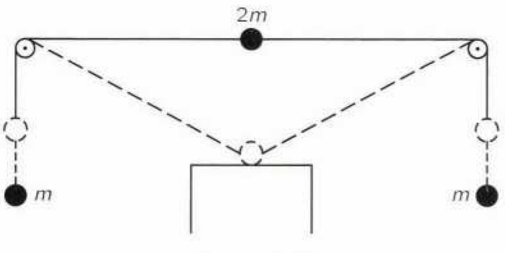


Figure 2-29

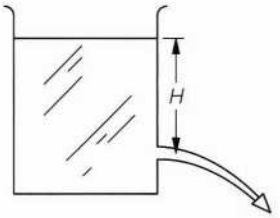


Figure 2-30

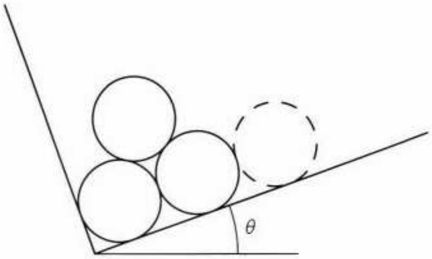


Figure 2-31

2.32 A tank of cross-sectional area A contains a liquid having density ρ. The liquid squirts freely from a small hole of area a distance H below the free surface of the liquid, as shown in Fig. 2-30. If the liquid has no internal friction (viscosity), with what speed v does it emerge?

2.33 Smooth, identical logs are piled in a stake truck. The truck is forced off the highway and comes to rest on an even keel lengthwise but with the bed at an angle θ with the horizontal, as shown in Fig. 2-31. As the truck is unloaded, the removal of the log shown dotted leaves the remaining three in a condition where they are just ready to slide, that is, if θ were any smaller, the logs would fall down. Find θ.

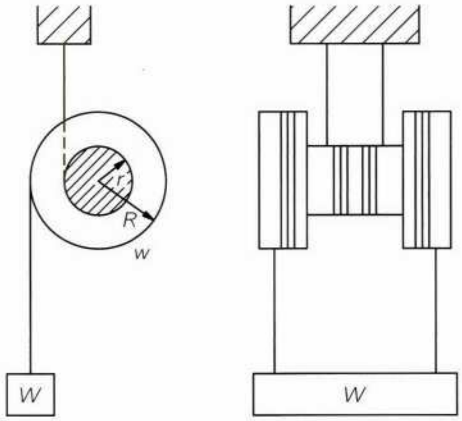


Figure 2-32

2.34 A spool of weight w and radii r and R is wound with cord, and suspended from a fixed support by two cords wound on the smaller radius; a weight W is then suspended from two cords wound on the larger radius, as shown in Fig. 2-32. W is chosen so that the spool is just balanced. Find W.

2.35 A suspension bridge is to span a deep gorge 54 m wide. The roadway will consist of a steel truss supported by six pairs of vertical cables spaced 9.0 m apart, as shown in Fig. 2-33. Each cable is to carry an equal share of the 4.80×10^4 kg weight. The two pairs of cables nearest the center are to be 2.00 m long. Find the proper lengths for the remaining vertical cables A and B and the maximum tension $T_{\max}$ in the two longitudinal cables, if the latter are to be at a 45° angle with the horizontal at their ends.

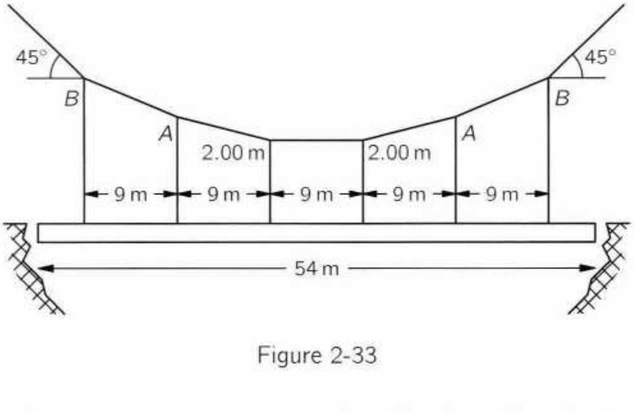


Figure 2-33

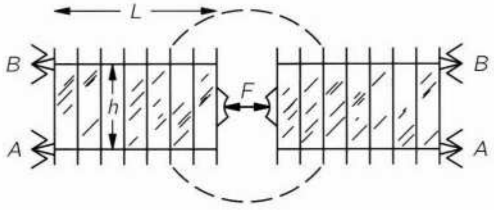


Figure 2-34

2.36 The insulating support structure of a Tandem Van de Graaff may be represented, as shown in Fig. 2-34: two blocks of about uniform density, length L, height h and weight W, supported from vertical bulkheads by pivot joints (A and B) and forced apart by a screw jack (F) at the center. Since the material of the blocks cannot support tension, the jack must be adjusted to give zero force on the upper pivot.

(a) What force F is required?

(b) What is the total force (magnitude and direction) $\boldsymbol{F}_A$ on one of the lower pivots A?

3

Kepler's Laws and Gravitation

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 7.

3.1 Some properties of the ellipse. The size and shape of an ellipse are determined by specifying the values of any two of the following quantities (as shown in Fig. 3-1):

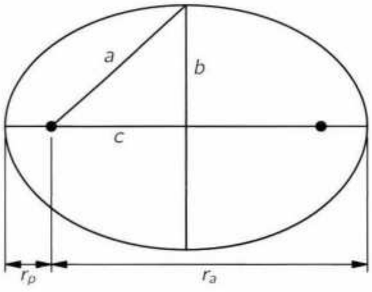


Figure 3-1

a : the semi-major axis
b : the semi-minor axis
c : the distance from the center to one focus
e : the eccentricity
r_p : the perihelion (or perigee) distance (the closest distance from a focus to the ellipse)
r_a : the aphelion (or apogee) distance (the farthest distance from a focus to the ellipse)

The relationships of these various quantities are

$$\begin{aligned} a^2 &= b^2 + c^2, \\ e &= c/a \quad \text{(definition of } e\text{)}, \\ r_p &= a - c = a(1-e), \\ r_a &= a + c = a(1+e). \end{aligned}$$

Show that the area of an ellipse is given by $A = \pi ab$.

3.2 The distance of the moon from the center of the earth varies from 363,300 km at perigee to 405,500 km at apogee, and its period is 27.322 days. A certain artificial earth satellite is orbiting so that its perigee height from the surface of the Earth is 225 km, and its apogee height is 710 km. The mean diameter of the Earth is 12,756 km. What is the sidereal period T of this satellite?

3.3 The eccentricity of the earth's orbit is 0.0167. Find the ratio $v_{\max}/v_{\min}$ of its maximum speed in its orbit to its minimum speed.

3.4 The radii of the earth and the moon are 6378 km and 1738 km, respectively, and their masses are in the ratio 81.3 to 1.000. Calculate the acceleration of gravity $g_{☾}$ at the surface of the moon. ($g_{♁} = 9.81\,\mathrm{m\,s^{-2}}$.)

3.5 In 1986, Halley's comet is expected to return on its seventh trip around the sun since the days in 1456 when people were so frightened that they offered prayers in the churches "to be saved from the Devil, the Turk, and the comet." In its most recent perihelion on April 19, 1910, it was observed to pass near the sun at a distance 0.60 AU.

(a) How far r_a does it go from the sun at the outer extreme of its orbit?

(b) What is the ratio $v_{\max}/v_{\min}$ of its maximum orbital speed to its minimum speed?

3.6 A satellite in a circular orbit near the earth's surface has a typical period of about 100 minutes. What should be the radius r of its orbit (in Earth radii, $r_{♁}$) for a period of 24 hours?

3.7 Consider two earth satellites of equal orbital radius, one of them in a polar orbit, the other in an orbit in the equatorial plane. Which satellite needed the larger booster rocket and why?

3.8 A true "Syncom" satellite rotates synchronously with the earth. It always remains in a fixed position with respect to a point P on the earth's surface.

(a) Consider the straight line connecting the center of the earth with the satellite. If P lies on the intersection of this line with the earth's surface, can P have any geographic latitude λ or what restrictions do exist? Explain.

(b) What is the distance r_s from the earth's center to a Syncom satellite of mass m? Express r_s in units of the earth-moon distance $r_{\text{♁☾}}$

Note: Consider the earth a uniform sphere. You may use $T_{\text{☾}} = 27$ days for the moon's period.

3.9 (a) Comparing data describing the earth's orbital motion about the sun with data for the moon's orbital motion about the earth, determine the mass of the sun $m_{\text{☼}}$ relative to the mass of the earth $m_{\text{♁}}$.

(b) Io, a moon of Jupiter, has an orbital period of revolution of 1.769 days and an orbital radius of 421,800 km. Determine the mass of Jupiter $m_{\text{♃}}$ in terms of the mass of the earth.

3.10 Two stars, a and b, move around one another under the influence of their mutual gravitational attraction. If the semi major axis of their relative orbit is observed to be R, measured in astronomical units (AU) and their period of revolution is T years, find an expression for the sum of the mass, $m_a + m_b$, in terms of the mass $m_{\text{☼}}$ of the sun.

3.11 If the attractive gravitational force between a very large central sphere M and a satellite m in orbit about it were actually $\boldsymbol{F} = -GMm\boldsymbol{R}/R^{(3+a)}$, (where $\boldsymbol{R}$ is the radial vector between them) how would Kepler's second and third law be modified? (In discussing the third law, assume a circular orbit.)

3.12 In making laboratory measurements of g, how precise does one have to be to detect diurnal variations Δg due to the moon's gravitation? For simplicity, assume that your laboratory is so located that the moon passes through zenith and nadir. Also, neglect earth-tide effects.

3.13 An eclipsing binary star system is one whose orbital plane nearly contains the line of sight, so that one star eclipses the other periodically. The relative orbital velocity of the two components can be measured from the Doppler shift of their spectral lines. If T is the observed period in days, and V is the orbital velocity in $\mathrm{km\,s^{-1}}$, what is the total mass M of the binary system in solar masses?

Note: The mean distance from the earth to the sun is 1.50×10^8 km.

3.14 A comet rounds the sun at a perihelion distance of $r_p = 1.00 \times 10^6$ km. At this point its velocity is $v = 500.0\,\mathrm{km\,s^{-1}}$.

(a) What is the radius of curvature R_c of the orbit at perihelion (in km)?

(b) For an ellipse with semi-major axis a and semi-minor axis b, the radius of curvature at perihelion is $R_c = b^2/a$. If you know R_c and r_p you should be able to write a relation involving a and only these quantities. Do so, and find a.

(c) If you were able to solve for a from the above information, you should be able to calculate the period T_c of the comet. Do so.

3.15 Using the idea that two mutually gravitating bodies each "fall" toward the other, and thus move about some fixed common point (their center of mass), show that their period in an orbit in which they remain a given fixed distance apart depends only upon the sum of their masses $M + m$ and not at all upon the ratio of their masses. This is also true for elliptical orbits. Assuming that the semi-major axes of the ellipses in which the bodies move are R and r, find the period T of their orbit.

3.16 How can one find the mass of the moon?

3.17 The trigonometric parallax of Sirius (i.e., the angle subtended at Sirius by the radius of the Earth's orbit) is 0.378 degrees arc. Using this and the data contained in Fig. 3-2, deduce as best you can the mass M of the Sirius system in terms of that of the sun, and

(a) assuming that the orbital plane is perpendicular to the line of sight, and

(b) allowing for the actual tilt of the orbit.

Is your value in part (b) above an upper or lower limit (or either)?

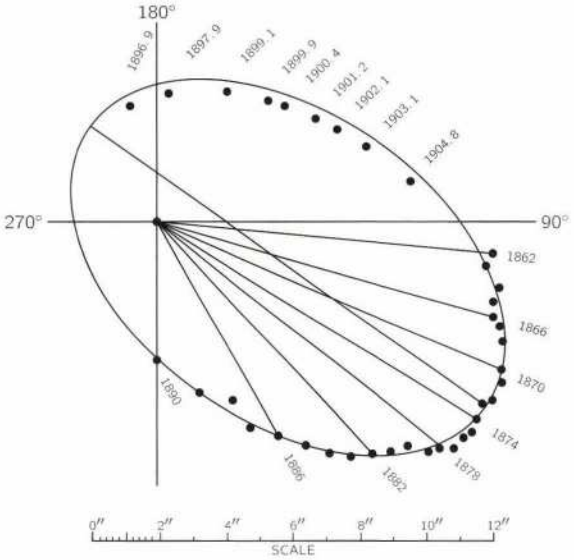


Figure 3-2

4

Kinematics

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 8.

4.1 (a) A body travels in a straight line with a constant acceleration. At $t = 0$, it is located at $x = x_0$ and has a velocity $v_x = v_{x0}$. Show that its position and velocity at time t are

$$\begin{aligned} x(t) &= x_0 + v_{x0}t + \frac{1}{2}at^2, \\ v_x(t) &= v_{x0} + at. \end{aligned}$$

(b) Eliminate t from the preceding equations, and thus show that, at any time,

$$v_x^2 = v_{x0}^2 + 2a(x - x_0).$$

4.2 Generalize the preceding problem to the case of three dimensional motion with constant acceleration components a_x, a_y, a_z, along the three coordinate axis. Show that

(a)

$$\begin{aligned} x(t) &= x_0 + v_{x0}t + \frac{1}{2}a_xt^2, \\ y(t) &= y_0 + v_{y0}t + \frac{1}{2}a_yt^2, \\ z(t) &= z_0 + v_{z0}t + \frac{1}{2}a_zt^2, \\ v_x(t) &= v_{x0} + a_xt, \\ v_y(t) &= v_{y0} + a_yt, \\ v_z(t) &= v_{z0} + a_zt. \end{aligned}$$

(b)

$$v^2 = v_x^2 + v_y^2 + v_z^2 = v_0^2 + 2\left[a_x(x - x_0) + a_y(y - y_0) + a_z(z - z_0)\right],$$

where

$$v_0^2 = v_{x0}^2 + v_{y0}^2 + v_{z0}^2.$$

4.3 An angle may be measured by the length of arc of a circle that the angle subtends, with the vertex of the angle at the center of the circle. If s is the arc length and R is the radius of the circle, as shown in Fig. 4-1, then the subtended angle θ, in radians, is

$$\theta = s/R.$$

(a) Show that, if $\theta \ll 1$ radian, $\sin\theta \approx \theta$, and $\cos\theta \approx 1$.

(b) With the above result, and the formulas for the sine and cosine of the sum of two angles, find the derivatives of $\sin x$ and $\cos x$, using the fundamental formula

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{y(x + \Delta x) - y(x)}{\Delta x}.$$

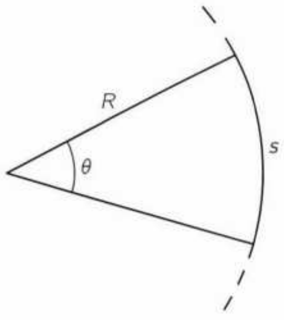


Figure 4-1

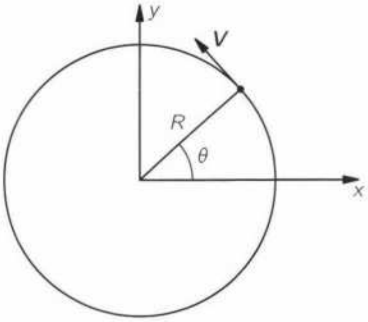


Figure 4-2

4.4 An object is moving counterclockwise in a circle of radius R at constant speed V, as shown in Fig. 4-2. The center of the circle is at the origin of rectangular coordinates (x, y), and at time $t = 0$ the particle is at $(R, 0)$. Show that

(a)

$$
\begin{aligned}
x &= R\cos\omega t, \\
y &= R\sin\omega t, \\
v_x &= -V\sin\omega t, \\
v_y &= V\cos\omega t, \\
a_x &= -\frac{V^2}{R}\cos\omega t, \\
a_y &= -\frac{V^2}{R}\sin\omega t, \\
a &= \frac{V^2}{R},
\end{aligned}
$$

(b)

$$
\begin{aligned}
\ddot{x} + \omega^2 x &= 0, \\
\ddot{y} + \omega^2 y &= 0,
\end{aligned}
$$

where $\omega = V/R =$ angular frequency.

4.5 A Skyhook balloon with a scientific payload rises at a rate of 1000 feet per minute. At an altitude of 30,000 feet the balloon bursts and the payload free-falls. (Such disasters *do* occur!)

(a) For what length of time t was the payload off the ground?

(b) What was the payload's speed v at impact?

Neglect air-drag.

4.6 Consider a train that can accelerate with an acceleration of $20\,\mathrm{cm\,s^{-2}}$ and slow down with a deceleration of $100\,\mathrm{cm\,s^{-2}}$. Find the minimum time t for the train to travel between two stations 2 km apart.

4.7 If you throw a small ball vertically upward in real air with drag, does it take longer to go up or come down?

4.8 Consider a point on the surface of the earth at the equator:

(a) What is its speed v relative to the center of the earth?

(b) What is its angular frequency ω?

(c) What is the ratio of its radial acceleration a due to angular motion and its gravitational acceleration g?

4.9 A Corporal rocket fired vertically was observed to have a constant upward acceleration of $2g$ during the burning of the rocket motor, which lasted for 50 seconds. Neglecting air resistance and variation of g with altitude,

(a) Draw a v-t diagram for entire flight of rocket.

(b) Calculate the maximum height attained $H_{\max}$.

(c) Calculate the total elapsed time T from the firing of the rocket to its return to Earth.

4.10 In a lecture demonstration a small steel ball bounces on a steel plate. On each bounce the downward speed of the ball arriving at the plate is reduced by a factor e in the rebound, i.e., $v_{\text{upward}} = ev_{\text{downward}}$.

If the ball was initially dropped from a height of 50 cm above the plate at time $t = 0$, and if 30 seconds later the silencing of a microphone sound indicated all bouncing had ceased, what was the value of e?

4.11 A projectile is fired over level terrain at an initial speed v_0, at an angle θ with the horizontal. (Neglect air resistance.)

(a) Find the maximum height attained $H_{\max}$ and the range R.

(b) At what angle should the above projectile be fired in order to attain the maximum range?

4.12 A champion archer hits a bullseye in a target mounted on a wall a distance L away and situated at a height h above his bow. Deduce the relation between the speed V at which the arrow left his bow, the arrow's initial angle θ with the horizontal, the height, and the distance to the target, whose solution the archer evidently knew.

Note: The archer did not neglect air resistance, but you may have to.

4.13 A boy throws a ball upward at an angle of 70° with the horizontal, and it passes neatly through an open window, 32 feet above his shoulder, moving horizontally.

(a) What speed v did the ball have as it left his hand?

(b) What was the radius of curvature of its path, R, as it passed over the windowsill?

Can you find the radius of curvature of its path at any given time?

4.14 A small pebble is lodged in the tread of a tire of radius R. If this tire is rolling at speed V without slipping on a horizontal road, and the pebble touches the road at time $t = 0$, where coordinates x (horizontal) and y (vertical) are zero, find equations for the x and y components of

(a) the position of the pebble,

(b) its velocity $\boldsymbol{v}$,

(c) and its acceleration $\boldsymbol{a}$,

as functions of the time.

4.15 The driver of a car is following a truck when he suddenly notices that a stone is caught between two of the rear tires of the truck. Being a safe driver (and a physicist too), he immediately increases his distance to the truck to 22.5 meters, so as not to be hit by the stone in case it comes loose. At what speed v was the truck traveling? (Assume the stone does not bounce after hitting the ground.)

4.16 A circus performer was devising a new act. He wanted to combine the Human Cannon Ball with a trapeze stunt. He had a cannon out of which he came with a muzzle velocity V. He wanted to get high enough so that he could grab the trapeze ($r = 2\,\text{m}$) and then continue on up to the platform located at $h = 20\,\text{m}$ above the floor, as shown in Fig. 4-3. (The trapeze should not go slack, i.e., his vertical velocity must be zero at both r and h).

(a) At what angle θ must the cannon be set?

(b) How far down the tent from the platform, x, should he put the cannon?

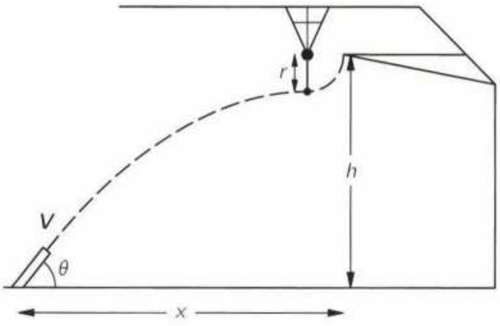


Figure 4-3

(c) What value of V must he choose?

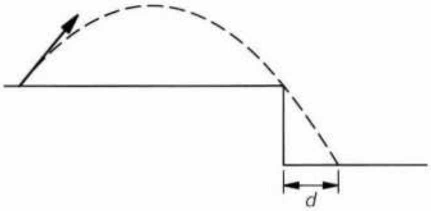


Figure 4-4

4.17 A mortar emplacement is set 27,000 ft horizontally from the edge of a cliff that drops 350 ft down from the level of the mortar, as shown in Fig. 4-4. It is desired to shell objects concealed on the ground behind the cliff. What is the smallest horizontal distance d from the cliff face that shells can reach if fired at a muzzle speed of $1000\,\mathrm{ft\,s^{-1}}$?

4.18 A Caltech freshman, inexperienced with suburban traffic officers, has just received a ticket for speeding. Thereafter, when he comes upon one of the "Speedometer Test" sections on a level stretch of highway, he decides to check his speedometer reading. As he passes the "0" start of the marked section, he presses on his accelerator and for the entire period of the test he holds his car at constant acceleration. He notices that he passes the 0.10 mile post 16 s after starting the test, and 8.0 s later he passes the 0.20 mile post.

(a) What speed v should his speedometer have read at the 0.20 mile post?

(b) What was his acceleration a?

4.19 On the long horizontal test track at Edwards AFB, both rocket and jet motors can be tested. On a certain day, a rocket motor, started from rest, accelerated constantly until its fuel was exhausted, after which it ran at constant speed. It was observed that this exhaustion of the rocket fuel took place as the rocket passed the midpoint of the measured test distance. Then a jet motor was started from rest down the track, with a constant acceleration for the entire distance. It was observed that both rocket and jet motors covered the test distance in exactly the same time. What was the ratio of the acceleration a_J of the jet motor to that of the rocket motor, a_R?

5

Newton's Laws

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 9. Exercises 5.1–5.13 should be solved using analytical methods. Exercises 5.14–5.17 should be solved using numerical methods.

5.1 Two blocks of mass $m_1 = 1\,\text{kg}$, $m_2 = 2\,\text{kg}$ on a horizontal surface, connected by a string, are being pulled by another string which is attached to a mass $m_3 = 2\,\text{kg}$ kg hanging over a pulley, as shown in Fig. 5-1. Neglect friction and the masses of the pulley and strings.

(a) Sketch free-body diagrams for all masses, showing the forces acting.

(b) Find the acceleration a of the masses.

(c) Find the tensions T_1 and T_2 in the strings.

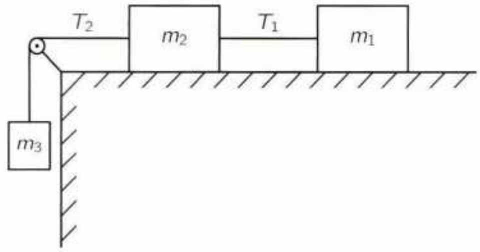


Figure 5-1

5.2 A mass m (kg) hangs on a cord suspended from an elevator which is descending with an acceleration of $0.1g$. What is the tension T in the cord in newtons?

5.3 Two objects of mass $m = 1\,\text{kg}$ each, connected by a taut string of length $L = 2\,\text{kg}$, move in a circular orbit with constant speed $V = 5\,\text{m}\,\text{s}^{-1}$ about their common center C in a zero-g environment, as shown in Fig. 5-2. What is the tension T in the string in newtons?

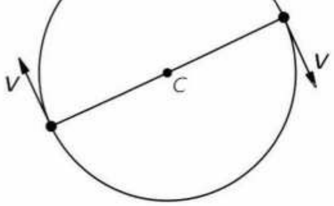


Figure 5-2

5.4 Referring to Fig. 5-3: What horizontal force F must be constantly applied to M so that M_1 and M_2 do not move relative to M? Neglect friction.

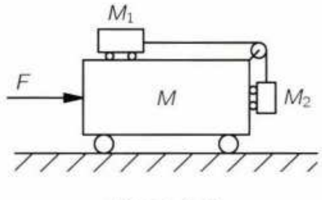


Figure 5-3

5.5 Referring to Fig. 5-4: What horizontal force F must be constantly applied to $M = 21\,\text{kg}$ so that $m_1 = 5\,\text{kg}$ does not move relative to $m_2 = 4\,\text{kg}$. Neglect friction.

5.6 In the system shown in Fig. 5-5, M_1 slides without friction on the inclined plane. $\theta = 30°$, $M_1 = 400\,\text{g}$, $M_2 = 200\,\text{g}$. Find the acceleration $\boldsymbol{a}$ of M_2 and the tension T in the cords.

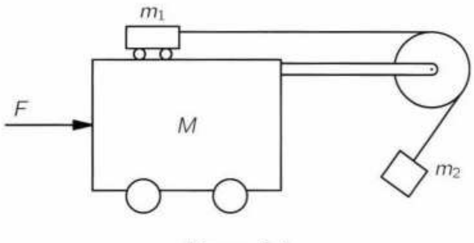


Figure 5-4

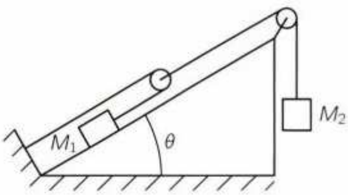


Figure 5-5

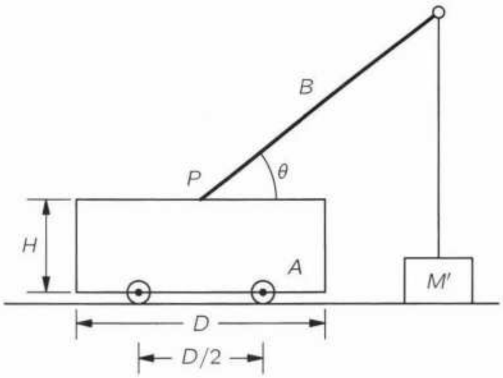


Figure 5-6

5.7 A simple crane is made of two parts, "A" with mass M_A, length D, height H, and distance $D/2$ between wheels of radius r; and part "B," a uniform rod or boom of length L and mass M_B. The crane is shown assembled in Fig. 5-6, with the pivot point P at midpoint of top of A. The center of gravity of A is midway between the wheels.

(a) With the rod or boom B set at angle θ with the horizontal, what is the maximum mass $M_{\max}$ that the crane can lift without tipping over?

(b) If there is a mass $M' = (4/5)M_{\max}$ at the end of the rope, what is the minimum time t necessary to raise this load M' a distance $(L \sin\theta)$ from the ground? (The angle θ remains fixed, and the mass of the rope may be neglected.)

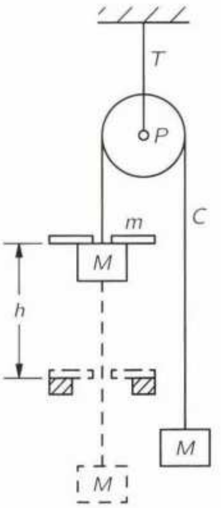


Figure 5-7

5.8 An early arrangement for measuring the acceleration of gravity, called Atwood's Machine, is shown in Fig. 5-7. The pulley P and cord C have negligible mass and friction. The system is balanced with equal masses M on each side as shown (solid line), and then a small rider m is added to one side. The combined masses accelerate through a certain distance h, the rider is caught on a ring and the two equal masses then move on with constant speed, v. Find the value of g that corresponds to the measured values of m, M, h, and v.

5.9 An elevator of mass M_2 has hanging from its ceiling a mass M_1, as shown in Fig. 5-8. The elevator is being accelerated upward by a constant force F. (F is greater than $(M_1 + M_2)g$.) The mass M_1 is initially a distance s above the elevator floor.

(a) Find the acceleration a_0 of the elevator.

(b) What is the tension T in the string connecting the mass M_1 to the elevator?

(c) If the string suddenly breaks, what is the acceleration a of the elevator immediately after, and what is the acceleration a' of mass M_1?

(d) How much time t does it take for M_1 to hit the bottom of the elevator?

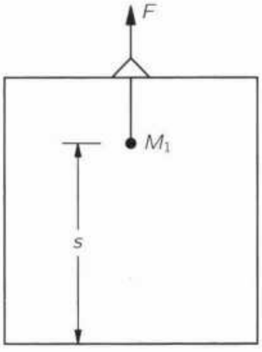


Figure 5-8

5.10 Given the system shown in Fig. 5-9, consider all surfaces frictionless. If $m = 150\,\text{g}$ is released when it is $d = 1\,\text{m}$ above the base of $M = 1650\,\text{g}$, how long after release, Δt, will m strike the base of M?

5.11 None of the identical gondolas on the Martian canal Rimini is quite able to support the load of both Paolo and Francesca, two affectionate marsupials who refuse to go in separate boats. The enterprising gondolier, Giuseppe, collects their fare by rigging them up from the mast as shown in Fig. 5-10, using the massless ropes and massless, frictionless pulleys characteristic of Martian construction. Giuseppe ferries them across before they hit either the mast or the deck. Assuming

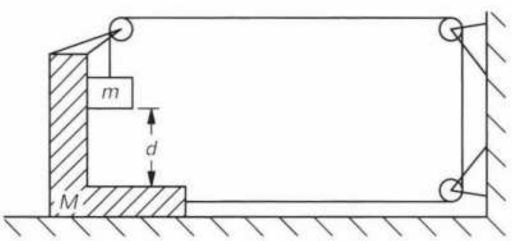


Figure 5-9

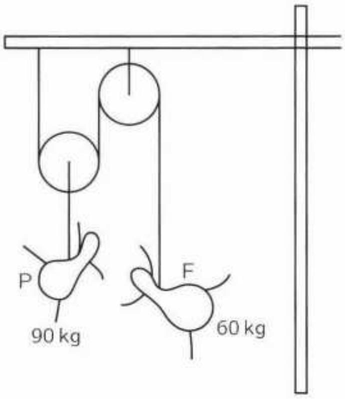


Figure 5-10

Paulo's mass is 90 kg and Francesca's is 60 kg, how much load W does Giuseppe save?

Hint: Remember that the tension in a massless cord that passes over a massless, frictionless pulley is the same on both sides of the pulley.

5.12 A painter working from a "bosun's" chair is hung down the side of a tall building, as shown in Fig. 5-11. Wishing to move in a hurry, the 180 lb painter pulls down on the fall rope so hard that he presses against the chair with a force of only 100 lb. The chair itself weighs 30.0 lb.

(a) What is the acceleration $\boldsymbol{a}$ of the painter and the chair?

(b) What is the total force F supported by the pulley?

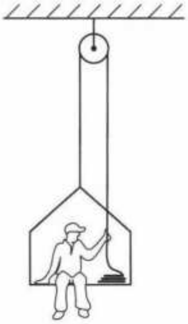


Figure 5-11

5.13 A space traveler about to leave for the moon has a spring balance and a 1.0 kg mass A, which when hung on the balance on the Earth gives the reading of 9.8 N. Arriving at the moon at a place where the acceleration of gravity is not known exactly but has a value of about one sixth the acceleration of gravity at the Earth's surface, he picks up a stone B which gives a reading of 9.8 N when weighed on the spring balance. He then hangs A and B over a pulley as shown in Fig. 5-12 and observes that B falls with an acceleration of $1.2\,\mathrm{m\,s^{-2}}$. What is the mass m_B of stone B?

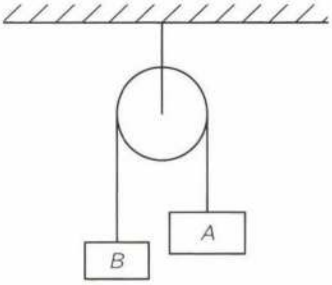


Figure 5-12

Use numerical methods to solve the following exercises.

5.14 A mass suspended from a spring hangs motionless, and is then given an upward blow such that it moves initially at unit speed. If the mass and spring constant are such that the equation of motion is $\ddot{x} = -x$, find the maximum height $x_{\max}$ attained by numerical integration of the equation of motion.

5.15 A particle of mass m moves along a straight line. Its motion is resisted by a force proportional to its velocity, $F = -kv$. It starts with speed $v = v_0$ at $x = 0$ and $t = 0$.

(a) Find x as a function of t by numerical integration.

(b) Find the time $t_{\frac{1}{2}}$ required to lose half its speed, and the maximum distance $x_{\max}$ attained.

Notes:

(a) Adjust the scales of x and t so that the equation of motion has simple numerical coefficients.

(b) Invent a scheme to attain good accuracy with a relatively coarse interval for Δt.

(c) Use dimensional analysis to deduce how $t_{\frac{1}{2}}$ and $x_{\max}$ should depend upon v_0, k, and m, and solve for the actual motion only for a single convenient value of v_0, say $v_0 = 1.00$ (in the modified x and t units).

5.16 A certain charged particle moves in an electric and a magnetic field according to the equations,

$$\frac{dv_x}{dt} = -2v_y,$$
$$\frac{dv_y}{dt} = 1 + 2v_x.$$

At $t = 0$ the particle starts at $x = 0$, $y = 0$ with velocity $v_x = 1.00$, $v_y = 0$. Determine the nature of the motion by numerical integration.

5.17 A shell is fired with a muzzle velocity $v = 1000\,\mathrm{ft\,s^{-1}}$ at an angle of 45° with the horizontal. Its motion is resisted by a force proportional to the cube of its velocity ($F = -kv^3$). The coefficient k is such that the resisting force is equal to twice the weight of the shell when $v = 1000\,\mathrm{ft\,s^{-1}}$. Find the approximate maximum height attained h_{max}, and the horizontal range R by numerical integration, and compare these with the values expected in the absence of resistance.

6

Conservation of Momentum

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 10.

6.1 When two bodies move along a line, there is a special system of coordinates in which the momentum of one body is equal and opposite to that of the other. That is, the total momentum of the two bodies is zero. This frame of reference is called the center-of-mass system (abbreviated CM). If the bodies have masses m_1 and m_2 and are moving at speeds v_1 and v_2, show that the CM system is moving at speed

$$v_{\mathrm{CM}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}.$$

6.2 Generalize Ex. 6.1 to any number of masses moving along a line, i.e., show that the speed of the coordinate system, in which the total momentum is zero, is given by

$$v_{\mathrm{CM}} = \frac{\sum m_i v_i}{\sum m_i}.$$

6.3 If T is the total kinetic energy of the two masses in Ex. 6.1, and T_{CM} is their total kinetic energy in the CM system, show that

$$T = T_{\mathrm{CM}} + \left(\frac{m_1 + m_2}{2}\right) v_{\mathrm{CM}}^2.$$

6.4 Generalize the result of Ex. 6.3 to any number of masses. Show that

$$T = T_{\mathrm{CM}} + \frac{\sum m_i}{2} v_{\mathrm{CM}}^2.$$

6.5 Two gliders with masses m_1 and m_2 are free to move on a horizontal air track. m_2 is stationary and m_1 collides with it perfectly elastically. They rebound with equal and opposite velocities. What is the ratio m_2/m_1 of their masses?

6.6 A neutron having a kinetic energy E collides head-on with a stationary nucleus of C^{12} and rebounds perfectly elastically in the direction from which it came. What is its final kinetic energy E'?

6.7 A projectile of mass $m = 10\,\mathrm{kg}$ is shot vertically upward from the earth with an initial velocity $v_p = 500\,\mathrm{m\,s^{-1}}$.

(a) Calculate the recoil velocity of the earth v_E.

(b) Calculate the ratio of the kinetic energy of the earth T_E to that of the projectile T_p at the moment of their separation.

(c) Sketch qualitatively the velocity and kinetic energy of the projectile and of the earth versus time.

Neglect air resistance and the orbital motion of the earth.

6.8 A particle of mass $m = 1.0\,\mathrm{kg}$, traveling at a speed $V = 10\,\mathrm{m\,s^{-1}}$, strikes a particle at rest of mass $M = 4.0\,\mathrm{kg}$ and rebounds in the direction from which it came, with a speed V_{F}. If an amount of heat $h = 20$ joules is produced in the collision, what is V_{F}? (Define all introduced quantities and state clearly from what physical laws your initial equations are derived.)

6.9 A machine gun mounted on the north end of a 10,000 kg, 5 m long platform, free to move on a horizontal air-bearing, fires bullets into a thick target mounted on the south end of the platform. The gun fires 10 bullets of mass 100 g each every second at a muzzle velocity of $500\,\mathrm{m\,s^{-1}}$. Does the platform move? If so, in what direction and at what speed v?

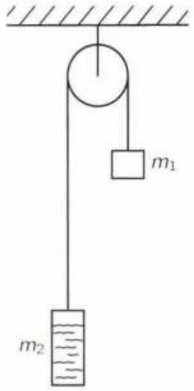


Figure 6-1

6.10 A mass m_1 is connected by a cable over a pulley to a container of water, which initially has a mass $m_2(t=0)=m_0$, as shown in Fig. 6-1. The system is then released and m_2 (with help of an internal pump) ejects water in the downward direction at a constant rate $dm/dt=r_0$ with a velocity v_0 relative to the container. Find the acceleration $\boldsymbol{a}$ of m_1, as a function of time. Neglect the masses of cable and pulley.

6.11 A toboggan slides down an essentially frictionless, snow covered slope, scooping up snow along the path. If the slope is 30° and the toboggan picks up 0.50 kg of snow per meter of travel, calculate its acceleration a at an instant when its speed is $4.0\,\mathrm{m\,s^{-1}}$ and its mass (including content) is 9.0 kg.

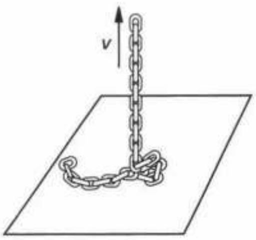


Figure 6-2

6.12 The end of a chain, of mass per unit length μ, at rest on a table top at $t=0$, is lifted vertically at a constant speed v, as shown in Fig. 6-2. Evaluate the upward lifting force F as a function of time.

6.13 The speed of a rifle bullet may be measured by means of a ballistic pendulum: The bullet, of known mass m and unknown speed V, embeds itself in a stationary wooden block of mass M, suspended as a pendulum of length L, as shown in Fig. 6-3. This sets the block to swinging. The amplitude x of swing may be measured and, using conservation of energy, the velocity of the block immediately after impact may be found. Derive an expression for the speed of the bullet in terms of m, M, L, and x.

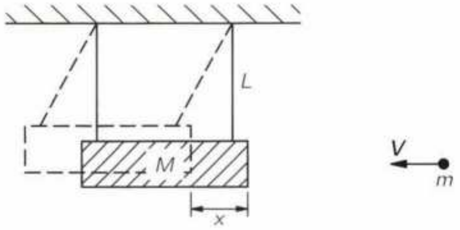


Figure 6-3

6.14 Two gliders A and A' are connected rigidly together and have a combined mass M and are separated by a distance $2L$. Another glider B of mass m, length L, is constrained to move between A and A', as shown in Fig. 6-4. All gliders move on a very long linear air track without friction. All collisions between (A, A') and B are perfectly elastic. Originally the whole system is at rest and glider B is in contact with glider A. A cap between A and B then is exploded, giving a total kinetic energy T to the system.

(a) Show the *qualitative* features of B's motion, i.e., position x on the track, velocity v with respect to the track, by sketching x and v as functions of time. Use the *same* time scale for both sketches.

(b) Calculate the period τ_0 in terms of T, L, m, and M.

Hint: The relative velocity of B with respect to (A, A') is

$$\boldsymbol{v}_{\mathrm{rel}}=\boldsymbol{v}_B-\boldsymbol{v}_{(A,A')}.$$

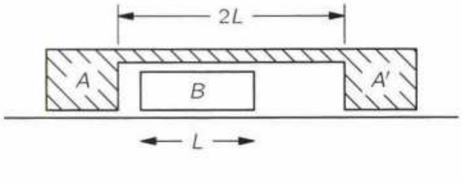


Figure 6-4

6.15 Two equally massive gliders, moving on a level air track at equal and opposite, velocities, $\boldsymbol{v}$ and $-\boldsymbol{v}$, collide almost elastically, and rebound with slightly smaller speeds. They lose a fraction $f \ll 1$ of their kinetic energy in the collision. If these same gliders collide with one of them initially at rest, with what speed will the second glider move after the collision? (This small residual speed Δv may easily be measured in terms of the final speed v of the originally stationary glider, and thus the elasticity of the spring bumpers may be determined.)

Note: If $x \ll 1$, $\sqrt{1-x}\approx 1-x/2$.

6.16 A rocket of initial mass $m = M_0$ ejects its burnt fuel at a constant rate $dm/dt = -r_0$ and at a velocity V_0 (relative to the rocket).

(a) Calculate the initial acceleration a of the rocket (neglect gravity).

(b) If $V_0 = 2.0\,\mathrm{km\,s^{-1}}$, at what rate r_0 must fuel be ejected to develop 10^5 kg-wt of thrust?

(c) Write a differential equation which connects the speed v of the rocket with its residual mass $m = M$, and solve the equation, if you can.

6.17 An earth satellite of mass 10 kg and average cross-sectional area $0.50\,\mathrm{m}^2$ is moving in a circular orbit at 200 km altitude where the molecular mean free paths are many meters and the air density is about $1.6 \times 10^{-10}\,\mathrm{kg\,m^{-3}}$. Under the crude assumption that the molecular impacts with the satellite are effectively inelastic (but that the molecules do not literally stick to the satellite but drop away from it at low relative velocity),

(a) Calculate the retarding force F_R that the satellite would experience due to air friction.

(b) How should such a frictional force vary with the satellite's velocity v? Would the satellite's speed decrease as a result of the net force on it? (Check the speed of a circular satellite orbit vs. height.)

7

Vectors

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 11.

Generalize Exs. 6.1 through 6.4 to three dimensional motion using vector notation.

7.1 If two bodies have masses m_1 and m_2 and are moving at velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, show that the CM system is moving at velocity

$$\boldsymbol{v}_{\text{cm}} = \frac{m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2}{m_1 + m_2}.$$

7.2 Show that for N bodies of masses m_i and velocities $\boldsymbol{v}_i$ the velocity of the coordinate system, in which the total momentum is zero, is given by

$$\boldsymbol{v}_{\text{cm}} = \frac{\sum\limits_{i=1}^{N} m_i\boldsymbol{v}_i}{\sum\limits_{i=1}^{N} m_i}.$$

7.3 If T is the total kinetic energy of the two masses in Ex. 7.1, and T_{CM} their total kinetic energy in the CM system, show that

$$T = T_{\text{CM}} + \left(\frac{m_1 + m_2}{2}\right)|\boldsymbol{v}_{\text{cm}}|^2.$$

7.4 Generalize the result of Ex. 7.3 to N masses. Show that

$$T = T_{\text{CM}} + \frac{\sum\limits_{i=1}^{N} m_i}{2}|\boldsymbol{v}_{\text{cm}}|^2.$$

7.5 A particle is initially at a point $\boldsymbol{r}_0$, and is moving under gravity with an initial velocity $\boldsymbol{v}_0$. Find the subsequent motion $\boldsymbol{r}(t)$.

7.6 You are given three vectors,

$$\begin{aligned}\boldsymbol{a} &= 3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k},\\ \boldsymbol{b} &= 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k},\\ \boldsymbol{c} &= \boldsymbol{i} + 3\boldsymbol{j}\end{aligned}$$

Find

(a) $\boldsymbol{a} + \boldsymbol{b}$

(b) $\boldsymbol{a} - \boldsymbol{b}$

(c) $\boldsymbol{a}_x$

(d) $\boldsymbol{a} \cdot \boldsymbol{i}$

(e) $\boldsymbol{a} \cdot \boldsymbol{b}$

(f) $(\boldsymbol{a} \cdot \boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a} \cdot \boldsymbol{b})\boldsymbol{c}$

7.7 A particle of mass 1 kg is moving in such a way that its position is described by the vector

$$\boldsymbol{r}(t) = t\boldsymbol{i} + (t + t^2/2)\boldsymbol{j} - (4/\pi^2)\sin(\pi t/2)\boldsymbol{k}.$$

(a) Find the position, velocity $\boldsymbol{v}(t)$, acceleration $\boldsymbol{a}(t)$, and kinetic energy $T(t)$ of the particle at $t = 0$ and $t = 1$ second.

(b) Find the force $\boldsymbol{F}(t)$ that will produce this motion.

(c) Find the radius of curvature $R(t)$ of the particle's path at $t = 1$ second.

7.8 A pilot flying at an air speed of 100 knots wishes to travel due north. He knows, from talking to the airport meteorologist, that there is a 25 knot wind from west to east at his flight altitude.

(a) In what direction should he head his plane?

(b) What will be the duration T of his flight, if his destination is 100 land miles away? (Neglect the time for landing and take-off, and note that 1 knot = 1.15 miles per hour.)

7.9 A cyclist rides at $10\,\mathrm{mi\,h^{-1}}$ due north and the wind, which is blowing at $6\,\mathrm{mi\,h^{-1}}$ from a point between N and E, appears to the cyclist to come from a point 15°E of N.

(a) Find the true direction of the wind.

(b) Find the direction in which the wind will appear to meet the cyclist on his return if he rides at the same speed.

7.10 A man standing on the bank of a river 1.0 mi wide wishes to get to a point directly opposite him on the other bank. He can do this in two ways:

(a) head somewhat upstream, so that his resultant motion is straight across,

(b) head toward the opposite bank and then walk up along the bank from the point downstream to which the current has carried him.

If he can swim $2.5\,\mathrm{mi\,h^{-1}}$ and walk $4.0\,\mathrm{mi\,h^{-1}}$, and if the current is $2.0\,\mathrm{mi\,h^{-1}}$ which is the faster way to cross, and by how much?

7.11 A motorboat that runs at a constant speed V relative to the water is operated in a straight river channel where the water is flowing smoothly with a constant speed R. The boat is first sent on a round trip from its anchor point to a point a distance d directly upstream. It is then sent on a round trip from its anchor point to a point a distance d away directly across the stream. For simplicity assume that the boat runs the entire distance in each case at full speed and that no time is lost in reversing course at the end of the outward lap. If t_V is the time the boat took to make the round trip in line with the stream flow, t_A the time the boat took to make the round trip across the stream, and t_L the time the boat would take to go a distance $2d$ on a lake.

(a) What is the ratio t_V/t_A?

(b) What is the ratio t_A/t_L?

7.12 Use vectors to find the great circle distance D between two points on the earth (radius = $r_{\oplus}$), whose latitudes and longitudes are (λ_1, ϕ_1) and (λ_2, ϕ_2).

Note: Use a system of rectangular coordinates with origin at the center of the earth, one axis along the earth's axis, another pointed toward $\lambda = 0$, $\phi = 0$, and the third axis pointed toward $\lambda = 0$, $\phi = 90°$ W. Let longitudes vary from 0° westward to 360°.

7.13 What is the magnitude and direction of the acceleration $\boldsymbol{a}$ of the moon at

(a) New moon?

(b) Quarter moon?

(c) Full moon?

Note:

$$\begin{aligned} R_{♁☼} &= 1.50 \times 10^8\,\text{km} \\ R_{♁☾} &= 3.85 \times 10^5\,\text{km} \\ M_{☼} &= 3.33 \times 10^5 M_{♁} \end{aligned}$$

7.14 Two identical 45° wedges M_1 and M_2, with smooth faces and $M_1 = M_2 = 8\,\text{kg}$, are used to move a smooth-faced mass $M = 384\,\text{kg}$, as shown in Fig. 7-1. Both wedges rest upon a smooth horizontal plane; one wedge is butted against a vertical wall, and to the other wedge a force $F = 592$ kg-wt is applied horizontally.

(a) What is the magnitude and direction of the acceleration $\boldsymbol{a}_1$ of the movable wedge M_1?

(b) What is the magnitude and direction of the acceleration $\boldsymbol{a}$ of the larger wedge M?

(c) What force F_2 does the stationary wedge M_2 exert on the heavy mass M?

Neglect friction.

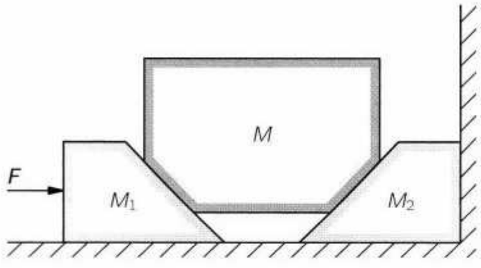


Figure 7-1

7.15 A mass m is suspended from a frictionless pivot at the end of a string of arbitrary length, and is set to whirling in a horizontal circular path whose plane is a distance H below the pivot point, as shown in Fig. 7-2. Find the period of revolution T of the mass in its orbit.

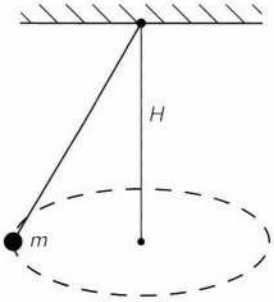


Figure 7-2

7.16 Two small, sticky, putty balls, a and b, each of mass 1 gram, travel under the influence of gravity with acceleration $-9.8\boldsymbol{k}\,\text{m}\,\text{s}^{-2}$. Given the initial conditions: at $t = 0$,

$$\begin{aligned} \boldsymbol{r}_a(0) &= 7\boldsymbol{i} + 4.9\boldsymbol{k}, \\ \boldsymbol{v}_a(0) &= 7\boldsymbol{i} + 3\boldsymbol{j}, \\ \boldsymbol{r}_b(0) &= 49\boldsymbol{i} + 4.9\boldsymbol{k}, \\ \boldsymbol{v}_b(0) &= -7\boldsymbol{i} + 3\boldsymbol{j}, \end{aligned}$$

find $\boldsymbol{r}_a(t)$ and $\boldsymbol{r}_b(t)$ for all times $t > 0$.

7.17 You are on a ship traveling steadily east at 15 knots. A ship on a steady course whose speed is known to be 26 knots is observed 6.0 miles due south of you; it is later observed to pass behind you, its distance of closest approach being 3.0 miles.

(a) What was the course of the other ship?

(b) What was the time T between its position south of you and its position of closest approach?

8

Non-Relativistic Two-Body Collisions in Three Dimensions

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 10 and 11.

The analysis of two-body collisions can often be simplified by the use of the CM system. Consider the general case of a non-relativistic two-body collision in the laboratory system. Two masses m_1 and m_2 with velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ collide. They may exchange mass during the collision, resulting in two masses m_3 and m_4 with velocities $\boldsymbol{v}_3$ and $\boldsymbol{v}_4$ after the collision, as shown in Fig. 8-1. The conservation laws for energy and momentum give the following relations:

Figure 8-1

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + Q = \tfrac{1}{2}m_3v_3^2 + \tfrac{1}{2}m_4v_4^2$$

$$m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2 = m_3\boldsymbol{v}_3 + m_4\boldsymbol{v}_4.$$

The value of Q determines the inelasticity of the collision process. This method of analysis in the laboratory system is often quite cumbersome and does not easily reveal possible systematics or simple relationships. In most cases, it is preferable to use the CM system, in which the collision is a linear one.

(i) Determine the CM velocity:

Before the collision:

$$\boldsymbol{v}_{\text{CM}} = \frac{m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2}{m_1 + m_2}.$$

After the collision:

$$\boldsymbol{v}'_{\text{CM}} = \frac{m_3\boldsymbol{v}_3 + m_4\boldsymbol{v}_4}{m_3 + m_4}.$$

Note that in all non-relativistic collisions

$$m_1 + m_2 = m_3 + m_4,$$

so that

$$\boldsymbol{v}'_{\text{CM}} = \boldsymbol{v}_{\text{CM}}.$$

In the following discussion we shall consider the *particular case* of

$$m_1 = m_3,$$
$$m_2 = m_4.$$

(ii) Find the velocities of m_1 and m_2 in the CM system, as shown in Fig. 8-2,

$$\boldsymbol{u}_1 = \boldsymbol{v}_1 - \boldsymbol{v}_{\text{CM}},$$
$$\boldsymbol{u}_2 = \boldsymbol{v}_2 - \boldsymbol{v}_{\text{CM}}.$$

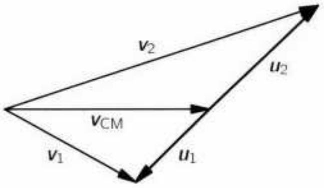


Figure 8-2

In the CM system, the momenta of the two masses are equal and opposite,

$$m_1\boldsymbol{u}_1 = -m_2\boldsymbol{u}_2,$$

that is, $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are *collinear* if the colliding bodies can be considered point masses. Also

$$\frac{|\boldsymbol{u}_1|}{|\boldsymbol{u}_2|} = \frac{m_2}{m_1}.$$

(iii) After the collision, the momenta in the CM system *again must be* equal and opposite, i.e.,

$$m_3\boldsymbol{u}_3 = -m_4\boldsymbol{u}_4,$$

$$\frac{|\boldsymbol{u}_3|}{|\boldsymbol{u}_4|} = \frac{m_4}{m_3}.$$

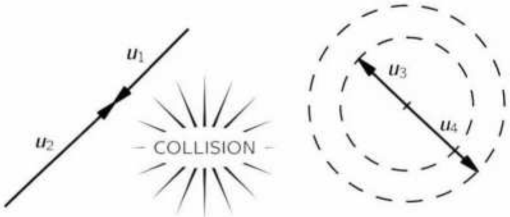


Figure 8-3

Note that in the CM system, the line of relative motion of the two masses may be rotated to a new direction by the collision, as shown in Fig. 8-3. The new direction is *not* defined by the laws of conservation of energy and momentum, but follows from the geometry of the interaction force and the initial relative motion. After the collision, the magnitudes of the velocities $\boldsymbol{u}_3$ and $\boldsymbol{u}_4$ may be larger, smaller, or equal to those of $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$, according to whether energy was released, absorbed, or unchanged in the collision. In the geometrical representation, the velocity vectors $\boldsymbol{u}_3$ and $\boldsymbol{u}_4$ must be collinear and their endpoints must fall on concentric spherical shells (circles in 2-D collisions) of radii

$$\frac{|\boldsymbol{u}_3|}{|\boldsymbol{u}_4|} = \frac{m_4}{m_3}$$

The magnitudes of $\boldsymbol{u}_3$ and $\boldsymbol{u}_4$ follow from the conservation of energy. In Ex. 7.3 it was shown that the total kinetic energy of two masses can be expressed as

$$T = T_{\mathrm{CM}} + \tfrac{1}{2}(m_1 + m_2)|\boldsymbol{v}_{\mathrm{CM}}|^2.$$

where

$$T_{\mathrm{CM}} = \tfrac{1}{2}m_1|\boldsymbol{u}_1|^2 + \tfrac{1}{2}m_2|\boldsymbol{u}_2|^2.$$

We know from conservation of momentum that

$$m_1|\boldsymbol{u}_1| = m_2|\boldsymbol{u}_2| = P,$$

therefore

$$T_{\mathrm{CM}} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\frac{P^2}{2}$$

Note that the "reduced mass" m_r of two masses m_1 and m_2 is defined

$$\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}.$$

In this notation

$$T_{\mathrm{CM}} = \frac{P^2}{2m_r}.$$

Before the collision:

$$T = T_{\mathrm{CM}} + \tfrac{1}{2}(m_1 + m_2)|\boldsymbol{v}_{\mathrm{CM}}|^2.$$

After the collision:

$$T' = T + Q = T'_{\mathrm{CM}} + \tfrac{1}{2}(m_3 + m_4)|\boldsymbol{v}'_{\mathrm{CM}}|^2$$

In non-relativistic collisions:

$$m_1 + m_2 = m_3 + m_4,$$

$$\boldsymbol{v}_{\mathrm{CM}} = \boldsymbol{v}'_{\mathrm{CM}},$$

therefore

$$T'_{\mathrm{CM}} = T_{\mathrm{CM}} + Q = T_{\mathrm{CM}}\left(1 + \frac{Q}{T_{\mathrm{CM}}}\right).$$

Also,

$$T'_{\mathrm{CM}} = \tfrac{1}{2}m_3|\boldsymbol{u}_3|^2 + \tfrac{1}{2}m_4|\boldsymbol{u}_4|^2,$$

and since

$$m_3|\boldsymbol{u}_3| = m_4|\boldsymbol{u}_4| = P',$$

it follows that

$$T'_{\mathrm{CM}} = \left(\frac{1}{m_3} + \frac{1}{m_4}\right)\frac{P'^2}{2} = \frac{P'^2}{2m'_r}.$$

In the particular case under discussion ($m_1 = m_2$, $m_3 = m_4$), we have $m_r = m'_r$, and consequently, from $T'_{\mathrm{CM}} = T_{\mathrm{CM}}\,(1 + Q/T_{\mathrm{CM}})$, it follows that

$$P'^2 = \left(1 + \frac{Q}{T_{\mathrm{CM}}}\right)P^2.$$

This expression gives the magnitude of the velocities in the CM system after the collision.

(1) *Elastic Collision*

$Q = 0$: kinetic energy unchanged in the collision and

$$P'^2 = P^2.$$

Therefore

$$|\boldsymbol{u}_3| = |\boldsymbol{u}_1|,$$
$$|\boldsymbol{u}_4| = |\boldsymbol{u}_2|.$$

(2) *Inelastic Collision*

$Q > 0$: kinetic energy released in the collision,
$Q < 0$: kinetic energy absorbed in the collision, and

$$P'^2 = \left(1 + \frac{Q}{T_{\mathrm{CM}}}\right)P^2.$$

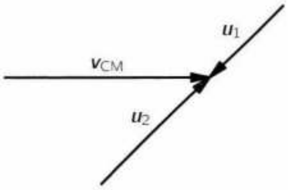
Therefore

$$|\boldsymbol{u}_3| = \left(1 + \frac{Q}{T_{\mathrm{CM}}}\right)^{1/2}|\boldsymbol{u}_1|,$$
$$|\boldsymbol{u}_4| = \left(1 + \frac{Q}{T_{\mathrm{CM}}}\right)^{1/2}|\boldsymbol{u}_2|.$$

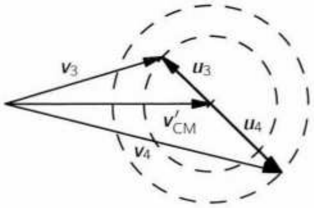
(iv) The velocities in the laboratory system after the collision are obtained simply by adding the CM velocity $\boldsymbol{v}'_{\mathrm{CM}}$ to $\boldsymbol{u}_3$ and $\boldsymbol{u}_4$, as shown in Fig. 8-4

$$\boldsymbol{v}_3 = \boldsymbol{v}'_{\mathrm{CM}} + \boldsymbol{u}_3,$$
$$\boldsymbol{v}_4 = \boldsymbol{v}'_{\mathrm{CM}} + \boldsymbol{u}_4.$$

Significant general and specific information on two-body collisions often can be directly deduced from the above geometrical representation of the scattering kinematics.



BEFORE



AFTER

Figure 8-4

8.1 Analogous to the above discussion, derive the results for a three-dimensional non-relativistic collision ($m_1 + m_2 = m_3 + m_4$) for the case $m_1, m_2 \neq m_3, m_4$, e.g., show that in the collision of two bodies with initial momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, the final momenta are given by

$$\boldsymbol{p}_3 = \boldsymbol{P}_3 + m_3\boldsymbol{v}_{\mathrm{CM}}$$
$$\boldsymbol{p}_4 = \boldsymbol{P}_4 + m_4\boldsymbol{v}_{\mathrm{CM}},$$

where $\boldsymbol{p}_i = m_i\boldsymbol{v}_i$ is the momentum of mass m_i in laboratory system, and $\boldsymbol{P}_i = \boldsymbol{p}_i - m_i\boldsymbol{v}_{\mathrm{CM}}$ is the momentum of mass m_i in the CM system, and

$$|\boldsymbol{P}_1| = |\boldsymbol{P}_2| = \sqrt{2m_r T_{\mathrm{CM}}},$$
$$|\boldsymbol{P}_3| = |\boldsymbol{P}_4| = \sqrt{2m'_r T'_{\mathrm{CM}}}.$$

8.2 A moving particle collides perfectly elastically with an equally massive particle initially at rest. Show that the two particles move at right angles to one another after the collision.

8.3 A moving particle of mass M collides perfectly elastically with a stationary particle of mass $m < M$. Find the maximum possible angle $\theta_{\max}$ through which the incident particle can be deflected.

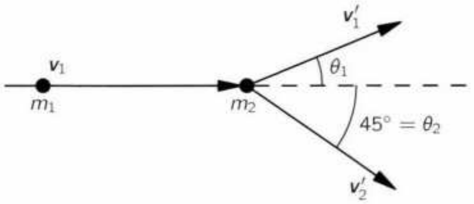


Figure 8-5

8.4 A particle of mass m_1 and velocity $\boldsymbol{v}_1$ collides perfectly elastically with another particle of mass $m_2 = 3m_1$ which is at rest ($\boldsymbol{v}_2 = 0$). After the collision, m_2 moves at angle $\theta_2 = 45^\circ$ with respect to the original direction of m_1, as shown in Fig. 8-5. Find θ_1, the final angle of motion of m_1, and v_1', v_2', the final velocities.

8.5 Two particles of equal mass m are shot at one another from perpendicular directions with equal speeds. After they collide, it is found that one particle was deflected 60° from its initial direction, towards the initial direction of the other particle, as shown in Fig. 8-6. Determine the angle α by which the second particle gets deflected towards the initial direction of the first if the collision is elastic.

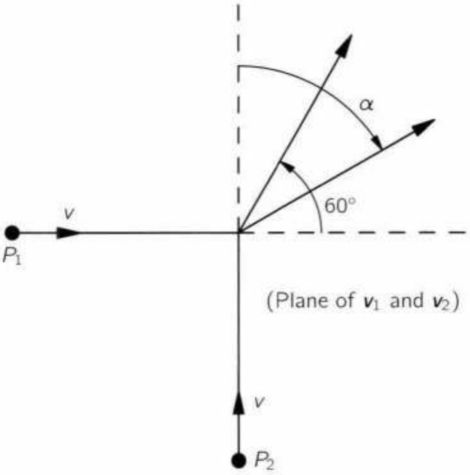


Figure 8-6

8.6 Two particles of equal mass are traveling on courses at right angles to each other with speeds of $v_1 = 8\,\mathrm{m\,s^{-1}}$ and $v_2 = 6\ \mathrm{s^{-1}}$, respectively. They collide elastically. After the collision, m_1 is observed to be traveling in a path that makes an angle $\theta = \arctan(1/2)$ with respect to the direction of its path before the collision, as shown in Fig. 8-7.

(a) What is the vector velocity $\boldsymbol{v}_{\mathrm{CM}}$ of the center of mass? Give Cartesian components.

(b) What are the magnitudes u_1, u_2 of the final velocities in the CM system?

(c) What is the final velocity $\boldsymbol{v}_1'$ of particle 1 in the lab. system?

8.7 A proton moving along the$x$-axis with a speed of $v_0 = 1.00 \times 10^7\,\mathrm{m\,s^{-1}}$ collides elastically with a stationary proton. After the collision, one proton moves in the xy-plane at an angle of 30° with the x-axis. Find the velocities $\boldsymbol{v}_1'$ and $\boldsymbol{v}_2'$ (speed and direction!) of both protons after the collision.

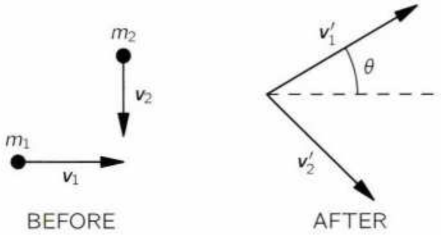


Figure 8-7

8.8 A proton moving along the x-axis with a speed of $v_0 = 1.00 \times 10^7\,\mathrm{m\,s^{-1}}$ collides elastically with a stationary beryllium (Be) nucleus. After the collision the Be nucleus is observed to move in the xy-plane at an angle 30° with the x-axis. Find:

(a) The speed v_2 of the Be nucleus in the lab system,

(b) The final velocity $\boldsymbol{v}_1'$ of the proton in the lab system,

(c) The final velocity $\boldsymbol{u}_1'$ of the proton in the CM system.

Note: Assume the relative masses of the Be nucleus and proton to be 9 : 1.

8.9 A circular air puck of mass 100 g and radius 2.00 cm is initially moving at a speed of $150\,\mathrm{cm\,s^{-1}}$ on a horizontal table, when it collides elastically with a stationary air puck of mass 200 g and radius 3.00 cm. At the instant of collision, the line joining the centers of the two pucks makes an angle of 60° with the original line of motion of the 100 g puck. If there is no friction, either with the table or between the pucks, find the velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ of each puck after the collision.

8.10 An object of mass m_1, moving with a linear speed v in a laboratory system, collides with an object of mass m_2 which is at rest in the laboratory. After the collision, it is observed that a fraction $|\Delta T/T|_{\mathrm{CM}} = 1 - \alpha^2$ of the kinetic energy in the CM system was lost in the collision. What was the fraction $|\Delta T/T|_{\mathrm{lab}}$ of energy lost in the *laboratory* system?

8.11 (a) A particle of mass m collides perfectly elastically with a stationary particle of mass $M > m$. The incident particle is deflected through a 90° angle. At what angle θ with the original direction of m does the more massive particle recoil?

(b) If in the collision a fraction $(1 - \alpha^2)$ of the CM energy is lost, what is the recoil angle of the originally stationary particle?

8.12 A proton with kinetic energy 1 MeV collides elastically with a stationary nucleus and is deflected through 90°. If the proton's energy is now 0.80 MeV, what was the mass M of the target nucleus in units of the proton mass m_p?

8.13 A puck of mass 1 kg moving at a speed of $v_1 = 6\,\mathrm{m\,s^{-1}}$ due N collides with a stationary puck of mass 2 kg. After the collision the 1 kg puck is moving at 45° NE of its original direction at a speed of $v_1' = 2\sqrt{2}\,\mathrm{m\,s^{-1}}$.

(a) What is the velocity $\boldsymbol{v}_2'$ of the 2 kg puck after impact?

(b) What fraction α of the kinetic energy was lost in the CM system?

(c) Through what angle θ was the 1 kg puck deflected in the CM system?

8.14 A "particle" of mass $m_1 = 2\,\mathrm{kg}$, which is moving with a velocity $\boldsymbol{v}_1 = (3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})\,\mathrm{m\,s^{-1}}$ collides inelastically with a second particle of mass $m_2 = 3\,\mathrm{kg}$, moving with a velocity $\boldsymbol{v}_2 = (-2\boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k})\,\mathrm{m\,s^{-1}}$.

(a) Find the velocity $\boldsymbol{v}$ of the composite particle.

(b) Find the total kinetic energy T_{CM} of the above particles in the CM system, *before* impact.

9

Forces

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 12.

9.1 The pressure $P + \Delta P$ inside a soap bubble is greater than the outside pressure P because of the surface tension. Show that in terms of the pressure differential the surface tension of a spherical soap bubble of radius R is given by the expression

$$\sigma = \frac{R}{4}\Delta P.$$

9.2 A 3 kg object has a motion given by $x = 6t^2 - 2t^3$ (x in meters, t in seconds). What is the force $\boldsymbol{F}$ (in newtons) acting on the object at $t = 4$ seconds.

9.3 Two masses, $m_1 = 4\,\text{kg}$ and $m_3 = 2\,\text{kg}$, are connected with cords of negligible weight over essentially frictionless pulleys to a third mass, $m_2 = 2\,\text{kg}$, as shown in Fig. 9-1. The mass m_2 moves on a long table with a coefficient of friction $\mu = 1/2$. What is the acceleration $\boldsymbol{a}$ of mass m_1 after the system is released from rest?

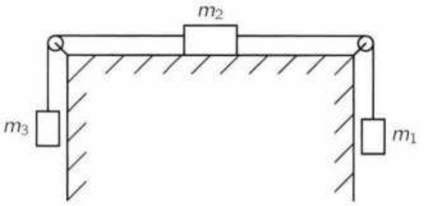


Figure 9-1

9.4 The interiors of two soap bubbles of equal radii are connected by a thin tube, as shown in Fig. 9-2. Describe what will happen in a realistic case.

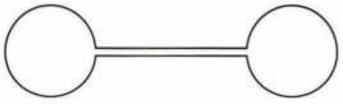


Figure 9-2

9.5 A rocket-powered high-speed sled running on rails employs pivot shoes between the sled and the rails, as shown in Fig. 9-3. Each shoe has replaceable rubbing pads at the heel and toe. The coefficient of friction between the rubbing pads and the rail is μ. The rate at which the rubbing pad material is worn off during the operation of the rocket sled is proportional to the friction force acting on the rubbing pad. If the shoe pivot point is at a given height h above the rail surface, at what horizontal distance, x, from the vertical centerline between the two rubbing pads should the pivot point P be placed so that the two rubbing pads will wear away at the same rate?

W : that portion of the weight of the rocket sled carried by the shoe in the diagram
H : horizontal component of force at the shoe pivot point
l : total length between centers of the two rubbing pads

9.6 Adjustable supports that can be slid up and down vertical posts are very useful in many applications. Such a support is shown in Fig. 9-4, with pertinent dimensions. If the coefficient of static friction between post and support is 0.30, and if a load 50 times the weight of the hanger is to be placed on the hanger at X, what is the minimum value of X for no slipping of the hanger?

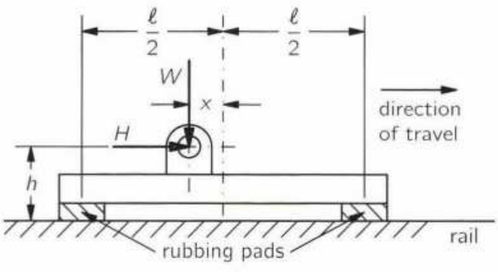


Figure 9-3

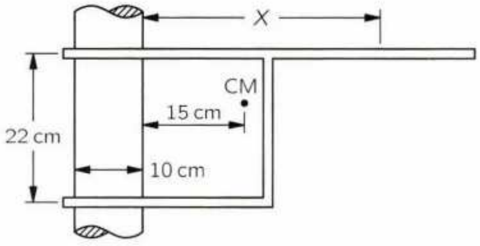


Figure 9-4

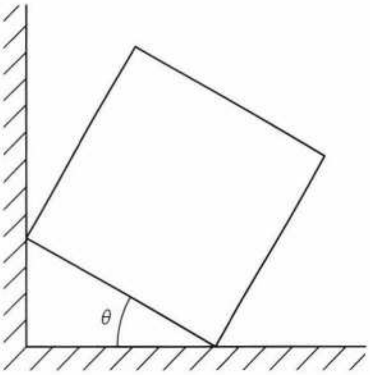


Figure 9-5

9.7 A cube of mass M rests tilted against the wall as shown in Fig. 9-5. There is no friction between the wall and the cube, but the friction between the cube and floor is just sufficient to keep the cube from slipping. When $0 < \theta < 45°$, find the minimum coefficient of friction μ_{min} as a function of θ. Check whether your answer is reasonable by noting values of μ for $\theta \to 0$ and $\theta \to 45°$ and by calculating θ for which $\mu = 1$.

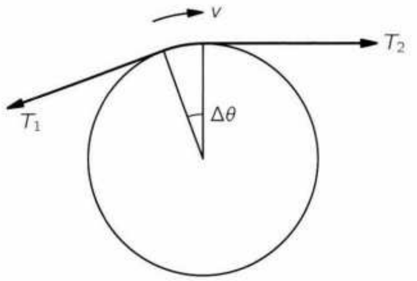


Figure 9-6

9.8 (a) A cord moving at a low speed v rubs against a round post and deviates from a straight line by a small angle $\Delta\theta \ll 1$ radian, as shown in Fig. 9-6. If the tension on one side of the post is $T + \Delta T$ and on the other side is T, what is the difference ΔT introduced by the friction?

(b) Integrate the preceding equation to find the ratio of tensions T_2/T_1 at the two ends of a cord wrapped around a circular post a finite angle α and pulled so as to slip.

9.9 A 5 g bullet is fired horizontally into a 3 kg wooden block resting on a horizontal surface. The coefficient of sliding friction between the block and surface is 0.2. The bullet remains embedded in the block, which is observed to slide 25 cm along the surface. What was the velocity v_0 of the bullet?

9.10 In their investigation at the scene of an automobile accident the police found, by measurement, that car A left skid marks 150 ft long before it collided with car B. It was also known that the coefficient of friction between rubber and the pavement at the scene of the accident was not less than 0.6. Find the speed v car A had just prior to the accident, and show that it must have exceeded the posted speed limit of 45 mph.

Note: 60 mph $= 88\,\text{ft}\,\text{s}^{-1}$ and acceleration due to gravity $= 32\,\text{ft}\,\text{s}^{-2}$.

9.11 An object rests at the base of a frictionless 20° incline 1.00 m long (slant). If the incline is accelerated along the table with an acceleration $a = 4.00\,\text{m}\,\text{s}^{-2}$, how much time t does it take the object to slide to the top of the slope?

9.12 A block of mass m slides on an inclined plane tilted at an angle θ with the horizontal. The coefficient of sliding friction is $\mu < \tan\theta$. Let $m = 1.00\,\text{kg}$, $\mu = 0.20$ and $\theta = 30°$. If the block is projected up the plane at an initial speed $3.00\,\text{m}\,\text{s}^{-1}$,

(a) how far up the plane, d, does it go?

(b) how much time t does it take to go up and come back to its starting point?

(c) how much energy ΔE is lost to heat in the process?

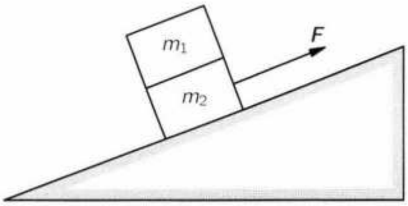


Figure 9-7

9.13 In the arrangement shown in Fig. 9-7, the inclined plane is 130 cm long and its upper end is 50 cm above the level of the lower end. The block m_2 rests on the plane, and has a mass of 60 g. The block m_1 has a mass 200 g. The coefficient of static friction between the two blocks is 0.50; the coefficient of sliding friction between the lower block and the plane is 0.33. A force $\boldsymbol{F}$ upward and parallel to the plane is applied to the lower block.

(a) What is the acceleration a of the lower block when the upper block just starts to slip on it?

(b) What is the maximum value of F before this slipping takes place?

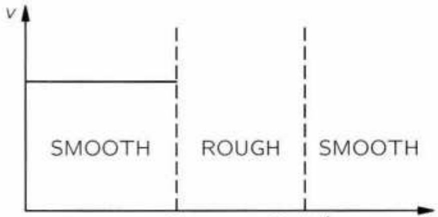


Figure 9-8

9.14 An ice puck of mass m is sliding without friction at velocity v_0 ($\text{cm}\,\text{s}^{-1}$), as shown in Fig. 9-8, when it meets a short strip of ice L (cm) wide, where there is a frictional force proportional to velocity $\boldsymbol{F} = -k\boldsymbol{v}$. Find an expression for the velocity v as a function of position x and complete the graph.

9.15 In a small increase ΔR of a soap bubble's radius, as shown in Fig. 9-9, work must be done against the surface tension σ, which leads to an increase ΔE in surface energy. The change in surface energy per change in surface area $\epsilon = \Delta E/\Delta A$ is called the *specific surface energy.* From the principle of virtual work find how the specific surface energy depends upon the radius R, over-pressure ΔP, etc., of the bubble. What is the numerical value of the ratio $|\sigma/\epsilon|$?

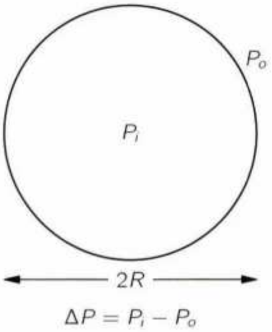


Figure 9-9

9.16 An air-conditioned school bus is approaching a railway crossing. One of the children has tied a hydrogen filled balloon to a seat. You observe that the anchor line of the balloon makes an angle of 30° with the vertical in the direction of motion, as shown in Fig. 9-10. Is the driver decelerating or accelerating the bus, and by how much? (Would a highway patrol officer commend the driver for his skill?)

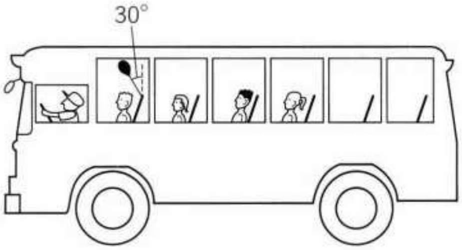


Figure 9-10

9.17 A particle of mass m and charge q moves at speed v in a plane at right angles to a magnetic field $\boldsymbol{B}$.

(a) Show that the particle moves in a circular path.

(b) Find the radius R of the circle.

(c) Find the time T required for the particle to go around the orbit.

This result is of importance in the operation of a cyclotron. Why?

9.18 A mass of 1000 g is supported by a cord 5.0 ft long fastened to a ring free to move on a horizontal rod, as shown in Fig. 9-11. The coefficient of static friction between the ring and the rod is 0.75. A second cord is fastened to the weight and passes over a pulley fastened to the rod 8-1/3 ft to the left of the ring. Weights W are attached to the other end of this cord until the ring just begins to slip. Find:

(a) The value of W when slipping just begins.

(b) The tension T in the five foot length of cord.

(c) The angle θ.

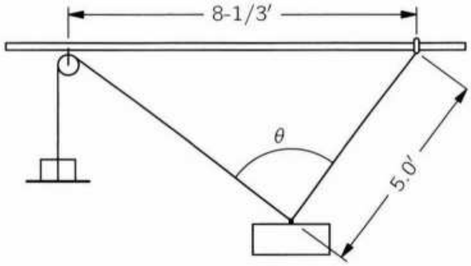


Figure 9-11

9.19 A side view of a simplified form of vertical latch is shown in Fig. 9-12. The lower member A can be pushed forward in its horizontal channel. The smooth sides of the channels have negligible friction, but at the interfaces of A and B, which are at 45° with the horizontal, there exists a static coefficient of friction μ. What is the minimum force F that must be applied horizontally to A to start motion of the latch, if B has a mass m?

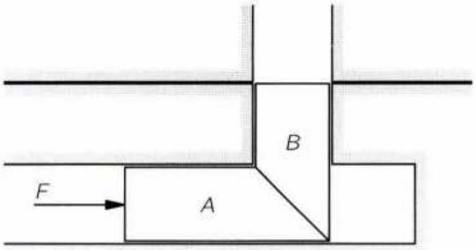


Figure 9-12

9.20 A particle of weight W rests on a rough inclined plane that makes an angle α with the horizontal, as shown in Fig. 9-13. If the coefficient of static friction $\mu = 2\tan\alpha$,

(a) Find the least horizontal force $H_{\min}$ acting transverse to the slope of the plane that will cause the particle to move.

(b) In what direction ϕ will it go, relative to $\boldsymbol{H}$?

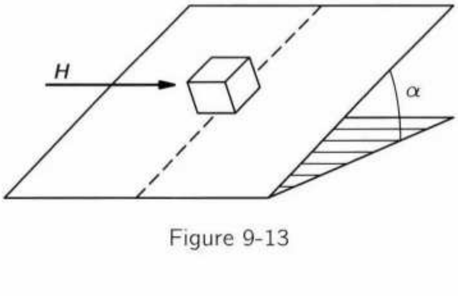


Figure 9-13

9.21 The gauge pressure inside a certain closed bottle of carbonated water is 3.00×10^5 Pa. The surface tension of water is $73\,\mathrm{mN\,m^{-1}}$. (Assume this is also the value for water in contact with CO_2 gas.) When the bottle is gently opened, a bubble will grow inside the liquid if it is initially "born" larger than a certain critical size R at a local nucleating center. Calculate this critical size for the conditions stated.

9.22 A particle of charge q and mass m moves in a combined electric and magnetic field E_y and B_z. (All other field components are zero.)

(a) Write the equations of motion of the particle.

(b) Apply a Galilean transformation to the coordinate system:

$$\begin{aligned} x' &= x - (E_y/B_z)t, \\ y' &= y, \\ z' &= z. \end{aligned}$$

(c) What do you conclude concerning the motion of a free particle in crossed electric and magnetic fields?

10

Potentials and Fields

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 13 and 14.

10.1 A mass m sliding on a frictionless horizontal surface at speed v_0 collides with a spring of force constant k and length x_0, as shown in Fig. 10-1. At what point x does the mass first come to rest? Neglect the mass of the spring.

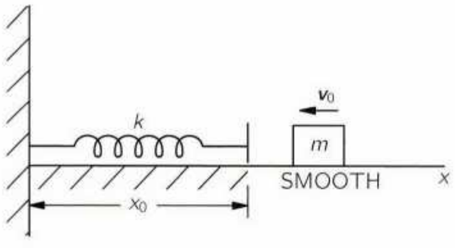


Figure 10-1

10.2 A certain spring has a force constant k. If it is stretched to a new equilibrium length within its linear range by a constant force F, show that it has the same force constant for displacement from the new equilibrium position.

10.3 A hollow spherical asteroid travels freely through space. There is a small particle of mass m in its interior. At what point in the interior will the particle be in equilibrium position.

10.4 The speed needed for a body to leave the Earth's gravitational field is (approximately) $7.0\,\mathrm{mi\,s^{-1}}$. If an interplanetary probe is given an initial speed of $8.0\,\mathrm{mi\,s^{-1}}$ just above the Earth's atmosphere, with what speed v relative to the Earth will it be traveling when it is at a distance of 10^6 mi from the Earth?

10.5 If the Earth carried a net charge of 1.00 C, what would its potential ϕ be?

10.6 A spherical shell of radius 0.50 m is (uniformly) charged to a potential of 10^6 V. Find the charge Q on the shell.

10.7 The maximum electric field strength that can be supported by dry air at atmospheric pressure without sparking (or corona discharge) is about $31\,\mathrm{kV\,cm^{-1}}$. Calculate the maximum potential ϕ to which an isolated, smooth sphere of diameter 20 cm can be raised (in air) without breakdown.

10.8 An object of mass 6.0 kg is free to move along the x-axis on a frictionless track. In each of the cases given it starts from rest at $x = 0$, $t = 0$.

(a) It moves 3.00 m under the action of a force $F = (3 + 4x)$ N in the x-direction (where x is in meters),

  (1) What velocity v does it acquire?

  (2) What is its acceleration a at that point?

  (3) What power P is being expended on it at that point?

(b) It moves for 3.00 s under the action of a force $F = (3 + 4t)$ N in the x-direction (where t is in seconds).

  (1) What velocity v does it acquire?

  (2) What is then its acceleration a?

  (3) What power P is being expended on it at that time?

In Ex. 10.9 through 10.11 a force $\boldsymbol{F} = (1.5y\boldsymbol{i} + 3x^2\boldsymbol{j} - 0.2(x^2+y^2)\boldsymbol{k})$ N acts upon a particle of mass 1.00 kg. At $t = 0$ the particle is located at $\boldsymbol{r} = 2\boldsymbol{i} + 3\boldsymbol{j}$ meters and is moving with a velocity $\boldsymbol{v} = 2\boldsymbol{j} + \boldsymbol{k}$ meters per second.

10.9 Find, at $t = 0$,

(a) The force $\boldsymbol{F}$ on the particle,

(b) The acceleration $\boldsymbol{a}$ of the particle,

(c) Its kinetic energy T, and

(d) The rate of change of kinetic energy, dT/dt.

10.10 Find, approximately, at $t = 0.01$,

(a) The location $\boldsymbol{r}$ of the particle,

(b) The velocity $\boldsymbol{v}$ of the particle, and

(c) Its kinetic energy T.

10.11 The particle moves from point $(0, -1, 0)$ to the point $(0, +1, 0)$ on a frictionless track under the action of the force $\boldsymbol{F}$ (plus a certain force of constraint). Find the work W done by the force $\boldsymbol{F}$ if the track is

(a) A straight track along the y-axis.

(b) A circular track in the yz-plane. Is this a conservative force?

10.12 A small, frictionless car coasts on an inclined track with a circular loop-the-loop of radius R at its lower end. From what height H above the top of the loop must the car start in order to traverse the loop without leaving the track?

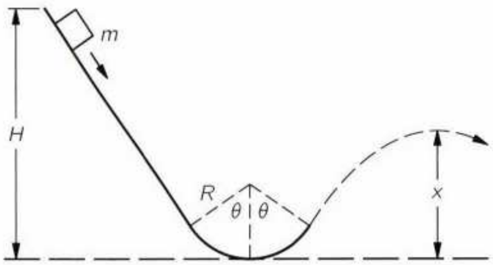


Figure 10-2

10.13 The lowest portion of a frictionless slide is part of a cylindrical surface of radius R subtending an angle θ on each side of the vertical as shown in Fig. 10-2. The slide begins at a vertical height H above its lowest point. A small object of mass m starts from rest at the upper end and slides down.

(a) What is the maximum height x of its trajectory above the lowest point after m leaves the slide?

(b) What force F does the object exert on the surface of the cylinder as it passes the lowest point of the slide?

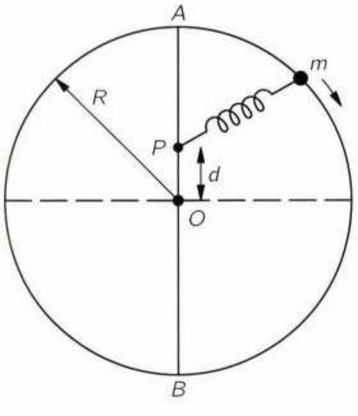


Figure 10-3

10.14 A small glider of mass m (kg) is connected to point P by means of a spring of negligible free (unstretched) length and of force constant k ($\mathrm{N\,m^{-1}}$). The point P is adjustable in position but once adjusted it does not move. The glider is free to move without friction on the outside surface of a circular hoop of radius R which stands in a vertical plane, as shown in Fig. 10-3. Let $\overline{\mathrm{OP}} = d$. If the mass starts from rest at the top of the hoop at point A and barely loses contact with the hoop as it passes the lowest point B, find d.

10.15 A flexible cable of length L which weighs M ($\mathrm{kg\,m^{-1}}$) hangs over a pulley of negligible mass, radius, and friction. Initially, the cable is just balanced. It is given a slight push to unbalance it, and it proceeds to accelerate. Find its speed v as the end flies off the pulley.

10.16 A particle starts from rest at the top of a frictionless sphere of radius R and slides on the sphere under the force of gravity. How far, d, below its starting point does it get before flying off the sphere?

10.17 A car with a mass of 3000 lb has a motor of 85 horsepower. To travel at a constant speed of 30 mph on the level the car must use 20 hp. Assuming that the frictional forces are the same, what is the steepest hill the car can climb at this same speed? (Specify the angle θ or some function of the angle that the slope of the hill makes with the horizontal.)

10.18 An automobile weighing 1000 kg is powered by an engine whose rated power is 120 kW. If the engine develops this power at a speed of $60\,\mathrm{km\,h^{-1}}$, what is the maximum acceleration a the car can have at this speed?

10.19 The 1960 World Records for the shot-put, the discus and the javelin were respectively 19.30 m, 59.87 m, and 86.09 m. The masses of the missiles involved were, respectively, 7.25 kg, 2 kg and 0.8 kg. Find the work done by each champion in making his record toss W_{shot}, W_{disc}, W_{jav}, assuming that each trajectory started at an elevation of 1.80 m above level ground and had an initial elevation of 45°. Neglect air resistance.

10.20 A certain spherical body of radius R and mass M has a uniform mass density throughout its volume. Find the gravitational potential ϕ and the gravitational field strength $\boldsymbol{g}$ as a function of the distance $\boldsymbol{r}$ from the center. Sketch your results graphically.

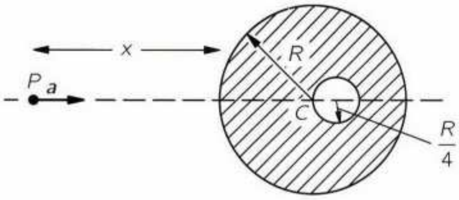


Figure 10-4

10.21 Find the gravitational acceleration $\boldsymbol{a}$ at a point P a distance x from the surface of a spherical mass of radius R and density ρ, which has a spherical cavity of radius $R/4$ whose center is situated at a distance $R/4$ beyond the center of the large sphere C, on the line PC produced, as shown in Fig. 10-4.

10.22 Consider a material of density ρ in the form of an extended (infinite) plate of thickness d. A spherical cavity of radius r (less than $d/2$) is cut at the origin of coordinates, as shown in Fig. 10-5.

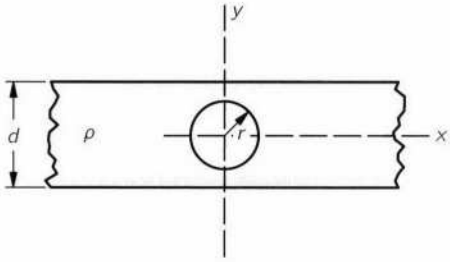


Figure 10-5

(a) What gravitational force $\boldsymbol{F}$ will act on a small mass m if located at *any* point y along the y-axis?

(b) Sketch the force as a function of y.

10.23 A small body of mass m moves under gravity in an elliptical orbit of eccentricity e and semi major axis a about a large mass M. (Assume that M remains stationary.) Evaluate the total energy E of m (kinetic plus potential). (Note that E does not depend upon e.)

10.24 (a) Deduce Kepler's third law for elliptical orbits, and give the relationship between the mass of a body M, the mass of its satellite m, the semi-major axis of the satellite's orbit, a, and the orbital period T.

(b) Show that all orbits having a given total energy E per unit mass will have the same period, and give the relationship between the period and E. (Assume $m \ll M$ for simplicity.)

10.25 Often, a capacitor consists of two(metallic) bodies, equally and oppositely charged. The capacitance C is then defined as the ratio of the charge on one body divided by the potential difference between them,

$$C = Q/(\phi_2 - \phi_1) \text{ farad},$$

with Q in coulombs and ϕ_1, ϕ_2 in volts.

Find the capacitance of a pair of concentric spherical shells, of radii A and B.

10.26 A 25 g weight hanger is attached to a spring of negligible mass whose force constant is $k = 15.3\,\mathrm{N\,m^{-1}}$. A mass $m = 50$ g is dropped from a height $h = 9.0$ cm onto the stationary weight hanger, with which it collides inelastically. What is the minimum height $H_{\min}$ attained by the mass m below its starting point?

10.27 Water (density 62.5 lb ft^{-3}) is pumped through a smooth hose whose nozzle has a cross-sectional area of 5.5 square inches. When the nozzle is aimed at an angle of 30° above the horizontal, the water stream is observed to have the apex of its trajectory 16 ft above the level of the nozzle. The pump inlet is connected to a large reservoir, and the water in the reservoir stands at an elevation 8.0 ft below the nozzle. If the over-all efficiency of the pump and the driving motor is 60 percent, what power P in kilowatts is being drawn from the electric line feeding the motor?

10.28 Estimate the pressure P at the center of the moon in atmospheres. ($1\,\text{atm} = 1.02 \times 10^5\,\text{Pa}$.) Use the following data for the moon:

$$\begin{aligned} \text{Mass } &= 7.0 \times 10^{22}\,\text{kg} \\ \text{Radius } &= 1740\,\text{km} \\ \text{Surface gravity } &= 160\,\text{cm}\,\text{s}^{-2} \\ \text{Mean density } &= 3.34\,\text{g}\,\text{cm}^{-3}. \end{aligned}$$

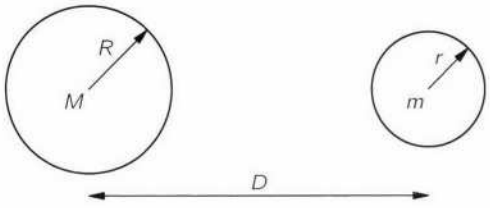


Figure 10-6

10.29 What is the minimum work W_{min} necessary to move a rock of mass m' from the earth's surface to the moon's surface, in terms of the mass and radius of the earth M and R, the mass and radius of the moon m and r, and the distance between their centers D? (See Fig. 10-6.)

10.30 A satellite of mass m moves in a circular orbit around an asteroid of mass M ($M \gg m$). If the asteroid's mass was suddenly* reduced to one-half its former value, what would happen to the satellite? Describe its new orbit.

10.31 (a) With what minimum speed v_{min} must an interstellar probe be launched from near the earth's surface in order to escape from the solar system with a residual speed of $10\,\text{mi}\,\text{s}^{-1}$ relative to the sun?

(b) The speed of the earth in its orbit is $18.5\,\text{mi}\,\text{s}^{-1}$. If it is desired to have the probe moving in a *prescribed direction* when it has escaped from the sun, what then is the *maximum* launching speed v_{max} that could be required?

10.32 It is desired to send a solar probe into an orbit with a perihelion distance of 0.010 AU and having the same period as Earth, so that data recorded during the flight may be transmitted to Earth one year after the launching date. With what speed v_0, and in what direction α relative to the earth-sun line, should the probe be launched from Earth?

Note: The orbital speed of the earth is $30\,\text{km}\,\text{s}^{-1}$.

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* How it could happen: The satellite is placed in orbit at a large distance from the asteroid to monitor the test of a nuclear device on the asteroid. The explosion expels half the asteroid's mass without directly affecting the distant satellite.

11

Units and Dimensions

11.1 What are the dimensions of

(a) the force constant of a spring?

(b) work?

(c) torque?

(d) surface tension?

(e) coefficient of friction?

(f) coefficient of viscosity?

(g) gravitational field?

(h) electric field?

(i) magnetic induction?

(j) E/B?

Note: Use the following symbols: M for mass, T for time, L for length, and Q for charge.

11.2 Show that the quantity $(\epsilon_0 c)^{-1}$ has the dimensions of resistance, and evaluate it numerically.

11.3 Moe and Joe, two cosmic physicists who grew up on different planets, meet at an interplanetary symposium on weights and measures to discuss the establishment of a universal system of units. Moe proudly describes the merits of the MKSA system, used in every civilized region of Earth. Joe equally proudly describes the beauties of the M′K′S′A′ system, used everywhere else in the solar system. If the constant factors relating the basic mass, length, and time standards of the two systems are μ, λ and τ, such that

$$m' = \mu m,$$
$$l' = \lambda l,$$
$$t' = \tau t,$$

what factors are needed to convert the units of velocity v, acceleration a, force F, and energy E between the two systems?

11.4 Use dimensional analysis to derive the dependence of the period T of the simple pendulum shown in Fig. 11-1 on its physical parameters.

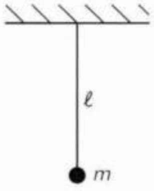

Figure 11-1

11.5 What is the numerical magnitude of $GM_{\odot}$ if lengths are measured in AU and times in years?

11.6 If a scale model of the solar system is made, using materials of the same respective average densities as the sun and planets, but reducing all linear dimensions by a scaling factor k, how will the periods T of revolution of the planets depend on k?

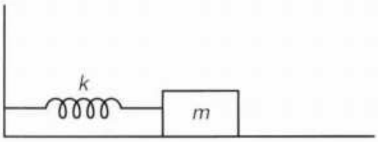

Figure 11-2

11.7 From dimensional analysis find the dependence on m, k, etc., of the period T of the mass-spring system shown in Fig. 11-2. (Neglect friction.)

11.8 A mass m is whirling in a circle with speed v at the end of a string of length l. (Neglect gravity.) Find how the tension T in the string and the radial acceleration a of the mass varies with these quantities.

11.9 A projectile of mass m is fired at an angle θ with the horizontal at an initial speed v. Find how the horizontal range R and time of flight T depend upon the relevant quantities.

11.10 A liquid drop of radius R, density ρ, may oscillate, with the surface tension σ providing the restoring force. Find how the period of oscillations T will depend upon these parameters.

11.11 A student discovers that he can tune the house piano by changing the tension in the wires. If the frequency ω is inversely proportional to length of the string, how does it depend on the length l, the tension T, and the linear density σ?

11.12 It is observed that water waves on a very deep ocean travel with a speed v that depends upon their wavelength λ, but not upon their amplitude. Find how the speed v should depend upon the wavelength and the density ρ of the water.

11.13 A box of volume V contains N particles of mass m, moving in various random directions with speed v. From dimensional analysis find how the pressure p of such a gas depends upon N, V, m, and v. Can you draw any conclusions concerning the nature of absolute temperature?

11.14 Show that Kepler's third law $r^3/t^2 = \text{const.}$ follows from the application of Newton's law $F = GMm/r^2$ to circular orbits. Use dimensional considerations.

Note: Here, r is the (constant) radial distance from the sun to a planet, t is the period of the planet's orbit, M is the mass of the sun, m is the mass of the planet, and F is the magnitude of the gravitational force between sun and planet.

12

Relativistic Kinematics and Dynamics, Mass and Rest Energy Equivalence

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Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 15 and 16.

12.1 Solve the Lorentz transformation for x, y, z, t in terms of x', y', z', t', between two inertial frames of reference with parallel axes having relative velocity V along x.

12.2 Write the Lorentz transformation found in Ex. 12.1 in differential form, and thus evaluate $dx/dt = v_x$ in terms of v'_x, V, etc.; do the same for $dy/dt = v_y$.

12.3 A particle in the S system moves along the x-axis with a velocity v_x and an acceleration a_x. What velocity and acceleration will it have in the S' system (with parallel axes) that is moving at velocity V along the x-axis (with respect to the S system)?

12.4 A stick of length $L = 5\,\mathrm{m}$ is at rest in a system S and oriented at an angle $\theta = 30^\circ$ with respect to the x-axis. What are the apparent length and orientation angle of this stick as measured by an observer in the (parallel) S' system, which moves at a speed of $v_x = c/2$ with respect to the S system?

12.5 A muon is formed high in the atmosphere and travels at a speed $v = 0.990c$ for a distance of 5.00 km before it decays.

(a) How long Δt does the muon "live," as measured by us, and as it would appear in its own frame of reference $\Delta t'$?

(b) What thickness of atmosphere L' does the muon traverse as measured in its reference frame?

12.6 Show that an electron has a rest energy $m_e c^2 = 0.511\,\mathrm{MeV}$.

12.7 A particle of mass m is caused to move along a line in such a way that its position is

$$x = \sqrt{b^2 + c^2 t^2} - b.$$

What force F must be applied to the particle to produce this motion?

12.8 The total electrical energy generated in the USA in 1965 amounted to $1.05 \times 10^{12}\,\mathrm{kWh}$.

(a) How much mass M was converted into energy in this process?

(b) If all of the mass change in the conversion of deuterium into helium were available (some is lost in neutrinos), what volume of heavy water per second H would be needed to supply the necessary deuterium?

Note: $M_{\mathrm{H}^2} = 2.0147\,\mathrm{u}$, $M_{\mathrm{He}^4} = 4.0039\,\mathrm{u}$.

12.9 The total power incident at the top of the earth's atmosphere from the sun is about $1.4\,\mathrm{kW\,m^{-2}}$. If this energy all arises from the conversion of ordinary hydrogen into helium, how much hydrogen H, in metric tons per second, does the sun "burn?" (Neglect the loss into neutrinos.)

12.10 (a) Evaluate the acceleration of gravity at the earth's surface g in units of light-year year^{-2}.

(b) If an isolated spaceship accelerates at such a rate that its occupants feel a constant acceleration equal to that of gravity at the earth's surface, and does so for a period of 5.00 years as measured by a stationary (unaccelerated) observer who is at rest with respect to the ship at $t = 0$, how far x has the ship gone, and how fast v is it traveling, at the end of this time?

13

Relativistic Energy and Momentum

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 16 and 17.

13.1 Show that the speed of a 1 GeV electron differs from c by one part in 8×10^6.

13.2 (a) Express the momentum p of a particle in terms of its kinetic energy T and rest energy mc^2.

(b) What is the speed v of a particle whose kinetic energy is equal to its rest energy?

13.3 The mass of a proton is $m_p = 938\,\text{MeV}$. In the cosmic radiation, protons having an energy of about $10^{10}\,\text{GeV}$ have been detected by indirect methods. Assume that a proton of this energy travels diametrically across a galaxy whose diameter is about 10^5 light-years. How much time Δt does this require, as measured in the proton's reference frame?

13.4 A particle of charge q, momentum p, is moving in a circle of radius R at right angles to a magnetic field B.

(a) If q is measured in units of electron charge, p is measured in MeV$/c$, and B is measured in gauss, what is the relation between p, B, and R? (Let $q = Zq_e$.)

(b) What is the radius of curvature of a proton of K.E. $= 60\,\text{GeV}$ in a $B = 0.3$ gauss field?

13.5 A cyclotron is being designed to accelerate protons to a kinetic energy of 150 MeV. The magnetic field strength is to be 1.00×10^4 gauss.

(a) What must be the minimum radius R_{min} of the magnet pole pieces?

(b) What frequency f must be used on the acceleration electrodes?

(c) By what fraction $\Delta f/f$ must the driving frequency be changed to allow for relativistic effects during the acceleration of a given particle?

13.6 A body of mass M, at rest in the laboratory, disintegrates into two parts of mass m_1 and m_2. Determine relativistically the *kinetic* energies T_1 and T_2 of the disintegration products.

13.7 A pion ($m_\pi = 273\,m_e$) at rest decays into a muon ($m_\mu = 207\,m_e$) and a neutrino ($m_\nu = 0$). Find the kinetic energy and momentum of the muon (T_μ, p_μ) and the neutrino (T_ν, p_ν) in MeV.

13.8 An excited atom of total mass m is at rest in a given coordinate system. It emits a photon, thereby losing energy ΔE. Taking into account the recoil of the atom, calculate the energy E_γ of the photon.

13.9 A particle of mass m, moving at speed $v = 4c/5$, collides inelastically with a similar particle at rest.

(a) What is the speed V of the composite particle?

(b) What is its mass M?

13.10 The Berkeley "bevatron" was designed to accelerate protons to sufficiently high energy to produce proton-antiproton pairs by the reaction

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + (\mathrm{p} + \bar{\mathrm{p}}).$$

The so-called threshold energy of this reaction corresponds to the situation when the four particles on the right move along together as a single particle of mass $M = 4m_p$. If the target proton is at rest before collision, what kinetic energy T must the bombarding proton have at threshold?

13.11 Calculate the threshold kinetic energy T_e for the production of a proton-antiproton pair by an electron-electron collision

$$\mathrm{e}^- + \mathrm{e}^- \rightarrow \mathrm{e}^- + \mathrm{e}^- + \mathrm{p}^- + \mathrm{p}^+.$$

You may use $m_e \approx 0.5\,\mathrm{MeV}$, $m_p \approx 1\,\mathrm{GeV}$. Compare this energy with the energy required in a p-p collision (as per Ex. 13.10).

13.12 A proton-antiproton pair may be created in the absorption of a photon (γ) by a proton at rest.

$$\gamma + \mathrm{p} \rightarrow \mathrm{p} + (\mathrm{p} + \bar{\mathrm{p}}).$$

What minimum energy E_γ must the photon have? (Express E_γ in terms of proton rest energy m_pc^2.) Compare this energy with the energy required in a p-p collision (as per Ex. 13.10), and in an e-e collision (as per Ex. 13.11).

13.13 A proton of mass m_p collides head-on with another proton at rest and produces a π-meson of mass $m_\pi \ll m_p$.

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{p} + \mathrm{p} + \pi.$$

(a) What is the minimum kinetic energy T_p of the incident proton?

(b) What is the kinetic energy T_π of the meson at threshold?

(c) Approximately what error is made by using non-relativistic expressions?

Give answers in terms of m_p and m_π. You may use non-relativistic expressions for kinetic energies and velocity transformations.

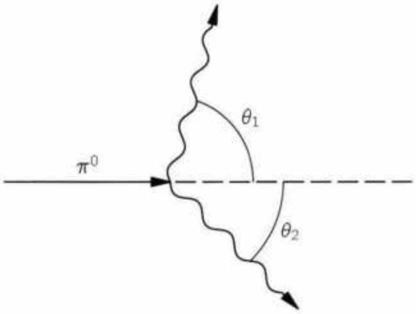

Figure 13-1

13.14 A π^0 meson may decay into two γ-rays* while at rest or in flight, as shown in Fig. 13-1: $\pi^0 \rightarrow \gamma + \gamma$.

Note: A γ-ray is a photon: $E_\gamma = p_\gamma c = h\nu$.

(a) If the decaying π^0 has a velocity $\boldsymbol{v}$ and mass m_π and a γ-ray is emitted at an angle θ with respect to the original direction of the π^0, find the γ-ray energy as function of m_π, v, and θ.

(b) What is the maximum and minimum energy, $E_{\max}$ and $E_{\min}$, an emitted γ-ray can have, and at what emission angles do these occur?

(c) Can you find a simple function of $E_{\max}$ and $E_{\min}$ which is independent of the velocity of the π^0, and what is its physical meaning?

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* The existence and the numerical value of the mass of the π^0 were inferred from measurements of this nature. Reference: A. G. Carlson, J. E. Hooper, and D. T. King, Phil. Mag. **41**, p. 701–724 (1950)

14

Rotation in Two Dimensions, The Center of Mass

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 18 and 19.

14.1 A rigid wheel of radius R is rolling without slipping on a horizontal surface. The plane of the wheel is vertical, and the axis of the wheel is moving horizontally with a speed V relative to the surface. If the axis of the wheel is parallel to the z-axis, V is in the positive x-direction, and θ is the angle through which the wheel has rotated since a certain point P on the rim was in contact with the ground, show that the instantaneous velocity (speed and direction) of the point P is given by

$$\boldsymbol{v} = V\left((1-\cos\theta)\boldsymbol{i} + \sin\theta\boldsymbol{j}\right).$$

14.2 The knowledge of the surface area (or volume) swept out by a plane curve (or area) can be used to find the center of mass of a thin, curved uniform wire or a thin uniform plane sheet.

(a) Show that the surface area A swept out by the plane curve C in its rotation through an angle α about the axis O which lies in the plane of C, as shown in Fig. 14-1(a), is equal to the length l of C, times the distance αr through which the CM moves,

$$A = \alpha r l,$$

(where r is the distance from the CM to O).

(b) Show that the volume V swept out by the plane area A in its rotation through an angle α about the axis O, as shown in Fig. 14-1(b), is equal to the area A times the distance through which the CM moved,

$$V = \alpha r A.$$

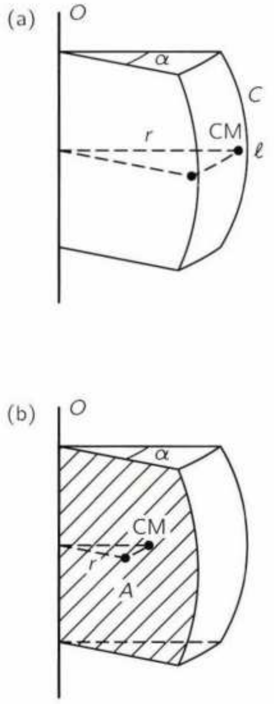

Figure 14-1

14.3 Show that the CM of any collection of particles moves as would a single particle having a mass equal to the sum of the masses of the individual particles, and subject to the vector sum of all forces acting on the separate particles,

$$M\ddot{\boldsymbol{R}} = \sum_i \boldsymbol{f}_i.$$

14.4 A force $\boldsymbol{F} = 30\boldsymbol{i} + 40\boldsymbol{j}$ newtons acts at the point at $\boldsymbol{r} = 8\boldsymbol{i} + 6\boldsymbol{j}$ meters. Find

(a) the torque τ about the origin,

(b) the magnitude (length) l of the lever arm of the force,

(c) the component of the force $F_\perp$ perpendicular to $\boldsymbol{r}$.

14.5 A yo-yo on a horizontal table is free to roll without slipping. If a horizontal force $\boldsymbol{F}$ is applied, as shown in Fig. 14-2, will it roll in or opposite to the direction of $\boldsymbol{F}$? Why?

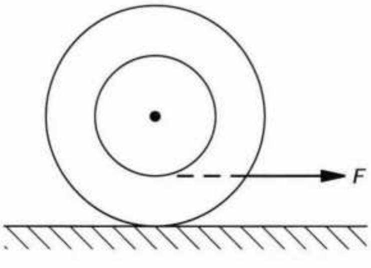

Figure 14-2

14.6 At what latitude λ is the tangential speed of a point due to the earth's rotation $200\,\mathrm{m\,s^{-1}}$ less than it is in Los Angeles (latitude 34°N)?

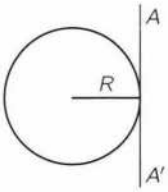

Figure 14-3

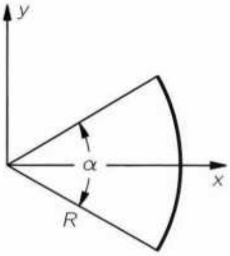

Figure 14-4

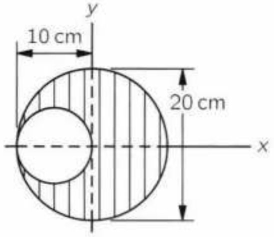

Figure 14-5

14.7 A circle of radius R tangent to an axis AA' as shown in Fig. 14-3, is revolved around the axis to generate a torus. Find the volume V of this torus.

14.8 A mass M and a mass $2M$ are rotating about their CM at angular velocity ω at a fixed distance R apart. What is their kinetic energy of rotation T?

14.9 (a) Find the CM of a thin uniform wire of length L, bent into a circular arc of radius R ($R > L/2\pi$). Use coordinates with origin at the center of the circle and with the x-axis passing through the center of the wire, as shown in Fig. 14-4.

(b) Use the result of part (a) above to find the CM of a circular sector made of uniform sheet metal, if the sector has a radius R and subtends an angle α at the center, as shown in the figure.

14.10 A disc of uniform density has a hole cut out of it, as shown in Fig. 14-5. Find the coordinates of the center of mass.

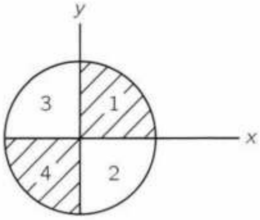

Figure 14-6

14.11 A solid cylinder has a density which varies by quadrants as shown in Fig. 14-6, with the numbers indicating relative densities. If the x-y axes are as indicated, what is the equation of the line drawn through the origin and through the center of mass?

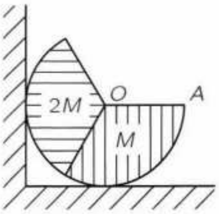

Figure 14-7

14.12 A cylinder of radius π cm and mass 3 kg is cut into thirds. The same thing is done to a second cylinder of radius π cm and mass 6 kg. A piece from one cylinder is glued to a piece from the other one giving the arrangement shown in Fig. 14-7, where the radius OA is horizontal. The floor has sufficient friction that slipping cannot occur, and the wall has negligible friction.

(a) What is the force F of the cylinder on the wall?

(b) How far x from the center along the radius OA would one have to place a point mass M so that the system would remain in equilibrium if the wall were removed?

Note: For the location of the CM of a sector of a circle, see Ex. 14.9.

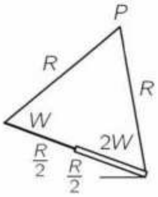

Figure 14-8

14.13 A rod of length R is made of two uniform pieces of equal length $R/2$ each, but one piece weighs twice as much as the other. The rod is suspended by cords of length R attached to each end and to a nail at P, as shown in Fig. 14-8. When the system comes to rest, what angle α does the rod make with the horizontal?

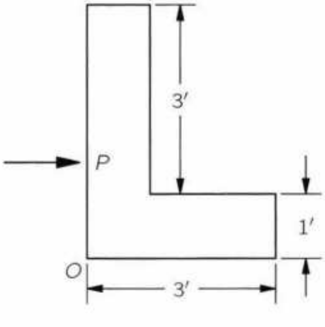

Figure 14-9

14.14 The L-shaped body shown in Fig. 14-9 is made of sheet metal of uniform thickness and rests on a frictionless, horizontal table. It is struck with a sudden blow at point P in the direction shown, and is observed to move away without rotating. How far from the vertex O was the blow applied?

14.15 From a square piece of uniform sheet metal with side length a an isosceles triangle is to be cut out from one edge as shown in Fig. 14-10, such that the remaining metal, when suspended from the apex P of the cut, will remain in equilibrium in any position. What is the altitude h of the cutout triangle?

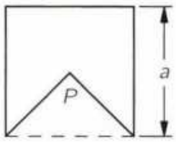

Figure 14-10

14.16 Masses M_1 and M_2 are placed at the opposite ends of a rigid rod of length L and negligible mass; the dimensions of M_1 and M_2 are negligible compared to L. The rod is to be set rotating about an axis perpendicular to it. Through what point x on this rod (measured from M_2) should the axis pass in order that the work required to set the rod rotating with an angular speed ω_0 shall be a minimum?

14.17 The essential elements of one form of simple speed governor are as shown in Fig. 14-11: A horizontal rod is mounted symmetrically to a vertical shaft and on the horizontal rod are freely sliding brake shoes; when the shaft turns, the brake shoes press against the inner surface of a stationary cylindrical brake drum. If the brake shoes are each of mass m and their thickness is negligible compared to the inner radius r of the brake drum, and if the coefficient of sliding friction between the shoes and the drum is μ, what is the power P required to turn the governor shaft in terms of m, r, μ, and f, the frequency of rotation of the shaft?

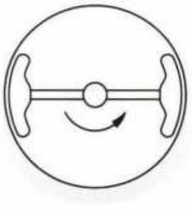

Figure 14-11

14.18 A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are now piled on as shown in Fig. 14-12, so that the sides form continuous planes, but the ends are offset at each brick from the previous brick by a distance L/a, where a is an integer. How many bricks, n, can be used in this manner before the pile topples over?

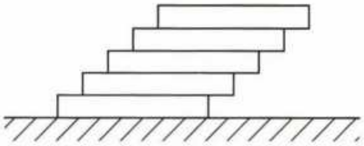

Figure 14-12

14.19 A rotating governor, as shown in Fig. 14-13, is to be designed to shut off power when the machine to which the governor is directly connected reaches a speed of 120 rpm. The operating collar C weighs 10.0 lb and slides without friction on the vertical shaft AB. C is so designed to shut off power when the distance $\overline{AC}$ reduces to 1.41 ft. If the four links of the governor framework are each 1.00 ft long between frictionless pivots and are relatively massless, what value should the masses M have so that the governor will operate as planned?

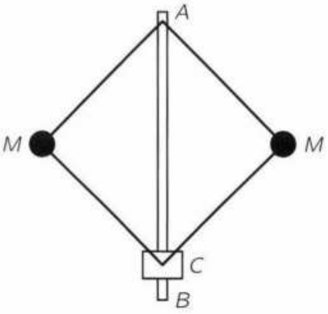

Figure 14-13

14.20 Four masses M in the same plane in field-free space connected by very light springs of spring constant k are spinning at angular velocity ω about an axis perpendicular to the plane and through the center of symmetry, as shown in Fig. 14-14. The springs have a relaxed length L.

(a) Assuming equilibrium, by how much, ΔL, are the springs extended?

(b) What condition determines whether stable equilibrium is possible?

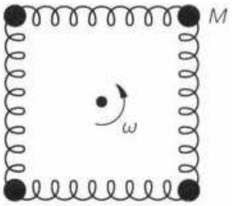

Figure 14-14

15

Angular Momentum, The Moment of Inertia

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 18 and 19.

15.1 A point mass moves under the influence of a central force—that is, a force whose line of action passes through a fixed point. Show that the angular momentum of the mass remains constant. Show also that this result is equivalent to Kepler's second law of planetary motion.

15.2 The statement $\boldsymbol{\tau} = d\mathbf{L}/dt$ is generally true for a rigid body if one considers all forces including inertial (pseudo) forces. If inertial forces are not included in the analysis, $\boldsymbol{\tau} = d\mathbf{L}/dt$ is still a correct relation for

(a) any fixed axis outside the body,

(b) any axis of fixed direction through the CM of the body,

(c) an axis about which the body is rotating at a given moment (instantaneous axis of rotation).

How many of the above statements can you prove?

15.3 A straight, uniform wire of length L and mass M is bent at its midpoint to form the angle θ, as shown in Fig. 15-1. What is its moment of inertia I for an axis passing through the point A, perpendicular to the plane determined by the bent wire?

Figure 15-1

15.4 A thin uniform trapdoor of mass m and width l is hinged at one edge to a level floor and stands vertically. If allowed to fall, with what angular speed ω will it strike the floor? Neglect friction in the hinge.

15.5 A mass m is hung from a string wound around a solid circular cylinder of mass M and radius r, pivoted on bearings of negligible friction, as shown in Fig. 15-2. Find the acceleration a of m.

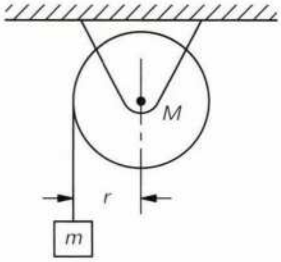

Figure 15-2

15.6 Calculate the moment of inertia I of the object shown in Fig. 15-3, about the axis A which is perpendicular to the plane of the figure. The object consists of 4 semicircles of radius a of uniform thin wire, of total mass M.

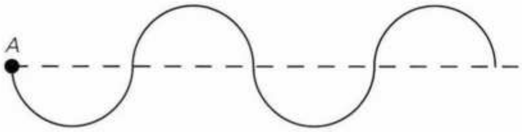

Figure 15-3

15.7 Calculate the moments of inertia I of the following rigid bodies, each of which has a mass m:

(a) a thin, straight uniform rod of length L, about a perpendicular axis through one end,

(b) a thin, straight, uniform rod of length L, about a perpendicular axis through its center,

(c) a thin-walled hollow circular cylinder of radius r, about its axis, and

(d) a solid circular cylinder of radius r, about its axis.

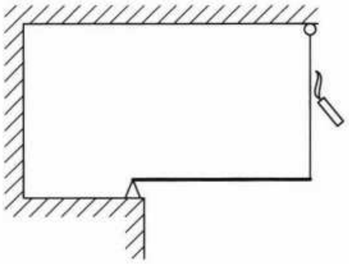

Figure 15-4

15.8 A horizontal thin rod of mass M, length L rests at one end on a support and is suspended by a string at the other end, as shown in Fig. 15-4. What force F is exerted by the rod on the support immediately after the string is burned?

15.9 Eight thin uniform rods, each of length L and mass m, are held in the form of a plane square by the framework of negligible mass shown dotted in Fig. 15-5(a). The square is set rotating freely about a frictionless axle through O, perpendicular to the plane of the framework, with an angular speed of ω_0 (rad s^{-1}). While thus rotating, an internal mechanism K, attached to the framework and with an unchanging moment of inertia about O of $(40/3)mL^2$, collapses the square to the cross shown in Fig. 15-5(b). How much work W was done by the mechanism in the collapsing process?

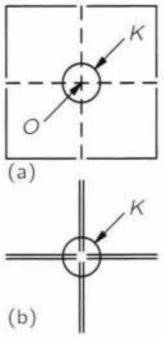

Figure 15-5

15.10 (a) Starting from rest, a symmetrical object rolls (without slipping) down an incline of height h. The moment of inertia of the object about its center of mass is I, the mass is M, and the radius of the rolling surface in contact with the incline is r. Determine the linear velocity V_0 of the center of mass at the bottom of the incline.

(b) Apply the general equation of part (a) above to determine the velocity of the center of mass if the object is

(1) a sphere,

(2) a disc,

(3) a disc of mass M_1 and outer radius R_1, with a spindle of mass m_2 and radius r_2 on which the disc rolls.

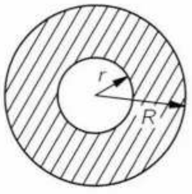

Figure 15-6

15.11 On an endless belt that is inclined at an angle θ with the horizontal, a uniform cylinder is placed, its axis horizontal and perpendicular to the edge of the belt. The surfaces are such that the cylinder can roll without slipping on the belt. How should the belt be caused to move so that, when released, the axis of the cylinder does not move?

15.12 Two visually indistinguishable cylinders of equal mass and identical outside dimensions roll down an inclined plane. One of the cylinders reaches the bottom sooner than the other.

(a) What do you conclude from that?

(b) Justify your model of the composition of the two cylinders by calculating the observed effect.

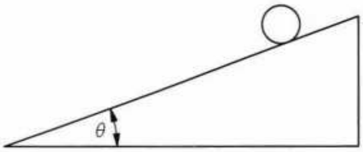

Figure 15-7

15.13 A spherical ball of radius R contains an empty spherical concentric hole of radius r $(r < R)$, as shown in Fig. 15-6. The mass per unit volume ρ is constant between r and R. Express the moment of inertia I of the ball about an axis passing through the center in terms of r, R, and ρ, and also in terms of r, R, and the total mass M.

15.14 A uniform, solid ball is placed at rest on an incline of slope angle θ, as shown in Fig. 15-7. What is the minimum value μ_0 of the coefficient of static friction μ between ball and incline so that the ball will roll down the incline without slipping?

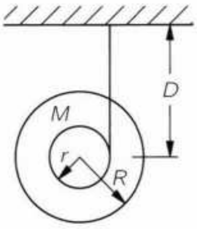

Figure 15-8

15.15 A yo-yo like spool consists of two uniform discs, each of mass M and radius R, and an axle of radius r and negligible mass. A thread wound around the axle is attached to the ceiling, and the spool is released from rest a distance D below the ceiling, as shown in Fig. 15-8.

(a) If there is to be no pendulum-like swinging motion, what angle θ should the thread make with the vertical as the spool is released?

(b) What is the downward acceleration a of the center of the spool?

15.16 The hoop H of radius r shown in Fig. 15-9 rolls without slipping down the incline. The starting height h is such that the hoop acquires a velocity just sufficient to "loop the loop"; i.e., the hoop just maintains contact with the circular track at point P. What is h?

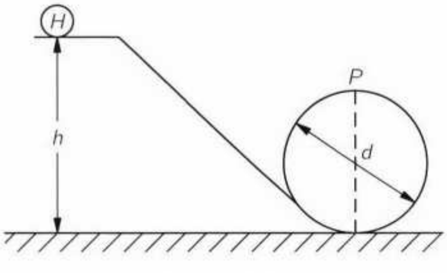

Figure 15-9

15.17 An air puck of mass m moves on the surface of a horizontal table, guided by a string attached to the puck and passing downward through a small hole in the table top. Initially the length of string above the table is r_1, and the puck is set moving at speed v_1 in a circular path of this radius. The string is then pulled downward through the hole until an amount r_2 remains above the table. Find

(a) the final speed v_2 of the puck,

(b) the work W required to pull the string through the hole from r_1 to r_2, and

(c) the magnitude of the force F needed to hold the radius constant, using the principle of conservation of energy.

15.18 A yo-yo of mass M, outer radius R, and moment of inertia I, on a horizontal table is free to roll without slipping. A force F is applied at the inner radius r at an angle α with the horizontal, as shown in Fig. 15-10.

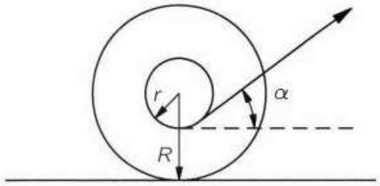

Figure 15-10

(a) Find the acceleration a of the yo-yo, if the yo-yo does not rise from the table top.

(b) How strong a force F at the angle α is needed in order to lift the yo-yo off the table?

15.19 Find the ratio h/r of the height of the cushion of a billiard table to the radius of the balls, as shown in Fig. 15-11, such that a ball that approaches the cushion with a pure rolling motion will rebound with a pure rolling motion even if the coefficient of friction between the ball and the table is negligible. Assume that the force exerted on the ball by the cushion during the impact is in the horizontal direction.

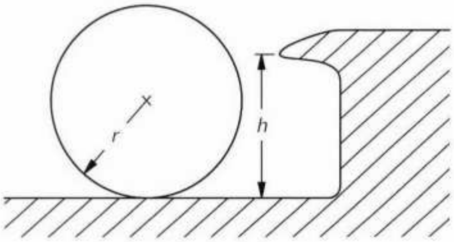

Figure 15-11

15.20 An irregular plate of metal of uniform thickness and mass M has its center of mass at point C, as shown in Fig. 15-12. The moment of inertia for an axis perpendicular to the sheet through point A is known to be I_A. Under what conditions on $r_1 = \overline{AC}$, $r_2 = \overline{BC}$, and $r_3 = \overline{AB}$ can one correctly express the moment of inertia for an axis through B, also perpendicular to the sheet, as

$$I_B = I_A + Mr_3^2?$$

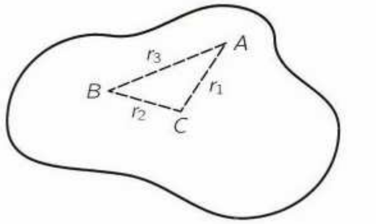

Figure 15-12

15.21 An adaptation of an inking arrangement for a printing press is as shown in Fig. 15-13. K is a firmly supported, but idling, inking roller of negligible moment of inertia; P is a driven press roll firmly supported and T is a transfer roll freely floating between K and P. T is a solid cylinder of radius r and mass M; it always rolls without slipping on both K and P, and the geometry is such that the line of centers TP is θ above the horizontal. What is the maximum angular acceleration $\alpha_{\max}$ that can be given to P without T losing contact with K?

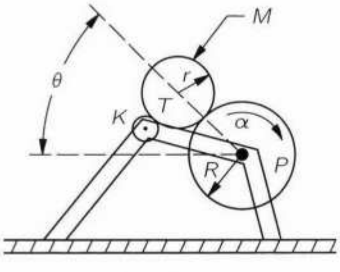

Figure 15-13

15.22 A thin uniform rod of length $2l$ rotating in a horizontal plane with angular velocity ω_0 about a fixed, vertical axis through its center is placed gently on a horizontal table having a coefficient of friction μ. How much time t is required for the rod to stop its rotation? Assume that the rod is uniformly supported by the table (each element of length is supported by the table just beneath it).

15.23 A uniform bowling ball of radius R and mass M is initially launched so that it is sliding with speed V_0 without rolling on an alley with a coefficient of friction μ.

(a) How far D does the ball go before it starts rolling without slipping?

(b) What then is its speed V?

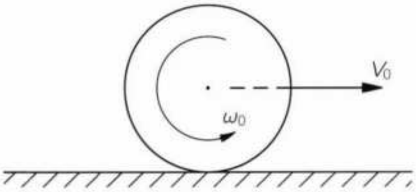

Figure 15-14

15.24 An amusing trick is to press a finger down on a marble, on a horizontal table top, in such a way that the marble is projected along the table as shown in Fig. 15-14, with an initial linear speed V_0 and an initial backward rotational speed ω_0 about a horizontal axis perpendicular to V_0. If the coefficient of sliding friction between marble and top is constant, and the marble has radius R,

(a) What relationship must hold between V_0, R, and ω_0 for the marble to slide to a complete stop?

(b) What relationship must hold between V_0, R, and ω_0 for the marble to skid to a stop and then start returning toward its initial position, with a final constant linear speed of $3V_0/7$?

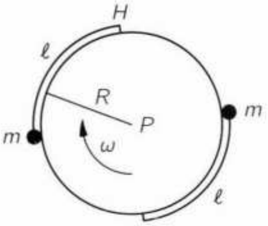

Figure 15-15

15.25 A uniform circular disc of radius R and mass M is arranged to spin freely with angular speed ω on a horizontal plane on a pivot P at its center. Pinned to its edge are two small masses m attached by cords of length l wrapped around its periphery, as shown in Fig. 15-15. While the disc is spinning, these masses are released simultaneously without disturbing the angular momentum of the system. Thereupon, the small masses fly off, their restraining cords being released from hooks H, H' when the cords extend radially outward. Find l, the length of these cords, such that the disc will be stopped by the action.

Note: This scheme has been used to reduce the spinning motion of satellite vehicles.

15.26 If Moe (coordinates x', y') rotates relative to Joe (coordinates x, y), who is at rest,

(a) Find the equations for the apparent force components that must act on a particle of mass m according to Moe.

(b) Show that these consist of the components of the true force $\boldsymbol{F}$ as seen by Joe, plus two pseudo forces: a radial centripetal force and a Coriolis force at right angles to the velocity.

Note: Assume that the transformation between coordinate systems is given by

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta, \\ y' &= -x\sin\theta + y\cos\theta, \end{aligned}$$

with $\theta = \omega t$.

15.27 Find the angular momentum of a planet of mass m moving in a circular orbit of radius R. Using this result, deduce that the distance of the moon from the earth will increase over a long period of time because of its tidal drag on the earth's rotation. Also, discuss the conservation of energy in the earth-moon system, from the standpoint of the possibility of escape of the moon.

16

Rotation in Three Dimensions

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 20.

16.1 Any three vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ may be thought of as defining a solid body having six faces, parallel in pairs—a parallelepiped (see Fig. 16-1). Show that the volume enclosed by such a figure is

$$V = |\boldsymbol{A} \cdot (\boldsymbol{B} \times \boldsymbol{C})|.$$

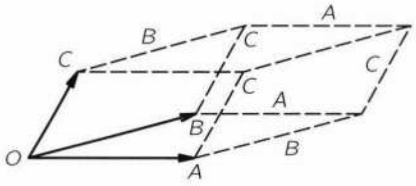

Figure 16-1

16.2 By writing the vectors in component form, or otherwise, prove the following vector equalities:

$$\begin{aligned}
\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) &= \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c} \\
(\alpha \boldsymbol{a}) \times \boldsymbol{b} &= \alpha(\boldsymbol{a} \times \boldsymbol{b}) \\
\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) &= (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c} \\
\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) &= \boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b}) \\
\boldsymbol{a} \times \boldsymbol{a} &= 0 \\
\boldsymbol{a} \cdot (\boldsymbol{a} \times \boldsymbol{b}) &= 0
\end{aligned}$$

16.3 A rigid body is rotating with an angular velocity $\boldsymbol{\omega}$ about a fixed axis. Show that the velocity of any point P in the body is $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$, where $\boldsymbol{r}$ is a vector from any point on the axis of rotation to the point P.

16.4 A collection of N particles with masses m_i, positions $\boldsymbol{r}_i$, and velocities $\boldsymbol{v}_i$ have a certain angular momentum

$$\boldsymbol{L} = \sum \boldsymbol{r}_i \times \boldsymbol{p}_i = \sum m_i \boldsymbol{r}_i \times \boldsymbol{v}_i.$$

On the other hand, as viewed in a coordinate system moving with their center of mass, suppose they have an angular momentum $\boldsymbol{L}_{\text{CM}}$. If $\boldsymbol{R}_{\text{CM}}$ and $\boldsymbol{V}_{\text{CM}}$ are the position and velocity of the CM, and $M = \sum m_i$ is the total mass of the particles, show that

$$\boldsymbol{L} = \boldsymbol{L}_{\text{CM}} + M\boldsymbol{R}_{\text{CM}} \times \boldsymbol{V}_{\text{CM}}.$$

16.5 A rigid body is rotated through an infinitesimal angle $\Delta\theta_1$ about a certain axis and is then rotated through an infinitesimal angle $\Delta\theta_2$ about some other axis intersecting the first axis at some point O. Show that the net displacement of any point in the body is the same as it would be if it were instead rotated through a single infinitesimal angle about some intermediate axis, and show how to find this axis and angle. Use this to prove that a rigid body subjected simultaneously to angular velocities about various axes moves as it would with a single angular velocity equal to their vector sum, treating each angular velocity as a vector of length ω directed along the axis of rotation.

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2.$$

16.6 A parallelepiped with one vertex at the origin, as shown in Fig. 16-1, has three adjacent vertices at the points $(10, -5, 3)$, $(3, -4, 7)$, and $(-5, -6, 3)$ meters, in rectangular coordinates (x, y, z). What is its volume V?

16.7 If the polar icecaps were to melt, what would happen to the earth's period of rotation? Explain.

16.8 How would you distinguish a hard-boiled egg from a raw egg (without cracking the shell)?

16.9 A jet airplane in which all the engines rotate in the direction of a right-handed screw advancing in the flight direction is executing a left turn. Does the gyroscopic effect of the engines tend to cause the airplane to

(a) roll right?

(b) roll left?

(c) yaw right?

(d) yaw left?

(e) pitch up?

(f) pitch down?

Why?

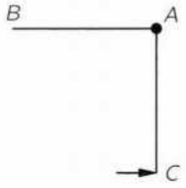

Figure 16-2

16.10 Two equal masses are connected by a flexible string of length l. An experimenter holds one mass in his hand and causes the other mass, initially at rest, to whirl around faster and faster in a horizontal circle about the held mass, until the whirling mass reaches angular velocity ω_0; he then releases the held mass.

(a) If the string breaks during the experiment, did it break before, or after he released the masses?

(b) If the string does not break, describe the motion of the masses subsequent to their release.

16.11 Two uniform, equal stiff rods AB and AC are freely hinged at A and placed on a smooth horizontal table with $AC \perp AB$, as shown in Fig. 16-2. A horizontal blow is delivered perpendicular to AC at C. Find the ratio of the resulting linear velocities of the centers of mass of the rods, $v_{\mathrm{AC}}/v_{\mathrm{AB}}$, immediately after this impulse.

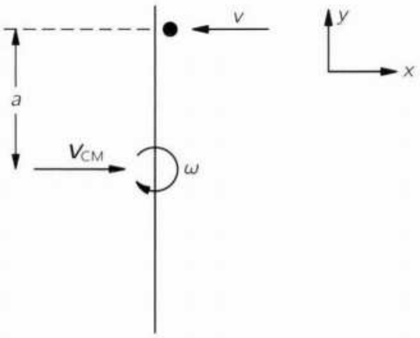

Figure 16-3

16.12 A thin uniform rod of mass M and length L and a puck of mass m slide without friction on a horizontal plane. At a certain instant the rod is perpendicular to the direction of its center of mass velocity, which is $\boldsymbol{V}_{\mathrm{CM}} = +V\boldsymbol{i}$ and has angular velocity around its center of mass $\boldsymbol{\omega} = -\omega\boldsymbol{k}$, as shown in Fig. 16-3. At this instant it is hit by the puck that was originally moving with velocity $\boldsymbol{v} = -v\boldsymbol{i}$. Find v and the distance a between the center of the rod and the point of contact that will leave the rod motionless after impact. Consider the collision perfectly elastic.

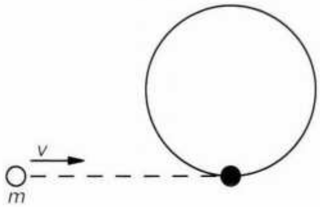

Figure 16-4

16.13 A thin circular wooden hoop of mass m and radius R rests on a horizontal frictionless plane. A bullet, also of mass m, moving with horizontal velocity v, strikes the hoop and becomes embedded in it, as shown in Fig. 16-4. Calculate

(a) the center of mass velocity V_{CM},

(b) the angular momentum L of the system about the CM,

(c) the angular velocity ω of the hoop, and

(d) the kinetic energy T of the system, before and after collision.

16.14 The four masses ($m_a = m_b = m_c = m_d/2$) shown in Fig. 16-5 lie on the corners of a rectangle on a frictionless horizontal surface. They are rigidly connected by rods of negligible weight. Another mass $m = m_a$ of velocity v_0 in the positive x-direction collides with m_a and sticks to it. Describe the motion of the object after collision.

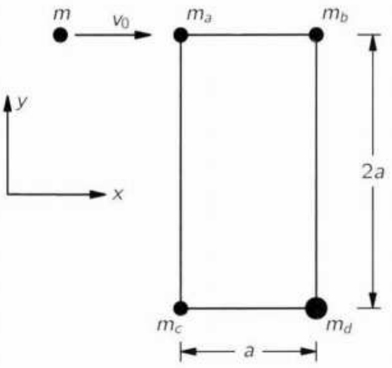

Figure 16-5

16.15 A uniform rod of length L and mass M is at rest on a frictionless horizontal surface. The rod receives an impulse $J = \int F dt$ of very short duration applied at right angles to the rod at a point P where $\overline{OP} = r$, as shown in Fig. 16-6.

(a) Just after the impulse, what is the velocity V_O of the center of mass O?

(b) What is the angular velocity ω about O?

(c) What is the instantaneous velocity V_A of the end point A?

(d) Determine the distance $\overline{AP}$ for which the velocity of the point A is zero just after the impact.

(e) If the rod is supported vertically from a pivot at A, at what point P should a blow be struck to set the rod in rotation about A without exerting an initial sideways force on the pivot?

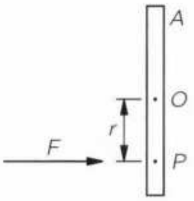

Figure 16-6

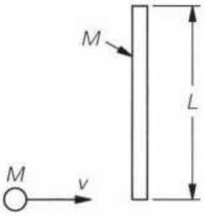

Figure 16-7

16.16 A thin rod of mass M and length L rests on a horizontal frictionless surface, as shown in Fig. 16-7. A small piece of putty, also of mass M, and with velocity v directed perpendicularly to the rod, strikes one end and sticks, making an inelastic collision of very short duration.

(a) What is the velocity V_{CM} of the center of mass of the system before and after the collision?

(b) What is the angular momentum L of the system about its center of mass just before the collision?

(c) What is the angular velocity ω (about the center of mass) just after the collision?

(d) What percentage of the kinetic energy is lost in the collision?

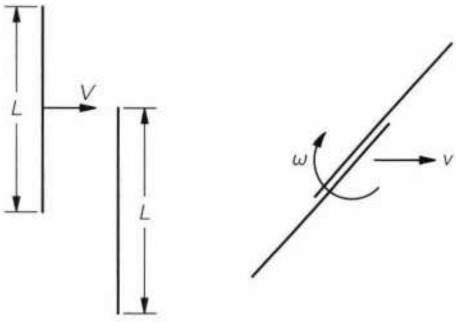

Figure 16-8

16.17 Two equal, rigid rods of length L and mass M are free to move without friction on a horizontal surface. Initially, one rod is stationary and the other is translating at speed V along a line perpendicular to the two rods, as shown in Fig. 16-8. The rods collide in such a way that the center of one meets one end of the other, and they henceforth stick together. Find the linear speed V_f and angular velocity ω_f of the composite rod after impact.

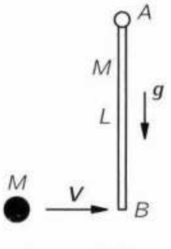

Figure 16-9

16.18 A thin uniform rod AB of mass M and length L is free to rotate in a vertical plane about a horizontal axle at end A, as shown in Fig. 16-9. A piece of putty, also of mass M, is thrown with velocity V horizontally at the lower end B while the bar is at rest. The putty sticks to the bar. What is the minimum velocity of the putty before impact that will make the bar rotate all the way around A?

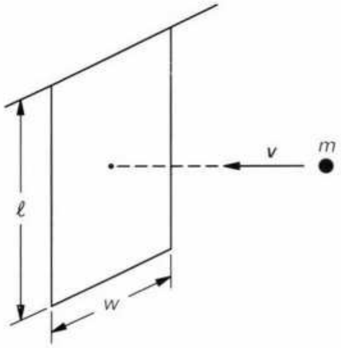

Figure 16-10

16.19 A thin rigid board of mass M, width w, and length l is suspended vertically from a frictionless horizontal axis at its top edge, as shown in Fig. 16-10. A bullet of mass m, traveling with velocity v perpendicular to the board, lodges in the center of the board.

(a) Just after impact, what is the speed v_f of the bullet?

(b) Through how large an angle θ will the system turn?

(c) What impulse J is felt by the bearings supporting the axis?

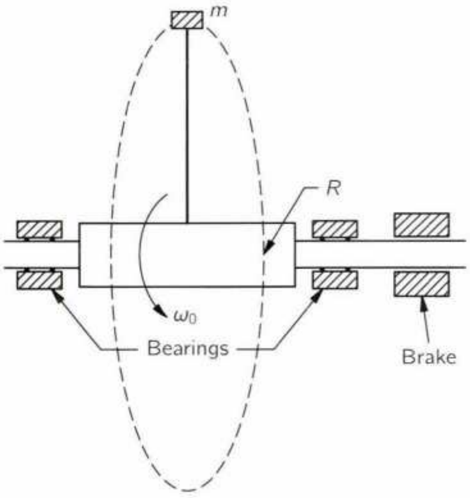

Figure 16-11

16.20 A horizontal spindle, of radius r and moment of inertia about its axis of I_0, has attached to it a cord which in turn is attached to a mass m whose dimensions are small compared to the other dimensions in the problem, as shown in Fig. 16-11. Originally the spindle is rotated about its horizontal axis with a constant angular speed ω_0, and the mass m swings with the same angular speed in a vertical circle of radius R. ω_0 is so large that the effect of gravity is negligible. At $t = 0$, a brake is actuated, stopping the motion of the spindle in a few degrees of rotation.

(a) What angular impulse J did the brake have to supply?

(b) When the cord has wound itself exactly ten times around the spindle, it breaks. What was its breaking strength F?

Warning: angular momentum is not conserved after the spindle is stopped: can you see why?

16.21 Two rods, each of length l, each with a mass m attached at its end, are clamped at an angle θ to a shaft, as shown in Fig. 16-12. (The shaft and rods are in the same plane.) What torque $\tau_{\max}$ must the bearings be able to withstand, if the clamping angle θ can be set anywhere from 0° to 90°, and the maximum angular velocity of the shaft is ω? (Neglect mass of rods and treat the m's as point masses.)

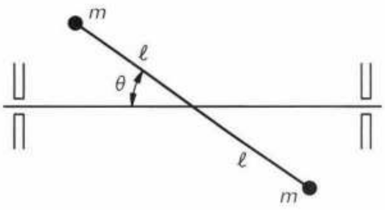

Figure 16-12

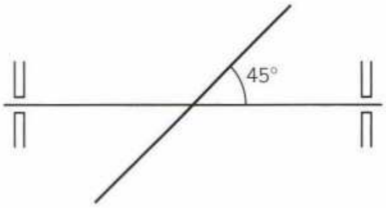

Figure 16-13

16.22 A uniform thin rod of mass M and length l is mounted at its center of mass on an axis inclined 45° to its length, as shown in Fig. 16-13.

(a) What angle θ does the angular momentum vector make with respect to the axis of rotation?

(b) What is the bearing torque τ of this rod rotating at angular velocity ω?

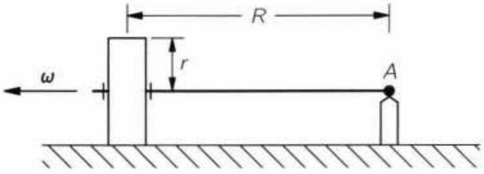

Figure 16-14

16.23 A thin solid wheel on a horizontal axle is constrained to travel in a circle of radius R on a horizontal table, as shown in Fig. 16-14. The axle pivots freely in all directions about the point A, which is fixed on the vertical driveshaft. If the mass of the wheel is m, its radius r, and its angular velocity about its axis is $\boldsymbol{\omega}$, with what force F does it press on the table? Use: $m = 1\,\text{kg}$, $R = 50\,\text{cm}$, $r = 10\,\text{cm}$, $\omega = 12{,}000\,\text{rad min}^{-1}$.

16.24 A turntable T_1, at rest, has mounted on it a turntable T_2 rotating with angular velocity ω_2, as shown in Fig. 16-15. At a certain time an internal clutch acts on the axle of T_2 to stop it with respect to T_1, but T_1 is free to revolve. T_1 alone has mass M_1 and moment of inertia I_1 about an axis A_1 through its center perpendicular to its plane; and T_2 has mass M_2 and moment of inertia I_2 about a similarly situated axis A_2; the distance between A_1 and A_2 is r. Find the angular velocity ω_1 of T_1 after T_2 stops.

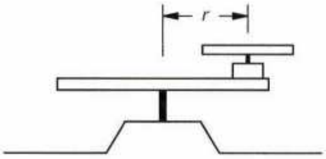

Figure 16-15

16.25 A man stands on a rotating platform at a distance R from the center. He throws a ball at speed V to another man diametrically opposite to him on the platform, also at radius R. If the platform turns with angular velocity ω,

(a) what is the radius of curvature r of the ball's trajectory, as seen in the rotating system?

(b) at what angle θ with respect to the diameter should the throw be aimed. (Show on a diagram.)

(c) what does the trajectory look like to a stationary observer?

Note: Assume $V \gg \omega R$, so centrifugal pseudo force may be ignored.

16.26 A certain satellite vehicle is approximately a uniform circular cylinder of mass m, radius a, and length L, with $L = 6a$. It is initially spinning at angular velocity ω_0 about its long axis, but because of small internal vibrations (due to a slight precessional motion at the start), energy is gradually transformed into heat. As a result, the satellite "slows down." Describe the only possible final state of rotation, and find the corresponding angular velocity ω_f, if as much energy as possible is transformed into heat. Assume that no outside influences are present.

16.27 If all the ice on earth were to melt, the height of mean sea level would increase by about 200 ft. Taking the mean latitude of the existing ice caps as 80°, and neglecting the irregular distribution of the oceans, by about how much time ΔT would the length of the day increase?

Note: Assume the earth is a sphere of radius 6370 km and moment of inertia $8.11 \times 10^{37}\,\mathrm{kg\,m^2}$.

16.28 Two equal masses m are fixed on a massless rod a distance $2r$ apart, and are attracted gravitationally by a mass M, situated at a distance $R \gg r$ from the center O of the rod. The rod makes an angle θ with R, as shown in Fig. 16-16. Find the approximate value of the torque τ on the rod about its center.

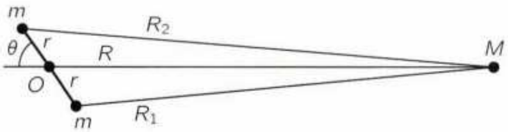

Figure 16-16

16.29 The elastic restoring torque exerted by a torsion fiber is proportional to the angle of twist: $\tau_{\text{fiber}} = -k\theta$.

(a) Show that the potential energy of such a fiber twisted through an angle θ is $U = (1/2)k\theta^2$.

(b) The deflecting torque exerted on a galvanometer coil is given by the expression

$$\tau = n\,AB\,i,$$

where

i = current through the coil,
n = number of turns of wire on the coil,
A = cross-sectional area of coil, and
B = the magnetic field produced by the permanent galvanometer magnet.

In a laboratory experiment, the charge on a capacitor is measured by discharging the capacitor through a galvanometer coil and noting the resulting maximum deflection. Here $|i| = |dq/dt|$, and the discharge takes place so quickly that the galvanometer coil does not appreciably move away from its initial $\theta = 0$ position during the time that current flows. Neglecting friction, show that the maximum "throw" of the galvanometer is proportional to the initial charge on the capacitor.

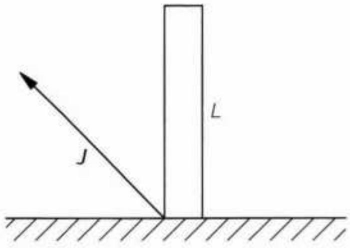

Figure 16-17

16.30 An upright rod of mass M and length L, as shown in Fig. 16-17, is given an impulse J at its base, directed at 45° upward from the horizontal, which sends the rod flying. What value(s) should J have so that the rod lands vertically again? (I.e., upright on the end at which J was applied.)

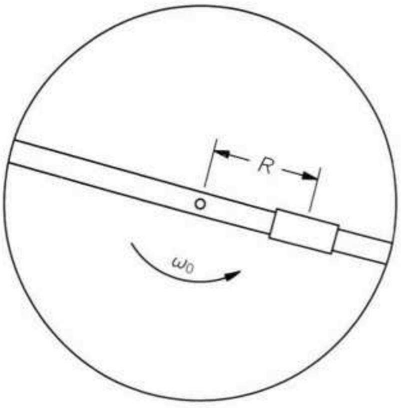

Figure 16-18

16.31 A turntable of moment of inertia I_0 rotates freely on a hollow vertical axis. A cart of mass m runs without friction on a straight radial track on the turntable. A cord attached to the cart passes over a small pulley and then downward through the hollow axis, as shown in Fig. 16-18. Initially the entire system is rotating at angular speed ω_0 and the cart is at a fixed radius R from the axis. The cart is then pulled inward by applying an excess force to the cord, and eventually arrives at radius r, where it is allowed to remain.

(a) What is the new angular velocity ω of the system?

(b) Show in detail that the difference in the energy of the system between the two conditions is equal to the work done by the centripetal force.

(c) If the cord is released, with what radial speed $\dot{r}$ will the cart pass the radius R?

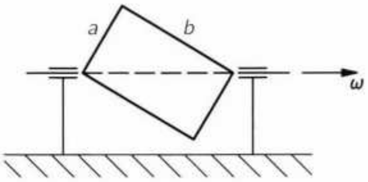

Figure 16-19

16.32 A thin rectangular plate of mass M, sides a, b, rotates about an axis along its diagonal with angular velocity ω, as shown in Fig. 16-19.

(a) What is the force F on the bearings?

(b) What is the kinetic energy T of the rotating plate?

16.33 A flywheel having the shape of a uniform thin circular plate of mass 10.0 kg and radius 1.00 m is mounted on a shaft passing through its CM but making an angle of $1^\circ 0'$ with its plane. If it rotates about this axis with angular velocity $25.0\,\mathrm{rad\,s^{-1}}$, what torque τ must be supplied by the bearings?

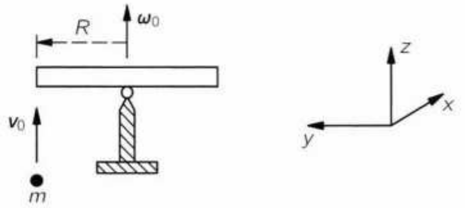

Figure 16-20

16.34 A uniform thin disc of radius R, mass M is mounted on a universal bearing permitting rotation about any axis. Initially it spins about a vertical axis (z-direction) with angular velocity ω_0, as shown in Fig. 16-20. A small mass m with velocity v_0 in the positive z-direction collides elastically with the rim of the wheel and rebounds in the negative z-direction.

(a) What is the direction of the angular momentum $\boldsymbol{L}$ of the disc after the collision?

(b) Describe the motion of the figure axis. Indicate by a sketch the trajectory of the point of intersection of the figure axis with a unit sphere as seen from the top.

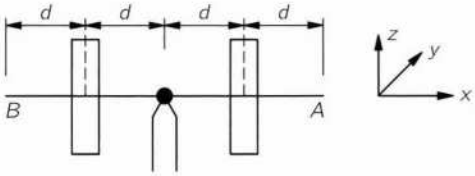

Figure 16-21

16.35 A pair of 10 cm radius, 2 kg disc flywheels with frictionless bearings are spinning at $1000\,\mathrm{rad\,s^{-1}}$ and are supported at a distance $d = 15\,\mathrm{cm}$ on either side of a universal bearing by a small diameter bar AB of mass $M = 1\,\mathrm{kg}$ and length $4d$, as shown in Fig. 16-21.

(a) If a ball of mass $m = 10\,\mathrm{g}$ is dropped from a height $h = 5\,\mathrm{cm}$ onto the tip A of the bar and rebounds upwards, give the components of the resulting angular momentum $\boldsymbol{L}$ of the flywheels and sketch the motion of the tip of the bar as seen from the $+x$ direction. Also, give the angular velocity Ω_n of this motion and the radius r of the circle described by the tip once circular motion is attained.

(b) If the same ball instead were attached to the tip A, then what would be the angular velocity of precession Ω_P, neglecting nutation? What would be the angular momentum L_P and the rotational kinetic energy T_P associated with Ω_P? How much potential energy ΔE is lost as the tip sinks below the xy-plane?

16.36 The moon and the sun both exert a torque upon the earth because of the earth's oblateness. Which body exerts the greater torque and by approximately what factor?

1st Hint: You may wish to make use of the accidental fact that the two bodies subtend almost equal angles in the sky as seen from Earth.

2nd Hint: The mean density of the sun is $1.41\,\mathrm{g\,cm^{-3}}$, and that of the moon is $3.34\,\mathrm{g\,cm^{-3}}$.

16.37 The equatorial radius of the earth is 6378.388 km while its polar radius is 6356.912 km. The specific gravity ρ at various depths D below the surface are shown in the table below (* denotes a discontinuity).

D (km)	ρ
0	2.60
30*	3.0
	3.3
100	3.4
200	3.5
400	3.6
1000	4.7
2000	5.2
2900*	5.7
	9.4
3500	10.2
5000*	11.5
	16.8
6000	17.1

Using these values, estimate

(a) the moment of inertia of the earth $I_{\text{♁}}$,

(b) its rotational angular momentum $L_{\text{♁}}$,

(c) its rotational kinetic energy $T_{\text{♁}}$,

(d) the time T required for the rotational axis to precess about the pole of the ecliptic due to the torques of the moon and the sun.

Note: The tilt of the earth's axis is 23.5°.

17

The Harmonic Oscillator, Linear Differential Equations

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 21.

17.1 Show that the small amplitude oscillations of a rigid body that is suspended a distance D above its center of mass, as shown in Fig. 17-1, are described by

$$\frac{d^2\theta}{dt^2} + \frac{MgD}{I}\theta = 0,$$

and that the period of oscillation T is given by

$$T = 2\pi\sqrt{\frac{I}{MgD}},$$

where M is the mass of the body and I is its moment of inertia about the suspension axis.

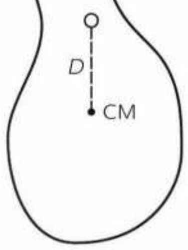

Figure 17-1

17.2 Which (if either) of the masses shown in Fig. 17-2 moves in simple harmonic motion (i.e., sinusoidal motion)?

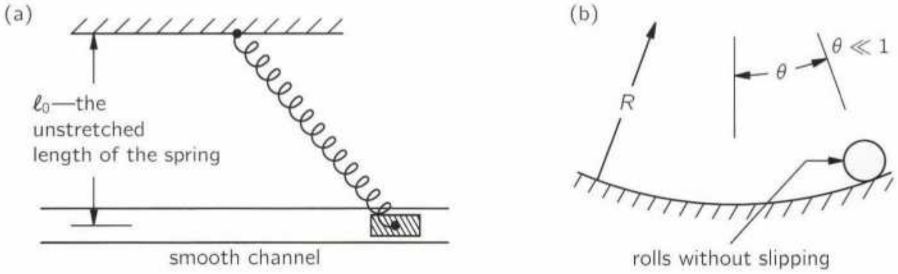

Figure 17-2

17.3 A uniform rod of mass M, length L swings as a pendulum with two horizontal springs of negligible mass and constants k_1 and k_2 acting at the bottom end, as shown in Fig. 17-3. Both springs are relaxed when the rod is vertical. What is the period T of small oscillations?

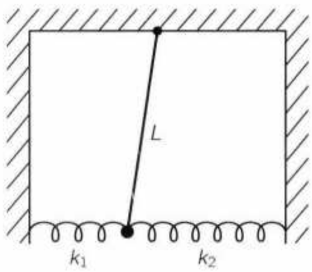

Figure 17-3

17.4 In terms of the maximum amplitude A, how far, x, from the equilibrium position is a simple, undamped mechanical harmonic oscillator when its kinetic energy is exactly equal to its potential energy?

17.5 Two particles A and B execute harmonic motion of the same amplitude (10 cm) on the same straight line. For particle A, $\omega_A = 20\,\mathrm{rad\,s^{-1}}$; for B, $\omega_B = 21\,\mathrm{rad\,s^{-1}}$. If at $t = 0$, they both pass through $x = 0$ in the positive x-direction (hence are then "in phase"),

(a) How far apart, Δx, will they be at $t = 0.350\,\mathrm{s}$?

(b) What is the velocity V of B relative to A at $t = 0.350\,\mathrm{s}$?

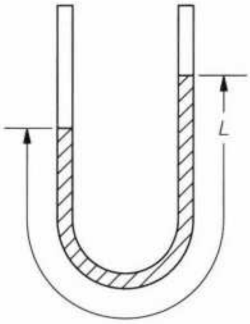

Figure 17-4

17.6 The vertical U-tube manometer shown in Fig. 17-4 has constant internal cross-section A and contains a total length of liquid L. Find the period of oscillation T of the liquid. Neglect friction and assume that the amplitude of oscillation is such that the two liquid surfaces remain within the straight vertical portions of the tube.

17.7 In its initial stages, a colony of bacteria grows at a rate proportional to the number of bacteria present. Write the differential equation which expresses this relationship.

17.8 A flat disc of radius R, mass M, is suspended at its rim with a torsion wire, as shown in Fig. 17-5. If the wire has a torsional constant K, what is the period T of torsional oscillation?

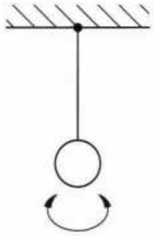

Figure 17-5

17.9 A frame made of stiff wire of uniform cross section and density consists of a semicircular arc ACB with its diameter AB, as shown in Fig 17-6. It is hung from a frictionless pin P passing through a hole at the midpoint of its diameter, and is set into vibration as a pendulum in its own plane. If the diameter of the frame $\overline{AB}$ is 50 cm, what is the period T of the oscillating motion for small arcs?

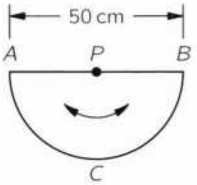

Figure 17-6

17.10 Consider an ideal wheel of mass M, and moment of inertia I_c about its frictionless axle. The wheel is suspended from a hanger of length d, of negligible mass and moment of inertia, which is free to move in the plane of the wheel about a pivot point at X, as shown in Fig. 17-7. The hanger and wheel are released from rest simultaneously when the hanger makes an angle θ_0 with the vertical through X. ($\theta_0 \ll 1$). For each of the two cases below, A and B,

(a) find the period T of the motion of the hanger,

(b) find the angular acceleration $\ddot{\theta}$ when $\theta = \theta_0$, and

(c) find the angular velocity $\dot{\theta}$ when $\theta = 0$.

(a) When the wheel is free to turn without friction about the axis C.

(b) When the wheel and hanger are locked together and constrained to move together about X as a rigid body.

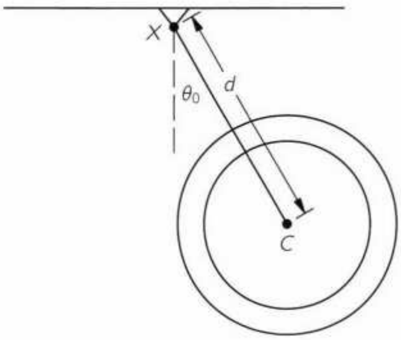

Figure 17-7

17.11 Two uniform, circular plane wheels of equal mass 1.00 kg are mutually pivoted about a horizontal axis A, perpendicular to both bodies and passing through their centers of mass, as shown in Fig. 17-8. The radii of gyration* R of the two bodies are also equal: $R = 0.20$ m. Body 2 is pivoted about a fixed horizontal axis B, a distance R from its CM. Initially, body 2 is motionless and body 1 is rotating at angular velocity ω_0 about the common axis. A small latch pin C, fixed in body 1, suddenly drops into a hole in body 2, stopping their relative motion. It is observed that the resulting pendular motion of the system has an amplitude of 90° each side of the vertical. Find ω_0.

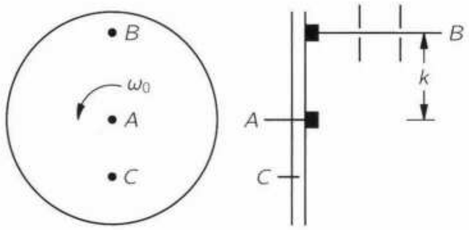

Figure 17-8

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* The radius of gyration R, about a given axis, of a mass M, having moment of inertia I about that axis, is defined to be $R = \sqrt{I/M}$.

17.12 An L-shaped uniform, rigid bar of mass M with legs each of length l hangs from its upper end A, pivoted to swing freely in its own plane, as shown in Fig. 17-9.

(a) At what angle θ_0 (with vertical) does it hang when at rest?

(b) If an impulse J is applied at the elbow in the direction shown, producing thereafter undamped, small oscillations, find $\theta(t)$ if J was applied at $t = 0$.

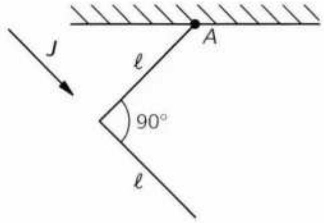

Figure 17-9

17.13 The gravitational force felt by a particle embedded in a solid uniform sphere, due to the mass of the sphere only, is directly proportional to the distance of the particle from the center of the sphere. If the earth were such a sphere, with a narrow hole drilled through it along a polar diameter, how much time T would it take for a body dropped in the hole to reach the surface at the opposite side of the earth?

17.14 A mass m moves in a straight line on a frictionless horizontal plane under the influence of two springs with spring constants k_1 and k_2. (The springs exert no force at the equilibrium point.) For the two systems shown in Fig. 17-10,

(a) Derive the equations of motion.

(b) Find the period of oscillation T.

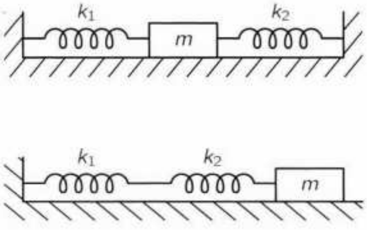

Figure 17-10

17.15 Two particles, of mass $3M/4$ and M, are connected by a massless spring of free length L and force constant k. These masses are initially at rest L apart on a horizontal frictionless table. A particle of mass $M/4$, moving with speed v along the line joining the two connected masses, collides with and sticks to the particle of mass $3M/4$. Find the amplitude A and period T with which the spring between the two masses vibrates.

17.16 Two unequal masses, $m_1 = m$ and $m_2 = 2m$, connected by a spring (spring constant K), rest on a frictionless table, as shown in Fig. 17-11. If the spring is compressed a distance d, with m_2 resting against the wall, and then the system is released from rest,

(a) Find how far, x, mass m_1 moves before m_2 starts moving.

(b) After m_2 has lost contact with the wall, what is the velocity of the center of mass V_{CM} and amplitude A of oscillation?

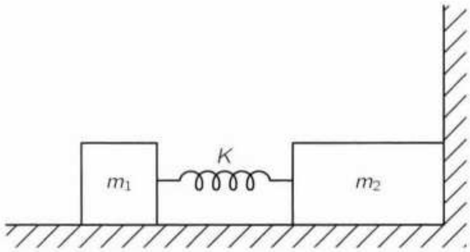

Figure 17-11

17.17 Two gliders of different mass, $M_1 \neq M_2$, are sliding along a horizontal air trough with velocity v_0. They are held together with a clamp which compresses a massless spring of force constant K between them, as shown in Fig. 17-12. The spring is displaced by an amount X from its uncompressed position. The clamp suddenly gives way (or is released) and the compressed spring forces the two masses apart. Find the final velocities v_1 and v_2 of the gliders.

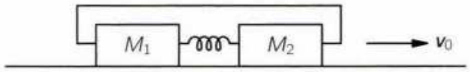

Figure 17-12

17.18 Given the undamped horizontal mechanical oscillator shown in Fig. 17-13, find the maximum amplitude of oscillation, $x_{\max}$, such that the upper mass will not slip on the lower mass. The coefficient of friction between the masses is μ.

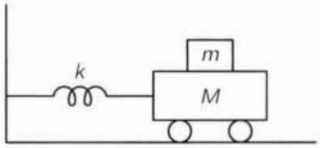

Figure 17-13

17.19 A 200 g mass oscillates on a horizontal guide under the influence of a light spring. Its maximum kinetic energy is 10^6 ergs, and its period is 1 s.

(a) What is the total system energy E?

(b) What is the force constant k of the spring?

(c) What is the amplitude x of the oscillation?

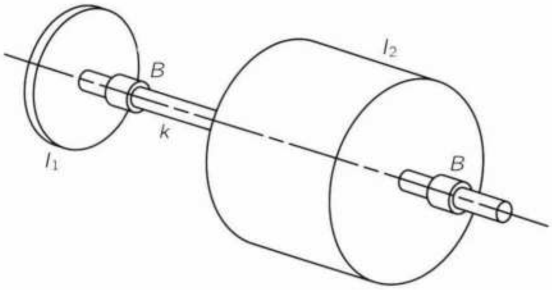

Figure 17-14

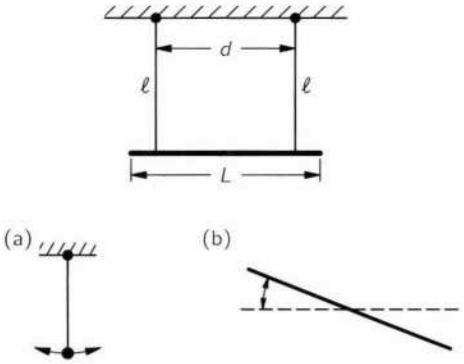

Figure 17-15

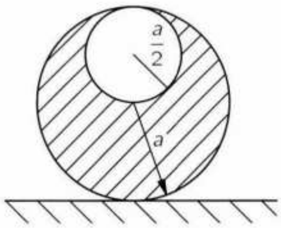

Figure 17-16

17.20 The rotating element of a jet engine shown in Fig. 17-14 consists of a turbine disc (with blades) having a moment of inertia I_1 and a compressor (with blades) having a moment of inertia I_2 connected together by a shaft of torsional stiffness k (the torque in the shaft is k times the relative angular displacement between turbine and compressor). ($B =$ bearings.)

(a) What is the frequency ν of torsional oscillation of the engine rotor?

(b) What is the ratio of the amplitudes A_1/A_2 of torsional oscillations of the turbine and compressor?

17.21 The position of the fluorescent spot on an oscilloscope screen is frequently determined by two harmonic voltages applied to the x and y axes, respectively,

$$x = A_x \cos(\omega t + \delta_x),$$
$$y = A_y \cos(\omega t + \delta_y).$$

Sketch and discuss the curve defined by the motion of the fluorescent spot for the following cases:

(a) $\delta_x = \delta_y$

(b) $\delta_x = \delta_y + \pi/2$ and $A_x = A_y$.

(c) $\delta_x = \delta_y + \pi/2$ and $A_x \neq A_y$,

(d) $\delta_x = \delta_y - \pi/4$ and $A_x = A_y$.

(e) $\delta_x = \delta_y - \alpha$ and $A_x \neq A_y$.

17.22 The bifilar pendulum shown in Fig. 17-15 consists of a rod of length L, mass M, suspended by two thin threads of length l which are separated by the distance $d < L$. Find the period of oscillation T for small amplitudes,

(a) if the rod swings like an ordinary pendulum, as shown in part (a) of the figure.

(b) if it oscillates about its CM, as shown in part (b) of the figure.

17.23 A solid circular cylinder of radius a has a hole of radius $a/2$ drilled through it parallel to its axis and $a/2$ from it, as shown in Fig. 17-16. The cylinder is placed on a horizontal plane on which it rolls without slipping. Find the period of oscillation T for small displacements from its equilibrium position.

17.24 A mass M, on small frictionless wheels, is allowed to oscillate transversely in a cylindrical trough of radius R, with an amplitude $A \ll R$, as shown in Fig. 17-17(a). Then two identical masses m, also mounted on small, frictionless wheels, are connected by a compressed, massless spring of spring constant K, are placed on a horizontal surface adjacent and parallel to the path of M, as shown in Fig. 17-17(b). At the instant M passes the bottom of the trough the cocked spring is released and the subsequent motion at quarter cycles for M is shown in Fig. 17-17(c). What is the value of m?

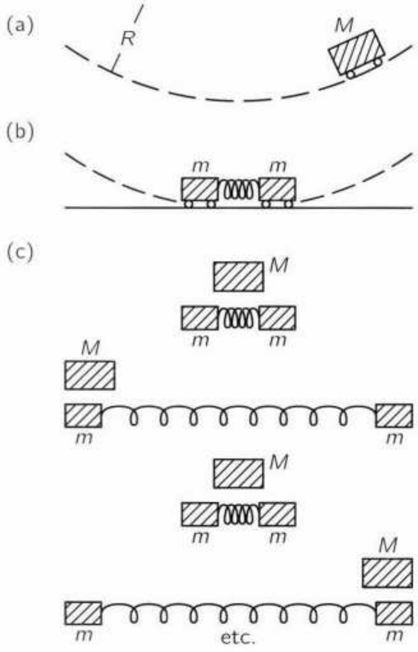

Figure 17-17

17.25 Two gliders in a frictionless air-trough have masses m_1 and m_2. They are connected by a massless spring of spring constant k, as shown in Fig. 17-18. They are pulled apart a distance A over the rest length of the spring and then released from rest.

(a) Find the oscillation period T of m_1 and m_2.

(b) Compare the period with the one of a single mass oscillator. Which physical concept, typical for relative motion of two bodies, do you discover?

(c) Find the energy of oscillation E.

(d) How do m_1 and m_2 share this energy?

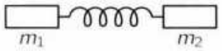

Figure 17-18

17.26 A simple pendulum consists of a mass M at the end of a massless rod of length L, freely pivoted so as to be able to swing in a full 360° arc. The period of the pendulum for small oscillations is $2\pi\,\mathrm{s}$. If the pendulum is carefully balanced at its top (unstable) equilibrium position and is then given a tiny push of $1\,\mathrm{mm\,s^{-1}}$, how much time T does it take the mass to move $10\,\mathrm{cm}$? (Assume the usual "linear system" approximations are valid over this distance, but not of course for the full circular swing.)

17.27 One end of a torsion rod is fastened to the center of a turntable which moves on frictionless bearings about its vertical axis of symmetry; and the other end of the rod is clamped at a point on the extended axis, as shown in Fig. 17-19. The torque constant of the rod is K, the total moment of inertia of the system is I. Initially the turntable executes an undamped angular simple harmonic motion of maximum amplitude θ_0. A dart of mass m falls vertically onto the turntable at a distance a from the axis when the turntable is passing through the equilibrium position; its needle point becomes firmly embedded, essentially instantaneously. Find θ_0', the new maximum amplitude. Neglect any spin of the dart about its axis, and assume its thickness is small compared with a.

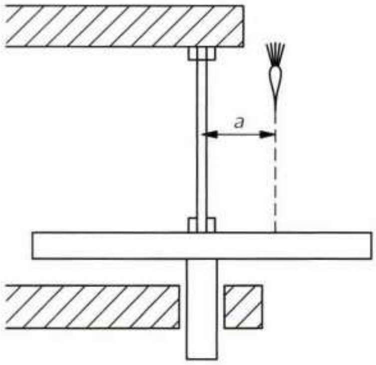

Figure 17-19

17.28 A certain rigid body of mass M is supported on a frictionless horizontal axis which lies a distance d from the CM. The moment of inertia about the axis of rotation is I.

(a) Write the differential equation which describes the variation of the angle θ with time, where θ is measured from the equilibrium position of the body.

(b) If the body undergoes small oscillations, so that $\sin\theta \approx \theta$, what is their period?

(c) If the moment of inertia of the rigid body about its CM is I_c. Find an expression for the period of small oscillations as a function of d and I_c, and thus show

   (1) that there are two values of d, say d_1 and d_2, which correspond to a given period,

   (2) that the period is $t = 2\pi(d_1 + d_2)/g$ in terms of d_1 and d_2,

   (3) that the period has a minimum value when $d = \sqrt{I_c/M}$ (the radius of gyration). Find this minimum period.

17.29 A certain linear spring has a free length D. When a mass m is hung on the end, it has a length $D+A$. While it is hanging motionless with mass m attached, a second mass m is dropped from a height A onto the first one, with which it collides inelastically. Find the period T, amplitude a, and the maximum height H (above the original equilibrium position) attained in the resulting motion.

17.30 A 20 g weight hanger with a 5 g weight on it is hung from a vertical spring of negligible mass. When the spring is displaced from equilibrium the system is found to vibrate in vertical simple harmonic motion with a period of $\pi/3$ s. If the 5 g weight is replaced by a 25 g weight, how far z can the spring be displaced from equilibrium before release if the weight is not to jump off the weight hanger?

18

Algebra

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 22.

The most general kind of number which satisfies the rules of elementary algebra is a *complex number*. Complex numbers may be written as a sum of a pure real (positive or negative) number and a pure *imaginary number*. An imaginary number is a real (positive or negative) number multiplied by $i = \sqrt{-1}$, the unit imaginary number. (The unit real number is $1 = \sqrt{+1}$.)

$$\text{(complex number) } u = \text{(real number) } x + \text{(imaginary number) } iy.$$

Any algebraic equation is still true if the sign of i is changed throughout. This is called taking the *complex conjugate*. If $u = x + iy$, then the complex conjugate of u, written u^*, is $u^* = x - iy$.

The rules of algebra, applied to complex numbers, show that

I. $(a + ib) + (c + id) = (a + c) + i(b + d)$.

II. $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$.

III. $|u| = \sqrt{uu^*} = \sqrt{x^2 + y^2}$ is called the *magnitude* of u.

A real number raised to an imaginary power is complex, and has unit magnitude. The real and imaginary parts behave like a sine and cosine function as the magnitude of the imaginary power increases. Specifically,

IV. $e^{i\theta} = \cos\theta + i\sin\theta$.

18.1 In the equation

$$u + iv = (a + ib)(c + id)$$

let $b/a = \tan\alpha$ and $d/c = \tan\beta$. Using Eq. II above, and formulas of trigonometry, show that

(a) $\sqrt{u^2 + v^2} = \sqrt{a^2 + b^2}\sqrt{c^2 + d^2}$,

(b) $v/u = \tan(\alpha + \beta)$.

18.2 Work Ex. 18.1 using Eq. IV above.

18.3 Show that

$$\cos\theta = (e^{i\theta} + e^{-i\theta})/2,$$
$$\sin\theta = (e^{i\theta} - e^{-i\theta})/2i.$$

18.4 Show that

$$(a + ib)/(c + id) = [ac + bd + i(bc - ad)]/(c^2 + d^2).$$

18.5 The quantities $\cosh\theta$ and $\sinh\theta$, defined as

$$\cosh\theta = (e^{\theta} + e^{-\theta})/2,$$
$$\sinh\theta = (e^{\theta} - e^{-\theta})/2,$$

are called the hyperbolic cosine and hyperbolic sine of θ. Show that

$$\cos i\theta = \cosh\theta,$$
$$\sin i\theta = i\sinh\theta,$$
$$\cosh^2\theta - \sinh^2\theta = 1.$$

18.6 Using the fundamental formula of differentiation

$$\frac{df}{dx}(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x},$$

show that

$$\frac{de^{\alpha x}}{dx} = \alpha e^{\alpha x}.$$

18.7 (a) By successive differentiation, or otherwise, show that e^x may be represented by the infinite series

$$e^x = 1 + x + x^2/2! + x^3/3! + \cdots .$$

(b) Show that $\cos x$ and $\sin x$ may be represented by the infinite series

$$\begin{aligned}\cos x &= 1 - x^2/2! + x^4/4! - x^6/6! \pm \cdots ,\\ \sin x &= x - x^3/3! + x^5/5! - x^7/7! \pm \cdots .\end{aligned}$$

(These series are of considerable value in calculating e^x, $\cos x$, and $\sin x$ for $x \ll 1$, although they do converge for all x.)

18.8 Find the complete algebraic solution of the equation

$$y = \sqrt[n]{1},$$

where n is an integer.

18.9 Using the properties of $e^{in\theta}$ and the binomial theorem, show that

$$\cos n\theta = \cos^n \theta - \frac{n(n-1)}{2!} \cos^{n-2} \theta \sin^2 \theta \pm \cdots .$$

18.10 (a) From the relation $e^{i(\theta+\phi)} = e^{i\theta} e^{i\phi}$, prove the trigonometric formulas giving the cosine and sine of the sum of two angles.

(b) Interpret geometrically the result of multiplying one complex number $Ae^{i\theta}$, by another complex number $Be^{i\phi}$.

18.11 From the following table of successive square roots of 11, find (to 3 places) $\log_{11} 2$ and $\log_{11} 7$.

Root r	$1/r$	$\sqrt[r]{11} = 11^{1/r}$
1	1.00000	11.0000
2	0.50000	3.3166
4	0.25000	1.8212
8	0.12500	1.3495
16	0.06250	1.1617
32	0.03125	1.0778
64	0.01563	1.0382
128	0.00781	1.0189

(Check your result by $\log_a N = \log_a b \log_b N$ where a and b are any two bases.)

19

Forced Oscillations with Damping

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 23, 24, and 25.

19.1 Sketch the transient "coasting" motion of

(a) an undamped oscillator,

(b) an underdamped oscillator,

(c) an overdamped oscillator,

all of which start from rest at the same time from the same displaced position.

19.2 What is the resonance frequency f of an LC circuit having 10 mH inductance and 1 pF capacitance?

19.3 Find the impedance Z of a 1.00 H inductance in series with a resistance of 377 Ω, at a frequency of 60 Hz.

19.4 A certain coil has a resistance of 20 Ω. When it is connected to a 60 Hz voltage source of 10 V rms amplitude, it is observed that a current of 0.3 A rms flows. Find the inductance L of the coil.

19.5 An inductance-capacitance series circuit is to be resonant for a frequency of 1.0×10^4 Hz. If $L = 7.6 \times 10^{-2}$ H, what should C be?

19.6 Find the impedance $\hat{Z}$ of an inductance L and a capacitance C as a function of angular frequency ω when they are connected

(a) in series.

(b) in parallel.

Discuss your answers qualitatively.

19.7 Two capacitors, C_1 and C_2 are connected

(a) in series.

(b) in parallel.

Find the effective capacitance C for these two cases.

19.8 Two inductances L_1 and L_2 are connected

(a) in series.

(b) in parallel.

Find the effective inductance L for these two cases.

19.9 A resonant circuit is composed of two inductances, L and $3L$, and two capacitors, C and $3C$, all connected

(a) in series.

(b) in parallel.

Find the resonant frequency ω_0 for these two cases.

19.10 The "time constant" of a resistance-capacitance series circuit is RC; that of an inductance-capacitance series circuit is $\sqrt{LC}$; from dimensional considerations what would you expect the time constant T of a resistance-inductance series circuit to be?

19.11 What is the approximate Q of a 1000 Hz tuning fork? (Use your experience.)

19.12 (a) Show that the differential equation of motion of a mass m on a spring whose force constant is k, and having a frictional force $-m\gamma v$ is

$$d^2x/dt^2 + \gamma dx/dt + \omega_0^2 x = 0,$$

where $\omega_0^2 = k/m$.

(b) Solve this equation (using complex variables) by assuming a solution of the form $x = e^{\alpha t}$ and thus show that the general solution when $\gamma < 2\omega_0$, is

$$x = e^{-\gamma t/2}\left(A\cos\omega t + B\sin\omega t\right),$$

with $\omega = \sqrt{\omega_0^2 - \gamma^2/4}$.

(c) What is the general solution if $\gamma > 2\omega_0$?

(d) At $t = 0$, the position and velocity of the mass m are $x = x_0$ and $\dot{x} = v_0$. Find the coefficients A and B.

19.13 The gliders used with a linear air track lose speed mainly because of the frictional drag of the thin air film which supports them. This viscous force is proportional to the velocity.

(a) Write and solve the differential equation of motion of a glider on a level track.

(b) Assuming the initial position and velocity of a glider moving in the x-direction are, respectively, x_0 and v_0, give the velocity of the glider as a function of time $v(t)$, and as a function of distance $v(x)$.

19.14 A certain glider on a tilted air track has a magnet embedded in it, and this magnet generates eddy currents in the track which react back on the magnet, giving a retarding force precisely proportional to the velocity: $F = -m\gamma v$. If the glider starts from rest, find (as a function of the angle of tilt of the track, and the drag coefficient γ of the magnet)

(a) the terminal velocity attained, v_∞,

(b) the velocity as a function of time $v(t)$,

(c) the position as a function of time $x(t)$.

19.15 A tourist on an ocean cruise inadvertently drops his camera of mass 1.0 kg from rest 20 m above the surface of the water. On impact the camera loses half its kinetic energy. After entering the water, it is subjected to a buoyant force equal to half its weight, and a drag force of $1/3\,\mathrm{kg\,s^{-1}}$ times v, its speed in the water. How far, d, below the surface is the camera 3.0 s after it enters the water?

19.16 The pivot point of a simple pendulum having a natural period of 1.00 s is moved laterally in a sinusoidal motion with an amplitude 1.00 cm and period 1.10 s. With what amplitude A should the pendulum bob swing after a steady motion is attained?

19.17 An object of mass 5.0 kg is found to oscillate with negligible damping when suspended from a spring which causes it to perform 10 complete cycles in 10.0 seconds. Thereafter, a certain small magnetic damping is applied, proportional to the velocity of motion and the amplitude decreases from 0.20 m to 0.10 m in 10 cycles.

(a) Write the equation of motion, with the coefficients of d^2x/dt^2, dx/dt, and x represented by numbers in SI units.

(b) What is now the period T of the motion?

(c) In how many cycles, N, (starting from 0.20 m amplitude) will the amplitude reach

(1) 0.05 m?

(2) 0.02 m?

(d) What is the approximate maximum rate of energy dissipation P by damping during the first cycle?

19.18 A capacitor of capacitance C is initially charged to a voltage V_0, and at $t = 0$ it is connected across a resistor of resistance R. Write the differential equation for V, the voltage across the capacitor, as a function of t, and solve it by assuming an exponential solution.

19.19 An uncharged capacitance C, "pure" inductance L, and variable resistance R are connected to a battery of potential difference V, as shown in Fig. 19-1. Write the equation pertaining to the circuit and graph the voltage across C as a function of time after the switch S is closed for various values of R.

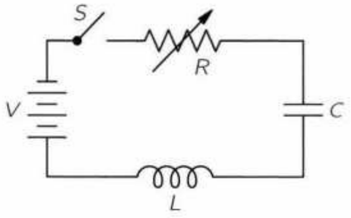

Figure 19-1

19.20 (a) Write down and solve the differential equations that describe the steady-state current that flows when a sinusoidal voltage of angular frequency ω is applied to

(1) an inductance L,

(2) a capacitance C.

(b) Use the results of part (a) to find the (complex) impedance

(1) Z_L of an inductance L,

(2) Z_C of a capacitance C.

19.21 Draw the phasor diagram (including $\hat{V}_{\text{in}} = \hat{V}_0$, $\hat{I}$, $\hat{V}_R$, $\hat{V}_L$, $\hat{V}_C$) for a series RLC circuit driven at resonance.

19.22 Draw phasor diagrams for the voltages and currents for a voltage $V_0 \cos \omega t$ applied to the following circuits:

(a) a resistance R and inductance L in series

(b) a resistance R and inductance L in parallel

(c) a resistance R and capacitance C in series

(d) a resistance R and capacitance C in parallel.

19.23 The circuit shown in Fig. 19-2 has the following characteristics:

$$\begin{aligned} V(t) &= V_0 \cos \omega t, \\ V_0 &= 10\,\text{V}, \\ \omega &= 25 \times 10^3\,\text{rad}\,\text{s}^{-1}, \\ L &= 4 \times 10^{-3}\,\text{H}, \\ C &= 4 \times 10^{-7}\,\text{F}, \\ R &= 160\,\Omega. \end{aligned}$$

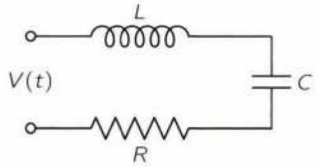

Figure 19-2

Consider the steady-state oscillations of this circuit.

(a) What is the amplitude of the current I_0?

(b) What is the phase shift δ of the current with respect to the applied driving voltage?

(c) Draw a phasor diagram, showing the voltage across the resistor, capacitance, and inductance.

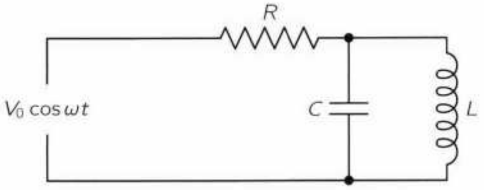

Figure 19-3

19.24 In the circuit shown in Fig. 19-3, the driving frequency is $\omega = 1/\sqrt{LC}$.

(a) What is the current I_R through R?

(b) What is the maximum current $I_{L,\max}$ through L?

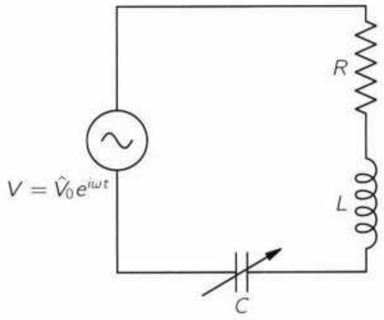

Figure 19-4

19.25 The RLC series circuit shown in Fig. 19-4 contains a generator supplying an alternating voltage of fixed frequency ω. The capacitor is variable.

(a) For a value $C = C_1$, the current I_1 is found to be in phase with the applied voltage. What is C_1, in terms of L and ω?

(b) The capacitance is then changed to $C = C_2$, so that the voltage is observed to lead the current I_2 by a phase angle of 45°. What is C_2 in terms of C_1, R, and ω?

(c) What was the ratio I_1/I_2?

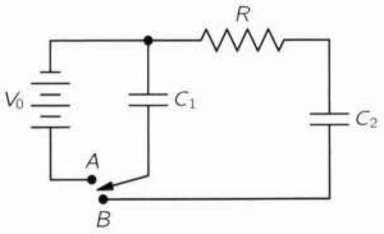

Figure 19-5

19.26 In the circuit shown in Fig. 19-5, originally the switch was at A, but at $t = 0$ it is thrown to B. After a long time,

(a) How much energy ΔE has been dissipated as heat in the resistor?

(b) What voltages V_1 and V_2, if any, remains on the capacitors?

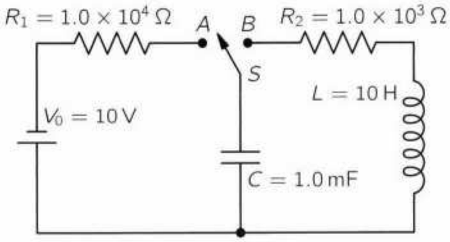

Figure 19-6

19.27 In the circuit shown in Fig. 19-6 with the switch S open at $t < 0$, the capacitor is uncharged.

(a) After the switch is closed at A, how much time t does it take the voltage across the capacitor to reach 8.0 V?

(b) At the instant the voltage across the C reaches 8.0 V, the switch is thrown to B. What is the initial value of the current I_0 that starts through L?

19.28 In the circuit shown in Fig. 19-7, the switch S is initially closed, and a steady current $I = V_0/R$ is flowing. At $t = 0$, S is suddenly opened. Find the maximum voltage $V_{\max}$ that is subsequently observed on the capacitor.

19.29 Given the circuit shown in Fig. 19-8, with the switch initially in position A. If the switch is suddenly changed to position B, what maximum voltage $V_{\max}$ will subsequently appear on the capacitor?

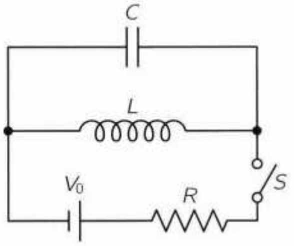

Figure 19-7

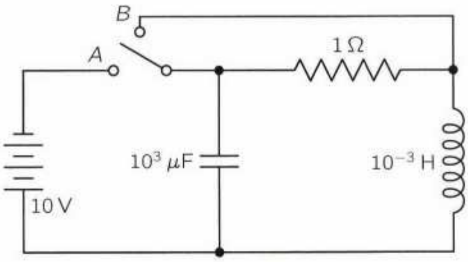

Figure 19-8

19.30 In order to suppress the 120 Hz "hum" from the power supply rectifier of an amplifier, a "smoothing filter" is used. In its simplest form, this consists of a resistance in series with a capacitance, as shown in Fig. 19-9. If the applied voltage has a DC component V_{DC} and a 120 Hz component of amplitude V_{AC}, find the corresponding voltages, V'_{DC} and V'_{AC}, at the terminals of the capacitor, for $R = 10^3\,\Omega$ and $C = 10\,\mu\mathrm{F}$.

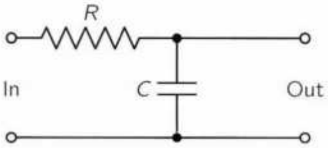

Figure 19-9

19.31 A spar buoy of uniform cross-section floats in a vertical position with a length L submerged when there are no waves on the ocean, as shown in Fig. 19-10.

(a) What is the amplitude A (with respect to the mean ocean surface) of the vertical motion of the buoy when there are sinusoidal waves of height h (crest to trough) and period T on the ocean? (Neglect fluid friction and non-vertical motions of the buoy.)

(b) If the undisturbed submerged length is 100 ft, the wave height 10 ft, and the wave period 5 seconds what is the amplitude of the buoy motion?

(c) What must the total length D of the buoy be so that the crests of the waves in part (b) above will just reach the top of the buoy?

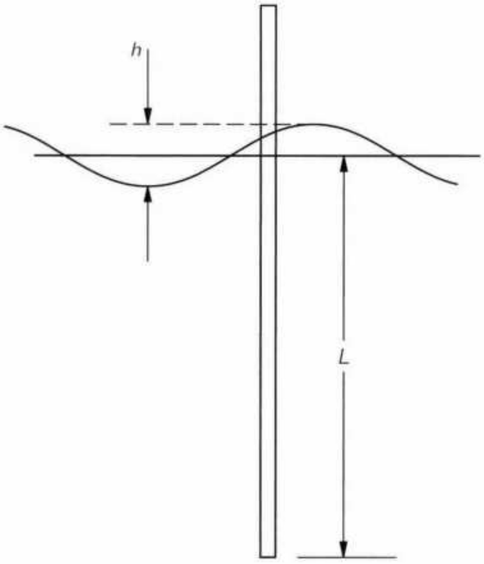

Figure 19-10

19.32 Fig. 19-11 shows the rotating element of a high speed centrifugal gas compressor. The impeller of mass M is rigidly mounted on a shaft of negligible mass and is located midway between the bearings B. When the compressor is not running the center of mass of the impeller is located eccentrically with respect to the centerline between the bearings by a small amount e (greatly exaggerated in the diagram). The elastic bending stiffness of the shaft is k (a transverse force F applied to the center of the shaft would produce a transverse displacement of $x = F/k$ at that point on the shaft).

(a) What is the circular frequency, ω_0, of bending vibrations of the system when the machine is not running?

(b) What is the bending deflection x of the shaft when the machine is running at angular speed ω?

(c) What is the critical speed ω_{cr} at which the compressor will fail?

(d) What is the critical speed ω'_{cr}, if the eccentricity is reduced by a factor of two?

(e) If, by rapid acceleration through the critical speed, a speed much greater than the critical speed is achieved without causing failure, where is the CM located with respect to the centerline between the bearings?

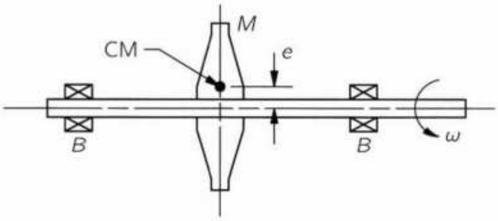

Figure 19-11

19.33 *Damped harmonic oscillator:* A mass m is suspended from a spring of force constant k in a medium which exerts a damping form $-m\gamma\, dx/dt$.

(a) For the case of underdamped motion find the complete solutions for the position $x = x(t)$ of m for all times $t > 0$ for the following driving forces:

(1) $F = \begin{cases} 0 & \text{for } t < 0 \\ F_0 & \text{for } t \geq 0 \end{cases}$

(2) no driving force, but at $t = 0$ an impulse $J = J_x$ is imparted to mass m.

(3) $F = \begin{cases} 0 & \text{for } t < 0 \\ F_0 \cos\omega_0 t & \text{for } t \geq 0 \end{cases}$

$\omega_0 = \sqrt{k/m}$

(b) If the oscillator is driven by a sinusoidal force $F = F_0 \cos\omega t$ and we consider long times, what is the frequency ω^* for which the amplitude reaches a maximum?

Note: Remember that the complete solution contains both steady-state and transient motion and that the initial conditions determine the constants of integration.

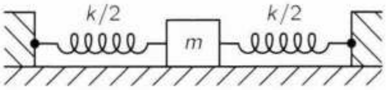

Figure 19-12

19.34 A mass m, attached to two identical horizontal springs of force constant $k/2$, as shown in Fig. 19-12, slides on a table top whose coefficient of friction is μ, assumed constant. The mass is pulled aside a distance A from the center point ($x = 0$) and is then released from rest.

(a) Write the differential equation of motion, and solve it for the time interval $0 < t < \pi\sqrt{m/k}$.

(b) Suppose that A is such that the mass comes to rest permanently at a distance B from the center ($x = \pm B$), after crossing the center point exactly N times.

  (1) What are the possible values of B?

  (2) Find A for $N = 0, 1, 2, \ldots$?

19.35 A particle of mass m and electric charge q is situated in an alternating electric field along the x-axis,

$$E = E_0 \cos \omega t.$$

The particle also experiences a force proportional to the *third derivative* of its x position,

$$F_\alpha = +\alpha d^3x/dt^3.$$

Find the amplitude A and phase δ of the oscillation of the particle in the steady state. This model gives an approximate description of a charged particle that scatters radiation.

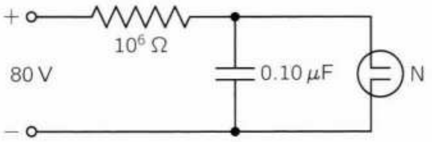

Figure 19-13

19.36 The circuit shown in Fig. 19-13 constitutes what is called a *relaxation oscillator*. It consists of a neon bulb connected across a capacitor that is charged through a resistor from a DC voltage source. The neon tube has infinite resistance as long as the voltage across it is less than 60 V. If the voltage attains or exceeds this value, the neon tube breaks down and then has negligible resistance, discharging the capacitor. The neon tube then "goes out", and returns to its infinite resistance state. If $C = 0.10\,\mu$F, $R = 10^6\,\Omega$, and $V_0 = 80$ V, find the frequency f at which the neon tube flashes.

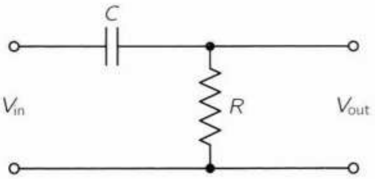

Figure 19-14

19.37 In many instances it is desirable to have an electronic circuit that will "differentiate" a function with respect to time. A simple circuit to accomplish this is shown in Fig. 19-14.

(a) Show that the output voltage of this circuit (if negligible current is allowed to flow into the output circuitry) is

$$V_{\text{out}}(t) = RC\,\frac{dV_{\text{in}}}{dt}(t),$$

provided $|V_{\text{out}}| \ll |V_{\text{in}}|$.

(b) Find V_{out} in the above circuit for the case in which $V_{\text{in}} = V_0 \cos \omega t$, and thus test the validity of the result of part (a) above, as a function of ω.

19.38 Invent a simple circuit that will "integrate" a function, and discuss its properties.

19.39 In electronic circuits it is often desired to provide a sinusoidal voltage of constant amplitude but variable phase. A circuit which accomplishes this is called a *phase-shifting network*. One example of such a network is shown in Fig. 19-15. Show that the voltage V_{out} measured between terminals A and B has half the amplitude of the input voltage V_{in} and a phase δ that may be adjusted between 0° and 180° by changing the resistance R'.

Hint: a phasor diagram is helpful.

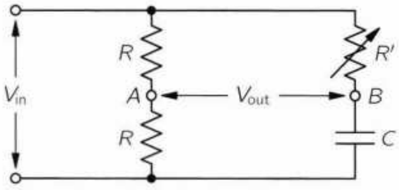

Figure 19-15

19.40 The ignition system of a gasoline engine is shown in Fig. 19-16. When the breaker points S are closed the battery voltage V_0 produces a current in the primary winding of the coil which has an inductance L and internal resistance R. The secondary winding of the coil has 100 times as many turns as the primary winding so that the voltage across the spark plug gap P is 100 times the voltage across the primary winding of the coil. The breaker points are opened by the distributor cam (not shown) each time the arm D is in contact with one of the spark plug circuits, as shown. C is the capacitance of the condenser, which is connected across the breaker points. Typical values of the circuit constants are $V_0 = 12\,\text{V}$, $C = 0.25\,\mu\text{F}$, $R = 6\,\Omega$, $L = 2\,\text{mH}$.

(a) If the engine has eight cylinders, is operating at 4,200 R.P.M., and the breaker points are open 30 % of the time, what is the current I in the primary of the coil at the instant the breaker points open? Assume that each time a spark occurs the currents in the system decay to zero before the breaker points close. Also, recall that each cylinder fires every other revolution of the engine.

(b) When the breaker points open what maximum voltage $V_{\max}$ would be produced across the spark plug gap, if a spark did not occur?

(c) How much time t elapses between the instant the breaker points open and the instant of maximum spark plug voltage?

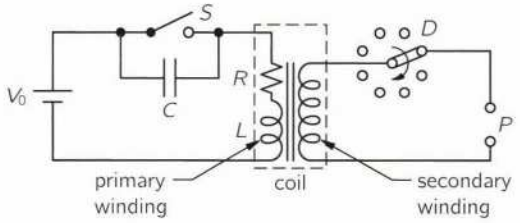

Figure 19-16

19.41 Calculate the speed V of an automobile at which an unbalanced wheel will begin to hop off the ground. Referring to Fig. 19-17: W_1 is the portion of the automobile weight supported by the wheel. W_2 is the unsprung weight (wheel + tire + brake assembly + etc.). W_3 is the unbalanced weight which is assumed to be very small compared to W_2 and to be located at the tire tread radius R. (For example, W_3 might be the weight of rubber worn off one spot on the tire during a "wheels-locked" panic stop). K is the stiffness constant of the suspension system given by $K = W_1/\delta_1$ where δ_1 is the static deflection of the wheel spindle with respect to the frame of the automobile. k is the stiffness constant of the tire, given by $k = (W_1 + W_2)/\delta_2$, where δ_2 is the static deflection of the wheel spindle with respect to the road. Neglect the influence of the shock absorbers and the small vertical motion of the automobile body (these influences turn out to be negligible in this problem for typical automobiles). Assume the following typical numerical values: $W_1 = 750\,\text{lb}$; $W_2 = 75\,\text{lb}$; $W_3 = 2\,\text{oz} = 0.125\,\text{lb}$; $R = 14$ inch; $\delta_1 = 6$ inch; $\delta_2 = 1.5$ inch.

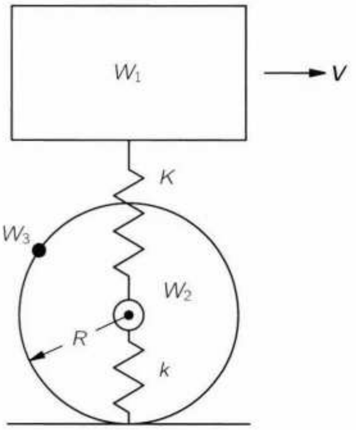

Figure 19-17

20

Geometrical Optics

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 26 and 27.

20.1 When we stand before an ordinary plane mirror, our image appears to be reversed, right for left; that is, the image of our right hand looks like the left hand of the "person" in the mirror. Why does the mirror not also reverse top for bottom? What is reversed by the mirror?

20.2 A man can walk at $5\,\mathrm{ft\,s^{-1}}$ on a sidewalk, but only $3\,\mathrm{ft\,s^{-1}}$ on the rough field shown in Fig. 20-1. He starts at point A, 140 ft from a wall, and goes to B, 120 ft south of the sidewalk along the wall.

(a) If the man walks on the sidewalk to point K, then walks across the field in a straight line to B, what must be the length of AK in order for him to reach B in the least time?

Note: It is legitimate to assume the "law of refraction" in solving this problem. However, if you have enough courage, you might try to solve it without such an assumption!

(b) What is his least time $t_{\min}$?

(c) What time t do alternate routes ACB and $AC'B$ require, if $\overline{CK} = \overline{KC'} = 10\,\mathrm{ft}$?

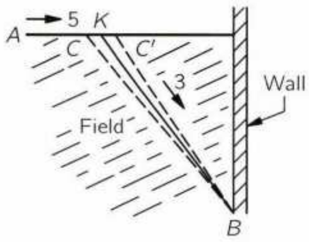

Figure 20-1

20.3 Light from source S sends a narrow beam perpendicular to a screen 1.0 m away, as shown in Fig. 20-2. The ray strikes the screen at P. If a Lucite slab of index of refraction 1.50 and thickness 0.20 m is inserted so that the beam strikes it at angle of incidence 30°,

(a) Find the lateral displacement of the ray, $\overline{PP'}$.

(b) Find the increase in time of path $SP' = t_0 + \Delta t$, over the time of the path in air $SP = t_0$.

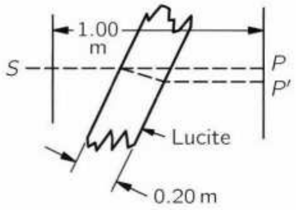

Figure 20-2

20.4 A hungry sportsman encounters a circular rain barrel 80 cm in diameter with 100 cm depth of water in it, and in passing he notices a fish inside adjacent to the wall and diametrically opposite him. The fish is part way from the bottom to the top of the barrel, and the ray from the fish to the man's eye emerges from the water at the axis of the barrel, making an angle of 60° with the vertical axis. If the man's eye is straight above the edge of the barrel, at what angle θ with this line of sight must he shoot his gun in order to hit the fish? (Neglect the deflection of the bullet as it enters the water.)

20.5 It is well known that when light goes from one transparent medium into another, not all of the light is refracted, but some is reflected (see Fig. 20-3) and very little, if any, is absorbed or scattered. What happens when a beam of light strikes the interface between two media, if the light beam is originally moving in the more dense medium? Discuss and sketch the situation for various angles θ.

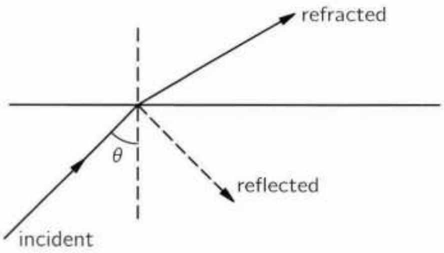

Figure 20-3

20.6 Two plane mirrors intersect each other so as to form a perfect internal right angle, with the line of intersection vertical. Make a sketch to explain why, in such a mirror, we "see ourselves as others see us."

20.7 Three mutually perpendicular mirrors intersect so as to form an internal right-angled corner. A light ray strikes one of the mirrors, and thence perhaps one or both of the other two. Show that, after all reflections have occurred (assuming the mirrors to be very large in extent) the ray is traveling exactly opposite to its original direction, but displaced laterally. Do you know of a practical application of such a "corner reflector"?

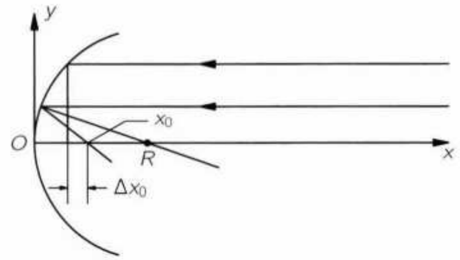

Figure 20-4

20.8 A bundle of light rays, parallel to and close to the axis, fall on a concave spherical mirror of radius R, as shown in Fig. 20-4.

(a) Find the position of the focal point $X_0 = X_0(R)$ for the case $y \ll R$.

(b) For $y/R = 0.2$ (note $y \ll R$ no longer holds) find the image point $X = X_0 + \Delta X_0$ and determine the value of $\Delta X_0/X_0$, the relative aberration of the mirror.

Hint: $(1 - a^2)^{-1/2} \approx 1 + a^2/2$.

20.9 The solar disk subtends an angle of approximately 32 minutes at the earth. Find the position x and diameter D of the solar image formed by a concave spherical mirror of radius $r = 400\,\text{cm}$.

20.10 The 200-inch Hale telescope, used in one of its several optical arrangements, has a focal length of 160 m. What should be the distance x between the focal planes for distant stars and

(a) the moon?

(b) an artificial earth satellite at a slant range of 300 km?

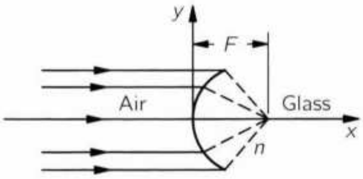

Figure 20-5

20.11 A parallel beam of light in air is to be brought to a point focus by a single refracting surface which bounds a region of index n, as shown in Fig. 20-5. (y is the normal distance from the axis.)

(a) Find the proper shape for this surface.

(b) Lenses are often made spherical—under what conditions is this acceptable?

20.12 The outer diameter of a piece of glass capillary tubing is D, and its index of refraction is n. When viewed from the side, the small capillary bore appears to have a diameter d'. What is its true diameter d?

20.13 A certain glass sphere has a radius of 2 cm and refractive index $n = 1.50$. If a point source of light is placed 12 cm from the center, where will the image be formed?

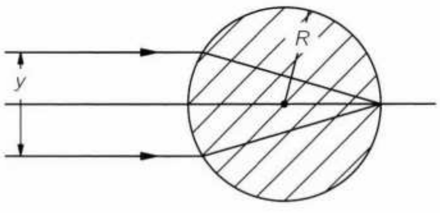

Figure 20-6

20.14 In the calibration of a photometric instrument it is necessary to determine that two parallel light rays are exactly a distance y apart. This is done by letting both rays fall normally and symmetrically on a glass rod, of radius R and index of refraction $n = 1.60$, and then adjusting the separation of the rays until they come exactly to a focus at the opposite circumference of the rod, as shown in Fig. 20-6. In terms of R, what is y?

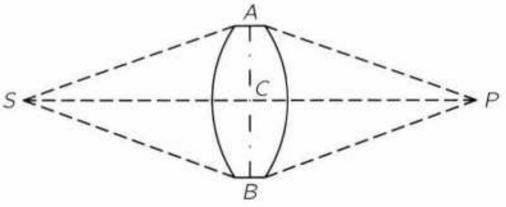

Figure 20-7

20.15 In Fig. 20-7 S is a source of light, P is its image produced by a lens, $\overline{SC} = \overline{CP} = 1.00\,\text{m}$, $\overline{AC} = \overline{BC} = 0.10\,\text{m}$. The lens ACB is 3.0 mm thick at its edge, with index of refraction $n = 1.6$. For the ray SCP to take just the same time as the rays SAP and SBP, how thick d should the lens be at C?

20.16 A lens of focal length F produces a real image of a distant object, and this image is viewed through a magnifying glass of focal length f. If the eye is focused at infinity while viewing, find the apparent angular magnification M of the system.

20.17 Two thin lenses L and L', of focal lengths f and f', are separated by a distance D. Find the equivalent focal length F of the combination, and the distances Δ, Δ' of the principle planes from the respective lenses L and L'.

20.18 The typical human eye can focus on objects lying between about 25 cm and infinity. A simple thin magnifying lens of focal length $f = +5\,\text{cm}$ is placed directly in front of the eye.

(a) Between what two limiting distances d should an object be placed to be seen clearly?

(b) Determine the angular magnification M for each of these positions.

20.19 A telephoto combination consists of a positive lens of focal length $f_1 = +30\,\text{cm}$ and a negative lens of focal length $f_2 = -10\,\text{cm}$. The separation between the two lenses is 27.5 cm, as shown in Fig. 20-8.

(a) Find the position x where a photographic plate should be placed in order to photograph an object 10 meters in front of the first lens.

(b) Make a ray diagram showing this arrangement.

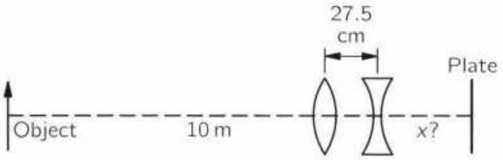

Figure 20-8

20.20 A simple astronomical telescope has an objective lens 4.0 cm in diameter with 10.0 cm focal length, and an eye lens with 2.0 cm focal length. The two lenses are placed on a common axis 12.0 cm apart. The telescope is to be used to observe stars as much as 5.7° (arctan 0.1) off-axis. How large must the radius r of the eye lens be in order to collect all the light from the objective?

Note: The cure is to place a "field lens" at the focal plane.

20.21 A luminous disk of diameter D is held perpendicular to the axis of a convex lens, and its real image is focused on a fixed screen, as shown in Fig. 20-9. When the disk is at a distance L_1 from the lens, the diameter of the image is d_1. The disk is now moved to a closer distance L_2 from the lens, and the screen is moved to refocus the image. The diameter of the new image is d_2.

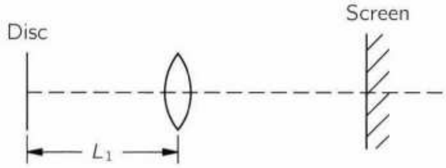

Figure 20-9

(a) Is $d_1 > d_2$ or $d_2 > d_1$?

(b) Must the screen be moved toward or away from the lens to form the image in the second case?

(c) What is the focal length f of the lens?

20.22 A certain beam of light is converging toward a focus at a certain point P, as shown in Fig. 20-10. It is desired to insert a single reflecting surface passing through a given axial point Q that will re-image the light to a new point focus at a given point P'. Find the shape of the required surface. Let the distance $\overline{QP'} = D$ and $\overline{QP} = d$. Note that there are two cases to be considered, in which the mirror intercepts the rays

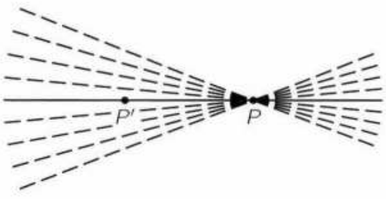

Figure 20-10

(a) before they pass through P, or

(b) after passing through P.

20.23 (a) Referring to Fig. 20-11: Establish that the equation of a parabolic curve of focal length f is $y = x^2/4f$.

(b) A bucket of fluid of density ρ is rotating uniformly with angular velocity ω about a vertical axis. Show that the upper surface of the fluid assumes a paraboloidal shape, and find the focal length f of the paraboloid.

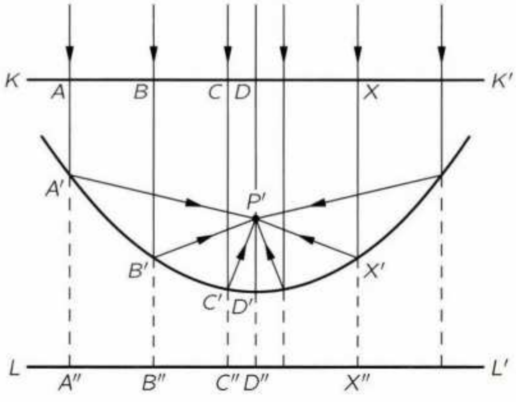

Figure 20-11

21

Electromagnetic Radiation: Interference

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 28 and 29.

21.1 Interpret the following two problems in complex numbers geometrically, and show that the absolute value of A in each case is as given:

(a)

$$A = re^{i\theta/2} + re^{-i\theta/2}$$
$$|A| = 2r\cos(\theta/2)$$

(b)

$$A = \sum_{n=0}^{N} re^{in\theta}$$
$$|A| = r\frac{\sin\left(\frac{N+1}{2}\theta\right)}{\sin(\theta/2)}$$

21.2 A charge q traverses a circular path of radius a at an angular velocity ω.

(a) Evaluate the electric field $\boldsymbol{E}(t)$ at a great distance R from the charge, at an angle θ with respect to the axis of the circular path.

(b) Find the intensity of the radiation $I(\theta)$ at a great distance R on the axis $(\theta = 0)$ and in the plane of the circle $(\theta = \pi/2)$.

Assume that $\omega a \ll c$ and $a \ll R$.

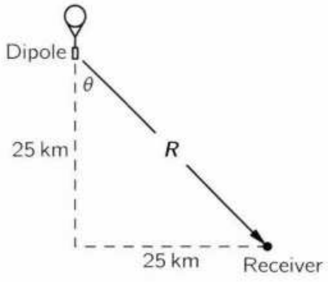

Figure 21-1

21.3 The power per unit area delivered by an electromagnetic wave is proportional to the mean-square electric field strength.

(a) If the total power radiated by an oscillating charge is P_{total}, how much power P falls on a unit area normal to the radius vector $\boldsymbol{R}$ at an angle θ with respect to the axis of oscillation?

(b) Evaluate P in $\mathrm{W\,m^{-2}}$ for a vertically oriented dipole suspended from a cosmic ray radiosonde balloon at an altitude of 25 km and at a horizontal distance of 25 km from the receiver, as shown in Fig. 21-1, if the transmitter is radiating 0.5 W total.

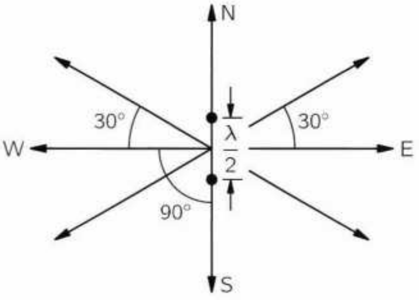

Figure 21-2

21.4 Two vertical antennas are arranged as shown in Fig. 21-2 and are driven in phase. The antennas are driven so that one would, if alone, radiate a certain intensity I_0 in all horizontal directions, and the other, an intensity $2I_0$. What should be the observed intensity I in the various directions shown in the figure?

21.5 Four identical dipole radiators are aligned parallel to one another and are equally spaced along a line at a distance 2.50 cm apart. They are driven at a frequency of 3.00×10^9 Hz and are phased so that, starting from one end, each successive dipole lags the preceding one by 90°. Find the intensity pattern of the radiation $I(\theta)$ at a great distance in the equatorial plane (perpendicular to the dipole axes), and sketch this function in the polar coordinates shown in Fig. 21-3. Such a diagram is called the *radiation pattern* or *lobe pattern* of an antenna system.

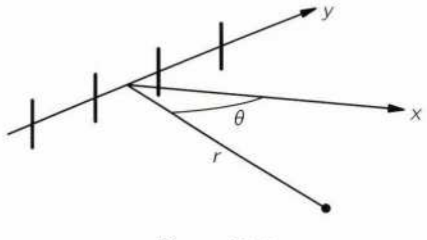

Figure 21-3

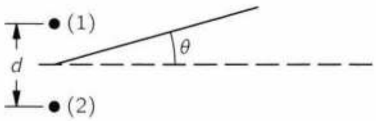

Figure 21-4

21.6 Two parallel dipoles are situated a distance $d = \lambda/2$ apart, and are oscillating with the same frequency and amplitude (s. Fig. 21-4).

(a) Find $I = I(\theta)$ (the lobe pattern) in the equatorial plane and sketch the pattern in a polar diagram if the oscillators are in phase.

(b) Find by what fraction of a period oscillator (2) must lag oscillator (1) so that an observer at $\theta = 210°$ will see maximum signal. In this case, for which value of θ will there be no signal?

(c) Find and sketch the lobe pattern implied in part (b) above.

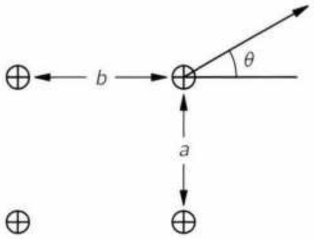

Figure 21-5

21.7 Four vertical dipoles are located at the corners of a horizontal rectangle of sides a, b as shown in Fig. 21-5. If they are driven in phase, at wavelength λ, what minimum values (> 0) should a, b have to produce maximum intensity in the direction $\theta = 30°$ far from the charges?

21.8 An observer is at distance R from two identical charges q that are both passing through the origin at $t = 0$. The first charge moves only along the z-axis, and $z(t) = d \sin \omega t$; the second charge moves only along the x-axis, and $x(t) = d \sin \omega t$. What is the magnitude of the electric field $E(t)$ at the two following points:

(a) $x = R/\sqrt{2}$, $y = 0$, $z = R/\sqrt{2}$,

(b) $x = 0$, $y = R$, $z = 0$.

Assume $R \gg c/\omega$ and $\omega d \ll c$.

21.9 A field engineer testing radiation patterns is flying in a helicopter at a ground speed of $120\,\mathrm{mi\,h^{-1}}$ at low altitude in a circular pattern of $2.0\,\mathrm{mi}$ radius about the mid-point between two vertical dipole transmitting antennae that lie in a north-south line. For the frequency being used for the test, the antennae are a half wavelength apart. Normally the antennae are operated in phase. However, the transmitter operator decided to play a joke on the field engineer, by changing the phase relation between the antennae at such a rate that no change in radiation intensity could be observed aboard the helicopter. If he started this change when the helicopter was due east of the antennae at what rate $d\alpha/dt$ was he changing the phase relation when the helicopter was $\theta°$ north of east of the antennae?

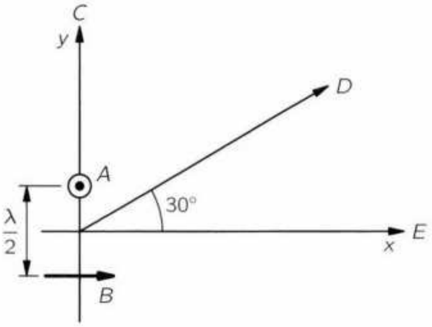

Figure 21-6

21.10 Two electric dipoles, A and B, are situated one-half wavelength apart and are perpendicular to one another and to the line joining their centers, as shown in Fig. 21-6. They are driven at the same frequency and phase, but dipole B has twice the amplitude of dipole A. Find the intensity I and the direction of the $\boldsymbol{E}$ vector of the radiation at a great distance from the dipoles in the directions C, D, and E (all in xy-plane) indicated in the figure.

21.11 A double line of N equally spaced oscillating dipoles is situated as shown in Fig. 21-7. All dipoles in row A are driven in the same phase, and all those in row B lag 90° in phase behind those of row A. Sketch the radiation pattern $I(\theta)$ in the equatorial plane at a great distance from the array.

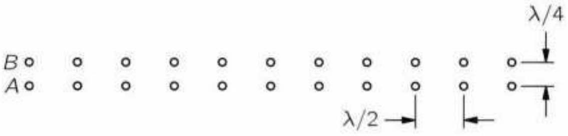

Figure 21-7

21.12 The conduction electrons in a long, straight, fine wire of length L are all oscillating along the wire with angular frequency ω, small amplitude a, and in the same phase. Find the magnitude of the electric field $E(t)$ at a great distance R ($R \gg L$), at an angle θ with respect to the wire.

22

Electromagnetic Radiation: Diffraction

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 30.

22.1 Consider n equally spaced dipole radiators, each of length a and separation d along a straight line, as shown in Fig. 22-1, oscillating along the line at the same amplitude A and frequency ω, but with successive phase shifts α. Each dipole radiator consists of a large number of atomic dipoles. Show that the diffraction pattern of the intensity at large distance at an angle θ is given by

$$I = I_0 \frac{\sin^2(\beta/2)}{(\beta/2)^2} \frac{\sin^2(n\phi/2)}{\sin^2(\phi/2)},$$

where

$$\phi = \alpha + \frac{2\pi d}{\lambda}\sin\theta,$$
$$\lambda = \frac{2\pi c}{\omega},$$
$$\beta = \frac{2\pi}{\lambda}a\sin\theta.$$

and I_0 is the intensity of a single dipole at the angle θ.

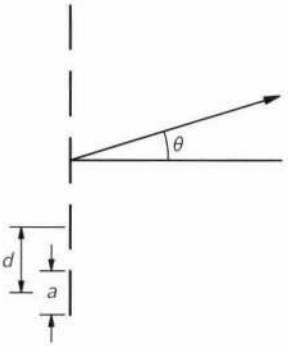

Figure 22-1

22.2 A certain spectral line has a wavelength of 5500 Å and a width of 10 Å. What is the Q of the atomic oscillator?

22.3 Under what conditions will an illuminated slit cast a "geometrical" shadow? (i.e., such that diffraction effects are negligible.)

22.4 An automobile with the customary two headlights (considered as point sources) is approaching from a distance on a straight road. The lights on the car are situated 120 cm apart.

(a) How far d from an observer would the car be when he could just be sure he was seeing two lights and not one, assuming the aperture of his iris is 0.5 cm and the effective wavelength of the light is 5500 Å?

(b) Would the fact that the light is "white" (a mixture of wavelengths) make it easier or harder to resolve the two sources?

22.5 The wavelengths of the D-lines of sodium are 5889.95 Å and 5895.92 Å, respectively. How long x must a grating with 600 lines/mm be in order to resolve these lines in the first order spectrum?

22.6 A point source of light L emitting a single wavelength λ is situated a *small* distance d above an ideal plane mirror. A screen stands at the end of the mirror at distance D from L, as shown in Fig. 22-2. ($D \gg d$) Find $I(z)/I_0$, the relative intensity of light on the screen as a function of z, the plane of the mirror being at $z = 0$. (*Note:* a mirror changes the phase of the light it reflects by 180°.)

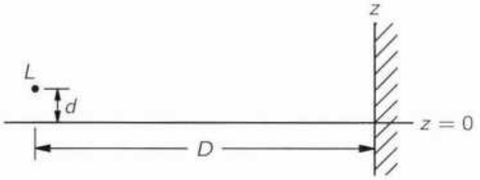

Figure 22-2

22.7 Parallel light of wavelength λ is normally incident from the left on a circular hole in a screen and the resultant intensity is observed at a point on the axis of the hole at $z = 10\lambda$ to the right.

(a) What radius r should the hole have to produce maximum intensity at the observation point?

(b) By what fraction $I/I_{\max}$ is the intensity reduced if the screen is removed?

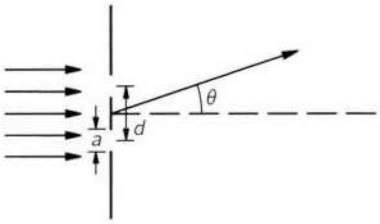

Figure 22-3

22.8 (a) Calculate the transmitted diffraction pattern $I(\theta)/I_{\max}$ for a screen, as shown in Fig. 22-3, having two parallel slits of width $a = 6\,\text{cm}$, separated by a distance $d = 12\,\text{cm}$, which is exposed to a parallel beam of $\lambda = 3\,\text{cm}$ microwave radiation.

(b) How many orders of principal maxima are there and for which values of θ?

(c) What is $I(\theta)/I_{\max}$ at the positions of the principal maxima?

22.9 The wavelengths of spectral lines are commonly measured to 0.001 Å using spectrographs whose resolving power may be only 0.010 Å. Are any basic laws of physics being violated in the process? Explain.

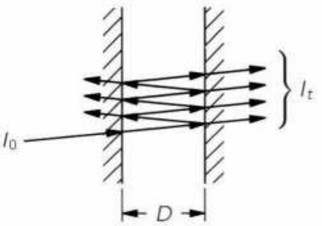

Figure 22-4

22.10 A Fabry-Perot interferometer consists of a pair of very accurately flat surfaces, parallel to each other at a distance D apart. The surfaces are coated so as to reflect a fraction R^2 of the intensity of the light incident on them normally, and to transmit a fraction T^2 of the intensity. Light of intensity I_0 and wavelength λ is incident upon one surface from the left (see Fig. 22-4). Part of this beam is transmitted directly through the system, but some of the light is reflected from the second surface, then from the first surface, and is thence transmitted. In general, the outgoing beam is made up of light which has been reflected 0, 2, 4, 6, ... times and transmitted through 2 films, all summed together. How should the relative transmitted intensity I_t/I_0 depend upon D, λ, R, and T?

Note: Narrow-band optical filters, called interference filters, operate on this same principle, but the two reflecting surfaces are made by high-vacuum coating a piece of glass with several layers, accurately controlled in thickness, or clear materials having various indexes of refraction.

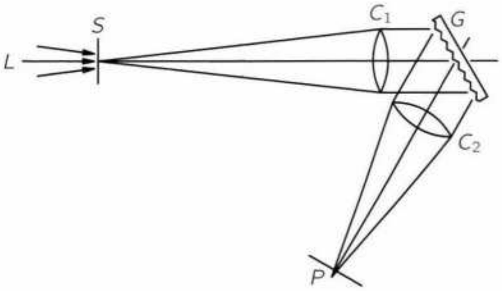

Figure 22-5

22.11 A common type of grating spectrograph is constructed as shown in Fig. 22-5. Light from a source L passes through a narrow slit S, thence through a collimator lens (or mirror) C_1 which renders it parallel (so that it strikes the grating as would plane waves from infinity). This parallel light is then diffracted by the grating G; the diffracted light which proceeds in a certain range of angular directions strikes another lens C_2, called the camera lens, and is focused in a plane P, where the spectrum appears as a band, perhaps crossed by narrow spectrum lines at various places. Let the length and width of the slit be h and w, the focal lengths of C_1 and C_2 be F_1 and F_2, the angles between the grating normal and the axes of C_1 and C_2 be θ_i and θ_d, and the number of lines per mm on the grating be N.

(a) How wide h' will the spectrum band appear at P?

(b) What wavelength(s) λ_m will appear on the axis of C_2 at P?

(c) How far apart D in the focal plane at P will two spectral lines appear whose wavelength differs by 1.00 Å? This quantity is often called the *dispersion* of the instrument.

(d) If the slit width w is much larger than the resolution $1.22\,\lambda F_1/A_1$ of the collimator lens, where A_1 is the aperture, how wide w' should a spectral line at P be?

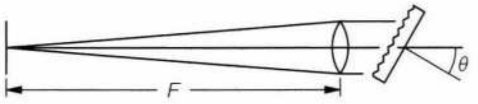

Figure 22-6

22.12 The spectrograph at the 150 ft solar tower telescope of the Mt. Wilson Observatory is of the Littrow type, shown schematically in Fig. 22-6. In this arrangement, a single lens acts as both the collimator and camera lens, and $\theta_i = -\theta_d$ (nearly). The spectrum is formed in a strip adjacent to the slit. The focal length of the Mt. Wilson instrument is $F = 23\,\text{m}$, and the grating has a ruled area $15\,\text{cm} \times 25\,\text{cm}$ with 600 lines/mm. The fifth order spectrum is commonly used.

(a) At what angle θ should the grating be tilted to bring the line $\lambda = 5250.218\,\text{Å}$ of neutral iron in coincidence with the entrance slit in the fifth order spectrum?

(b) What other wavelengths in the range 3600 Å–7000 Å will also be coincident with the slit?

(c) Suggest a simple way to remove the unwanted orders, leaving only the fifth order.

(d) What is the dispersion d of the instrument at fifth order $\lambda = 5250\,\text{Å}$?*

(e) What is the minimum $\Delta\lambda$ which can theoretically be resolved at fifth order $\lambda = 5250\,\text{Å}$ by this instrument?

22.13 When the grooves of a diffraction grating are shaped in such a way as to throw most of the incident radiation into a particular direction, the grating is said to be blazed for this direction. Suppose it were possible to shape the grooves perfectly in a sawtooth shape as shown in Fig. 22-7, each groove surface being tilted at a certain angle θ_b.

(a) Use the notion of the diffracted beam being the radiation emitted by oscillators in the material, which are driven in phase with the incoming radiation, to deduce in what direction the diffracted beam would be most intense if $\theta_i = 0$. (Assume white light.)

(b) Estimate the approximate angular range over which the blaze would extend.

Figure 22-7

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* Note that, although $\theta_i = -\theta_d$ at $\lambda = 5250\,\text{Å}$, θ_i is fixed while θ_d depends upon λ.

23

Electromagnetic Radiation: Refraction, Dispersion, Absorption

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Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 31.

23.1 Is red light or blue light deflected most by a prism, and for what physical reason?

23.2 Find the index of refraction n of aluminum for x-rays of wavelength 1.56×10^{-8} cm. Assume that all of the electrons in aluminum have natural frequencies very much less than the frequency of the x-rays.

23.3 The index of refraction of the ionosphere for radio waves of frequency 100 MHz is $n = 0.90$. Find the density ρ of electrons per cubic centimeter in the ionosphere.

23.4 Noting that the electric field of a light wave traveling through a medium of refractive index n is $E = E_0 e^{i\omega(t-nz/c)}$,

(a) Show that, if $n = n' - in''$, $E = E_0 e^{-n''\omega z/c} e^{i\omega(t-n'z/c)}$.

(b) Use Eq. (31.20) from Vol. I,

$$n - 1 = \frac{Nq_e^2}{2\epsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 + i\gamma\omega}$$

to find the rate at which the intensity I of a beam of radiation whose frequency is exactly equal to the natural frequency ω_0 of an atom is attenuated.

23.5 In Section 31-5 of Vol. I, it was deduced that the instantaneous energy flux of a wave is $S = \epsilon_0 c E^2$ watts per square meter:

(a) Find the total rate P at which energy is radiated by an electron which is oscillating with amplitude x_0 and angular frequency ω.

(b) Compare the energy radiated per cycle with the stored energy $m\omega^2 x_0^2/2$, and thus find the damping constant γ_R. This process is called *radiation damping*.

(c) An excited atom gives out radiation having a certain wavelength λ. Calculate theoretically the expected breadth $\Delta\lambda$ of the spectral line, if the breadth arises solely from radiation damping. (Think of the atom as being a tiny damped oscillator having a large Q.)

Electromagnetic Radiation: Radiation Damping, Scattering

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 32.

24.1 What is the wavelength dependence of the scattering of light by free electrons?

24.2 A beam of light is passing through a region containing N scatterers per unit volume, each of which scatters the light with a cross section σ. Show that the intensity of light remaining in the beam, as a function of the distance x traversed, is

$$I = I_0 e^{-N\sigma x}.$$

24.3 Using the scattering cross section formula, Eq. (32.19) in Vol. I,

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{\omega^4}{(\omega^2 - \omega_0^2)^2},$$

and the formula for the index of refraction of a gas, Eq. (31.19) in Vol. I, show that the quantity $N\sigma$ can be written as

$$N\sigma = \frac{2}{3\pi}\frac{(n-1)^2}{N}\left(\frac{2\pi}{\lambda}\right)^4,$$

where N is the number of atoms per unit volume of gas. (This was one of the first ways used to estimate Avogadro's number, using the scattering of the blue sky.)

24.4 The inner corona of the sun (called the K-corona) is sunlight that has been scattered by free electrons. The apparent brightness of the K-corona at one solar radius from the sun's limb is about 10^{-8} that of the solar disc (per unit area). Estimate the number of free electrons N_e per cm^3 near the sun.

24.5 By what percentage f is the blue light ($\lambda = 4500$ Å) of the sun attenuated in going through the atmosphere when the sun is

(a) at the zenith?

(b) 10° from the horizon?

24.6 A short straight piece of copper wire placed in the beam of electromagnetic waves sent out by a radar antenna will "scatter" some of the wave. The electric field of the wave sets up a motion of the electrons in the wire and this motion results in the radiation of the "scattered" wave. For a short piece of wire (length $\ll \lambda$) we can assume that the average displacement of the electrons in the wire is along the axis of the wire and is proportional to the component of the electric field parallel to the wire. That is, if there are N electrons in the wire and d is their average instantaneous displacement, then $d = \chi E_{\parallel}$, where $E_{\parallel}$ is the component of the electric field of the wave parallel to the wire. We would like to know (in terms of χ and N):

(a) What is the maximum scattering cross-section σ_{max} of the wire, if the incident radiation has amplitude E_0 and frequency ω?

(b) How does the scattering cross-section σ depend on the orientation θ of the wire?

24.7 A new radiation is discovered (called "X-rays" because they are new and mysterious!), which are suspected of being transverse waves like light. Scattering of these rays by electrons in matter has been observed. How could you prove they are transverse waves and can be polarized?

24.8 Show that if the equation of motion of a charged oscillator is assumed to be

$$md^2x/dt^2 + \omega_0^2 x - (2e^2/3c^3)d^3x/dt^3 = F(t),$$

the third-derivative term will correctly describe the rate of loss of energy by radiation (the radiation resistance) at any frequency.

Hint: Assume $F(t) = A\cos\omega t$ and find the amount of power absorbed from the driving source.

24.9 Interstellar space is believed to be populated by clouds of tiny dust grains composed of carbon, ice, and small amounts of other elements. Estimate the minimum mass per unit area ($\mathrm{g\,cm^{-2}}$) of such dust needed to obscure our view of stars behind it by, say a factor 100 (i.e., 5.0 stellar magnitudes). Note that the dust grains may remove starlight by scattering as well as by simple absorption.

25

Electromagnetic Radiation: Polarization

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 33.

25.1 Is the light obliquely reflected from a lake (e.g., light of the rising sun or moon) polarized with its electric field in a vertical or horizontal plane?

25.2 Photographers often use a yellow filter or polaroid filter to make the clouds stand out against the sky. Explain the physical reasons for these techniques in both cases.

25.3 Two polaroid sheets are set with their axes at right angles. A third sheet is then inserted between them with its axis at an angle θ with respect to the axis of the first polaroid. If unpolarized light of intensity I_0 is normally incident upon this combination, what intensity I_t should be transmitted? Assume the polaroids are ideal (50 % transmission of incident unpolarized light and no transmission through crossed polaroids).

25.4 Discuss the intensity and polarization of the radiation emitted by an electron moving at constant speed in a circular path, particularly for points on the axis of the circle and in the plane of the circle.

25.5 Assume that when a beam of plane polarized light strikes a polaroid sheet, a fraction α^2 of the intensity is transmitted if the polaroid axis is parallel to the polarization axis, and a fraction ϵ^2 is transmitted if the two axes are at right angles. (If the polaroid were ideal, α^2 would be unity and ϵ^2 would be zero.) Unpolarized light of intensity I_0 is normally incident on a pair of polaroid sheets with an angle θ between their axes. What relative intensity I_t/I_0 should be transmitted? (Ignore reflection effects.)

25.6 A vacationing Caltech freshman, strolling with a girlfriend, sees the moon, 10° above the horizon, reflected in a calm lake. Nostalgically recalling *The Feynman Lectures on Physics*, Chapter 33, he attempts to calculate how bright the image should appear, compared with the moon itself. He assumes the radiation from the moon to be (nearly) unpolarized. What result should he expect to get? Show that the reflected intensity $I_R/I_{☾}$ approaches 100 percent for grazing incidence.

25.7 Show that Brewster's angle (the angle of incidence i for which the reflected ray is linearly polarized) is such that $\tan i = n$, where n is the index of refraction.

25.8 If light falls perpendicularly on the plane facet of a diamond (index of refraction $n = 2.40$),

(a) What fraction f of the incident radiation is reflected?

(b) What is Brewster's angle β for diamond?

25.9 Consider plane polarized light traveling through a series of n polarizers, each rotated clockwise through an angle θ/n with respect to the one before it and the first polarizer rotated at an angle θ/n clockwise with respect to the initial plane of polarization.

(a) Find the intensity I_t of the light that passes through the polarizers, if the initial intensity is I_0.

(b) What is this light's polarization?

25.10 Linearly polarized light is sent through a quarter-wave plate followed by a polaroid plate. The direction of polarization makes an angle θ with the optical axis of the quarter-wave plate. As the *polaroid is rotated*, one observes maxima I_{max} and minima I_{min} of the emerging light intensity. What is the ratio $I_{\text{max}}/I_{\text{min}}$ for $\theta < 45°$? Assume normal incidence.

25.11 A piece of crystal quartz, whose indices of refraction are $n_o = 1.553$ (ordinary) and $n_e = 1.544$ (extraordinary) is ground to a thin sheet, 0.12 mm thick, in such a way that its optic axis is parallel to the faces of the sheet. If it is now sandwiched between crossed polaroids so that its optic axis is at 45° with those of the polaroids, what wavelength(s) of visible light (4000–7500 Å) will be transmitted through the system with maximum intensity?

25.12 The indices of refraction of crystalline quartz for light of wavelength 600 millimicrons are $n_o = 1.544$ and $n_e = 1.553$, for the ordinary and extraordinary rays, respectively. If a crystal of quartz is cut parallel to its optic axis, one may take advantage of the maximum difference in speed of the ordinary and extraordinary rays as they enter normally and progress through the crystal.

(a) What thickness of crystal d is required to shift the relative phases of these two rays by 90°, for light of the above wavelength?

(b) Suppose the indices of refraction for light of wavelength 410 millimicrons are $n_o = 1.557$ and $n_e = 1.567$, and that the crystal of quartz is cut as a quarter-wave-plate for wavelength 600 millimicrons. Describe fully the state of polarization of emergent light $\boldsymbol{E}$ of this shorter wavelength which is linearly polarized before entry into the crystal.

25.13 You are given a polished plate of black obsidian and are asked to measure the index of refraction of the material. How would you proceed, and what precision would you expect to attain?

26

Electromagnetic Radiation: Relativistic Effects

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 34.

26.1 A disc of radius A rolls without slipping on a horizontal plane. Find the equations of the path followed by a point at a radius $R \leq A$ from the center of the disc in terms of A, R, and the angle θ through which the disc has turned. Let x be measured from the center of the disc vertically and z be measured horizontally. Show that

$$\begin{aligned} z &= A\theta + R\sin\theta, \\ x &= R\cos\theta. \end{aligned}$$

26.2 Discuss the polarization properties of synchrotron radiation and bremsstrahlung.

26.3 If you were to make measurements of the electromagnetic radiation emitted by an astronomical object, like the Crab Nebula, how would you distinguish between synchrotron radiation and bremsstrahlung?

26.4 Assume that you are looking along the x-axis of your coordinate system at a small section of the Crab Nebula, and observe synchrotron radiation polarized in the z-direction. What is the direction of the magnetic field? Discuss.

26.5 Consider an electron circling with relativistic velocity in a uniform magnetic field. Sketch the electric field strength as a function of time (over at least two cycles) for an observer looking

(a) in a direction along the magnetic field,

(b) in a direction perpendicular to the magnetic field.

26.6 Is the radiation pressure of a given light beam greater on a black or on a mirror surface? Explain.

26.7 Bradley (1728) observed the aberration of light by which stars appear to be displaced in the sky because of the earth's motion in its orbit. The telescope must be pointed "forward" a maximum of 20.5″ arc for stars near the pole of the ecliptic. If one considers the velocity of light as known, 3.00×10^8 m/s, to what value of the radius of Earth's orbit R does this observation lead?

26.8 As suggested in Section 34-8 of Vol. I, derive the expression $\sin\theta = v/c$ for the aberration of starlight, using the Lorentz transformation.

26.9 A man in a rocket ship traveling at $0.5c$ observes a star at exactly 90° relative to his direction of motion. A certain line in the spectrum of the star appears to be at frequency ν_0. He now reverses the ship and retraces his path at the same speed.

(a) At what angle θ_1 does he see the star now?

(b) What is the frequency ν_1 of the spectral line?

26.10 A space probe is moving radially away from an observer. The observer directs a radar beam of frequency ν_0 towards the probe and finds the frequency of the returning reflected beam to be ν. What is the speed v of the space vehicle relative to the observer? (Assume that in the rest frame of the vehicle the beam is reflected without change in frequency.)

26.11 Spectrograms of the radiation coming from opposite ends of the sun's equatorial diameter show a shift of 0.1 Å for the H_α-line 6564.7 Å. What is the peripheral speed v of the sun at its equator?

26.12 The D-lines of sodium (laboratory wavelength, 589.0 millimicrons) are observed to be shifted to 588.0 millimicrons in the spectrum from a certain star. What is the star's velocity $\boldsymbol{v}$ relative to the observer? (Is the non-relativistic Doppler formula sufficiently accurate in this case?)

26.13 The Caltech astronomer M. Schmidt measured the wavelengths of certain spectral lines from a distant quasar and found that they were red-shifted by $\Delta\lambda \approx 2\lambda$. With what speed v is the quasar presumably receding?

26.14 If $z = ct$ in Ex. 26.1, find the transverse acceleration d^2x/dt^2 of the point. This is the retarded acceleration needed for calculating the radiation from a particle moving in a circular path of radius R.

(a) Express the result in terms of the observable quantities R, v (the speed of the particle in its path), and x (the apparent transverse position of the particle at the time of observation).

(b) Find the ratio of the maximum and minimum radiation intensities $I_{\max}/I_{\min}$ that will be observed as the particle moves toward and away from the observer in its circular path.

26.15 An electron initially at rest at $x = 0$, $z = 0$ is given (for times $t > 0$) an acceleration $\boldsymbol{a}$ in the xz-plane at an angle θ with respect to the z-axis, as shown in Fig. 26-1. Calculate (relativistically) the electric field $\boldsymbol{E}(t)$ at a point $z = R_0$, where R_0 is a "large" distance. Express your answer in terms of $\boldsymbol{a}$, the true acceleration, θ, and $\beta = v/c$, where v is the true speed of the electron.

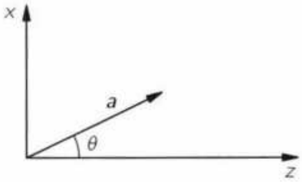


Figure 26-1

26.16 A disc of area A and thickness d is illuminated by normally incident light of wavelength λ and intensity I_0. Calculate the force F due to radiation pressure for the three following cases.

(a) The disc is opaque and non-reflecting.

(b) The disc has a refractive index $n = 1.5$. (Answer accurate to 10 percent is sufficient.)

(c) The disc has a complex refractive index $n = 1 - in''$. (n'' is real.)

26.17 A spherical balloon of radius 30 m is made almost perfectly reflecting on the outside to avoid solar heating. When the balloon is moving freely in space at 1.0 AU from the sun, what force F results from the reflection of solar radiation?

26.18 In one proposed means of space propulsion, a thin sheet of highly reflecting plastic film would be used as a radiation pressure "sail." A plane square sheet $100\,\mathrm{m} \times 100\,\mathrm{m}$ is available, and the mass of the spaceship is 10^3 kg. If the spaceship initially travels in a circular orbit of 1 AU radius about the sun,

(a) Describe how to use the "sail" to increase the mean radius of the orbit.

(b) Find at what rate dR/dt the orbit radius will grow. (*Hint:* Maximize the power transferred to the spacecraft by radiation pressure.)

26.19 Assume that interplanetary space is populated by small grains of "dust," of mean density ρ and of roughly spherical shape of radius R.

(a) Show that, for any size dust grain, the ratio of the gravitational attraction toward the sun to the radiation pressure away from the sun is independent of the distance from the sun.

(b) Using the fact that the solar radiation intensity at the earth's orbit is roughly $1374\,\mathrm{W\,m^{-2}}$, and assuming the absorption cross-section to be πR^2, find for what radius R the radiation pressure and gravitational attraction will just balance.

(c) Considering the results of Chapter 32 of Vol. I, can the effective cross-section of a dust grain be appreciably greater than πR^2?

27

Quantum Behavior: Waves, Particles, and Photons

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 37 and 38.

27.1 Compare qualitatively the x-ray diffraction patterns of two crystals having the same size and shape unit cells, but different atomic arrangements within their unit cells.

27.2 A parallel beam of electrons with momentum p_0 passes through a slit of width W. What is the approximate angular spread θ of the beam after the slit?

27.3 (a) Thermal neutrons have kinetic energy $T \approx (1/40)\,\text{eV}$. What is the wavelength λ associated with such neutrons?

(b) What is the wavelength associated with electrons of kinetic energy

(1) 1 keV?

(2) 1 MeV?

(c) What wavelength would be associated with an object of mass 50 g traveling at $1000\,\text{m}\,\text{s}^{-1}$?

(d) Which has a shorter wavelength, a 1 MeV x-ray or an electron having a *total* energy $E = 1\,\text{MeV}$?

27.4 Light of wavelength 4100 Å produces electrons of 1.0 eV maximum kinetic energy in a certain photoelectric cell. What is the longest wavelength λ_{max} to which the cell will respond?

27.5 The energy (kinetic plus potential) of an electron in the ground state of a hydrogen atom is $-13.6\,\text{eV}$. If a free electron is captured by a proton to form a hydrogen atom in the ground state,

(a) what is the wavelength λ of the photon emitted in this process?

(b) does the emitted radiation fall into the visible, infrared, or ultraviolet region of the spectrum?

27.6 A muon μ^- and a proton may form a (hydrogen-like) "muonic atom." Estimate the Bohr radius R for this atom in angstroms. ($m_{\mu^-} = 206\,m_e$)

27.7 It is desired to estimate the minimum possible energy of a harmonic oscillator, for which the energy is determined by $E = p^2/2m + \beta x^2/2$. According to the uncertainty principle, the momentum p can be reduced only at the cost of an increase in the mean displacement, so the condition of minimum energy implies a compromise between these two contributions. What should the minimum energy E_{min} be?

27.8 A spectral line ($\lambda = 5000\,\text{Å}$) due to an electron transition from an excited state to the ground state of an atom is observed to have a spread in wavelength of 0.01 Å. On the average, for how long a time τ does the atom exist in the excited state?

27.9 Free neutrons are observed to decay into protons and electrons. A conceivable model for a neutron would be an electron-proton bound state. If the neutron has an estimated radius of 1×10^{-15} m, evaluate the approximate kinetic energy T (in MeV) for a bound electron in such a model. Discuss your opinion of this model.

27.10 For small time intervals the rest energy of a particle may be indeterminate because of the uncertainty principle. For example, inside the atomic nucleus a proton may emit a (virtual) π^0-meson ($m_{\pi^0} = 270\, m_e$) which is then reabsorbed within a short time. Estimate the size R of a nucleus by considering the distance a π^0 may travel before it is reabsorbed (within the nucleus). Make sure your arguments are self-consistent.

27.11 Section 32-3 of Vol. I discusses how an excited atom would radiate away its energy at a certain rate, which has the effect both of limiting the "lifetime" of an excited state and of introducing a finite width to the corresponding spectral line. Show that these effects, interpreted as uncertainties in the energy and the time of measurement of a photon (or of momentum and position) are consistent with the uncertainty principle.

27.12
(a) Check by your own dimensional analysis the "Bohr radius" of the hydrogen atom.

(b) Show by the uncertainty principle that the energy needed to remove an electron from its associated proton in hydrogen is on the order of a few electron volts.

27.13
(a) An x-ray of wavelength 500 Å is absorbed by a hydrogen atom in its ground state. Find the kinetic energy T of the ejected electron in electron volts.

(b) What is the minimum frequency f_{min} of x-ray radiation that will ionize unexcited hydrogen?

27.14 An orchardist found it was easy to set two trees in line, but harder to set three. However, by practice and careful surveying, he set out 64 small trees on a square E-W, N-S grid, 8 trees to a row, and 8 rows, with a $6.0\,\text{m} \times 6.0\,\text{m}$ basic square. Standing at one corner of his orchard, he observed 3 lines of 8 trees each, counting the tree in the corner where he stood, 2 lines of 4 each, and 4 lines of 3 each.

(a) What was the least angle θ_{min} between two adjacent lines of these nine lines?

(b) What was the greatest distance d_{max} between two successive trees in any one of these lines?

(c) In an "infinite orchard" set out on this basic grid, each of these lines would appear from the air as one of a set of parallel lines well populated with trees. The distance between adjacent lines of any set could be considered as its "grating space." Find the grating space d for successive sets from the south front of the orchard to the 45° line.

27.15 The sodium and chlorine atoms of an NaCl crystal are alternately spaced at the corners of a cubic lattice whose Na-to-Cl closest spacing is $d = 2.82$ Å.

(a) Find the largest five interplanar spacings d for the NaCl crystal.

(b) Find at what angles θ first-order Bragg reflections should occur for these planes, if x-rays of wavelength 1.50 Å were used.

27.16 In the ultraviolet spectrum of hydrogen one observes a series of lines known as the Lyman series. The wavelengths of the three longest-wavelength lines of this series are 1216 Å, 1026 Å, and 973 Å. Compute the wavelengths λ of three other possible lines in the spectrum of hydrogen that could be "predicted" on the basis of this information alone, together with the Ritz combination principle.

Note: two of these lines are in the visible region (Balmer series), and one is a line in the infrared (the first line of the Paschen series).

27.17 A photon of wavelength $\lambda = 3$ Å (x-ray) is scattered at an angle of 90° by a free electron, which was originally at rest, as shown in Fig. 27-1. What is the kinetic energy T of the recoil electron?

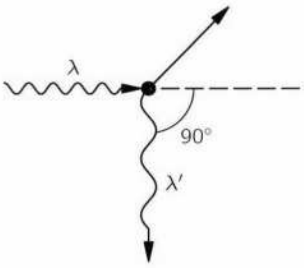


Figure 27-1

27.18 A "monochromatic" neutron beam of "thermal" $(1/40)$ eV neutrons impinges on a crystal with lattice spacing $d = 1.2$ Å between planes parallel to the surface.

(a) At what angles θ does Bragg reflection occur?

(b) Below what critical energy E_c will diffraction no longer take place?

27.19 Consider electron diffraction by a crystal of Bragg plane spacing $d = 1.2$ Å. The diffraction setup allows you to use electron beams of energy

(I) $E_k = 10.0$ keV,

(II) $E_k = 0.5$ MeV.

(a) At what angle θ does the 1st order diffraction maximum occur in each case?

(b) If you were to use this setup for a very accurate determination of the crystal spacing d, which beam would you use and why?

27.20 A certain atom returns from an excited state E_1 to the ground state E_0 by the emission of a photon. The atom emits this photon over a typical time of 10^{-8} s.

(a) If the emitted photon has a wavelength of 5000 Å, what is the half-width $\Delta\lambda$ of the wave train representing this photon?

(b) The wavelength of the photon is to be determined by means of a perfect diffraction grating placed perpendicular to the direction of propagation. If the grating spacing is such that the first order maximum is at 45°, how wide w (in meters) must the grating be to permit a measurement of the half-width $\Delta\lambda$ found in part (a) above?

(c) What is the energy difference $(E_1 - E_0)$ between the excited state and the ground state of the atom, in eV?

27.21 In a simple, non-relativistic model of the hydrogen atom, it is assumed that:

(1) The force between the electron and the proton is given by

$$\boldsymbol{F} = -\frac{e^2}{4\pi\epsilon_0 r^3}\,\boldsymbol{r}.$$

where $\boldsymbol{r}$ is the radius vector between them.

(2) The electron moves in a circular orbit about the proton such that the product of the electron's momentum and the radius of the orbit (angular momentum) is given by

$$pr = n\hbar,$$

where $n = 1, 2, 3, \ldots$, and $\hbar =$ Planck's constant$/2\pi$.

(a) Compute the radius r_n of the n^{th} orbit and the angular velocity ω_n of the electron in the n^{th} orbit.

(b) What are the electron's kinetic energy T_n, potential energy U_n and total energy E_n in the the n^{th} orbit?

(c) An atomic electron can lose energy by making a quantum jump from a higher orbit n_i to a lower orbit n_f while emitting a photon. What does this model imply about the energies ΔE that a photon emitted by a hydrogen atom can have?

28

Kinetic Theory of Gases

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 39.

28.1 How should the pressure P of a gas vary with N, the number of atoms per unit volume, and $\langle v \rangle$, the average speed of an atom? (Should P be proportional to N and/or to $\langle v \rangle$, or should it vary more, or less, rapidly than linearly?)

28.2 If an ideal gas is compressed adiabatically, then $PV^\gamma = \text{constant}$. On the other hand, under all conditions, $PV/T = \text{constant}$. Combine these to deduce the relationships during an adiabatic compression,

(a) between P and T,

(b) between V and T.

28.3 (a) Two samples of gas, A and B, of the same initial volume V_0, and at the same initial absolute pressure P_0, are suddenly compressed adiabatically, each to one half its initial volume. How does the final pressure of each sample (P_A, P_B) compare with its initial pressure, if γ_A is 5/3 (monatomic) and γ_B is 7/5 (diatomic)?

(b) Find the ratio of work W_A/W_B required to perform the two compressions described.

28.4 Two containers of volume $V_1 = V_2 = V$ are connected by a small tube with a valve, as shown in Fig. 28-1. Initially, the valve is closed and the two volumes contain monatomic gas at pressures P_1 and P_2 and temperatures T_1 and T_2, respectively. After the valve is opened, what will be the final pressure P_f and temperature T_f inside the joint volume? (Neglect heat lost from the system.)

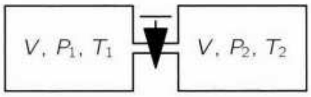


Figure 28-1

28.5 (a) Imagine a tall vertical column of gaseous or liquid fluid whose density varies with height. Show that the pressure as a function of height follows the differential equation $dP/dh = -\rho(h)g$.

(b) Solve this differential equation for the case of an ideal gaseous atmosphere of molecular weight μ, in which the temperature is constant as a function of h.

28.6 A cylinder with a leakless, frictionless piston contains $1.0\,\text{m}^3$ of a monatomic gas ($\gamma = 5/3$) at gauge pressure 1 atmosphere ($1.01 \times 10^5\,\text{N}\,\text{m}^{-2}$). The gas is slowly compressed at constant temperature until the final volume is only $0.4\,\text{m}^3$. How much work W must be done to accomplish this compression?

28.7 A bicycle pump is being used to inflate a tire to a pressure of $50\,\text{lb}\,\text{in}^{-2}$ gauge pressure, starting with air at atmospheric pressure, $14.7\,\text{lb}\,\text{in}^{-2}$ at 20 °C (293 K). If $\gamma = 1.40$ for air, at what temperature T is the air as it leaves the pump? Neglect heat losses to the walls of the pump.

28.8 A 50 liter tank is connected to a 15 liter tank through a short tube containing a pressure release valve that will only allow gas to pass from the larger tank to the smaller tank if the pressure in the larger tank exceeds the pressure in the smaller tank by 88 cm of Hg. If at 17 °C the larger tank contains gas at atmospheric pressure and the smaller tank is evacuated, what will be the pressure P in the smaller tank when both tanks are at 162 °C?

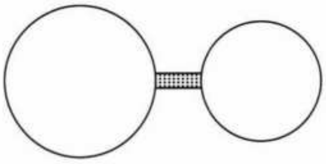


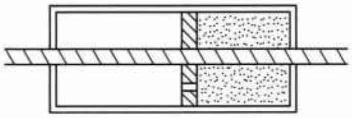
Figure 28-2

28.9 Two bulbs of volumes $200\,\text{cm}^3$ and $100\,\text{cm}^3$, as shown in Fig. 28-2, are connected by a short tube containing an insulating porous plug that permits equalization of pressure but not of temperature between the bulbs. The system is sealed at $27\,°\text{C}$ when it contains oxygen under a pressure of 760 mmHg. The small bulb is immersed in an ice bath at $0\,°\text{C}$ and the large bulb is placed in a steam bath at $100\,°\text{C}$. What is the final pressure P inside the system? Neglect thermal expansion of the bulbs.

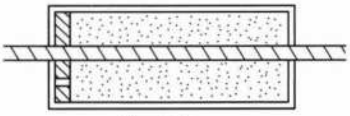
28.10 A mole of ideal monatomic gas in a non-insulated container with a movable piston is originally at P_1, V_1, and $T_1 = 27\,°\text{C}$. Then the gas is slowly heated using a total of 8.31 watt-hours of energy and at the same time allowed to expand at constant pressure to a new state P_2, V_2, and T_2. From evaluation of the new energy content of the gas and the work done by the gas during expansion, find

(a) T_2,

(b) V_2/V_1.



Initial State



Final State

Figure 28-3

28.11 Helium gas is contained in one half of each of two identical containers, and the other half of each container is evacuated. The two halves of each container are separated by a piston which has a small stopcock through it. (See Fig. 28-3). Two experiments are now performed:

(a) The stopcock is opened in one piston and the gas is allowed to flow through to the other side until equilibrium is reached. The piston is then very slowly moved to one end of the container.

(b) The piston of the other container is very slowly allowed to move into the evacuated end of the container, and then the stopcock is opened.

Compare quantitatively the final state (T_f, P_f) of the gas in the two containers, assuming the helium was initially at temperature T_0, pressure P_0.

Note: Use $\gamma = 5/3$ for helium. Ignore heat loss through walls, and friction.

28.12 An atmosphere in which the pressure and density as a function of height satisfy the relation $P\rho^{-\gamma} = \text{constant}$ is called an adiabatic atmosphere.

(a) Show that the temperature of such an atmosphere decreases linearly with height, and find the constant of proportionality. This temperature gradient dT/dh is called the *adiabatic lapse rate.*

(b) Find the adiabatic lapse rate of the earth's atmosphere.

(c) Use an argument based on energy considerations to show that an atmosphere having less or more temperature gradient than the adiabatic lapse rate will be stable or unstable against convection, respectively.

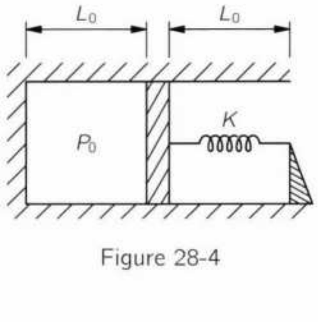


Figure 28-4

28.13 A cylinder, filled with argon, is equipped with a spring loaded piston of mass m and area A. At equilibrium the argon is at total pressure P_0, the piston is at distance L_0 from either end of the system (as shown in Fig. 28-4), and the spring (constant K) is compressed by x_0 (its free length being $L_0 + x_0$). Find the angular frequency ω of small oscillations ($x \ll L_0$) of the piston if the gas compresses isothermally.

Hint: You may use $1/(1+x) \approx 1 - x$ for $x \ll 1$.

28.14 At ordinary temperature nitrogen tetroxide is partially dissociated into nitrogen dioxide as follows:

$$\text{N}_2\text{O}_4 \leftrightarrow 2\text{NO}_2.$$

Into an evacuated flask of $250\,\text{cm}^3$ volume, 0.90 g of liquid N_2O_4 at $0\,°\text{C}$ is introduced. When the temperature in the flask has risen to $27\,°\text{C}$ the liquid has all vaporized and the pressure is 960 mmHg. What percentage f_d of the nitrogen tetroxide has dissociated?

29

Principles of Statistical Mechanics

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 39 and 40.

29.1 What is the root-mean-square velocity $v_{\rm rms}$ of an object of mass m in a gas of temperature T?

29.2 Sketch the Maxwellian distributions $f(v_x)$ vs. v_x, and $f(v)$ vs. v, for two different temperatures T_1 and T_2, with $T_2 > T_1$.

29.3 Consider the molecules of a certain diatomic gas to be rigid dumbbells, free to rotate and move around, but not to vibrate. What is the (classical) specific heat C_V at constant volume?

29.4 The molar heat capacity at constant volume of a substance $C_{V,m}$ is the amount of energy needed to raise the temperature of one mole of the substance by one degree, while holding the volume constant. What is the molar heat capacity at a constant volume of

(a) an ideal monatomic gas?

(b) an ideal diatomic gas?

29.5 In the atmospheric pressure law

$$n = n_0 e^{-mgh/kT},$$

$kT/mg = RT/\mu g = h_0$ is called the *scale height*, where μ is the molecular mass. Evaluate the scale height for the earth's atmosphere and the sun's atmosphere, given $\mu_{\text{♁}} = 29\,\mathrm{g\,mol^{-1}}$, $T_{\text{♁}} = 300\,\mathrm{K}$, $\mu_{\text{☼}} = 1.5\,\mathrm{g\,mol^{-1}}$, $T_{\text{☼}} = 5500\,\mathrm{K}$, $g_{\text{☼}} = 2.7\times10^2\,\mathrm{m\,s^{-2}}$.

29.6 Consider a long, closed cylinder of length L, cross section A, standing upright in the earth's gravitational field. It is filled with an isothermal, homogeneous, diatomic gas of N molecules, each having mass m. What is the pressure at the two ends of the gas tube?

Note: Obviously, $P(h=0) - P(h=L) = Nmg/A$.

29.7 An isothermal atmosphere in a constant gravity field confined in an infinitely tall column of uniform cross-sectional area A contains equal numbers N of two types of molecules, one of mass m_1 and one of mass m_2 $(m_2 > m_1)$.

(a) Find the fraction $f_1(h)$ of molecules that have mass m_1 as a function of height above the ground $(h = 0)$.

(b) What are the maximum and minimum values of $f_1(h)$ and at what values of h do they occur?

(c) If you are at a distance above the ground where about 63.2 percent $(1 - 1/e)$ of the molecules of m_2 are below you, what fraction f of the molecules m_1 are below you at the same elevation?

29.8 The Maxwellian distribution law is of the general form $dN/dv = Av^2e^{-bv^2}$. This may be transformed to $y = x^2e^{-x^2}$.

(a) Graph this equation for $0 \le x \le 3.0$ to show how an increasing $y = x^2$ curve is suppressed by the exponential.

(b) Find its maximum ordinate.

(c) See how closely the area under your curve comes to

$$\int_0^\infty x^2 e^{-x^2}\, dx.$$

29.9 According to the Maxwell-Boltzmann distribution law, the fraction of molecules in a given volume having speed v in the range dv is given by

$$f(v)\, dv = Av^2 e^{-mv^2/2kT}\, dv.$$

From this distribution, calculate the fraction $F(E)\, dE$ having kinetic energy E in the range dE.

29.10 The speed distribution function for a group of N particles is given by

$$\begin{aligned} f(v)\, dv &= kv\, dv \quad (0 < v < V) \\ f(v)\, dv &= 0 \qquad\qquad (v > V) \end{aligned}$$

(a) Find k.

(b) Find the average speed $\langle v \rangle$ and the root-mean-square speed v_{rms}.

29.11 In a gas at thermal equilibrium, what fraction f of the molecules striking a surface have kinetic energies greater than

(a) average?

(b) three times average?

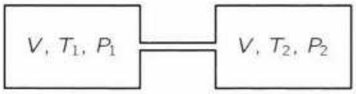


Figure 29-1

29.12 Two containers of equal volume ($V_1 = V_2 = V$) are connected by a small "pinhole" tube, as shown in Fig. 29-1. The containers are kept at constant temperatures T_1 and T_2, respectively. All gas molecules have the same mass m. Find $P_1/P_2 = f(T_1, T_2)$, the ratio of the pressures inside the containers as function of T_1 and T_2 only.

29.13 A furnace at temperature T contains n atoms of mass m per unit volume, some of which escape through a small hole of area A. (The hole is small enough so that equilibrium inside the oven is not disturbed).

(a) What is the number dn of atoms leaving through the area A per second, having speeds between v and $v + dv$?

(b) What is the root mean square velocity $v_{\text{rms}}^{(e)}$ of the escaping atoms? (If different from the root mean square velocity inside the furnace, $v_{\text{rms}}^{(i)}$, explain why.)

29.14 Na^{23} vapor in a gas discharge tube emits a strong yellow line at $\lambda = 5890\,\text{Å}$. If the vapor is at room temperature, estimate roughly how wide, $\Delta\lambda$, this line will appear, due to Doppler shifts caused by thermal motion. (Useful information: for Na^{23}, $mc^2 \approx 23 \times 10^9\,\text{eV}$.)

29.15 The observability of an absorption spectrum series depends on the number of atoms in the lowest energy state for the series. If the lowest energy state for the Lyman series of hydrogen is $-13.6\,\text{eV}$, and the lowest energy state for the Balmer series is $-3.4\,\text{eV}$, at what temperature T of the gas will the number of atoms available to produce a Balmer absorption spectrum equal $1/e$ times the number of atoms available to produce Lyman absorption? This is the basis for one way to estimate the temperature of a star.

29.16 The ground state of the hydrogen atom has four substates which have the same, or very nearly the same, energy. Similarly, the first excited state, 10.2 eV higher, has 16 such substates. What is the ratio N_0/N_1 of the numbers of atoms in these two states at the surface of the sun, where $T = 5700\,\mathrm{K}$?

29.17 In a radiometer, the molecules of a gas at low pressure bombard a set of thin, light vanes which are black on one side and shiny on the other. When radiation strikes these vanes, the absorbed energy is carried away principally by the molecules striking the blackened side of each vane, and the vanes turn as a result of this unbalanced force.* Consider a vessel in which there are N molecules of mass m per unit volume, at an absolute temperature T. A thin vane of unit area inside the vessel is absorbing radiant energy at a rate of 11 watts, and this energy is being carried off (isotropically) by the molecules striking one side of the vane. Estimate roughly the unbalanced force F on the vane for air at room temperature.

29.18 (a) Air at NTP is flowing at speed v through a smooth pipe of constant cross-sectional area A. As the air passes a wire grid that offers negligible resistance to the flow it is heated; the energy input is W watts. It ultimately emerges from the tube at a speed v'. Write equations for conservation of mass, energy, and momentum as the air traverses the tube, and thus find:

(1) v',

(2) the final temperature T',

(3) the thrust F.†

(b) Approximate the thrust F and discuss the performance of an aircraft jet engine in light of part (a) above, if the engine consumes 100 kg of air and 2.00 kg of kerosene per second. The heat of combustion of kerosene is about $4.65 \times 10^7\,\mathrm{J\,kg^{-1}}$. What complications might invalidate your result?

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* The blackened side is rougher than the shiny side, which causes the molecules striking the rough side to become trapped for a short time before rebounding, while the molecules striking the shiny side rebound at once. The trapped molecules therefore tend to come to thermal equilibrium with the vane, while those bouncing directly away do not.

† This is basically a jet engine. *Hint:* For simplicity, you may consider a "low-efficiency" engine with equal intake and exhaust pressures.

30

Applications of Kinetic Theory: Equipartition

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 41 and 42.

30.1 Calculate the following:

(a) the temperature T for which kT is equal to 1 electron volt,

(b) the value of kT in electron volts for room temperature,

(c) the wavelength λ of a photon corresponding to a quantum jump of 1 eV.

30.2 Activation energies, heats of vaporization, heats of formation or dissociation, etc., are commonly expressed in Joules per mole or in electron volts per atom. How many Joules per mole is 1 eV/atom?

30.3 Over the temperature range 0–300 °C the heat of vaporization of mercury changes by only 3 percent, and is about 0.61 eV/atom on the average. How much error will one make in calculating the vapor density of mercury at 0 °C, if the heat of vaporization at 300 °C is used instead of the correct 0 °C value?

Moral: A small percentage error in a large exponent can have a large effect.

30.4 For two black-body sources, at temperatures $T_1 = 2000\,\mathrm{K}$ and $T_2 = 4000\,\mathrm{K}$, find the relative intensity I_1/I_2 of light with wavelength $\lambda = 0.31$ microns.

30.5 (a) Plot the vapor density of mercury vs. inverse temperature $1/T$ on semi-log paper (data are available in the Handbook of Chemistry and Physics), and from this plot deduce the heat of vaporization of mercury. Check your results against the tabulated value.

(b) Do the same for water.

Note: Chemists use an energy unit called a *kilocalorie*. 1 kcal = 4186 J.

30.6 The distribution law for black-body radiation is:

$$I(\omega) = \frac{\hbar\omega^3 d\omega}{\pi^2 c^2 \left(e^{\hbar\omega/kT} - 1\right)}$$

By changing the variable from ω to $z = \hbar\omega/kT$, show that:

(a) The total radiation intensity, integrated over all frequencies, is proportional to the fourth power of the absolute temperature.

(b) The frequency ω_m at which $I(\omega)$ has its maximum value is proportional to the absolute temperature.

30.7 Consider a quantized oscillator (e.g., an atom) which can have the following four energy states:

$$E$$
$$E + \Delta E - \epsilon$$
$$E + \Delta E$$
$$E + \Delta E + \epsilon$$

(For simplicity, you may consider ϵ negligible, saying that the oscillator has one energy state E and three energy states $E + \Delta E$.)

(a) If you have N_0 such oscillators, what is the specific heat C_V of the system at temperature T?

(b) What is C_V for $T \to \infty$? Give a physical explanation.

31

Applications of Kinetic Theory: Transport Phenomena

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 43.

31.1 The transfer of what molecular physical quantity gives rise to

(a) heat conduction?

(b) viscosity?

31.2 The cross section for collision of slow neutrons in hydrogen is about $20 \times 10^{-24}\,\text{cm}^2$. What is the mean free path l of such neutrons in hydrogen at NTP?

31.3 The "diameter" of an oxygen molecule is roughly $3\,\text{Å}$. Estimate the mean free path l and the mean time between collisions τ for oxygen gas at NTP.

31.4 A "Dewar" (e.g., Thermos Bottle), as shown in Fig. 31-1, is a double-walled evacuated chamber used for thermal insulation. Assume the requirement for good insulation in a Dewar with wall separation D is that $l > 10D$, where l is the mean free path of the residual gas in the Dewar. If $D = 1\,\text{cm}$ and the residual gas is oxygen,

(a) what pressure P would be allowable in the Dewar at room temperature?

(b) What is the ratio of the thermal conductivities κ_1/κ_2 of the Dewar gas for the two pressures $P_1 = 400\,\text{mmHg}$ and $P_2 = 200\,\text{mmHg}$?

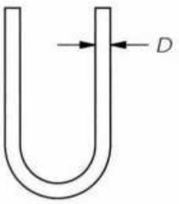

Figure 31-1

31.5 Two gases, A and B, are at density ρ_A and ρ_B at a certain temperature T_0. A particular ion is observed to have a mobility μ_a in gas A and μ_b in gas B. What mobility μ would you expect the ion to have in a mixture of these gases, at density $\rho_A + \rho_B$ and temperature T_0?

31.6 When a temperature gradient exists in a material, an energy flow proportional to the temperature gradient results. (Ignore convection.) The coefficient of proportionality, reduced to a unit area and unit temperature gradient, is called the thermal conductivity, κ. Thus $dE/dt = \kappa A\, dT/dx$. Show that, in the absence of convection, the thermal conductivity of a gas is

$$\kappa = \frac{kn_0vl}{\gamma - 1} = \frac{1}{\gamma - 1}\frac{kv}{\sigma}.$$

where n_0 is the number of gas molecules per unit volume, v is their average velocity, l is their mean free path, σ is their cross-section for collision, and $(\gamma - 1)kT$ is the average energy of a molecule at temperature T.

Hint: Interpret thermal conductivity as a transport of internal (heat) energy U across a plane, from one mean free path on either side.

31.7 When a velocity gradient exists in a fluid, such that the velocity changes with distance at right angles to the flow direction, a drag results, called *viscosity*. In a gas, this is due to the transport of momentum across a plane, from roughly one mean free path on either side. If the flow is in the x-direction and there is a gradient of v_x in the y-direction, then the drag force per unit area on a plane perpendicular to y is

$$F/A = \eta\, dv_x/dy.$$

Show that, for a gas the coefficient of viscosity η is approximately,

$$\eta = n_0 vml = vm/\sigma,$$

where n_0 is the number of gas molecules per unit volume, v is their average velocity, m is their mass, l is their mean free path, and σ is their cross-section for collision.

31.8 Notice that the thermal conductivity and the viscosity of a gas are both independent of the pressure.

(a) Devise a suitable modification of the formulas for the energy transfer dE/dt between two surfaces whose temperature are T and $T + \Delta T$, situated a distance D apart, if (mean free path) $l \gg D$. (See Ex. 31.6.)

(b) Do the same for the momentum transfer (drag force per unit area) F/A between two such surfaces, moving at speeds U and $U + \Delta U$. (See Ex. 31.7.)

31.9 A certain vessel contains 10^{24} molecules of a gas for which the mean free path is l. For approximately what path length L will the probability be 1/2 that no single molecule in the container goes farther than L before it suffers its next collision?

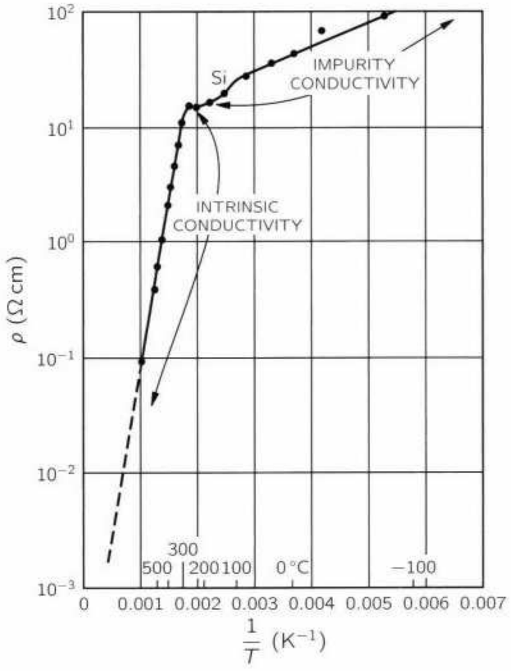

Figure 31-2

31.10 The resistivity of nearly pure silicon as a function of temperature is shown in Fig. 31-2. Make a quantitative deduction concerning the nature of the current flow in this substance above and below 300 °C.

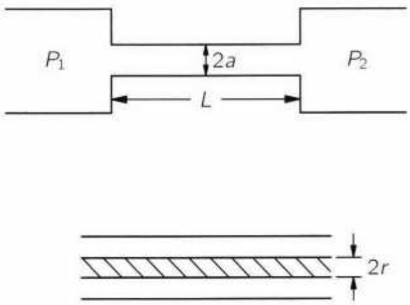

Figure 31-3

31.11 In "laminar" flow the viscosity of a liquid alone determines the speed with which it can be transported through a pipe between two reservoirs at different pressure ($P_1 > P_2$). In Fig. 31-3, the liquid flows through a cylindrical pipe of radius a, length L. ($L \gg l =$ mean free path). The liquid has coefficient of viscosity η. Show that in the steady state the amount of liquid flowing per second from reservoir 1 to reservoir 2 is given by

$$V = \frac{\pi a^4}{8\eta}\,\frac{P_1 - P_2}{L}.$$

Hint: the forces acting on a "thread" of liquid must sum to zero. This gives $v = v(r)$. (See figure.)

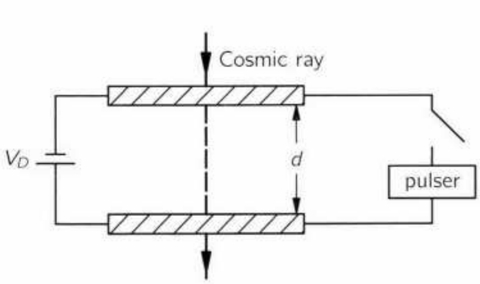

Figure 31-4

31.12 Fig. 31-4 shows a single gap spark chamber with gap spacing d between two aluminum plates. A charged cosmic ray particle (singly) ionizes the neon gas molecules along its path. At time t later, a high voltage pulse is applied between the plates and a spark is observed along the particle path. In order to keep the chamber free of spurious electrons and ions which exist from earlier cosmic rays, a DC voltage V_D is applied between the plates. This "clearing voltage" also sweeps out the electrons and ions from the track of interest.

(a) At what voltage, V_D, will the space be completely cleared

(1) of electrons,

(2) of neon ions,

when the pulse occurs?

(b) A spark chamber with spacing $d = 0.63\,\text{cm}$ is observed to give no tracks for an applied V_D of 80 V when $t = 0.4 \times 10^{-6}\,\text{s}$. Are the electrons essential to the formation of a spark? (Justify your answer.)

Useful information: Geometric cross section of neon $\approx 4 \times 10^{-16}\,\text{cm}^2$, density of neon $\approx 3 \times 10^{19}$ molecules/cm^3, $m_e c^2 \approx 5 \times 10^5\,\text{eV}$, $m_{\text{neon}} c^2 \approx 2 \times 10^{10}\,\text{eV}$, $kT \approx (1/40)\,\text{eV}$ at room temperature.

32

Thermodynamics

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 44.

32.1 An ideal reversible Carnot engine is run backwards to serve as a refrigerator. If it takes in heat at 250, K and ejects it at 350 K to the outside, for every Joule of heat removed from the box, how many joules, E, are ejected to the outside?

32.2 If a refrigerator is to be maintained at $-3\,^\circ$C, and the outside air is at $27\,^\circ$C, what minimum amount of work W_{min} has to be done to remove a joule of heat from inside the refrigerator?

32.3 Two reversible engines operate on Carnot cycles between the same minimum and maximum volume, maximum and minimum pressure, maximum and minimum temperature. One engine uses helium as the working substance, the other uses air. Which engine delivers more work per cycle?

32.4 In a modern steam power plant using superheated steam, the temperature in the steam generator is $600\,^\circ$C. The intake river water used to cool the condenser is at $20\,^\circ$C. What maximum efficiency ϵ_{max} could such a plant have?

32.5 An insulated container with a movable, frictionless piston of mass M and area A, contains N grams of helium gas in a volume V_1, as shown in Fig. 32-1. The external pressure is P. The gas is *very slowly* heated by an internal heating coil until the volume occupied by the gas is $2V_1$. What are

(a) the work W done by the gas,

(b) the heat ΔQ supplied to the gas?

(c) the change ΔU in the internal energy of the gas,

(d) the initial temperature T_i and the final temperature T_f of the gas.

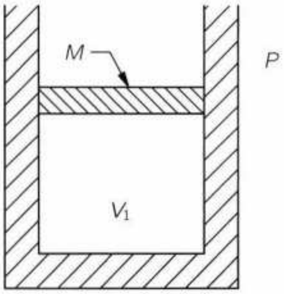

Figure 32-1

32.6 An ideal gas with coefficient γ, is initially at the condition $P_0 = 1\,\text{atm}$, $V_0 = 1$ liter, $T_0 = 300\,\text{K}$. It is then:

1. Heated at constant V until $P = 2\,\text{atm}$.
2. Expanded at constant P until $V = 2$ liters.
3. Cooled at constant V until $P = 1\,\text{atm}$.
4. Contracted at constant P until $V = 1$ liter.

(a) Draw a P-V diagram for this process.

(b) What work W is done per cycle?

(c) What is the maximum temperature T_{max} the gas attains?

(d) What is the total heat input ΔQ in steps 1 and 2?

(e) What is the combined change ΔS in entropy during steps 1 and 2?

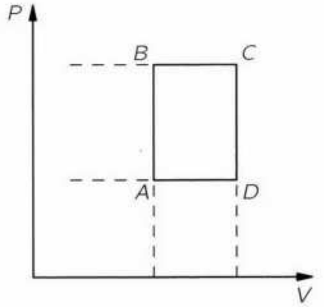

Figure 32-2

32.7 An ideal engine operates with a perfect gas having coefficient $\gamma = 4/3$, in the cycle shown in Fig. 32-2 ($A \to B \to C \to D$). At A, $P = 1\,\text{atm}$, $V = 22.4$ liters, $T = 300\,\text{K}$. At C, $P = 2\,\text{atm}$, $V = 33.6$ liters, $T = 900\,\text{K}$.

(a) How much work W is done per cycle?

(b) What is the temperature T_B at B?

(c) How much heat input $\Delta Q_{A\to B}$ is required for $A \to B$?

(d) How much heat input $\Delta Q_{B\to C}$ is required for $B \to C$?

(e) If the engine works from heat reservoirs at 900 °C and 300 °C only, what is the maximum efficiency ϵ_{max} with which the engine may be operated using the given cycle?

(f) What is the maximum efficiency for any engine working between these temperatures?

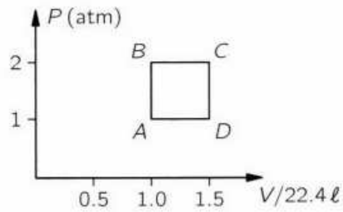

Figure 32-3

32.8 A sample of ideal gas with $\gamma = 4/3$ is taken successively from condition A (1 atm pressure, 22.4 liter volume, at 300 K) to condition C (2 atm, 33.6 liter, 900 K) by two routes, ABC and ADC, as shown in Fig. 32-3.

(a) Show that the change ΔS in entropy is the same by both paths.

(b) Compute ΔS.

32.9 In an ideal reversible engine employing 28 g nitrogen as working substance ($\gamma = 7/5$) in a cyclic operation *abcd* without valves, the temperature of the source is 400 K, of the sink 300 K. The initial volume of gas at point a is 6.0 liters and the volume at point c is 18.0 liters.

(a) At what volume V_b should the cylinder be changed from heat input (isothermal expansion) to isolation and adiabatic expansion (from V_b to V_c)?

(b) At what volume V_d should the adiabatic compression begin?

(c) How much heat $\Delta Q_{a\to b}$ is put in on the $V_a \to V_b$ part of the cycle?

(d) How much heat $\Delta Q_{c\to d}$ is extracted during the $V_c \to V_d$ part?

(e) What is the efficiency ϵ of the engine?

(f) What change ΔS in entropy per gram occurs in the working substance during $a \to b$ and $c \to d$?

Hint: You should find that in a Carnot cycle for an ideal gas the expansion ratios V_b/V_a and V_c/V_d are equal.

32.10 One mole of gas in a container is initially at a temperature 127 °C. It is suddenly expanded to twice its initial volume without heat exchange with the outside. Then it is slowly compressed, holding the temperature constant, to its original volume. The final temperature is found to be −3 °C.

(a) What is the coefficient γ of the gas?

(b) What change ΔS in entropy, if any, has occurred?

32.11 A gas of coefficient γ in a cylinder of volume V_0 at temperature T_0 and pressure P_0 is compressed slowly and adiabatically to volume $V_0/2$. After being allowed to come to temperature equilibrium (T_0) at this volume, the gas is then allowed to expand slowly and isothermally to its original volume V_0. In terms of P_0, V_0, T_0, what is the net amount of work W the piston does on the gas?

32.12 Translate the ideal Carnot cycle *abcd* on a P-V diagram between T_1 and T_2 and (P_a, V_a), (P_c, Vc) into a T-S (temperature-entropy) diagram with corresponding points $ABCD$.

32.13 The first earth settlers on the moon will have great problems in keeping their living quarters at a comfortable temperature. Consider the use of Carnot engines for climate control. Assume that the temperature during the moon-day is 100 °C, and during the moon-night is −100 °C. The temperature of the living quarters is to be kept at 20 °C. The heat conduction rate through the walls of the living quarters is 0.5 kW per degree of temperature difference. Find the power P_{day} which has to be supplied to the Carnot engine during the day, and the the power P_{night} which must be supplied at night.

32.14 Two "identical" bodies of constant heat capacity C_p, originally at temperatures T_1 and T_2 are used as reservoirs for a Carnot engine operating in infinitesimal reversible cycles, as shown in Fig. 32-4. If the bodies remain at constant pressure and undergo no phase changes,

(a) show that after the engine comes to rest, the final temperature $T_f = \sqrt{T_1 T_2}$,

(b) find the total work W done by the engine.

Hint: Recall that ΔQ is related to ΔT and consider what happens to the entropy in a reversible cycle.

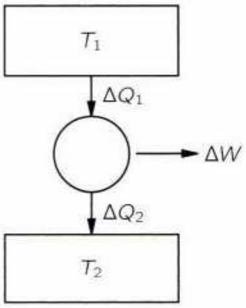

Figure 32-4

32.15 A careless experimenter left the valve of a tank of helium slightly open over the weekend. The gas, originally at 200 atm slowly escaped isothermally at 20 °C. What change ΔS in entropy per kg of gas occurred?

32.16 The ideal gas turbine engine cycle consists of adiabatic compression from A to B, addition of heat at constant pressure from B to C, adiabatic expansion from C to D, and rejection of heat at constant pressure from D to A, all carried out in a reversible manner. (See Fig. 32-5.) Assume that the working fluid in the engine is an ideal gas with ratio of specific heats γ, and that $\dot{N}$ moles pass through the engine per unit time. Assume further that the maximum gas temperature T_C is fixed by the maximum temperature that the hot parts of the engine can withstand and that the minimum gas temperature, T_A, is fixed by the ambient air temperature.

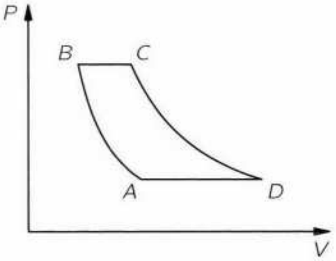

Figure 32-5

(a) Find the power $\dot{W}$ produced by the engine as a function of the pressure ratio $p = P_B/P_A$.

(b) At what pressure ratio p_m is the power a maximum?

(c) What is the maximum power $\dot{W}_m$?

(d) What is the efficiency at maximum power ϵ_m?

(e) What is the volume flow rate $\dot{V}_A$ of air ($\gamma = 7/5$) at atmospheric conditions ($P_A = 1.013 \times 10^5$ Pa, $T_A = 300$ K) in such an engine, if it produces 1 megawatt of power and the maximum temperature is $T_C = 1200$ K?

(f) What is the efficiency ϵ_e of the engine in part (e) above?

(g) What is the pressure ratio p_e of the engine in part (e)?

33

Illustrations of Thermodynamics

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 45 and 46.

33.1 How does the total radiation intensity in a black body cavity depend on temperature?

33.2 In a molecular gas, the number of molecules in a closed container is independent of temperature. Is the same true for a "photon gas"? Explain.

33.3 Calculate the specific heat C_V of a "photon gas" at constant volume V and temperature T.

33.4 A DC current of 10 A flows through a resistor of 10 Ω which, is kept at the constant temperature of 10 °C.

(a) What is the rate of entropy change dS_R/dt of the resistor (in $\mathrm{cal\,K^{-1}\,s^{-1}}$)?

(b) What contribution dS_U/dt is made to the entropy change of the universe?

33.5 For an ideal gas, whose internal energy depends only upon temperature T, show that the difference between the molar heat capacities at constant pressure and at constant volume is equal to the gas constant R: $C_{P,m} - C_{V,m} = R$.

33.6 The latent heat of vaporization of water is about $2.44 \times 10^6\,\mathrm{J\,kg^{-1}}$, and the vapor density at 100 °C is $0.598\,\mathrm{kg\,m^{-3}}$. Use the Clausius-Clapeyron Equation to find the rate of change of the boiling temperature with altitude, dT/dz, near sea level in $\mathrm{^\circ C\,km^{-1}}$. Assume the temperature of the air is 300 K.

33.7 The top layers of the water in a lake are initially at 0 °C. A cold breeze starts to blow, keeping the surface of the lake at a temperature ΔT *below* 0 °C.

(a) Find the rate of increase of the thickness of the ice dz/dt, assuming that the heat loss required to lower the temperature of the ice which has already formed is much less than that required to form new ice. (Assume also that the temperature in the ice changes linearly with depth.)

(b) Using the result of part (a), calculate the thickness of the ice z one hour after freezing begins if $\Delta T = 10\,^\circ\mathrm{C}$.

Use: thermal conductivity of ice: $\kappa = 2.0\,\mathrm{W\,m^{-1}\,K^{-1}}$, latent heat of ice formation: $L = 3.3 \times 10^5\,\mathrm{J\,kg^{-1}}$.

33.8 Two volumes of gas are separated by a stopcock, which is opened at time $t = 0$. $V_2 = 2V_1$. (See Fig. 33-1.) What is the change in entropy ΔS of (a) the gas, (b) the surroundings, and (c) the universe, after a very long time for the two following cases:

(1) V_1 contains 1 mol of helium and V_2 contains 2 mol of Argon,

(2) V_1 contains 1 mol of helium and V_2 contains 2 mol of helium.

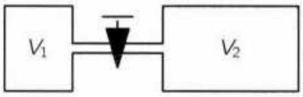

Figure 33-1

33.9 Calculate the change in entropy ΔS when radiation (photon gas) is:

(a) Expanded isothermally from volume V_1 to V_2.

(b) Heated at constant volume raising the temperature from T_1 to T_2.

(c) Use the results in parts a and b above to find the entropy $S(V, T)$ for a photon gas that is initially at volume V_0 and temperature T_0.

33.10 Sunlight beats down perpendicularly on a large black paved field in Equatorial Africa. If the surface radiates as a black body, what maximum temperature T does it attain? (Solar constant: $1395\,\mathrm{W\,m^{-2}}$).

33.11 The sun radiates approximately like a black body of temperature 5700 K. If a perfectly black copper sphere is irradiated by sunlight at a distance of one astronomical unit from the sun, what equilibrium temperature T will it attain? (The sun's diameter subtends an angle of 0.50° at the earth.)

33.12 Black body radiation in an isothermal cavity of volume V_0 is compressed at constant temperature until the volume is decreased to $V_0/2$. By what factor f is the radiation pressure changed?

33.13 A thin spherical shell of negligible heat capacity and radius R has a black interior surface and an exterior surface with absorption coefficient A for black body radiation. The shell floats in empty space and at time $t = 0$ contains a "photon gas" at temperature T_0. Find the temperature T for times $t \geq 0$.

33.14 (a) Consider a thin black spherical shell of radius R, containing an ideal monatomic gas at temperature T_0. If the sphere contracts adiabatically to radius $R/2$, by what factor f would the radiation intensity change as seen by a very distant observer if you neglect gravitational changes of the gas in the compression?

(b) If in part (a) above you included gravitational effects, would f be larger or smaller and why?

33.15 Consider a thin, black spherical shell (of negligible heat capacity) of radius R containing black body radiation at temperature T_0. If the sphere contracts adiabatically to radius $R/2$, by what factor f will the radiation intensity change as seen by a very distant observer?

33.16 A blackened solid copper cylinder 2.0 cm long and $10.0\,\mathrm{cm^2}$ in the base area is suspended by a thin insulating fiber at the center of an evacuated hollow spherical cavity 25.0 cm in radius, and having blackened walls. The walls of the cavity are maintained at a constant temperature of 27 °C.

(a) If the copper cylinder is in thermal equilibrium with the cavity, at what rate dE/dt is it radiating energy to the cavity walls? Give answer in $\mathrm{J\,s^{-1}}$.

(b) If the cylinder is heated to 150 °C and then allowed to cool, what is its rate of temperature decrease dT/dt as it passes through temperature $T =$ 127 °C? (Assume that at any instant all parts of the copper are at the same temperature.)

Use $\rho_{\mathrm{Cu}} = 8950\,\mathrm{kg\,m^{-3}}$, $C_P(\mathrm{Cu}) = 390\,\mathrm{J\,kg^{-1}\,K^{-1}}$.

33.17 The density at the center of the sun is about $80\,\mathrm{g\,cm^{-3}}$ and the central temperature is about $13 \times 10^6\,\mathrm{K}$. The matter is composed almost entirely of protons and electrons. Find the gas pressure P_G and the radiation pressure P_R at the center of the sun.

33.18 At 0 °C, the specific volume of saturated water vapor is $206\,\mathrm{m^3\,kg^{-1}}$. What is the latent heat of vaporization L in $\mathrm{J\,kg^{-1}}$ at this temperature? (Determine dp/dT from tables, calculate L, compare with tabular value.)

33.19 (a) Use a thermodynamic argument to show that if a substance expands when it freezes, its freezing temperature must decrease with increasing pressure.

(b) Estimate the lowest temperature of the ice on a skating rink for which ice skating would be possible.

33.20 If a certain object absorbs a fixed fraction A of all radiation incident upon its surface and reflects the rest, show that at temperature T it emits an amount $A\sigma T^4$ per unit area, where σ is the Stefan-Boltzmann constant for blackbody radiation derived in Section 45-3, Vol. I of *The Feynman Lectures on Physics*

$$\sigma = \frac{k^4\pi^2}{60\hbar^3c^3} = 5.67\times10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}.$$

33.21 A black body of radius r at temperature T is surrounded by a thin shell of radius R, black on both sides. Find by what factor f this radiation shield reduces the rate of cooling of the body. (Consider space between sphere evacuated, with no thermal conduction losses.)

33.22 A black spherical shell at a certain position in interplanetary space has equilibrium temperature T_0. If the same object were copper-plated would its new equilibrium temperature be higher, lower, or equal to T_0? Typical values for the reflectivity of copper as function of wavelength are:

λ (micron)	0.305	0.385	0.450	0.550	0.600	0.700	1.00	3.0	9.0
$R(\lambda)$	0.25	0.29	0.37	0.48	0.72	0.83	0.90	0.97	0.98

$$R(\lambda) = \frac{I_{\text{reflected}}}{I_{\text{incident}}}$$

Consider the sun a black body radiator of 5700 K. Also, see Ex. 33.20.

34

The Wave Equation, Sound

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 47.

34.1 The compressive elastic property of a liquid is measured by M, the bulk modulus, where M is defined by

$$dP = -M\,\frac{dV}{V}.$$

dV/V is the fractional change in volume brought about by a change in external pressure dP. By dimensional analysis, determine the speed v of sound propagation in a liquid as a function involving M.

34.2 Two identical ropes of small mass hanging from a support are stretched by weights of 1 kg and 2 kg, respectively. What is the ratio v_1/v_2 of the wave velocities for transverse waves along the two ropes?

34.3 Find the ratio of the speed of sound in helium v_{He} to that in hydrogen v_{H} at the same temperature.

34.4 Two whistles, A and B, of the same length are blown. A is blown with air cooled to liquid air temperature ($-180\,^\circ\mathrm{C}$), and B is blown with heated air. One whistle gives a pitch exactly one octave above the other (twice the frequency). What should be the temperature T of the air blowing whistle B?

34.5 Show that $u = Ae^{i(\omega t - kx)}$ satisfies the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\,\frac{\partial^2 u}{\partial t^2},$$

provided ω and k satisfy the relation $\omega = vk$.

34.6 A uniform, perfectly flexible string of density σ ($\mathrm{kg\,m^{-1}}$) is stretched with a tension T.

(a) Derive the wave equation governing the lateral displacement y.

(b) Deduce the speed v of propagation of disturbances along the string.

Assume that $\partial y/\partial x \ll 1$ at all points and times, and consider only vibration in a plane.

Note: the component of the string tension in the lateral direction is very nearly given by $T\,\partial y/\partial x$.

34.7 A crude musical instrument is constructed by stretching a wire of negligible mass under tension T between two points and firmly attaching a mass m to the wire at a distance x from one end, as shown in Fig. 34-1. The mass is displaced from equilibrium by a small distance A ($A \ll x$, $A \ll l - x$), and then allowed to vibrate.

(a) Find the frequency ν of the sound.

(b) Write the equation for the displacement of the mass from equilibrium as a function of time, $y(t)$.

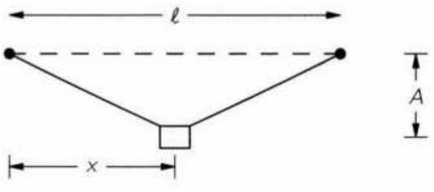

Figure 34-1

(c) As x is varied, what are the minimum and maximum frequencies available? Neglect the effects of gravity.

34.8 A sound wave passes through a gas of density ρ_0 and pressure P_0. The displacement of the gas molecules is described by the equation $\xi = \xi_m \cos(\omega t - kx)$.

(a) Find the equation for the pressure P in the gas as a function of x and t.

(b) Find the kinetic energy of motion T due to the sound wave in a volume of gas with area A in the yz-plane and of length λ along the x-axis.

34.9 A light, flexible diaphragm is located at a node of an organ pipe in which the sound level is 120 dB at a frequency of 100 Hz. The medium is air at NTP.

(a) What is the amplitude χ_m of the diaphragm's motion (in cm)?

(b) What is the amplitude of the temperature change ΔT in the gas?

34.10 If one inhales helium and speaks, the voice sounds unnatural and high in pitch.

(a) If all your resonant cavities ("the empty parts of your head") were filled with helium instead of air, by about what factor f would every resonant frequency be increased?

(b) If you were to sing a tune, what effect would the helium have on the key in which you sing? Discuss.

34.11 Pinch a single length of rubber band about 5 cm long between the fingernails of your two hands; twang it to observe the pitch; then stretch it 2×, 3×, 4×, 5× its original length without changing the mass of band between fingernails, twanging it as you proceed. Discuss the results observed. Why doesn't a violin string do the same thing if you increase its length?

34.12 Consider a steady plane sound wave of frequency 1000 Hz in which the pressure peaks are ±0.1 Pa from the prevailing atmospheric pressure of 1×10^5 Pa.

(a) What change in density ρ_e accompanies such a wave?

(b) What is the maximum particle displacement χ_m?

(c) What is the intensity I of the wave (in $\mathrm{W\,m^{-2}}$)?

Note: Take the velocity of sound as $340\,\mathrm{m\,s^{-1}}$.

35

Linear Wave Systems: Beats, Modes

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapters 46 and 49.

35.1 If you write the wave equation as

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

What are the phase velocity v_{ph} and group velocity v_g in terms of c?

35.2 If in a medium the phase velocity differs from the group velocity, we speak of dispersion. Do we find dispersion in

(i) an audible sound wave in air?

(ii) a stretched copper wire?

(iii) a water wave?

35.3 The phase velocity of a water wave of wavelength λ is, neglecting surface tension and the effects of finite depth,

$$v_{ph} = \sqrt{g\lambda/2\pi}.$$

(a) Show that the group velocity v_g is one half the phase velocity.

(b) What are the group and phase velocities of a wave of wavelength 1000 m?

35.4 Six gliders of mass m each, connected by identical springs, are constrained to move in a linear air trough, as shown in Fig. 35-1. How many independent modes of oscillation are there?

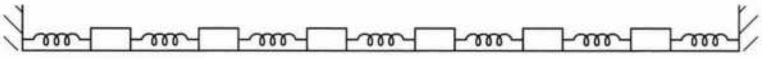

Figure 35-1

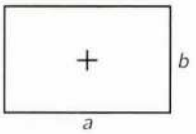

Figure 35-2

35.5 Write the solution of the wave equation, $u = u(x, y, t)$, describing the vibrational mode of the rectangular plate shown in Fig. 35-2, which is clamped at the edge. Assume that vibrations propagate in this plate at speed c, and that the amplitude of the vibrations is A.

35.6 Use the idea of infinitely long, periodic wave trains moving in opposite directions to deduce what will happen if an ideal, uniform stretched string of length L, held firmly at both ends, is pulled aside a distance A (normal to its length) at its midpoint and then released. Sketch a few representative views of the appearance of the string at various times throughout a half-cycle of the motion.

35.7 If surface tension in included, the phase velocity of a surface wave on a liquid of density ρ and surface tension T is

$$v_{ph} = \left(\frac{2\pi T}{\lambda\rho} + \frac{g\lambda}{2\pi}\right)^{1/2}.$$

if the depth is sufficiently great. Find the group velocity v_g of such a wave.

35.8 Find the phase velocity v_{ph} of ripples of wavelength 1.0 cm

(a) in water (surface tension 70 dyne cm^{-1}),

(b) in alcohol (surface tension 26 dyne cm^{-1}).

35.9 For ripples on water that advance with *minimum* speed, find

(a) the wavelength λ,

(b) the frequency ν,

(c) the speed c.

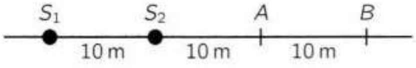

Figure 35-3

35.10 Two sources of sound S_1 and S_2 are situated 10 m apart, as shown in Fig. 35-3. On the same line at A and B are two observers. S_1 emits a steady plane wave whose excess pressure is given by $p_e = A\cos(200t)$, and S_2 emits a steady plane wave for which $p_e = A\cos(210t)$. The distance intervals, $\overline{S_1S_2}$, $\overline{S_2A}$ and $\overline{AB}$ are each 10 m. For parts (a) and (b) below, take the speed of sound as 340 m/sec.

(a) How many beats per second ν does each observer hear?

(b) What is the time interval Δt between the maximum pulses heard by A and those heard by B?

(c) If there were dispersion in the medium such that the wave of higher frequency propagated at 341 m/sec and that of lower propagated at 340 m/sec, with what speed v would beats formed by the two sounds advance from A to B?

35.11 A long diesel freight train is traveling uphill at a speed of 5.0 m s^{-1} on a straight track. As it approaches a tunnel in a sheer vertical wall, the engineer gives a long steady blast of the horn, whose principal frequency is 340 Hz. Both the horn itself and its echo from the wall are heard

(a) by the engineer,

(b) by a worker on the ground near the caboose.

How many beats per second ν does each person hear?

35.12 Show that the function

$$f(x,y,z,t) = Ae^{i\omega t}\sin(l\pi x/a)\,\sin(m\pi y/b)\,\sin(n\pi z/c),$$

where

$$\omega^2 = v^2\pi^2(l^2/a^2 + m^2/b^2 + n^2/c^2),$$

and l, m, n are integers ≥ 1,

(a) satisfies the three-dimensional wave equation (with propagation velocity v),

(b) is equal to zero at $x = 0$ and $x = a$, $y = 0$ and $y = b$, and $z = 0$ and $z = c$, and

(c) oscillates sinusoidally in time.

(d) If $a : b : c = 1 : 2 : 3$ what is the lowest frequency ω_0?

(e) Evaluate the lowest ten frequencies in terms ω_0. List them in order of increasing frequency and plot them roughly on a vertical scale.

35.13 On a straight air track (negligible friction) gliders of mass m_1 and m_2 are attached to two opposite walls by springs of stiffness k_1 and k_2, respectively, with

$$k_1/m_1 = k_2/m_2 = \omega_0^2.$$

and are also attached to one another by a spring of stiffness k, as shown in Fig. 35-4.

(a) Write down the equations of motion of the two gliders.

(b) Substitute $x = Ae^{i\omega t}$ and $y = Be^{i\omega t}$ into the above equations and find the frequencies and amplitudes of the two normal modes of vibration.

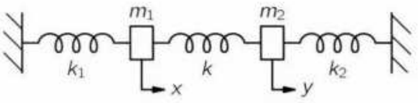

Figure 35-4

35.14 Three pendulums, all of length l, are symmetrically arranged as shown in Fig. 35-5. The center bob, of mass $2m$, is connected by ideal springs of force constant k to the outer bobs, each of mass m. What are the frequencies ω and relative displacements (x_1, x_2, x_3) of the pendulums when they are vibrating in each of the three normal modes of the system? Consider only vibrations in the plane of the figure.

Caution: A little physical insight will save a lot of algebra in this problem.

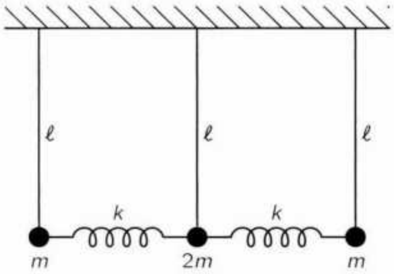

Figure 35-5

35.15 Two identical cylinders of moment of inertia I are hung by identical wires of torsion constant K as shown in Fig. 35-6. No pendulum-like motion is allowed. How many modes of oscillation are there and what are their periods T?

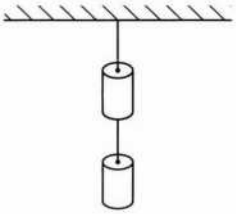

Figure 35-6

35.16 Two identical masses m are suspended in identical springs with force constant k, as shown in Fig. 35-7. Find the angular frequencies ω of the normal modes. No pendulum-like motions are allowed.

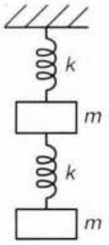

Figure 35-7

35.17 A thin rod of mass M, length L, moves on a frictionless horizontal table. Its CM is constrained to move on the straight line a–a', as shown in Fig. 35-8. Two identical springs (spring constant K) are connected to the endpoints of the rod, and operating parallel to a–a'. For small amplitudes,

(a) find the frequencies of the fundamental modes of oscillation.

(b) describe the motion of the fundamental modes.

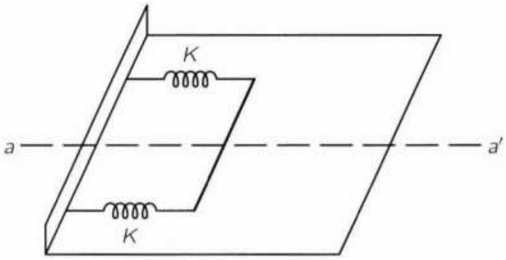

Figure 35-8

35.18 In a standing wave on a stretched string, which can be described by $y = A\sin(kx)\cos(\omega t)$, each element of mass σdx along the wave can be treated as an infinitesimal harmonic oscillator undergoing oscillations about $y = 0$. Determine the total energy E contained in one wavelength λ of the standing wave.

35.19 A nonrelativistic electron with mass m is confined between two plates separated by a distance L.

(a) *Estimate* the minimum possible kinetic energy $T_{\min}$ using the uncertainty principle,

(b) Calculate this energy, using the condition that the wave function describing the electron is a standing sine wave with nodes at the boundaries.

(c) What is the average momentum $\langle p \rangle$ of the electron? (*Careful:* consider magnitude *and* direction.)

36

Fourier Analysis of Waves

Refer to *The Feynman Lectures on Physics*, Vol. I, Chapter 50.

36.1 Give the Fourier expansions of the functions

(a) $y = \text{const}$,

(b) $y = \sin x \ (0 \le x \le 2\pi)$.

36.2 Starting from the Fourier analysis of the square wave graphed in Fig. 36-1,

$$f(x) = \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right),$$

Show that

(a) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \pm \cdots = \pi/4$,

(b) $1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots = \pi^2/8$,

(c) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots = (4/3)\left(1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \cdots\right) = \pi^2/6$.

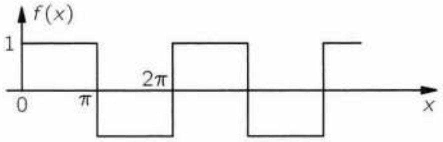

Figure 36-1

36.3 (a) Analyze the periodic function $g(x)$, one cycle of which is shown in Fig. 36-2, into its Fourier components, and show that your result agrees with the result obtained by integrating the function of Ex. 36.2.

(b) Use the result of part (a) above to show that

(1) $1 + 1/3^4 + 1/5^4 + 1/7^4 + \cdots = \pi^4/96$,

(2) $\sum_{n=1}^{\infty} 1/n^4 = \frac{2^4}{2^4-1}(1 + 1/3^4 + 1/5^4 + \cdots) = \pi^4/90$.

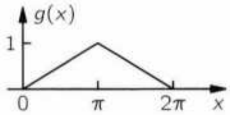

Figure 36-2

36.4 In Chapter 45 of Vol. I, one needs to evaluate

$$\int_0^\infty \frac{x^3 dx}{e^x - 1}.$$

You can now do this by multiplying numerator and denominator by e^{-x} and expanding

$$\frac{1}{1 - e^{-x}} = 1 + e^{-x} + e^{-2x} + \cdots,$$

and carrying out the integrals term by term. Thus you should obtain

$$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \int_0^\infty u^3 e^{-u}\, du \times \left(1 + 1/2^4 + 1/3^4 + \cdots\right)$$
$$= 6 \times \pi^4/90 = \pi^4/15.$$

Try it.

36.5 A guitar string is plucked at the center of its length, as shown in Fig. 36-3. Calculate the amplitudes of the first three harmonics A_1, A_2, A_3, relative to the amplitude of the fundamental A_0.

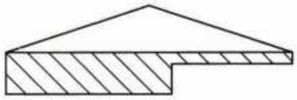

Figure 36-3

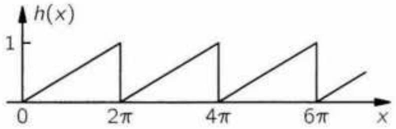

Figure 36-4

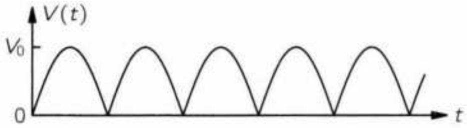

Figure 36-5

36.6 Find the Fourier series which represents the sawtooth function $h(x)$ used in the horizontal sweep circuit of an oscilloscope, as plotted in Fig. 36-4.

36.7 A string of length L has a linear mass density σ and is under tension S. At time $t = 0$, the shape is given by

$$y(x) = 3\sin\frac{\pi x}{L} + \sin\frac{3\pi x}{L}.$$

(a) What is the period of oscillation T?

(b) What is the shape at one-half period? ($t = T/2$)

36.8 A full-wave rectifier is a device which transforms a sine wave of amplitude V_0 into the form shown in Fig. 36-5.

(a) Evaluate the average value of $V(t)$. This will be the DC output voltage.

(b) Find the amplitude A_2 of the second harmonic component of the output voltage.

36.9 A certain transformer yields an output voltage proportional to

$$V_{\text{out}} = V_{\text{in}} + \epsilon(V_{\text{in}})^3.$$

Analyze the effect on V_{out} introduced by the cubic term,

(a) on an input sine wave $\sin x$,

(b) on two or more input sine waves of different frequency added together.

Exercises for Volume II

Introduction

The set of exercises included here is a collection of problems given to Sophomores at Caltech during the years 1962–64, i.e., during the first two years that the course was revised using Professor Feynman's lectures in physics. The exercises were presented first either as homework exercises or as examination problems and thus vary greatly in difficulty. The order in which they are arranged here is roughly, but not at all strictly, in order of difficulty within each chapter. As with the exercises for Volume I, this set is not a "final" set and must be revised and added to as the course evolves.

The idea for about one-half the exercises was suggested by R. P. Feynman. The rest of the problems were contributed by those people teaching sophomore physics: J. Blue, T. Caughey, G. Chapline, M. Clauser, R. Dashen, R. Dolen, R. Griffith, F. Henyey, W. Karzas, R. Kavanagh, P. Peters, J. Pine, M. Plesset, M. Sands, I. Tammaru, A. Title, and C. H. Wilts.

A first editing of most of the problems was done by C. H. Wilts and myself after the 1962–63 school year. Although most problems are original, or at least original versions of "standard" problems, some problems were taken directly from: Introduction to Electricity and Optics, Second Edition, by N. H. Frank, McGraw-Hill 1950; and Physics for Students of Science and Engineering, by D. Halliday and R. Resnick, Wiley 1960. We thank the authors and publishers for permission to publish their problems.

The typing, both in the initial, generally hectic, stages, and in the final form was done by Mrs. F. L. Warren and is gratefully acknowledged.

G. Neugebauer

37

Electromagnetism

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 1.

37.1 (a) What would a proton's mass M be if the gravitational force between two protons at rest were to equal the electric force? How does this compare with its actual mass m_p?

(b) What would be the electric force F between two dimes placed at opposite ends of a 10 meter lecture table if their nuclear and electronic charges were unbalanced by about 1%? Can you think of some object whose "weight" equals that force? (Assume that a dime is made from 2.5×10^{-3} kg of pure silver.)

37.2 (a) Make a rough estimate of the work W_{U} that must be done against electric forces to assemble a uranium nucleus from two equal halves.

(b) What about the work W_{He} required to assemble two deuterons to make a helium nucleus?

Express your answers both in joules per atom and in kilowatt hours per pound.

37.3 In copper there is one "conduction" electron for each copper atom. When a current of10 amperes flows through a piece of No. 10 gauge copper wire (31.8 feet per pound), what is the average speed $\langle v\rangle$ of the conduction electrons? How big is $\langle v\rangle^2/c^2$? (*Remember:* the ratio of "magnetic" effects to "electric" effects is about this small.)

37.4 In a certain region of space, there is a uniform electric field $\boldsymbol{E}$ of 10,000 volts per centimeter in the $+x$ direction. There is in the same region a uniform magnetic field $\boldsymbol{B}$ in the $+y$ direction. A beam of muons with velocity $v = c/3$ travels through this region *on a straight line* in the $+z$ direction, as shown in Fig. 37-1. Given that a muon has a mass 207 times the electron mass and a charge equal in magnitude to the electronic charge,

(a) What is the strength of the magnetic field?

(b) Can you tell from this experiment if the charge on the muons is + or −?

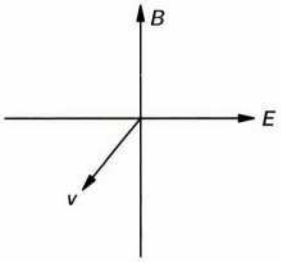

Figure 37-1

37.5 In a certain region of space there is a uniform magnetic field $\boldsymbol{B}$, such that $B_x = 0$, $B_y = 0$, and $B_z = B_0$. The field is constant in time, and there are no currents or electric fields in the region of space we consider. A particle of mass m and positive charge $+q$ is started at $x = 0$, $y = 0$, $z = 0$ with a velocity $\boldsymbol{v}$ in the $+x$ direction.

(a) Sketch and describe quantitatively in terms of B_0, m, v, and q the path of the particle. (Assume $v/c \ll 1$.)

(b) Suppose, that $B_x = 0$, $B_y = 0$, but $B_z = B_0 + ax$ with $a > 0$. For ax always small compared to B_0, but not completely negligible, show on a sketch the qualitative behavior of the particle trajectory. (See Charpak *et al.*, Physical Review Letters, Vol. 6, 128 (1961) for the use of a similar field in an important experiment.)

(c) Show that the field postulated in part (b) is inconsistent with one of Maxwell's equations if the field fills a finite volume of space and, as above, you assume there are no currents or electric fields in the volume.

37.6 A particle with a mass m and a positive charge q is at a point $x = z = 0$, $y = a$, and is moving with a low velocity

$$\boldsymbol{v} = v_0\,\boldsymbol{e}_x.$$

The charge is influenced by a negative charge $-Q$ fixed at the origin and by a uniform magnetic field B_0 in the $+z$ direction.

(a) How large must B_0 be such that the moving particle describes a circle of radius a about the stationary one?

(b) If the magnitude of the magnetic field strength were different than this, explain why the speed of the particle is a function of the radial distance only.

(c) Sketch qualitatively several cycles of the trajectory followed by the particle if it were released from the point $x = z = 0$, $y = a$ with zero velocity.

38

Differential Calculus of Vector Fields

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 2.

38.1 A copper wire of radius a has a concentric insulating sheath with outer radius b. The wire carries an electric current that raises its temperature to T_1 while the outside of the insulation remains at T_2, near room temperature.

(a) What is $\boldsymbol{\nabla} T$ inside the insulation? Give answer in terms of a, b, T_1 and T_2.

(b) How big is the temperature difference $(T_1 - T_2)$ if a current of 20 A is sent through No. 10 gauge copper wire covered with a layer of rubber 0.2 cm thick whose thermal conductivity is $1.6 \times 10^{-3}\,\mathrm{W\,cm^{-1}\,K^{-1}}$? (The conductor in No. 10 gauge copper wire has a diameter of 0.13 cm and a resistance of $1\,\Omega/1000\,\mathrm{ft}$.)

38.2 Show by direct computation that

(a) $\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{A}) = 0$.

(b) $\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{A}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$.

38.3 If $\boldsymbol{R}$ is the vector from the origin to the point (x, y, z) show that

(a) $\boldsymbol{\nabla} \cdot \boldsymbol{R} = 3$.

(b) $\boldsymbol{\nabla} \times \boldsymbol{R} = 0$.

Show that, except at $R = 0$,

(c) $\boldsymbol{\nabla} \cdot (\boldsymbol{R}/R^3) = 0$.

(d) $\boldsymbol{\nabla} \times (\boldsymbol{R}/R^3) = 0$.

(e) $\boldsymbol{\nabla}(1/R) = -\boldsymbol{R}/R^3$.

(f) From part (b) above and Eq. (2.46) in Vol. II we know that $\boldsymbol{R}$ can be written as $\boldsymbol{R} = \boldsymbol{\nabla}\varphi$. What is φ?

38.4 Maxwell's equations are

(1) $\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho/\epsilon_0$,

(2) $\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$,

(3) $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$,

(4) $c^2\, \boldsymbol{\nabla} \times \boldsymbol{B} = \partial \boldsymbol{E}/\partial t + \boldsymbol{j}/\epsilon_0$,

and the conservation of charge can be written

(5) $\boldsymbol{\nabla} \cdot \boldsymbol{j} = -\partial \rho/\partial t$.

(a) Show that Eq. (3) above is consistent with the divergence of Eq. (2).

(b) Show that Eq. (5) above follows from taking the divergence of Eq. (4) (i.e., the validity of Maxwell's equations requires that charge is conserved).

(c) Show that in empty space ($\boldsymbol{j} = 0$, $\rho = 0$) $\boldsymbol{E}$ satisfies the wave equation

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0.$$

Hint: take the curl of Eq. (2).

(d) Show that in empty space, $\boldsymbol{B}$ satisfies the same equation,

$$\nabla^2 \boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = 0.$$

(e) Show that Eq. (2) implies that $\boldsymbol{E}$ can be written as

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t},$$

where $\boldsymbol{A}$ is defined through $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$.

(f) Why can $\boldsymbol{B}$ be written as $\boldsymbol{\nabla} \times \boldsymbol{A}$?

38.5 The velocity of a solid object rotating about an axis is a field $\boldsymbol{v}(x, y, z)$. Show that

(a) $\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$.

(b) $\boldsymbol{\nabla} \times \boldsymbol{v} = 2\boldsymbol{\omega}$, where $\boldsymbol{\omega}$ = angular velocity.

38.6 (a) Prove by direct computation that if $\boldsymbol{A}$ is a constant vector and $\boldsymbol{R}$ is the radius vector, then

$$\boldsymbol{\nabla} \times (\boldsymbol{A} \times \boldsymbol{R}) = 2\boldsymbol{A}.$$

(b) For vectors we know that

$$\boldsymbol{B} \times (\boldsymbol{A} \times \boldsymbol{C}) = \boldsymbol{A}(\boldsymbol{B} \cdot \boldsymbol{C}) - (\boldsymbol{B} \cdot \boldsymbol{A})\boldsymbol{C}.$$

which might lead us to think that

$$\boldsymbol{\nabla} \times (\boldsymbol{A} \times \boldsymbol{R}) = \boldsymbol{A}(\boldsymbol{\nabla} \cdot \boldsymbol{R}) - (\boldsymbol{\nabla} \cdot \boldsymbol{A})\boldsymbol{R} = 3\boldsymbol{A}. \qquad \text{(False)}$$

Why does the substitution of $\boldsymbol{\nabla}$ for $\boldsymbol{B}$ give an incorrect result?

38.7 A long steel shaft is subjected to a fancy heat treatment. At a particular time t, while it is cooling, the temperature distribution $T(x)$ is as shown in part (a) of Fig. 38-1, and isotherms for every 10 °C interval are shown in part (b). We assume throughout that the temperature depends only on x, the distance from one end of the bar.

(a) At the points A, B, and C draw arrows whose direction and magnitude are representative of the direction and magnitude of $\boldsymbol{\nabla} T$.

(b) At which of the five labeled points is the divergence of the heat flow $\boldsymbol{h}$ largest?

(c) At how many of the five labeled points is $\boldsymbol{\nabla} \times \boldsymbol{h} = 0$?

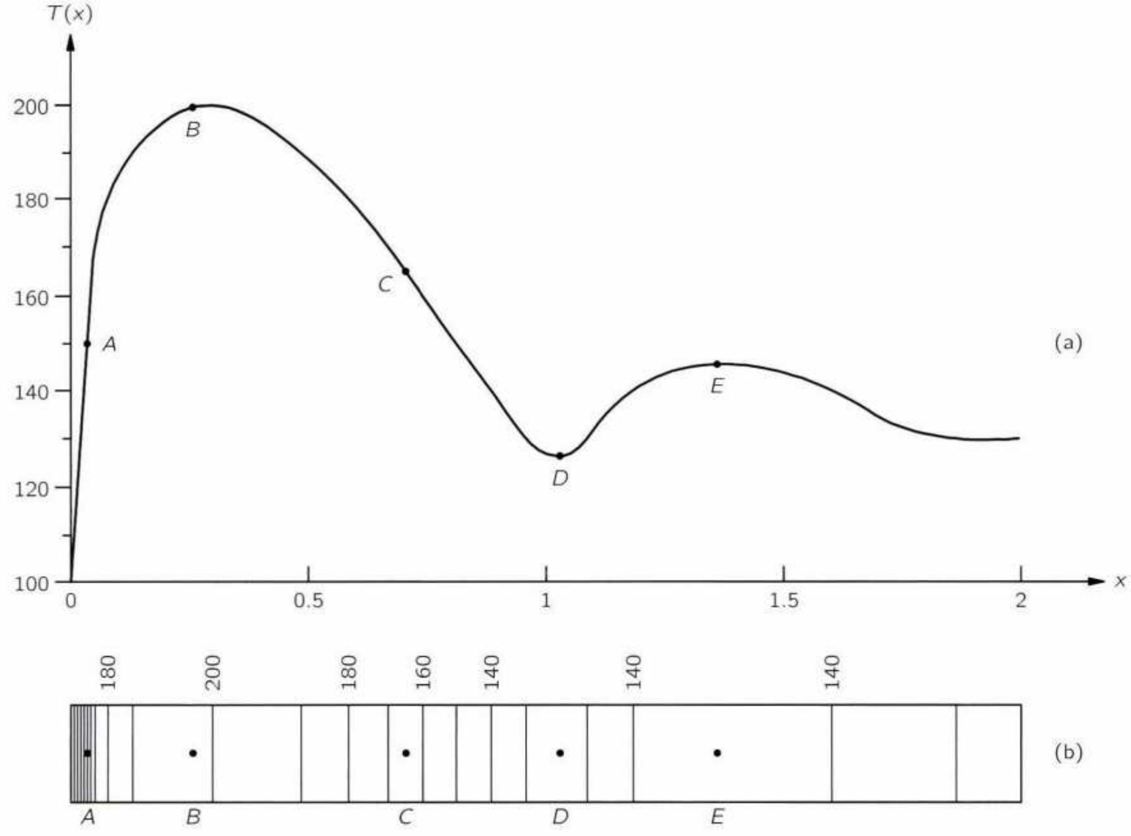
T(x)
200
180
160
140
120
100
0
0.5
1
1.5
2
x
A
B
C
D
E
(a)
180
200
180
160
140
140
140
A
B
C
D
E
(b)

Figure 38-1

39

Vector Integral Calculus

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 3.

39.1 (a) Maxwell's equations have been given in the form of a statement in Chapter 1 of Vol. II and in their differential form in Chapter 2. Show that the two forms are equivalent.

(b) If ρ is the charge per unit volume and $\boldsymbol{j}$ is the electric current density, show that the equation

$$\boldsymbol{\nabla}\cdot\boldsymbol{j}=-\frac{\partial\rho}{\partial t}$$

is equivalent to a statement of the conservation of charge.

39.2 A layer of radioactive material is deposited on the surface of a sphere. The material emits α particles of high energy. Suppose that the particles are emitted only radially outward from the surface of the sphere. It would appear that this flow of charged particles constitutes a current. Is a magnetic field produced by this current?

39.3 The field of a point charge located at the origin is of the form

$$\boldsymbol{E}=\frac{K}{r^3}\,\boldsymbol{r},$$

where $\boldsymbol{r}=x\boldsymbol{e}_x+y\boldsymbol{e}_y+z\boldsymbol{e}_z$, $r=(x^2+y^2+z^2)^{1/2}$, and K is a constant.

(a) Calculate the flux of $\boldsymbol{E}$ through the spherical surface S of radius a with origin at the center.

(b) Use Gauss's theorem to connect the flux of $\boldsymbol{E}$ through the spherical surface with the integral of $\boldsymbol{\nabla}\cdot\boldsymbol{E}$ over the volume. Can you explain your result?

(c) Calculate the line integral of the vector $\boldsymbol{E}$ around the path s in the xy-plane shown in Fig. 39-1. Use Stokes's theorem to verify the result.

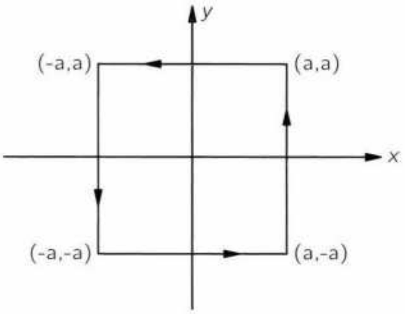

Figure 39-1

39.4 (a) Using the results of Ex. 38.3 part (a), find a (useless) formula for the volume V of a region in terms of an integral over its surface S.

(b) Check your result in part (a) for a sphere and a rectangular block.

40

Electrostatics

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 4.

40.1 With reference to Fig. 40-1:

(a) Find the potential ϕ at the point P a distance r from a line of charge $(l_1 + l_2)$ meters long, which contains a charge density of λ coulombs/meter.

(b) Compare your answer to part (a) with the expected potential if $r \gg (l_1 + l_2)$.

(c) Check your answer to part (a) in the limit $r \ll (l_1 + l_2)$ by comparing the electric field $\boldsymbol{E}$ derived from ϕ with the field derived using Gauss's law.

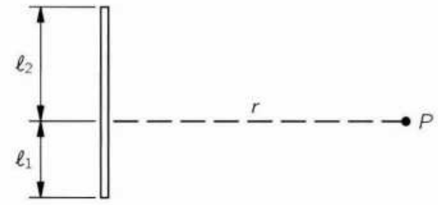

Figure 40-1

40.2 Calculate the electric field $\boldsymbol{E}$ at the point P, a distance r from the center and on the axis of a thin disk of radius R with a uniform surface charge density σ, as shown in Fig. 40-2.

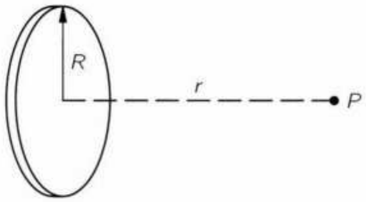

Figure 40-2

40.3 A charge q' is placed on the inner of the two concentric metal spheres, as shown in Fig. 40-3; a charge q is placed on the outer sphere.

(a) Sketch the radial component of the electric field E_r as a function of the radial distance.

(b) Sketch the potential relative to infinity as a function of the radial distance.

(c) What is the potential ϕ at the surface of the inner sphere?

(d) If the center sphere is moved off the center of the outer sphere, explain what happens to the field for $r_b < r < r_c$ and for $r > r_c$.

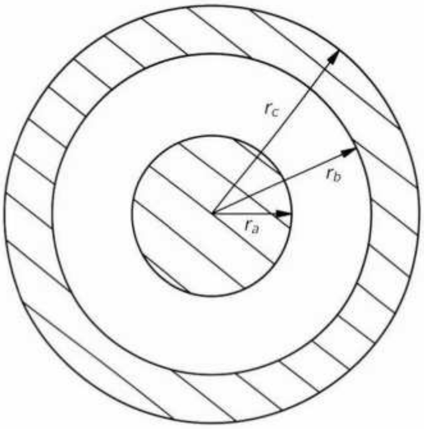

Figure 40-3

41

Applications of Gauss' Law

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 5.

41.1 Show that the electric potential ϕ has the following interesting property: the average value of ϕ over an imaginary spherical surface is the same as the value of ϕ at the center of the sphere if there are no charges inside the sphere. Can you think of an application in which this property is useful?

41.2 Find the electric field $\boldsymbol{E}$ inside but far from the ends of an extremely long cylinder that has a uniform charge density ρ throughout. Note the difference between this result and the field inside a uniformly charged sphere.

41.3 Two large, flat metal plates are held parallel to each other and separated by a distance d. They are connected together at their edge by a metal strip. A thin plastic sheet carrying a surface charge σ per unit area is placed between the plates at a distance $d/3$ from the upper plate, as shown in Fig. 41-1. Calculate $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ the electric field near the upper and lower plates, respectively.

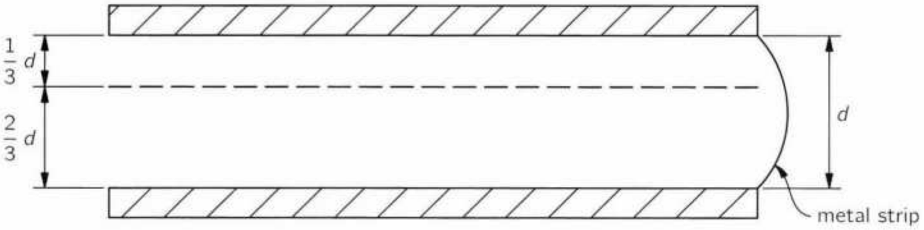

Figure 41-1

41.4 Find the general formula for the x-component of the electric field at x, E_x, if the charge density ρ varies only with x throughout all space.

41.5 In a certain electronic tube, electrons are emitted from a hot plane metal surface, and collected by a plane metal plate parallel to the emitter, at the distance d away, as shown in Fig. 41-2. (The distance d is small compared with the lateral dimensions of the plates.) The electric potential between the plates is given by $\phi = kx^{4/3}$ where x is the distance from the emitter.

(a) What is the surface charge density σ_e on the emitter?

(b) What is the surface charge density σ_c on the collector?

(c) What is the volume charge density $\rho(x)$ for $0 < x < d$?

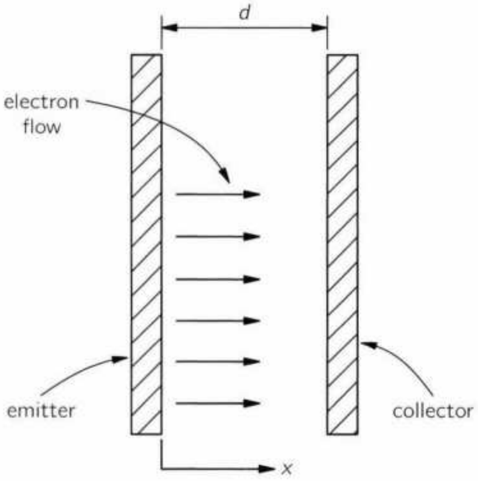

Figure 41-2

41.6 Consider a conductor which has a charge distribution σ coulombs/m^2 on its surface; σ need not be a constant. Show that the force on the charge in a little element of area dA is normal to the surface and is given by $(\sigma^2/2\epsilon_0)\,dA$. (The factor of 1/2 is correct. Explain.)

41.7 The maximum field strengths that can exist at the surface of a conductor in a vacuum before field emission takes place is about 10^8 V/m. If the conductor is made of copper (weighing $8.9\,\mathrm{g\,cm^{-3}}$), and the surface charge producing such a field is negative,

(a) compare the number of excess electrons per unit area N_e to the number of atoms per unit area N_a.

(b) compare the force F_e on an electron in this field to the force F_a acting on an electron a typical atomic distance from a proton ($\approx 0.5\,\text{Å}$).

41.8 The negative muon is a particle that has the same electrical charge as the electron but a mass 207 times larger. When a negative muon is stopped in matter, it is attracted by a nucleus and may replace one of the atomic electrons and form a "muonic" atom. Because of its larger mass, the muon will approach closer to the nucleus than the electron and, for heavy nuclei, will even be inside the nucleus in the lowest energy state. The muon does not interact with nuclear matter through nuclear forces, but only through electrical forces and sees the nucleus as a sphere of uniform charge. From other experiments in nuclear physics it is found that the radius of nuclei can be given by $R = R_0 A^{1/3}$ where R_0 is approximately 1.2×10^{-15} m and A is the number of protons plus neutrons.

Consider a model of the muonic atom where the muon oscillates back and forth along a line through the center of a lead nucleus.

(a) What is the natural frequency ω of these oscillations?

(b) What is the energy difference between the two lowest states of your model of the muonic atom? (Recall from Chapter 41 of Vol. I that the quantum energy states of a harmonic oscillator are separated by an energy difference $\hbar\omega$.)

(c) It is observed experimentally that when the muonic atoms are formed in lead 6 MeV X-rays are emitted. How would you interpret this radiation?

41.9 Imagine that the earth were of uniform density and that a tunnel was drilled along a diameter.

(a) If an object were dropped into the tunnel show that it would oscillate with a period P equal to the period of a satellite orbiting the earth just at the surface.

(b) Calculate P.

41.10 It is known that the earth gives off about 8×10^{20} joules per year in heat energy. Before studying thermal models in detail in order to explain this, it is useful to consider some models which are clearly over-simplifications but which do give order of magnitude estimates. As an example, consider the possibility that the heat is all produced by radioactive materials uniformly distributed in the earth that decay, giving off particles whose kinetic energy is converted into heat. It is estimated that the temperature at the center of the earth is roughly $2500\,^\circ\mathrm{C}$ and that the thermal conductivity of typical materials is about $0.03\,\mathrm{W\,cm^{-1}\,K^{-1}}$. Is the model described above consistent with these estimates?

41.11 Two long concentric conducting cylinders are insulated from each other and charged. Far from the ends, the inner cylinder has a net charge density of $+\lambda_1$, and the outer one a net charge density of $+\lambda_2$ coulombs per unit length. The inner cylinder has inner and outer radii r_1 and r_2, while the outer cylinder has radii r_3 and r_4.

(a) Find $\boldsymbol{E}(r)$

  (1) at a point near the middle (i.e., where end effects can be neglected),

  (2) just outside the outer cylinder.

(b) Find the potential difference $\Delta\phi$ between the two cylinders.

(c) Describe qualitatively any changes in the fields and potentials if

  (1) r_1 is decreased.

(2) r_2 is increased.

(3) the outside cross-section of the inner cylinder is made square with sides $2r_2$ (assuming $\sqrt{2}\, r_2 < r_3$).

42

The Electric Field in Various Circumstances

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 6.

42.1 Use the method of images to find the force $\boldsymbol{F}$ on a charge Q located distances a and b from two semi-infinite conducting planes at right angles to each other, as shown in Fig. 42-1.

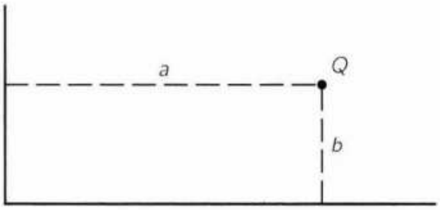

Figure 42-1

42.2 A particle with electric charge q is released (from rest) at the distance x_0 from the surface of a large, grounded, conducting plate. The particle is attracted by the plate, and moves toward it.

(a) What is the kinetic energy of the particle as a function of its distance x from the plate? (Neglect any energy loss by radiation.)

(b) Do you see anything unphysical about your answer to part (a) above?

(c) A real metal plate can be thought of as an ideal conducting plane only for distances down to the atomic spacing, that is, to about 1 Å. Estimate the kinetic energy with which an electron would arrive at a conducting plate, if released at rest 1 cm from the plate. Give your answer in electron-volts.

42.3 A rectangular box of insulating plastic 1 cm by 10 cm by 100 cm is filled with a uniform charge density ρ coulombs per cm^3. Consider a straight line that is perpendicular to the 10 cm by 100 cm face and passes through the center of the box. Sketch roughly a graph of the potential ϕ along this line as a function of the distance from the center. Consider the range of distances from 0.001 cm (i.e., inside the box) to distances much larger than 100 cm; a log-log plot of ϕ vs. distance is appropriate. On the same graph sketch a curve that gives the magnitude of the electric field $\boldsymbol{E}$.

42.4 The earth is continually bombarded by high energy cosmic rays that come from outside the solar system. It has been determined by high altitude measurements from balloons and satellites that the cosmic rays are made up almost completely of protons, although there are also a few percent alpha-particles, heavier nuclei, and electrons. The mean energy of the bombarding protons is a few billion electron volts; the intensity of the protons arriving at the earth's atmosphere is about one proton per cm^2 per sec.

One might wonder how long it would take for the charge arriving at the earth in cosmic rays to raise the earth's potential to the point that the protons could no longer reach the earth. How does the time T required to accomplish this compare to the approximate 5 billion year age of the earth? If the time is shorter than the earth's age, one is faced with the question: "Why are cosmic rays still coming?"

42.5 (a) Find the capacitance per unit length C_l of a long cylindrical capacitor made up of a conducting cylinder of radius a inside and co-axial with a cylinder of radius b.

(b) Qualitatively, what would happen if there were imperfections in the construction which, in effect, created a sharp outward protuberance on the outer wall?

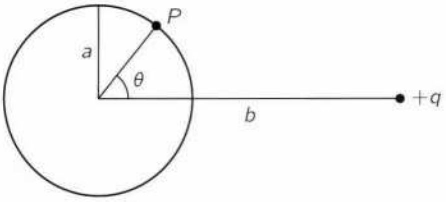

Figure 42-2

42.6 A charge $+q$ is located a distance b from the center of an uncharged, insulated, conducting sphere of radius a, as shown in Fig. 42-2.

(a) What is the potential ϕ of the sphere?

(b) What is the surface charge density $\sigma(\theta)$ induced on the sphere at the point P shown in the figure.

(c) If the sphere is raised to a potential V, what is the magnitude of the force $\boldsymbol{F}$ between the charge and the sphere?

42.7 In Chapter 6 of Vol. II it is demonstrated qualitatively that it is possible to find the field outside a sphere with a surface charge density varying as $\cos\theta$ superposing the fields of two slightly offset oppositely charged spheres. Carry out the process quantitatively and find the field both *outside and inside* a sphere with a surface charge density $\sigma(\theta) = \sigma_0 \cos\theta$, where σ_0 is a constant and θ is the polar angle.

42.8 The field of a dipole $\boldsymbol{p}$ is given in Eqs. (6.14) and (6.15) of Vol. II.

(a) Find the radial and tangential components of the dipole field $\boldsymbol{E}$ at a point (r, θ, φ) (spherical coordinates).

(b) Show that the electric field of a dipole points in the same direction at all points on any given straight line which passes through the dipole.

(c) At any given distance from the dipole, what are the directions and relative strengths of $\boldsymbol{E}$ for points lying at angles of 0, $\pi/4$, and $\pi/2$ relative to the direction of $\boldsymbol{p}$?

42.9 Consider a dipole placed in an electric field which, before the dipole was inserted, was uniform and of strength E_0.

(a) If the dipole moment $\boldsymbol{p}$ points in the direction of the external field there will be an equipotential surface which *encloses* the dipole. Show that this surface is a sphere.

(b) Find the strength of the dipole moment for which the sphere will have radius a.

(c) Sketch the electric field outside the sphere.

(d) If a thin conducting shell at the same potential were made coincident with the equipotential surface, how would the fields be changed?

(e) What would be the surface charge density $\sigma(\theta)$ on the sphere (where θ is the polar angle)?

(f) What would be the dipole moment of this charge density?

(g) How would you utilize the results of the above?

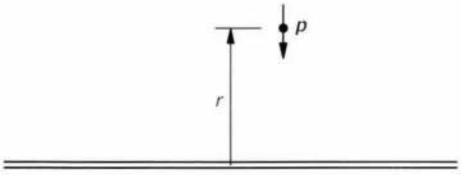

Figure 42-3

42.10 A particle with an electric dipole moment $\boldsymbol{p}$ is placed at the distance r from a long wire which has a charge λ per unit length (λ is a constant). The dipole moment lies in the plane defined by the wire and the particle, as shown in Fig. 42-3.

(a) What is the force $\boldsymbol{F}$ and the torque $\boldsymbol{\tau}$ on the particle,

  (1) if $\boldsymbol{p}$ is normal to the wire?
  (2) if $\boldsymbol{p}$ is parallel to the wire?

(b) Show that in general the force $\boldsymbol{F}$ on a dipole $\boldsymbol{p}$ in an electric field $\boldsymbol{E}$ is $\boldsymbol{F} = \boldsymbol{\nabla}(\boldsymbol{p} \cdot \boldsymbol{E})$.

42.11 Find the potential ϕ at a distance r from a very large sheet of dipoles. Assume that there are N dipoles per unit area and that each dipole has dipole moment $\boldsymbol{p}$ pointing normal to the surface.

42.12 An electric charge $+q$ is distributed uniformly on a thin ring of radius a. The ring is placed in the yz-plane with its center at the origin. A charge $-q$ is placed at the origin, as shown in Fig. 42-4.

(a) Find the potential ϕ at the point $P = (x, 0, 0)$.

(b) What is the electric field $\boldsymbol{E}$ at P?

(c) How does the electric field vary with x for $x \gg a$?

(d) How does the field vary at large distances compare with the field of a dipole? Can you explain?

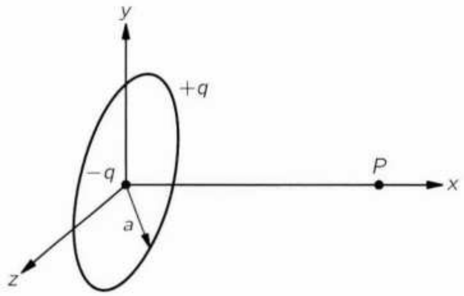

Figure 42-4

42.13 A parallel plate capacitor with a capacitance of 100 pF (1 pF $= 10^{-12}$ farad) and a separation of 1 cm is charged with a battery to a potential difference of 10 volts. The battery is then disconnected from the capacitor. Blue light is shined on the bottom plate, which causes electrons with kinetic energies ranging from 0 to 1.5 eV to be emitted. The battery voltage is such that the electrons are attracted to the upper plate. The total current which goes to the upper plate is shown in Fig. 42-5 as a function of time.

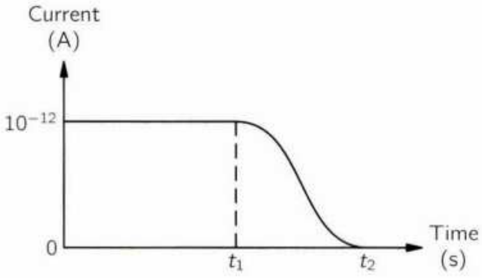

Figure 42-5

(a) How long will it take for the potential difference between the plates to become zero (time t_1 in the figure)?

(b) What is the potential difference V at times very much larger than t_2?

(c) If the plate separation was doubled *before* the capacitor was charged, how would your answer to part (a) change?

(d) If the plate separation were doubled *after* the capacitor was fully charged and after the battery was removed, how would your answer to part (a) change?

42.14 An insulating rod 1 meter long and 1 centimeter in radius has its axis along the x-axis and its ends at $x = -0.5$ meter and $x = +0.5$ meter. It has a total volume charge density given by $\rho = ar^2$, where r is the distance to the axis of the rod and a is a positive constant, $2\,\mathrm{C\,m^{-5}}$.

(a) Find the magnitude of the electric field $\boldsymbol{E}$ at the four points $y = 0$, 0.5, 1.0, and 2.0 cm; $x = z = 0$. In this part assume the rod is infinite in length.

(b) Make a good estimate of the potential at the origin ($x = y = z = 0$) relative to zero potential at infinity. Assign an uncertainty to your estimate and justify it.

(c) Will the potential at the point $x = 0.5$ meter; $y = z = 0$ be greater than, less than, or equal to the potential at the origin?

43

The Electric Field in Various Circumstances (continued)

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 7.

43.1 (a) Show that the equipotentials produced by two parallel line charges of opposite sign are circular cylinders. Assume a density of $+\lambda$ and $-\lambda$ coulombs/meter on two lines separated by a distance d.

(b) From the result in part (a) above find the capacitance per unit length C_l of two parallel wires of radius r with their centers separated a distance d. Assume that $d \gg 2r$.

(c) Show that if $x, y \gg d$ the potential can be obtained from the complex function

$$f(z) \equiv U + iV = \frac{1}{z} = \frac{1}{x + iy}.$$

44

Electrostatic Energy

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 8.

44.1 Do you agree with the statement made in Chapter 8 of Vol. II that a nucleus which contains Z protons distributed more or less uniformly through the volume of a sphere of radius r has an electrostatic energy of about the value given below?

$$U = \frac{3}{5}\,Z(Z-1)\,\frac{e^2}{r}?$$

44.2 A radio tuning capacitor has a maximum capacity of 100 pF. By rotation of the moving plates, the capacity can be reduced to 10 pF. Assume the capacitor is charged to a potential difference of 300 volts at maximum capacity. The tuning knob is then rotated to minimum capacity, while the charge on the plates remains constant.

(a) What is the final value of the potential difference V_f?

(b) How much mechanical work W is done in rotating the knob?

44.3 Two capacitors with capacitance C_1 and C_2 are initially charged with charges Q_1 and Q_2.

(a) Show that except in special cases the stored electrostatic energy U decreases when the two capacitors are joined together in parallel.

(b) Where does the lost energy appear?

(c) Find the conditions under which they can be joined without loss of energy.

44.4 (a) Show that when a dipole of dipole moment $\boldsymbol{p}$ is placed in an electric field $\boldsymbol{E}$, the electrostatic energy is given by

$$U = -\boldsymbol{p}\cdot\boldsymbol{E}.$$

(b) Calculate the torque exerted on the dipole by the field $\boldsymbol{E}$. Do the calculation directly as well as by using the above energy relation.

(c) Is the energy the same if the dipole were formed from two charges placed sequentially in the field? If not, calculate the difference; if so, justify your reasoning physically.

44.5 Show that the plates of a parallel-plate capacitor on which there is a charge Q attract each other with a force $F = Q^2/(2\epsilon_0 A)$, where A is the the area of the plates.

Hint: Consider the work necessary to increase the plate separation from x to $x+dx$.

44.6 The π meson (or pion) is a particle that is found in all three charged states; there are positive, negative and neutral pions. The mass (times c^2) of charged pions is 139.6 MeV while the mass of neutral pions is 135.0 MeV. In one model of the pion, the mass difference is attributed to electrostatic energy. If one further assumes that pions are represented as spheres and that the charge of a charged pion is uniformly distributed throughout the sphere, it is possible to calculate the "radius" of the pion. Under these assumptions calculate the radius r_π of the pion. (Is your result compatible with other estimates of nuclear dimensions?)

44.7 A metal spherical shell of inner radius a and outer radius b is located with its center at the origin. There is a small hole cut at one point of the shell.

(a) If there is no net charge on the shell, how much work W_a is required to bring a charge q_1 from infinity, through the hole, and to the origin?

(b) How much work W_b is required if the shell is given a total charge q_2?

45

Dielectrics

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 10.

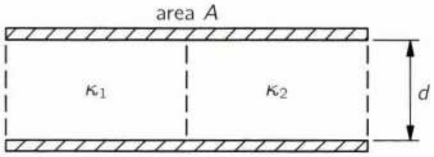

Figure 45-1

45.1 A parallel-plate capacitor is filled as shown in Fig. 45-1 with two dielectrics of equal size but unequal dielectric constants κ_1 and κ_2.

(a) Use a simple argument (ignoring edge effects) to show that the capacitance is given by

$$C = \frac{\epsilon_0 A}{d}\frac{\kappa_1 + \kappa_2}{2}.$$

where A is the area of the plates and d is their separation.

(b) Give a more detailed proof, using Eq. (10.26) of Vol. II,

$$\boldsymbol{\nabla}\cdot(\kappa \boldsymbol{E}) = \rho_{\text{free}}/\epsilon_0.$$

(c) Can you think of another proof, using the energy density given in Eq. (8.30) of Vol. II?

Note: for space filled with a dielectric, the electrostatic energy density is $u = \kappa\epsilon_0 E^2/2$.

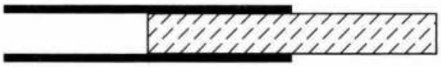

Figure 45-2

45.2 A parallel plate capacitor has square plates 20 cm on a side and a plate separation of 1 cm. It is charged to a potential difference of 10 V and then disconnected from the battery. A large square sheet of dielectric slightly less than 1 cm thick is inserted between the plates so that a 10 cm by 20 cm area of the capacitor is filled with dielectric, as shown in Fig. 45-2. The dielectric constant is 4.0.

(a) What is the magnitude of the attractive force F between the plates?

(b) What is the dipole moment per unit volume P in the dielectric at positions well inside the plates (so that edge effects can be neglected)?

(c) Suppose the dielectric sheet consisted of material of dielectric constant 4.0 which had embedded in it a uniform distribution of small metal spheres. Would the potential difference between the plates be greater than, less than, or equal to, that for the original homogeneous dielectric?

45.3 A parallel-plate capacitor of separation d has a capacitance C_0 in air. An insulating slab of dielectric constant κ, thickness $t < d$, and area equal to that of the plates is inserted into the capacitor, the slab faces being parallel to those of the capacitor. Neglecting end effects, prove that the capacitance is now

$$C = \frac{C_0}{1 - ((\kappa - 1)/\kappa)(t/d)}.$$

45.4 An isolated metal sphere of radius a has a free charge Q on its surface. The sphere is covered with a uniform dielectric layer having inner radius a, outer radius b, and dielectric constant κ.

(a) Calculate the polarization surface charge σ_{pol} on the inner and outer surfaces of the dielectric.

(b) What is the volume density of polarization charge ρ_{pol} inside the dielectric?

45.5 A parallel-plate capacitor is connected to a battery which maintains a potential difference V between its plates. A slab of dielectric constant κ is inserted between the plates, completely filling the space between them.

(a) Show that the battery does an amount of work $W_{\mathrm{batt}} = qV(\kappa - 1)$ during the insertion process, if q is the charge on the capacitor plates before the slab is inserted.

(b) How much work W_{mech} is done by mechanical forces on the slab when it is inserted between the plates? Is this work done on, or by, the agent inserting the slab?

(c) If instead of inserting a dielectric between the capacitor's plates we change the plate separation until the capacitance is the same as when the dielectric slab is inserted, what then is the relationship between W_{batt} and W_{mech}?

45.6 Two coaxial metal pipes of radii a and b ($a < b$) are lowered vertically into a large oil bath. If a voltage V is applied between the pipes, show that the top level of the oil between the pipes rises a height

$$H = \frac{V^2(\kappa - 1)\epsilon_0}{\ln{(b/a)}\rho(b^2 - a^2)g},$$

where κ is the dielectric constant and ρ is the density of the oil.

45.7 Show that, when an electric field line cuts through a surface separating two dielectrics of dielectric constants κ_1 and κ_2 it makes angles θ_1 and θ_2 with the normal to the surface in the two media, given by the relation $\kappa_1 \cot\theta_1 = \kappa_2 \cot\theta_2$.

Hint: assume there are no free charges on the surface.

46

Inside Dielectrics

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 11.

46.1 Consider a sphere of dielectric material with radius a which is uniformly polarized with a volume polarization density $\boldsymbol{P}$ as shown in Fig. 46-1. Find the electric field inside and outside this sphere.

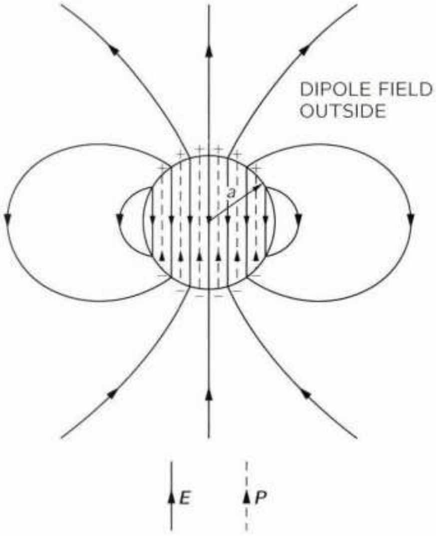

Figure 46-1

46.2 The dielectric constant of helium at 0 °C and 1 atm pressure is 1.000074. Find the dipole moment p induced in each helium atom when the gas is in an electric field of intensity $100\,\mathrm{V\,m^{-1}}$.

46.3 Water vapor is a polar gas whose dielectric constant exhibits an appreciable temperature dependence. The following table gives experimental data on this effect.

T (K)	P (cmHg)	$(\kappa - 1) \times 10^5$
393	56.49	400.2
423	60.93	371.7
453	65.34	348.8
483	69.75	328.7

(a) Assuming that water vapor obeys the ideal gas law, calculate the molecular polarizability α as a function of inverse temperature $1/T$ and plot it.

(b) From the slope of the curve, deduce a value for the permanent dipole moment p_0 of the H_2O molecule.

46.4 (a) Consider a system consisting of two atoms separated by a fixed distance a, each atom having polarizability α. Find the relation between a and α for such a system to be ferroelectric.

(b) If you find part (a) too easy, consider a line of oxygen atoms, regularly spaced with a distance a between each atom. Suppose also that there is a titanium atom half-way in between successive oxygen atoms, as shown in Fig. 46-2. Let the polarizability of oxygen be α_{O}, titanium, α_{T}. Find the conditions on α_{O} and α_{T} such that the system is a ferroelectric.

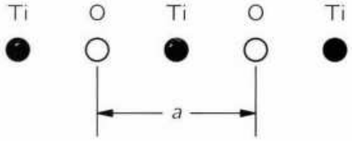

Figure 46-2

Note:

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \approx 1.20$$

46.5 A "dielectric" material consists of a number of brass spheres of diameter d, with centers spaced $3d$ apart, in a regular lattice. Assuming that each sphere is influenced only by the imposed external electric field (i.e., neglecting the effect of neighboring spheres or the redistribution of induced charges), find the dielectric constant κ for this material.

47

Electrostatic Analogs

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 12.

47.1 A ring of radius a is made of copper wire whose diameter b is much less than a. The ring is embedded in the center of a plastic sphere whose radius R is *very* large compared with the radius a of the ring. An alternating magnetic field induces a current in the ring. The current heats the wire, generating thermal energy at the rate W (joules per second). The temperature at the outside surface of the large sphere is T_R. In the steady state, what is the temperature T_0 at the center of the ring, if the thermal conductivity of the plastic equals K?

47.2 In Ex. 41.10, one simple thermal model of the earth was considered. Another (again over-simplified) model is that inside the earth there is a spherical core of extremely high conductivity. Find what the radius a of this core must be if its temperature is 2500 °C, if the thermal conductivity of the surrounding Earth is 0.03 W/cm K, and if 8×10^{20} joules of heat energy are given off by the earth each year.

47.3 (a) For certain geometries and physical conditions, it is useful to write the potential in the form $\varphi = f(r)\cos\theta = f(r)(z/r)$, where $r^2 = x^2 + y^2 + z^2$. (This form was used, for example, in Chapter 12 of Vol. II to solve the problem of the flow of "dry water" around a sphere.) If $f(r)$ is expanded in a power series

$$f(r) = \sum_{n=-\infty}^{\infty} b_n r^n,$$

only two of the coefficients b_n can be non-zero if φ is to satisfy Laplace's equation. Find these two.

(b) For the analogous two dimensional problems, the potential may be written in the form

$$\varphi = g(\rho)\cos\theta = g(\rho)\,\frac{z}{\rho},$$

with

$$g(\rho) = \sum_{n=-\infty}^{\infty} c_n \rho^n,$$

where $\rho^2 = y^2 + z^2$. Find which values of c_n can be non-zero if φ is to satisfy Laplace's equation.

Note: use Cartesian coordinates to do your calculations.

47.4 Two thin water pipes are separated by a distance d and go normal to and through a large wall of thickness t, as shown in Fig. 47-1. The thermal conductivity of the wall is K while the temperature far from the pipes is T_0. Hot water which gives off $+W$ watts to the wall flows in the pipe at $x = +d/2$ while cold water which absorbs W watts from the wall flows in the other pipe. Consider this as a two dimensional problem and neglect the finite size of the pipes. Find the temperature T_P at a point P located at $x = 100d$, $y = 100d$. Let $T_0 = 20$ °C, $d = 50$ cm, $K = 0.03$ W/cm K, $W = 200$ watts and $t = 10$ cm. Make suitable approximations in evaluating your answer.

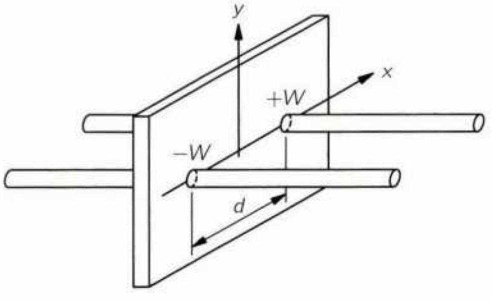

Figure 47-1

48

Magentostatics

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 13.

48.1 Four long No. 10 copper wires are parallel to each other, their cross section forming a square 20 cm on edge. A 20 amp current is set up in each wire in the direction shown in Fig. 48-1.

(a) What are the magnitude and direction of the magnetic field $\boldsymbol{B}$ at the center of the square?

(b) What is the magnitude and direction of the force per meter $\boldsymbol{F}$ acting on the lower left wire?

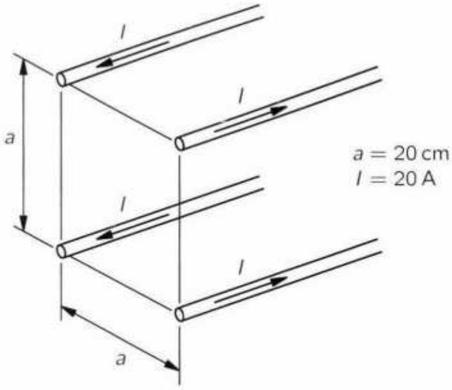

Figure 48-1

48.2 A long, solid dielectric cylinder of radius a is *permanently* polarized so that the polarization is everywhere radially outward, with a magnitude proportional to the distance from the axis of the cylinder, i.e., the dipole moment per unit volume is $\boldsymbol{P} = P_0\,\boldsymbol{r}/2$. The cylinder is rotated with constant angular velocity ω about its axis. What is the magnetic field $\boldsymbol{B}$ on the axis of the cylinder, at points not too close to the ends?

48.3 A long coaxial cable consists of two concentric conductors with the dimensions shown in Fig. 48-2. There are equal and opposite currents I in the conductors. It may be assumed that the currents are uniformly distributed in the conductors.

(a) Find the magnetic field $\boldsymbol{B}$ at radial distance r within the conductor ($r < a$).

(b) Find $\boldsymbol{B}$ between the two conductors ($a < r < b$).

(c) Find $\boldsymbol{B}$ within the outer conductor ($b < r < c$).

(d) Find $\boldsymbol{B}$ outside the cable ($r > c$).

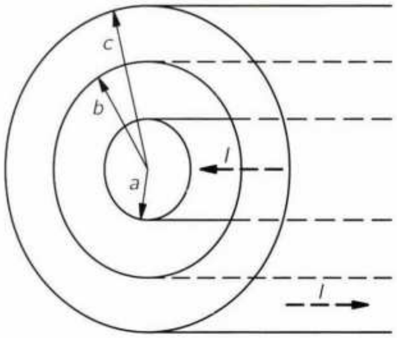

Figure 48-2

48.4 A long wire carries the current I_1, while a rectangular loop of wire whose length and width are l and w respectively carries the current I_2. The wire lies in the plane of the loop, parallel to its length, at distance a away, as shown in Fig. 48-3.

(a) What is the force $\boldsymbol{F}_{\text{loop}}$ on the loop?

(b) What is the force $\boldsymbol{F}_{\text{wire}}$ on the wire?

(c) What is the torque $\boldsymbol{\tau}_{\text{loop}}$ on the loop?

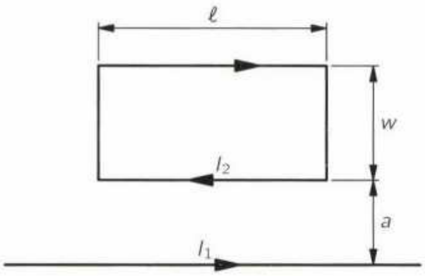

Figure 48-3

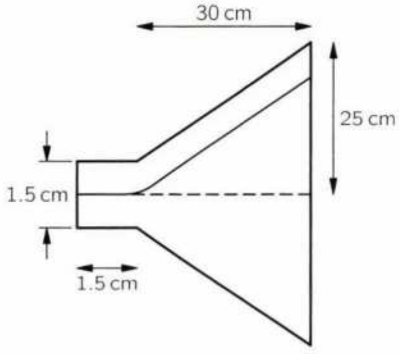

Figure 48-4

48.5 The scan of a TV picture tube is generated by deflecting the electron beam with a magnetic field that comes from a set of deflecting coils attached around part of the neck of the picture tube. The electrons are typically emitted from the gun (before deflection) with an energy of around 3.0 keV; they are further accelerated after deflection. Using the dimensions shown in Fig. 48-4,

(a) estimate the magnitude of the magnetic field $\boldsymbol{B}$ at the electron beam at the time of maximum deflection.

(b) estimate the number of ampere-turns nI in the coil at the same time.

You may neglect the post deflection acceleration in making your estimate. (In what direction would your answer change if you were to include it?)

48.6 A very long conducting rod of radius a has an off-center hole of radius b whose axis is parallel to but off-set by a distance d from the axis of the rod, as shown in Fig. 48-5. A uniform current density $+j$ flows in the conductor. What is the magnitude of the magnetic field B at the axis of the hole, far from the ends?

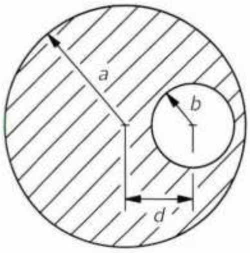

Figure 48-5

The Magnetic Field in Various Situations

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 14.

49.1 In a plastic film factory, a wide belt of thin plastic material is traveling between two successive rollers with the speed v, as shown in Fig. 49-1. In the manufacturing process, the film has accumulated a uniform surface electric charge σ.

(a) What is the vector potential $\boldsymbol{A}$ near the surface of the belt in the middle of a large flat span? (Near point P of the figure.)

(b) What is the magnetic field $\boldsymbol{B}$ in the same region?

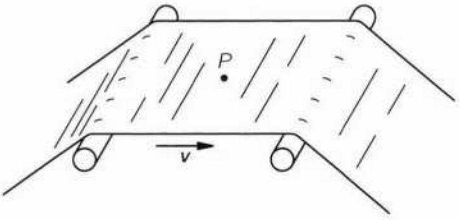

Figure 49-1

49.2 A wire of the shape shown in Fig. 49-2 carries a current I. What is the magnetic field $\boldsymbol{B}$ at the center of the semicircle arising from

(a) each straight segment of length l?

(b) the semicircular segment of length πr?

(c) the entire wire?

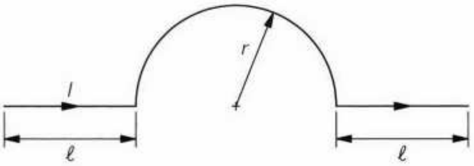

Figure 49-2

49.3 In practical magnetic structures, uniform magnetic fields are frequently necessary. The uniformity of the field produced by Helmholtz coils, or two co-axial loops which carry currents in the same direction, is one of their most important characteristics. Assume that the coils have a radius a, have axes on the x-axis, carry a current I each, and are separated by a distance b, as shown in Fig. 49-3.

(a) Find the magnetic field at a point P on the axis of the loops and a distance x from the mid-point O.

(b) Expand the expression for the field in a power series retaining terms to order x^2.

(c) What relationship must exist between a and b such that the x^2 terms vanish? What is the significance of this?

(d) Show that the field created by the coils to the order and under the condition established in part (c) is given by

$$B_x = \frac{8I}{5^{3/2}a\epsilon_0 c^2}.$$

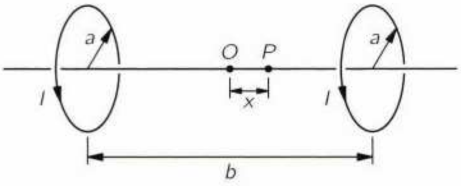

Figure 49-3

49.4 A square loop of wire of edge a carries a current I.

(a) Using the Biot-Savart law show that the magnitude of the magnetic field for a point on the axis of the loop and a distance x from its center is given by

$$B = \frac{4a^2 I}{\pi\epsilon_0 c^2 (4x^2 + a^2)(4x^2 + 2a^2)^{1/2}}.$$

(b) Find the same result using the vector potential.

Hint: leave the vector potential in its integral form and use

$$\frac{\partial}{\partial x}\int_a^b f(x,y)\,dy = \int_a^b \frac{\partial}{\partial x} f(x,y)\,dy,$$
$$\lim_{x\to c}\int f(x,y)\,dy = \int \left[\lim_{x\to c} f(x,y)\right] dy.$$

49.5 Use the vector potential to calculate the magnetic field at any point on the axis of a circular loop of radius a which carries a current I.

49.6 Consider a conducting sphere of radius a which is raised to a potential V. If the sphere is rotated about one axis with an angular velocity ω, show that the magnetic field outside corresponds to that of a dipole with moment

$$\mu = \epsilon_0 \omega V \left(\frac{4}{3}\pi a^3\right).$$

Show also that the field inside is given by

$$B_{\text{int}} = \frac{2\omega V}{3c^2}.$$

These results are exact everywhere inside and outside. If you cannot show this, derive the first result for distances much larger than a, and the second result at the center.

49.7 It has been pointed out that the rotation of the earth could possibly be measured by measuring the potential difference between the center and edge of a charged conducting cylinder placed at the North Pole with its axis pointing to the center of the earth.

(a) Assuming that the cylinder is very long compared to its radius and that measurements are performed near its center so edge effects can be neglected, show that the potential difference can be written in the form

$$V = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{v}{c}\right)^2,$$

where v is the velocity of a point at the outside edge of the cylinder and λ is the surface charge per unit length.

(b) See if you can visualize reasonable parameters of the equipment necessary to measure this effect.

Note: assume that the smallest potential difference we can measure is on the order of 10^{-6} volts and that the maximum magnitude of the electric field in vacuum is about 10^8 volts per meter.

50

The Vector Potential

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 15.

50.1 A superconducting metal has the property that inside the superconductor, both $\boldsymbol{B}$ and $\boldsymbol{E}$ are zero.

(a) For the $\boldsymbol{E}$ field we found that the boundary condition at the surface of the superconductor is that $\boldsymbol{E}$ must be normal to the surface, or in other words, that the tangential component must be zero. What is the analogous boundary condition for the magnetic field $\boldsymbol{B}$?

A small current loop with its magnetic moment $\boldsymbol{\mu}$ oriented at an angle θ from the normal, is located a distance d from an infinite, superconducting sheet, as shown in Fig. 50-1.

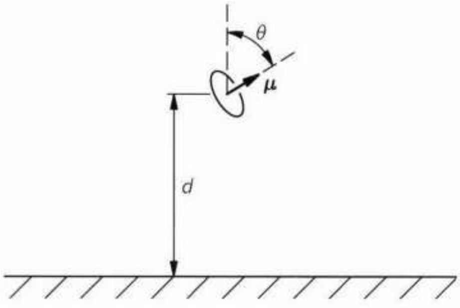

Figure 50-1

(b) Outline in a few sentences a method for finding the magnetic field everywhere, assuming that you already know what the field of a magnetic dipole is.

(c) Find the torque $\boldsymbol{\tau}$ on the dipole as a function of angle θ.

(d) Deduce the equilibrium angles. Which are stable, which unstable?

(e) Find the force, as a function of θ, towards or away from the superconductor.

51

The Laws of Induction

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 17.

51.1 Fig. 51-1 shows a uniform magnetic field $\boldsymbol{B}$ confined to a cylindrical volume of radius r. The field $\boldsymbol{B}$ is decreasing in magnitude at a constant rate of 100 gauss/s. What is the instantaneous acceleration (direction and magnitude) experienced by an electron placed at P_1, at P_2, and at P_3? Assume a is 5.0 cm.

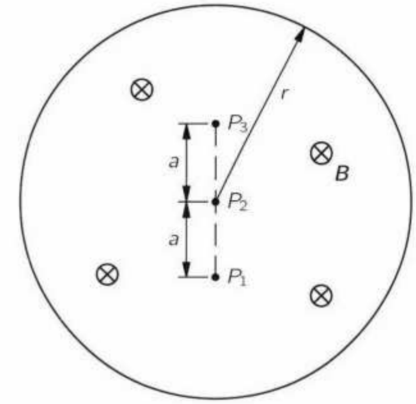

Figure 51-1

51.2 A stiff wire bent into a semicircle of radius r is rotated with a frequency f in a uniform magnetic field as shown in Fig. 51-2. If the internal resistance of the meter M is R_M and the remainder of the circuit has negligible resistance,

(a) what are the amplitude V_0 and frequency ω_V of the induced voltage?

(b) what are the amplitude I_0 and frequency ω_I of the induced current?

Note: Assume that the current is small so that the field produced by the current is small compared to the field $\boldsymbol{B}$.

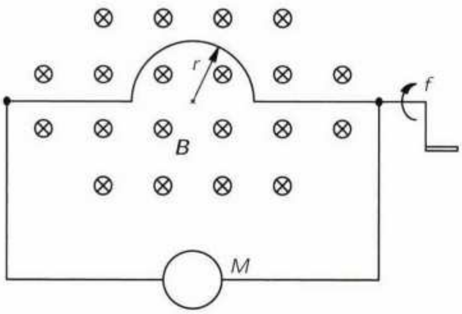

Figure 51-2

51.3 A small circular loop (1) of radius a carries the steady current I. Another loop (2), also of radius a, is placed on the axis through the center of loop (1) and at the distance R, with $R \gg a$. The planes of the two loops are parallel, as shown in Fig. 51-3. Loop (2) is now rotated at the angular velocity ω about one of its diameters. If the circuit of loop (2) is open so that no current flows in it, what is the emf $\mathcal{E}$ generated?

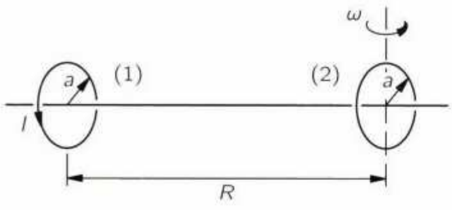

Figure 51-3

51.4 A metal wire of mass m slides without friction on two rails spaced a distance d apart, as shown in Fig. 51-4. The track lies in a vertical uniform magnetic field $\boldsymbol{B}$.

(a) A constant current I flows from generator G along one rail, across the wire, and back down the other rail. Find the velocity $\boldsymbol{v}(t)$ (speed and direction) of the wire as a function of time assuming it to be at rest at $t = 0$.

(b) The generator is replaced by a battery with constant emf $\mathcal{E}$. The velocity of the wire now approaches a constant final value. What is this terminal speed? How, as a function of time, does the speed v approach this value?

Hint: you may assume that the internal resistance of the circuit is purely that of the battery, equal to R.

(c) What is the current I in part (b) above when the terminal speed has been reached?

Note: assume that the field due to the current I is much smaller than the field $\boldsymbol{B}$.

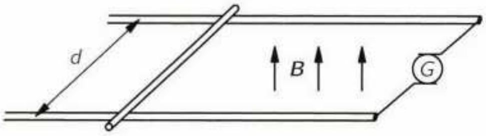

Figure 51-4

51.5 A circuit contains two coils of inductances $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively, connected in series. These coils have mutual inductance $\mathfrak{M}$.

(a) Find the resultant inductance $\mathcal{L}$ of the circuit.

(b) If the wires to one coil are reversed how will the inductance change?

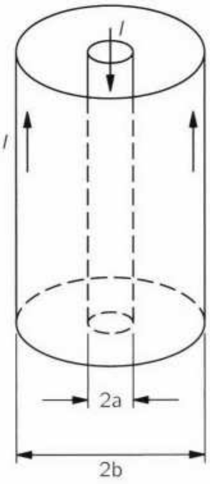

Figure 51-5

51.6 A "coax" cable is made up of a wire that is surrounded by a coaxial conducting cylinder carrying the return current. The inner wire has radius a and the outer shell has radius b, as shown in Fig. 51-5.

(a) Show that if the current in the inner wire is assumed to flow on the surface of the wire, the self-inductance per unit length of such a cable is

$$\mathfrak{L} = \frac{\ln(b/a)}{2\pi\epsilon_0 c^2}.$$

(b) Calculate the self-inductance per unit length if the current is uniformly distributed throughout the central wire. Compare your results to see the importance of your assumption concerning the current distribution.

51.7 A toroidal coil of N turns has a square cross section, each side of the square being of length a. The toroid has an inner radius b (and an outer radius $b + a$).

(a) Show that the self-inductance is

$$\mathfrak{L} = \frac{N^2 a}{2\pi\epsilon_0 c^2} \ln\left(1 + \frac{a}{b}\right).$$

(b) Express in similar terms the mutual inductance $\mathfrak{M}$ of the system formed by the coil and a long, straight wire along the axis of symmetry of the coil. Assume the conductors closing the circuit of which the long straight wire is a part are situated far from the coil, so that their influence may be neglected.

(c) Find the ratio of the self-inductance of the coil to the mutual inductance of the system.

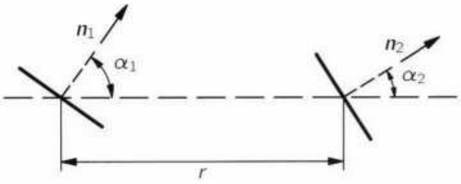

Figure 51-6

51.8 Two plane current loops, each with area A and carrying current I, are placed a distance r apart. The normals to the current loops, $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ make angles α_1 and α_2 with the line joining the loops. The vectors $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ and the line joining the centers are coplanar, as shown in Fig. 51-6.

(a) find the mutual inductance $\mathfrak{M}$ of this system of current loops. Assume the radius of each loop is much smaller than the distance between loops.

(b) using this expression for $\mathfrak{M}$, find the magnitude and direction of the force $\boldsymbol{F}$ between the two loops.

(c) how would the force be different if the current were reversed in one or both of the current loops?

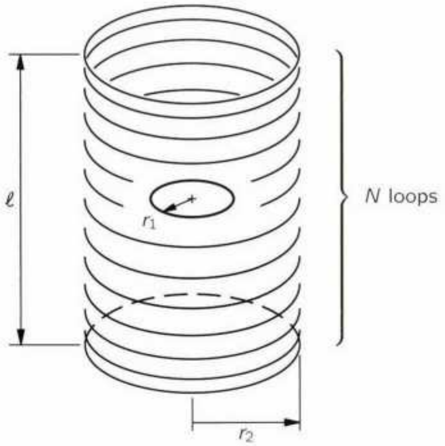

Figure 51-7

51.9 Consider a single loop coil of radius r_1 centered and coaxial with a solenoid coil of length l made up of N loops of radius r_2 where $r_2 \gg r_1$, as shown in Fig. 51-7. If the single loop is called circuit 1 and the solenoid is called circuit 2, find both $\mathfrak{M}_{12}$ and $\mathfrak{M}_{21}$.

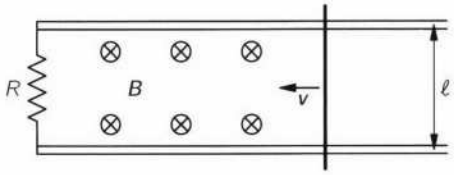

Figure 51-8

51.10 A wire moves with a velocity v while resting on two rails connected at one end through a resistance R, as shown in Fig. 51-8. There is a uniform $\boldsymbol{B}$ field (perpendicular to the page, and going into it).

(a) If $v = 100\,\mathrm{cm\,s^{-1}}$, $l = 10\,\mathrm{cm}$, $B = 0.1\,\mathrm{Wb\,m^{-2}}$ and $R = 10\,\Omega$, what is the current I through R? (Neglect the field caused by the current).

(b) Discuss how your estimate of the induced current would change if the field arising from the induced current were not neglected.

(c) Discuss how your answers would change if the magnet which is producing $\boldsymbol{B}$ were moving with the same velocity v as the wire.

(d) Is the self-inductance of the circuit around which the induced current flows increasing, decreasing, or unchanging with time?

51.11 Consider two coaxial loops of radius a which are separated by a distance d; $d \gg a$. A current $I = K_0 t^2$ is sent through one loop (1), as shown in Fig. 51-9; the resistance of the other loop (2) is R.

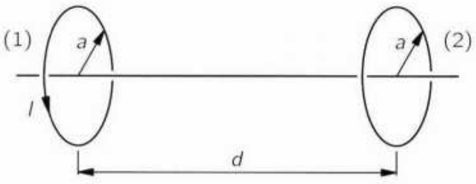

Figure 51-9

(a) Neglecting self-inductance, what is the torque τ on loop (2)?

(b) Show that, if the self-inductance is neglected, the force on loop (2) is

$$F = \frac{24\pi^4 a^8 K_0^2 t^3}{(4\pi\epsilon_0 c^2)^2 d^7 R}.$$

(c) In what direction is the force?

(d) In what way (qualitatively) is the true force and torque different from your estimate; i.e., how does the self-inductance affect the torque and force?

(e) Explain what would happen in parts (a) and (b) above if loop (2) were rotated 90° about an axis normal to the common axis of the loops.

52

Solutions of Maxwell's Equations in Free Space

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 20.

52.1 It is sometimes convenient to consider complex solutions of differential equations.

(a) If we assume the fields vary sinusoidally in time and with the coordinate x (no y- nor z-dependence), show that each component of, for example,

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{+i(\omega t - kx)},$$

satisfies the wave equation. (Remember that the actual field is found by taking the real part of this expression.)

(b) Convince yourself that the real part of $\boldsymbol{E}$ corresponds to a plane wave traveling along the x-axis. In what direction is it going?

(c) Show that the operation $\boldsymbol{\nabla}$ when applied to a function like that in part (a) becomes

$$\boldsymbol{\nabla} = \boldsymbol{e}_x \frac{\partial}{\partial x} = \boldsymbol{e}_x(-ik),$$

where $\boldsymbol{e}_x$ is a unit vector along the x-axis and i is $\sqrt{-1}$; i.e. show that we can replace the $\boldsymbol{\nabla}$ operation by a simple multiplication.

(d) What similar statement can you make about the time derivative applied to a function like that in part (a)?

(e) Using the results of part (c) above write down (by inspection) how Maxwell's equations appear when applied to fields which vary sinusoidally with x and t. What relationship must exist between k and ω?

(f) If the field has the form $\boldsymbol{E} = \boldsymbol{E}_0 e^{+i(\omega t + kx)}$ how do your answers change?

52.2 A plane electromagnetic wave of frequency ω is reflected from a mirror traveling with a velocity v in the same direction as the wave.

(a) Using Maxwell's equations calculate the frequency ω' of the reflected wave as seen by a stationary observer.

(b) Compare this result with that obtained in Section 34-6 of Vol. I using relativity per se.

53

Solutions of Maxwell's Equations with Currents and Charges

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 21.

53.1 Carry out the details of deriving Eq. (21.26) in Vol. II.

53.2 Eq. (21.1) in Vol. II gives an equation for calculating the electric field due to moving charges. Consider a dipole made up of a positive and negative charge oscillating about the origin along the z-axis, as shown in Fig. 53-1; i.e., the motion of the positive charge is $z_+ = (d/2)\cos\omega t$ and that of the negative charge is $z_- = -(d/2)\cos\omega t$. The dipole moment is then defined to be $\boldsymbol{p} = d\cos\omega t\,\boldsymbol{e}_z$. Show that the equation mentioned above can be used to calculate the entire electric field of the dipole,

$$
\begin{aligned}
E_\varphi &= 0,\\
E_\theta &= \frac{p}{4\pi\epsilon_0}\sin\theta\left[\left(-\frac{\omega^2}{c^2r}+\frac{1}{r^3}\right)\cos\omega\left(t-\frac{r}{c}\right)-\frac{\omega}{cr^2}\sin\omega\left(t-\frac{r}{c}\right)\right],\\
E_r &= \frac{p}{2\pi\epsilon_0}\cos\theta\left[\frac{1}{r^3}\cos\omega\left(t-\frac{r}{c}\right)-\frac{\omega}{cr^2}\sin\omega\left(t-\frac{r}{c}\right)\right].
\end{aligned}
$$

Assume the point P is at a distance $r \gg d$ from the dipole.

Hints: $\boldsymbol{e}_{r_+} \approx +\boldsymbol{e}_r$, $d(\boldsymbol{e}_{r_+})/dt$ and $d^2(\boldsymbol{e}_{r_+})/dt^2$ are vectors nearly in the $\boldsymbol{e}_\theta$ direction.

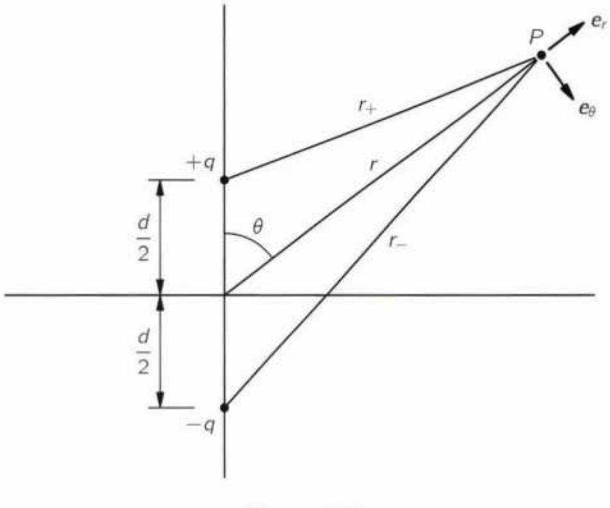

Figure 53-1

53.3 From the symmetry of Maxwell's equations and the form of the electric and magnetic field of an oscillating electric dipole,* deduce the field of an oscillating magnetic dipole. The near field must resemble the field of a dipole formed by a small current loop of radius a ($a \ll c/\omega$) and current $i = i_0\cos\omega t$.

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* The electric fields of an oscillating electric dipole were given in Ex. 53.2; the corresponding magnetic field is found using Eq. (21.1) in Vol. II: $c\boldsymbol{B} = \boldsymbol{e}_{r'} \times \boldsymbol{E}$.

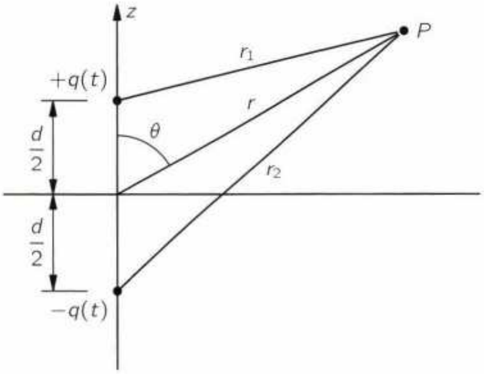

Figure 53-2

53.4 In Ex. 53.2, an oscillating dipole was made up of two moving charges. Another way of producing a dipole is as follows: the dipole is made up of two conducting balls joined by a wire of length d. An oscillating current in the wire is set up which establishes a net charge $\pm q(t)$ at the ends, but leaves the wire neutral; $q(t)$ can be represented as the real part of $Q_0 e^{i\omega t}$.

At any field point P, a distance $r \gg d$ from the dipole, as shown in Fig. 53-2, the integral for the retarded potential gives an exact expression for ϕ,

$$\phi = \frac{Q_0}{4\pi\epsilon_0}\left(\frac{\cos\omega(t - r_1/c)}{r_1} - \frac{\cos\omega(t - r_2/c)}{r_2}\right).$$

(a) Assuming that $\omega d/2c \ll 1$ show that

$$\phi \approx \frac{Q_0 d\cos\theta}{4\pi\epsilon_0 r}\left(\frac{1}{r}\cos\omega(t - r/c) - \frac{\omega}{c}\sin\omega(t - r/c)\right).$$

(b) Further show that

$$A_z \approx -\frac{Q_0\omega d\,\sin\omega(t - r/c)}{4\pi\epsilon_0 c^2 r}.$$

(c) Convince yourself that these potentials give the same electric and magnetic radiation fields (that part of the fields which is proportional to $1/r$) as were found previously.

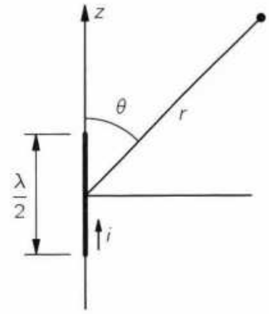

Figure 53-3

53.5 An antenna to be used at a frequency $\omega = 2\pi c/\lambda$ is made up of two collinear wires each one-quarter wavelength long, and is driven at their junction by a sinusoidal voltage of the appropriate frequency. The resulting current distribution in the wires is to a high degree of approximation sinusoidal,

$$i = -i_0\sin\omega t\,\cos\frac{2\pi z}{\lambda},$$

with $-\lambda/4 < z < \lambda/4$. (See Fig. 53-3.) To find the radiation from this antenna it can be regarded as a superposition of many dipoles, each located at a point z, of length Δz with strength varying from dipole to dipole.

(a) Show that the proper dipole strength to use is

$$\Delta p = \left(\frac{i_0}{\omega}\cos\frac{2\pi z}{\lambda}\cos\omega t\right)\Delta z.$$

(b) Show that at large distances ($r \gg c/\omega$), the field of the entire antenna is

$$E_\theta = \frac{i_0}{2\pi\epsilon_0 cr}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\cos\omega\left(t - \frac{r}{c}\right)$$

$$B_\varphi = \frac{1}{c}E_\theta$$

(c) Make a rough polar plot of E_θ vs. θ for both this case and a single dipole and compare.

53.6 A particle with charge q moves in a circle of radius a with a uniform speed v, as shown in Fig. 53-4. Consider the fields when the particle is at point P:

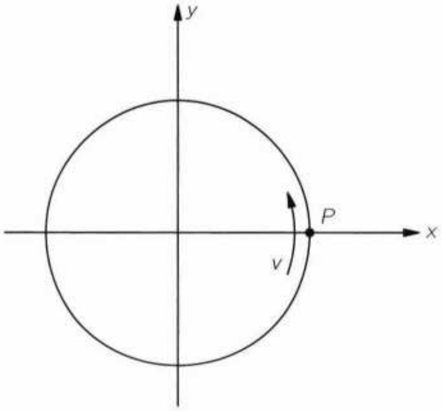

Figure 53-4

(a) Find the potential ϕ at the center of the circle.

(b) Find the vector potential $\boldsymbol{A}$ at the center.

(c) Determine the electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ at the center using the potentials and Eqs. (18.19) and (18.16) in Vol. II.

(d) What is the direction of the electric field with respect to the radius vector to point P?

(e) Also calculate these fields using Eq. (21.1) in Vol. II.

Note: this problem is relativistic. The speed v is not necessarily small compared to c.

54

AC Circuits

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 22.

54.1 (Game) A network in the shape of a cubical frame has terminals at each vertex and $1\,\Omega$ resistances along each edge. Find the effective resistance between all possible sets of terminals.

54.2 (a) Find the current I in the circuit shown in Fig. 54-1.

(b) What is the current I if there is a mutual inductance $\mathfrak{M}$ which couples the two inductances?

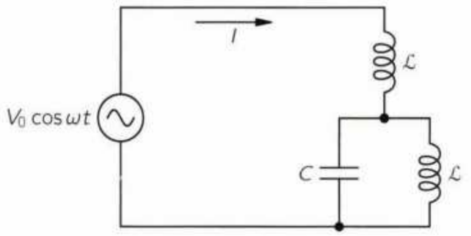

Figure 54-1

54.3 A crossover network for a hi-fi system is shown in Fig. 54-2. The effective resistance of each speaker is R.

(a) Show that if $R^2 = \mathcal{L}/2C$, the input impedance Z (the impedance seen by the generator) is purely resistive and equal to R.

(b) Show that the crossover frequency is given by $\omega_c^2 = 1/\mathcal{L}C$. The crossover frequency ω_c is defined as the frequency for which each speaker receives one-half of the total power.

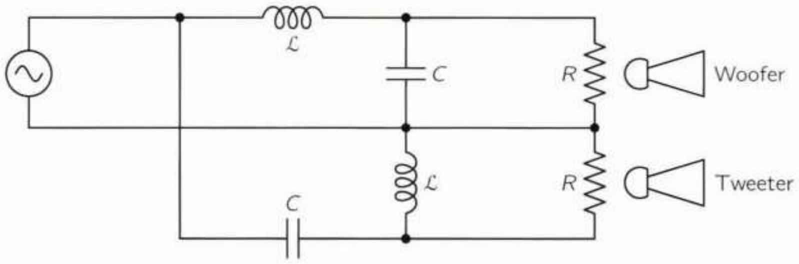

Figure 54-2

54.4 (a) Show that for the circuit shown in Fig. 54-3 the potential difference (voltage) from point a to point b has a magnitude that is independent of ω.

(b) Draw a sketch of the phase of this potential difference as a function of ω.

(c) What would be the effect on the magnitude of the voltage across a–b and on its phase if the source had an internal resistance of $R/10$?

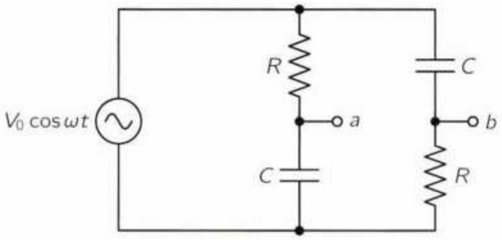

Figure 54-3

54.5 A simple parallel circuit is shown in Fig. 54-4.

(a) Draw a rough graph showing the amplitude of the current I as a function of frequency ω for selected values of L, C, and R.

(b) If $R \gg \sqrt{L/C}$ compare the figure of merit* Q of this circuit to that of a circuit containing the same elements in series, but in which $R \ll \sqrt{L/C}$. Specifically consider the parallel circuit with $R = K\sqrt{L/C}$ and the series circuit with $R = (1/K)\sqrt{L/C}$.

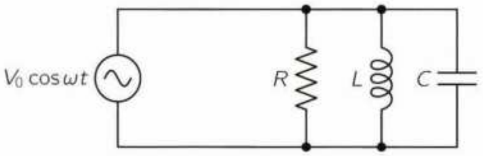

Figure 54-4

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* Recall that $Q = \omega_0/\Delta\omega$, where ω_0 is the resonant frequency and $\Delta\omega$ is the width of the frequency response curve. (See Chapter 23 of Vol. I.) For the parallel circuit take $\Delta\omega$ between points where $|I| = \sqrt{2}I_{\text{min}}$. For the series circuit, take $\Delta\omega$ between points where $|I| = I_{\text{max}}/\sqrt{2}$.

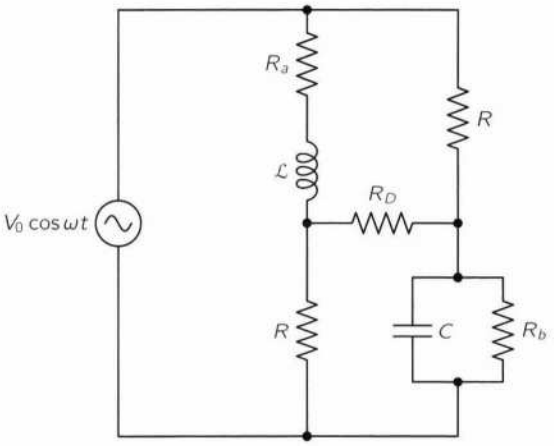


Figure 54-5

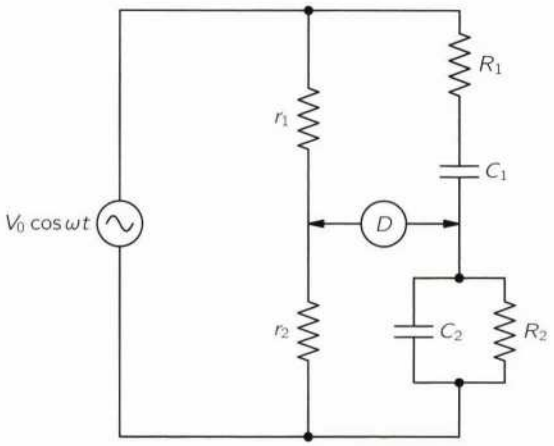


Figure 54-6

54.6 The circuit shown in Fig. 54-5 is a bridge used for measuring inductances. The generator has an alternating emf at the frequency ω. When the bridge is balanced the current through the detector R_D is zero. Find $\mathcal{L}$ in terms of R and C.

54.7 The circuit shown in Fig. 54-6 is a Wien bridge, frequently used in RC oscillators. It is said to be balanced when no current flows through the detector D. Show that balance requires that both of the following equations be simultaneously satisfied:

$$\frac{r_1}{r_2} = \frac{R_1}{R_2} + \frac{C_2}{C_1},$$
$$\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}.$$

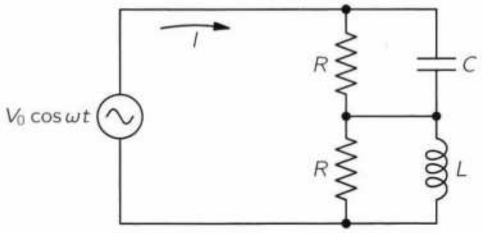


Figure 54-7

54.8 A voltage source $V(t) = V_0 \cos\omega t$ is applied to the circuit shown in Fig. 54-7.

(a) Show that if R, L, and C are selected such that $RC = L/R$ the current I is independent of frequency.

(b) What is the phase difference ϕ between the applied voltage and the voltage across the capacitor-resistor pair (for $RC = L/R$)?

54.9 The circuit shown in Fig. 54-8 is arranged so that the connection to point P_3 could be made to any of the points $P_0, P_1, P_2, \ldots, P_n$.

(a) Find an expression for the average power $\langle P \rangle$ dissipated in R if the connection is made to point P_m, where $0 < m < n$.

(b) If $R = 1{,}000\,\Omega$, $L = 10\,\text{H}$, $C = 20\,\mu\text{F}$, $\omega = 100\,\text{rad/sec}$,

  (1) for what value of m is the power a maximum?

  (2) for $m = 2$ and $V_0 = 100\,\text{V}$, what are the maximum instantaneous voltages $(V_{P_0P_2})_{\max}$ between points P_0 and P_2, and $(V_R)_{\max}$ across R?

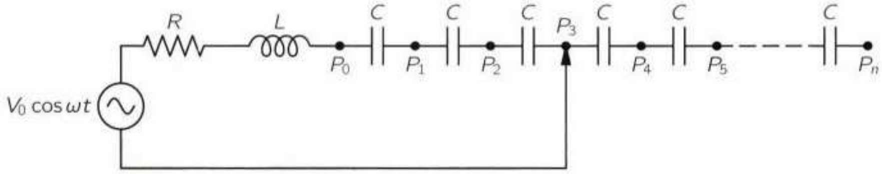
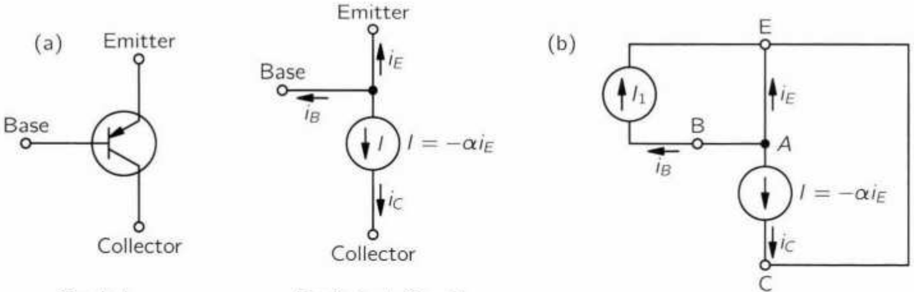


Figure 54-8

Supplement: Transistors

A small signal equivalent for an ideal transistor is shown in Fig. 54-9(a). The source I is an ideal current source and α is a constant characteristic of the transistor. By "a small signal equivalent circuit" we mean one that is equivalent to the transistor *only* for small changes of the voltages and currents around the DC operating voltages and currents of the transistor. Such a circuit does not tell us anything about the appropriate DC voltages at which the transistor should operate. Resistors, which are essential to set these DC voltages, will be *omitted* from the figures in this supplement and the exercises to follow, for the sake of simplicity.



Symbol

Equivalent Circuit

Figure 54-9

First, we will show that transistors are essentially current amplifiers. Let us connect a current source between the base and the emitter and calculate the current in the collector, as shown in Fig. 54-9(b). By examining point A we see that $i_E + i_B + i_C = 0$, and since $i_C = -\alpha i_E$, we have $-i_C/\alpha + i_B + i_C = 0$, or

$$\frac{i_C}{i_B} = \frac{\alpha}{1-\alpha}.$$

Thus the quantity $\beta = \alpha/(1-\alpha)$ is the current gain between collector and base when the emitter is the common element (this configuration is usually called "grounded emitter"). Typically α is around 0.98 to 0.99; thus β runs between 50 and 100. Since α is so close to unity, we will make the approximation

$$\beta \approx \frac{1}{1-\alpha}.$$

54.10 The *emitter follower configuration* is shown in part (a) of Fig. 54-10 and the corresponding equivalent circuit is shown in part (b). R_L represents the load driven by the emitter follower.

(a) Calculate the voltage V_L across the load if a sinusoidal oscillator (represented by voltage $V_0 \cos\omega t$ with a series internal resistance R_I) is connected between the base and the collector (see figure).

(b) Show that if $R_I/R_L \ll \beta$ the ratio V_L/V_0 is unity, independent of β.

(c) Compare V_L/V_0 to the ratio V_L'/V_0' when the oscillator is connected directly across R_L, given $R_I = 100\,\Omega$, $R_L = 10\,\Omega$ and $\beta = 100$.

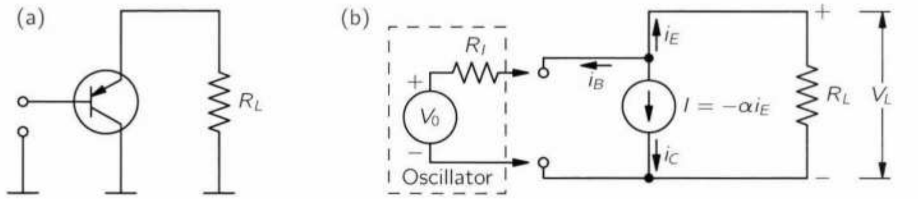


Figure 54-10

(d) Replace R_L by a capacitor C_L. Calculate and plot $|V_L/V_0|$ as a function of frequency for the case when the oscillator is connected to C_L through the emitter follower, and $|V'_L/V'_0|$ for the case when it is connected directly across C_L. (Plot both ratios on the same graph and take $\beta \approx 10$.)

54.11 The *common emitter configuration* is shown with its corresponding equivalent circuit in Fig. 54-11(a).

(a) Calculate the ratio V_L/V_0.

(b) As you will find out, the ratio V_L/V_0 is proportional to β; this is usually undesirable since β depends strongly on temperature and varies widely from transistor to transistor. At the expense of losing gain by adding a resistor R_E in series with the emitter, as shown in Fig. 54-11(b), the dependence on β is greatly reduced. Show that this is so and that the condition for V_L/V_0 to be fairly independent of β is that $R_I/R_E \ll \beta$. (Notice that $\alpha = 1/(1 + 1/\beta) \approx 1$, so α depends very weakly on β. Thus α is a stable parameter for transistors.)

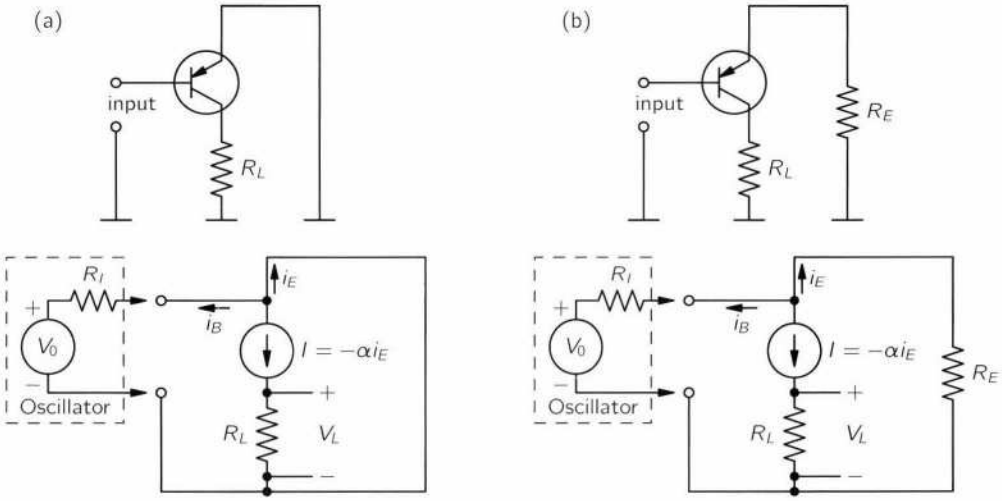


Figure 54-11

54.12 The *common base configuration* is shown in Fig. 54-12. This circuit is very useful when it is desirable to have a unit current gain and to increase the source impedance.

(a) Calculate the current gain I_L/I_0.

(b) Calculate the impedance Z_{out} seen by R_L (that is, remove R_L and calculate the ratio V_L/I_L).

(c) If the circuit is driven by the sinusoidal oscillator used in Ex. 54.10 give the equivalent circuit as seen by R_L, as shown in Fig. 54-13, and calculate the value of the source in terms of V_0, R_I, and α.

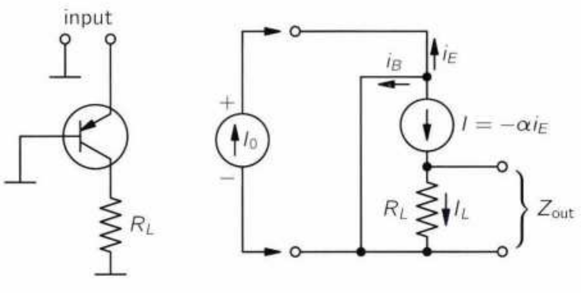


Figure 54-12

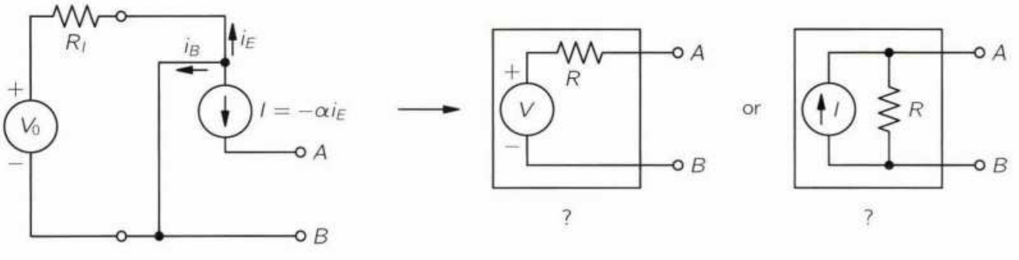


Figure 54-13

Summary: the characteristics of the three basic transistor configurations are summarized in the following table:

Configuration	Voltage gain	Current gain	$Z_{\text{out}}/Z_{\text{in}}$
Emitter follower	≈ 1	usually > 1	< 1
Common emitter	usually > 1	usually > 1	
Common base	usually > 1	≈ 1	usually > 1

54.13 Once excited the circuit shown in part (a) of Fig. 54-14 would keep oscillating forever at its resonant frequency. However, there are always energy losses due to finite resistance in the wires as well as electromagnetic radiation. A physical circuit would look much more like the one shown in part (b) of the figure. Due to the presence of resistor R, oscillations would not be maintained in such a circuit. On the other hand, if we add a "*negative resistor*" $-R$ in series with R, as shown in part (c) of the figure, then the oscillations would be maintained. The negative resistance provides the energy dissipated in R. As mentioned in Section 22-8 of Vol. II, circuits can be made with transistors (and other devices—tubes, tunnel diodes, etc.) whose effective impedance has a negative real part. These circuits are useful for making oscillators. Part (d) of Fig. 54-14 shows a circuit whose terminals A and B can present a negative resistance. (The transformer is an ideal transformer whose turn ratio is $1:n$. The input and output voltages are given by $V_{\text{out}} = nV_{\text{in}}$ and $i_{\text{out}} = i_{\text{in}}/n$.)

(a) Calculate the impedance Z_{AB} between A and B.

(b) Give the condition for the resistance between A and B to be negative, and indicate the appropriate polarity of the transformer.

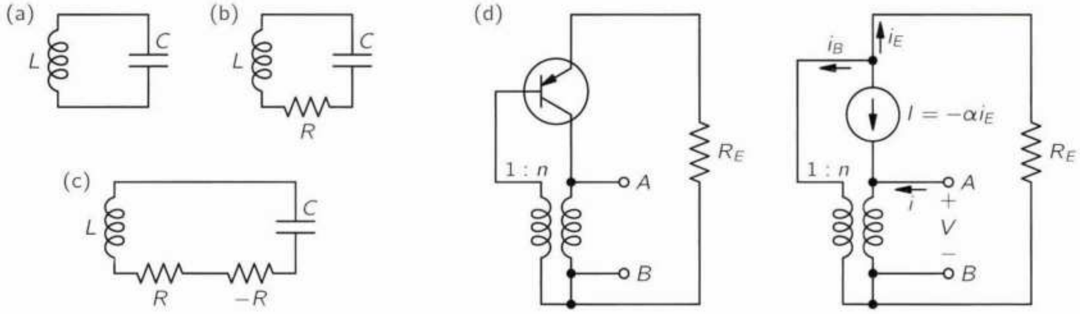


Figure 54-14

55

Cavity Resonators

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 23.

55.1 (a) Find the approximate resonant frequency ω of the cavity shown in Fig. 55-1. Assume $d \ll a$ and $d \ll (b - a)$.

(b) What are the main effects which you have neglected in part (a) above?

(c) If the cavity is cooled uniformly (i.e., so the whole cavity is at the same temperature) does thermal contraction lead to an increase, a decrease, or no change in the resonant frequency?

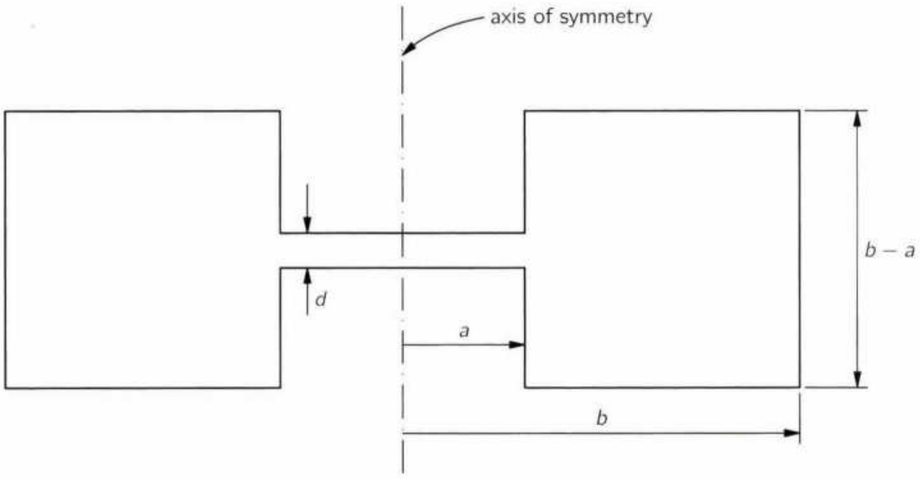


Figure 55-1

56

Waveguides

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 24.

56.1 A transmission line has inductance L_0 per unit length and capacitance C_0 per unit length. If the voltage and current are changing slowly (corresponding to transmission of signals with wavelength long compared to the line spacing),

(a) show that the governing equations are

$$\frac{\partial V}{\partial x} = -L_0 \frac{\partial I}{\partial t},$$
$$\frac{\partial I}{\partial x} = -C_0 \frac{\partial V}{\partial t}.$$

(b) Hence show that I and V both satisfy the wave equations

$$\frac{\partial^2 I}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 I}{\partial t^2},$$
$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 V}{\partial t^2},$$

with

$$v^2 = \frac{1}{L_0 C_0}.$$

Note: the assumption regarding slowly varying signals is not necessary, but the proof of this is beyond the scope of this chapter.

56.2 The characteristic impedance of a transmission line is $z_0 = \sqrt{L_0/C_0}$ where L_0 is the inductance per unit length and C_0 is the capacitance per unit length. Show that for a transmission line consisting of two thin strips of width b and a distance a apart ($a \ll b$),

$$z_0 \approx \frac{1}{\epsilon_0 c}\,\frac{a}{b}.$$

56.3 A cavity is made by putting conducting plates across the ends of a section of a cylindrical coaxial line of length l and central conductor radius a.

(a) Find the frequency ω_1 of the lowest mode for which the electric field is always radial.

(b) Give an expression for the n^{th} mode of the electric field $\boldsymbol{E}$.

(c) Compare the resonant frequency ω_1 to $\omega_0 = 1/\sqrt{LC}$ where L and C are the inductance and capacitance of a length l of the coaxial line.

56.4 A rectangular waveguide made of perfectly conducting material has sides of length a and b as shown in Fig. 56-1. The ends of a section of length l are covered with plates of conducting material; i.e., the waveguide is effectively a resonant cavity. If the electric field is given by the real part of

$$\boldsymbol{E}(x, y, z, t) = E_0(x, z)e^{i\omega t}\boldsymbol{e}_y,$$

(a) what is $E_0(x, z)$ for the cavity mode with the lowest resonant frequency?

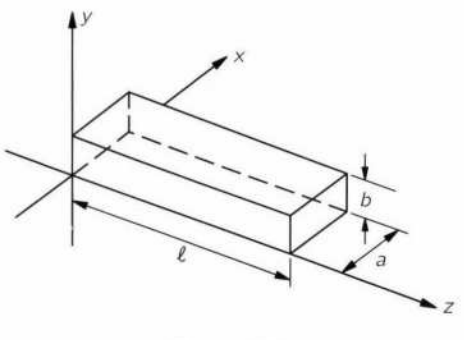


Figure 56-1

(b) what is this lowest resonant frequency ω_0 of the cavity?

56.5 A coaxial cable is composed of two concentric conducting cylinders. One end ($x = 0$) is connected to a voltage generator which produces a voltage $V(t) = V_0 \cos\omega t$. The other end of the cable ($x = l$) is covered with a conducting plate. The inductance per unit length is L_0 and the capacitance is C_0. The length of the cable is $l = 5\pi c/2\omega$, where c is the velocity of light.

(a) Sketch the voltage between the conductors as a function of the distance x.

(b) Specify the values of x for which the magnitude of the voltage is maximum.

(c) Write an expression for the forward going and reflected traveling waves, $V_f(x,t)$ and $V_r(x,t)$, which make up the voltage across the conductors.

(d) What is the current $I(x,t)$ at $x = 0$, $x = l/2$, and $x = l$?

(e) If the voltage source is an ideal generator whose shaft turns with an angular velocity ω, what average torque $\langle\tau\rangle$ must be applied to the generator?

56.6 A transmission line is terminated at $x = l$ by an impedance Z_T, as shown shown in Fig. 56-2.

(a) Show that the "sending end" impedance ($x = 0$) is given by

$$Z_S = iZ_0\,\frac{\tan(\omega l\sqrt{LC}) - i(Z_T/Z_0)}{1 + i(Z_T/Z_0)\tan(\omega l\sqrt{LC})}$$

where $Z_0 = \sqrt{L/C}$ is the characteristic impedance for the line.

What is Z_S if

(b) $Z_T = 0$?

(c) $Z_T = \infty$?

(d) $Z_T = Z_0$?

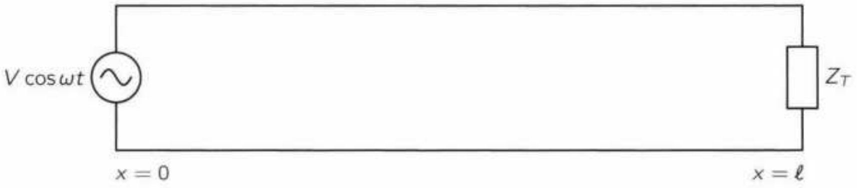


Figure 56-2

56.7 A transmission line with a characteristic impedance Z_1 is connected to a transmission line with a characteristic impedance Z_2. If the system is being driven by a voltage V_{incident} from a generator connected to the first line (Z_1) show that the "reflection coefficient" is given by

$$\frac{V_{\text{reflected}}}{V_{\text{incident}}} = \frac{Z_2 - Z_1}{Z_1 + Z_2},$$

while the "transmission coefficient" is given by

$$\frac{V_{\text{transmitted}}}{V_{\text{incident}}} = \frac{2Z_2}{Z_1 + Z_2}.$$

56.8 At JPL's Goldstone Tracking Station the electronics cage is separated from the feed of the 85 foot receiver antenna by about 40 feet of wave guide. The inside dimensions of the wave guide are 5.75 by 11.5 inches. If a 960 MHz carrier is used, compare the signal velocity v_g with the velocity in free space c.

56.9 The electric fields inside of waveguides which are described in Chapter 24 of Vol. II have the property that the component of the electric field in the direction of propagation is zero; i.e., the electric field is transverse. (Modes of propagation such as these are therefore called TE, or transverse electric, modes.) There are also modes called TM modes in which there is no magnetic field in the direction of propagation. For the rectangular wave guide shown in Figs. 24-3 and 24-4 in Vol. II, the vector potential of the TM modes is given by

$$A = \boldsymbol{e}_z \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\, e^{i(\omega t - k_z z)},$$

where

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}.$$

(a) Satisfy yourself that the magnetic fields found from this are really transverse.

(b) Show that the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields satisfy the wave equation and the proper boundary conditions.

Hint: We require that

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t},$$
$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A},$$

where

$$\boldsymbol{\nabla} \cdot \boldsymbol{A} = -\frac{1}{c^2}\frac{\partial \phi}{\partial t}.$$

(c) Show that the mn-mode is not propagated if

$$\omega < c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

57

Electrodynamics in Relativistic Notation

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 25. In the following exercises the units are such that $c = 1$.

57.1 Write in four-vector form:

(a) $(\phi^2 - \boldsymbol{A}^2)$.

(b) $(\boldsymbol{A} \cdot \boldsymbol{j} - \rho\phi)$.

57.2 In the Compton effect, a stationary electron is hit by a photon, resulting in a change of momentum of each of them, as shown in Fig. 57-1. Find the energy of the emitted photon E^{γ_2} in terms of its incident energy E^{γ_1} and the angle of deviation from its initial path θ.

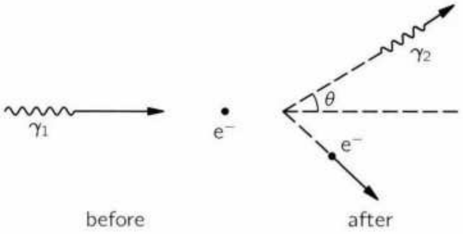


Figure 57-1

57.3 A positron (e^+) can be made by bombarding a stationary electron (e^-) with a photon (γ),

$$\gamma + e^- \to e^- + e^+ + e^-.$$

What is the minimum photon energy E^γ? Use four-vectors and invariant combinations of them wherever possible.

57.4 An electron-positron pair can be produced by a photon through the reaction

$$\gamma + e^- \to e^- + (e^+ + e^-).$$

It is impossible, however, for the reaction

$$\gamma \to e^+ + e^-$$

to occur for a single isolated photon even though the photon energy is larger than twice the electron rest mass and charge is conserved. Using four-vectors show that this is true.

57.5 A particle of mass m_a at rest is struck by another particle of mass m_b and momentum p_b. After a totally inelastic collision they coalesce to form a single new particle.

(a) What is its mass m_c and velocity v_c?

(b) Compare your results with the values that would be calculated non-relativistically.

58

Lorentz Transformations of the Fields

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 26. In the following exercises the units are such that $c = 1$.

58.1 Write out and evaluate

$$\nabla_\mu F_{\mu\nu}$$

for the electromagnetic field tensor $F_{\mu\nu}$.

58.2 (a) Find the four-vector f_μ whose three-vector part is

$$\rho \boldsymbol{E} + \boldsymbol{j} \times \boldsymbol{B}.$$

(b) What is the physical meaning of both time and space components of this four-vector?

58.3 (a) Show that $\boldsymbol{E}^2 - \boldsymbol{B}^2$ and $\boldsymbol{E} \cdot \boldsymbol{B}$ are invariant under Lorentz transformations.

Note: if $\boldsymbol{E}$ and $\boldsymbol{B}$ form an acute angle in one frame, they do so in all frames.

(b) For what important physical phenomenon are both of these invariants equal to zero?

58.4 If $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields at a certain point in space in a given frame of reference, determine the velocity of another frame in which the electric and magnetic fields will be parallel. There are many frames which have this property—if we have found one of them then the same property will be had by any other frame moving relative to the first with a velocity parallel to the common direction of $\boldsymbol{E}$ and $\boldsymbol{B}$. Therefore, we have a choice, and it is sufficient and convenient to find the frame which has a velocity $\boldsymbol{v}$ perpendicular to both fields.

58.5 In Chapter 26 of Vol. II the fields due to a charged particle moving with uniform velocity were obtained by the transformation of the potentials of a stationary charge to a moving frame. The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ were obtained from A_μ in the usual way. Now, find the fields by starting with the fields from a stationary charge and using the transformation laws of the fields.

58.6 Show that the electric and magnetic fields of a charge moving with uniform velocity $\boldsymbol{v}$ can be written

$$\boldsymbol{E} = \frac{q\boldsymbol{r}}{4\pi\epsilon_0 r^3} \frac{1 - v^2}{(1 - v^2 \sin^2\theta)^{3/2}}$$

$$\boldsymbol{B} = \frac{q}{4\pi\epsilon_0} \frac{\boldsymbol{v} \times \boldsymbol{r}}{r^3} \frac{1 - v^2}{(1 - v^2 \sin^2\theta)^{3/2}}$$

where $\boldsymbol{r}$ is the radius vector from the present position of the particle to the observer and θ is the angle between $\boldsymbol{r}$ and $\boldsymbol{v}$).

58.7 A very long straight wire carries a current I produced by electrons moving at velocity $\boldsymbol{v}$. Stationary positive ions in the wire make the total charge density vanish.

(a) Find the fields outside the wire in a frame stationary with respect to the wire.

(b) Transform the fields to a frame moving with the electrons.

Hint: In Chapter 13 of Vol. II the electric field observed from this moving frame was obtained by another method; see Eq. (13.28).

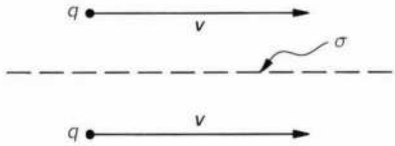


Figure 58-1

58.8 Two electrons with equal velocities $\boldsymbol{v}$ are moving side by side the distance a apart. Midway between them is an infinite sheet of fixed positive charges with a surface charge density σ, as shown in Fig. 58-1.

(a) How large must σ be in order that the electrons maintain the separation a?

(b) Compare the charge density σ_h needed if the electrons have an energy of 500 MeV to the density σ_l needed if they are moving at a very low velocity.

58.9 If f_μ is the four vector force acting on a particle, and u_μ is the four-vector velocity, show that

$$f_\mu u_\mu = 0.$$

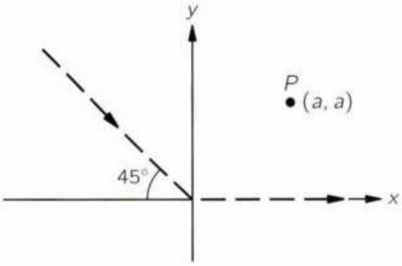


Figure 58-2

58.10 A particle of charge q moves in the xy-plane at constant speed v, along the trajectory shown by the dashed line in Fig. 58-2. (It scatters at the origin). The speed remains constant throughout. At $t = t_1$ it is at $x = a$, $y = 0$.

(a) The point P is at $x = y = a$. Find the electric field at P at time t_1 if $v/c = 0.5$ (c is speed of light).

(b) If in part (a) above the particle trajectory before the scattering were down the y-axis, how would your answer change?

59

Field Energy and Field Momentum

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 27.

59.1 Using the technique used to derive Eq. (27.11) in Vol. II, find equivalent expressions for

(a) $\boldsymbol{\nabla} \times (\boldsymbol{A} \times \boldsymbol{B})$

(b) $\boldsymbol{\nabla}(\boldsymbol{A} \cdot \boldsymbol{B})$

59.2 How many megatons of energy U are contained in the magnetic field of the earth external to the earth? Assume that the earth's field is a dipole with a strength of about 2/3 gauss at the equator. A megaton is the energy released by the explosion of 1-million tons of TNT or 4.2×10^{15} J. In view of your answer, consider how much you think a one megaton hydrogen bomb exploded high in the atmosphere would disturb the earth's field.

59.3 For a long straight wire of resistance R per unit length, calculate the flux of the Poynting vector $\boldsymbol{S}$ at the surface of the wire when the wire carries a current I. Compare this with the heating calculated using Ohm's law.

59.4 A long coaxial cable is made of two perfectly conducting concentric cylinders of radii a and b $(a < b)$. One end of the cable is connected to a battery whose terminal voltage is V, and the other end is connected to a resistance R so that there is a current $I = V/R$. Compute, using the Poynting vector $\boldsymbol{S}$, the rate of energy flow dU/dt.

59.5 The average power radiated by a broadcasting station is 10 kW.

(a) What is the magnitude of the average Poynting vector $|\langle \boldsymbol{S} \rangle|$ at points on the surface of the earth 10 km distant?

Note: at this distance, the waves can be considered plane. It is reasonable to assume that the power is radiated by a quarter wavelength $(\lambda/4)$ antenna above a perfectly conducting plane.

(b) Find the maximum electric and magnetic intensities, $|\boldsymbol{E}_{\text{max}}|$ and $|\boldsymbol{B}_{\text{max}}|$.

59.6 The fields of the lowest TE mode of the rectangular wave guide shown in Figs. 24-3 through 24-6 of Vol. II are given by

$$\boldsymbol{E} = E_0 \sin \frac{\pi x}{a} \cos(\omega t - k_z z)\boldsymbol{e}_y$$

$$\boldsymbol{B} = -E_0 \frac{k_z}{\omega} \sin \frac{\pi x}{a} \cos(\omega t - k_z z)\boldsymbol{e}_x - E_0 \frac{\pi}{\omega a} \cos \frac{\pi x}{a} \sin(\omega t - k_z z)\boldsymbol{e}_z$$

(a) Show that the solution given above satisfies the boundary conditions for the problem.

(b) Calculate the Poynting vector $\boldsymbol{S}$ and the energy density u.

(c) Calculate the average rate of energy flow $\langle dU/dt \rangle$ across any plane perpendicular to the z-axis.

(d) Calculate the average energy density $\langle u \rangle_{\text{time avg}}$ in the wave guide.

(e) Use the results of parts (c) and (d) above to calculate the average velocity with which the energy is propagated, v_g. Show that this result is the same as the group velocity, Eq. (24.27) in Vol. II.

59.7 (a) Find the rate of energy flow per unit area from an oscillating dipole with moment $\boldsymbol{p}\cos\omega t$.

Hint: keep only the radiation terms - i.e., those which drop off as $1/r$.

(b) By integrating over the area of a large sphere centered on the dipole, show that the average power radiated is

$$\langle P\rangle = \frac{1}{3}\frac{p^2}{(4\pi\epsilon_0 c^2)}\frac{\omega^4}{c}.$$

59.8 A plane light wave is incident upon a free electron. The electron oscillates under the influence of the $\boldsymbol{E}$ field. Calculate the ratio of the average energy radiated per unit time by the electron $\langle P_{\text{rad}}\rangle$ to the average light energy incident per unit area per unit time $\langle P_{\text{incident}}\rangle$.

Hint: assume that the light wave is of low frequency and neglect the effect of the $\boldsymbol{B}$ field of the wave on the electron.

59.9 A dust particle in the solar system experiences two forces: the gravitational force of the sun, and the radiation force of light directed away from the sun. Since the gravity force is proportional to the volume of the particle and the radiation force is proportional to its cross section area, there will be a particle size for which these two forces are balanced.

(a) Assuming a spherical dust particle with density $\approx 10^3\,\text{kg}\,\text{m}^{-3}$ that absorbs all the radiation incident upon it, find the radius a for which the forces balance.

(b) An explanation for why a comet's tail points away from the sun has been based on the above phenomenon, assuming that the tail consists of small particles, perhaps even gas molecules. Is it a reasonable theory?

Note: the sun's mass is 2×10^{30} kg, and it radiates energy at the rate of 4×10^{26} W.

59.10 An "air-core" toroid of mean radius R and cross-sectional area πr^2 is wound with N turns of wire ($r \ll R$). A current with the time dependence $I(t) = Kt$ is turned on at $t = 0$.

(a) Directly from the magnetic field, find the energy stored in the magnetic field $U_B(t)$.

(b) Find the direction and magnitude of the Poynting vector at a point just inside the toroid $\boldsymbol{S}(t)$.

(c) Using the Poynting vector, find the rate of change of the field energy inside the toroid dU/dt. Check your answer with that of part (a) above.

59.11 The "paradox" posed in Section 17-4 of Vol. II in connection with Faraday's law has evoked considerable interest. By choosing a simple geometry, you can explicitly resolve it. Consider a magnetic dipole of moment $\boldsymbol{\mu}$ at the center of a thin non-conducting spherical shell uniformly coated with charge.

(a) If the sphere is charged to a potential V, show that after the current in the dipole is turned off the sphere will have angular momentum

$$\boldsymbol{L} = \frac{2}{3}\frac{\boldsymbol{\mu}V}{c^2}.$$

(Neglect the magnetic field caused by rotation of the charged sphere.)

(b) Compare $\boldsymbol{L}$ in part (a) above with the angular momentum stored in the fields before the current is turned off.

(c) Do you think it would be feasible to make this into a sophomore laboratory experiment?

(d) Would it work with a metal sphere?

Hint: In using Faraday's law it is most convenient to calculate magnetic flux with the vector potential of a dipole, as given by Eq. (14.34) in Vol. II.

60

Electromagnetic Mass

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 28.

60.1 If the rest mass of the electron m_e is identified with the electrostatic energy of its charge and if the charge is uniformly distributed in the volume of a sphere, calculate the radius a. Compare with the result given by Eq. (28.2) in Vol. II.

60.2 It is well known that an electron in addition to charge q_e and mass m_e has an angular momentum (spin) and a magnetic moment related according to

$$\frac{\text{angular momentum}}{\text{magnetic moment}} = \frac{m_e}{q_e}.$$

This is correct to about 0.1 %. Suppose the mass is given by Eq. (28.4) in Vol. II,

$$m_e = \frac{2}{3}\frac{e^2}{ac^2},$$

with $e^2 = q_e^2/4\pi\epsilon_0$.

(a) Take a uniformly charged spherical shell with charge q and radius a and place a magnetic dipole of strength μ at the center. Show that the angular momentum of the electromagnetic field is

$$L = \frac{2}{3}\frac{q\mu}{4\pi\epsilon_0 c^2}\frac{1}{a}.$$

(b) Find the ratio of angular momentum to magnetic moment and compare with the value m/q quoted above.

(c) Given that μ_z for an electron is $(\hbar q_e/2m_e)$, calculate the maximum surface velocity $v_{\max}$ of the spinning electron to give this magnetic moment. Make any comment you feel suitable.

Note: The quantity $4\pi\epsilon_0 c\hbar/q_e^2 = 1/\alpha$ has the approximate value 137.

61

The Motion of Charges in Electric and Magnetic Fields

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 29.

61.1 A charged particle (charge q, mass m) is initially at rest at the origin. It is acted upon by a constant electric field in the x-direction.

(a) Calculate the velocity $\boldsymbol{v}$ and position (x, y, z) of the particle as a function of time (relativistically).

(b) If the particle has an initial velocity v_0 in the y-direction, how is your answer modified?

61.2 In a proton cyclotron the protons travel in circular paths in a uniform magnetic field $\boldsymbol{B}$.

(a) Find the "cyclotron frequency" ω, the angular velocity of the proton as a function of its charge q_p, mass m_p and the magnitude of the field B, at low energies.

(b) How will ω change as the total energy increases?

(c) At what *kinetic* energy T has ω changed by 1 percent?

61.3 At $t = 0$ a particle with mass m, charge q is located at the origin at rest. There is a uniform $\boldsymbol{E}$ field in the y-direction and a uniform $\boldsymbol{B}$ field in the z-direction.

(a) Find the subsequent motion of the particle, $x(t)$, $y(t)$, $z(t)$ assuming non-relativistic motion. What restriction on $\boldsymbol{E}$ and $\boldsymbol{B}$ does this imply?

(b) Can you suggest what the relativistic motion would be like? What happens if $E/B > c$?

(c) If we put a plate in the xz-plane at $y = 0$ and another parallel one at $y = d$ with potential difference $V_0 = Ed$ and apply a magnetic field parallel to the plates, we have what is called a magnetron. If electrons are emitted from the negative cathode essentially at rest, how strong must the magnetic field B be so that the electrons can't reach the positive anode?

61.4 The principle of alternating-gradient focusing can be illustrated by the optical analog shown in Fig. 61-1. Even though the lenses have equal focal lengths, the combination has a converging action under certain circumstances.

(a) For parallel incoming light, determine l as a function of d.

(b) Under what conditions is the image real or virtual?

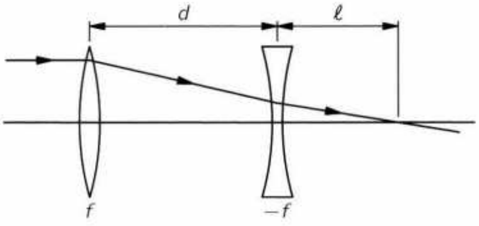


Figure 61-1

62

Refractive Index of Dense Materials

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 32.

62.1 Show that in a non-polar material, the square of the index of refraction n at low frequencies is equal to the dielectric constant κ.

62.2 At a frequency of about 6 MHz, the ionosphere becomes transparent. Estimate the electron density ρ in the ionosphere using the free electron model.

62.3 An electric field applied to a metal is held constant for a long time, and then is suddenly turned off. Using the free electron model of a metal show that the relaxation time (i.e., the time for the drift velocity to drop to $1/e$ of its initial value) is equal to 2τ, twice the mean time between collisions.

62.4 In a metal there are plane-wave solutions to Maxwell's equations with the form

$$E_x = E_0 e^{i(\omega t - kz)},$$

where k is a complex number. For low frequencies

$$k = (1-i)\sqrt{\frac{\sigma\omega}{2\epsilon_0 c^2}},$$

where σ is the conductivity of the metal.

(a) Write an expression for the magnetic field $\boldsymbol{B}$ associated with such a wave.

(b) What is the angle θ between $\boldsymbol{E}$ and $\boldsymbol{B}$?

(c) What is the ratio of the peak value of $\boldsymbol{B}$ to the peak value of $\boldsymbol{E}$ at any given value of z?

(d) What is the phase difference ϕ between $\boldsymbol{E}$ and $\boldsymbol{B}$?

Hint: If the maximum of $\boldsymbol{E}$ occurs at t_1 and the maximum of $\boldsymbol{B}$ occurs at t_2, the phase difference is defined as $\pm\omega(t_1 - t_2)$.

62.5 Equation (32.50) in Vol. II suggests that the ultraviolet cut-off of a metal (i.e., the value of ω at which index n changes from real to imaginary) is quite sharp. Experiments show that this cut-off is not sharply defined. Show by means of a better approximation for n^2 that this experimental result is really in agreement with the theory.

63

Reflection from Surfaces

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 33.

63.1 (a) Determine the transmission coefficient I_t/I_i for a plane electromagnetic wave passing through three dielectric media as shown in Fig. 63-1.

(b) Show that if $n_2 = \sqrt{n_1 n_3}$ and $l = \lambda_2/4$ the transmission ratio is unity. (This is the reason for "coating" lenses in good cameras and binoculars.)

(c) In binoculars to be used with ordinary white light with median wavelength $\lambda_1 = 5500\,\text{Å}$, what should be the thickness l of the coating?

Note: assume the index of refraction of the glass is 1.5.

(d) If it is only possible to coat one side of a lens, does it matter which side is coated? Why?

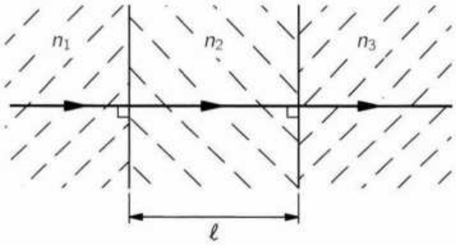


Figure 63-1

63.2 A beam of light with a wavelength 4500 Å (in vacuum) is incident on a prism and totally reflected through 90°, as shown in Fig. 63-2. The index of refraction of the prism is 1.6.

(a) Compute the distance d beyond the long side of the prism at which the electric field strength is reduced to $1/e$ of its value just at the surface, assuming the light is polarized so that $\boldsymbol{E}$ is perpendicular to the plane of incidence.

(b) Is your answer changed if $\boldsymbol{E}$ lies in the plane of incidence?

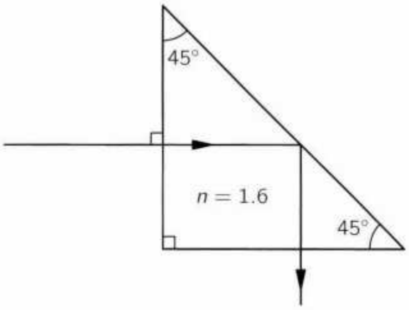


Figure 63-2

64

The Magnetism of Matter

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 34.

64.1 A charged particle of mass m moves in a plane perpendicular to a uniform magnetic field $\boldsymbol{B}$.

(a) Show that if $\boldsymbol{B}$ changes slowly in time, the magnetic moment $\boldsymbol{\mu}$ produced by the orbital motion remains constant.

(b) What, precisely, do we mean by "slowly" in part (a)?

65

Paramagnetism and Magnetic Resonance

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 35.

65.1 In a low-energy cyclotron, protons go once around their circular orbit in a time T of about 0.13 microseconds. A proton nuclear magnetic resonance experiment with the protons in the *same* magnetic field shows a resonance at 21 MHz. Find the g-value of the proton from these data.

65.2 Derive Eq. (35.9) from Vol. II in the manner suggested in the text. Can you reconcile this derivation with the proof in Chapter 34 that there can be no paramagnetism based strictly on classical physics?

65.3 A paramagnetic salt contains 10^{22} ions per cm^3, with magnetic moment of 1 Bohr magneton. It is placed in a field of uniform magnetic induction of 10,000 gauss ($1\,\text{Wb}\,\text{m}^{-2}$). Calculate the percentage of excess parallel spins $(N_{\text{up}} - N_{\text{down}})/N$ at both room temperature (300 K) and at liquid helium temperature (4.2 K).

65.4 Derive an equation for quantum mechanical paramagnetism for particles of spin one, following the derivation in Chapter 35 of Vol. II for spin one-half.

66

Ferromagnetism

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 36.

66.1 A uniformly magnetized sphere has a total magnetic moment of $\mu = (4/3)\pi a^3 M$, where a is the radius, and M the magnetization.

(a) Calculate the equivalent surface current per unit length $\boldsymbol{K}$ which can replace this sphere as far as external effects are concerned.

(b) Show that this current distribution has the same total magnetic moment.

66.2 The magnet frame shown in part (a) of Fig. 66-1 is wrapped by 2,150 turns of wire which carries a current of 5 A. The iron has a uniform width (out of the page) of 28 cm and has the B vs. H curve shown in part (b) of the figure.

(a) Make an estimate of how big a magnetic field B_{gap} will be obtained in the air gap.

(b) What are the main effects you have neglected?

Hint: since the B-H curve is empirical and non-linear do not be surprised if the problem cannot be solved analytically or exactly.

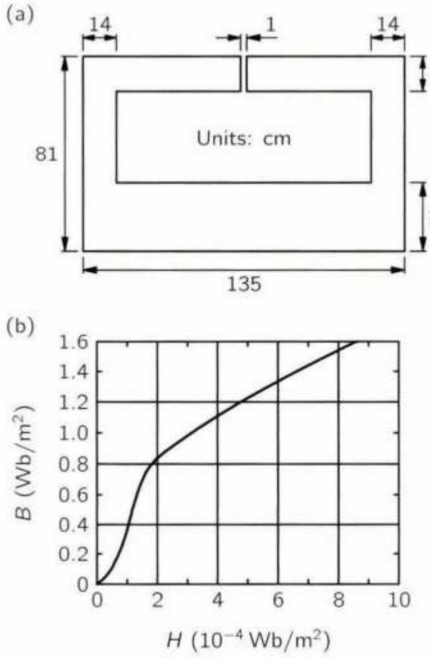


Figure 66-1

66.3 A magnetic flux is produced in an air gap by use of a bar of permanent magnet material and pole pieces of soft iron, as shown in part (a) of Fig. 66-2. The characteristics of the permanent magnet material are shown in part (b). The material is first magnetized to point P by passing a large current through an external coil. Assuming the soft iron to have infinite permeability and neglecting fringing of the flux, find the flux density in the gap B_{gap} after the current has been shut off.

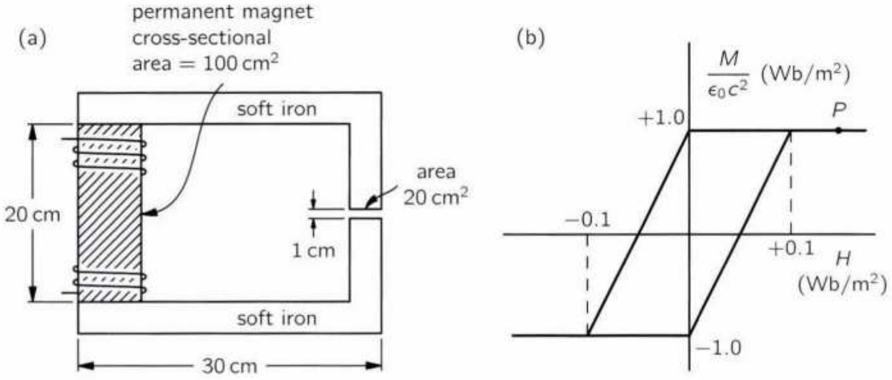


Figure 66-2

66.4 A very long cylindrical iron rod is permanently magnetized with a uniform magnetization $\boldsymbol{M}$ which is pointing along the axis of the cylinder.

(a) Neglecting any end effects, find $\boldsymbol{B}$ and $\boldsymbol{H}$ in the iron.

(b) If there is a long needle-shaped cavity lined up along the axis, what is $\boldsymbol{B}_{\text{cav}}$ and $\boldsymbol{H}_{\text{cav}}$ in the center of the cavity?

67

Elasticity

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 38. Use the following data for these exercises:

	specific gravity	Young's modulus (10^{11} dynes/cm^2)
steel	7.83	20.01
aluminum	2.71	6.96

67.1 In many applications in space technology it is important to use materials which have a maximum strength to weight ratio.

(a) Compare the radii r_{Al} and r_{steel} of a solid circular aluminum strut and a steel strut of equal stiffness and equal length L.

Note: the stiffness is defined as the ratio of the applied lateral force to the resultant displacement.

(b) How do the masses of these struts m_{Al} and m_{steel} compare?

67.2 An aluminum beam of length L with a square cross-section is held rigidly on one end as shown in Fig. 67-1. A mass m is attached to the free end of the rod. Find the natural frequency ω of vibration of this system. Assume that the beam has a square cross-section with sides a, that the mass of the rod is much smaller than m, and that the mass m can be considered a point mass.

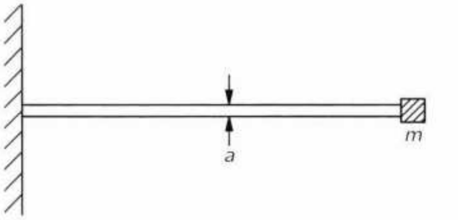


Figure 67-1

67.3 In Chapter 47 of Vol. I the speed of sound in a *fluid* is found in terms of the rate of change of pressure with density.

(a) Show that for *longitudinal* waves in a *solid* (plane, compressional waves) the phase velocity is given by

$$V_{\text{long}}^2 = \frac{(1-\sigma)Y}{(1-2\sigma)(1+\sigma)\rho},$$

where ρ, Y, σ are, respectively, the density, Young's modulus and Poisson's ratio of the solid material. This velocity applies to longitudinal waves in "infinite" medium. In this case the motion of each particle is always parallel to the direction of the wave; when the material is compressed by the wave there can be no sideways motion such as occurs in a rod which gets fatter when it is compressed.

(b) How large do you think the dimensions of a block should be for the formula in part (a) to be applicable?

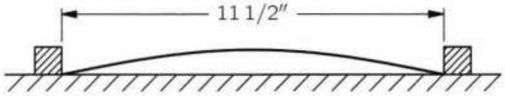


Figure 67-2

67.4 A 12″ steel ruler 1/2″ wide by 1/32″ thick is wedged between two blocks put on a table 11 1/2″ apart as shown in Fig. 67-2. Neglecting the weight of the ruler (≈ 0.05 lb),

(a) into what kind of a curve is the rule bent?

(b) what is the force F against the blocks?

67.5 Determine the buckling load P for a beam of length L, Young's modulus Y, clamped at one end and free at the other, as shown in Fig. 67-3. The beam has a rectangular cross-section with a thickness t and a width w.

Figure 67-3

68

The Flow of Dry Water

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 40.

68.1 (a) Prove to your own satisfaction the statement made in Chapter 40 of Vol. II that if a fluid cannot support a shear stress, the pressure is the same in any direction.

(b) As a mathematical exercise show the quite useful vector identity used in Chapter 40

$$(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = \frac{1}{2}\boldsymbol{\nabla}(\boldsymbol{v}\cdot\boldsymbol{v}) + (\boldsymbol{\Omega}\times\boldsymbol{v}),$$

where $\boldsymbol{\Omega} = (\boldsymbol{\nabla}\times\boldsymbol{v})$.

68.2 The liquid in a cylinder of circular cross-section rotates with a constant angular velocity ω about the axis; particles a distance r from the axis rotate with a speed $v = \omega r$.

(a) Find the shape of the surface at the top of the liquid.

(b) Show that, as was pointed out in Chapter 40, the circulation per unit area, i.e., $\boldsymbol{\nabla}\times\boldsymbol{v}$, is twice the angular velocity at which the water is rotating.

68.3 A sphere of radius a and mass m moves through "dry" water at a constant velocity v.

(a) Show that the total kinetic energy of the ball plus the fluid is:

$$\frac{1}{2}\left(m + \frac{M}{2}\right)v^2,$$

where M is the mass of the fluid displaced by the ball.

(b) What is the total momentum $\boldsymbol{p}_{\text{total}}$ of the ball plus the fluid?

69

The Flow of Wet Water

Refer to *The Feynman Lectures on Physics*, Vol. II, Chapter 41.

69.1 If a ball of radius a is dragged with a constant velocity v through a viscous liquid slowly enough so that the flow is laminar, the force applied is a measure of the viscous force of the liquid on the ball. Although you *can* figure out this force exactly, it is interesting to find the form of the force law from dimensional arguments after noting on what parameters the force should depend. Do so. Can you see by qualitative physical arguments *why* the parameters enter the way they do?

69.2 If a viscous fluid flows in a small pipe, the flow can be considered to be laminar, i.e., sheets of fluid in cylindrical tubes are flowing past each other. For a pipe of radius a the velocity profile across the pipe will look roughly as shown in Fig. 69-1.

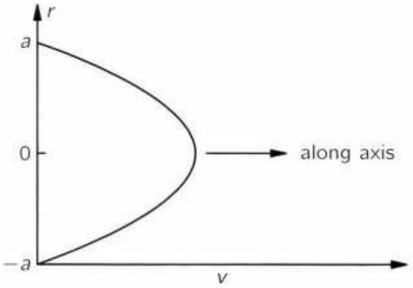


Figure 69-1

(a) Show that if r is the radial distance from the center of the pipe, η is the viscosity of the fluid, and there is a pressure drop $(P_1 - P_2)/L$ per unit length of pipe, then the velocity is given by

$$v(r) = \frac{1}{4\eta}\,\frac{P_1 - P_2}{L}\,(a^2 - r^2).$$

In exact analogy to Ohm's law, the discharge rate of fluid Q from such a pipe can be related to the pressure differential $\Delta P = P_1 - P_2$ by an equation

$$\Delta P = QR,$$

where R is the "resistance" of the pipe.

(b) Find the resistance R of a pipe of radius a and length L.

(c) What would be the analog to a capacitor?

(d) Can you think of any reasons why analogies like these might be useful?

69.3 A large, shallow tray is partly filled with water (an "incompressible" liquid with viscosity η). A thin flat sheet of wood floats on the-water with its bottom surface at the height d above the bottom of the tray. The other dimensions of the sheet are both much larger than d. The sheet is moved horizontally at a slow speed v. What is the rate of energy dissipation dU/dt in a unit volume of the water near the middle of the sheet?

Exercises for Volume III

Introduction

The present set of exercises is designed to accompany Vol. III of "Feynman Physics." Like the set which goes with Vol. II, it includes homework and exam problems used at Caltech during the years 1963–64. Again I have tried to arrange the problems roughly in order of difficulty within each chapter.

Even more than the preceding set, this set does not represent a final effort; it must grow as the course evolves. As a matter of fact, these problems have been typed before Vol. III was published in its final form. I hope that any discrepancies in notation which will, therefore, certainly exist will be taken as a further indication of the preliminary nature of these problems.

Most of the problems were written up by M. Sands, R. P. Feynman, J. Pine, and myself. The ideas for about three-fourths were suggested by R. P. Feynman. A preliminary editing was done by C. Wilts and myself in the summer of 1963. The final reading and correction of this collection was done by I. Tammaru.

Again it is my pleasure to thank Mrs. F. L. Warren for typing these problems in all the various stages of preparation.

G. Neugebauer

70

Probability Amplitudes

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 3.

70.1 The imaginary electron interference experiment described in Chapter 3 of Vol. III is illustrated in Fig. 70-1. From the interference pattern P_{12} drawn in the figure, one can estimate the wavelength λ to be associated with the amplitude functions ϕ_1 and ϕ_2. Call the distance between the centers of the slits d, and the distance from the wall to the backstop L. Let x be the distance from the center of the backstop to the first minimum of P_{12}, and get any other dimensions needed by measuring on the figure.

(a) What would you expect for λ if $L \gg d$?

(b) Taking the curves given for P_1 and P_2 in the figure, compute what you would expect for the magnitude of P_{12} at the center of the pattern, at the first maximum away from the center, and at the first two minima in the interference pattern. Compare with the curve for P_{12} given in the figure. (*Is the figure accurate?*)

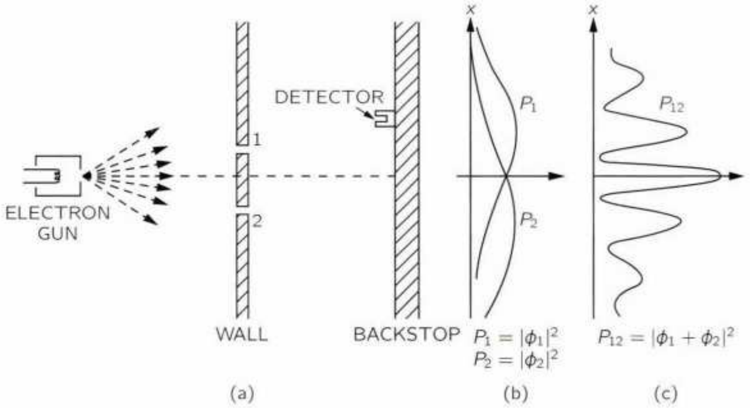


Figure 70-1

70.2 Consider the two-slit electron interference experiment of Ex. 70.1, and assume that the distances from the gun to the wall and from the wall to the backstop are very large in comparison with the distance between the slits. Assume also that the width of the slits is very small compared with their separation. Answer the following questions as quantitatively as you can.

(a) What happens to the interference pattern of P_{12} if the gun is moved upward the distance D?

(b) What happens to the pattern if the distance between the slits is doubled?

(c) What happens to the pattern if slit 1 is made twice as wide as slit 2?

70.3 Vertically polarized monochromatic light is incident on a polaroid whose axis for transmitting light is tilted at an angle θ to the vertical. Classically, what is the ratio of the transmitted intensity to the incident intensity? What does the polaroid do for the case of a single incident photon?

70.4 A beam of 20,000 eV electrons passes through a thin polycrystalline gold foil and then strikes a photographic plate. The plate shows blackening in the form of rings concentric with the axis of the electron beam. Why? Predict the ring diameters, for a distance of 10 cm between gold foil and photographic plate.

70.5 If one considers the standard double slit diffraction experiment of Ex. 70.1, it is possible to show that the entire pattern on the screen can be predicted from a knowledge of the amplitudes for the electrons to be at the slits.

(a) If a_1 and a_2 are the two complex numbers giving the amplitudes for getting electrons at slits 1 and 2, what is the formula for the relative intensity distribution on the screen $I(x)$, where x is the distance from the center point? Make the approximation that x and the separation between the two slits d are both small compared to the distance between the slits and the viewing screen L.

(b) If the pattern depends only on the amplitudes at slits 1 and 2, how does the electron "know" what wavelengths to use behind the slit in determining the pattern?

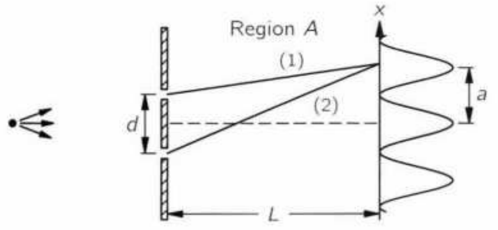


Figure 70-2

70.6 In the diffraction experiment shown in Fig. 70-2, the source emits particles of momentum p_0, mass m, and velocity v.

(a) What is the spacing a between the central maximum and the neighboring maxima? Assume $L \gg d$, $L \gg a$.

(b) If some influence alters the phase for the upper path (1) by $\delta\phi_1$ and for the lower path (2) by $\delta\phi_2$, show that the central maximum is displaced by a distance S given by:

$$S = +(\delta\phi_1 - \delta\phi_2)\,\frac{L}{d}\,\frac{\hbar}{p_0}.$$

Thus, if $(\delta\phi_1 - \delta\phi_2)$ is the same for all paths giving rise to the interference pattern, then the whole pattern is displaced and we may say that the particles have been deflected upward a distance S.

(c) Suppose that in Region A the particle has a small potential energy which is a function only of its vertical position. Then the momentum of a particle at height x above the center line, $p(x)$, will differ slightly from $p(0)$, its value on the center line. Show that

$$p(x) = p(0) + \frac{m}{p(0)}\,(V(0) - V(x)),$$

or, for the case where $V(x)$ varies slowly,

$$p(x) = p(0) + \frac{Fx}{v},$$

where F is the negative of the gradient of the potential $-\partial V/\partial x$.

(d) Under the circumstances in part (c) above, the momenta on paths (1) and (2) will differ, and also the wavelengths.

(1) Show that the phase difference between the upper and lower paths is:

$$(\delta\phi_1 - \delta\phi_2) = \frac{d}{2v}\,\frac{F}{\hbar}\,L.$$

(Note that the average vertical spacing between the two routes is $d/2$.)

(2) Show that the pattern is displaced upwards by $\frac{1}{2}(F/m)t^2$, where $t = L/v$ is the classical time to go from slit to screen. Comment.

70.7 Electrons (spin one-half) are emitted from a source S placed in front of a screen which contains two slits as shown in Fig. 70-3. Assume that when an electron reaches a slit, it goes through with amplitude α for "unflipped" spin and amplitude β for "flipped" spin. Assume further that it is impossible to distinguish which slit the electron passed through.

(a) If all electrons are emitted with spin up, calculate the intensity distribution P on the screen at point x in terms of α, β, and amplitudes such as $\langle 1 \,|\, S\rangle$ and $\langle x \,|\, 1\rangle$.

(b) How does this distribution differ from the case in which all electrons emitted are spin down and all other conditions are similar?

(c) If the electrons are emitted with spins randomly up or down, again find how this case differs from part (a) above, assuming all other conditions unchanged.

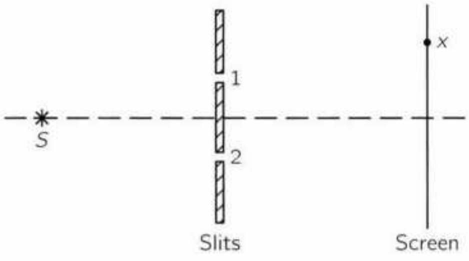


Figure 70-3

70.8 Surprisingly, large interference effects can occur even when one of the interfering possibilities is not very probable. In the two hole diffraction experiment, if one hole is stopped down so that the probability of getting through is reduced by a factor of 100, show that the arrival probability at a maximum of the pattern is still about 50 percent higher than at a minimum.

70.9 The diameter of the nearest stars is too small to be "seen" with the best telescopes (the angle subtended is less than the resolution of the telescope). The diameter of a star was first measured by Michelson using an optical interferometer. The method just barely works for the nearest stars. In *Nature* **178**, 1046 (1956), Brown and Twiss proposed a new method, called "intensity correlation," for such measurements, and tested their method on the star Sirius. They took two parabolic reflectors (old searchlight mirrors) each with a photomultiplier tube at the focus. The outputs of the multipliers were fed by coax cables to a circuit that measured the average value of the *product* of the two currents (a so-called "correlator"). From the variation of this product with the separation of the two mirrors they determined the angle subtended by the star.

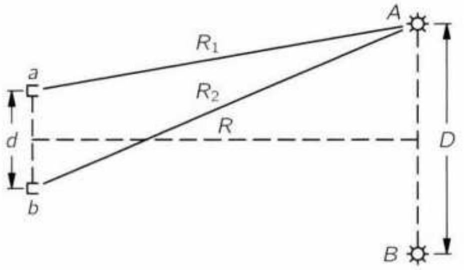


Figure 70-4

There were at the time many physicists who said that the method couldn't work. The argument was that since light came in photons which went either to one mirror or the other, there could be no correlation in the two currents. You can show that this argument is wrong by considering the following idealized experiment. There are two small sources—say two light bulbs—A and B, at a large distance from two photomultiplier tubes a and b with the geometry shown in Fig. 70-4. Counters are attached to the detectors a and b that measure the number of photons per second p_1 and p_2 arriving at each counter. The counters a and b are also connected to a "coincidence" circuit that measures p_{12} the counting rate for the *simultaneous* appearance (within some small time τ) of two photoelectrons.

Let $\langle a \,|\, A\rangle$ be the amplitude for a photon to arrive at a from A in any particular resolving time interval. Then $\langle a \,|\, A\rangle$ is $ce^{i\alpha_1}$ where c is a complex constant and α_1 is k times the distance R_1 from A to a. Similarly $\langle b \,|\, A\rangle = ce^{i\alpha_2}$ with $\alpha_2 = kR_2$ where R_2 equals the distance from A to b.

(a) Show that the coincidence counting rate p_{12} is proportional to

$$2 + \cos 2k(R_2 - R_1).$$

(b) How can this result be used to measure D if R is known? Ignore the fact that the actual process must be represented by a superposition of such models because light comes from all areas on the star's surface and not just from two points on the limbs.

71

Identical Particles

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 4.

71.1 A broadcasting transmitter radiates 1000 kW at a frequency of 1.0 MHz.

(a) What is the energy E (in electron volts) of each quantum radiated?

(b) How many quanta N are emitted in each period of oscillation of the electromagnetic field? (The high coherence of these quanta is possible because they are Bose "particles.")

71.2 In a black body, $E(\omega)$ the *energy* per unit volume in the radiation with frequencies between ω and $\omega + \Delta\omega$, is given by Planck's Radiation Formula,

$$\Delta E(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3 (e^{\hbar\omega/kT} - 1)} \Delta\omega.$$

(a) What is the behavior of $E(\omega)$ for small ω? For large ω?

(b) At what frequency $\omega_{E_{\max}}$ is there the most energy per unit frequency interval?

(c) At what wavelength $\lambda_{E_{\max}}$ is there the most energy per unit wavelength interval?

(d) Estimate the temperature T of the sun by assuming that its maximum energy radiation occurs in the middle of the visible spectrum ($\approx 5000\,\text{Å}$).

71.3 Estimate the strength of the magnetic field B required to make the spins of the two electrons in a helium atom line up in the same direction. (Approximate the helium atom by a harmonic oscillator with a natural frequency corresponding to optical light. "Ground state helium" will have two electrons in the lowest level, with opposite spins. Because of the exclusion principle, one will have to go to the next level up if both are to have the same spin).

71.4 In a particular system, suppose "transitions" can occur between certain energy levels, as shown in Fig. 71-1. That is, we assume the population or number of atoms in the energy levels is changed with an accompanying emission or absorption of quanta. It is given that the two excited states and ground state are in thermal equilibrium with themselves when the total system is bathed in radiation of frequency $\hbar\omega = \Delta E$. Direct transitions with energy $2\Delta E$ are forbidden.

(a) Solve for N_1/N_0 and N_2/N_1 in terms of $n(\omega)$, the number of quanta.

(b) Derive a simple relation for $n(\omega)$, the number of photons (bosons), involving $\Delta E/kT$ only.

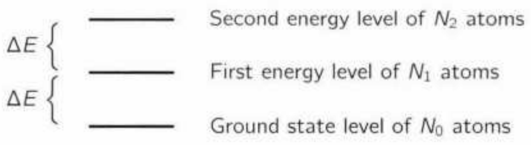


Figure 71-1

(c) Find approximate expressions for $n(\omega)$ in the limit,

(1) $\hbar\omega \gg kT$,

(2) $\hbar\omega \ll kT$.

71.5 Before neutrons were discovered it was thought that nuclei contained protons and electrons. Show that the N^{14} atom (atom of nitrogen with nucleus of mass near 14 times the proton) would be a Bose particle. Experiments (spectrum of the N_2 molecule) showed that this atom was a Fermi particle. This was the first evidence for a new nuclear particle. Show how the neutron hypothesis resolves the dilemma.

71.6 In a laser a number of similar atoms are raised to an excited state. The presence of a small amount of some light of one kind then induces emission until all atoms contribute like an avalanche and thus create very many photons all of exactly the same wavelength and direction.

(a) Explain how the atoms can be "trained" to all emit in the same direction.

(b) Could you expect that some day one could develop a similar device for neutrinos (massless particles with spin one-half)?

71.7 (a) Show that if two non-identical particles do not interact, the probability that one goes from a to b while the second goes from c to d is the product of two factors $P_{ac\to bd} = P_{a\to b}P_{c\to d}$, where $P_{a\to b}$ is the probability the first would go from a to b if the second were not present and $P_{c\to d}$ is the probability the second would go from c to d if the first were not present.

(b) Is the restriction to nonidentical particles in part (a) above essential?

71.8 Deuterium is a Bose particle which has spin one; thus a beam of deuterons can be in one of the three states $+1$, 0, -1. An experiment is performed in which deuterons are scattered from deuterons. What is the probability $P(\theta)$ of detecting deuterons as a function of the scattering angle θ between the incident and target deuterons in the center of mass system. Assume that the incident deuterons are unpolarized, the deuterons are not broken up and reassembled in the scattering process, there is no change of spins during the scattering, and that $f(\theta)$ is the amplitude for a deflection θ.

71.9 Let $f_1(\theta)$ be the amplitude for the scattering of a π-meson from a proton and $f_2(\theta)$ be the amplitude for the scattering from a neutron. What would you guess for the *probability* that a π-meson would be scattered from a helium nucleus at the angle θ in terms of P_1 and P_2 the probabilities for scattering from protons and neutrons? (Assume the π-meson interacts with only one of the particles in the nucleus, i.e. higher order scattering processes may be neglected.) Consider two cases:

(a) That the recoil of the proton or neutron after the scattering breaks up the nucleus.

(b) That the recoil is so small that the nucleus remains intact.

(c) Can you say which case gives more scattering?

Note: your answer depends upon assumptions used in describing process (b).

71.10 A beam of neutrons is incident on a neutron target in a neutron scattering experiment. A detector is set up so as to detect neutrons which are scattered at an angle θ in the center of mass system. There is an amplitude f for a particle in the incident beam to be scattered into the detector without any change in spin. There is also an amplitude g for a particle in the beam to scatter into the detector with a spin flip (interchanging its spin direction with the particle from the target.) If one assumes that f and g are independent of θ, what probability for a count in the detector would one predict, if:

(a) the incident and target neutrons both have spins in the $+z$ direction.

(b) the beam of neutrons have their spins aligned in the $+z$ direction and the target neutrons have spins aligned in the $-z$ direction.

(c) the incident beam is unpolarized while the target neutrons are polarized in the $+z$ direction.

(d) both the incident and target neutrons are unpolarized.

(e) What would the answer to part (a) be if the target was polarized protons and the scattering amplitudes for neutron-proton scattering equaled those for neutron-neutron scattering? Assume the detector has the same efficiency for detecting neutrons and protons.

Hint: the amplitude g for spin exchange only applies when the two initial spins are different, i.e. when "spin exchange" is meaningful.

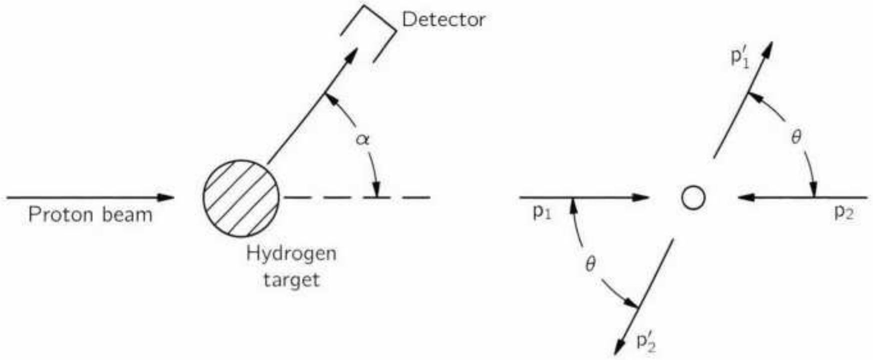


Figure 71-2

71.11 A beam of non-relativistic protons is passed through a thin target of liquid hydrogen, and as shown in Fig. 71-2, scattered protons are counted at some angle α with respect to the incident beam direction. The scattering can be analyzed in the center-of-mass system as shown in the figure. Two protons p_1 and p_2 approach with equal velocities; after the collision two protons p'_1 and p'_2 leave the collision at the angle θ. If we define the z-axis as perpendicular to the scattering plane, each proton can have J_z (z-component of spin angular momentum) either $\pm\hbar/2$. We say the spin is either "up" or "down." Suppose both protons have spin "up," and that the amplitude that p_1 is scattered into a detector at θ is $f(\theta)$. Since we cannot tell *which* proton is detected, the amplitude that *a* proton will appear at the angle θ is $f(\theta) - f(\pi - \theta)$. The minus sign appears because protons are "fermions." We can then say that the probability that a proton is observed in the detector is

$$|f(\theta) - f(\pi - \theta)|^2.$$

Suppose now that p_1 has spin "up" and that p_2 has spin "down," and that the amplitude that p_1 is scattered into the detector with *no* change of spin is $f'(\theta)$ and that the amplitude for scattering *with* a spin flip is $g(\theta)$ because the scattering amplitude depends on the relative spin orientations. In this case the amplitude that an "up" proton will arrive at the detector can be written $f'(\theta) - g(\pi - \theta)$.

(a) What is the relation between θ and α?

(b) What is the amplitude for the "up"–"down" case that a proton will be scattered into the detector with spin "down"?

(c) Suppose that a "natural" beam of unpolarized protons is scattered from a "natural" (unpolarized) target, and that the detector cannot distinguish between the two polarizations. What is the scattering *probability* for the angle θ?

(d) Show that if $f' = f$ and $g = 0$, the scattering of protons of random spins is equivalent to a mixture of "pure fermion" scattering for which the amplitude is $f(\theta) - f(\pi - \theta)$ and "pure boson" scattering for which the amplitude is $f(\theta) + f(\pi - \theta)$,

$$P(\theta) = A|f(\theta) - f(\pi - \theta)|^2 + B|f(\theta) + f(\pi - \theta)|^2.$$

(e) Find A and B in part (d) above.

71.12 Suppose N electrons are in a very large box of volume V in a condition to give the least possible energy.

(a) If we disregard the interaction between the electrons show that each mode of the box is occupied by just two electrons, provided that the momentum of the mode $\hbar k = p$ has a magnitude less than $p_{\max}$ where

$$N = \frac{1}{(2\pi\hbar)^3} \int_0^{p_{\max}} V \cdot 2 \cdot 4\pi p^2 \, dp.$$

(b) What is the energy U of all the electrons? Express this internal energy U in terms of the volume of the box.

(c) Find the pressure exerted by this so-called "degenerate electron" gas. Show the pressure-volume reaction is of the form $PV^\gamma =$ constant.

(d) Find γ.

71.13 Matter in white dwarf stars is so highly compressed that the theory of Ex. 71.12 applies to them. If ρ is the density of the material, $\rho/2M_p$ is the number of protons per cubic meter, where M_p is the mass of a proton and we suppose the nuclei have about as many neutrons as protons. (So set $N/V = \rho/2M_p$ in the equations of Ex. 71.12.)

The equations of equilibrium of a star of such material held together by gravitation appear in a book on astrophysics as

$$\begin{aligned} P &= A\rho^{5/3}, \\ dP/dr &= -G\rho M(r)/r^2, \\ dM(r)/dr &= 4\pi\rho r^2, \end{aligned}$$

where r the is radial distance from the center of the star, $M(r)$ is the mass inside the sphere of radius r, P is pressure, and A is a numerical constant.

(a) Can you explain the reason for these equations?

(b) Can you supply a formula, or a numerical value, for the constant A?

You may assume that all pressure is exerted by the degenerate electrons, the nuclei having virtually no effect (Why?).

72

Spin One

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 5.

72.1 Prove the statement made in Section 5-6 of Vol. III, that if the (spin 1) Stern-Gerlach apparatus C can be broken into two parts, A and B, then

$$\langle \chi \,|\, C \,|\, \phi \rangle = \sum_k \langle \chi \,|\, B \,|\, k \rangle \langle k \,|\, A \,|\, \phi \rangle$$

where $k = -,\, 0,\, +$ are spin 1 base states.

72.2 Three "improved" Stern-Gerlach apparatuses, described in Chapter 5 of Vol. III, which separate a beam according to the value of the z-component of spin but which bring the beam together spatially, are put in series with each other, and a beam of spin one particles is sent through. The first and third are lined up with the same orientation while the middle is set at an arbitrary angle. In the notation of Chapter 5, this would appear as shown in Fig. 72-1.

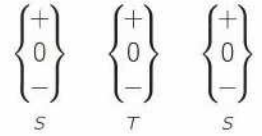


Figure 72-1

(a) If one slot of T is "open," does the proportion of the beam found in each of the three states of the final S system depend on the input state, i.e., on the proportion of the beam in $+S$, $0S$, or $-S$. Why?

(b) What if two slots of T are "open"?

(c) What if three slots of T are "open"?

72.3 A set of three "improved-type" Stern-Gerlach experiments is set up for spin one particles as shown in as shown in Fig. 72-2. The three apparatuses are placed along a straight line, but the T apparatus is rotated about this line by an angle of 9° with respect to the two S apparatuses. A beam of spin one particles enters from the left. The beam which leaves the first S apparatus has an intensity of N_1 particles per second.

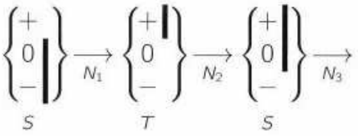


Figure 72-2

(a) What is N_2, the intensity of the beam leaving the T apparatus?

(b) What is the intensity N_3 of the beam that leaves the last S apparatus?

(c) What are N_2 and N_3 if all the "stops" are removed from the T apparatus?

72.4 Consider a sequence of modified Stern-Gerlach apparatuses S, T, S' used with particles of spin 1, as shown in Fig. 72-3. (T is rotated by 90° with respect to S and S. Assume the incident beam is unpolarized.)

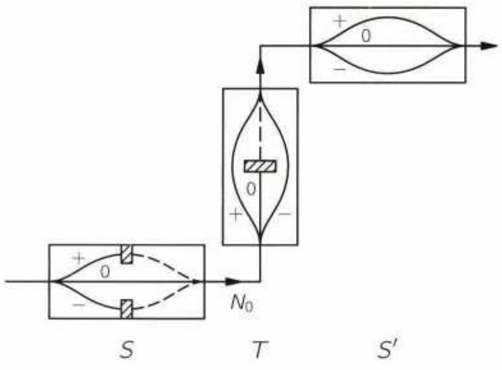


Figure 72-3

(a) For N_0 particles coming out of the S apparatus, find the expected number of particles emerging from S' in the states $|+S'\rangle$ and $|0S'\rangle$. (Call them $N_{+S'}$ and $N_{0S'}$.)

(b) Suppose we had available "transparent detectors," which could be placed into the $+$ and $-$ beams of the T apparatus. Let these detectors have the property that when a particle passes through one of them a signal is generated, without the spin state of the particle being changed. Furthermore the momentum of the particle is not appreciably changed, in the sense that its trajectory through the T apparatus can be considered to remain the same as if the detector were not present. With detectors present in the $+$ and $-$ states of T (the 0 state remains blocked), what is the probable number of counts recorded for N_{+T}, and N_{-T}, and for $N_{+S'}$, and $N_{0S'}$, if a total of N_0 particles emerge from the S apparatus?

(c) How would the answer for $N_{+S'}$, in the experiment described in part (b) above change, if it was discovered after the experiment that no counts had been recorded for N_{+T} and N_{-T} because the signals from the detectors had not been recorded?

(d) If each detector is only 50 % efficient (i.e., 50 % of the time there is no interaction between the particle and the detector) what is the answer for $N_{+S'}$ and $N_{0S'}$?

(e) If the blocks from the + and − states of S are removed and N_0 particles are sent into S, what are $N_{+S'}$ and $N_{0S'}$? (The detectors in T are removed also.)

73

Spin One-Half

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 6.

73.1 Imagine that atoms with a spin of one-half are filtered by a sequence of two "improved" Stern-Gerlach apparatuses. It is supposed that each apparatus is arranged to permit only one beam to go through, as indicated in Fig. 73-1. For each of the eight arrangements shown in (a)–(h) consider that N unpolarized atoms enter at P. Give the expected number of atoms to arrive at Q.

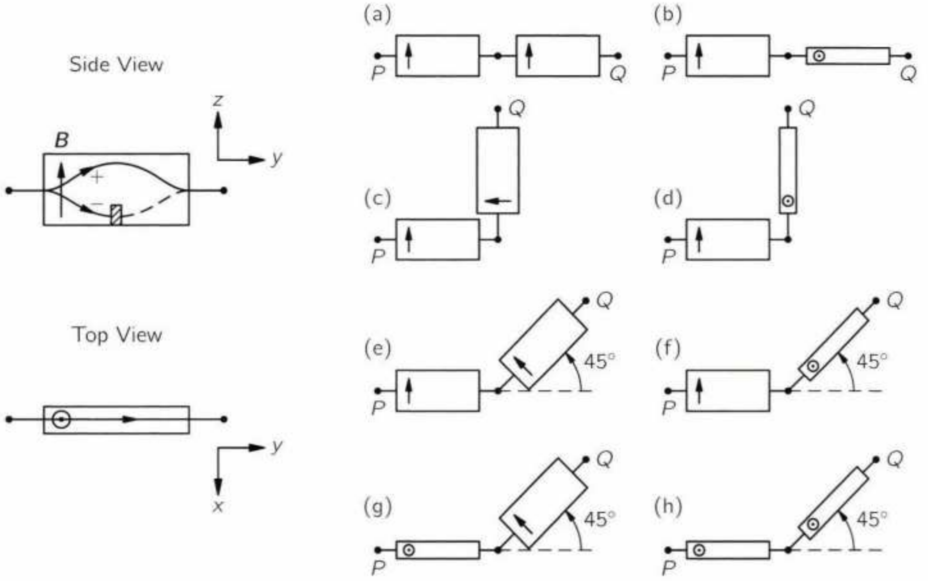


Figure 73-1

73.2 A spin one-half particle enters some apparatus with amplitudes a and b to have spin up and down along the z-axis.

(a) Show that the probability for the particle to arrive at any given point in the apparatus is necessarily of the form $|aX + bY|^2$ where X and Y are some complex numbers characteristic of the apparatus.

In terms of X, Y what is the probability to arrive at the given point if

(b) the incoming particle has spin up in z? Down?

(c) the incoming particle has spin up along the x-axis? Down?

(d) the incoming particle has spin up along axis with polar angles θ, ϕ?

There are several ways to imagine that the incoming particles are in "random" spin states:

(I) For some experiments the electrons are up along z, for others down, a coin being flipped each time to decide which.

(II) The same circumstance as (I) above, except they are either up or down along x.

(III) Each is oriented in some direction θ, ϕ but all directions are chosen at random (so we average over solid angle $\sin\theta\, d\theta\, d\phi/4\pi$).

For each of the above "random" circumstances,

(e) find the average probability $\langle P \rangle$ to arrive in the apparatus at any given point, showing it is equal in all circumstances.

(f) suppose spin one-half particles are coming out of a hole, being prepared by one of the methods (I), (II), (III) on the other side. Can you think of any way at all that, by observations on your side of the hole, you could tell whether method (I), (II), or (III) actually being used?

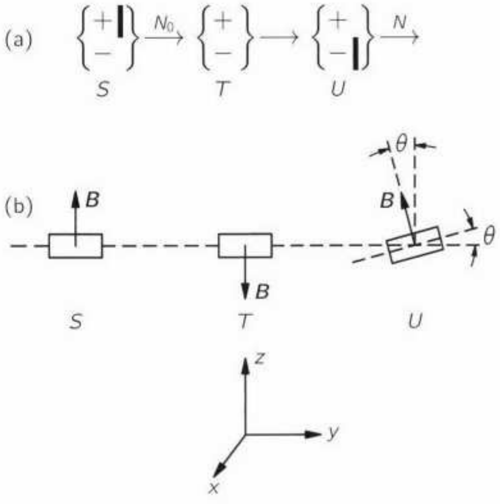


Figure 73-2

73.3 Three modified spin one-half Stern-Gerlach apparatuses S, T and U are in series as shown part (a) of Fig. 73-2.

(a) Express N, the number of spin one-half particles coming out of U, in terms of N_0, the number coming out of S, and in terms of objects like $\langle +T \mid +U \rangle$, etc.

Now consider the same apparatus as above, but with the $\boldsymbol{B}$-field orientations given in part (b) of Fig. 73-2. In particular, the $\boldsymbol{B}$-field of the T-apparatus is rotated anti-parallel to the $\boldsymbol{B}$-field of the S-apparatus, and the $\boldsymbol{B}$-field of the U-apparatus is rotated an angle θ from the z-axis.

(b) Find $\langle +T \mid -S \rangle$ and $\langle -T \mid -S \rangle$ explicitly.

(c) Find explicitly $\langle +U \mid -S \rangle$ using, as an exercise, only the transformation table for rotations around the z- and the y-axes.

(d) Find limiting forms of your answer in part (c) for

(1) $\theta = 0$.
(2) $\theta = \pi$.

(e) Explain your answer for $\theta = \pi$ in part (d) upon comparison with $\langle +T \mid -S \rangle$ of part (b).

73.4 A piece of calcite splits a beam of light (going in the z-direction) into two, corresponding to x and y. A single photon entering a calcite S has an amplitude to be in one or the other of these two beams, x, y. A similar calcite (turned backwards) can be used to bring the two beams together again, etc., just as in the Stern Gerlach apparatus. Another calcite T can have its axis tilted by angle θ in the xy-plane, splitting into states x', y', or xT, yT. Find the amplitudes $\langle xT \mid xS \rangle$, $\langle yT \mid yS \rangle$, etc., from your knowledge of the classical theory of polarized light—by making sure that in each case the intensity in a beam of a large number of photons will agree with the classical result. Consider rotations only about the propagation axis z, for light cannot be brought to rest (rotations about the other axes can be described by their effect on the direction of propagation, rather than on the polarization—in this way light, which is a two state system is very different in its transformation properties from an electron, which is also a two state system.)

73.5 Find all four of the elements of the $\langle j \mid A \mid i \rangle$ where i and j are x and y for the following pieces of apparatus through which light may be passed:

(a) A x-y calcite splitter and restorer with beam y blocked.

(b) A calcite splitter and restorer set at angle θ with beam y blocked.

(c) Polaroid set at axis x to pass.

(d) Polaroid set at axis θ in xy-plane to pass.

(e) A x-y calcite splitter and restorer with a block of glass in beam x which delays the phase of that beam by angle θ.

(f) A x-y calcite splitter and restorer with the same glass in both beams.

(g) A calcite splitter and restorer set at 45° with a glass in the x beam delaying the phase by 90°.

(h) A quarter-wave plate.

(i) A birefringent material, at axis x (give general formula in terms of thickness).

(j) A sugar solution which turns the plane of polarization to the right by angle θ.

(k) A device which splits the beam into x, y beams, changes the x beam to the y direction (by putting it through sugar water turning its polarization by 90°) and puts the two beams back together again.

(l) Show that you can make perpetual motion with the device in part (k) above.

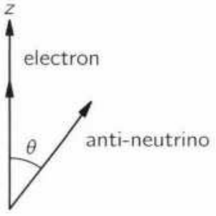


Figure 73-3

73.6 According to the theory of β-decay in a certain kind of nuclear decay (in which the nucleus suffers no change of angular momentum or parity, called "Fermi allowed") an electron moving along the z-axis with velocity v is emitted with amplitude $\sqrt{1-v/c}\,\sin\theta/2$ with spin up along z, and with amplitude $\sqrt{1+v/c}\,\cos\theta/2$ with spin down along z. (Here θ is the angle, from z, that the anti-neutrino is emitted, as shown in Fig. 73-3. Incidentally, anti-neutrinos always have their spin along their direction of motion).

(a) What is the probability that the spin is up along z? Down?

(b) What is the probability that the spin is in the direction* $+x$ (if the neutrino is in the xz-plane)? The direction $-x$?

(c) What is the probability in the directions $\pm y$?

(d) If, as is usual, the neutrino is not observed (average over all anti-neutrino directions) what is the answer to part (a)?

73.7 In problem 73.6, making a Lorentz transformation of velocity v along z to bring the electron to rest does not change the numerical values of the amplitudes to have spin up and down along this axis. (Can you think of a reason why this might be so?) However, the apparent direction of the anti-neutrino is, of course, altered. Show that the amplitudes in problem 73.6 mean that the electron spin is lined up opposite to the direction (and hence opposite to the spin) of the anti-neutrino in the system in which the electron is at rest. (This occurs because the nucleus lost no angular momentum.)

---

* Strictly speaking, this refers to the coordinate system moving with the electron. Just use regular formulas for combining amplitudes.

74

The Dependence of Amplitudes on Time

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 7.

74.1 A particle of spin one in a magnetic field $\boldsymbol{B}$ in the z direction has three states (labeled $+$, 0, $-$) with energies $+\mu B$, 0 and $-\mu B$ respectively.

(a) Show quantum mechanically that in an inhomogeneous magnetic field a beam of such particles would be split into three beams and find the laws giving the deflection $\Delta\theta$, assumed small (in terms of the length of the field L, the initial momentum of the particles p_0, etc.).

(b) Show quantum mechanically that such a particle will "precess" (use the coefficients of Section 5-7 in Vol. III, to make an argument like that of Section 7-5.).

(c) Suggest at least two independent ways in which μ might be measured experimentally.

75

The Hamiltonian Matrix

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 8.

Remarks on the definition of base states: Our original definition of the "+" and "−" states was in terms of the deflection relative to the direction of $\boldsymbol{B}$ and $\nabla \cdot \boldsymbol{B}$ (which were both in the $+z$-direction) in a Stern-Gerlach apparatus. Since, in the symmetry plane, $\boldsymbol{F} = -\nabla \cdot U = \mu_z(\partial B/\partial z)\boldsymbol{e}_z$, we see that the S-G apparatus measured the component of the magnetic moment $\boldsymbol{\mu}$, not angular momentum (or spin) $\boldsymbol{J}$. But the angular momentum $\boldsymbol{J}$ is in many ways a more fundamental quantity than $\boldsymbol{\mu}$, and we prefer to define our base states according to the value of $\boldsymbol{J}$ instead. Hence, we have to redefine the base states in those systems where $\boldsymbol{\mu}$ and $\boldsymbol{J}$ are antiparallel (such as the electron, and atomic systems). But we do not have to change our rotation matrices, because these were derived from properties of 3-dimensional space, and we had a choice regarding the relative phases of some elements. The convention of specifying the base states by their value of J_z is actually the one used later on in the text. In the tables of Chapter 12 in Vol. III, for example, the states are labeled by the sign of the number $m = J_z/\hbar$. In Section 7-5 the negative muon is considered to be in a $|+\rangle$ state relative to the x-axis when its spin points in the $+x$-direction, even though its magnetic moment would then point in the $-x$-direction.

With this convention the direction of $\boldsymbol{\mu}$ relative to $\boldsymbol{J}$ does not enter the problem when the coordinates are rotated, but it must now be taken into account when we are asking about the force and the energy.

For a particle in a uniform magnetic field, for example, the base states in a coordinate system in which the z-axis is parallel to $\boldsymbol{B}$ are also stationary states (their energy is constant). The time dependence of the amplitudes is given by $e^{-iEt/\hbar}$, where $E = U = -\boldsymbol{\mu} \cdot \boldsymbol{B} = \mu_z B$. It is clear that for a state of given J_z, the sign of the exponent will depend on whether $\boldsymbol{\mu}$ and $\boldsymbol{J}$ are parallel or antiparallel.

As an example, for the electron we have $\boldsymbol{\mu} = -(g_e/m)\boldsymbol{J}$ $(g_e = 2)$, whereas, for the positron $\boldsymbol{\mu} = +(g_e/m)\boldsymbol{J}$. The possible values of J_z are $\pm\hbar/2$, so the time dependence of the amplitudes are as given in Table 75-1, where $(t, B\boldsymbol{e}_z)$ denotes the fact that our "apparatus" consists of a wait of time interval t in the magnetic field $\boldsymbol{B} = B\boldsymbol{e}_z$.

Table 75-1

$(t, B\boldsymbol{e}_z)$	$\|+\rangle$	$\|-\rangle$
$\langle +\|$	$e^{i\gamma t}$	0
$\langle -\|$	0	$e^{-i\gamma t}$

$$\gamma = \begin{cases} +g_e B/2m & \text{for the positron} \\ -g_e B/2m & \text{for the electron} \end{cases}$$

Note, as was pointed out in a lecture, that the signs in the exponents of Eq. (7.36) in Vol. III should be reversed, because the negative muon, like the electron, has antiparallel $\boldsymbol{\mu}$ and $\boldsymbol{J}$.

In the following problems we arc considering spin 1/2 particles with $|\mu_z| = \mu$ but the relative direction of $\boldsymbol{\mu}$ and $\boldsymbol{J}$ is unspecified. For your answers to match those given in the appendix, you should assume in your solutions (as we have, in ours) that $\boldsymbol{\mu}$ and $\boldsymbol{J}$ are parallel. (If they should be antiparallel in some given situation, you can still find the right answer by replacing μ with $-\mu$.)

75.1 A beam of spin one-half particles with magnetic moment μ is sent into a Stern-Gerlach filter which passes only particles in the $|+\rangle$ state (spin up) with respect to the z-axis. The particles then spend the time T in a uniform magnetic field $\boldsymbol{B}_0$ which is parallel to the x-axis. After leaving the uniform field the particles enter a second Stern-Gerlach filter which passes only particles in the $|-\rangle$ state (spin down) with respect to the z-axis.

(a) What is the smallest value of B_0 for which all of the particles will get through the second filter?

(b) If the particles spend only half as long in the same field, what is the probability P that they will get through the second filter?

75.2 A beam of spin one-half particles with magnetic moment μ is sent into a Stern-Gerlach filter which passes only particles in the $|+\rangle$ state (spin up) with respect to the z-axis. The beam then goes into a magnetic field which is at 45° with respect to the z-axis in the xz-plane. At a time t later, what are the probabilities $P_{+x}(t)$ and $P_{+y}(t)$ that the particles will be found, respectively, with $J_x = \hbar/2$ or with $J_y = \hbar/2$?

75.3 A spin one-half particle has its spin pointing in the $+z$ direction at time $t = 0$. The particle is located in an apparatus such that the amplitude per unit time to go from the plus to minus z-state is the same as the amplitude to go from minus to plus z-state and both are equal to i times some positive constant $(A/\hbar)$, i.e., $H_{12} = H_{21} = -A$. Further $H_{11} = H_{22}$ and may be taken equal to zero.

(a) What is the probability $P_{+z}(t)$ for finding the particle in the $+z$ state at time $t > 0$?

(b) Find the two combinations of amplitudes to be in the $+z$ and $-z$ states which correspond to stationary states. What are the energies of these states?

(c) At any time t, there exists an axis along which the probability of spin up is unity. In what direction is this axis?

(d) Can you think of a physical piece of equipment that would have this effect?

76

The Ammonia Maser

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 9.

76.1 In Chapter 9 of Vol. III, the probability of an ammonia molecule making a transition from state $|\,II\rangle$ to state $|\,I\rangle$ by shining microwaves on the molecule was calculated; state $|\,II\rangle$ has a lower energy than state $|\,I\rangle$ so this corresponds to the absorption of energy by radiation.

(a) Redevelop these ideas to find the probability $P(I \to II)$ *per unit time* for inducing emission by the ammonia molecule.

(b) How does the probability of absorption $P(II \to I)$ compare to that of (stimulated) emission $P(I \to II)$?

(c) How are these probabilities related to the Einstein A and B coefficients defined in Chapter 42 of Vol. I?

(d) Find the rate $A_{I,II}$ of spontaneous emission by the ammonia molecule.

77

Other Two-State Systems

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 10.

77.1 Protons (magnetic moment μ) in a water sample are exposed to a uniform magnetic field. Now although the magnitude of the field $\boldsymbol{B}$ is constant, the field changes its direction in time for a nuclear magnetic resonance (NMR) experiment:

$$\begin{aligned} B_x &= B\sin\theta\cos\omega t \\ B_y &= -B\sin\theta\sin\omega t \\ B_z &= B\cos\theta \end{aligned}$$

Initially ($t = 0$) we are given that the spin of the protons are along the magnetic field in the $+1/2$ state. Assume θ, the angle from the z-axis in spherical coordinates, is very small.

(a) What value must ω have for resonance?

(b) What is the probability $P_{-z}(t)$ for the particle at time t to have a spin down along the z-axis, for ω at resonance?

77.2 A spin one-half particle (magnetic moment μ) is placed in a large magnetic field $\boldsymbol{B}_0$. An oscillating magnetic field $2\boldsymbol{B}_n\cos\omega t$ whose magnitude is much smaller than $\boldsymbol{B}_0$ is applied in a direction normal to $\boldsymbol{B}_0$. If the spins of the particle were initially lined up in a direction opposite to $\boldsymbol{B}_0$, what is the probability $P_{\parallel}(t)$ of having the spin lined parallel to $\boldsymbol{B}_0$ after a time t?

78

More Two-State Systems

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 11.

78.1 (a) Show that the Pauli spin matrices can be treated as the components of a vector $\boldsymbol{\sigma}$ which obeys the following rules:

$$\boldsymbol{\sigma} \times \boldsymbol{\sigma} = 2i\boldsymbol{\sigma}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{\sigma} = 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) Find σ_x, σ_y, and σ_z.

78.2 Carbon dioxide is a linear molecule (OCO) which likes to pick up an extra electron and become a negative ion. Imagine that the electron would have the energy E_{O} if it were attached to either oxygen atom, or the energy E_{C} if it were attached to the carbon atom. The stationary states need, however, have neither of these energies, because there is some small chance for the electron to jump between an oxygen atom and the carbon atom. (Assume that the chance to jump directly from oxygen to oxygen is negligible.)

(a) Find the possible energy levels of the CO_2 ion in terms of E_{O}, E_{C}, and one other parameter.

(b) Give a physical description of each of the stationary states, in the case that the energies E_{O} and E_{C} are equal.

78.3 In the methane molecule, four hydrogen atoms are placed in the four corners of a tetrahedron with a single carbon atom in the center. In the methane ion, an electron is missing from one of the four bonds, thus leaving a "hole" which can "jump" from any of the H atoms to another. This is an example of a four state system. By using symmetry arguments to reduce the number of different Hamiltonian matrix elements to a minimum, predict the number of different energy levels you would expect to observe from the electronic structure of the methane ion. Neglect the rotational and vibrational interactions of the atoms. Express the separation of the levels in terms of the fewest possible matrix elements you can.

78.4 Consider six atoms spaced equally around a ring, as shown in Fig. 78-1. Add one extra electron, and define base states $|\,1\rangle$, $|\,2\rangle$, … $|\,6\rangle$, where $|\,1\rangle$ means the electron is at atom 1, $|\,2\rangle$ means it is at atom 2, etc. Assume that the extra electron has a definite amplitude per unit time $-iA/\hbar$ to jump from any atom to either one of the two nearest neighbors, but zero amplitude to jump further.

(a) Show that $|\,I\rangle$ is a stationary state if the amplitudes $C_k = \langle k\,|\,I\rangle$, with $|\,k\rangle$ being the k^{th} base state, are all equal to $(1/\sqrt{6})\exp\left(-\frac{i}{\hbar}\,E_I t\right)$.

(b) Find E_I in part (a) above.

(c) How many other stationary states are there?

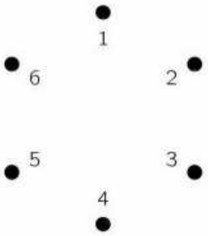


Figure 78-1

It can be shown that if ψ is a stationary state the amplitudes $C_k = \langle k \mid \psi \rangle$ are related in the following way, provided δ is suitably chosen:

$$
\begin{aligned}
C_2 &= C_1 e^{i\delta} \\
C_3 &= C_2 e^{i\delta} \\
C_4 &= C_3 e^{i\delta} \\
C_5 &= C_4 e^{i\delta} \\
C_6 &= C_5 e^{i\delta}
\end{aligned}
$$

(d) What are the allowed values of δ?

(e) Find the energy level diagram for the system and give the spacings between levels.

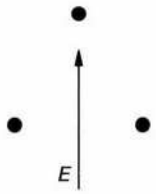


Figure 78-2

78.5 A molecule is made up of three similar atoms placed in the corners of an equilateral triangle. In the negative ion of this molecule, an additional electron is added which is able to jump from any of the three atoms to another.

(a) Take the Hamiltonian matrix element to jump from either atom to another to equal $-A$ and calculate the energy spacings of the molecular ion.

(b) An electric field is applied to the ion in the plane of the ion and pointing to one apex as shown in Fig. 78-2. If the strength of the field is such that the potential energy of the electron at this apex is increased by $\epsilon A = 0.01A$ above the potential energy at the other corners, how much and in what way are the energy spacings changed?

79

The Hyperfine Splitting in Hydrogen

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 12.

79.1 Calculate the amount of splitting in the $j = 1$ level of a hydrogen atom which is placed

(a) in interstellar space where the magnetic field is of the order of 10^{-5} gauss,

(b) at the surface of the earth where the magnetic field is about 1/2 gauss,

(c) in the largest magnetic fields which have been produced in the laboratory, i.e., fields of about 100,000 gauss.

Express your answers both in frequency and wavelength.

80

Propagation in a Crystal Lattice

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 13.

80.1 Consider an infinite line of atoms with equal spacing b, and assume that an electron can be attached to any given atom in two configurations i and j with different energies E_i and E_j, i.e., assume that a suitable set of base states are:

$$\begin{aligned}\left|\begin{matrix}\text{electron at atom } x_n\\ \text{in configuration } i\end{matrix}\right\rangle &= |\,x_n, i\rangle,\\ \left|\begin{matrix}\text{electron at atom } x_n\\ \text{in configuration } j\end{matrix}\right\rangle &= |\,x_n, j\rangle.\end{aligned}$$

Assume further that the electron can jump from one atom to its nearest neighbor with amplitudes:

$$\begin{aligned}&-\frac{A_{ii}}{i\hbar} \text{ to go from } |\,x_n, i\rangle \text{ to } |\,x_{n+1}, i\rangle \text{ or } |\,x_{n-1}, i\rangle,\\ &-\frac{A_{jj}}{i\hbar} \text{ to go from } |\,x_n, j\rangle \text{ to } |\,x_{n+1}, j\rangle \text{ or } |\,x_{n-1}, j\rangle,\\ &-\frac{A_{ji}}{i\hbar} \text{ to go from } |\,x_n, i\rangle \text{ to } |\,x_{n+1}, j\rangle \text{ or } |\,x_{n-1}, j\rangle,\\ &-\frac{A_{ij}}{i\hbar} \text{ to go from } |\,x_n, j\rangle \text{ to } |\,x_{n+1}, i\rangle \text{ or } |\,x_{n-1}, i\rangle.\end{aligned}$$

Consider the case when $A_{ij} = A_{ji} = -A$ and $A_{jj} = A_{ii} = -B$.

(a) Follow the procedure described in Chapter 13 of Vol. III to find the allowed values of the energy of such a system.

(b) Describe the band structure in the cases that $|E_i - E_j| \ll 2B$, and $|E_i - E_j| \gg 2B$.

Check your answer with the solution found in Chapter 13.

80.2 Consider an infinite line of atoms composed of two kinds of atoms, "a" types and "b" types, as shown in Fig. 80-1. Let the amplitude for an electron to be found on the n^{th} "a" type be C_n^a, while the amplitude to be found on the n^{th} "b" type is C_n^b. Assume that the energy of an electron on an "a" atom is $E_0 + \Delta E$ while the energy of an electron on a "b" atom is $E_0 - \Delta E$: further assume that the Hamiltonian matrix elements equal $-A$ for jumps to nearest neighbors. The spacing between atoms is b.

Figure 80-1

(a) Calculate and plot roughly the energy of a stationary state as a function of the electron's wave number k. (You will have two energies for a given value of k.)

(b) What limits can you put on k, in order to include every state exactly once?

80.3 Scattering from an impurity: Referring to the example in Section 13-6 of Vol. III, let the atom at $n = 0$ be different in a different way. Let $H_{00} = E_0$, $H_{01} = H_{10} = H_{0(-1)} = H_{(-1)0} = -B$, where $B \neq A$.

(a) Find the amplitude of transmission γ, and the amplitude of reflection (scattering) β.

(b) Verify that $|\beta|^2 + |\gamma|^2 = 1$.

80.4 For both the exercise 80.3 and for the example in Section 13-6 of Vol. III, $\gamma = 1+\beta$. It is easy to verify that $\gamma = 1+\beta$ is also true in the general case, which combines the two. "Conservation of particles" therefore gives, for the general scattering in one dimension:

$$|\beta|^2 + |1+\beta|^2 = 1.$$

(a) Show that this requires that $\mathrm{Re}\,[\beta/(1+\beta)] = 0$.

(b) Show that β may be written:

$$\beta = ie^{i\eta}\sin\eta,$$

where η is real.

The quantity η is called the "scattering phase shift" and tells both the phase and magnitude of the scattered wave. (True in the three-dimensional as well as in the one-dimensional case.)

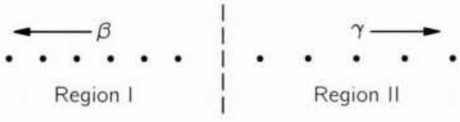


Figure 80-2

80.5 Consider the one-dimensional analog of an interface where an infinite crystal changes its properties, as shown in Fig. 80-2. Let the particles be incident from the left, as in Section 13-6 of Vol. III. In region I we have the parameters E_0, $-A$, b, and in region II, E_0', $-A'$, b'. The amplitude analogous to A and A' that applies between the two atoms on either side of the interface, $n = 0$ and $n = +1$, is B. (Assume A, A', B, are all real.)

(a) Show that $\gamma = (B/A')(1+\beta)$ at the interface between atoms $n = 0$ and $n = +1$.

(b) Find β in terms of A, A', B, kb, $k'b'$.

(c) Show that if $k'b'$ is imaginary $|\beta| = 1$. What does this mean physically? (What values of $E - E_0'$ produce complete reflection?)

(d) Verify conservation of particles by showing that

$$|\beta|^2 + \frac{v_g'/b'}{v_g/b}\,|\gamma|^2 = 1,$$

where v_g and v_g' are the group velocities in the two regions. Can you explain the factor multiplying $|\gamma|^2$?

81

Semiconductors

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 14.

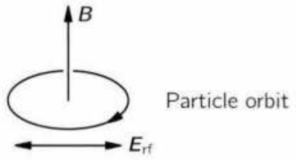


Figure 81-1

81.1 Cyclotron resonance experiments are usually performed as shown schematically in Fig. 81-1, where $B = B_0$, a static magnetic field in the z-direction, and $E_{\text{rf}} = E_0 \cos \omega t$, along the x-axis. The resonance at ω_c, the cyclotron resonance frequency, is detected by a change in the power absorbed from the E_{rf} field. From elementary considerations of a particle orbit in a uniform magnetic field,

$$\omega_c = \frac{qB}{m^*},$$

where m^* is the effective mass. Assume throughout that m^* does not depend on the direction of the particle motion. For the equation of motion of an electron (or hole) in a semiconductor, take:

$$m^* \left(\frac{d\boldsymbol{v}}{dt} + \frac{1}{\tau} \boldsymbol{v} \right) = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}),$$

where τ is the mean time between collisions. (See Sections 32-1 and 32-6 in Vol. II.)

(a) Let $v_x = v_0 e^{i\omega t}$ and $E_x = E_0 e^{i\omega t}$, and show that

$$\frac{v_x}{E_x} = \frac{q\tau}{m^*} \left[\frac{1 + i\omega\tau}{1 + (\omega_c^2 - \omega^2)\tau^2 + 2i\omega\tau} \right].$$

(b) Explain why power absorption is proportional to $\operatorname{Re}[v_x/E_x]$ in part (a) above.

(c) Describe how both τ and m^* can be obtained from cyclotron resonance data.

(d) Note that detection of a resonance requires $\omega_c \tau > 1$. What does this signify physically?

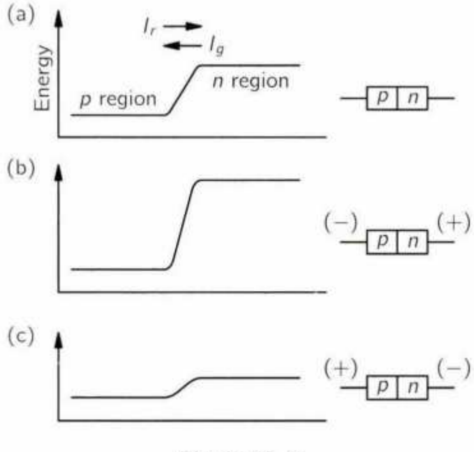


Figure 81-2

81.2 The energy diagram for holes in a typical "p-n" junction (as in a diode) is shown in part (a) of Fig. 81-2, when no external voltage is applied. In equilibrium there is a "thermal generation" hole current I_g of holes which diffuse from the n to the p region and just equals a "recombination" current I_r of holes which goes from the p to the n region. When a "reverse voltage bias" is applied, the energy diagram is as in part (b) of the figure, and when a "forward bias" is applied it changes to that shown in part (c).

(a) By considering the currents first in equilibrium and then with bias voltages, show that the net hole current has the form

$$I(\text{holes}) = I_g(e^{qV_e/kT} - 1)$$

where V_e is the voltage applied to the junction.

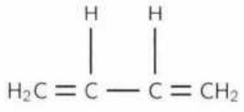
(b) What is the relationship for the total current?

82

The Independent Particle Approximation

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 15.

82.1 The structure of the butadiene molecule can be represented as shown in Fig. 82-1. If we remove (theoretically) the four electrons of the double bonds, and then add them separately we can treat the problem in terms of the independent particle model. Specifically, we can consider a four pit system with the usual energies E_0 and Hamiltonian elements $-A$.



$$\mathrm{H_2C=\overset{H}{\overset{|}{C}}-\overset{H}{\overset{|}{C}}=CH_2}$$

Figure 82-1

(a) What wavelength radiation λ is emitted when butadiene molecules make transitions from the first excited to the ground state? Assume $A = 1\,\text{eV}$.

(b) In singly ionized butadiene, only three of the electrons of the double bonds are present. What can you say about how these electrons are distributed in the molecule?

82.2 Estimate the energy needed to break a benzene ring by using molecular orbital theory (in the independent particle approximation) to calculate the energy difference ΔE between configurations (a) and (b) shown in Fig. 82-2. Find the answer in electron volts by utilizing the fact that the transition from the first excited state of benzene to the ground state produces radiation of wavelength about 2,000 Å.

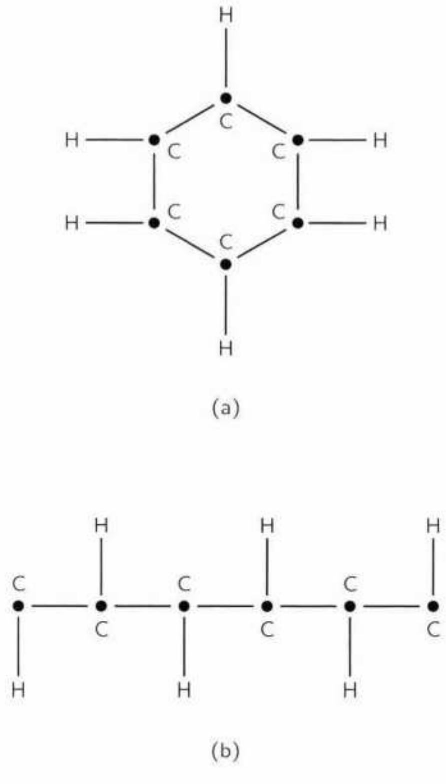


Figure 82-2

82.3 A ferromagnetic material at very low temperatures can be discussed in terms similar to the spin waves discussed in Chapter 15 of Vol. III. Specifically, for any mode K with an energy $E_K \approx Ab^2K^2$, there is a probability distribution, based on thermodynamics, for finding either none, one, two, three, etc., down spins in a ferromagnet which at zero temperature has all the spins aligned up.

(a) Show that for mode K the mean number of atoms with down spins is proportional to

$$\frac{1}{e^{E_K/kt}-1}.$$

(b) If these ideas are extended to three dimensions, then $E_K \approx Ab^2(K_x^2 + K_y^2 + K_z^2)$ and the mean total number of down spins per unit volume is given by

$$\frac{\text{Number of down spins}}{\text{volume}} = \int \frac{d^3K/(2\pi)^3}{e^{E_K/kt}-1}.$$

Show why this is so.

(c) In the limit as T goes to zero, the magnetization goes to the saturation magnetization, M_{sat}. Show that at low temperatures the ratio of the magnetization to the saturation magnetization can be written as

$$\begin{aligned}\frac{M}{M_{\text{sat}}} &= 1 - \text{const}\, T^{3/2}\\ &= 1 - \left(\frac{kT}{4\pi A}\right)^{3/2}\left[\frac{4}{\sqrt{\pi}}\int_0^\infty \frac{x^2\,dx}{e^{x^2}-1}\right]\end{aligned}$$

(d) Evaluate the integral in part (c) above by expanding the integrand in a series.

83

The Dependence of Amplitudes on Position

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 16.

83.1 Consider in one dimension the motion of a particle of mass m bound in a square well potential, as shown in Fig. 83-1, with

$$V = \begin{cases} V_0, & x < 0 \text{ or } x > a \\ 0, & 0 < x < a. \end{cases}$$

Note: for simplicity, let $V_0 \to \infty$.

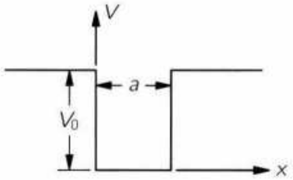


Figure 83-1

(a) For the stationary state of lowest energy E_0, $\psi_0(x,t) = u_0(x)e^{-iE_0t/\hbar}$. We must have $u_0(x) = 0$ just outside the well (i.e., at $x = -\epsilon$ or $x = a + \epsilon$). Why?

(b) Solve the Schrödinger Equation inside the well, subject to the condition stated in part (a).

(c) Find E_0 and sketch $u_0(x)$. ($u_0(x)$ need not be normalized.)

(d) Find the energy difference between the lowest state and the first excited state.

(e) For the lowest state, roughly sketch the probability distribution for finding the particle with momentum p in the range dp. No exact integrations are required. Don't worry about normalization. But, be sure to indicate the scale along the momentum axis.

83.2 Consider the motion of a particle of mass m in a one dimensional potential well, as shown in Fig. 83-2(a), defined by:

$$V = \begin{cases} \infty, & x < 0 \\ 0, & 0 < x < a \\ V_0, & x > a \end{cases}$$

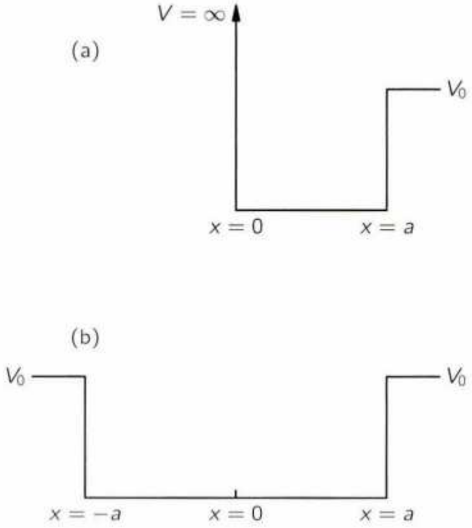


Figure 83-2

(a) Find the value of V_0 such that the energy of the particle in the ground state for this well is 10% less than the ground state energy for a well with $V_0 \to \infty$.

(b) Let V_0 be equal to the value found for part (a) above, in the potential well shown in Fig. 83-2(b), defined by:

$$V = \begin{cases} V_0, & x < -a \\ 0, & -a < x < a \\ V_0, & x > a \end{cases}$$

Without detailed calculation, give the energy of the first excited state for this well.

83.3 Consider the one-dimensional problem of a particle of mass m bound in a rectangular potential well:

$$V = \begin{cases} V_0, & |x| > a \\ 0, & |x| < a \end{cases}$$

(a) Show that the following two equations are obtained by requiring that the wave functions are solutions of the Schrödinger equation which fulfill the required boundary conditions:

$$\alpha \cot(\alpha a) = -\beta,$$

or

$$\alpha \tan(\alpha a) = +\beta,$$

where

$$\alpha = +\sqrt{\frac{2mE}{\hbar^2}},$$
$$\beta = +\sqrt{\frac{2m(V_0 - E)}{\hbar^2}}.$$

(b) If $V_0 a^2 = 4\hbar^2/2m$, estimate the energies of the ground state and first excited state. Sketch the wave functions of these states.

(c) How many bound states are there if $V_0 a^2 < \pi^2\hbar^2/8m$?

83.4 In Chapter 16 of Vol. III the momentum spread associated with a Gaussian wave function was found. In general, the spatial width will not remain constant in time, however, but will spread out:

$$\psi(x) = Ke^{-[a(t)x^2 + c(t)]}.$$

(a) Using the Schrödinger equation, show that for a free particle,

$$\frac{1}{a(t)} = \frac{1}{a_0} + \frac{2i\hbar}{m}t,$$

where $a_0 = a(0)$.

(b) What is $c(t)$?

(c) If the wave function describes an electron initially confined to a region 1.0 Å wide, how far will it be spread in 1 second?

(d) Transform the wave function $\psi(x)$ to momentum space $\phi(p)$, and find the probability of finding the particle with a definite momentum p.

(e) How does the width Δp of the momentum probability function vary with time?

(f) Show that the momentum spread found here is in agreement with the "velocity spread" found directly from the time dependence of the spatial wave function.

84

Angular Momentum

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 18.

84.1 A certain excited state of an atom has spin one and can lose its energy by emitting a photon and going to a state of spin zero. Consider an excited atom whose component of angular momentum along some z-axis is zero, and let $A(\theta)$ be the amplitude that it will emit a right-hand-circularly polarized photon into a small solid angle $\Delta\Omega$ in a direction at the angle θ with respect to the z-axis. How does $A(\theta)$ depend on θ?

84.2 The X, a spin 1/2 even parity particle, decays via the following scheme:

$$\mathrm{X} \to \mathrm{Y} + \gamma$$

where the Y is a spin 1/2 even parity particle. If the X were polarized with its spin up along the $+z$-axis, it could decay with the end products moving along the z-axis in eight ways, with amplitudes a through h, as shown in Fig. 84-1 (the wiggly arrows indicate photons moving in the direction of the arrow, and the arrow on the Y indicates the direction of its spin). Assume the angle between the polarization of the X and the momentum of the Y equals θ.

(a) Which of the amplitudes a–h are necessarily zero?

(b) Calculate the angular distribution of Y's polarized with their spins pointing in the same direction as their direction of motion ("positive helicity") $f^+(\theta)$ when a group of X's polarized up along the z-axis decays.

(c) Calculate the angular distribution of all Y's regardless of their polarizations $f(\theta)$ when a group of X's polarized up along the z-axis decays.

(d) Careful experiments have failed to show any deviation from a uniform angular distribution in this decay. What might be the physical reason for this?

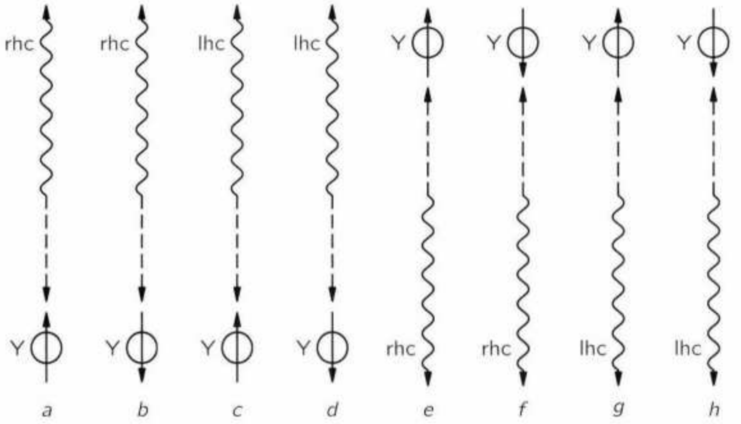


Figure 84-1

84.3 The reaction

$$\gamma + \mathrm{p} \to \mathrm{p}^* \to \mathrm{p} + \pi^0$$

is commonly studied at the CIT synchrotron. In this reaction p^* is an excited state of the proton which decays into a proton and π^0. Over a certain range of photon energies the p^* is found to have a total angular momentum $j = 3/2$. Assume that a beam of right hand circularly polarized photons at the right energy to give a p^* with $j = 3/2$ is incident in the $+z$ direction on unpolarized protons. The angular distribution for this reaction can be analyzed as follows: The photon plus proton have the amplitude a to form the p^* in the state $|\, j = 3/2, m = +1/2\rangle$ and the amplitude b to form the p^* in the state $|\, 3/2, +3/2\rangle$. The excited state of the proton, p^*, now decays into a neutral pion with zero spin and a proton moving in opposite directions. Let c be the amplitude for the proton to go along the $+z$-axis with its spin up and let d be the amplitude for it to go along the $-z$-axis with its spin up.

(a) Explain why only $m = +3/2$ and $m = +1/2$ are allowed for the p^* and why only $m' = +1/2$ and $m' = -1/2$ are allowed for the final state. (The m' refers to the direction of emission.)

(b) Predict the angular distribution of π^0's in terms of a, b, c and θ. (Assume $c = d$, and give a reason why that should be so.)

84.4 Consider elastic scattering of $\pi+$ mesons by an unpolarized proton target. (The meson has spin zero; parity is conserved.) It is hypothesized that the scattering is dominated by a process in which the proton is excited to a state with $j = 3/2$ by absorption of the meson. ($j = 3/2$ is achieved by a combination of proton spin and meson-proton orbital angular momentum.) The meson is then re-emitted with the proton returning to its ground state. Show that such a hypothesis predicts an angular distribution of scattered mesons proportional to $(1 + 3\cos^2\theta)$.

84.5 The ground state of an atom has spin zero and even parity. The first excited state has spin one and unknown parity. Assume a supply of atoms in the first excited state, all with $m = +1$ in the z-direction, and consider photons emitted in transitions to the ground state.

(a) If photons are detected regardless of their polarization, investigate whether their angular distribution can be used to determine the parity of the first excited state.

(b) Show that measurement of the angular distributions of x'- and y'-polarized photons can determine the unknown parity. (The z'-axis is taken along the direction of photon emission, and is in the xz-plane.)

85

The Hydrogen Atom and the Periodic Table

Refer to *The Feynman Lectures on Physics*, Vol. III, Chapter 19.

85.1 Hydrogen gas prepared in the $3d$ state is allowed to decay to the ground state by emission of radiation. Describe the emission spectrum of this radiation. (How many lines does it have? What are their wavelengths and relative intensities?)

85.2 (a) Use a classical argument to show that the distance r between the nucleus and the electron in a hydrogen atom in the ground state can not exceed $R = 2r_B$ (where $r_B = \hbar^2/me^2$ is the Bohr radius).

(b) Use quantum mechanics to find the probability $P(r > R)$ that the electron in a hydrogen atom in the ground state can be found *further* from the nucleus than the distance $R = 2r_B$.

85.3 The interaction of slow muons with matter has been described as consisting of three steps: First, due to electrostatic attraction, the muon enters a Bohr orbit around the nucleus (such an arrangement is called a "muonic atom"). Second, the muon's orbit decays with emission of radiation, until the muonic atom is in its ground state. Third, the muon decays via weak interaction with the nucleus.

Consider a muonic atom of Radium G (^{206}Pb, $Z = 82$):

(a) Find the ground state energy E_0.

(b) Find the wavelength $\lambda_{1\to 0}$ of photons emitted in a transition from the first excited state to the ground state.

(c) Find the characteristic distance r'_B of the muon orbit in the ground state.

(d) Find an approximate expression for the probability $P(r < R)$ that the muon in a muonic atom in the ground state can be found within some distance R about the nucleus.

(e) Compare the probabilities for an electron and for a muon to be "inside" the nucleus.

Note: for the electron, the characteristic distance $r_B \gg R$ when R is the nuclear radius; in this case you may assume that within the nucleus, the electron's wave function $\psi(r)$ does not vary significantly from its value at the center of the nucleus $\psi(0)$.

Note: Use $m_\mu = 207m_e$ for the muon's mass.

85.4 A student passes light from a tube of incandescent hydrogen gas through a diffraction grating, resulting in a number of discrete spectral lines.

(a) One of the lines measured has a wavelength of 4430 Å, with a width of 150 Å. This corresponds to an energy and uncertainty of $E = (2.80 \pm 0.09)\,\text{eV}$. Which transition between H energy levels could produce this line? What energy is required to excite the hydrogen atom from the ground state into the level that produces this transition?

(b) The line of part (a) is a member of the Balmer series. The Balmer series is the series of spectral lines produced by a transition from a higher energy state into the $n = 2$ state of H. What are the shortest and longest wavelengths of emission in the Balmer series?

(c) In many astrophysical applications, such as the study of spectral absorption lines in stellar atmospheres, H spectral lines are found alongside spectral lines of other elements. One such set of spectral lines belong to singly ionized helium atoms. What is the ground state energy of singly ionized He?

(d) What is the shortest wavelength spectral line that can be produced from a transition into the $n = 4$ state of singly ionized He? How does this compare to the shortest wavelength line in the Balmer series?

Appendix

A

Units and Dimensions

Units and Dimensions	Symbol	Value	Remarks
MASS [M]			
kilogram	kg	1 kg	SI unit
gram	g	10^{-3} kg	cgs unit
pound	lb	0.4536 kg	
LENGTH [L]			
meter	m	1 m	SI unit
centimeter	cm	10^{-2} m	cgs unit
millimeter	mm	10^{-3} m	
micron	μ	10^{-6} m	
Ansgstrom	Å	10^{-10} m	
kilometer	km	10^{3} m	
inch	in	2.54 cm	*Exact.*
foot	ft	0.3048 m	*Exact.*
mile	mi	1.6093 km	
astronomical unit	AU	1.497×10^{8} km	☼–♁ distance
light year	ly	9.5×10^{12} km	
parsec	pc	206265 AU	3.263 ly or 31×10^{12} km
TIME [T]			
second	s	1 s	SI and cgs unit
millisecond	ms	10^{-3} s	
microsecond	μs	10^{-6} s	
nanosecond	ns	10^{-9} s	
minute	$^{\mathrm{m}}$ or min	60 s	$1^{\mathrm{m}} = 60^{\mathrm{s}}$
hour	$^{\mathrm{h}}$ or h	3600 s	$1^{\mathrm{h}} = 60^{\mathrm{m}}$
day (solar)	$^{\mathrm{d}}$ or d	8.64×10^{4} s	$1^{\mathrm{d}} = 86400^{\mathrm{s}}$
year	$^{\mathrm{y}}$ or y	3.16×10^{7} s	$1^{\mathrm{y}} = 365.25^{\mathrm{d}}$
VELOCITY [LT^{-1}]			
$\mathrm{m\,s^{-1}}$	$\mathrm{m\,s^{-1}}$	$1\,\mathrm{m\,s^{-1}}$	SI unit
$\mathrm{mi\,h^{-1}}$	$\mathrm{mi\,h^{-1}}$	$(22/15)\,\mathrm{ft\,s^{-1}}$	mph
ACCELERATION [LT^{-2}]			
$\mathrm{m\,s^{-2}}$	$\mathrm{m\,s^{-2}}$	$1\,\mathrm{m\,s^{-2}}$	SI unit
FORCE [MLT^{-2}]			
newton	N	$1\,\mathrm{kg\,m\,s^{-2}}$	SI unit
dyne	dyn	10^{-5} N	cgs unit
MOMENTUM/IMPULSE [MLT^{-1}]			
$\mathrm{kg\,m\,s^{-1}}$	$\mathrm{kg\,m\,s^{-1}}$	1 N s	SI unit
ENERGY/WORK [$ML^{2}T^{-2}$]			
joule	J	$1\,\mathrm{kg\,m^{2}\,s^{-2}}$	SI unit
erg	erg	10^{-7} J	cgs unit
electron volt	eV	1.602×10^{-19} J	
POWER [$ML^{2}T^{-3}$]			
watt	W	$1\,\mathrm{J\,s^{-1}}$ or 1 V A	SI unit
CHARGE [Q]			
coulomb	C	1 A s	SI unit
electron charge	q_e	1.602×10^{-19} C	

Units and Dimensions	Symbol	Value	Remarks
CURRENT $[QT^{-1}]$			
ampere	A	1 A	SI unit
milliampere	mA	10^{-3} A	
microampere	μA	10^{-6} A	
POTENTIAL $[ML^2T^{-2}Q^{-1}]$			
volt	V	$1\,\mathrm{J\,C^{-1}}$	SI unit
ELECTRIC FIELD $[MLT^{-2}Q^{-1}]$			
volt per meter	$\mathrm{V\,m^{-1}}$	$1\,\mathrm{N\,C^{-1}}$	SI unit
MAGNETIC FIELD $[MT^{-1}Q^{-1}]$			
tesla	T	$1\,\mathrm{N\,s\,m^{-1}\,C^{-1}}$	SI unit
weber per m^2	$\mathrm{Wb\,m^{-2}}$	1 T	
gauss	gauss	10^{-4} T	
gamma	γ	10^{-9} T	
RESISTANCE $[ML^2T^{-1}Q^{-2}]$			
ohm	Ω	$1\,\mathrm{V\,A^{-1}}$	SI unit
INDUCTANCE $[ML^2Q^{-2}]$			
henry	H	$1\,\mathrm{V\,s\,A^{-1}}$	SI unit
CAPACITANCE $[M^{-1}L^{-2}T^2Q^2]$			
farad	F	$1\,\mathrm{C\,V^{-1}}$	SI unit
microfarad	μF	10^{-6} F	
picofarad	pF or $\mu\mu$F	10^{-12} F	

B

Physical Constants and (rounded) Values

Refer e.g. to: C. W. Allen, *Astrophysical Quantities*, The Athlone Press, 1963.
E. R. Cohen, *Il Nuovo Cimento Series 10*, **6**, 110 (1957).
Physics Today **17**, 48 (1964).

1. Astrophysical and Geophysical

Sun:	
Mass ($m_{☼}$)	1.99×10^{30} kg $= 3.33 \times 10^{5}\, m_{♁}$
Equatorial radius	6.96×10^{8} m
Mean density	$1.41\ \mathrm{g/cm^3}$
Earth:	
Mass ($m_{♁}$)	5.98×10^{24} kg
Equatorial radius	6.38×10^{6} m
Mean density	$5.52\ \mathrm{g/cm^3}$
Angular velocity	$7.29 \times 10^{-5}\ \mathrm{rad\,s^{-1}}$
Mean orbital speed	$29.77\ \mathrm{km\,s^{-1}}$
Land area	$1.48 \times 10^{14}\ \mathrm{m^2}$ $\approx 29\,\%$ of surface
Ocean area	$3.63 \times 10^{14}\ \mathrm{m^2}$ $\approx 71\,\%$ of surface
Mean ocean depth	3770 m
Moon:	
Mass ($m_{☾}$)	7.34×10^{22} kg $= (1/81.31)\, m_{♁}$
Equatorial radius	1.74×10^{6} m
Mean density	$3.34\ \mathrm{g/cm^3}$
Sun-Earth distance:	
Mean	$1.50 \times 10^{11}\ \mathrm{m} = 1\ \mathrm{AU}$
r_P	1.47×10^{11} m
r_A	1.52×10^{11} m
Earth-Moon distance:	
Mean	3.84×10^{8} m
r_P	3.63×10^{8} m
r_A	4.06×10^{8} m
Speed of light:	$c = 2.9979 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Universal gravitational constant:	$G = 6.670 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Solar constant at 1 AU:	$1347\ \mathrm{W\,m^{-2}}$
Acceleration of gravity: (at earth's surface; varies with latitude and altitude)	$g = 9.81\ \mathrm{m\,s^{-2}}$

2. Atomic and Nuclear

Planck's constant:	$h = 6.626 \times 10^{-34}\ \mathrm{J\,s}$ $= 4.135 \times 10^{-15}\ \mathrm{eV\,s}$

	$\frac{h}{2\pi} = \hbar = 1.0544 \times 10^{-34}\,\mathrm{J\,s}$ $= 6.58 \times 10^{-16}\,\mathrm{eV\,s}$
Bohr radius:	$\frac{4\pi\epsilon_0\hbar^2}{m_e q_e^2} = \frac{\hbar^2}{m_e e^2} = a_0 = 5.29 \times 10^{-9}\,\mathrm{cm}$ $= 0.529\,\text{Å}$
Classical electron radius:	$\frac{e^2}{m_e c^2} = \alpha^2 a_0 = r_0 = 2.82 \times 10^{-15}\,\mathrm{m}$ $= 2.82\,\mathrm{fermi}$
Fine structure constant:	$\frac{e^2}{\hbar c} = \alpha = \frac{1}{137}$
$-E_0$ for hydrogen:	$\frac{e^2}{2a_0} = 13.6\,\mathrm{eV}$
Nuclear "radius": (A is the atomic mass number, i.e., the number of protons plus the number of neutrons)	$\approx 1.3 \times 10^{-13}\,\mathrm{cm}\,A^{1/3}$ $\approx 1.3 \times 10^{-5}\,\text{Å}\,A^{1/3}$
Atomic "radius":	$\approx 1\,\text{Å}$
Molecular "radius":	$\approx 1.5\,\text{Å}$
Electron rest mass:	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$ $m_e c^2 = 0.51\,\mathrm{MeV}$
Proton rest mass:	$m_p = 1.67252 \times 10^{-27}\,\mathrm{kg}$ $= 1836\,m_e$ $m_p c^2 = 938.26\,\mathrm{MeV}$
Neutron rest mass:	$m_n = 1.67482 \times 10^{-27}\,\mathrm{kg}$ $m_n c^2 = 939.55\,\mathrm{MeV}$
Atomic mass unit ($\equiv \frac{1}{12}$ mass of C^{12}):	$1\,\mathrm{amu} = 1.66 \times 10^{-27}\,\mathrm{kg}$ $(1\,\mathrm{amu}) \times c^2 = 931\,\mathrm{MeV}$
Compton wavelength for electron:	$\frac{h}{m_e c^2} = 2\pi\alpha a_0 = \lambda_{C_e} = 2.43 \times 10^{-12}\,\mathrm{m}$
Free electron scattering cross section:	$\frac{8\pi}{3}\,r_0^2 = 6.65 \times 10^{-29}\,\mathrm{m}^2$

3. Macroscopic

ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
$1/4\pi\epsilon_0$	$9.00 \times 10^{9}\,\mathrm{m\,F^{-1}}$
μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
$\mu_0\epsilon_0 c^2$	1
$1/\epsilon_0 c$	$377\,\Omega$
Resistivity of copper:	$\approx 10^{-8}\,\Omega\,\mathrm{cm}$
Refractive index:	
n_{water}	1.33
n_{glass}	≈ 1.5
n_{air}	1.0003
Speed of sound (air at STP):	$331\,\mathrm{m\,s^{-1}}$
Density of air (STP $= 0\,^\circ\mathrm{C}$ at $1\,\mathrm{atm}$):	$1.293\,\mathrm{kg\,m^{-3}}$ $\approx 10^{-3}\,\mathrm{g\,cm^{-3}}$
Standard atmosphere:	$1\,\mathrm{atm} = 1.013 \times 10^{5}\,\mathrm{N\,m^{-2}}$ $= 760\,\mathrm{mmHg}$ $\approx 14.7\,\mathrm{lbs\,in^{-2}}$
Density of water ($20\,^\circ\mathrm{C}$):	$1.00 \times 10^{3}\,\mathrm{kg\,m^{-3}}$
Avogadro's number:	$N_0 = 6.025 \times 10^{23}$ molecules/mol

Boltzmann's constant: $k = 1.38 \times 10^{-23}\,\mathrm{J/K}$
$= 8.62 \times 10^{-5}\,\mathrm{eV/K}$

Gas constant: $R = 8.31\,\mathrm{J/(mol\,K)}$

Molar volume at STP: $22.41 \times 10^{3}\,\mathrm{cm^3/mol}$

Stefan-Boltzmann constant: $\dfrac{\pi^2}{60}\dfrac{k^4}{\hbar^3 c^2} = \sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2\,K^4}$

$$kT = 1\,\mathrm{eV} \text{ for } T = 11{,}600\,\mathrm{K}$$
$$\approx \frac{1}{40}\,\mathrm{eV} \text{ at room temperature}$$

$1\,\mathrm{eV/atom} = 9.652 \times 10^{4}\,\mathrm{J/mol} = 23.1\,\mathrm{kcal/mol}$

$1\,\mathrm{hp} = 746\,\mathrm{W}$

4. Numerical constants

$1^\circ = 1.745 \times 10^{-2}\,\mathrm{rad}$

$1' = 2.9089 \times 10^{-4}\,\mathrm{rad}$

$1'' = 4.8481 \times 10^{-6}\,\mathrm{rad}$

$e = 2.71828\ldots$

$\pi = 3.14159\ldots$

$\log_{10} e = 0.434$

$\ln 2 = 0.693$

C

Answers

Chapter 1

1.9 (a) $n_G \approx 10^{19}\,\mathrm{cm}^{-3}$, $n_L \approx 10^{22}\,\mathrm{cm}^{-3}$
  (b) $m \approx 10^{-23}\,\mathrm{g}$
  (c) $l \approx 10^{-5}\,\mathrm{cm}$
  (d) $P \approx 10^{-7}\,\mathrm{atm}$

1.10 $A \approx 1.4 \times 10^{-14}\,\mathrm{cm}^2$

1.11 $N_A = 6.02 \times 10^{23}\,\mathrm{mol}^{-1}$

1.12 (a) $N_H \approx 2.7 \times 10^{19}\,\mathrm{cm}^{-3}$
  (b) $N_A \approx 6.1 \times 10^{23}\,\mathrm{mol}^{-1}$

1.13 $N_A \approx 10^{24}\,\mathrm{mol}^{-1}$

1.14 $N_g \approx 0.7 \times 10^{19}\,\mathrm{cm}^{-3}$

Chapter 2

2.5 $W = (3/\sqrt{2})\,\text{kg-wt}$

2.6 $F_W = (\tan\alpha)\,\text{kg-wt}$, $F_P = (1/\cos\alpha)\,\text{kg-wt}$

2.7 $W_2 = 0.25\,\text{kg-wt}$

2.8 $A = \frac{1}{2}(1+\sqrt{3})\,\text{kg-wt}$, $B = \sqrt{3/2}\,\text{kg-wt}$

2.9 $T_1 = T_2 = (50/\sqrt{2})\,\text{lb-wt}$

2.10 (a) The motion of W is downward
  (b) $F = W(y/x)$, in tension

2.11 $F = W\sqrt{h(2R-h)}/(R-h)$

2.12 (a) $F_B = 0$ (except gravity)
  (b) $\tau_O = FD$
  (c) $\tau_P = FD$

2.13 $F = 20.7\,\mathrm{N}$ bearing 45°, applied 0.34 m left of point O

2.14 $v = \sqrt{2gD\,\dfrac{W_1 - W_2}{W_1 + W_2}\,\sin\theta}$

2.15 $v = \sqrt{gD(\sin\phi - \sin\theta)}$

2.16 (a) $a = \frac{1}{2}(1 - 1/\sqrt{2})g$
  (b) M_2 will move downward
  (c) $t_1 = \sqrt{H/a}$
  (d) M_1 will not strike the pulley

2.17 $T = \dfrac{L}{x}\left(W + \dfrac{w}{2}\right)\tan\theta$

2.18 (a) $F_R = 45\,\text{lb-wt}$
  (b) $F_h = 45\,\text{lb-wt}$, $F_v = 90\,\text{lb-wt}$

2.19 $\theta = 30^\circ$

2.20 $\alpha = 30^\circ$

2.21 $F = 0.6W$

2.22 $T = 2\,\text{ton-wt}$

2.23 $T = 265\,\text{g-wt}$, $\alpha = 79.1^\circ$

2.24 $T = \sqrt{3}\,W/4$

2.25 $\theta = 30^\circ$

2.26 $T = Wh/2\pi r$

2.27 $W = 4w/\sin\theta$

2.28 $F_1 = W/3$, $F_2 = 2W/3$, $F_{DF} = 4W/3\sqrt{3}$

2.29 (a) AC, CE, EG, BC, EF, and ED could be replaced with cables
  (b) $F_{BD} = W/2$, $F_{DE} = 5W/12$

2.30 $W = 3w/4$

2.31 $v = 196\,\mathrm{cm\,s}^{-1}$

2.32 $v = \sqrt{2gH}$

2.33 $\theta = \arctan(1/3\sqrt{3}) \approx 10.9^\circ$

2.34 $W = wr/(R-r)$

2.35 $A = 5\,\mathrm{m}$, $B = 11\,\mathrm{m}$, $T_{\max} \approx 34 \times 10^3\,\text{kg-wt}$

2.36 (a) $F = WL/h$
  (b) $\boldsymbol{F}_A = W\sqrt{1 + (L/h)^2}$, bearing $\arctan(h/L)$

Chapter 3

3.2 $T \approx 1.6\,\mathrm{h}$

3.3 $v_{\max}/v_{\min} = 1.033$

3.4 $g_{\text{☾}} = 1.6\,\mathrm{m\,s}^{-2} \approx g_{\text{♁}}/6$

3.5 (a) $r_a \approx 35.2\,\mathrm{AU}$
  (b) $v_{\max}/v_{\min} = 59$

3.6 $r \approx 5.9\,r_{\text{♁}}$

3.8 (a) $\lambda = 0$
  (b) $r_s = r_{\text{♁☾}}/9$

3.9 (a) $m_{\text{☼}}/m_{\text{♁}} = 3.33 \times 10^5$
  (b) $m_{\text{♃}}/m_{\text{♁}} = 318$

3.10 $m_a + m_b = (R^3/T^2)m_{\text{☼}}$

3.11 Kepler's second law would be unchanged. Kepler's third law would become $T^2 \propto R^{(3+a)}$, where T is the orbital period of the satellite.
  Note: $T^2 = (4\pi^2/GM)R^{(3+a)}$

3.12 $\Delta g/g = 7 \times 10^{-6}$

3.13 $M = (1.02 \times 10^{-7}\,\mathrm{d}^{-1}\,\mathrm{km}^{-3}\,\mathrm{s}^3)\,TV^3\,m_{\text{☼}}$

3.14 (a) $R_c = 1.88 \times 10^6\,\mathrm{km}$
  (b) $a = r_p^2/(2r_p - R_c) = 8.33 \times 10^6\,\mathrm{km}$
  (c) $T_c = 2\pi a^{3/2} R_c/v r_p \approx 4.8\,\mathrm{d}$

3.15 $T^2 = \dfrac{4\pi^2(R+r)^3}{G(M+m)}$

3.17 (a) $M \approx 3.1\,m_{\text{☼}}$

Chapter 4

4.5 (a) $t = 1843.8\,\text{s}$
 (b) $v \approx 1385\,\text{ft}\,\text{s}^{-1}$

4.6 $t \approx 155\,\text{s}$

4.7 It takes longer to come down.

4.8 (a) $v \approx 465\,\text{m}\,\text{s}^{-1}$
 (b) $\omega \approx 7.3 \times 10^{-5}\,\text{s}^{-1}$
 (c) $a/g \approx 3.5 \times 10^{-2}$

4.9 (b) $H_{\max} = 46\,\text{mi}$
 (c) $T = 2.7 \times 10^2\,\text{s}$

4.10 $e \approx 0.98$

4.11 (a) $H_{\max} = (v_0^2/2g)\sin^2\theta$, $R = (v_0^2/g)\sin 2\theta$
 (b) $\theta = \pi/4$

4.12 $V = \dfrac{L}{\cos\theta}\sqrt{\dfrac{g}{2(L\tan\theta - h)}}$

4.13 (a) $v \approx 48\,\text{ft}\,\text{s}^{-1}$
 (b) $R \approx 8.5\,\text{ft}$

4.14 (a) $x = Vt - R\sin\dfrac{Vt}{R}$, $y = R(1 - \cos\dfrac{Vt}{R})$
 (b) $v_x = V(1 - \cos\dfrac{Vt}{R})$, $v_y = V\sin\dfrac{Vt}{R}$
 (c) $a_x = \dfrac{V^2}{R}\sin\dfrac{Vt}{R}$, $a_y = \dfrac{V^2}{R}\cos\dfrac{Vt}{R}$

4.15 $v = 14.8\,\text{m}\,\text{s}^{-1}$

4.16 (a) $\theta = \arctan 3$
 (b) $x = 14\,\text{m}$
 (c) $V = 19.8\,\text{m}\,\text{s}^{-1}$

4.17 $d \approx 202\,\text{ft}$

4.18 (a) $v = 52.5\,\text{mi}\,\text{h}^{-1}$
 (b) $a = 2.75\,\text{ft}\,\text{s}^{-2}$

4.19 $a_J/a_R = 8/9$

Chapter 5

5.1 (b) $a = 2g/5$
 (c) $T_1 = (2/5)$ kg-wt, $T_2 = (6/5)$ kg-wt

5.2 $T = 8.8m\,\text{N}$

5.3 $T = 25\,\text{N}$

5.4 $F = \dfrac{M_2}{M_1}(M + M_1 + M_2)g$

5.5 $F = 392\,\text{N}$

5.6 $\boldsymbol{a} = g/9$ upward, $T = 222$ g-wt

5.7 (a) $M_{\max} = \dfrac{(M_A + M_B)D - 2M_B L\cos\theta}{4L\cos\theta - D}$
 (b) $t = \sqrt{\dfrac{8L\sin\theta}{g}}$

5.8 $g = \dfrac{v^2(2M + m)}{2mh}$

5.9 (a) $a_0 = \dfrac{F - (M_1 + M_2)g}{M_1 + M_2}$
 (b) $T = \dfrac{M_1}{M_1 + M_2}F$
 (c) $a = g - F/M_2$, $a' = g$
 (d) $t = \sqrt{2M_2 s/F}$

5.10 $\Delta t = 0.9\,\text{s}$

5.11 $W = 2.7$ kg-wt

5.12 (a) $\boldsymbol{a} = g/3$ up
 (b) $F = 280$ lb-wt

5.13 $m_B \approx 5.8\,\text{kg}$

5.14 $x_{\max} = 1$

5.17 $h_{\max} \approx 5 \times 10^3\,\text{ft}$, $R \approx 1.6 \times 10^4\,\text{ft}$

Chapter 6

6.5 $m_2/m_1 = 3$

6.6 $E' = 0.71E$

6.7 (a) $v_E = 8.4 \times 10^{-22}\,\text{m}\,\text{s}^{-1}$
 (b) $T_E/T_p = 1.7 \times 10^{-24}$

6.8 $V_F = 3.66\,\text{m}\,\text{s}^{-1}$

6.9 The platform moves to the north at $v = 5 \times 10^{-4}\,\text{m}\,\text{s}^{-1}$.

6.10 $\boldsymbol{a}(t) = \dfrac{(m_1 - m_0)g + r_0(v_0 + gt)}{m_1 + m_0 - r_0 t}$ downward, for $0 < t < \dfrac{m_0}{r_0}$.

6.11 $a = 4.0\,\text{m}\,\text{s}^{-2}$

6.12 $F = \mu v(v + gt)$

6.13 $V = x\left(\dfrac{m + M}{m}\right)\sqrt{\dfrac{g}{L}}$

6.14 (b) $\tau_0 = 2L\sqrt{\dfrac{Mm}{2T(M + m)}}$

6.15 $\Delta v \approx vf/4$

6.16 (a) $a = r_0 V_0/M_0$
 (b) $r_0 = 490\,\text{kg}\,\text{s}^{-1}$
 (c) $v = -V_0\displaystyle\int_{M_0}^{M}\frac{dm}{m}$

6.17 (a) $F_R = 5.1 \times 10^{-3}\,\text{N}$
 (b) $F_R \propto v^2$

Chapter 7

7.5 $\boldsymbol{r}(t) = \boldsymbol{r}_0 + \boldsymbol{v}_0 t + \frac{1}{2}\boldsymbol{g}t^2$

7.6 (a) $5\boldsymbol{i} + \boldsymbol{j}$
 (b) $\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$
 (c) 3
 (d) 3
 (e) 3
 (f) $15\boldsymbol{i} - 18\boldsymbol{j} + 9\boldsymbol{k}$

7.7 (c) $R(1) \approx 4.6\,\text{m}$

7.8 (a) He should head his plane 14.5° west of north.
 (b) $T = 53.9\,\text{min}$

7.9 (a) The wind blows from 40.5° NE.
 (b) The wind appears to the cyclist to blow from 35.6° SE on his return.

7.10 Method 2 is faster, by 4.0 min.

7.11 (a) $t_V/t_A = V/\sqrt{V^2 - R^2}$
 (b) $t_A/t_L = t_V/t_A$

7.12 $D = r_{\text{♁}}\arccos[\sin\lambda_1\sin\lambda_2 + \cos\lambda_1\cos\lambda_2\cos(\phi_1 - \phi_2)]$

7.13 (a) $\boldsymbol{a} \approx 3.2 \times 10^{-3}\,\text{m}\,\text{s}^{-2}$, toward sun
 (b) $\boldsymbol{a} \approx 7.2 \times 10^{-3}\,\text{m}\,\text{s}^{-2}$, bearing $\approx 24°$
 (c) $\boldsymbol{a} \approx 8.5 \times 10^{-3}\,\text{m}\,\text{s}^{-2}$, toward sun

7.14 (a) $\boldsymbol{a}_1 = 2g$, rightward

(b) $\boldsymbol{a} = \sqrt{2}g$, bearing 45°

(c) $F_2 = 272\,\text{kg-wt}$

7.15 $T = 2\pi\sqrt{H/g}$

7.16 For $t \le 3$, $\boldsymbol{r}_a(t) = (7 + 7t)\boldsymbol{i} + 3t\boldsymbol{j} + 4.9(1 - t^2)\boldsymbol{k}$, $\boldsymbol{r}_b(t) = (49 - 7t)\boldsymbol{i} + 3t\boldsymbol{j} + 4.9(1 - t^2)\boldsymbol{k}$. For $t \ge 3$, $\boldsymbol{r}_a(t) = \boldsymbol{r}_b(t) = 28\boldsymbol{i} + 3t\boldsymbol{j} + 4.9(1 - t^2)\boldsymbol{k}$.

7.17 (a) The course of the other ship was due North.

(b) $T = 0.17\,\text{h}$

Chapter 8

8.3 $\theta_{\max} = \arcsin{(m/M)}$

8.4 $\theta_1 = \arctan 3$, $v_1' = v\sqrt{10}/4$, $v_2' = v\sqrt{2}/4$

8.5 $\alpha = 120°$

8.6 (a) $\boldsymbol{v}_{\text{CM}} = (4\boldsymbol{i} - 3\boldsymbol{j})\,\text{m}\,\text{s}^{-1}$

(b) $u_1 = u_2 = 5\,\text{m}\,\text{s}^{-1}$

(c) $\boldsymbol{v}_1' = (4\boldsymbol{i} + 2\boldsymbol{j})\,\text{m}\,\text{s}^{-1}$

8.7 $\boldsymbol{v}_1' = \sqrt{3}\,v_0/2$ bearing −30°, $\boldsymbol{v}_2' = v_0/2$ bearing +60°

8.8 (a) $v_2 = \sqrt{3} \times 10^6\,\text{m}\,\text{s}^{-1}$

(b) $\boldsymbol{v}_1' = \sqrt{73} \times 10^6\,\text{m}\,\text{s}^{-1}$, bearing 113°

(c) $\boldsymbol{u}_1' = 9 \times 10^6\,\text{m}\,\text{s}^{-1}$, bearing 120°

8.9 $\boldsymbol{v}_1 = 50\sqrt{7}\,\text{cm}\,\text{s}^{-1}$, bearing $\arctan(-\sqrt{3}/2) = -41°$, $\boldsymbol{v}_2 = 50\,\text{cm}\,\text{s}^{-1}$, bearing +60°

8.10 $\left|\dfrac{\Delta T}{T}\right|_{\text{lab}} = \dfrac{(1 - \alpha^2)m_2}{m_1 + m_2}$

8.11 (a) $\theta = \arctan\sqrt{\dfrac{M - m}{M + m}}$

(b) $\theta = \arctan\sqrt{\dfrac{\alpha^2 M^2 - m^2}{(M + m)^2}}$

8.12 $M = 9m_p$

8.13 (a) $\boldsymbol{v}_2' = \sqrt{5}\,\text{m}\,\text{s}^{-1}$, bearing $\arctan\frac{1}{2}$ NW

(b) $\alpha = 3/4$

(c) $\theta = 90°$

8.14 (a) $\boldsymbol{v} = (2\boldsymbol{j} + 2\boldsymbol{k})\,\text{m}\,\text{s}^{-1}$

(b) $T_{\text{CM}} = 30\,\text{J}$

Chapter 9

9.2 $\boldsymbol{F} = -108\boldsymbol{e}_x\,\text{N}$

9.3 $\boldsymbol{a} = g/8$ downward

9.4 See Ex. 9.1.

9.5 $x = \mu h$

9.6 $X = 32\,\text{cm}$

9.7 $\mu_{\min} = t\frac{1}{2}(\cot\theta - 1)$

9.8 (a) $\Delta T = \mu T\,\Delta\theta$

(b) $T_2/T_1 = e^{\mu\alpha}$

9.9 $v_0 = 595\,\text{m}\,\text{s}^{-1}$

9.10 $v = 51.8\,\text{mph}$

9.11 $t = 2.2\,\text{s}$

9.12 (a) $d = 0.68\,\text{m}$

(b) $t = 1.1\,\text{s}$

(c) $\Delta E = 2.3\,\text{J}$

9.13 (a) $a = 75\,\text{cm}\,\text{s}^{-2}$

(b) $F = 200\,\text{g-wt}$

9.14 For $0 \le x < x_0$, $v(x) = v_0$. For $x_0 \le x < x_0 + L$, $v(x) = v_0 - \dfrac{k}{m}(x - x_0)$. For $x \ge x_0 + L$, whenever $v_0 > kL/m$, $v(x) = v_0 - \frac{k}{m}\,L$.

9.15 $\epsilon = (R/4)\Delta P$, $|\sigma/\epsilon| = 1$

9.16 The bus is accelerating at $(g/\sqrt{3})\,\text{m}\,\text{s}^{-2}$.

9.17 (b) $R = mv/qB$

(c) $T = 2\pi m/qB$

9.18 (a) $W = 600\,\text{g-wt}$

(b) $T = 800\,\text{g-wt}$

(c) $\theta = 90°$

9.19 $F = mg(1 + \mu)/(1 - \mu)$

9.20 (a) $H_{\min} = \sqrt{3}W\sin\alpha$

(b) $\phi = 60°$

9.21 $R = 4.9 \times 10^{-5}\,\text{cm}$

9.22 (a) $F_x = qv_y B_z$, $F_y = q(E_y - v_x)B_z$, $F_z = 0$

(b) $F_x' = qv_y' B_z$, $F_y' = -qv_x' B_z$

(c) The motion of a free particle in crossed electric and magnetic fields is cycloidal.

Chapter 10

10.1 $x = x_0 - v_0\sqrt{m/k}$

10.3 The particle will be in equilibrium anywhere in the interior.

10.4 $v \approx v_\infty = 3.9\,\text{mi}\,\text{s}^{-1}$

10.5 $\phi = 1.4 \times 10^3\,\text{V}$

10.6 $Q = 5.6 \times 10^{-5}\,\text{C}$

10.7 $\phi = 310\,\text{kV}$

10.8 (a) $v = 3.0\,\text{m}\,\text{s}^{-1}$, $a = 2.5\,\text{m}\,\text{s}^{-2}$, $P = 45\,\text{W}$

(b) $v = 4.5\,\text{m}\,\text{s}^{-1}$, $a = 2.5\,\text{m}\,\text{s}^{-2}$, $P = 67.5\,\text{W}$

10.9 (a) $\boldsymbol{F} = (4.5\boldsymbol{i} + 12\boldsymbol{j} - 2.6\boldsymbol{k})\,\text{N}$

(b) $\boldsymbol{a} = (4.5\boldsymbol{i} + 12\boldsymbol{j} - 2.6\boldsymbol{k})\,\text{m}\,\text{s}^{-2}$

(c) $T = 2.5\,\text{J}$

(d) $dT/dt = 21.4\,\text{W}$

10.10 (a) $\boldsymbol{r} = (2.00\boldsymbol{i} + 3.02\boldsymbol{j} + 0.1\boldsymbol{k})\,\text{m}$

(b) $\boldsymbol{v} = (0.045\boldsymbol{i} + 2.12\boldsymbol{j} + 0.97\boldsymbol{k})\,\text{m}\,\text{s}^{-1}$

(c) $T = 2.71\,\text{J}$

10.11 (a) $W = 0$

(b) $W = 0$ and the force is conservative.

10.12 $H = R/2$

10.13 (a) $x = H\sin^2\theta + R\cos^2\theta - R\cos^3\theta$

(b) $F = mg(2H/R + 1)$

10.14 $d = mg/k - R/5$

10.15 $v = \sqrt{gL/2}$

10.16 $d = R/3$

10.17 $\theta = \arcsin 0.27$

10.18 $a = 7.2\,\text{m}\,\text{s}^{-2}$

10.19 $W_{\text{shot}} \approx 625\,\text{J}$, $W_{\text{disc}} \approx 570\,\text{J}$, $W_{\text{jav}} \approx 330\,\text{J}$

10.20
$$\phi(r) = \begin{cases} -\dfrac{GM}{2R^3}(3R^2 - r^2) & \text{for } r \le R \\ -\dfrac{GM}{r} & \text{for } r > R \end{cases}$$

$$\boldsymbol{g}(\boldsymbol{r}) = \begin{cases} -\dfrac{GM}{R^3}\,\boldsymbol{r} & \text{for } r \le R \\ -\dfrac{GM}{r^3}\,\boldsymbol{r} & \text{for } r > R \end{cases}$$

10.21 $\boldsymbol{a} = -\frac{4\pi}{3}\rho G\left(\frac{R^3}{(R+x)^2} - \frac{(R/4)^3}{(5R/4+x)^2}\right)$

10.22 $F_x = F_z = 0$ and

$$F_y = \begin{cases} m(-2\pi\rho dG + 4\pi r^3\rho G/3y^2) & \text{for } y > d/2, \\ m(-4\pi\rho Gy + 4\pi r^3\rho G/3y^2) & \text{for } d/2 > y > r, \\ m(-4\pi\rho Gy + 4\pi\rho Gy/3) & \text{for } r > y \geq 0. \end{cases}$$

10.23 $E = -GmM/2a$

10.24 (a) $T^2 = 4\pi^2 a^3/GM$

(b) $T^2 = \pi^2 G^2 M^2/2(E/m)^3$

10.26 $H_{\min} = 19.2\,\mathrm{cm}$

10.27 $P = 25\,\mathrm{kW}$

10.28 $P = 4.60 \times 10^4\,\mathrm{atm}$

10.29 $W_{\min} = GMm'\left(\frac{1}{R} - \frac{1}{D-r}\right) + Gmm'\left(\frac{1}{D-R} - \frac{1}{r}\right)$

10.30 The satellite would escape on a parabolic orbit.

10.31 (a) $v_{\min} = 11.8\,\mathrm{mi\,s^{-1}}$

(b) $v_{\max} = 47.1\,\mathrm{mi\,s^{-1}}$

10.32 $v_0 = 40.8\,\mathrm{km\,s^{-1}}$, $\alpha = 40.9^\circ$

Chapter 11

11.1 (a) MT^{-2}

(b) ML^2T^{-2}

(c) ML^2T^{-2}

(d) MT^{-2}

(e) 1 (dimensionless)

(f) $ML^{-1}T^{-1}$

(g) LT^{-2}

(h) $MLT^{-2}Q^{-1}$

(i) $MT^{-1}Q^{-1}$

(j) LT^{-1}

11.2 $(\epsilon_0 c)^{-1} = 377\,\Omega$

11.3 $v' = \frac{\lambda}{\tau}v$

$a' = \frac{\lambda}{\tau^2}a$

$F' = \frac{\mu\lambda}{\tau^2}F$

$E' = \frac{\mu\lambda^2}{\tau^2}E$

11.4 $T \propto \sqrt{l/g}$

11.5 $GM_{☼} = 4\pi^2$

11.6 T is independent of k.

11.7 $T \propto \sqrt{m/k}$

11.8 $T \propto mv^2/l$, $a \propto v^2/l$

11.9 $R \propto (v^2/g)f(\theta)$, $T \propto (v/g)f'(\theta)$, for functions f and f' that depend only on θ.

11.10 $T \propto \sqrt{m/\sigma}$

11.11 $\omega \propto (1/l)\sqrt{T/\sigma}$

11.12 $v \propto \sqrt{g\lambda}$

11.13 $p \propto f(N)mv^2/V$ for a function f that depends only on N.

11.14 GM is constant, and the units of $GM = Fr^2/m$ are L^3T^{-2}. The only length and time involved are the radial distance r and the orbital period t. Thus, $r^3/t^2 = \text{const}$.

Chapter 12

12.1 $x = \gamma(x' + \beta ct')$

$y = y'$

$z = z'$

$t = \gamma(t' + \beta x'/c)$

with $\beta = V/c$ and $\gamma = 1/\sqrt{1-\beta^2}$.

12.2 $v_x = \frac{v'_x + V}{1 + \beta v'_x/c}$

$v_y = \frac{v'_y/\gamma}{1 + \beta v'_x/c}$

12.3 $v'_x = \frac{v_x - V}{1 - \beta v_x/c}$

$a'_x = \frac{a_x/\gamma^3}{(1 - \beta v_x/c)^3}$

12.4 $L' \approx 4.5\,\mathrm{m}$, $\theta' \approx 33.7^\circ$

12.5 (a) $\Delta t = 16.8\,\mu\mathrm{s}$, $\Delta t' = 2.37\,\mu\mathrm{s}$

(b) $L' = 0.71\,\mathrm{km}$

12.7 $F = mc^2/b$

12.8 (a) $M \approx 42\,\mathrm{kg}$

(b) $H \approx 2\,\mathrm{cm^3\,s^{-1}}$

12.9 $H \approx 6.3 \times 10^8\,\mathrm{t\,s^{-1}}$

12.10 (a) $g = 1.03\,\mathrm{ly\,y^{-2}}$

(b) $x = 4.15\,\mathrm{ly}$, $v = 0.982c$

Chapter 13

13.2 (a) $pc = T\sqrt{1 + 2mc^2/T}$

(b) $v/c = \sqrt{3}/2$

13.3 $\Delta t = 5\,\mathrm{min}$

13.4 (a) $pc = 3 \times 10^{-2}\,BRZ$

(b) $R = 6.8 \times 10^3\,\mathrm{km}$

13.5 (a) $R_{\min} = 1.8\,\mathrm{m}$

(b) $f = 15\,\mathrm{MHz}$

(c) $\Delta f/f = 0.14$

13.6 $T_1 = \frac{(M - m_1)^2 - m_2^2}{2M}c^2$

$T_2 = \frac{(M - m_2)^2 - m_1^2}{2M}c^2$

13.7 $T_\mu = 4.1\,\mathrm{MeV}$

$T_\nu = 29.7\,\mathrm{MeV}$

$p_\mu = p_\nu = 29.7\,\mathrm{MeV}/c$

13.8 $E_\gamma = \Delta E(1 - \Delta E/2mc^2)$

13.9 (a) $V = c/2$

(b) $M = (4/\sqrt{3})m$

13.10 $T = 6m_pc^2 = 5.6\,\mathrm{GeV}$

13.11 $T_e = 4004\,\mathrm{GeV}$

13.12 $E_\gamma = 4m_pc^2 = 3.8\,\mathrm{GeV}$

13.13 (a) $T_p \approx 2m_\pi c^2 = 279\,\mathrm{MeV}$

(b) $T_\pi \approx m_\pi^2 c^2/2m_p = 10.4\,\mathrm{MeV}$

13.14 With $\beta = v/c$,

(a) $E_\gamma = \dfrac{m_\pi c^2\sqrt{1-\beta^2}}{2(1-\beta\cos\theta)}$

(b) $E_{\max} = \dfrac{m_\pi c^2(1+\beta)}{2\sqrt{1-\beta^2}}$ when $\theta = 0$,

$E_{\min} = \dfrac{m_\pi c^2(1-\beta)}{2\sqrt{1-\beta^2}}$ when $\theta = \pi$

(c) $\dfrac{\sqrt{1-\beta^2}}{2}(E_{\max}+E_{\min}) = \dfrac{m_\pi c^2}{2}$ $(= E_\gamma$, when $v = 0)$

Chapter 14

14.4 (a) $\tau = 140\,\mathrm{N\,m}$
(b) $l = 2.8\,\mathrm{m}$
(c) $F_\perp = 14\,\mathrm{N}$

14.6 $\lambda \approx 66.6^\circ$

14.7 $V = 2\pi^2R^3$

14.8 $T = M\omega^2R^2/3$

14.9 (a) $x = (2R^2/L)\sin(L/2R)$, $y = 0$
(b) $x = (4R/3\alpha)\sin(\alpha/2)$

14.10 $x = 1.7\,\mathrm{cm}$, $y = 0$

14.11 $y = x/2$

14.12 (a) $F = 8.1\,\mathrm{N}$
(b) $x = (3\sqrt{3}/2)\,\mathrm{cm}$

14.13 $\alpha = \arctan(1/6\sqrt{3})$

14.14 $\overline{OP} = 1.5'$

14.15 $h = a(3-\sqrt{3})/2$

14.16 $x = M_1L/(M_1+M_2)$

14.17 $P = 16m\mu\pi^3f^3r^2$

14.18 $n = a$

14.19 $M = 4.0\,\mathrm{lb}$

14.20 (a) $\Delta L = M\omega^2L/(2k - M\omega^2)$
(b) Stable equilibrium is possible only if $M\omega^2/k < 2$.

Chapter 15

15.3 $I = ML^2/12$

15.4 $\omega = \sqrt{3g/l}$

15.5 $a = mg/(m + M/2)$

15.6 $I = 22Ma^2$

15.7 (a) $I = mL^2/3$
(b) $I = mL^2/12$
(c) $I = mr^2$
(d) $I = mr^2/2$

15.8 $F = Mg/4$

15.9 $W = 6mL^2\omega_0^2$

15.10 (a) $V_0 = r\sqrt{2Mgh/(I+Mr^2)}$

15.11 The belt should be accelerated at the rate $2g\sin\theta$.

15.13 $I = \dfrac{8}{15}\pi\rho(R^5 - r^5) = \dfrac{2}{5}M\dfrac{R^5-r^5}{R^3-r^3}$

15.14 $\mu_0 = (2/7)\tan\theta$

15.15 (b) $a = g/(1 + R^2/2r^2)$

15.16 $h = 3d/2 - 3r$

15.17 (a) $v_2 = (r_1/r_2)v_1$
(b) $W = \frac{1}{2}mv_1^2(r_1^2/r_2^2 - 1)$
(c) $F = mv^2/r$

15.18 (a) $a = FR(R\cos\alpha - r)/(I + MR^2)$

15.19 $h/r = 7/5$

15.20 $r_3^2 + r_1^2 = r_2^2$

15.21 $\alpha_{\max} = (2g/R)\cos\theta$

15.22 $t = 2l\omega_0/3\mu g$

15.23 (a) $D = 12V_0^2/49\mu g$
(b) $V = (5/7)V_0$

15.24 (a) $V_0 = (2/5)R\omega_0$
(b) $V_0 = (1/4)R\omega_0$

15.25 $l = R(\sqrt{1 + M/4m} - 1)$

15.26 (a) $F'_x = F_x\cos\theta + F_y\sin\theta + 2m\omega\dot{y}' + m\omega^2x'$
$F'_y = F_y\cos\theta - F_x\sin\theta - 2m\omega\dot{x}' + m\omega^2y'$

Chapter 16

16.6 $V = 406\,\mathrm{m}^3$

16.9 The gyroscopic effect of the engines causes the airplane to (e) pitch up.

16.10 (a) If the string breaks, it happens before the masses are released.

(b) If the string does not break, the velocity of the CM of the two masses will be $l\omega_0/2$, while the masses rotate about their CM with angular velocity ω_0.

16.11 $v_{\mathrm{AC}}/v_{\mathrm{AB}} = -7/2$

16.12 $v = \frac{1}{2}(M/m - 1)V - L^2\omega^2/24V$
$a = \omega L^2/12V$

16.13 (a) $V_{\mathrm{CM}} = v/2$
(b) $L = mvR/2$
(c) $\omega = v/3R$
(d) $T_{\mathrm{before}} = mv^2/2$, $T_{\mathrm{after}} = mv^2/3$

16.14 The velocity of the CM of the object is $v_0/6$, and it rotates about its CM at angular velocity $2v_0/15a$.

16.15 (a) $V_O = J/M$
(b) $\omega = 12Jr/ML^2$
(c) $V_A = (J/M)(1 - 6r/L)$
(d) $\overline{AP} = 2L/3$
(e) $\overline{AP} = 2L/3$ (again)

16.16 (a) $V_{\mathrm{CM}} = v/2$ (before and after)
(b) $L = MvL/4$
(c) $\omega = 6v/5L$
(d) 20 % of the kinetic energy is lost.

16.17 $V_f = V/2$, $\omega_f = 6V/7L$

16.18 $V = \sqrt{8gL}$

16.19 (a) $v_f = \dfrac{l}{2}\sqrt{\dfrac{g}{L}\dfrac{m+M}{2(m/4+M/3)}}$

16.20 (a) $J = I_0\omega_0$
(b) $F = mR^2\omega_0^2/(R - (1+20\pi)r)$

16.21 $\tau_{\max} = ml^2\omega^2$

16.22 (a) $\theta = 45^\circ$
(b) $\tau = (Ml^2/24)\omega^2$

16.23 $F = 89.8\,\mathrm{N}$

16.24 $\omega_1 = I_2\omega_2/(I_1 + I_2 + M_2r^2)$

16.25 (a) $r = V/2\omega$
(b) $\theta \approx 2\omega R/V$

16.26 The satellite rotates about an axis perpendicular to its length (passing through its CM), with angular velocity $\omega_f = 2\omega_0/13$.

16.27 $\Delta T \approx 1\,\mathrm{ms}$

16.28 $\tau \approx (2GMm/R^3)r^2\sin(2\theta)$

16.30 $J = M\sqrt{\pi g L n/3}$, with $n \in \mathbb{Z}$.

16.31 (a) $\omega = \dfrac{I_0 + mR^2}{I_0 + mr^2}\,\omega_0$
(c) $\dot{r} = \omega_0\sqrt{\dfrac{I_0 + mR^2}{I_0 + mr^2}(R^2 - r^2)}$

16.32 (a) $F = \dfrac{M\omega^2 ab(a^2 - b^2)}{12(a^2 + b^2)^{3/2}}$
(b) $T = \dfrac{M\omega^2 a^2 b^2}{12(a^2 + b^2)}$

16.33 $\tau \approx 27\,\mathrm{N\,m}$

16.34 (a) $L_x/L_z = 4mv_0/MR\omega_0$
(b) $\omega_x/\omega_0 = 8mv_0/MR\omega_0$

16.35 (a) $\boldsymbol{L} = (20\boldsymbol{e}_x + 0.0119\boldsymbol{e}_y)\,\mathrm{J\,s}$
$\Omega_n = 154\,\mathrm{s}^{-1}$
$r = 1.77 \times 10^{-2}\,\mathrm{cm}$
(b) $\Omega_P = 1.47 \times 10^{-3}\,\mathrm{s}^{-1}$
$L_P = 1.91 \times 10^{-4}\,\mathrm{J\,s}$
$T_P = 1.40 \times 10^{-7}\,\mathrm{J}$
$\Delta E \approx 2.8 \times 10^{-7}\,\mathrm{J}$

16.36 $\tau_{☾} \approx 2.2\tau_{☼}$

16.37 (a) $I_{⛢} = 8.11 \times 10^{37}\,\mathrm{kg\,m^2}$
(b) $L_{⛢} = 5.91 \times 10^{33}\,\mathrm{kg\,m^2\,s^{-1}}$
(c) $T_{⛢} = 2.16 \times 10^{29}\,\mathrm{J}$
(d) $T = 25{,}725\,\mathrm{y}$

Chapter 17

17.3 $T = 2\pi\sqrt{\dfrac{2LM}{3gM + 2L(k_1 + k_2)}}$

17.4 $x = A/\sqrt{2}$

17.5 (a) $\Delta x = 2.18\,\mathrm{cm}$
(b) $V = -49.4\,\mathrm{cm\,s^{-1}}$

17.6 $T = \pi\sqrt{2L/g}$

17.8 $T = \pi\sqrt{MR^2/K}$

17.9 $T \approx 1.4\,\mathrm{s}$

17.10 (a) $T_{\mathrm{A}} = 2\pi\sqrt{\dfrac{d}{g}}$, $T_{\mathrm{B}} = 2\pi\sqrt{\dfrac{I_c + Md^2}{Mgd}}$
(b) $\ddot{\theta}_{\mathrm{A}} = -\dfrac{g}{d}\sin\theta_0$, $\ddot{\theta}_{\mathrm{B}} = -\dfrac{Mgd}{I_c + Md^2}\sin\theta_0$
(c) $\dot{\theta}_{\mathrm{A}} = \sqrt{\dfrac{2g}{d}(1 - \cos\theta_0)}$, $\dot{\theta}_{\mathrm{B}} = \sqrt{\dfrac{2Mgd}{I_c + Md^2}(1 - \cos\theta_0)}$

17.11 $\omega_0 = 28\,\mathrm{s}^{-1}$

17.12 (a) $\theta_0 = \arctan(1/3)$
(b) $\theta(t) = \theta_0 - \dfrac{J}{M\sqrt{gl}}\sqrt[4]{\dfrac{288}{125}}\sin\left(\sqrt{\dfrac{g}{l}}\sqrt[4]{\dfrac{9}{10}}\,t\right)$

17.13 $T \approx 42\,\mathrm{min}$

17.14 (b) $T_A = 2\pi\sqrt{m/(k_1 + k_2)}$, $T_B = 2\pi\sqrt{m(k_1 + k_2)/k_1 k_2}$

17.15 $A = (v/4)\sqrt{M/2k}$, $T = 2\pi\sqrt{M/2k}$

17.16 (a) $x = d$
(b) $V_{\mathrm{CM}} = (d/3)\sqrt{k/m}$, $A = d\sqrt{2/3}$

17.17 $v_1 = v_0 - X\sqrt{\dfrac{K}{M_1(1 + M_1/M_2)}}$
$v_2 = v_0 + X\sqrt{\dfrac{K}{M_2(1 + M_2/M_1)}}$

17.18 $x_{\max} = (m + M)\mu g/k$

17.19 (a) $E = 10^6\,\mathrm{ergs}$
(b) $k = 7.8 \times 10^3\,\mathrm{g\,s^{-2}}$
(c) $x = 16\,\mathrm{cm}$

17.20 (a) $\nu = (1/2\pi)\sqrt{k(1/I_1 + 1/I_2)}$
(b) $A_1/A_2 = -I_2/I_1$

17.22 (a) $T = 2\pi\sqrt{l/g}$
(b) $T = (2\pi L/\sqrt{3}d)\sqrt{l/g}$

17.23 $T = \pi\sqrt{29a/g}$

17.24 $m = KR/2g$

17.25 (a) $T = 2\pi\sqrt{m_1 m_2/k(m_1 + m_2)}$
(b) You discover *reduced mass.*
(c) $E = \frac{1}{2}kA^2$
(d) $E_1/E_2 = m_2/m_1$

17.26 $T \approx 5.3\,\mathrm{s}$

17.27 $\theta_0' = \sqrt{I/(I + ma^2)}\,\theta_0$

17.29 $T = 2\pi\sqrt{2A/g}$, $a = A\sqrt{2}$, $H = A(\sqrt{2} - 1)$

17.30 $z = 49\,\mathrm{cm}$

Chapter 18

18.8 $y = \cos(2k\pi/n) + i\sin(2k\pi/n)$, with $k = 0, 1, 2, \ldots, n - 1$

18.11 $\log_{11} 2 = 0.289$, $\log_{11} 7 = 0.811$

Chapter 19

19.2 $f = 10^4/2\pi\,\mathrm{Hz}$

19.3 $Z \approx 377\sqrt{2}\,\Omega$

19.4 $L \approx 71\,\mathrm{mH}$

19.5 $C \approx 3.3 \times 10^{-3}\,\mu\mathrm{F}$

19.6 (a) $\hat{Z} = i\left(\omega L - \dfrac{1}{\omega C}\right)$
(b) $\hat{Z} = \dfrac{i\omega L}{1 - \omega^2 LC}$

19.7 (a) $C = C_1 C_2/(C_1 + C_2)$
(b) $C = C_1 + C_2$

19.8 (a) $L = L_1 + L_2$
(b) $L = L_1 L_2/(L_1 + L_2)$

19.9 (a) $\omega_0 = 1/\sqrt{3LC}$
(b) $\omega_0 = 1/\sqrt{3LC}$ (again)

19.10 $T = L/R$

19.11 $Q \approx 10^4$

19.12 (c) $x = Ae^{-\left(\gamma/2 + \sqrt{\gamma^2/4 - \omega_0^2}\right)t} + Be^{-\left(\gamma/2 - \sqrt{\gamma^2/4 - \omega_0^2}\right)t}$
(d) $A = x_0$
$B = \dfrac{2v_0 + \gamma x_0}{\sqrt{4\omega_0^2 - \gamma^2}}$

19.13 (b) $v(t) = v_0 e^{-(\gamma/m)t}$

$$v(x) = v_0 - \frac{\gamma}{m}(x - x_0)$$

19.14 (a) $v_\infty = g\sin(\theta)/\gamma$

(b) $v(t) = v_\infty[1 - e^{-\gamma t}]$

(c) $x(t) = v_\infty[t + (e^{-\gamma t} - 1)/\gamma]$

19.15 $d = 42.8\,\mathrm{m}$

19.16 $A = 5.75\,\mathrm{cm}$

19.17 (a) $5d^2x/dt^2 + 0.693dx/dt + 20\pi^2 x = 0$

(b) $T = 1.00006\,\mathrm{s}$

(c) (1) $N = 20$; (2) $N = 33$

(d) $P \approx 1.1\,\mathrm{W}$

19.18 $V = V_0 e^{-t/RC}$

19.19 *Hint:* undamped, damped and over-damped oscillations.

19.20 (b) (1) $Z_L = i\omega L$; (2) $Z_C = 1/i\omega C$

19.23 (a) $I_0 = (1/16)\,\mathrm{A}$

(b) $\delta = 0$

19.24 (a) $I_R = 0$

(b) $I_{L,\max} = V_0\sqrt{C/L}$

19.25 (a) $C_1 = 1/\omega^2 L$

(b) $C_2 = C_1/(1 - \omega RC_1)$

(c) $I_1/I_2 = \sqrt{2}$

19.26 (a) $\Delta E = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}V_0^2$

(b) $V_1 = V_2 = \dfrac{C_1}{C_1 + C_2}V_0$

19.27 (a) $t \approx 16.1\,\mathrm{s}$

(b) $I_0 = 0$

19.28 $V_{\max} = (V_0/R)\sqrt{L/C}$

19.29 $V_{\max} = 10\sqrt{2}\,\mathrm{V}$

19.30 $V'_{\mathrm{DC}} = V_{\mathrm{DC}}$, $|V'_{\mathrm{AC}}| \approx |V_{\mathrm{AC}}|/7.6$

19.31 (a) $A = h/(2 - 8\pi^2 L/gT^2)$

(b) $A = 1.3\,\mathrm{ft}$

(c) $D = 106.3\,\mathrm{ft}$

19.32 (a) $\omega_0 = \sqrt{k/M}$

(b) $x = e\omega^2/(\omega_0^2 - \omega^2)$ in direction of e.

(c) $\omega_{\mathrm{cr}} = \omega_0$

(d) $\omega'_{\mathrm{cr}} = \omega_0$

(e) The CM is located on the centerline between the bearings.

19.33 (a) With $\omega_0 = \sqrt{k/m}$ and $\omega_\gamma = \sqrt{\omega_0^2 - \gamma^2/4}$,

$$(1)\quad x = \frac{F_0}{k} + \left(x_0 - \frac{F_0}{k}\right)e^{-\gamma t/2} \times \left(\cos\omega_\gamma t + \frac{\gamma}{2\omega_\gamma}\sin\omega_\gamma t\right)$$

$$(2)\quad x = \frac{J}{m\omega_\gamma}e^{-\gamma t/2}\sin\omega_\gamma t$$

$$(3)\quad x = \frac{F_0}{\gamma\sqrt{km}}\sin\omega_0 t + e^{-\gamma t/2}\left[x_0\cos\omega_\gamma t - \frac{1}{\omega_\gamma}\left(\frac{F_0}{m\gamma} - \frac{\gamma}{2}x_0\right)\sin\omega_\gamma t\right]$$

(b) $\omega^* = \sqrt{\omega_0^2 - \gamma^2/2}$

19.34 (a) The equation of motion for $0 < t < \pi\sqrt{m/k}$ is $m\ddot{x} + kx - mg\mu = 0$, with solution $x = (A - d)\cos(\omega t) + d$, where $\omega^2 = k/m$ and $d = mg\mu/k$.

(b) (1) $0 \le B \le d$; (2) $A = B + 2Nd$, or $A = 2(N+1)d - B$

19.35 $A = \dfrac{qE_0}{\omega^2\sqrt{m^2 + \alpha^2\omega^2}}$

$$\delta = \pi + \arctan\frac{\alpha\omega}{m}$$

19.36 $f \approx 7.2\,\mathrm{Hz}$

19.40 (a) $I = 2\,\mathrm{A}$

(b) $V_{\max} = 16.9\,\mathrm{kV}$

(c) $t = 35\,\mu\mathrm{s}$

19.41 $V = 46.5\,\mathrm{mph}$

Chapter 20

20.2 (a) $\overline{AK} = 50\,\mathrm{ft}$

(b) $t_{\min} = 60.0\,\mathrm{s}$

(c) $t = 60.1\,\mathrm{s}$

20.3 (a) $\overline{PP'} = 0.0387\,\mathrm{m}$

(b) $\Delta t = 0.11t_0$

20.4 $\theta \approx 11.2^\circ$

20.8 (a) $X_0 = R/2$

(b) $\Delta X_0/X_0 = -0.02$

20.9 $x = 200\,\mathrm{cm}$, $D = 1.86\,\mathrm{cm}$

20.10 (a) $x = 6.7 \times 10^{-5}\,\mathrm{m}$

(b) $x = 8.5 \times 10^{-2}\,\mathrm{m}$

20.11 (a) $y = \pm\sqrt{2Fx(1 - 1/n) - x^2(1 - 1/n^2)}$

20.12 $d = d'/n$

20.13 The image will be formed 4 cm from the center.

20.14 $y = 1.92R$

20.15 $d = 2.0\,\mathrm{cm}$

20.16 $M = F/f$

20.17 $\dfrac{1}{F} = \dfrac{1}{f} + \dfrac{1}{f'} - \dfrac{D}{ff'}$

$$\Delta = \frac{fD}{f + f' - D} \qquad \text{(toward } L' \text{ from } L\text{)}$$

$$\Delta' = \frac{f'D}{f + f' - D} \qquad \text{(toward } L \text{ from } L'\text{)}$$

20.18 (a) $4.167\,\mathrm{cm} < d < 5\,\mathrm{cm}$

(b) $M(4.167\,\mathrm{cm}) = 6$, $M(5\,\mathrm{cm}) = 5$

20.19 (a) $x = 5.2\,\mathrm{cm}$

20.20 $r = 1.6\,cm$

20.21 (a) $d_2 > d_1$

(b) The screen must be be moved away from the lens.

(c) $f = L_1d_1/(D + d_1) = L_2d_2/(D + d_2)$

20.22 With P at the origin, x horizontal, along the focal axis, and y vertical (the normal distance from the focal axis to the mirror),

(a) (hyperbola)

$$y^2 = \frac{4Dd}{(D - d)^2}\left[\left(x - \frac{D + d}{2}\right)^2 - \frac{(D - d)^2}{4}\right]$$

(b) (ellipse)

$$(2d + D)^2y^2 + 4d(d + D)\left(x - \frac{D}{2}\right)^2 = 4d(d + D)\left[(d + D)d - \frac{D^2}{4}\right]$$

20.23 (b) $f = g/2\omega^2$

Chapter 21

21.2 (a) $\boldsymbol{E}(t) = \dfrac{qa\omega^2}{4\pi\epsilon_0 c^2 R}\big[\cos\left(\omega(t - R/c)\right)\boldsymbol{e}_x + \sin\left(\omega(t - R/c)\right)\cos\theta\,\boldsymbol{e}_y\big]$

(b) $I(0) = \left(\dfrac{qa\omega^2}{4\pi\epsilon_0 c^2 R}\right)^2$, $I(\pi/2) = I(0)/2$

21.3 (a) $P = \dfrac{3}{8\pi}\dfrac{\sin^2\theta}{R^2}P_{\text{total}}$

(b) $P = 2.4 \times 10^{-11}\,\text{W}\,\text{m}^{-2}$

21.4 $I = \begin{cases} 5.8 I_0 & \text{E, W} \\ 0.17 I_0 & \text{N, S} \\ 3.0 I_0 & \text{other directions shown in figure} \end{cases}$

21.5 $I(\theta) = I_0 \dfrac{\sin^2[\pi(1 + \sin\theta)]}{\sin^2\left[\frac{\pi}{4}(1 + \sin\theta)\right]}$

21.6 (a) $I(\theta) = 2I_0[1 + \cos(\pi\sin\theta)]$

(c) $I(\theta) = 2I_0[1 + \cos(\pi/2 + \pi\sin\theta)]$

21.7 $a = 2\lambda$, $b = 2\lambda/\sqrt{3}$

21.8 (a) $E(t) = 0$

(b) $E(t) = \dfrac{\sqrt{2}q\omega^2 d}{4\pi\epsilon_0 c^2 R}\sin\left(\omega(t - R/c)\right)$

21.9 $d\alpha/dt = -(1/120)\cos\theta\,\text{Hz}$

21.10 C: $I = 5I_A$, $\boldsymbol{E}$ bearing 26.5° w/r to x-axis
D: $I = I_A$, $\boldsymbol{E}$ left-hand circularly polarized
E: $I = I_A$, $\boldsymbol{E}$ normal to xy-plane

21.11 $I(\theta) = 2I_0\left[1 + \cos\left(\dfrac{\pi}{2}(1 - \cos\theta)\right)\right]\dfrac{\sin^2\left(\frac{N\pi}{2}\sin\theta\right)}{\sin^2\left(\frac{\pi}{2}\sin\theta\right)}$

21.12 $E(t) = \dfrac{qa\omega^2}{4\pi\epsilon_0 c^2 R}\cos\left(\omega(t - R/c)\right)\sin\theta$,
where q is the total charge of the oscillating electrons.

Chapter 22

22.2 $Q \approx 5.5 \times 10^6$

22.4 (a) $d = 9.1\,\text{km}$

22.5 $x = 1.6\,\text{mm}$

22.6 With $r_1 = \sqrt{(z - d)^2 + D^2}$ and $r_2 = \sqrt{(z + d)^2 + D^2}$,
$\dfrac{I(z)}{I_0} = \dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} - \dfrac{2}{r_1 r_2}\cos\left(\dfrac{2\pi}{\lambda}(r_1 - r_2)\right)$.

22.7 (a) $r = \sqrt{10.25}\lambda$

(b) $I/I_{\max} \approx 1/4$

22.8 (a) $\dfrac{I(\theta)}{I_{\max}} = \dfrac{\sin^2(2\pi\sin\theta)\,(1 + \cos(8\pi\sin\theta))}{8\pi^2\sin^2\theta}$

(b) There are 6 orders of principal maxima (see part (c) below).

(c)

θ (rad)	0	0.23	0.44	0.60	0.84	1.20
$I(\theta)/I_{\max}$	1	0.44	0.01	0.006	0.045	0.002

22.10 $\dfrac{I_t}{I_0} = \dfrac{T^4}{1 - 2R^2\cos(4\pi D/\lambda) + R^4} = \dfrac{T^4}{2R^2(1 - \cos(4\pi D/\lambda)) + T^4}$

22.11 (a) $h' = h\dfrac{F_2}{F_1}$

(b) $\lambda_m = \dfrac{10^7\,\text{Å}}{mN}|\sin\theta_i - \sin\theta_d|$

(c) $D = 10^{-7}\,\text{mm}\,\dfrac{mNF_2}{\cos\theta_d}$

(d) $w' = w\dfrac{F_2}{F_1}\dfrac{\cos\theta_i}{\cos\theta_d}$

22.12 (a) $\theta = 51.9°$

(b) $\lambda = 3750\,\text{Å}, 4370\,\text{Å}, 6560\,\text{Å}$

(d) $d = 5.6\,\text{mm}\,\text{Å}^{-1}$

(e) $\min\Delta\lambda = 7 \times 10^{-3}\,\text{Å}$

Chapter 23

23.2 $n = 0.9999916$

23.3 $\rho \approx 10^7\,\text{cm}^{-3}$

23.4 (b) $I = I_0 e^{-Nq^2/\epsilon_0 m\gamma c}$, where I_0 is the intensity of the incident radiation.

23.5 (a) $P = q_e^2\omega^4 x_0^2/12\pi\epsilon_0 c^3$

(b) $\gamma_R = q_e^2\omega^2/6\pi\epsilon_0 mc^3$

(c) $\Delta\lambda = (2\pi c/\omega^2)\gamma_R = 1.2 \times 10^{-4}\,\text{Å}$

Chapter 24

24.4 $N_e \approx 10^7\,\text{cm}^{-3}$

24.5 (a) $f = 1.5\,\%$

(b) $f = 9\,\%$

24.6 (a) $\sigma_{\max} = \dfrac{N^2\chi^2 q_e^2\omega^4}{6\pi\epsilon_0^2 c^4}\left(\dfrac{E_\parallel}{E_0}\right)^2$

(b) $\sigma(\theta) = \sigma_{\max}\cos^2\theta$

Chapter 25

25.3 $I_t = (I_0/8)\sin^2 2\theta$

25.4 Regarding intensity, see Ex. 21.2, part (b). Along the axis of the circle the radiation is circularly polarized; in the plane of the circle, it's linearly polarized.

25.5 $I_t/I_0 = \frac{1}{2}(\alpha^4 + \epsilon^4)\cos^2\theta + \alpha^2\epsilon^2\sin^2\theta$

25.6 $I_R/I_{\leftmoon} \approx 34.5\,\%$, and girlfriend leaves in disgust!

25.8 (a) $f = 17\,\%$

(b) $\beta = 67.4°$

25.9 (a) $I_t = I_0\cos^{2n}(\theta/n)$

25.10 $I_{\max}/I_{\min} = \cot^2\theta$

25.12 (a) $d = 1.67 \times 10^{-2}\,\text{mm}$

(b) $\boldsymbol{E}$ is elliptically polarized:

$$E_x = E_0\cos\phi\cos\omega t,$$
$$E_y = E_0\sin\phi\cos(\omega t + 1.4\pi),$$

where ϕ is the direction of polarization of the incident light.

Chapter 26

26.7 $R = 1.5 \times 10^8\,\text{km}$

26.9 (a) $\tan\theta_1 = 0.75$

(b) $\nu_1 = 1.67\nu_0$

26.10 $v/c = (\nu_0 - \nu)/(\nu_0 + \nu)$

26.11 $v/c = 7.6 \times 10^{-6}$

26.12 $\boldsymbol{v} = 510\,\text{km}\,\text{s}^{-1}$, approaching observer

26.13 $v/c \approx 0.8$

26.14 (a) $\dfrac{d^2x}{dt^2} = -\dfrac{v^2 x}{R^2}\dfrac{1 + vR/cx}{(1 + vx/cR)^3}$

(b) $\dfrac{I_{\max}}{I_{\min}} = \dfrac{(1 + v/c)^4}{(1 - v/c)^4}$

26.15 $\boldsymbol{E}(t, R_0) = -\dfrac{q_e}{4\pi\epsilon_0 c^2 R_0}\dfrac{a\sin\theta}{(1-\beta\cos\theta)^3}\,\boldsymbol{e}_x$, for $t \geq R_0/c$.

26.16 (a) $F = I_0 A/c$
(b) $F \approx 0.16(I_0 A/c)[1 - \cos(6\pi d/\lambda)]$
(c) $F = (I_0 A/c)[1 - e^{-4\pi n'' d/\lambda}]$

26.17 $F = 1.3 \times 10^{-2}\,\mathrm{N}$

26.18 (b) $dR/dt = 360\,\mathrm{m\,s^{-1}}$

26.19 (b) $R \approx (6 \times 10^{-4}\,\mathrm{kg\,m^{-2}})/\rho$

Chapter 27

27.2 $\theta \approx \arcsin(h/p_0 W)$

27.3 (a) $\lambda = 1.8\,\text{Å}$
(b) (1) $\lambda = 0.39\,\text{Å}$; (2) $\lambda = 8.7 \times 10^{-3}\,\text{Å}$
(c) $\lambda = 1.3 \times 10^{-25}\,\text{Å}$
(d) The X-ray has a shorter wavelength than the electron.

27.4 $\lambda_{\max} = 6150\,\text{Å}$

27.5 (a) $\lambda = 910\,\text{Å}$
(b) The emitted radiation is ultraviolet.

27.6 $R \approx 2.6 \times 10^{-3}\,\text{Å}$

27.7 $E_{\min} = h\sqrt{\beta/m}$

27.8 $\tau \approx 8 \times 10^{-10}\,\mathrm{s}$

27.9 $T \approx 620\,\mathrm{MeV}$

27.10 $R \approx 5 \times 10^{-13}\,\mathrm{cm}$

27.13 (a) $T = 11.3\,\mathrm{eV}$
(b) $f_{\min} = 3.3 \times 10^{15}\,\mathrm{Hz}$

27.14 (a) $\theta_{\min} = 7.1^\circ$
(b) $d_{\max} = 21.6\,\mathrm{m}$
(c) $d = 1.9\,\mathrm{m}, 2.7\,\mathrm{m}, 3.3\,\mathrm{m}, 4.3\,\mathrm{m}$

27.15 (b) $\theta = 15.7^\circ, 22.2^\circ, 27.4^\circ, 32.2^\circ, 36.5^\circ$

27.16 $\lambda = 6560\,\text{Å}, 4860\,\text{Å}, 18{,}840\,\text{Å}$

27.17 $T = 33\,\mathrm{eV}$

27.18 (a) $\theta \approx \pi/4$
(b) $E_c = 2.1 \times 10^{-21}\,\mathrm{J}$

27.19 (a) $\theta_I = 3^\circ$, $\theta_{II} = 0.3^\circ$

27.20 (a) $\Delta\lambda = 8.3 \times 10^{-4}\,\text{Å}$
(b) $w = 3\sqrt{2}\,\mathrm{m}$
(c) $E_1 - E_0 = 2.46\,\mathrm{eV}$

27.21 (a) $r_n = \dfrac{4\pi\epsilon_0 n^2\hbar^2}{me^2}$

$$\omega_n = \frac{me^4}{(4\pi\epsilon_0)^2 n^3\hbar^3}$$

(b) $T_n = \dfrac{me^4}{2(4\pi\epsilon_0)^2 n^2\hbar^2}$

$$U_n = -2T_n$$
$$E_n = -T_n$$

(c) $\Delta E \propto \left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$

Chapter 28

28.2 (a) $P^{\gamma-1}/T^\gamma = \text{const}$
(b) $TV^{\gamma-1} = \text{const}$

28.3 (a) $P_A = 3.17P_0$, $P_B = 2.64P_0$
(b) $W_A/W_B = 1.1$

28.4 $P_f = \dfrac{1}{2}(P_1 + P_2)$

$$T_f = \frac{T_1T_2(P_1+P_2)}{T_1P_2 + T_2P_1}$$

28.5 (b) $P = P_0 e^{-\mu gh/RT}$

28.6 $W = 1.82 \times 10^5\,\mathrm{J}$

28.7 $T = 173\,^\circ\mathrm{C}$

28.8 $P = 200\,\mathrm{mmHg}$

28.9 $P = 842\,\mathrm{mmHg}$

28.10 (a) $T_2 = 1740\,\mathrm{K}$
(b) $V_2/V_1 = 5.8$

28.11 (a) $T_f = T_0$, $P_f = P_0/2$
(b) $T_f = T_0/2^{2/3}$, $P_f = P_0/2^{5/3}$

28.12 (a) $\dfrac{dT}{dh} = -\dfrac{\gamma-1}{\gamma}\dfrac{\mu g}{R}$, where μ is the mean molar mass of the atmospheric gas and R is the ideal gas constant.
(b) On Earth, $dT/dh \approx -0.01\,\mathrm{K\,m^{-1}}$

28.13 $\omega = \sqrt{(P_0A/L_0 + K)/m}$

28.14 $f_d = 31\,\%$

Chapter 29

29.3 $C_V = \frac{5}{2}R$

29.4 (a) $C_{V,m} = \frac{3}{2}R$ (monatomic)
(b) $C_{V,m} = \frac{5}{2}R$ (diatomic)

29.5 $h_{♁} = 8.8\,\mathrm{km}$, $h_{☼} = 113\,\mathrm{km}$

29.6 $P(h=0) = \dfrac{Nmg}{A}\dfrac{e^{mgL/kT}}{e^{mgL/kT} - 1}$

$$P(h=L) = \frac{Nmg}{A}\frac{1}{e^{mgL/kT} - 1}$$

29.7 (a) $f_1(h) = m_1/\left(m_1 + m_2 e^{-(m_1+m_2)gh/kT}\right)$
(c) $f = 1 - e^{-m_1/m_2}$

29.9 $F(E)\,dE = A\sqrt{2E/m^3}\,e^{-E/kT}\,dE$

29.10 (a) $k = 2N/V^2$
(b) $\langle v\rangle = 2V/3$, $v_{\mathrm{rms}} = V/\sqrt{2}$

29.11 (a) $f = 0.55$
(b) $f = 0.06$

29.12 $P_1/P_2 = \sqrt{T_1/T_2}$

29.13 (a) $dn = A\dfrac{n}{4}vf(v)\,dv$, with

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$$

(b) $v_{\mathrm{rms}}^{(e)} = \sqrt{4kT/m}$, $v_{\mathrm{rms}}^{(i)} = \sqrt{3kT/m}$

29.14 $\Delta\lambda \approx 0.012\,\text{Å}$

29.15 $T = 1.2 \times 10^5\,\mathrm{K}$

29.16 $N_0/N_1 \approx 10^{-9}$

29.17 $F \approx (W/3)\sqrt{2\pi m/kT}$ (depends on approximation used)

29.18 (a) With P being the intake/exhaust pressure, and γ the heat capacity ratio for air,

(1) $v' = -\dfrac{P}{(\gamma-1)\rho v} + \sqrt{\left(\dfrac{P}{(\gamma-1)\rho v} + v\right)^2 + \dfrac{2W}{\rho v A}}$

(2) $T' = (v'/v)T$

(3) $F = \rho v A(v' - v)$

(b) $F \approx 5.3 \times 10^4\,\mathrm{N}$

Chapter 30

30.1 (a) $T = 11{,}600\,\mathrm{K}$
(b) $kT = (1/40)\,\mathrm{eV}$
(c) $\lambda = 12{,}400\,\text{Å}$

30.2 $96{,}520\,\mathrm{J\,mol^{-1}} = 1\,\mathrm{eV/atom}$

30.3 One will err by approximately 100 %!

30.4 $I_1/I_2 \approx 10^{-5}$

30.7 (a) $C_V = \dfrac{3N_0k(\Delta E/kT)^2 e^{\Delta E/kT}}{(3 + e^{\Delta E/kT})^2}$

Chapter 31

31.2 $l = 1.8 \times 10^3\,\mathrm{cm}$

31.3 $l \approx 10^{-7}\,\mathrm{m}$, $\tau \approx 0.2 \times 10^{-9}\,\mathrm{s}$

31.4 (a) $P = 11 \times 10^{-4}\,\mathrm{mmHg}$
(b) $\kappa_1/\kappa_2 = 1$

31.5 $\mu = \mu_a\mu_b/(\mu_a + \mu_b)$

31.8 With the number of gas molecules per unit volume n_0, their average velocity v, their mass m, and the heat capacity ratio γ of the gas,

(a) $\dfrac{dE}{dt} = \dfrac{1}{4}\dfrac{n_0 v k\Delta T}{\gamma - 1}$

(b) $\dfrac{F}{A} = \dfrac{1}{4} n_0 v m \Delta U$

31.9 $L \approx 56l$

31.12 (a) With

$$V_{\text{clear}}(m) = \frac{d^2}{lcq_e}\frac{\sqrt{3mc^2kT}}{t},$$

where l is the mean free path of the gas, the space will be completely cleared

(1) of electrons at voltage $V_D = V_{\text{clear}}(m_e)$,

(2) of neon ions at voltage $V_D = V_{\text{clear}}(m_{\text{Ne}})$.

(b) By comparing $t = 0.4 \times 10^{-6}\,\mathrm{s}$ to the times needed to completely clear the chamber of electrons ($t_e \approx 4 \times 10^{-7}\,\mathrm{s}$), and neon ions ($t_{\text{Ne}} \approx 8 \times 10^{-5}\,\mathrm{s}$), it is apparent that the electrons are essential to the formation of a spark.

Chapter 32

32.1 $E = 1.4\,\mathrm{J}$

32.2 $W_{\min} = (1/9)\,\mathrm{J}$

32.3 The helium engine delivers more work per cycle.

32.4 $\epsilon_{\max} = 66\,\%$

32.5 (a) $W = (P + Mg/A)V_1$
(b) $\Delta Q = \frac{5}{2}W$
(c) $\Delta U = \frac{3}{2}W$,
(d) $T_i = 4W/NR$, $T_f = 2T_i$

32.6 (b) $W = 101\,\mathrm{J}$
(c) $T_{\max} = 1200\,\mathrm{K}$
(d) $\Delta Q = 102.3\,\mathrm{J} \cdot (2\gamma + 1)/(\gamma - 1)$
(e) $\Delta S = 0.23\,\mathrm{J\,K^{-1}} \cdot (\gamma + 1)/(\gamma - 1)$

32.7 (a) $W = 1.1 \times 10^3\,\mathrm{J}$
(b) $T_B = 600\,\mathrm{K}$
(c) $\Delta Q_{A\to B} = 6.8 \times 10^3\,\mathrm{J}$
(d) $\Delta Q_{B\to C} = 9.1 \times 10^3\,\mathrm{J}$
(e) $\epsilon_{\max} = 7\,\%$
(f) $\epsilon_{\max} = 66\,\%$

32.8 (b) $\Delta S = 27.8\,\mathrm{J\,K^{-1}}$

32.9 (a) $V_b = 8.8\,\mathrm{L}$
(b) $V_d = 12.3\,\mathrm{L}$
(c) $\Delta Q_{a\to b} = 1.25 \times 10^3\,\mathrm{J}$
(d) $\Delta Q_{c\to d} = 0.95 \times 10^3\,\mathrm{J}$
(e) $\epsilon = 25\,\%$
(f) $\Delta S = 0.11\,\mathrm{J\,K^{-1}\,g^{-1}}$

32.10 (a) $\gamma = 1.57$
(b) $\Delta S = -1.4\,\mathrm{cal\,K^{-1}}$

32.11 $W = P_0V_0\left(\dfrac{2^{\gamma-1} - 1}{\gamma - 1} - \ln 2\right)$

32.13 $P_{\text{day}} = 10.9\,\mathrm{kW}$, $P_{\text{night}} = 24.6\,\mathrm{kW}$

32.14 (b) $W = C_p(T_1 + T_2 - 2T_f)$

32.15 $\Delta S = 11 \times 10^3\,\mathrm{J\,K^{-1}}$

32.16 (a) $\dot{W} = \dot{N}R\dfrac{\gamma}{\gamma - 1}\Big(T_C + T_A - T_C p^{-(1-1/\gamma)} - T_A p^{-(1-1/\gamma)}\Big)$

(b) $p_m = \left(\dfrac{T_C}{T_A}\right)^{\gamma/2(\gamma-1)}$

(c) $\dot{W}_m = \dot{N}R\dfrac{\gamma}{\gamma - 1}\left(\sqrt{T_C} - \sqrt{T_A}\right)^2$

(d) $\epsilon_m = 1 - \sqrt{T_A/T_C}$

(e) $\dot{V}_A = 2.8\,\mathrm{m^3\,s^{-1}}$

(f) $\epsilon_e = 50\,\%$

(g) $p_e = 11.3$

Chapter 33

33.3 $C_V = (16\sigma/c)VT^3$, where σ is the Stefan-Boltzmann constant (derived in Section 45-3 of Vol. I).

33.4 (a) $dS_R/dt = 0.85\,\mathrm{cal\,K^{-1}\,s^{-1}}$
(b) $dS_U/dt = dS_R/dt$

33.6 $dT/dz = -2.9\,\mathrm{K\,km^{-1}}$

33.7 (a) $dz/dt = (\kappa/L\rho)(\Delta T/z)$ where ρ is the density of ice.
(b) $z(t = 1\,\mathrm{h}) = 2.1\,\mathrm{cm}$

33.8 (1) (a) $\Delta S_{\text{gas}} \approx 16\,\mathrm{J\,K^{-1}}$
(b) $\Delta S_{\text{surroundings}} = 0$
(c) $\Delta S_{\text{universe}} = \Delta S_{\text{gas}}$
(2) (a) $\Delta S_{\text{gas}} = 0$
(b) $\Delta S_{\text{surroundings}} = 0$
(c) $\Delta S_{\text{universe}} = 0$

33.9 (a) $(\Delta S)_T = (16\sigma/3c)T^3(V_2 - V_1)$
(b) $(\Delta S)_V = (16\sigma/3c)V(T_2^3 - T_1^3)$
(c) $S(V, T) = S(V_0, T_0) + (16\sigma/3c)(VT^3 - V_0T_0^3)$

33.10 $T = 122\,^\circ\mathrm{C}$

33.11 $T = 270\,\mathrm{K}$

33.12 $f = 1$

33.13 $T = T_0 e^{-(3Ac/16R)t}$

33.14 (a) $f = 64$
(b) f would be larger.

33.15 $f = 4$

33.16 (a) $dE/dt = 1.95 \times 10^{-4}\,\mathrm{J\,s^{-1}}$
(b) $dT/dt = 0.06\,\mathrm{K\,s^{-1}}$

33.17 $P_G = 1.7 \times 10^{16}\,\mathrm{Pa}$, $P_R = 7.2 \times 10^{-12}\,\mathrm{Pa}$

33.21 $f = R^2/(R^2 + r^2)$

Chapter 34

34.1 $v \propto \sqrt{M/\rho}$, where ρ is the density of the liquid.

34.2 $v_1/v_2 = 1/\sqrt{2}$

34.3 $v_{\mathrm{He}}/v_{\mathrm{H}} = 0.77$

34.4 $T = 99\,^\circ\mathrm{C}$

34.6 (a) $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{\sigma}{T}\dfrac{\partial^2 y}{\partial t^2}$
(b) $v = \sqrt{\dfrac{T}{\sigma}}$

34.7 (a) $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{T}{m}\dfrac{l}{x\,(l-x)}}$
(b) $y(t) = A\cos(2\pi\nu t)$
(c) $\nu_{\max} = \infty$

$$\nu_{\min} = \frac{1}{\pi}\sqrt{\frac{T}{ml}}$$

34.8 (a) $P(x,t) = P_0 - (\omega^2\xi_m/k)\sin(\omega t - kx)$
(b) $T = \frac{1}{4}\rho_0 A\lambda\omega^2\xi_m^2$

34.9 (a) $\chi_m = 1.05 \times 10^{-2}\,\mathrm{cm}$
(b) $\Delta T = 2.18 \times 10^{-2}\,\mathrm{K}$

34.10 (a) $f = 2.95$

34.12 (a) $\rho_e = 8.7 \times 10^{-7}\,\mathrm{kg\,m^{-3}}$
(b) $\chi_m = 3.6 \times 10^{-8}\,\mathrm{m}$
(c) $I = 1.1 \times 10^{-5}\,\mathrm{W\,m^{-2}}$

Chapter 35

35.1 $v_{ph} = v_g = c$

35.3 (b) $v_g = 19.7\,\mathrm{m\,s^{-1}}$, $v_{ph} = 39.5\,\mathrm{m\,s^{-1}}$

35.4 There are 6 modes of oscillation.

35.5 $u(x,y,t) = Ae^{i\omega t}\sin\left(\dfrac{\pi}{a}x\right)\sin\left(\dfrac{\pi}{b}y\right)$,

$$\omega = c\sqrt{\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}}$$

35.7 $v_g = \dfrac{1}{2}\left(v_{ph} + \dfrac{4\pi T/\rho\lambda}{v_{ph}}\right)$

35.8 (a) $v_{ph} = 24.4\,\mathrm{cm\,s^{-1}}$
(b) $v_{ph} = 18.3\,\mathrm{cm\,s^{-1}}$

35.9 (a) $\lambda = 1.7\,\mathrm{cm}$
(b) $\nu = 13.6\,\mathrm{Hz}$

35.10 (a) $\nu = 1.6$ beats per second.
(b) $\Delta t = 0.03\,\mathrm{s}$
(c) $v = 363\,\mathrm{m\,s^{-1}}$

35.11 (a) $\nu_{\text{engineer}} \approx 10$ beats per second.
(b) $\nu_{\text{worker}} = \nu_{\text{engineer}}$

35.12 (d) $\omega_0 = (7/6)(v\pi/a)$

35.13 (a) $\dfrac{d^2x}{dt^2} + \omega_0^2 x + k(x-y)/m_1 = 0$
$\dfrac{d^2y}{dt^2} + \omega_0^2 y + k(y-x)/m_2 = 0$
(b) (1) $\omega^2 = \omega_0^2$, with $A/B = 1$
(2) $\omega^2 = \omega_0^2 + k/m_1 + k/m_2$, with $A/B = -m_2/m_1$

35.14 (1) $\omega = \sqrt{g/l}$, with $x_1 = x_2 = x_3$
(2) $\omega = \sqrt{g/l + k/m}$, with $x_1 = -x_3$ and $x_2 = 0$
(3) $\omega = \sqrt{g/l + 2k/m}$, with $x_1 = x_3$ and $x_2 = -x_3$

35.15 There are two modes of oscillation, having periods (1) $T = T_0/\sqrt{2.62}$; (2) $T = T_0/\sqrt{0.38}$, with $T_0 = 2\pi\sqrt{I/K}$.

35.16 There are two normal modes, having frequencies (1) $\omega = \omega_0\sqrt{0.38}$; (2) $\omega = \omega_0\sqrt{2.62}$, with $\omega_0 = \sqrt{k/m}$.

35.17 (a) There are two fundamental modes, having frequencies (1) $\omega = \omega_0\sqrt{2}$; (2) $\omega = \omega_0\sqrt{6}$, with $\omega_0 = \sqrt{k/m}$.

35.18 $E = \frac{1}{4}\sigma\omega^2 A^2\lambda$

35.19 (a) $T_{\min} \approx h^2/2mL^2$
(b) $T_{\min} = h^2/8mL^2$
(c) $\langle p\rangle = 0$

Chapter 36

36.3 (a) $g(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2}\left(\cos x + \dfrac{1}{9}\cos 3x + \dfrac{1}{25}\cos 5x + \cdots\right)$

36.5 $A_1/A_0 = 1$, $A_2/A_0 = 0$, $A_3/A_0 = 1/9$

36.6 $h(x) = \dfrac{1}{2} - \dfrac{1}{\pi}\left(\sin x + \dfrac{1}{2}\sin 2x + \dfrac{1}{3}\sin 3x + \cdots\right)$

36.7 (a) $T = 2L\sqrt{\sigma/S}$
(b) $y(x, t = T/2) = -y(x, t = 0)$

36.8 (a) $\langle V\rangle = 2V_0/\pi$
(b) $A_2 = 4V_0/15\pi$

36.9 (a) $V_{\text{out}} = (1 + 3\epsilon/4)\sin x - (\epsilon/4)\sin 3x$

Chapter 37

37.1 (a) $M = 1.9 \times 10^{-9}\,\mathrm{kg} = 1.1 \times 10^{18} \cdot m_p$
(b) $F = 9.9 \times 10^{13}\,\mathrm{N} \approx 1.1 \times 10^{10}$ ton-wt

37.2 Two models may be used to approximate the energy, one in which the halves are two hemispheres forming a sphere, and the other in which the halves are adjacent spheres. The distance between the centers of charge are different in these cases, and the resulting energies differ by a factor of about 1.5. We use the second model, concentrating the charges at the center of two equal spheres.
(a) $W_{\mathrm{U}} = 4.1 \times 10^{-11}\,\mathrm{J/atom} = 1.3 \times 10^{7}\,\mathrm{kWh\,lb^{-1}}$
(b) $W_{\mathrm{He}} = 7.6 \times 10^{-14}\,\mathrm{J/atom} = 1.4 \times 10^{6}\,\mathrm{kWh\,lb^{-1}}$

37.3 $\langle v\rangle = 1.4 \times 10^{-4}\,\mathrm{m\,s^{-1}}$, $\langle v\rangle^2/c^2 = 2.2 \times 10^{-25}$

37.4 (a) $B = 10^{-2}\,\mathrm{Wb\,m^{-2}} = 100$ gauss
(b) You can not determine the charge of the muons from this experiment because the direction of *both* the electric and magnetic forces change with a change of sign in charge.

37.5 (a) The particle's path is a circle with radius $R = mv/qB_0$.

(c) According to Maxwell's equations, the circulation of $\boldsymbol{B}$ around any closed path is zero. Assume $\boldsymbol{B}$ fills a finite volume around the origin and consider a rectangular path in the xz-plane with one corner at the origin $(0, 0, 0)$ and the diagonally opposite corner at $(\Delta x, 0, \Delta z)$. The circulation of $\boldsymbol{B}$ around this rectangle is $(B_0 + a\,\Delta x)\Delta z - B_0\,\Delta z = a\,\Delta x\,\Delta z \neq 0$.

37.6 (a) $B_0 = \dfrac{mv_0}{aq} - \dfrac{Q}{4\pi\epsilon_0 a^2 v_0}$

(b) The magnetic force is always perpendicular to $\boldsymbol{v}$ and does not change its magnitude. The electric field can be derived from a potential, so that the change in kinetic energy due to the electric force depends only on the distance r between the charges. The change in kinetic energy equals $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$, and since this depends only on r, v is a function of r.

Chapter 38

38.1 (a) $\boldsymbol{\nabla}T$ is radially symmetric and points inward. For a cylinder of radius r,

$$\boldsymbol{\nabla}T = \frac{\partial T}{\partial r}\,\boldsymbol{e}_r = -\frac{(T_1 - T_2)}{\ln(b/a)}\,\frac{\boldsymbol{e}_r}{r},$$

where $\boldsymbol{e}_r$ is a unit radius vector.

(b) $T_1 - T_2 = 1.2\,\mathrm{K}$

38.3 Noting that $\boldsymbol{R} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$,

(a) $\boldsymbol{\nabla}\cdot\boldsymbol{R} = \partial x/\partial x + \partial y/\partial y + \partial z/\partial z = 1 + 1 + 1 = 3$

(b) $\boldsymbol{\nabla}\times\boldsymbol{R} = (\partial z/\partial y - \partial y/\partial z)\boldsymbol{e}_x + (\partial x/\partial z - \partial z/\partial x)\boldsymbol{e}_x + (\partial y/\partial x - \partial x/\partial y)\boldsymbol{e}_x = 0$ (because each term $= 0$).

(c) $\boldsymbol{\nabla}\cdot(\boldsymbol{R}/R^3) = \dfrac{\partial}{\partial x}\dfrac{x}{R^3} + \dfrac{\partial}{\partial y}\dfrac{y}{R^3} + \dfrac{\partial}{\partial z}\dfrac{z}{R^3}$

$$= \left(\frac{1}{R^3} - \frac{3x^2}{R^5}\right) + \left(\frac{1}{R^3} - \frac{3y^2}{R^5}\right) + \left(\frac{1}{R^3} - \frac{3z^2}{R^5}\right)$$

$$= \frac{3}{R^3} - \frac{3(x^2 + y^2 + z^2)}{R^5} = \frac{3}{R^3} - \frac{3R^2}{R^5} = 0$$

(d) $\boldsymbol{\nabla}\times(\boldsymbol{R}/R^3)$

$$= \left(\frac{\partial}{\partial y}\frac{z}{R^3} - \frac{\partial}{\partial z}\frac{y}{R^3}\right)\boldsymbol{e}_x + \left(\frac{\partial}{\partial z}\frac{x}{R^3} - \frac{\partial}{\partial x}\frac{z}{R^3}\right)\boldsymbol{e}_y + \cdots$$

$$= \left(\frac{3yz}{R^5} - \frac{3yz}{R^5}\right)\boldsymbol{e}_x + \cdots = 0$$

(e) $\boldsymbol{\nabla}(1/R)$

$$= \frac{\partial}{\partial x}\frac{1}{(x^2 + y^2 + z^2)^{1/2}}\,\boldsymbol{e}_x + \frac{\partial}{\partial y}\frac{1}{R}\,\boldsymbol{e}_y + \frac{\partial}{\partial z}\frac{1}{R}\,\boldsymbol{e}_z$$

$$= -\frac{1}{2}\frac{2x}{(x^2 + y^2 + z^2)^{3/2}} - \frac{\partial}{\partial y}\frac{y}{R^3} - \frac{\partial}{\partial z}\frac{z}{R^3}$$

$$= -\frac{x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z}{R^3} = -\boldsymbol{R}/R^3$$

(f) $\varphi = \boldsymbol{R}^2/2 + \mathrm{const}$

38.4 (a) Taking the divergence of both sides of Eq. (2),

$$\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\boldsymbol{E}) = \boldsymbol{\nabla}\cdot\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}(\boldsymbol{\nabla}\cdot\boldsymbol{B}),$$

but the divergence of a curl is always zero [see Eq. (2.59d) in Vol. II], so the left-hand side is 0, which is consistent with $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$.

(b) The divergence of the left-hand side of Eq. (4) is

$$\boldsymbol{\nabla}\cdot(c^2\boldsymbol{\nabla}\times\boldsymbol{B}) = c^2\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}) = 0,$$

which must equal the divergence of its right-hand side, so

$$0 = \boldsymbol{\nabla}\cdot\left(\frac{\partial\boldsymbol{E}}{\partial t} + \frac{\boldsymbol{j}}{\epsilon_0}\right) = \frac{\partial}{\partial t}\,\boldsymbol{\nabla}\cdot\boldsymbol{E} + \frac{\boldsymbol{\nabla}\cdot\boldsymbol{j}}{\epsilon_0} = \frac{\partial}{\partial t}(\rho/\epsilon_0) + \frac{\boldsymbol{\nabla}\cdot\boldsymbol{j}}{\epsilon_0}.$$

(c) Using Eq. (2.59e) from Vol. II, the curl of the left-hand side of Eq. (2) is

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{E}) = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{E}) - \nabla^2\boldsymbol{E},$$

which must equal the curl of its right-hand side,

$$\boldsymbol{\nabla}\times\left(-\frac{\partial\boldsymbol{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\boldsymbol{B}) = -\frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{\partial\boldsymbol{E}}{\partial t} + \frac{\boldsymbol{j}}{\epsilon_0}\right),$$

while in empty space $\boldsymbol{\nabla}\cdot\boldsymbol{E} = \rho/\epsilon_0 = 0$ and $\boldsymbol{j} = 0$.

(d) For $\boldsymbol{j} = 0$, the curl of Eq. (4) gives

$$c^2\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{B}) = \frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\boldsymbol{E}).$$

Applying Eq. (2.59e) from Vol. II to the left-hand side and Eq. (2) to the right-hand side,

$$c^2\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{B}) - c^2\nabla^2\boldsymbol{B} = -\frac{\partial^2\boldsymbol{B}}{\partial t^2},$$

but from Eq. (3), $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$.

(e) Using Eq. (2.59b) from Vol. II and Eq. (2),

$$\boldsymbol{\nabla}\times\left(-\boldsymbol{\nabla}\phi - \frac{\partial\boldsymbol{A}}{\partial t}\right) = -\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\phi) - \boldsymbol{\nabla}\times\left(\frac{\partial\boldsymbol{A}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\boldsymbol{A}) = -\frac{\partial\boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times\boldsymbol{E}.$$

(f) Eq. (3) says that $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$. By Eq. (2.51) from Vol. II, $\boldsymbol{B}$ can therefore be written as $\boldsymbol{\nabla}\times\boldsymbol{A}$.

38.5 Let z be the axis of rotation, then

$$\boldsymbol{v} = v\left[-\frac{y}{\sqrt{x^2 + y^2}}\boldsymbol{e}_x + \frac{x}{\sqrt{x^2 + y^2}}\boldsymbol{e}_y\right]$$

$$= \omega(-y\boldsymbol{e}_x + x\boldsymbol{e}_y) \text{ for constant radius } \boldsymbol{r}.$$

(a) $\boldsymbol{\nabla}\cdot\boldsymbol{v} = \omega\left[-\dfrac{\partial y}{\partial x} + \dfrac{\partial x}{\partial y}\right] = 0$

(b) $\boldsymbol{\nabla}\times\boldsymbol{v} = \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -\omega y & \omega x & 0 \end{vmatrix}$

$$= 0\boldsymbol{e}_x + 0\boldsymbol{e}_y + (\omega - (-\omega))\boldsymbol{e}_z = 2\omega\boldsymbol{e}_z.$$

38.6 (a) With $\boldsymbol{A} = A_x\boldsymbol{e}_x + A_y\boldsymbol{e}_y + A_z\boldsymbol{e}_z$ and $\boldsymbol{R} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$,

$$\boldsymbol{A}\times\boldsymbol{R} = (zA_y - yA_z)\boldsymbol{e}_x + (xA_z - zA_x)\boldsymbol{e}_y + (yA_x - xA_y)\boldsymbol{e}_z$$

$$\boldsymbol{\nabla}\times(\boldsymbol{A}\times\boldsymbol{R}) = \left[\frac{\partial}{\partial y}(\boldsymbol{A}\times\boldsymbol{R})_z - \frac{\partial}{\partial z}(\boldsymbol{A}\times\boldsymbol{R})_y\right]\boldsymbol{e}_x + \cdots$$

$$= [A_x - (-A_x)]\,\boldsymbol{e}_x + \cdots$$

$$= 2\,[A_x\boldsymbol{e}_x + A_y\boldsymbol{e}_y + A_z\boldsymbol{e}_z] = 2\boldsymbol{A}.$$

(b) The proof that $\boldsymbol{B}\times(\boldsymbol{A}\times\boldsymbol{C}) = \boldsymbol{A}(\boldsymbol{B}\cdot\boldsymbol{C}) - (\boldsymbol{B}\cdot\boldsymbol{A})\boldsymbol{C}$ requires that $A_x(B_xC_x) = C_x(A_xB_x)$ but $A_x(\partial x/\partial x) \neq x(\partial A_x/\partial x)$. The correct equation is

$$\boldsymbol{\nabla}\times(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{A}(\boldsymbol{\nabla}\cdot\boldsymbol{B}) - (\boldsymbol{\nabla}\cdot\boldsymbol{A})\boldsymbol{B} + (\boldsymbol{B}\cdot\boldsymbol{\nabla})\boldsymbol{A} - (\boldsymbol{A}\cdot\boldsymbol{\nabla})\boldsymbol{B}.$$

38.7 (b) $\boldsymbol{\nabla}\cdot\boldsymbol{h}$ is largest at point B; its absolute magnitude is largest at point D.

(c) $\boldsymbol{\nabla}\times\boldsymbol{h} = 0$ everywhere. ($\boldsymbol{h}$ is proportional to $\boldsymbol{\nabla}T$ and the curl of a gradient is always zero.)

Chapter 39

39.1 (b) Integrate both sides of the equation over a fixed volume V enclosed by a surface S and use Gauss's theorem:

$$\int_V \boldsymbol{\nabla}\cdot\boldsymbol{j}\,dV = -\int_V \frac{\partial\rho}{\partial t}\,dV,$$
$$\int_S \boldsymbol{j}\cdot\boldsymbol{n}\,da = -\frac{d}{dt}\int_V \rho\,dV,$$

or [Net current leaving V] = −[Rate of change of total charge within V], which says "charge is conserved."

39.2 No magnetic field is produced.

39.3 (a) $\displaystyle\int_S \boldsymbol{E}\cdot\boldsymbol{n}\,da = 4\pi K$

(b) *Hint:* To avoid the singularity of $\boldsymbol{\nabla}\cdot\boldsymbol{E}$ at the origin, replace the point charge with an "equivalent" uniformly charged sphere, and take limits as the radius of the sphere goes to 0.

(c) $\boldsymbol{E}\cdot\boldsymbol{ds}$ is antisymmetric about the origin, so corresponding pieces of the line integral on opposite sides of the origin cancel, thus

$$\oint \boldsymbol{E}\cdot\boldsymbol{ds} = 0.$$

On the other hand, with Stoke's theorem,

$$\oint_\Gamma \boldsymbol{E}\cdot\boldsymbol{ds} = \int_S (\boldsymbol{\nabla}\times\boldsymbol{E})\cdot\boldsymbol{n}\,da,$$

for any surface S bounded by path Γ, but $\boldsymbol{\nabla}\times\boldsymbol{E} = 0$ in electrostatics, so the line integral is again zero.

39.4 (a) $V = \dfrac{1}{3}\displaystyle\int_S \boldsymbol{R}\cdot\boldsymbol{n}\,da$, where $\boldsymbol{n}$ is the unit normal to the surface element da.

(b) (1) For a sphere of radius r, $\boldsymbol{R}\cdot\boldsymbol{n} = r$, so

$$V = \frac{r}{3}\int_S da = \frac{r}{3}(4\pi r^2) = \frac{4}{3}\pi r^3.$$

(2) For a rectangular block with edge lengths L_1, L_2, L_3, consider one face, say with edges L_1 and L_2; on this face $\boldsymbol{R}\cdot\boldsymbol{n} = L_3/2$, so the integral of part (a) evaluates to $(1/3)(L_3/2)(L_1L_2) = L_1L_2L_3/6$. By symmetry the integral is the same over each of the 6 faces and thus $V = L_1L_2L_3$.

Chapter 40

40.1 (a) $\phi(P) = \dfrac{\lambda}{4\pi\epsilon_0}\ln\left(\dfrac{l_2+\sqrt{r^2+l_2^2}}{-l_1+\sqrt{r^2+l_1^2}}\right)$

(b) For $r \gg (l_1+l_2)$,

$$\phi(P) \approx \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{r+l_2}{r-l_1}\right) \approx \frac{\lambda(l_1+l_2)}{4\pi\epsilon_0 r} = \frac{q}{4\pi\epsilon_0 r},$$

where q is the total charge.

(c) For P very close to the line of charge (but not near the ends) symmetry dictates that the field should be approximately radial. Thus for $r \ll (l_1+l_2)$,

$$\phi(P) \approx \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{2l_2}{r^2/2l_1}\right) = \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{4l_1l_2}{r^2}\right),$$

so

$$\boldsymbol{E}(P) = -\frac{\partial\phi}{\partial r}\,\boldsymbol{e}_r \approx \frac{\lambda}{2\pi\epsilon_0 r}\,\boldsymbol{e}_r.$$

Applying Gauss's law to a cylindrical surface surrounding a length Δz of the line of charge,

$$\int_S \boldsymbol{E}\cdot\boldsymbol{n}\,da \approx 2\pi r\,\Delta z\,E_r = \frac{\lambda\,\Delta z}{\epsilon_0}.$$

Thus, $\boldsymbol{E}(P) \approx (\lambda/2\pi\epsilon_0 r)\boldsymbol{e}_r$, which checks.

40.2 $\boldsymbol{E}(P) = \dfrac{\sigma}{2\epsilon_0}\left(1-\dfrac{r}{\sqrt{r^2+R^2}}\right)\boldsymbol{e}_r$

40.3 (a) Use $E_r = \begin{cases} 0 & \text{for } r<r_a \\ q'/4\pi\epsilon_0 r^2 & \text{for } r_a<r<r_b \\ 0 & \text{for } r_b<r<r_c \\ (q'+q)/4\pi\epsilon_0 r^2 & \text{for } r>r_c \end{cases}$

(c) $\phi(r_a) = \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{(q+q')}{r_c} - q'\left(\dfrac{1}{r_b}-\dfrac{1}{r_a}\right)\right]$

(d) The region $r_b<r<r_c$ is inside a conductor where the field is always 0. The fields in the region $r>r_c$, outside the outer conductor, do not change.

Chapter 41

41.1 With the origin at the center of a sphere S having radius r, and using spherical coordinates (r,θ,ψ), the average value of ϕ over S is

$$\begin{aligned}\langle\phi(r)\rangle_S &= \frac{1}{4\pi r^2}\int_S \phi(r,\theta,\psi)\,da \\ &= \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\phi(r,\theta,\psi)\sin\theta\,d\theta\,d\psi\end{aligned}$$

So,

$$\begin{aligned}\frac{d\langle\phi(r)\rangle_S}{dr} &= \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\frac{\partial\phi}{\partial r}\sin\theta\,d\theta\,d\psi \\ &= -\frac{1}{4\pi r^2}\int_S \boldsymbol{E}\cdot\boldsymbol{n}\,da = 0,\end{aligned}$$

because there are no charges inside the sphere. Thus, $\langle\phi(r)\rangle_S = \text{constant} = \lim_{r\to 0}\langle\phi(r)\rangle_S = \phi(0)$.

41.2 $\boldsymbol{E}(r) = (\rho r/2\epsilon_0)\boldsymbol{e}_r$ at radial distance r from the *axis* of the cylinder. For a sphere, $\boldsymbol{E}(r) = (\rho r/3\epsilon_0)\boldsymbol{e}_r$ at radial distance r from the *center* of the sphere.

41.3 With the plates parallel to the xy-plane, $\boldsymbol{E}_1 = (2\sigma/3\epsilon_0)\boldsymbol{e}_z$, $\boldsymbol{E}_2 = -(\sigma/3\epsilon_0)\boldsymbol{e}_z$.

41.4 $E_x(x) = \displaystyle\int_{-\infty}^{x}\frac{\rho(\xi)\,d\xi}{2\epsilon_0} - \int_x^{\infty}\frac{\rho(\xi)\,d\xi}{2\epsilon_0}$

41.5 (a) $\sigma_e = 0$

(b) $\sigma_c = \frac{4}{3}k\epsilon_0 d^{1/3}$

(c) $\rho(x) = -\frac{4}{9}\epsilon_0 k x^{-2/3}$

41.6 A charge density σ on a surface element da produces a field $\sigma/2\epsilon_0$ normal to da. When the element is part of the surface of a metal the field is σ/ϵ_0 outside the metal, and zero inside the metal. By the principle of superposition the charges on the rest of the metal (other than those on da) must produce a field $\sigma/2\epsilon_0$. This field exerts a force on da normal to da and equal in magnitude to $\sigma\,da\cdot\sigma/2\epsilon_0 = (\sigma^2/2\epsilon_0)\,da$.

41.7 (a) $N_c/N_a \approx 1/3000$

(b) $F_c/F_a \approx 1/6000$

41.8 (a) $\omega = 1.7 \times 10^{22}\,\mathrm{s}^{-1} (f = \omega/2\pi = 2.7 \times 10^{21}\,\mathrm{Hz})$

(b) $\hbar\omega = 1.8 \times 10^{-12}\,\mathrm{J} = 11\,\mathrm{MeV}$

(c) 11 MeV is about the same energy as that of the x-rays emitted when a meson stops in lead and forms a muonic atom, which suggests that the muons are trapped in the first excited state and then make a transition to the lowest state.

41.9 (a) An object of mass m in a tunnel along a diameter of Earth would feel, at a distance r from Earth's center, a force toward the center,

$$F = \frac{GMmr^3/R^3}{r^2} = \frac{mg}{R}r$$

where M is Earth's mass and R is its radius. Thus the object executes harmonic motion according to the equation $m\ddot{r} = -(mg/R)r$, having angular frequency $\omega = \sqrt{g/R}$. A satellite orbiting just at Earth's surface must have an angular frequency given by $m\omega^2 R = mg$, so $\omega = \sqrt{g/R}$ in this case also.

(b) $P = 1.4\,\mathrm{h}$

41.10 The model predicts the temperature at the surface of the earth to be about $-50{,}000\,^\circ\mathrm{C}$, a good deal colder than is the case.

41.11 (a) (1) $\boldsymbol{E}(r) = \dfrac{\lambda_1}{2\pi\epsilon_0 r}\boldsymbol{e}_r$ for $r_2 \le r \le r_3$; (2) $\boldsymbol{E}(r_1) = \dfrac{\lambda_1 + \lambda_2}{2\pi\epsilon_0 r_4}\boldsymbol{e}_r$, where $\boldsymbol{e}_r$ is a unit radial vector.

(b) $\Delta\phi = \dfrac{\lambda_1}{2\pi\epsilon_0}\ln\dfrac{r_3}{r_2}$

(c) (1) The fields and potentials are unchanged. (2) The field will be zero for $r < r_2$ and the same as before for $r > r_2$. $\Delta\phi$ will decrease. (3) The field inside the square will vanish. The field between the cylinders will be distorted (the field lines must remain perpendicular at both surfaces). The field for $r > r_3$ is unchanged.

Chapter 42

42.1 The force is attractive:

$$F_x = -\frac{Q^2}{4\pi\epsilon_0}\left[\frac{1}{4a^2} - \frac{a}{4(a^2+b^2)^{3/2}}\right]$$

$$F_y = -\frac{Q^2}{4\pi\epsilon_0}\left[\frac{1}{4b^2} - \frac{b}{4(a^2+b^2)^{3/2}}\right]$$

42.2 (a) $\text{K.E.} = \dfrac{q^2}{16\pi\epsilon_0}\left(\dfrac{1}{x} - \dfrac{1}{x_0}\right)$

(b) Unphysical because it goes to infinity for $x \to 0$. Not correct at small x because of surface structure.

(c) $\text{K.E.} = 3.6\,\mathrm{eV}$

42.4 $T = 2.6 \times 10^6\,\mathrm{s}$ (one month)

42.5 (a) $C_l = \dfrac{2\pi\epsilon_0}{\ln(b/a)}$

(b) C_l would decrease in the region of the protrusion.

42.6 (a) $\phi = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{b}$

(b) $\sigma(\theta) = \dfrac{q}{4\pi}\left[\dfrac{1}{ab} - \dfrac{b^2 - a^2}{a(a^2 + b^2 - 2ab\cos\theta)^{3/2}}\right]$

(c) $F = \dfrac{a}{b^2}qV - \dfrac{ab}{(b^2 - a^2)^2}\dfrac{q^2}{4\pi\epsilon_0}$

42.7 Outside a sphere of uniform charge the field is the same as if its total charge were concentrated at its center. Thus, the field outside the two spheres of radius radius a and charge density $\pm\rho$ is a dipole field with moment $p = (4/3)\pi a^3 \rho d$, where d is the distance between the centers of the spheres. Inside a sphere of uniform charge density $\pm\rho$ the field at radius r is $\boldsymbol{E}^{\pm} = (\pm\rho/3\epsilon_0)\boldsymbol{e}_r$. The dipole field is the sum $\boldsymbol{E} = \boldsymbol{E}^{+} + \boldsymbol{E}^{-}$, which is anti-parallel to the dipole. By similar triangles $E/E^{+} = d/r$, thus $\boldsymbol{E} = -(\rho/3\epsilon_0)d\,\boldsymbol{e}_z$, where $\boldsymbol{e}_z$ is a unit vector parallel to the dipole. But $\sigma_0 = \rho d$. Thus, the dipole field outside the spheres has moment $p = (4/3)\pi a^3 \sigma_0$, while the field inside the spheres is $\boldsymbol{E} = -(\sigma_0/3\epsilon_0)\,\boldsymbol{e}_z$.

42.8 (a) $E_r = \dfrac{p}{2\pi\epsilon_0 r^3}\cos\theta$

$E_\theta = \dfrac{p}{4\pi\epsilon_0 r^3}\sin\theta$

(b) The points along a straight line which passes through the dipole have the same θ and φ coordinates (only r varies). The direction of the electric field is determined by the ratio $E_\theta/E_r = \frac{1}{2}\tan\theta$, so it is the same for all such points.

(c) For $\theta = 0$, $\pi/4$, and $\pi/2$ the angles between $\boldsymbol{E}$ and $\boldsymbol{p}$ are respectively 0, arctan 3, and π, while the relative strengths of $\boldsymbol{E}$ in these directions are $4 : \sqrt{10} : 2$.

42.9 (a) Use superposition: Taking the dipole and the field in the x-direction, the total potential at (x, y, z) is

$$\phi = -E_0 x + \frac{px}{4\pi\epsilon_0(x^2 + y^2 + z^2)^{3/2}}.$$

Setting $\phi = 0$ and rearranging yields,

$$x^2 + y^2 + z^2 = \left(\frac{p}{4\pi\epsilon_0 E_0}\right)^{2/3},$$

which is the equation of a sphere with radius $a = (p/4\pi\epsilon_0 E_0)^{1/3}$.

(b) $p = 4\pi\epsilon_0 E_0 a^3$

(d) The fields wouldn't change.

(e) $\sigma(\theta) = 3\epsilon_0 E_0 \cos\theta$

(f) $p = 4\pi\epsilon_0 E_0 a^3$ (The is just like the example worked in Section 6-4 of Vol. II, with the same moment used there implicitly.)

(g) Any problem that has a similar "potential" basis could be solved similarly.

42.10 (a) (1) $\boldsymbol{F} \approx (p\lambda/2\pi\epsilon_0 r^2)\boldsymbol{e}_r$, and $\boldsymbol{\tau} = 0$. (2) $\boldsymbol{F} = 0$ and $\boldsymbol{\tau} = (p\lambda/2\pi\epsilon_0 r)$ perpendicular to the plane of the wire and particle.

(b) Let $\boldsymbol{r}$ be the radius vector to the dipole's negative charge $-q$, and let $\boldsymbol{d}$ be the displacement from its negative charge to its positive charge $+q$. Then the total force on the dipole in an electric field $\boldsymbol{E}$ is $\boldsymbol{F} = q[\boldsymbol{E}(\boldsymbol{r}+\boldsymbol{d}) - \boldsymbol{E}(\boldsymbol{r})]$. For small displacements $\boldsymbol{d}$, $\boldsymbol{E}(\boldsymbol{r}+\boldsymbol{d}) - \boldsymbol{E}(\boldsymbol{r}) = (\partial\boldsymbol{E}/\partial x)d_x + (\partial\boldsymbol{E}/\partial y)d_y + (\partial\boldsymbol{E}/\partial z)d_z = \boldsymbol{\nabla}(\boldsymbol{d}\cdot\boldsymbol{E})$. Thus $\boldsymbol{F} = q\boldsymbol{\nabla}(\boldsymbol{d}\cdot\boldsymbol{E}) = \boldsymbol{\nabla}(q\boldsymbol{d}\cdot\boldsymbol{E}) = \boldsymbol{\nabla}(\boldsymbol{p}\cdot\boldsymbol{E})$.

42.11 Outside the sheet the electric field is 0 because the fields of the positive and negative charges cancel, so the potential is constant (independent of r). Taking the potential to be zero in the middle of the sheet, $\phi = Np/2\epsilon_0$ on one side and $\phi = -Np/2\epsilon_0$ on the other side.

42.12 (a) $\phi(P) = \dfrac{q}{4\pi\epsilon_0}\left(\dfrac{1}{\sqrt{x^2+a^2}} - \dfrac{1}{x}\right)$

(b) $\boldsymbol{E}(P) = \dfrac{q}{4\pi\epsilon_0}\left(\dfrac{x}{(x^2+a^2)^{3/2}} - \dfrac{1}{x^2}\right)\boldsymbol{e}_x$

(c) For $x \gg a$, $\boldsymbol{E} \approx -\dfrac{3qa^2}{8\pi\epsilon_0 x^4}\,\boldsymbol{e}_x$.

(d) A dipole at the origin would produce a field $E \propto 1/x^3$ (as opposed to $E \propto 1/x^4$). We have here a contribution from a "higher moment." One way to see this (and to see that the dipole moment of the given configuration is zero) is to notice we have, in effect *canceling dipoles*: every diameter of the ring is like two equal and opposite dipoles. The "higher moment" is, in fact, a *quadrupole* moment.

42.13 (a) $t_1 = 1000\,\mathrm{s}$

(b) $V = 1.5\,\mathrm{V}$

(c) Capacitance is halved, so there is half as much charge on the plates, and thus $t_1 = 500\,\mathrm{s}$.

(d) Charge is unchanged, so t_1 is unchanged.

42.14 (a)

y (cm)	0	0.5	1.0	2.0
E (V/m)	0	$\dfrac{(0.005)^3}{2\epsilon_0}$	$\dfrac{(0.01)^3}{2\epsilon_0}$	$\dfrac{(0.01)^3}{4\epsilon_0}$

(b) $\phi(0,0,0) \approx \dfrac{(0.01)^4}{8\epsilon_0}(3 + 8\ln 10)\,\mathrm{V}$

(This is an upper bound.)

(c) $\phi(0.5,0,0) < \phi(0,0,0)$

Chapter 43

43.1 (a) Assume the lines of charge density $\pm\lambda$ are parallel to the z-axis, intersecting the x-axis at $x = \pm d/2$. The potential at a distance r from a line of charge density λ is $-(\lambda/2\pi\epsilon_0)\ln r$. Thus the total potential at a point in the xy-plane is

$$\phi = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{\sqrt{(x-d/2)^2+y^2}}{\sqrt{(x+d/2)^2+y^2}}\right).$$

Curves of constant potential must therefore satisfy

$$(x-d/2)^2 + y^2 = c\left((x+d/2)^2+y^2\right)$$

for some constant c. This can be rewritten as

$$\left(x - \left(\frac{1+c}{1-c}\right)\frac{d}{2}\right)^2 + y^2 = \frac{cd^2}{(1-c)^2}$$

which is the equation of a circle.

(b) $C_l \approx \dfrac{\pi\epsilon_0}{\ln(d/r)}$

(c) For $x, y \gg d$,

$$\frac{\sqrt{(x-d/2)^2+y^2}}{\sqrt{(x+d/2)^2+y^2}} \approx \sqrt{\frac{x^2+y^2+xd}{x^2+y^2-xd}} \approx \sqrt{1 + \frac{2xd}{x^2+y^2}}.$$

Taking the logarithm of the right-hand side,

$$\frac{1}{2}\ln\left(1 + \frac{2xd}{x^2+y^2}\right) \approx \frac{xd}{x^2+y^2}.$$

Noting that $1/z = 1/(x+iy) = (x-iy)/(x^2+y^2)$,

$$\phi = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{\sqrt{(x-d/2)^2+y^2}}{\sqrt{(x+d/2)^2+y^2}}\right) \approx \frac{\lambda}{2\pi\epsilon_0}\frac{xd}{x^2+y^2} = \frac{\lambda d}{2\pi\epsilon_0}\operatorname{Re}\left(\frac{1}{z}\right).$$

Chapter 44

44.1 You *should* agree!

44.2 (a) $V_f = 3000\,\mathrm{V}$

(b) $W = 4.05 \times 10^{-5}\,\mathrm{J}$

44.3 (a)
$$\Delta U = \frac{(Q_1+Q_2)^2}{2(C_1+C_2)} - \left(\frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}\right) = -\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)\left(\frac{Q_1}{C_1} - \frac{Q_2}{C_2}\right)^2 < 0$$

(b) Most of the lost energy goes into heat generated by the current that flows as the charges readjust themselves (a tiny fraction is radiated).

(c) $\Delta U = 0$ only when $V_1 = V_2$, where $V_1 = Q_1/C_1$ and $V_2 = Q_2/C_2$ are the initial voltages across the capacitors. (No current flows when the voltages across the capacitors are equal.)

44.4 (a) Let $\boldsymbol{r}$ be the radius vector to the dipole's negative charge $-q$ and let $d\boldsymbol{e}_z$ be the displacement from the dipole's negative charge to its positive charge, so that the dipole moment is $\boldsymbol{p} = qd\boldsymbol{e}_z$. Let the potential ϕ of the electric field $\boldsymbol{E}$ be zero at infinity. Then in moving the dipole from infinity we do work $U = -q\phi(\boldsymbol{r}) + q\phi(\boldsymbol{r} + d\boldsymbol{e}_z) \approx q(d\,\partial\phi/\partial z) = (qd)(-E_z) = -\boldsymbol{p}\cdot\boldsymbol{E}$.

(b) For $\boldsymbol{p} = q\boldsymbol{d}$, $\tau = q(\boldsymbol{d}/2)\times\boldsymbol{E} + (-q)(-\boldsymbol{d}/2)\times\boldsymbol{E} = \boldsymbol{p}\times\boldsymbol{E}$, which equals $pE\sin\theta$ when the angle between $\boldsymbol{E}$ and $\boldsymbol{p}$ equals θ. On the other hand, $U = -\boldsymbol{p}\cdot\boldsymbol{E} = -pE\cos\theta$, and the torque equals $\partial U/\partial\theta = pE\sin\theta$.

(c) The energy is not the same if the dipole is formed from two charges placed sequentially in the field. In that case the charges exert forces on each other, and there is an extra energy term $-q^2/4\pi\epsilon_0 d$, corresponding to the work required to assemble the dipole.

44.5 The energy in the capacitor is $U = Q^2/2C$, while it's capacitance is $C = \epsilon_0 A/x$, where x is the plate separation. Thus $U = (Q^2/2\epsilon_0 A)x$. The force on the plates is $F = -\partial U/\partial x = -Q^2/2\epsilon_0 A$, which is negative, implying that the force is attractive.

44.6 $r_\pi \approx 2 \times 10^{-14}\mathrm{cm}$

44.7 (a) $W_a = \dfrac{q_1^2}{8\pi\epsilon_0}\left(\dfrac{1}{b} - \dfrac{1}{a}\right)$

(b) $W_b = W_a + \dfrac{q_1q_2}{4\pi\epsilon_0 b}$

Chapter 45

45.1 (a) Neglecting end effects, the electric field between the plates is uniform and perpendicular to the plates, so the total capacitance is $C = (Q_1+Q_2)/V = C_1 + C_2$, where Q_1 and Q_2 are the free charges on each half-plate, C_1 and C_2 are the capacitances of each half-plate, and V is the potential between the plates. The area of each half-plate is $A/2$, so $C_1 = \kappa_1\epsilon_0(A/2)/d$ and $C_2 = \kappa_2\epsilon_0(A/2)/d$. Thus $C = C_1 + C_2 = (\epsilon_0 A/d)(\kappa_1+\kappa_2)/2$.

(b) If the plates are kept at potential difference V, then

$$V = \phi_{\text{lower}} - \phi_{\text{upper}} = -\int_{\text{upper}}^{\text{lower}}\boldsymbol{E}\cdot d\boldsymbol{s} = E_1 d = E_2 d,$$

since the line integral may be chosen in either region. If we now relate the electric field (of magnitude V/d) to the surface charge on the plates, we can compute the total charge Q and thus get the total capacitance C. To do this we use Eq. (10.26) from Vol. II, $\nabla\cdot(\kappa\boldsymbol{E}) = \rho_{\text{free}}/\epsilon_0$, to get

$$\int_S \kappa\boldsymbol{E}\cdot\boldsymbol{n}\,da = \int_V \frac{\rho_{\text{free}}}{\epsilon_0}\,dV = \frac{Q_{\text{free}}}{\epsilon_0}.$$

In the metal $\boldsymbol{E} = 0$, so $\kappa_1 E_1\,da = (\sigma_1)_{\text{free}}/\epsilon_0\,da$. Thus $(Q_1)_{\text{free}} = (\sigma_1)_{\text{free}}(A/2) = \epsilon_0\kappa_1 E_1(A/2) = (\epsilon_0 A/d)(\kappa_1/2)V$. Therefore $C = [(Q_1)_{\text{free}} + (Q_2)_{\text{free}}]/V = (\epsilon_0 A/d)\,(\kappa_1+\kappa_2)/2$.

45.2 (a) $F = 7.1\times10^{-8}\,\text{N}$

(b) $P = 1.1\times10^{-8}\,\text{C}\,\text{m}^{-2}$ (downward, assuming the top plate is positive)

(c) The presence of metal spheres in the dielectric would reduce the potential difference between the plates.

45.3 If the field in the empty region between the plates is E_0 then the field in the dielectric is reduced, $E_d = E_0/\kappa$. The voltage between the plates is the sum of the voltages across the empty and dielectric-filled regions, $V = E_0(d-t)+E_d t$. So the charge on the plates is $Q = CV = C(E_0(d-t) + (E_0/\kappa)t)$. However, if the dielectric is removed, the voltage between the plates becomes $V_0 = E_0 d$, and $Q = C_0 V_0 = C_0 E_0 d$. Thus $C_0 E_0 d = C(E_0(d-t)+(E_0/\kappa)t)$, and solving for C gives the desired result.

45.4 (a) $\sigma_{\text{pol}}(\text{inner surface}) = -\left(\dfrac{\kappa-1}{\kappa}\right)\dfrac{Q}{4\pi a^2}$

$\sigma_{\text{pol}}(\text{outer surface}) = \left(\dfrac{\kappa-1}{\kappa}\right)\dfrac{Q}{4\pi b^2}$.

(b) Inside the dielectric, $\rho_{\text{pol}} = 0$.

45.5 (a) If the battery delivers a charge Δq to the capacitor at the potential V, the work done is $W_{\text{batt}} = \Delta q\,V$. Before the dielectric is inserted, the capacitance is $C = q/V = \epsilon_0 A/d$, where d is the distance between the plates. After the dielectric is inserted, the capacitance becomes $C' = q'/V = \kappa\epsilon_0 A/d$ (see Eq. (10.10) in Vol. II), so $q' = \kappa q$. Thus, $W_{\text{batt}} = (q'-q)V = qV(\kappa-1)$.

(b) $W_{\text{mech}} = -\frac{1}{2}qV(\kappa-1)$. This work is done *on* the agent.

(c) In any process where the potential difference between the plates of a capacitor is maintained while the capacitance is changed, $W_{\text{batt}} = -2W_{\text{mech}}$.

45.6 When equilibrium has been reached the battery can be disconnected without effect, so we consider only work done by the electric force and gravity: $\Delta U_{\text{grav}} = -\Delta U_{\text{el}}$. The center of mass of the displaced oil is at height $H/2$, thus for a small displacement of the top level of the oil ΔH, the work done by gravity is $\Delta U_{\text{grav}} = \Delta(mgH/2) = \pi(b^2-a^2)H\rho g\Delta H$ (assuming the lower level of the oil does not change appreciably). Since the charge Q on the pipes remains constant, while their capacitance C changes with the presence of oil between them, the work done by the electric force is $\Delta U_{\text{el}} = \Delta(Q^2/2C) = -(Q^2/2C^2)\Delta C = -(V^2/2)\Delta C$. The capacitance per unit length between coaxial cylindrical conductors with a dielectric between them is $C_l = 2\pi\epsilon_0\kappa/\ln(b/a)$ (compare with Ex. 42.5), so $\Delta C = 2\pi\epsilon_0(\kappa-1)\,\Delta H/\ln(b/a)$. Putting this all together we have

$$\pi(b^2-a^2)H\rho g\,\Delta H = (V^2/2)2\pi\epsilon_0(\kappa-1)\,\Delta H/\ln(b/a),$$

and solving for H gives the desired result.

45.7 Let the normal and tangential components of the electric fields in the two regions, with respect to their boundary, be (E_{1n}, E_{1t}) and (E_{2n}, E_{2t}), such that $\cot\theta_1 = E_{1n}/E_{1t}$ and $\cot\theta_2 = E_{2n}/E_{2t}$. Consider a cylindrical volume of cross-sectional area A and height h with its axis normal to the boundary and intersecting it. Apply Gauss's theorem to Eq. (10.26) from Vol. II, $\nabla\cdot(\kappa\boldsymbol{E}) = \rho_{\text{free}}/\epsilon_0$, to see that the total flux of $\kappa\boldsymbol{E}$ through the surface of this volume must equal the charge density of the free charges within it (divided by ϵ_0). With h arbitrarily small the flux through the surface of the volume is $\kappa_1 E_{1n}A - \kappa_2 E_{2n}A$; since there are no free charges on the boundary, this must equal zero, so $\kappa_1 E_{1n} = \kappa_2 E_{2n}$. Now consider a small rectangular loop of length l parallel to the boundary and height h normal to the boundary and intersecting it. Apply Stokes theorem to Maxwell's equation for electrostatics, $\nabla\times\boldsymbol{E} = 0$, to see that the circulation of $\boldsymbol{E}$ around the loop must equal zero. With h arbitrarily small the circulation around the loop is $E_{1t}l - E_{2t}l$, so $E_{1t} = E_{2t}$. Hence, $\kappa_1 E_{1n}/E_{1t} = \kappa_2 E_{2n}/E_{2t}$, which is the same as $\kappa_1\cot\theta_1 = \kappa_2\cot\theta_2$.

Chapter 46

46.1 Taking the sphere at the origin and $\boldsymbol{P}$ in the $+z$-direction,

$$\boldsymbol{E}_{\text{outside}} = \frac{a^3P}{3\epsilon_0 r^5}\left(3xz\boldsymbol{e}_x + 3yz\boldsymbol{e}_y + (3z^2-r^2)\boldsymbol{e}_z\right),$$
$$\boldsymbol{E}_{\text{inside}} = -\frac{P}{3\epsilon_0}\,\boldsymbol{e}_z,$$

where r is the distance from the center of the sphere to the field point (x, y, z).

46.2 $p = 2.44\times10^{-39}\text{C}\,\text{m}$

46.3 (a) Use $\alpha = (\kappa-1)kT/P$.

(b) $p_0 = 6.4\times10^{-30}\,\text{C}\,\text{m}$

46.4 (a) $\alpha = 2\pi a^3$

(b) $\alpha_{\text{T}} \approx \dfrac{2.62a^3 - \alpha_{\text{O}}}{1 + 18.33\alpha_{\text{O}}/a^3}$

46.5 $\kappa = 1 + \pi/54$

Chapter 47

47.1 $T_0 = \dfrac{W}{4\pi K}\left(\dfrac{1}{a} - \dfrac{1}{R}\right) + T_R$

47.2 $a \approx 6250\,\text{km}$

47.3 (a) $b_n \neq 0$ for $n = 1$ and $n = -2$ only

(b) $c_n \neq 0$ for $n = 1$ and $n = -1$ only

47.4 $T_P \approx 20.53\,^\circ\text{C}$

Chapter 48

48.1 Taking x horizontal and y vertical,

(a) $\boldsymbol{B} = (8\times10^{-5})\boldsymbol{e}_y\,\text{Wb}\,\text{m}^{-2}$

(b) $\boldsymbol{F} = (-6\boldsymbol{e}_x + 2\boldsymbol{e}_y)10^{-4}\,\text{N}\,\text{m}^{-1}$

48.2 $\boldsymbol{B} = 0$

48.3 $\boldsymbol{B}$ at radius r is tangent to the circle of radius r, in the counterclockwise direction (with respect to the figure), and its magnitude is

(a) $B = \dfrac{Ir}{2\pi\epsilon_0 c^2 a^2}$ for $r < a$,

(b) $B = \dfrac{I}{2\pi\epsilon_0 c^2 r}$ for $a < r < b$,

(c) $B = \dfrac{I}{2\pi\epsilon_0 c^2 r}\left(\dfrac{c^2-r^2}{c^2-b^2}\right)$ for $b < r < c$,

(d) $B = 0$ for $r > c$.

48.4 (a) $\boldsymbol{F}_{\text{loop}} = \dfrac{I_1 I_2}{2\pi\epsilon_0 c^2}\, l \left(\dfrac{1}{a} - \dfrac{1}{a+w}\right)$, up (with respect to the figure)

(b) $\boldsymbol{F}_{\text{wire}} = -\boldsymbol{F}_{\text{loop}}$

(c) $\boldsymbol{\tau}_{\text{loop}} = 0$

48.5 (a) $B \approx 7.9 \times 10^{-3}\,\text{Wb}\,\text{m}^{-2}$

(b) $nI \approx 1.6 \times 10^4$ ampere-turns

48.6 $B = dj/2\epsilon_0 c^2$ (the same as it would be without the hole)

Chapter 49

49.1 With the origin at P, taking y in the direction of v, and z perpendicular to the sheet (up, with respect to the figure),

(a) $\boldsymbol{A} = -\dfrac{\sigma v z}{2\epsilon_0 c^2}\, \boldsymbol{e}_y$

(b) $\boldsymbol{B} = \dfrac{\sigma v}{2\epsilon_0 c^2}\, \boldsymbol{e}_x$

49.2 At the center of the semicircle

(a) $\boldsymbol{B}$ from the straight segments $= 0$,

(b) $\boldsymbol{B}$ from the curved segment $= (I/4\epsilon_0 c^2 r)$ into the page (with respect to the figure),

(c) $\boldsymbol{B}$ from the entire wire is the sum of $\boldsymbol{B}$ from each of its parts, which equals $\boldsymbol{B}$ from the curved segment.

49.3 With the $+x$-axis going to the right (with respect to the figure),

(a)
$$\boldsymbol{B}(x) = \frac{Ia^2}{2\epsilon_0 c^2}\left[\frac{1}{\left(a^2 + (b/2 - x)^2\right)^{3/2}} + \frac{1}{\left(a^2 + (b/2 + x)^2\right)^{3/2}}\right]\boldsymbol{e}_x$$

(b) With $r^2 = a^2 + b^2/4$

$$\boldsymbol{B}(x) \approx \frac{Ia^2}{2\epsilon_0 c^2}\left[\frac{2}{r^3} + \left(\frac{15b^2}{4r^7} - \frac{3}{r^5}\right)x^2\right]\boldsymbol{e}_x$$

(c) The x^2 terms vanish when $r = \sqrt{5}a/2 \to a = b$, that is to say, when the loops are separated by a distance equal to their radius. Then the field near the center is constant in x, on the axis, except for terms in $(x/r)^3$ and higher.

(d) With $r = \sqrt{5}a/2$,

$$\boldsymbol{B} = \frac{Ia^2}{2\epsilon_0 c^2}\left[\frac{2}{r^3}\right]\boldsymbol{e}_x = \frac{Ia^2}{\epsilon_0 c^2 (\sqrt{5}a/2)^3}\,\boldsymbol{e}_x$$
$$= \frac{8I}{5^{3/2} a \epsilon_0 c^2}\,\boldsymbol{e}_x$$

49.5 With the origin at the center of the loop and z along its axis, you should get the following result:

$$\boldsymbol{B}(z) = \frac{Ia^2}{2\epsilon_0 c^2 (a^2 + z^2)^{3/2}}\,\boldsymbol{e}_z$$

49.7 (a) The magnetic field $\boldsymbol{B}$ inside the cylinder is that of a solenoid with a surface current density σv, where $\sigma = \lambda/2\pi a$ is the surface charge density, and $v = \omega a$ (ω being the angular velocity of the cylinder). Thus $\boldsymbol{B}$ is parallel to the axis of the cylinder, and

$$B = \frac{\sigma v}{\epsilon_0 c^2} = \frac{\lambda\omega}{2\pi\epsilon_0 c^2}.$$

(See Section 13-5 in Vol. II.) The potential difference V between the center of the cylinder and its edge is the work done per unit charge q displaced from radius $r = 0$ to $r = a$ against the force $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$, so

$$V = \int_0^a \frac{\boldsymbol{F}}{q}\,dr = \int_0^a (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})\,dr$$

Inside the cylinder $\boldsymbol{E} = 0$, $\boldsymbol{v} \times \boldsymbol{B} = Bv$, and $v = \omega r$, so

$$V = \int_0^a \frac{\lambda\omega}{2\pi\epsilon_0 c^2}\,\omega r\,dr = \frac{\lambda\omega^2}{2\pi\epsilon_0 c^2}\left(\frac{a^2}{2}\right) = \frac{\lambda}{4\pi\epsilon_0}\left(\frac{v}{c}\right)^2.$$

(b) Just outside the cylinder, the magnitude of the electric field is $E = \sigma/\epsilon_0$, so $\lambda = 2\pi a\sigma = 2\pi a(\epsilon_0 E)$. Substituting $2\pi a(\epsilon_0 E)$ for λ and ωa for v into the result obtained for part a above, and rearranging, we find that

$$a_{\min}^3 = \frac{2c^2 V}{E_{\max}\omega^2},$$

where $\omega = 2\pi/(24 \cdot 60 \cdot 60\,\text{s})$ is the angular velocity of the earth. With $V = 10^{-6}\,\text{V}$ and $E_{\max} = 10^8\,\text{V}\,\text{m}^{-1}$, this yields $a \approx 7 \times 10^3\,\text{m}$, so the experiment is not feasible.

Chapter 50

50.1 (a) The normal component of $\boldsymbol{B}$ must be zero.

(b) We have to satisfy $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$ and $\boldsymbol{\nabla}\times\boldsymbol{B} = \boldsymbol{j}/\epsilon_0 c^2$ in the region outside the superconductor, plus the boundary condition on the normal component $B_n = 0$ at the surface of the superconductor. To find the field outside the superconductor we place an "image dipole" inside the superconductor a depth d below the surface, oriented at an angle $(\pi - \theta)$ from vertical. The boundary condition $B_n = 0$ is then automatically satisfied. The image dipole has no effect outside the superconductor, where $\boldsymbol{\nabla}\times\boldsymbol{B}_{\text{image}} = \boldsymbol{\nabla}\cdot\boldsymbol{B}_{\text{image}} = 0$. Hence the field outside the superconductor is that of the image dipole plus the original dipole.

(c) Taking the y-axis normal to μ, parallel to the superconducting sheet, and into the page in Fig. 50-1,

$$\boldsymbol{\tau} = -\frac{\mu^2 \sin 2\theta}{64\pi\epsilon_0 c^2 d^3}\,\boldsymbol{e}_y$$

(d) Stable equilibria: $\theta = \pi/2,\ 3\pi/2$. Unstable equilibria: $\theta = 0,\ \pi$.

(e) $F = \dfrac{3\mu^2}{64\pi\epsilon_0 c^2 d^4}(3 + \cos 2\theta)$ *away* from the superconductor

Chapter 51

51.1 Magnitude of acceleration at P_2: 0, at P_1 and P_3: $4.4 \times 10^7\,\text{m}\,\text{s}^{-2}$, going to the right at P_1 and to the left at P_3 (with respect to the figure).

51.2 (a) $V_0 = \pi^2 r^2 f B$, $\omega_V = 2\pi f$

(b) $I_0 = V_0/R_m$, $\omega_I = \omega_V$

51.3 $\mathcal{E} = \dfrac{\pi a^4 I\omega}{2\epsilon_0 c^2 R^3}\sin\omega t$

51.4 (a) $\boldsymbol{v}(t) = \dfrac{IBd}{m}\,t$ away from the generator

(b) $v(t) = \dfrac{\mathcal{E}}{Bd}\left(1 - e^{-B^2 d^2 t/mR}\right)$, $v_{\text{terminal}} = \dfrac{\mathcal{E}}{Bd}$

(c) $I_{\text{terminal}} = 0$

51.5 (a) $\mathfrak{L} = \mathfrak{L}_1 + \mathfrak{L}_2 - 2\mathfrak{M}$

(b) $\mathfrak{L} = \mathfrak{L}_1 + \mathfrak{L}_2 + 2\mathfrak{M}$

51.6 (a) The energy per unit length of the cable is $U = \mathfrak{L}I^2/2$, where $\mathfrak{L}$ is the inductance per unit length. So $\mathfrak{L} = (2/I^2)U$. The magnetic field between the wire and the conducting cylinder is just the field of the wire, whose magnitude is $B = I/(2\pi\epsilon_0 c^2 r)$—elsewhere the field is 0. The total energy of the cable is $(\epsilon_0 c^2/2)\int B^2\,dV$ taken over space, so the energy per unit length is $U = (\epsilon_0 c^2/2)\int B^2\,da$ taken over a plane normal to the cable. Thus, with r as the radius of the cable,

$$\begin{aligned}\mathfrak{L} &= \left(\frac{2}{I^2}\right)\frac{\epsilon_0 c^2}{2}\int_a^b \left(\frac{I}{2\pi\epsilon_0 c^2 r}\right)^2 2\pi r\,dr\\ &= \frac{1}{2\pi\epsilon_0 c^2}\int_a^b \frac{1}{r}\,dr = \frac{\ln(b/a)}{2\pi\epsilon_0 c^2}.\end{aligned}$$

(b) $\mathfrak{L} = \dfrac{\ln(b/a) + 1/4}{2\pi\epsilon_0 c^2}$.

51.7 (a) Let the current I be flowing in the windings of the toroid, and consider a circular path s, perpendicular to, and a distance $a < r < b$ from the toroid's axis of symmetry. If B is the magnitude of the magnetic field along this path, then $\int B\,ds = 2\pi r B = NI/\epsilon_0 c^2$. Thus, $B = NI/2\pi\epsilon_0 c^2 r$. The total magnetic energy U in the the toroid is the integral $(\epsilon_0 c^2/2)\int B^2\,dV$ taken over it's volume. With $dV = a(2\pi r)dr$,

$$\begin{aligned}U &= \frac{\epsilon_0 c^2}{2}\,2\pi a\int_b^{b+a}\left(\frac{NI}{2\pi\epsilon_0 c^2 r}\right)^2 r\,dr\\ &= \frac{N^2 I^2 a}{4\pi\epsilon_0 c^2}\ln\left(1 + \frac{a}{b}\right).\end{aligned}$$

On the other hand, the energy in a coil with self-inductance $\mathfrak{L}$ carrying a current I is $U = \mathfrak{L}I^2/2$. Equating these two expressions for U and solving for $\mathfrak{L}$ gives the desired result.

(b) $\mathfrak{M} = \dfrac{Na}{2\pi\epsilon_0 c^2}\ln\left(1 + \dfrac{a}{b}\right)$

(c) $\mathfrak{L}/\mathfrak{M} = N$

51.8 (a) $\mathfrak{M} = \dfrac{A^2}{8\pi\epsilon_0 c^2 r^3}\left[3\cos(\alpha_1 + \alpha_2) + \cos(\alpha_1 - \alpha_2)\right]$

(b) $\boldsymbol{F} = -(3\mathfrak{M}/r)I^2\boldsymbol{e}_r$

(c) The direction of $\boldsymbol{F}$ is reversed if (only) one of the currents is reversed.

51.9 $\mathfrak{M}_{12} = \mathfrak{M}_{21} = \dfrac{N\pi r_1^2}{\epsilon_0 c^2 l\sqrt{1 + 4r_2^2/l^2}}$

51.10 (a) $I = 1.0\,\mathrm{mA}$

(b) The flux through the loop is decreasing, thus by Lenz's law the induced EMF tends to increase the magnitude of $\boldsymbol{B}$, resulting in a greater rate of change in flux, which requires additional induced current. Thus, the estimate of the induced current would increase if the field arising from the induced current were not neglected.

(c) If the magnet were moving with the wire, it would, by symmetry, be the same as if the wire and the magnet were both fixed in place while the rails moved. You'd get the same answers for the induced current.

(d) The self-inductance of the circuit is Φ/I where Φ is the magnetic flux through the area spanned by the circuit; it is decreasing, because the current is constant, the magnetic field is constant, while the area enclosed by the loop is decreasing.

51.11 (a) $\tau = 0$

(c) $\boldsymbol{F}$ lies along the loops' axis; loop (2) is repelled by loop (1).

(d) Self-inductance reduces the induced current in loop (2), so the force on (2) is reduced, while the torque remains 0.

(e) There would be no flux from loop (1) through loop (2), thus no induced current in (2), and so no force and no torque on (2).

Chapter 52

52.1 (a) Let $\boldsymbol{E}_0 = E_x\boldsymbol{e}_x + E_y\boldsymbol{e}_y + E_z\boldsymbol{e}_z$ and consider the x-component of $(\nabla^2 - \partial^2/\partial(ct)^2)\boldsymbol{E}$,

$$\begin{aligned}&\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_x e^{i(\omega t - kx)}\\ &= \left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)E_x e^{i(\omega t - kx)}\\ &= \left(-k^2 + \frac{\omega^2}{c^2}\right)E_x e^{i(\omega t - kx)}.\end{aligned}$$

This is a solution of the wave equation if $k^2 = \omega^2/c^2$, i.e., if $k = \pm\omega/c$. The y- and z-components are exactly the same.

(b) The wave goes toward $+x$ if $k > 0$, toward $-x$ if $k < 0$.

(c) By definition, $\boldsymbol{\nabla} = \boldsymbol{e}_x\partial/\partial x + \boldsymbol{e}_y\partial/\partial y + \boldsymbol{e}_z\partial/\partial z$. If there is no y- or z-dependence of the fields, then $\boldsymbol{\nabla} = \boldsymbol{e}_x\partial/\partial x$. If the field components are of the form Ae^{-ikx}, then $\boldsymbol{\nabla}Ae^{-ikx} = \boldsymbol{e}_x\partial(Ae^{-ikx})/\partial x = -ikAe^{-ikx}\boldsymbol{e}_x$, so $\boldsymbol{\nabla} = -ik\boldsymbol{e}_x$.

(d) $\partial/\partial t = i\omega$

(e)

$$\begin{aligned}\boldsymbol{\nabla}\cdot\boldsymbol{E} = \rho/\epsilon_0 &\to -ikE_x = \rho/\epsilon_0\\ \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0 &\to B_x = 0\\ \boldsymbol{\nabla}\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} &\to k\boldsymbol{e}_x\times\boldsymbol{E} = \omega\boldsymbol{B}\\ c^2\boldsymbol{\nabla}\times\boldsymbol{B} = \boldsymbol{j}/\epsilon_0 + \frac{\partial\boldsymbol{E}}{\partial t} &\to -ikc^2\boldsymbol{e}_x\times\boldsymbol{B} = \boldsymbol{j}/\epsilon_0 + i\omega\boldsymbol{E}\end{aligned}$$

The relation between k and ω depends on what $\boldsymbol{j}$ and ρ are. In free space where $\boldsymbol{j} = \rho = 0$, $k = \pm\omega/c$.

(f) In all the solutions above, $k \to -k$.

52.2 (a) $\omega' = \omega(1 - v/c)/(1 + v/c)$

Chapter 53

53.3 Using the notation and variables given in Ex. 53.2, and replacing p by μ where $\mu = \pi a^2 i_0$ is the magnetic moment of the dipole,

$$\begin{aligned}B_\varphi &= 0,\\ B_\theta &= \frac{1}{c^2}\,(E_\theta \text{ electric dipole}),\\ B_r &= \frac{1}{c^2}\,(E_r \text{ electric dipole}),\\ E_\varphi &= -(B_\varphi \text{ electric dipole}),\\ E_\theta &= E_r = 0.\end{aligned}$$

53.4 (c) The fields produced by the dipole are

$$\begin{aligned}\boldsymbol{E} &= -\frac{Q_0 d\omega^2\sin\theta}{4\pi\epsilon_0 c^2 r}\cos\omega(t - r/c)\boldsymbol{e}_\theta + O\left(\frac{1}{r^2}\right),\\ \boldsymbol{B} &= \frac{1}{c}\boldsymbol{e}_r\times\boldsymbol{E}\\ &= -\frac{Q_0 d\omega^2\sin\theta}{4\pi\epsilon_0 c^3 r}\cos\omega(t - r/c)\boldsymbol{e}_\phi + O\left(\frac{1}{r^2}\right).\end{aligned}$$

These results agree with those of Ex. 53.2, with $p = Q_0 d$.

53.5 (a) Considering a short segment of wire of length Δz to be a dipole of the type considered in Ex. 53.4, the charge on each end of the dipole is $q = \int i\,dt$ so the dipole moment is

$$\begin{aligned}\Delta p &= \left(\int -i_0 \sin\omega t \cos\frac{2\pi z}{\lambda}\,dt\right)\Delta z\\ &= \left(\frac{i_0}{\omega}\cos\frac{2\pi z}{\lambda}\cos\omega t\right)\Delta z.\end{aligned}$$

(c) The total dipole moment of the antenna is $p = \int_{-\lambda/4}^{\lambda/4} \Delta p/\Delta z\,dz = (2i_0 c/\omega^2)\cos\omega t$. A single dipole with this moment produces a field

$$\boldsymbol{E}' = -\frac{i_0}{2\pi\epsilon_0 cr}\sin\theta\cos\omega(t - r/c)\boldsymbol{e}_\theta + O\left(\frac{1}{r^2}\right).$$

So we compare $f(\theta) = \cos\left(\frac{\pi}{2}\cos\theta\right)/\sin\theta$ with $g(\theta) = \sin\theta$:

	θ	0°	30°	60°	90°
half-wave antenna	$f(\theta)$	0	0.418	0.816	1
single dipole	$g(\theta)$	0	0.500	0.867	1

Note: $\lim_{\theta\to 0} f(\theta)/g(\theta) = \pi/4$.

53.6 (a) $\phi = \dfrac{q}{4\pi\epsilon_0 a}$

(b) $\boldsymbol{A} = \dfrac{qv}{4\pi\epsilon_0 c^2 a}\left(\sin\frac{v}{c}\,\boldsymbol{e}_x + \cos\frac{v}{c}\,\boldsymbol{e}_y\right)$

(c)
$$\begin{aligned}\boldsymbol{E} = -\frac{q}{4\pi\epsilon_0 a^2}\Bigg(&\left[\left(1-\frac{v^2}{c^2}\right)\cos\frac{v}{c} + \frac{v}{c}\sin\frac{v}{c}\right]\boldsymbol{e}_x\\ &+ \left[\frac{v}{c}\cos\frac{v}{c} - \left(1-\frac{v^2}{c^2}\right)\sin\frac{v}{c}\right]\boldsymbol{e}_y\Bigg)\end{aligned}$$

$$\boldsymbol{B} = \frac{qv}{4\pi\epsilon_0 c^2 a^2}\boldsymbol{e}_z$$

Chapter 54

54.1 Between adjacent terminals $R = (7/12)\,\Omega$. Between square diagonal terminals $R = (3/4)\,\Omega$. Between cube diagonal terminals $R = (5/6)\,\Omega$.

54.2 (a) $I = \dfrac{1-\omega^2\mathfrak{L}C}{\omega\mathfrak{L}[2-\omega^2\mathfrak{L}C]}V_0\sin\omega t$

(b) $I = \dfrac{1-\omega^2\mathfrak{L}C}{\omega(\mathfrak{L}\pm\mathfrak{M})[2-\omega^2(\mathfrak{L}\mp\mathfrak{M})C]}V_0\sin\omega t$

54.5 (a) Use $|I| = E_0\sqrt{\dfrac{1}{R^2} + \dfrac{(\omega^2 LC-1)^2}{\omega^2L^2}}$.

(b) In both cases $Q = 2K$.

54.6 $\mathfrak{L} = R^2C$

54.8 (a)
$$\begin{aligned}\hat{I} &= \hat{V}/[Z_{\mathrm{RC}} + Z_{\mathrm{RL}}]\\ &= \hat{V}/\left[\frac{1}{1/R + i\omega C} + \frac{1}{1/R + 1/(i\omega L)}\right]\\ &= \frac{\hat{V}}{R}\left[\frac{1 + R(i\omega C + 1/(i\omega L)) + R^2C/L}{2 + R(i\omega C + 1/(i\omega L))}\right]\end{aligned}$$

Clearly, if $R^2C/L = 1$, then $\hat{I} = \hat{V}/R$ independent of ω.

(b) $\phi = -\arctan(\omega RC)$

54.9 (a) $\langle P\rangle = \dfrac{(V_0^2R)/2}{R^2 + (\omega L - m/\omega C)^2}$

(b) (1) $m = 2$

(2) $(V_{P_0P_2})_{\max} = 2V_0/(\omega RC)$, and $(V_R)_{\max} = V_0$

54.10 (a) $\dfrac{V_L}{V_0} = \dfrac{1}{(1-\alpha)(R_I/R_L)+1}$

(b) With $\beta \approx 1/(1-\alpha)$, the answer to part a above, becomes

$$\frac{V_L}{V_0} = \frac{1}{(R_I/R_L)/\beta + 1}.$$

Thus if $R_I/R_L \ll \beta$, then $V_L/V_0 \approx 1$.

(c) $\dfrac{(V_L/V_0)}{(V_L'/V_0')} \approx 10$

(d) Use

$$\frac{V_L}{V_0} = \frac{1}{1 + j\omega R_IC_L/\beta}$$

and

$$\frac{V_L'}{V_0'} = \frac{1}{1 + j\omega R_IC_L}.$$

54.11 (a) $\dfrac{V_L}{V_0} = -\beta\dfrac{R_L}{R_I}$

(b) Substituting $i_0 = (i_ER_E - V_0)/R_I$ into $i_0 + i_E(1-\alpha) = 0$, and rearranging,

$$i_E\left(\frac{R_E}{R_I} + (1-\alpha)\right) = \frac{V_0}{R_I}.$$

Since $V_L = -\alpha i_ER_L$ and $\alpha \approx 1$,

$$\begin{aligned}\frac{V_L}{V_0} &= -\alpha\frac{R_L}{R_E}\frac{1}{1 + (1-\alpha)(R_I/R_E)}\\ &\approx -\alpha\frac{R_L}{R_E}\frac{1}{1 + (R_I/R_E)/\beta},\end{aligned}$$

thus if $R_I/R_E \ll \beta$, $V_L/V_0 \approx -\alpha(R_L/R_E)$.

54.12 (a) $I_L/I_0 = \alpha \approx 1$

(b) $Z_{\mathrm{out}} = \infty$

(c) The equivalent circuit is a current source, $I = \alpha V_0/R_I$.

54.13 (a) $Z_{AB} = \dfrac{n^2R_E}{\alpha}\dfrac{1}{n + 1/\beta}$

(b) For Z_{AB} to be negative, we must have $n < 0$ and $|n| > 1/\beta$. The meaning of a negative n is that the polarity of the transformer (that is, of one of the windings) must be inverted.

Chapter 55

55.1 (a) $\omega = \dfrac{c}{a}\sqrt{\dfrac{2d}{(b-a)\ln(b/a)}}$

(c) If the temperature is decreased, thermal contraction will lead to an increase of frequency.

Chapter 56

56.2 For length Δx, the capacitance is $C_0\,\Delta x \approx \epsilon_0(b\,\Delta x)/a$, thus $C_0 \approx \epsilon_0 b/a$. We know that $c = 1/\sqrt{L_0C_0}$, so $z_0 = \sqrt{L_0/C_0} = \sqrt{L_0C_0/C_0^2} = 1/(cC_0) \approx a/\epsilon_0 cb$.

56.3 (a) $\omega_1 = \pi c/l$

(b) Taking the z-axis along the coaxial line, and letting $E_0 = |\boldsymbol{E}|$ at time $t = 0$ in the middle of the line ($z = l/2$), at the surface of the central conductor (radial distance $r = a$),

$$\boldsymbol{E} = E_0\frac{a}{r}\sin(k_nz)e^{i\omega_nt}\boldsymbol{e}_r,$$

with $k_n = n\pi/l$ and $\omega_n = n\pi c/l$.

(c) $\omega_1/\omega_0 \approx \pi$

56.4 (a) $E_0(x,z) = E_0\sin\dfrac{\pi x}{a}\sin\dfrac{\pi z}{l}$

(b) $\omega_0 = \pi c\sqrt{\dfrac{1}{a^2} + \dfrac{1}{l^2}}$

56.5 (a) Use $V(x,t) = V_0 \cos(\omega t) \cos(kx)$ with $k = 5\pi/2l$.

(b) The magnitude of the voltage is maximum at $x = 0$, $2l/5$, and $4l/5$ (whenever $\cos(\omega t) > 0$).

(c)
$$V_f(x,t) = \frac{1}{2} V_0 \cos(\omega t - kx)$$
$$V_r(x,t) = \frac{1}{2} V_0 \cos(\omega t + kx)$$

(d)
$$I(0,t) = 0$$
$$I(l/2,t) = -\frac{1}{\sqrt{2}} \frac{V_0}{z_0} \sin(\omega t)$$
$$I(l,t) = \frac{V_0}{z_0} \sin(\omega t)$$

with $z_0 = \sqrt{L_0/C_0}$

(e) $\langle \tau \rangle = 0$

56.6 (b) $Z_S = iZ_0 \tan(\omega l \sqrt{LC})$

(c) $Z_S = -iZ_0 \cot(\omega l \sqrt{LC})$

(d) $Z_S = Z_0$

56.7 Consider an arbitrary incident waveform V_{inc} in line 1, its reflection from the juncture with line 2, V_{ref} (also in line 1), and the transmitted waveform in line 2, V_{trans}. At the juncture of the lines the voltages and the currents must match. Thus, at the juncture,

$$V_{\text{inc}} + V_{\text{ref}} = V_{\text{trans}}$$
$$I_{\text{inc}} - I_{\text{ref}} = I_{\text{trans}}.$$

(The incident and reflected currents are added with opposite sign because the corresponding waves move in opposite directions.) Dividing these equations by V_{inc} while substituting $I_{\text{inc}} = V_{\text{inc}}/Z_1$, $I_{\text{ref}} = V_{\text{ref}}/Z_1$ and $I_{\text{trans}} = V_{\text{trans}}/Z_2$, produces a pair of linear equations,

$$1 + (V_{\text{ref}}/V_{\text{inc}}) = (V_{\text{trans}}/V_{\text{inc}}),$$
$$1 - (V_{\text{ref}}/V_{\text{inc}}) = \frac{Z_1}{Z_2}(V_{\text{trans}}/V_{\text{inc}}).$$

whose simultaneous solution yields the stated results.

56.8 $v_g/c \approx 0.845$

56.9 (a) $\boldsymbol{B} = \boldsymbol{\nabla} \times A = (\partial A_z/\partial y)\boldsymbol{e}_x - (\partial A_z/\partial x)\boldsymbol{e}_y$, since $\boldsymbol{A} = A_z \boldsymbol{e}_z$. Thus, $\boldsymbol{B} \cdot \boldsymbol{e}_z = 0$.

(c) The mn-mode will not be propagated if k_z is pure imaginary, because then $e^{-ik_z z}$ is either $e^{+|k_z|z}$, which increases without bound and is unphysical, or $e^{-|k_z|z}$ which diminishes exponentially. For k_z to be pure imaginary, we must have $(\omega/c)^2 - (m\pi/a)^2 - (n\pi/b)^2 < 0$ which is true whenever $\omega < c\sqrt{(m\pi/a)^2 + (n\pi/b)^2}$.

Chapter 57

57.1 (a) $A_\mu A_\mu$

(b) $-A_\mu j_\mu$

57.2 $$E^{\gamma_2} = \frac{E^{\gamma_1}}{1 + (E^{\gamma_1}/m_e)(1 - \cos\theta)}$$

57.3 $E^\gamma = 4m_e$

57.4 If the reaction $\gamma \to e^+ + e^-$ were possible then $p_\mu^\gamma = p_\mu^+ + p_\mu^-$. Squaring both sides and simplifying, using $(p_\mu^\gamma)^2 = 0$ and $(p_\mu^\pm)^2 = m_e$, yields $0 = m_e^2 + p_\mu^+ p_\mu^-$, which must hold in any coordinate system. Consider the system in which e^- is at rest, i.e., $p_\mu^- = (m_e, \boldsymbol{0})$; then $0 = m_e^2 + m_e E^+$ or $E^+ = -m_e$, which is unphysical.

57.5 (a) Relativistically, the total energy of particle b is $E_b = \sqrt{m_b^2 + p_b^2}$, and

$$m_c = \sqrt{m_a^2 + 2m_a E_b + m_b^2}$$
$$v_c = \frac{p_b}{m_a + E_b}$$

(b) Non-relativistically,

$$m_c = m_a + m_b$$
$$v_c = \frac{p_b}{m_a + m_b}$$

Chapter 58

58.1 Assuming the Lorenz gauge $\nabla_\mu A_\mu = 0$, and additionally using Eq. (25.22) from Vol. II,

$$\nabla_\mu F_{\mu\nu} = \nabla_\mu \nabla_\mu A_\nu - \nabla_\mu \nabla_\nu A_\mu = \Box^2 A_\nu - \nabla_\nu(\nabla_\mu A_\mu) = \frac{j_\nu}{\epsilon_0}.$$

58.2 (a) $f_\mu = j_\nu F_{\mu\nu} = (\boldsymbol{j} \cdot \boldsymbol{E}, \rho\boldsymbol{E} + \boldsymbol{j} \times \boldsymbol{B})$.

(b) The time component of f_μ is the power per unit volume expended by the fields on the charges, and the space components give the force per unit volume.

58.3 (a) Let $F_{\mu\nu}$ be the electromagnetic field tensor corresponding to $\boldsymbol{E}$ and $\boldsymbol{B}$. Then $\boldsymbol{B}^2 - \boldsymbol{E}^2 = (1/2)F_{\mu\nu}F_{\mu\nu}$ and $\boldsymbol{E} \cdot \boldsymbol{B} = F_{xt}F_{zy} + F_{yt}F_{xz} + F_{zt}F_{yx} = (1/8)\epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}$ (where $\epsilon^{\alpha\beta\gamma\delta}$ is the Levi-Civita symbol); both are scalar products of 4-tensors, and therefore Lorentz-invariant.

(b) These quantities are both zero for electromagnetic radiation propagating in free space.

58.4 $$\frac{\boldsymbol{v}}{1 + v^2} = \frac{\boldsymbol{E} \times \boldsymbol{B}}{\boldsymbol{E}^2 + \boldsymbol{B}^2}$$

58.7 Taking z in the wire parallel to $\boldsymbol{v}$, and the field point in the xy-plane.

(a) In a frame stationary with respect to the wire,

$$\boldsymbol{E} = 0,$$
$$\boldsymbol{B} = \frac{I}{2\pi\epsilon_0 c^2 r}\, \boldsymbol{e}_\phi,$$

where $\boldsymbol{r}$ is the radius vector to the field point and $\boldsymbol{e}_\phi$ is perpendicular to the radius vector and to the wire ($\boldsymbol{e}_\phi = -\sin\phi\, \boldsymbol{e}_x + \cos\phi\, \boldsymbol{e}_y$).

(b) In a frame stationary with respect to the electrons,

$$E'_z = B'_z = 0,$$
$$\boldsymbol{E}' = \frac{\gamma v I}{2\pi\epsilon_0 c^2 r}\, \boldsymbol{e}_r,$$
$$\boldsymbol{B}' = \frac{\gamma I}{2\pi\epsilon_0 c^2 r}\, \boldsymbol{e}_\phi,$$

with $\gamma = 1/\sqrt{1 - v^2}$.

58.8 (a) $\sigma = (q_e/2\pi a^2)\sqrt{1 - v^2}$

(b) $\sigma_h/\sigma_l \approx 1/1000$

58.9 $u_\mu = \gamma(1, \boldsymbol{v})$ and $f_\mu = \gamma(\boldsymbol{F} \cdot \boldsymbol{v}, \boldsymbol{F})$. Hence

$$f_\mu u_\mu = f_t u_t - \boldsymbol{f} \cdot \boldsymbol{u} = \gamma^2(\boldsymbol{F} \cdot \boldsymbol{v} - \boldsymbol{F} \cdot \boldsymbol{v}) = 0$$

58.10 (a) $$\boldsymbol{E} = \frac{q}{2\sqrt{3}\pi\epsilon_0 a^2} \boldsymbol{e}_y$$

(b) The answer would not change.

Chapter 59

59.2 $U = 960$ megatons

59.3 Along the wire there is a voltage drop IR per unit length, which implies there is an $\boldsymbol{E}$ field at the surface of the wire of strength IR (in the direction of the current). At the surface of the wire the $\boldsymbol{B}$ field has magnitude $I/2\pi a\epsilon_0 c^2$, where a is the radius of the wire. Since $\boldsymbol{E}$ and $\boldsymbol{B}$ are perpendicular, $\boldsymbol{S} = \epsilon_0 c^2 \boldsymbol{E} \times \boldsymbol{B} = -(I^2 R/2\pi a)\boldsymbol{e}_r$. The flux of $\boldsymbol{S}$ into the surface of the wire per unit length equals $2\pi a|\boldsymbol{S}| = I^2 R$, the same as the heating per unit length.

59.4 In cylindrical coordinates (z, ϕ, r) with z along the axis of the cable, $\boldsymbol{B} = I\boldsymbol{e}_\phi/2\pi\epsilon_0 c^2 r$ and $\boldsymbol{E} = V\boldsymbol{e}_r/\ln(b/a)r$. So, $\boldsymbol{S} = IV\boldsymbol{e}_z/2\pi\ln(b/a)r^2$. The total rate of energy flow is $dU/dt = \int_a^b |\boldsymbol{S}| 2\pi r\, dr = (IV/\ln(b/a)) \int_a^b 1/r\, dr = IV$.

59.5 (a) $|\langle \boldsymbol{S} \rangle| = 2.6 \times 10^{-5}\,\mathrm{W\,m^{-2}}$

(b) $|\boldsymbol{E}_{\max}| = 0.14\,\mathrm{V\,m^{-1}}$, $|\boldsymbol{B}_{\max}| = 5 \times 10^{-10}\,\mathrm{Wb\,m^{-2}}$

59.6 (a) The boundary conditions are that the normal component of $\boldsymbol{B}$ and the tangential component of $\boldsymbol{E}$ are zero along the conducting surfaces. Since $\sin(\pi x/a) = 0$ for $x = 0$ and $x = a$, these conditions are satisfied.

(b) $$\boldsymbol{S} = \epsilon_0 c^2 E_0^2 \left(\frac{k_z}{\omega} \sin^2 \frac{\pi x}{a} \cos^2(\omega t - k_z z)\boldsymbol{e}_z - \frac{\pi}{4\omega a} \sin \frac{2\pi x}{a} \sin 2(\omega t - k_z z)\boldsymbol{e}_x \right)$$

$$u = \epsilon_0 E_0^2 \left(\sin^2 \frac{\pi x}{a} \cos^2(\omega t - k_z z) + \frac{\pi^2 c^2}{4\omega^2 a^2} \left(\cos \frac{2\pi x}{a} - \cos 2(\omega t - k_z z) \right) \right)$$

(c) $\langle dU/dt \rangle = \epsilon_0 c^2 E_0^2 \dfrac{k_z}{\omega} \dfrac{ab}{4}$

(d) $\langle u \rangle_{\text{time avg}} = \dfrac{\epsilon_0 E_0^2}{2} \left(\sin^2 \dfrac{\pi x}{a} + \dfrac{\pi^2 c^2}{2\omega^2 a^2} \cos \dfrac{2\pi x}{a} \right)$

(e) $$v_g = \frac{\text{avg flow rate}}{\text{avg stored per unit length}} = \frac{\langle dU/dt \rangle}{\int_0^b \int_0^a \langle u \rangle_{\text{time avg}}\, dx\, dy} = \frac{c^2 k_z}{\omega}$$

But, $k_z = \sqrt{\omega^2/c^2 - \pi^2/a^2}$, so $v_g = c\sqrt{1 - \pi^2 c^2/a^2\omega^2} = c\sqrt{1 - (\omega_c/\omega)^2}$, with $\omega_c = \pi c/a$.

59.7 With $\boldsymbol{p}$ at the origin, oriented along the z-axis, and using spherical coordinates for the field point (r, θ, ϕ),

$$\boldsymbol{S} = \frac{p^2\omega^4}{16\pi^2\epsilon_0 c^3 r^2} \sin^2\theta \cos^2\omega(t - r/c)\, \boldsymbol{e}_r.$$

(a) $\dfrac{\text{flow rate}}{\text{unit area}} = \boldsymbol{S} \cdot \boldsymbol{e}_r = \dfrac{p^2\omega^4}{16\pi^2\epsilon_0 c^3 r^2} \sin^2\theta \cos^2\omega(t - r/c)$

(b) $$\langle P \rangle = 2\pi r^2 \int_0^\pi \langle \boldsymbol{S} \rangle \cdot \boldsymbol{e}_r \sin\theta\, d\theta = 2\pi r^2 \int_0^\pi \left[\frac{p^2\omega^4}{16\pi^2\epsilon_0 c^3 r^2} \sin^2\theta \frac{1}{2} \boldsymbol{e}_r \right] \cdot \boldsymbol{e}_r \sin\theta\, d\theta = \frac{p^2\omega^4}{16\pi\epsilon_0 c^3} \int_0^\pi \sin^3\theta\, d\theta = \frac{p^2\omega^4}{12\pi\epsilon_0 c^3}$$

59.8 $\dfrac{\langle P_{\text{rad}} \rangle}{\langle P_{\text{incident}} \rangle} = \dfrac{8\pi}{3} \left(\dfrac{e^2}{m_e c^2} \right)^2$, with $e^2 = q_e^2/4\pi\epsilon_0$

59.9 (a) $a \approx 6000$ Å

59.10 (a) $U_B(t) = (N^2 K^2 r^2/4\epsilon_0 c^2 R)t^2$

(b) $\boldsymbol{S}(t) = -(N^2 K^2 r/8\pi^2\epsilon_0 c^2 R^2)t\, \boldsymbol{e}_r$

(c) The surface area of the toroid $= (2\pi R)(2\pi r)$, so $dU/dt = \int \boldsymbol{S} \cdot \boldsymbol{n}\, da = 4\pi^2 Rr\boldsymbol{S}(t) = (N^2 K^2 r^2/2\epsilon_0 c^2 R)\, t$. $dU/dt = d(U_B + U_E)/dt = dU_B/dt$, which is consistent because E inside the toroid is unchanging and thus the energy stored in the electric field is constant.

Chapter 60

60.1 Using Eq. (8.7) from Vol. II for a solid sphere of charge, $mc^2 = U_{\text{elec}} = \frac{3}{5} Q^2/(4\pi\epsilon_0 a) = \frac{3}{5} e^2/a$, which gives $a = \frac{3}{5} e^2/(mc^2) \approx 1.7 \times 10^{-15}\,\mathrm{m}$. On the other hand, using Eq. (28.2) from Vol. II for a spherical shell of charge, $U_{\text{elec}} = \frac{1}{2} e^2/a$, gives $a = \frac{1}{2} e^2/(mc^2) \approx 1.4 \times 10^{-15}\,\mathrm{m}$.

60.2 (b) $\dfrac{L}{\mu} = \dfrac{2}{3} \dfrac{q}{4\pi\epsilon_0 c^2} \dfrac{1}{a} = \dfrac{m_e}{q_e}$, with m_e as given.

(c) $v_{\max} = \frac{3}{2}\hbar/(ma)$. However, if we set $a = \frac{1}{2} e^2/(mc^2)$, as per Ex. 60.1, then $v_{\max} = 3\hbar c^2/e^2 \approx 3(137)c$, or about 400 times the speed of light!

Chapter 61

61.1 Using $\alpha = (qE/mc)$, $\beta = (\alpha/\gamma)$, $\gamma = 1/\sqrt{1 - v_0^2/c^2}$,

(a) $$v_x = \frac{\alpha c t}{\sqrt{1 + \alpha^2 t^2}}, \quad v_y = v_z = 0, \quad x = \frac{c}{\alpha} \left(\sqrt{1 + \alpha^2 t^2} - 1 \right), \quad y = z = 0$$

(b) $$v_x = \frac{\beta c t}{\sqrt{1 + \beta^2 t^2}}, \quad v_y = \frac{v_0}{\sqrt{1 + \beta^2 t^2}}, \quad v_z = 0, \quad x = \frac{c}{\beta} \left(\sqrt{1 + \beta^2 t^2} - 1 \right), \quad y = \frac{v_0}{\beta} \ln \left(\beta t + \sqrt{1 + \beta^2 t^2} \right), \quad z = 0$$

61.2 (a) At low energies, $\omega \approx q_p B/m_p$.

(b) The frequency decreases as the energy increases.

(c) $T = m_p c^2/100 \approx 9.4\,\mathrm{MeV}$

61.3 (a) $$x = \frac{E}{B} t - \frac{E}{B\omega_c} \sin\omega_c t, \quad y = \frac{E}{B\omega_c} (1 - \cos\omega_c t), \quad z = 0,$$

with $\omega_c = qB/m$, the cyclotron frequency. This solution is valid only if $E/B \ll c$.

(b) For the case $E/B < c$, the motion is a cycloid, as per part (a) above, only it is stretched out in the x-direction by the factor $\gamma = 1/\sqrt{1 - (E/Bc)^2}$. For $E/B > c$, the solution is given in part (b) of Ex. 61.1 if we transform to the coordinate system S' moving with velocity $V = Bc^2/E$ in the $+x$-direction, such that $\boldsymbol{B}' = 0$, and then identify our $+x$ and $+y$ axes with, respectively, the $-y$ and $+x$ axes in Ex. 61.1, choosing $v_0 = -Bc^2/E$ as the initial velocity of the particle.

(c) Relativistically,

$$B = \frac{\sqrt{2mV_0/q + (V_0/c)^2}}{d}$$

Non-relativistically (letting $c \to \infty$),

$$B = \frac{\sqrt{2mV_0/q}}{d}$$

61.4 (a) $l = f(f-d)/d$

(b) Real image for $l > 0$ (or $f > d$), virtual image for $l < 0$ (or $f < d$).

Chapter 62

62.1 By definition, $\boldsymbol{D} = \epsilon_0\boldsymbol{E} + \boldsymbol{P} = \kappa\epsilon_0\boldsymbol{E}$. Thus $\kappa = (E + P/\epsilon_0)/E$. For a transverse wave of angular frequency ω and wave number k propagating in the z-direction, Eq. (32.23) in Vol. II gives $-k^2E + (\omega^2/c^2)E = -(\omega^2/\epsilon_0c^2)P$, which can be rearranged $n^2 = (E + P/\epsilon_0)/E$, with the index of refraction written $n = kc/\omega$. Thus $n^2 = \kappa$. [The approximations used here are just those used to derive Eq. (32.23)].

62.2 $\rho \approx 4.5 \times 10^{11}\,\mathrm{m}^{-3}$

62.3 With $E = 0$, $\gamma = 1/\tau$ and $\omega_0 = 0$ (free electron model), Eq. (32.1) in Vol. II becomes $0 = m(\dot{v} + v/\tau)$, with solution $v = v_0e^{-t/\tau}$; the relaxation time is τ. However, with $E \neq 0$, (but neglecting B), the force on the electrons is $q_eE = m(\dot{v} + v/\tau)$ or, taking the derivative, $\ddot{v} + \dot{v}/\tau = (q_e/m_e)\dot{E}$. Since we are taking $B = 0$, $c^2(\boldsymbol{\nabla}\times\boldsymbol{B}) = \boldsymbol{j}/\epsilon_0 + \dot{\boldsymbol{E}} = 0$. Thus, $\dot{E} = -j/\epsilon_0 = -Nq_ev/\epsilon_0$. Hence, $\ddot{v} + \dot{v}/\tau = (q_e/m_e)(-Nq_ev/\epsilon_0) = -\omega_p^2v$, where ω_p is the plasma frequency. Thus $\ddot{v} + \dot{v}/\tau + \omega_p^2v = 0$. Setting $v = v_0e^{i\omega t}$ one finds $-\omega^2 + i\omega/\tau + \omega_p^2 = 0$, with solutions $\omega = i/2\tau \pm \sqrt{\omega_p^2 - (1/2\tau)^2}$. The relaxation time is the reciprocal of the imaginary part of ω, which is 2τ (since $\omega_p > 1/\tau$).

62.4 (a) $\boldsymbol{B} = (kE_x/\omega)\boldsymbol{e}_y$

(b) $\theta = 90°$

(c) $|\boldsymbol{B}|_{\max}/|\boldsymbol{E}|_{\max} = \sqrt{\sigma/\omega\epsilon_0c^2}$

(d) $\phi = 45°$

62.5 From Eq. (32.42) in Vol. II,

$$\begin{aligned} n^2 &= 1 + \frac{\sigma/\epsilon_0}{i\omega(1+i\omega\tau)} = 1 - \frac{\sigma/\epsilon_0}{\omega^2\tau(1-i/\omega\tau)} \\ &\approx 1 - \frac{\sigma/\epsilon_0}{\omega^2\tau}(1+i/\omega\tau) \\ &= 1 - \left(\frac{\omega_p}{\omega}\right)^2 - i\left(\frac{\omega_p^2}{\omega^3\tau}\right), \end{aligned}$$

where ω_p is the plasma frequency. Thus n goes from real to imaginary when ω changes by more than $1/\tau$ around ω_p. (To be more precise, the phase of n is $3\pi/8$ when $\omega = \omega_p - 1/2\tau$, and $\pi/8$ when $\omega = \omega_p + 1/2\tau$.)

Chapter 63

63.1 (a) $\dfrac{I_t}{I_i} = \dfrac{1 + (r_{12}r_{23})^2 - r_{12}^2 - r_{23}^2}{1 + (r_{12}r_{23})^2 + 2r_{12}r_{23}\cos(4\pi l/\lambda_2)}$, where $r_{ij} = (n_i - n_j)/(n_i + n_j)$.

(b) For $I_t/I_i = 1$ we must have $-r_{12}^2 - r_{23}^2 = 2r_{12}r_{23}\cos(4\pi l/\lambda_2)$, or $|r_{12} + r_{23}e^{i4\pi l/\lambda_2}| = 0$, which requires that $r_{12} = r_{23}$, and $4\pi l/\lambda_2 = n\pi$ with n any odd integer. $r_{12} = r_{23}$ is the same as $(n_1 - n_2)/(n_1 + n_2) = (n_2 - n_3)/(n_2 + n_3)$, which is satisfied when $n_2 = \sqrt{n_1n_3}$. While $4\pi l/\lambda_2 = n\pi$ is satisfied when $n = 1$ and $l = \lambda_2/4$.

(c) $l \approx 1100\,\text{Å}$

(d) It does not make any difference for a fairly flat lens, where normal incidence can be assumed, since the equation for (I_t/I_i) in part (a) above is symmetrical in (subscripts) 1 and 3.

63.2 (a) $d \approx 1350\,\text{Å}$

(b) No. (The answer does not depend on the polarization of $\boldsymbol{E}$.)

Chapter 64

64.1 (b) We must assume that the particle travels practically in a circular orbit, which requires that $|dB/dt| \ll qB^2/2\pi m$.

Chapter 65

65.1 $g = 5.5$

65.3 $(N_{\text{up}} - N_{\text{down}})/N \approx 0.22\,\%$ at room temperature, $16\,\%$ at liquid helium temperature.

Chapter 66

66.1 With the center of the sphere at the origin and using spherical coordinates,

(a) $\boldsymbol{K} = M\sin\theta\,\boldsymbol{e}_\phi$

(b) $$\mu = \int_0^\pi K\pi(a\sin\theta)^2a\,d\theta = \int_0^\pi M\pi a^3 sin^3\theta\,d\theta = \frac{4}{3}\pi a^3M.$$

66.2 (a) $B_{\text{gap}} \approx 1.2\,\mathrm{Wb\,m^{-2}}$

(b) This estimate assumes that the flux of $\boldsymbol{B}$ is constant through any cross-section of the yoke, and it ignores end effects in the gap between the pole pieces.

66.3 $B_{\text{gap}} \approx 0.80\,\mathrm{Wb\,m^{-2}}$

66.4 (a) $\boldsymbol{B} = \boldsymbol{M}/\epsilon_0c^2$, $\boldsymbol{H} = 0$

(b) $\boldsymbol{B}_{\text{cav}} = \boldsymbol{H}_{\text{cav}} = 0$

Chapter 67

67.1 (a) $r_{\text{Al}}/r_{\text{steel}} = 1.30$

(b) $m_{\text{Al}}/m_{\text{steel}} = 0.59$

67.2 $\omega = (1.32 \times 10^5)\cdot a^2/\sqrt{mL^3}$

67.3 (a) Eq. (38.20) in Vol. II gives the pressure p that must be applied to the ends of a bar of length L to change its length by ΔL while its width and height remain constant, $p = [(1-\sigma)Y/(1+\sigma)(1-2\sigma)]\Delta L/L$. When compressional waves are being transmitted through a substance, say in the z-direction, the material at any given location experiences only the local compressional strain $\partial L/\partial z$. The pressure experienced at z is thus

$$p(z) = \frac{(1-\sigma)Y}{(1+\sigma)(1-2\sigma)}\frac{\partial L}{\partial z}.$$

The net force acting on a volume element $\Delta x\,\Delta y\,\Delta z$ is $F(z+\Delta z) - F(z) = \Delta x\,\Delta y(\partial p/\partial z)\Delta z$, which must equal the mass of the element times its acceleration, $(\rho\,\Delta x\,\Delta y\,\Delta z)(\partial^2L/\partial t^2)$, where ρ is the density of the material. Thus, $\partial p/\partial z = \rho\partial^2L/\partial t^2$. Substituting $p(z)$, as given above, produces the wave equation with wave velocity $V_{\text{long}} = \sqrt{(1-\sigma)Y/(1-2\sigma)(1+\sigma)\rho}$,

$$\frac{\partial^2L}{\partial z^2} - \frac{1}{V_{\text{long}}^2}\frac{\partial^2L}{\partial t^2} = 0.$$

(b) Because $p(z)$ depends on the width and height of the block not changing, those dimensions should be large compared to the wavelength of the longitudinal waves.

67.4 (a) The ruler is bent into a half-wave of a sine having wavelength $\approx 23''$.

(b) $F \approx 2.8\,\text{lb}$

67.5 $P = \dfrac{\pi^2}{48}\dfrac{Ywt^3}{L^2}$

Chapter 68

68.2 Taking z on the axis of the cylinder, and r on any radius,

(a) $z = (\omega^2/2g)r^2$

(b) (See Ex. 38.5.) The velocity field is $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$, where $\boldsymbol{\omega} = \omega\boldsymbol{e}_z$, $\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y$. Therefore $\boldsymbol{v} = \omega(-y\boldsymbol{e}_x + x\boldsymbol{e}_y)$. Thus,

$$\boldsymbol{\nabla} \times \boldsymbol{v} = \omega \begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & x & 0 \end{vmatrix} = \omega\boldsymbol{e}_z(1+1) = 2\boldsymbol{\omega}.$$

68.3 (b) $\boldsymbol{p}_{\text{total}} = m\boldsymbol{v}$

Chapter 69

69.1 With unit symbols M for mass, D for distance, T for time. For steady flow of an incompressible fluid (small Reynold's number) we may neglect the fluid density, so the parameters that determine the drag force on the ball are its radius a [units: D], it's velocity v [units: D/T], and the viscosity of the fluid η [units: $M/(DT)$]. We seek integral powers x, y, and z such that the viscous force on the ball $F \propto a^x v^y \eta^z$. The units of force are MD/T^2, so we must have $M^1D^1T^{-2} = [D]^x[D/T]^y[M/(DT)]^z = M^zD^{x+y-z}T^{-y-z}$, with unique solution $x = y = z = 1$. Therefore $F \propto av\eta$.

69.2 (b) $R = 8\eta L/\pi a^4$

69.3 $dU/dt = \eta v^2/d^2$

Chapter 70

70.1 (a) $\lambda = 2dx/L$

70.2 (a) If the gun is moved upward the distance D, then P_1, P_2 and P_{12} all move down by $D(L/L')$, where L is the distance from the wall to the backstop and L' is the distance from the electron gun to the wall.

(b) If the distance between the slits is doubled, then the distances between the pattern's maxima/minima are halved.

(c) If slit 1 is made twice as wide as slit 2, then more electrons get through slit 1 than slit 2, so P_1 is is twice as high as P_2, which makes P_12 asymmetric, larger to the left, and shifts the pattern slightly to the left.

70.3 Classically, the transmitted field is reduced by a factor $\cos\theta$ relative to the incident field, and since intensity is proportional to the square of the field, the transmitted intensity is reduced by $\cos^2\theta$. Quantum mechanically, the amplitude for a photon to be transmitted (rather than absorbed) by the polaroid is reduced by a factor $\cos\theta$, so (assuming photons are always transmitted when the axis of the polaroid is vertical) the probability for a photon to be transmitted becomes $\cos^2\theta$, while the probability of it being absorbed becomes $\sin^2\theta$.

70.4 The innermost ring's diameter will be approximately 0.36 cm, and the next one will be about 0.72 cm.

70.5 (a) With incident electrons having momentum $\hbar k$, $a_1 = |a_1|e^{i\phi_1}$ and $a_2 = |a_2|e^{i\phi_2}$,

$$I(x) = \frac{1}{L^2}\Big(|a_1|^2 + |a_2|^2 + 2|a_1||a_2|\cos\left[\frac{kdx}{L} + (\phi_1 - \phi_2)\right]\Big).$$

(b) The amplitude for an electron to get to x is $\phi = \langle x\,|\,1\rangle a_1 + \langle x\,|\,2\rangle a_2$. As discussed in Vol. III, Chapter 7, states with definite energy E have time-dependence $e^{-iEt/\hbar}$. Since the amplitudes $\langle x\,|\,1\rangle$ and $\langle x\,|\,2\rangle$ have no time-dependence, that of a_1 and a_2 determine E. $E^2 = m^2c^4 + p^2c^2$ determines the momentum p, and that determines the wavelength $\lambda = h/p$.

70.6 (a) $a = \lambda L/d = hL/p_0 d$

(c) The total energy of the particles must be conserved, so $p^2(x)/2m + V(x) = p^2(0)/2m + V(0)$, or $p(x) = \sqrt{p^2(0) + 2m(V(0) - V(x))}$. When $V(x)$ varies slowly, $V(x) - V(0)$ will be small, so $p(x) \approx p(0) + (m/p(0))(V(0) - V(x))$; furthermore, $V(x) - V(0) \approx (\partial V/\partial x)x = -Fx$. Thus $p(x) \approx p(0) + (m/p(0))Fx$, which is the same as $p(x) \approx p(0) + Fx/v$, since the initial velocity of the particles is $v = p(0)/m$.

(d) (1) Approximating both paths to have length L, the difference in phase between the paths $\Delta\phi \approx \Delta kL = (\Delta p/\hbar)L$, where Δk is the difference in wave numbers and Δp is the corresponding difference in momenta. The average difference in height of the two paths is $d/2$, thus, using the given result for part (c), the average difference in momenta is $\langle\Delta p\rangle = F(d/2)/v$. Therefore $\Delta\phi \approx (dF/2v\hbar)L$.

(d) (2) Substituting the result from part (1) above into the given result for part (b), and using $p_0 = mv$, the displacement of the pattern is

$$S = +(dF/2v\hbar)L\,\frac{L}{d}\,\frac{\hbar}{mv} = \frac{1}{2}\frac{F}{m}\left(\frac{L}{v}\right)^2 = \frac{1}{2}\frac{F}{m}t^2.$$

70.7 Let the amplitude for the particle to get from S to x (without regard to spin) be $\phi = \langle x\,|\,1\rangle\langle 1\,|\,S\rangle + \langle x\,|\,2\rangle\langle 2\,|\,S\rangle$. Let the total probability for an electron to get through a slit (without regard to spin) be $\gamma^2 = \alpha^2 + \beta^2$. Denote the intensity of spin up and spin down electrons at x as P_+ and P_- respectively, so the total intensity of electrons at x is $P = P_+ + P_-$, then

(a) $P_+ = \alpha^2\phi^2$, $P_- = \beta^2\phi^2$, $P = \gamma^2\phi^2$.

(b) $P_+ = \beta^2\phi^2$, $P_- = \alpha^2\phi^2$, $P = \gamma^2\phi^2$.

(c) $P_+ = P_- = \frac{1}{2}\gamma^2\phi^2$, $P = \gamma^2\phi^2$.

70.8 For a probability to be reduced by a factor of 100 the corresponding probability amplitude must be reduced by a factor of 10, thus the amplitude for a particle to arrive at some point on the screen through the stopped down hole is reduced by a factor of 10. If the total amplitude to arrive through the unstopped hole is written $ae^{i\phi_1}$, the amplitude to arrive through the stopped down hole can be written $0.1ae^{i\phi_2}$. The maximum and minimum arrival probabilities occur when these amplitudes add with the same and with opposite phase, respectively. Thus the ratio of the maximum to minimum arrival probabilities is $|a + 0.1a|^2/|a - 0.1a|^2 = 1.21/0.81 \approx 1.49$.

70.9 (a) The coincidence counting rate p_{12} is proportional to the probability of a coincidence at the counters. There are four ways to have a coincidence: a photon from source A goes to counter a while a photon from source B goes to counter b, a photon from source A goes to counter b while a photon from source B goes to counter a, two photons from source A go to the counters, or two photons from source B go to the counters. The first two ways are indistinguishable so their amplitudes add, the last two ways are distinguishable, so their probabilities add. Thus, the probability of a coincidence in the counters is

$$P = |\langle a\,|\,A\rangle\langle b\,|\,B\rangle + \langle b\,|\,A\rangle\langle a\,|\,B\rangle|^2 + |\langle a\,|\,A\rangle\langle b\,|\,A\rangle|^2 + |\langle a\,|\,B\rangle\langle b\,|\,B\rangle|^2.$$

It is given that $\langle a\,|\,A\rangle = ce^{ikR_1}$ and $\langle b\,|\,A\rangle = ce^{ikR_2}$. By symmetry, $\langle b\,|\,B\rangle = \langle a\,|\,A\rangle$ and $\langle a\,|\,B\rangle = \langle b\,|\,A\rangle$. Substituting these amplitudes into the formula for P given above and rearranging yields

$$P = 2|c|^4(2 + \cos 2k(R_2 - R_1)).$$

(b) If R is known then D may be found by measuring p_{12} for various separations d. As a function of d, p_{12} varies cosinusoidally.

Chapter 71

71.1 (a) $E = 4.14 \times 10^{-9}\,\mathrm{eV}$

(b) $N = 1.51 \times 10^{27}$ quanta/cycle

71.2 (a) $\dfrac{\Delta E(\omega)}{\Delta\omega} = \dfrac{kT}{\pi^2c^3}\omega^2 + O(\omega^3)$ for small ω,

$\dfrac{\Delta E(\omega)}{\Delta\omega} \approx \dfrac{\hbar\omega^3}{\pi^2c^3}e^{-\hbar\omega/kT}$ for large ω.

(b) $\omega_{E_{\max}} = (3.69 \times 10^{11}\,\mathrm{s}^{-1}\,\mathrm{K}^{-1}) \cdot T$

(c) $\lambda_{E_{\max}} = (2.9 \times 10^7\,\text{Å}\,\mathrm{K})/T$

(d) $T = 5800\,\mathrm{K}$

71.3 $B = 2.12 \times 10^8$ gauss

71.4 (a) $N_1/N_0 = N_2/N_1 = n(\omega)/\,(n(\omega) + 1)$

(b) $n(\omega) = 1/\left(e^{\Delta E/kT} - 1\right)$

(c) (1) $n(\omega) \approx e^{-\Delta E/kT}$; (2) $n(\omega) \approx kT/\Delta E$

71.5 Proton, neutrons and electrons are all Fermi particles; an atom containing only these particles is a boson if the total number of particles is even, and a fermion if the total number of particles is odd. The N^{14} atom has 7 orbital electrons around a nucleus of charge +7. If the nucleus contained only protons and electrons then it would have to contain 14 protons and 7 electrons (to have the right mass number and charge). Including orbital electrons, the *atom* would have 28 fermions in all, so it would be a boson. On the other hand, if the nucleus has only protons and neutrons, then it has 7 of each, so that the *atom* has 21 fermions in all, making it a Fermi particle. Note that the N^{14} *nucleus* is the other way around: Fermi under the first hypothesis and Bose under the second.

71.6 (a) If an atom is excited it has a certain transition rate (probability per unit time) of emission of a photon of any given kind (frequency, direction, polarization). If there are n photons of one kind already present, the rate of emission of that kind of photon is increased by the factor $n + 1$. This explains the "avalanche" effect. (The atoms are not really "trained" to emit photons all of the same kind; it is the surrounding sea of photons of this kind that magnifies the rate.)

(b) Neutrinos are fermions, of which no two can be in the same state (i.e., if there is one neutrino of a certain kind, then the probability that another of this kind will be emitted is reduced to zero). Therefore a neutrino "laser" such as described could not exist.

71.7 (a) $P_{a\to b} = |\langle b\,|\,a\rangle|^2$ and $P_{c\to d} = |\langle d\,|\,c\rangle|^2$. When the particles are not identical the amplitude of the joint events ($a \to b$ and $c \to d$) is the product of their amplitudes, $\langle bd\,|\,ac\rangle = \langle b\,|\,a\rangle\langle d\,|\,c\rangle$. Thus $P_{ac\to bd} = |\langle b\,|\,a\rangle\langle d\,|\,c\rangle|^2 = |\langle b\,|\,a\rangle|^2|\langle d\,|\,c\rangle|^2 = P_{a\to b}P_{c\to d}$.

(b) When the particles are identical the joint events ($a \to b$ and $c \to d$) can not be distinguished from ($a \to d$ and $c \to b$), so their respective amplitudes add (possibly with opposing phase), thus $P_{ac\to bd} = |\langle b\,|\,a\rangle\langle d\,|\,c\rangle \pm \langle d\,|\,a\rangle\langle b\,|\,c\rangle|^2 = P_{a\to b}P_{c\to d} + P_{a\to d}P_{c\to b} \pm 2\,\mathrm{Re}\,[\langle b\,|\,a\rangle^*\langle d\,|\,c\rangle^*\langle d\,|\,a\rangle\langle b\,|\,c\rangle]$.

71.8 $P(\theta) = |f(\theta)|^2 + \frac{2}{3}\,\mathrm{Re}\,[f(\theta)^*f(\pi - \theta)] + |f(\pi - \theta)|^2$

71.9 (a) $P(\theta) = 4P_1(\theta) + 4P_2(\theta)$

(b) $P(\theta) = 4P_1(\theta) + 4P_2(\theta) + 8\,\mathrm{Re}\,[f_1^*(\theta)f_2(\theta)]$

(c) The answer depends on the relative phase of f_1 and f_2; if f_1 and f_2 have the same or similar phase (which is probably the case) then case (b) gives more scattering.

71.10 (a) 0

(b) $2|f - g|^2$

(c) $|f - g|^2$

(d) $|f - g|^2$

(e) $2|f|^2$

71.11 (a) $\alpha \approx \theta/2$

(b) $g(\theta) - f'(\pi - \theta)$

(c) $P(\theta) = \dfrac{1}{2}|f(\theta) - f(\pi - \theta)|^2 + \frac{1}{2}|f'(\theta) - g(\pi - \theta)|^2 + \frac{1}{2}|g(\theta) - f'(\pi - \theta)|^2$

(e) $A = 3/4,\ B = 1/4$

71.12 (b) $U = \dfrac{3(3\pi^2)^{2/3}\hbar^2}{10m_e}\,\dfrac{N^{5/3}}{V^{2/3}}$

(c) $P = \dfrac{(3\pi^2)^{2/3}\hbar^2}{5m_e}\left(\dfrac{N}{V}\right)^{5/3}$

(d) $\gamma = 5/3$

71.13 (a) $P = A\rho^{5/3}$ follows from the solution to Ex. 71.12. $dM(r)/dr = 4\pi\rho r^2$ just gives the amount of mass dM in a spherical shell of radius r and thickness dr. $dP/dr = -G\rho M(r)/r^2$ follows, assuming spherical symmetry, from the star's hydrostatic equilibrium under gravity, which requires that $-\boldsymbol{\nabla}P - \rho\boldsymbol{\nabla}\phi = 0$ [See Vol. II, Eq. (40.1)], where $\phi = -GM(r)/r$ is the gravitational potential per unit mass.

(b) $A = \dfrac{(3\pi^2)^{2/3}\hbar^2}{5m_e}(2M_p)^{-5/3}$

Chapter 72

72.1 If $C = A \cdot B$ then we can insert a wide-open T apparatus between A and B without any effect, $C = A \cdot T \cdot B$. The amplitude for the beam in state ϕ to pass through A emerging in the state kT and to come out of B in state χ is $\langle\chi\,|\,B\,|\,k\rangle\langle k\,|\,A\,|\,\phi\rangle$. The processes corresponding to the three states of T are indistinguishable and therefore the amplitude $\langle\chi\,|\,C\,|\,\phi\rangle$ for the beam in state ϕ to go through C and emerge in state χ is the sum of their amplitudes:

$$\langle\chi\,|\,C\,|\,\phi\rangle = \sum_k \langle\chi\,|\,B\,|\,k\rangle\langle k\,|\,A\,|\,\phi\rangle.$$

72.2 (a) No.
(b) Yes.
(c) Yes.

72.3 (a) $N_2 = \frac{3}{4}N_1$
(b) $N_3 = \frac{1}{16}N_1$
(c) $N_2 = N_1$ and $N_3 = 0$

72.4 (a) $N_{+S'} = N_{-S'} = 0$ and $N_{0S'} = N_0$
(b) $N_{+T} = N_{-T} = \frac{1}{2}N_0$, $N_{+S'} = N_{-S'} = \frac{1}{4}N_0$ and $N_{0S'} = \frac{1}{2}N_0$
(c) No change.
(d) $N_{+S'} = N_{-S'} = \frac{1}{8}N_0$ and $N_{0S'} = \frac{3}{4}N_0$
(e) $N_{+S'} = N_{-S'} = \frac{1}{6}N_0$ and $N_{0S'} = \frac{1}{3}N_0$

Chapter 73

73.1 (a) $N/2$
(b) $N/4$
(c) $N/4$
(d) $N/4$
(e) $(N/4)(1 + 1/\sqrt{2})$
(f) $N/4$
(g) $(N/4)(1 + 1/\sqrt{2})$
(h) $N/2$

73.2 (a) Let X and Y be the amplitudes for arriving at the given point with spin up and spin down respectively. Then the total amplitude to arrive at the given point is $aX + bY$, so the probability is $|aX + bY|^2$.
(b) $P = |X|^2$ for up in z, $P = |Y|^2$ for down in z
(c) $P = \frac{1}{2}|X + Y|^2$ for up in x, $P = \frac{1}{2}|X - Y|^2$ for down in x
(d) $P = |(\cos\theta/2)X - ie^{i\phi}(\sin\theta/2)Y|^2$
(e) $\langle P\rangle = \frac{1}{2}(|X|^2 + |Y|^2)$

73.3 (a) $N = N_0|\langle +U\,|\,-S\rangle|^2$
(b) $\langle +T\,|\,-S\rangle = 1$, $\langle -T\,|\,-S\rangle = 0$
(c) $\langle +U\,|\,-S\rangle = i\sin\theta/2$
(d) (1) $i\sin 0/2 = 0$; (2) $i\sin\pi/2 = i$
(e) The difference between amplitudes 1 and i is an irrelevant phase factor; all probabilities remain the same.

73.4 Calcite splits light into two beams linearly polarized in perpendicular directions, transmitting each with about the same intensity. When linearly polarized light of intensity I_0 passes through a polarizer with its axis turned at angle θ to the light's polarization, the transmitted intensity is $I = I_0\cos^2\theta$ (Malus' law). From this we can guess immediately that $\langle iT\,|\,jS\rangle = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

The same result can be derived by considering the electric field $\boldsymbol{E}$ incident on the first calcite, which is split by it into perpendicular linearly polarized fields E_x and E_y, each of which passes through a second calcite, having corresponding axes x' and y' rotated by angle θ, thus producing fields $E_{x'} = E_x\cos\theta + E_y\sin\theta$ and $E_{y'} = -E_x\sin\theta + E_y\cos\theta$. To check: For $\theta = 0$ we have $\langle iT\,|\,jS\rangle = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ and for $\theta = \pi/2$ we have $\langle iT\,|\,jS\rangle = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$, etc. from which we can see that a beam of pure xS will be passed in the x-channel of T, but blocked in the y-channel if $\theta = 0$ and vice versa if $\theta = \pi/2$, etc. We can derive Malus' law from considering probabilities, such as, for example, $|\langle xT\,|\,xS\rangle| = \cos^2\theta$.

73.5 Let $x \to \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $y \to \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Then
(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}$
(c) Same as (a).
(d) Same as (b).
(e) $\begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix}$
(f) $e^{i\theta}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
(g) $\frac{1}{2}\begin{pmatrix} 1+i & -1+i \\ -1+i & 1+i \end{pmatrix}$
(h) Same as (e) with $\theta = \pi/2$: $\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$
(i) $\begin{pmatrix} e^{i\omega t n_x/c} & 0 \\ 0 & e^{i\omega t n_y/c} \end{pmatrix}$,
where ω is the frequency of the incident light, n_x and n_y are indexes of refraction along the x and y axes, and t is the thickness of the material.
(j) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$
(k) $\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$

73.6 (a) $P_{+z} = (1 - v/c)\sin^2\theta/2$, $P_{-z} = (1 + v/c)\cos^2\theta/2$
(b) $P_{\pm x} = \dfrac{1}{2}\left(1 + \dfrac{v}{c}\cos\theta \pm \sqrt{1 - v^2/c^2}\sin\theta\right)$
(c) $P_{\pm y} = \dfrac{1}{2}\left(1 + \dfrac{v}{c}\cos\theta\right)$
(d) If the neutrino is not observed $P_{\pm z} = \frac{1}{2}(1 \mp v/c)$.

Chapter 74

74.1 (a) You should find that

$$\Delta\theta \approx \frac{m\mu_z}{p_0^2}\,\frac{\partial B_z}{\partial z}\,L,$$

where m is the mass of the particle, and μ_z is quantized and may have the values $+\mu$, 0, $-\mu$.

Chapter 75

75.1 (a) $B_0 = \pi\hbar/2\mu T$
(b) $P = 1/2$

75.2 $P_{+x}(t) = \dfrac{1}{2}\left(1 + \sin^2\dfrac{\mu B}{\hbar}\,t\right)$

$P_{+y}(t) = \dfrac{1}{2}\left(1 + \dfrac{1}{\sqrt{2}}\sin\dfrac{2\mu B}{\hbar}\,t\right)$

75.3 (a) $P_{+z}(t) = \cos^2(At/\hbar)$
(b)
$$C_I = \frac{1}{\sqrt{2}}(C_{+z} + C_{-z}) = \frac{1}{\sqrt{2}}e^{+iAt/\hbar}, \qquad E = -A$$
$$C_{II} = \frac{1}{\sqrt{2}}(C_{+z} - C_{-z}) = \frac{1}{\sqrt{2}}e^{-iAt/\hbar}, \qquad E = +A$$
(c) $\alpha = 2At/\hbar$, $\beta = \pi$, $\gamma =$ any (Euler angles)

Chapter 76

76.1 Where $g(\omega_0)$ is the intensity of radiation at the resonant frequency ω_0, and μ is the electric dipole moment of the ammonia molecule,

(a) $P(I \to II \text{ by stimulated emission}) = (\pi\mu^2/\epsilon_0\hbar^2 c)g(\omega_0)$

(b) $P(II \to I \text{ by absorption})$
$= P(I \to II \text{ by stimulated emission})$

(c) $B_{I,II} = B_{II,I} = P(II \to I)/g(\omega_0)$ (and see below)

(d) $A_{I,II} = (\hbar\omega_0^3/\pi^2c^2)B_{I,II}$

Chapter 77

77.1 (a) $\omega = (2\mu B/\hbar)\cos\theta$

(b) $P_{-z}(t) = \sin^2[(\mu B/\hbar)\sin\theta\, t]$

77.2 With $\omega_0 = 2\mu B_0/\hbar$, and $\omega_n = 2\mu B_n/\hbar$,

$$P_{\parallel}(t) = \frac{\omega_n^2}{(\omega-\omega_0)^2+\omega_n^2}\sin^2\left(\frac{t}{2}\sqrt{(\omega-\omega_0)^2+\omega_n^2}\right).$$

Chapter 78

78.1 (a) *Hint:* Use Table 11-2 in Vol. III.

(b) $\sigma_x\sigma_y\sigma_z = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$

78.2 (a) Either $E = E_{\text{O}}$, or $E = (1/2)(E_{\text{O}} + E_{\text{C}} \pm \sqrt{(E_{\text{O}} - E_{\text{C}})^2 + 8A^2})$, where $-iA/\hbar$ is the amplitude per unit time for the electron to jump between O and C atoms.

(b) Either $E = E_{\text{O}}$ and there is zero amplitude for the electron to be on the C atom, or $E = E_{\text{O}} \pm A\sqrt{2}$ and the probability of the electron being on the C atom equals the total probability that the electron is on *either* O atom.

78.3 $$\begin{vmatrix} A_0 - A_1 - E & 0 & 0 & 0 \\ 0 & A_0 - A_1 - E & 0 & 0 \\ 0 & 0 & A_0 - A_1 - E & 0 \\ A_1 & 2A_1 & 3A_1 & A_0 + 3A_1 - E \end{vmatrix} = 0,$$

where $-iA_1/\hbar$ and $-iA_0/\hbar$ are, respectively, the amplitudes per unit time for the "hole" to jump and not to jump. $E_I = A_0 - A_1$, $E_{II} = A_0 + 3A_1$, $\Delta E = |E_I - E_{II}| = 4A_1$.

78.4 (b) $E_I = E_0 + 2A$, where E_0 is the energy of the extra electron at any of the atoms.

(c) There are 6 stationary states.

(d) $\delta = 0°, 60°, 120°, 180°, 240°, 300°$

(e) Energy levels: $E_0 \pm A$, $E_0 \pm A$, $E_0 \pm 2A$; spacings between levels (transition energies): A, $2A$, $3A$, $4A$.

78.5 (a) Energy levels: $E_0 + A$, $E_0 + A$, $E_0 - 2A$, where E_0 is the energy of the extra electron at any of the atoms; spacings between levels: $3A$.

(b) Energy levels: $E_0 + (1+2\epsilon/3)A$, $E_0 + A$, $E_0 - (2-\epsilon/3)A$; spacings between levels: $(2\epsilon/3)A$, $(3-\epsilon/3)A$, $(3+\epsilon/3)A$

Chapter 79

79.1 (a) $f_{I,II} = 28\,\text{Hz}$, $\lambda_{I,II} = 10^9\,\text{cm}$; $f_{I,III} = 14\,\text{Hz}$, $\lambda_{I,III} = 2\times10^9\,\text{cm}$

(b) $f_{I,II} = 1.4\,\text{MHz}$, $\lambda_{I,II} = 2\times10^4\,\text{cm}$; $f_{III,II} = 0.7\,\text{MHz}$, $\lambda_{III,II} = 4\times10^4\,\text{cm}$

(c) $f_{I,II} = 2.8\times10^5\,\text{MHz}$, $\lambda_{I,II} = 10^{-1} cm$; $f_{I,III} = 1130\,\text{MHz}$, $\lambda_{I,III} = 27\,\text{cm}$

Chapter 80

80.1 (a) Let the spatial dependence of the amplitudes for configurations i and j be e^{ik_ix} and e^{ik_jx}, respectively. Then the allowed energies of the system are:

$$E = \frac{1}{2}[E(i) + E(j)] \pm \frac{1}{2}\sqrt{[E(i) - E(j)]^2 + 16A^2\cos k_ib\cos k_jb}$$

with

$$E(i) = E_i - 2B\cos k_ib \quad\text{and}\quad E(j) = E_j - 2B\cos k_jb.$$

Note: if $A = 0$, so that the i and j configurations are decoupled, this just gives the two solutions $E = E(i)$ and $E = E(j)$.

(b) The general case where $k_i \neq k_j$ is hard to analyze, so consider the case of localized particles (wave-packets) for which a characteristic wave number $k = k_i = k_j$ can be associated with a velocity (see Section 13-3, Vol. III). Then $E = E_0 - 2B\cos kb \pm \frac{1}{2}\sqrt{|E_i - E_j|^2 + 16A^2\cos^2 kb}$. When $A = 0$, we have two cosine curves of amplitude $2B$ separated by $|E_i - E_j|$. When $A \neq 0$ the curves are repelled from each other (except where $kb = \pi/2$). If $|E_i - E_j| \ll 2B$ then the two bands overlap; if $|E_i - E_j| \gg 2B$ they they don't overlap.

80.2 (a) $E = E_0 \pm \sqrt{(\Delta E)^2 + 4A^2\cos^2 kb}$

(b) $-\pi/2b < k \leq \pi/2b$

80.3 (a) $\beta = -\dfrac{G}{G - iB^2\sin kb}$, with $G = \left(A^2 - B^2\right)\cos kb$

$\gamma = 1 + \beta$

(b) β is of the form $\beta = -d/(d - ie)$, where d and e are real. Thus, $|\beta|^2 = d^2/(d^2+e^2)$ and $|\gamma|^2 = |1+\beta|^2 = |(d - ie - d)/(d - ie)|^2 = e^2/(d^2+e^2)$. Hence, $|\beta|^2 + |\gamma|^2 = 1$.

80.4 (a) $1 = |\beta|^2 + |1+\beta|^2 = 1 + 2|\beta|^2 + 2\,\text{Re}\,[\beta]$, so $|\beta|^2 + \text{Re}\,[\beta] = 0$. Hence,

$$\text{Re}\left[\frac{\beta}{1+\beta}\right] = \text{Re}\left[\frac{\beta(1+\beta^*)}{|1+\beta|^2}\right] = \frac{\text{Re}\,[\beta] + |\beta|^2}{|1+\beta|^2} = 0.$$

(b) Let $\beta = ce^{i\delta}$ with c and δ real. $|\beta|^2 + \text{Re}\,[\beta] = 0$ implies $c^2 + c\cos\delta = 0$, so $c = -\cos\delta$. Let $\eta = \delta - \pi/2$. Then $c = -\cos(\eta + \pi/2) = \sin\eta$ and $e^{i\delta} = e^{i(\eta+\pi/2)} = ie^{i\eta}$. Hence, $\beta = i\sin\eta e^{i\eta}$.

80.5 (b) $\beta = -\dfrac{P - e^{ikb}}{P - e^{-ikb}}$, with $P = \dfrac{B^2}{AA'}e^{ik'b'}$

(c) If $k'b'$ is imaginary then P is real and β is of the form $-(r+is)/(r-is)$ with r and s real. Hence $|\beta| = 1$. The amplitudes a_n go like $e^{-|k'b'|n}$, which means there is no net transmission of particles into region II. The corresponding energy is $E = E_0' - 2A'\cosh|k'b'|$, so $E < E_0' - 2A'$; i.e., the energy is below the "transmission band" in region II.

Chapter 81

81.1 (b) The average power is

$$\langle P\rangle = \frac{1}{2}\text{Re}\left[qE_x^*v_x\right] = \frac{1}{2}q\,\text{Re}\left[E_x^*E_x\frac{v_x}{E_x}\right] = \frac{1}{2}qE_0^2\,\text{Re}\left[\frac{v_x}{E_x}\right].$$

81.2 (a) The solution is outlined in Section 14-5 of Vol. III: $I_g = CN_p(\text{n side})$ and $I_r = CN_p(\text{p side})e^{-qV/kT}$ for some constant of proportionality C. When a reverse-voltage bias V_e is applied the recombination current becomes $I_r' = CN_p(\text{p side})e^{-q(V-V_e)/kT} = I_g e^{qV_e/kT}$. The thermal generation current is not altered. Therefore the net hole current (to the right, with respect to the figure) is $I_g(e^{qV_e/kT} - 1)$.

(b) $I_{\text{total}} = I_0(e^{qV_e/kt} - 1)$, where I_0 is the maximum magnitude of the current through the junction under a reverse voltage bias ($V \ll 0$, with no breakdown).

Chapter 82

82.1 $\lambda \approx 10^{-6}\,\text{m}$

82.2 $\Delta E \approx 3.1\,\text{eV/molecule}$

82.3 (a) If the probability for finding no down spins in mode K is C, the probability of finding n down spins is $Ce^{-nE_K/kT}$. The mean number of atoms with spin down in mode K is then

$$\bar{n}_K = \frac{\sum_{n=0}^{\infty} nCe^{-nE_K/kT}}{\sum_{n=0}^{\infty} Ce^{-nE_K/kT}} = \frac{Ce^{E_k/kT}/(e^{E_k/kT}-1)^2}{Ce^{E_k/kT}/(e^{E_k/kT}-1)}$$
$$= \frac{1}{e^{E_K/kT}-1}.$$

(b) If the only states are those in which all the down spins belong to the same mode, the mean total number of down spins at temperature T is $\sum_K \bar{n}_K$, which equals $\int \bar{n}_K\, d^3K/(2\pi)^3$ per unit volume, $d^3K/(2\pi)^3$ being the number of modes per unit volume with K in the range of d^3K.

(d)
$$\frac{4}{\sqrt{\pi}} \int_0^{\infty} \frac{x^2\,dx}{e^{x^2}-1} = \frac{4}{\sqrt{\pi}} \sum_{n=1}^{\infty} \int_0^{\infty} x^2 e^{-nx^2}\,dx$$
$$= \frac{4}{\sqrt{\pi}} \sum_{n=1}^{\infty} \frac{\sqrt{\pi}}{4} \frac{1}{n^{3/2}}$$
$$= \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} = \zeta(3/2) \approx 2.61$$

Chapter 83

83.1 (a) For $\psi(x,t) = u_0(x)e^{-iE_0t/\hbar}$, the Schrödinger equation gives $d^2u_0/dx^2 = (2m/\hbar^2)[V(x) - E_0]u_0(x)$. In the regions where $V(x) = V_0$ (with $V_0 \to \infty$), $|u_0(x)|$ would become infinite as $|x|$ approaches ∞ and would not describe a bound particle.

(b)
$$u_n(x) = A\sin\left(\frac{\sqrt{2mE_n}}{\hbar}\,x\right)$$
$$E_n = \frac{\pi^2\hbar^2}{2ma^2}(n+1)^2$$
with constant A and $n = 0, 1, 2, \ldots$

(c) $E_0 = \pi^2\hbar^2/2ma^2$, and

$$u_0(x) = \begin{cases} A\sin(\pi x/a) & \text{for } 0 \le x \le a \\ 0 & \text{for } x < 0 \text{ and } x > a \end{cases}$$

(d) $E_1 - E_0 = 3\pi^2\hbar^2/2ma^2$

83.2 (a) $V_0 \approx 35\pi^2\hbar^2/2ma^2$

(b) The wave function $u(x)$ of the first excited state for this well should have one node, and be antisymmetric about $x = 0$. We can satisfy Schrödinger's equation and the boundary conditions with the same energy $E = (0.9)\pi^2\hbar^2/2ma^2$ as in part (a) above, if

$$u(x) = \begin{cases} u_a(x) & \text{for } x \ge 0 \\ -u_a(-x) & \text{for } x < 0 \end{cases}$$

where $u_a(x)$ is the wave function applicable in part (a) above when $x > 0$.

83.3 (b) $E_0 \approx 0.27V_0$, and $E_1 \approx 0.90V_0$

(c) If $V_0a^2 < \pi^2\hbar^2/8m$ there is only one bound state.

83.4 (a) Applying Schrödinger's equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$$

to $\psi(x) = Ke^{-[a(t)x^2 + c(t)]}$ and simplifying,

$$-i\hbar\left(\frac{da}{dt}x^2 + \frac{dc}{dt}\right) = -\frac{\hbar^2}{2m}\left(-2a + 4a^2x^2\right).$$

This must hold for all x, so the coefficients of the powers of x can be equated. In particular for x^2,

$$-i\hbar\frac{da}{dt} = -\frac{\hbar^2}{2m}4a^2,$$

with solution

$$\frac{1}{a(t)} = \frac{1}{a_0} + \frac{2i\hbar}{m}t.$$

(b) $c(t) = c_0 + \frac{1}{2}\ln[a_0/a(t)]$ where $c_0 = c(0)$.

(c) It will spread to a region approximately 2300 km wide.

(d) $|\phi(p)|^2 = K^2\,\dfrac{\pi}{a_0}\,e^{-2c_0 - p^2/2\hbar^2a_0}$

(e) $\Delta p = \sqrt{a_0}\hbar$. (It does not vary with time.)

(f) Substituting $c(t)$ and $a(t)$ from parts (a) and (b) above into $\psi(x)$, squaring and simplifying,

$$|\psi(x)|^2 = K^2|a(t)/a_0|e^{-2c_0 - x^2/(1/2a_0 + 2a_0\hbar^2t^2/m^2)}$$

The width of the spatial wave function is thus $\Delta x = \sqrt{1/4a_0 + a_0\hbar^2t^2/m^2}$. For large t, $\Delta x \approx (\hbar\sqrt{a_0}/m)t = (\Delta p/m)t = \Delta vt$.

Chapter 84

84.1 $A(\theta) \propto \sin\theta$

84.2 (a) $a = c = d = e = f = g = 0$

(b) $f^+(\theta) = |b|^2\sin^2(\theta/2)$

(c) $f(\theta) = |b|^2\sin^2(\theta/2) + |h|^2\cos^2(\theta/2)$

(d) Parity conservation implies $b = \pm h$, so $f(\theta) = |b|^2$.

84.3 (a) The initial components of angular momentum along the z-axis are $m = 1$ for the photons and $m = \pm 1/2$ for the initial protons, thus the only angular momentum states available to p^* are $m = 1 + 1/2 = 3/2$ and $m = 1 - 1/2 = 1/2$. Along the direction of emission the total angular momentum of the final state is just that of the final proton, so $m' = \pm 1/2$.

(b) $c = d$ because strong interactions conserve parity. The angular distribution of pions (and protons) is

$$f(\theta) = \frac{1}{2}|c|^2\left(\frac{3}{4}(|b|^2 - |a|^2)\sin^2\theta + |a|^2\right).$$

84.5 (a) The angular distribution of photons when polarization is not detected is proportional to $(1+\cos^2\theta)$, regardless of whether the parity of the initial state is even or odd. Hence, the parity of the first excited state cannot be determined.

(b) The angular distribution of x'-polarized photons is isotropic (does not depend on θ) if the initial state has even parity, but it is proportional to $\cos\theta$ if the initial state has odd parity. Hence, in this case, the parity of the first excited state is easily determined.

Chapter 85

85.1 The emission spectrum has two lines of equal intensity at wavelengths 1210 Å and 6560 Å.

85.2 (a) The energy of the ground state is the Rydberg energy $E = -me^4/2\hbar^2 = -e^2/2r_B$. Classically the maximum possible distance R of the electron is that at which the kinetic energy is 0, i.e., the potential energy equals the total energy, $-e^2/R = -e^2/2r_B$, so $R = 2r_B$.

(b) $P(r > R) = 0.238$

85.3 (a) $E_0 = 18.9\,\text{MeV}$

(b) $\lambda_{1\to 0} = 8.7 \times 10^{-4}$ Å

(c) $r'_B = 3.1 \times 10^{-5}$ Å

(d) $P(r \le R) = 1 - (1 + 2s + 2s^2)e^{-2s}$, with $s = R/r'_B$.

(e) $P(e \text{ in nucleus}) \approx 1.8 \times 10^{-6}$; $P(\mu \text{ in nucleus}) \approx 0.83$.

85.4 (a) The $n = 5 \to n = 2$ transition has an energy $E_{5\to 2} = 2.86\,\text{eV}$, which is within the given range. An energy of 13.06 eV is required to transition from the ground state into the $n = 5$ state.

(b) Shortest wavelength transition is the $n = \infty \to n = 2$ transition, with $\lambda = 3650$ Å. Longest wavelength from the $n = 3 \to n = 2$ transition, with $\lambda = 6560$ Å.

(c) The ground state energy of He is $E = -4E_R = -54.4\,\text{eV}$.

(d) The shortest wavelength transition into the $n = 4$ state is $n = \infty \to n = 4$, with $\lambda = 3650$ Å. Note that this is the same as the shortest wavelength of the Balmer series.

SCIENCE

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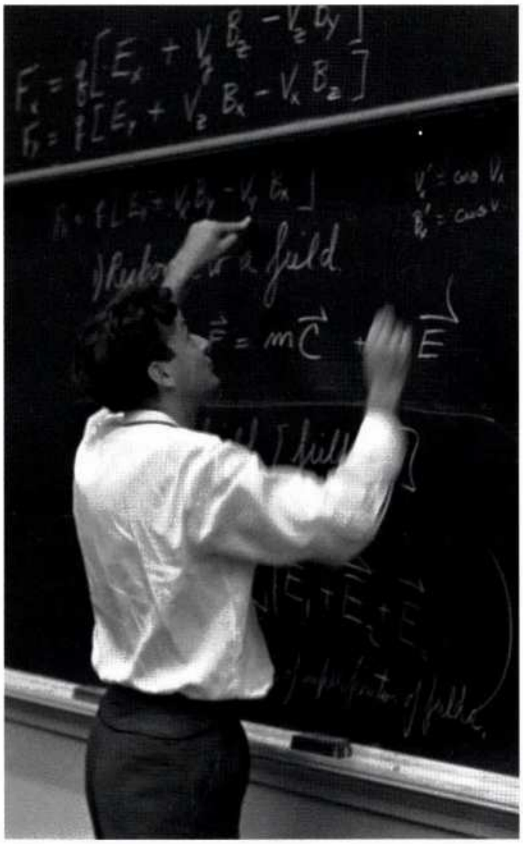

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