University of Ibadan

Department of Physics

PHY102 – 2016/17 Session (2017): Exercise 1.

1) $\int (u+at)dt = ?$ where u and a are constants

2) $\int_0^t [u+at']dt' = ?$ where u and a are constants

3) $\int \sin(at)dt = ?$ where a is a constant

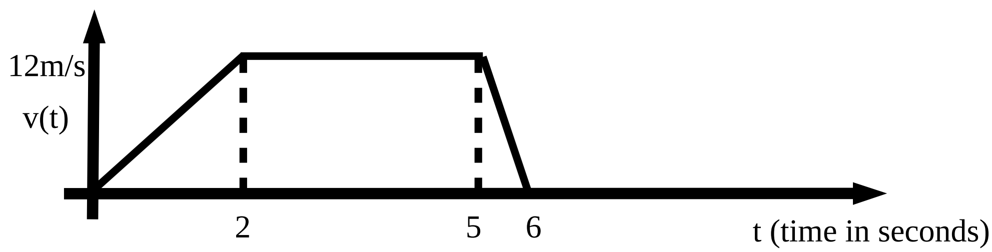
4) Consider a sphere of radius R whose density ρ is not constant but varies as the cube of the distance (r) from its centre: $\rho(r) = kr^3$, k is a constant. Determine the mass of the sphere in terms of k and R.

5) Is cos(x+y) = cos(x) + cos(y)? (Give a proof or counter-example)

6) The velocity-time graph of a car is as shown below with 12m/s being the maximum speed attained. (a) Tell a story describing the acceleration and speed of the car.

(b) Determine the total distance traveled by the car in the six (6) seconds shown.

(c) Determine the acceleration of the car in the last one second.



6) Einstein's relation is actually $E = \sqrt{m^2c^b + (pc)^n}$ where E is the energy, m is mass, c is the speed of light and p is momentum (momentum is mass times velocity). Determine the values of b and n.

7) In physics, each particle of momentum p has a wave corresponding to it. The wavelength λ of this wave is related to p by de Broglie's expression $p = h/\lambda$, where, h is called the Planck's constant. Determine

a) the dimension of Planck's constant

b) the SI units of Planck's constant.

8) (Farai): The volume of liquid flowing per unit time depends on the coefficient of viscosity η, radius r of the pipe and the pressure gradient $\frac{P}{l}$. Using their dimensions and noting that $\eta = \frac{Fl}{Av}$ where F is a force, A is an area and v is a velocity, determine the expression for volume flowing per unit time.

HW1 Solutions

(1) $\int (u+at)\,dt = \int (ut^0 + at^1)\,dt = \frac{ut^{0+1}}{0+1} + \frac{at^{1+1}}{1+1} + c = \underline{\underline{ut + \frac{1}{2}at^2 + C}}$

(2) $\int_0^t (u+at')\,dt' = \left[ut' + \frac{1}{2}at'^2 + c\right]_{t'=0}^{t'=t} = \left(ut + \frac{1}{2}at^2 + c\right) - \left(u0 + \frac{1}{2}a0^2 + c\right)$

$= \underline{\underline{ut + \frac{1}{2}at^2}}$

(3) Since $\frac{d(\cos at)}{dt} = -a\sin at$ then: $-\frac{1}{a}\frac{d(\cos at)}{dt} = \sin at$

$\Rightarrow \int \sin at\,dt = -\frac{1}{a}\cos at$ (i.e., what should you differentiate in order to get $\sin at$?)

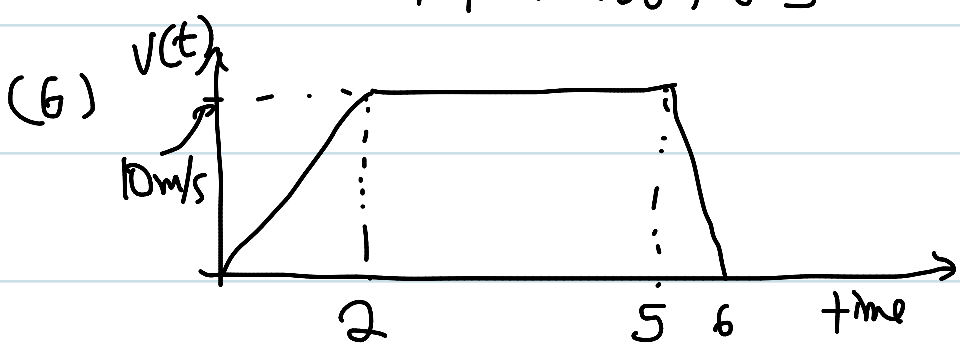
(4) See ~~last page for solution to Q4~~

(5) No: $\cos(x+y) \neq \cos(x) + \cos(y)$

Proof by counter example: $\underbrace{\cos \pi/2}_{=1} \neq \underbrace{\cos \pi/6}_{=\frac{\sqrt{3}}{2} = 0.866} + \underbrace{\cos \pi/3}_{=0.5}$ i.e. $\cos(90°) \neq \cos 30° + \cos 60°$

$1 \neq 0.866 + 0.5$

(6)



a) Starting from rest, the car accelerates uniformly for the first two seconds until it attains a speed of 10m/s. It then continues at this speed for the next three seconds. At this point it then decelerates uniformly to rest in the last one second.

b) Total distance travelled $= \int_0^6 v\,dt$ = Area under curve.

1st two seconds, Area $= \frac{1}{2}$ base × height $= \frac{1}{2} \times 2s \times 10m/s = 10m$

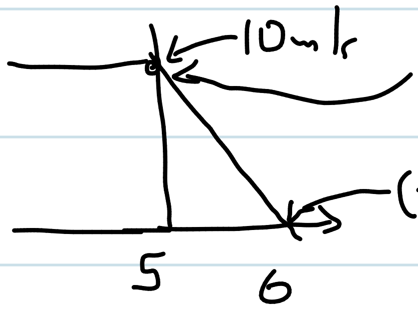
area of triangle

From 2s to 5s, Area = Length × Breadth $= 3s \times 10m/s = 30m$

Last 1s, Area $= \frac{1}{2}$ base × height $= \frac{1}{2} \times 1s \times 10m/s = 5m$

$\therefore$ total area $= 10m + 30m + 5m = \underline{\underline{45m}} =$ total distance traveled

6c) Acceleration in last one second:



$(x_1, y_1) = (5s, 10m/s)$

$(x_2, y_2) = (6s, 0m/s)$

$$\Rightarrow a = \frac{\Delta V}{\Delta t} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(0-10)m/s}{(6-5)s}$$

$$= \frac{-10m/s}{1s} = \underline{\underline{-10m/s^2}}$$

6) $E = \sqrt{m^2c^b + (pc)^n}$

$\Rightarrow E^2 = m^2c^b + (pc)^n \Rightarrow E^2, m^2c^b, (pc)^n$ have the same dimension. The dimension of energy is ML^2T^{-2} { from kinetic ENERGY $= \frac{1}{2}mv^2 \Rightarrow [E] = (M)(LT^{-1})^2 = ML^2T^{-2}$

or from Work $\Leftrightarrow$ ENERGY $\Rightarrow [E] = [\text{force} \times \text{dist}] = (MLT^{-2})L$
$= ML^2T^{-2}$ }

$\therefore [E^2] = (M^2L^2T^{-2})^2 = M^2L^4T^{-4}$

$[m^2c^b] = M^2(LT^{-1})^b = M^2L^bL^{-b}$

compare $\Rightarrow \boxed{b = 4}$

since LT^{-1} is the dimension of c (speed)

$[(pc)^n] = (\underbrace{MLT^{-1}}LT^{-1})^n = (ML^2T^{-2})^n = \underbrace{M^nL^{2n}T^{-2n}}$

p = momentum = mass × velocity
$\Rightarrow [p] = M(LT^{-1})$

compare with
$[E^2] = M^2L^4T^{-4}$

$\Rightarrow n = 2, 4 = 2n,$
$-2n = -4 \Rightarrow \boxed{n = 2}$

$\rightarrow \begin{pmatrix} b = 4 \\ n = 2 \end{pmatrix} \Rightarrow E = \sqrt{m^2c^4 + (pc)^2}$

7) a) $p = \frac{h}{\lambda} \Rightarrow p\lambda = h. \Rightarrow [p\lambda] = [h]$

momentum = mass x velocity (See Q6 above)

wavelength

$\Rightarrow (MLT^{-1})L = [h]$

$\Rightarrow [h] = ML^2T^{-\cancel{2}1}$

$= ML^2T^{-1}$

b) From $[h] = ML^2T^{-\cancel{2}2}$, the SI Unit of h is: $kg\, m^2 s^{-\cancel{2}1} = kg\, m^2 s^{-1}$

$= ML^2T^{-1}$

for M for L for T

⑧ $\frac{d\,\text{Volume}}{dt} = k\eta^x r^y \left(\frac{P}{L}\right)^z \Rightarrow \left[\frac{d\,\text{Volume}}{dt}\right] = \left[k\eta^x r^y \left(\frac{P}{L}\right)^z\right]$ ——— (1)

$[\eta] = \left[\frac{fl}{Av}\right] = \frac{(MLT^{-2})(L)}{(L^2)(LT^{-1})} = ML^{-1}T^{-1}$

velocity

$[r] = L$; $\left[\frac{P}{L}\right] = \left[\frac{\text{Force}/\text{Area}}{L}\right] = \frac{MLT^{-2}/L^2}{L} = ML^{-2}T^{-2}$

$\left[\frac{d\,\text{Volume}}{dt}\right] = \frac{L^3}{T} = L^3T^{-1}$; $[k] = 1$

k = dimensionless constant

Using these in Eq(1) gives:

$L^3T^{-1} = (ML^{-1}T^{-1})^x L^y (ML^{-2}T^{-2})^z = M^{x+z} L^{-x+y-2z} T^{-x-2z}$

Equating indices (powers) gives:

$x + z = 0 \longrightarrow \Rightarrow x = -z$

$-x + y - 2z = 3$ ——— (*)

$-x - 2z = -1 \Rightarrow +z - 2z = -1 \Rightarrow -z = -1 \Rightarrow z = 1 \Rightarrow x = -1$

since $x = -z$ since $x = -z$

Using $z = 1$ and $x = -1$ in Eq (*) above gives: $-(-1) + y - 2(1) = 3$

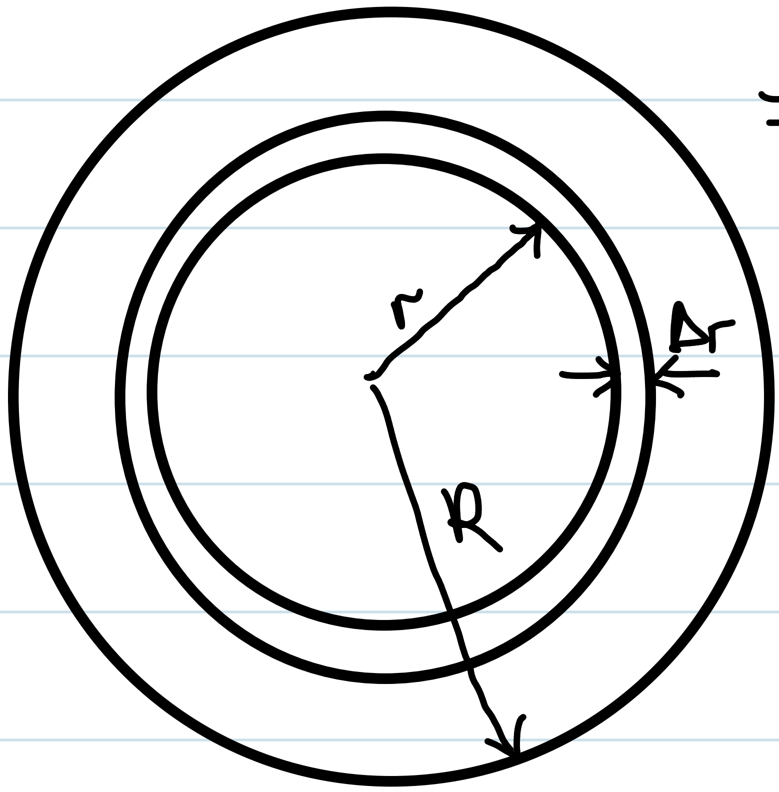
$\Rightarrow 1 + y - 2 = 3 \Rightarrow y - 1 = 3 \Rightarrow y = 4$

$\Rightarrow x = -1, \; y = 4, \; z = 1$

$$\Rightarrow \frac{dVolume}{dt} = k\,\eta^{x}\, r^{y} \left(\frac{P}{L}\right)^{z} = k\,\eta^{-1} r^{4} \left(\frac{P}{L}\right)^{1}$$

Q4

Mass = Volume X density.



Volume of small shell is ΔV

= surface area of shell $\times \Delta r$

$= \frac{4}{3}\pi r^2 \Delta r$

$\therefore \underbrace{\Delta M}_{\text{mass of small shell}} = (\Delta V) \times \rho$ ← density

$= \left(\frac{4}{3}\pi r^2 \Delta r\right)\rho$

$$\therefore \text{Total Mass, } M = \int_{r=0}^{r=R} \left(\frac{4}{3}\pi r^2 dr\right) \rho$$

$$= \frac{4}{3}\pi \int_0^R r^2 \, (kr^3)\, dr$$

$$= \frac{4}{3}\pi \int_0^R k r^5 \, dr = \frac{4\pi k}{3}\left[\frac{r^6}{6}\right]_0^R$$

$$= \frac{2k\pi}{9} R^6 = \text{Mass of Sphere!}$$

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Department of Physics

PHY102 First Semester 2016/17 Session (2017): HW 2. Due Date: N/A

Take the acceleration due to gravity (g) to be 9.8 m/s^2

References: (1) Halliday and Resnick (2) Farai (3) etc.

1) A van starts from rest and moves with a constant acceleration of 4m/s^2. Determine its speed and distance traveled in the first 12 s.

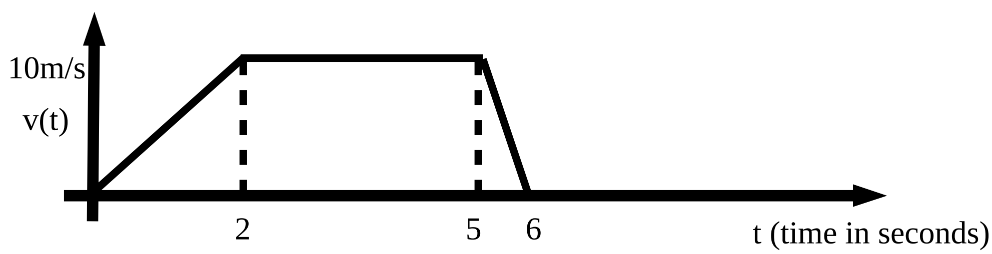
2) The velocity-time graph of a car is as shown below with 10m/s being the maximum speed attained. (a) Plot the position x(t) as a function of time t.

(b) Determine the average speed of the car over the six (6) seconds shown.

(c) Determine the total distance traveled in the six seconds (Hint: Area under graph)

(c) Plot the acceleration a(t) as a function of time t.

(d) Plot the position-time graph of the car



3) A ball dropped from a building hits the ground after 11s. Determine (a) its speed upon hitting the ground (b) the height from which it was dropped.

4) 5.2 seconds after seeing a lightning from a cloud, you hear its thunder. How far is the cloud from you assuming the speed of sound is 340m/s.

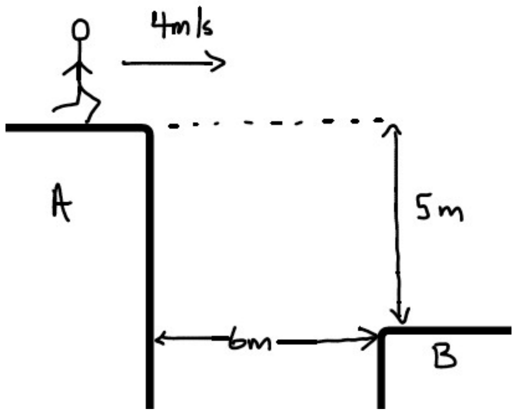
5) A stone is dropped by a bird which was 320m above the ground and rising vertically at 10m/s. Determine: (a) the maximum height reached by the stone (b)the position and velocity of the stone 4 s after it was dropped (c) the time taken for the stone to hit the ground.

6) In order to determine the height or depth of a cliff, a hiker (climber) drops a stone from the top of the cliff and measures the time between the moment the stone was dropped and the instant s/he hears the stone hit the ground. If this time is 3.3 seconds and the speed of sound in air is 340 m/s, determine the height of the cliff.

7) The acceleration due to gravity on the moon is about one-sixth that on the earth. A man can throw a stone 10 m straight up on earth. How high can he throw the stone on the moon assuming the (initial) speeds of throw are the same in both cases?

8) A canon ball is fired with a velocity of 150 m/s at an angle of 40° to the horizontal. (a) Determine the position of the bullet after 10 s. Was it moving up or down at this time? (b) When does the canon ball land and how far from the point of release?

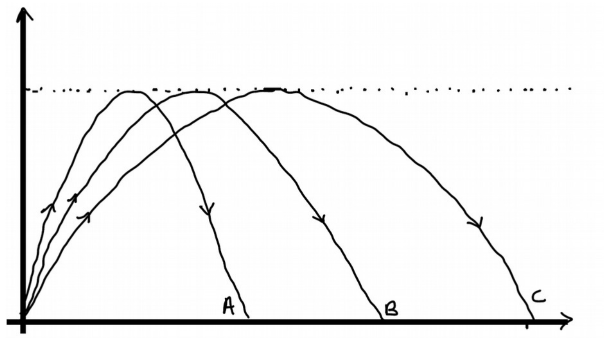
9) Consider the situation shown below. Building B is 6m from A, horizontally. If the stuntman jumps, will he land on the second building (B) or not? What should be his minimum horizontal speed in order to land on B?



10) A long-jump athlete jumped 8.5 m. Estimate her angle of takeoff. (Hint, you will need to assume her initial speed u. Assume she is a good 100 m sprinter with a record of about 10 s)

11) Assuming air resistance is zero, rank the projectile paths shown according to (a) total time of flight, (b) initial vertical velocity, (c) initial horizontal velocity (d) initial

speed. In each case, rank in ascending order



12) A fan with rotating blades of radius r = 0.2 m makes 2400 revolutions per minute. Determine, for the tip of a blade, (a) the distance traveled in one revolution (b) the speed (c) the acceleration (d) the period of motion.

13) Consider a train which goes at constant speed around a bend (which forms a part of a circle). If the maximum legally allowed acceleration experienced by passengers is 0.050g (i.e., 0.050 X 9.8m/s^2), determine (a) the smallest radius of curvature of the bend that is allowed if the speed of the train is 200 km/h (b) the maximum speed of the train if the radius of curvature of the bend is 0.5 km.

14) (a) Determine your centripetal acceleration due to the earth's spin, assuming you are at the equator. (b) At what latitude is the centripetal acceleration equal to 0.8cm/s^2? (c)Determine the period of rotation of the earth in order to have a centripetal acceleration of 10.0 m/s^2 at the equator.

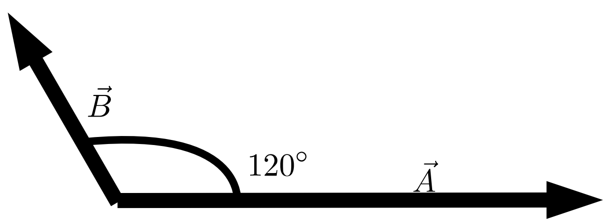
15) (Farai) The engine rotating a shaft is shut off when the angular speed of the shaft is 1,800 rpm. It stops rotating 15s later. Determine (a) the constant angular acceleration (deceleration) (b) the total angular displacement before coming to rest after the engine has been shut off, assuming constant angular deceleration.

16) A man walked 2 km north and then 1.5 km north-west. Using a geometrical approach, determine his displacement from the initial position.

17) A woman walked a distance 30 m and then a distance of 40m. Show how her displacement vectors may be added to give a net (resultant) displacement of magnitude (a) 10 m (b) 50 m (c) 70 m

18) Three vectors $\vec{A}, \vec{B}, \vec{C}$ each have the same magnitude (10 N) lie in the xy plane and make angles $45°, 90°,$ and $120°$, respectively, with the positive x direction.
* Using a geometrical approach determine (a) $\vec{A}+\vec{C}$ (b) $\vec{A}+\vec{B}-\vec{C}$
* Using an analytical approach determine (a) $\vec{A}+\vec{C}$ (b) $\vec{A}+\vec{B}-\vec{C}$ (c) $\vec{A}+\vec{B}+\vec{C}$

19) Determine (a) $\vec{A}\times\vec{B}$ (b) $\vec{A}\cdot\vec{B}$ and (c) $\vec{A}-2\vec{B}$ for the vectors $\vec{A}$ and $\vec{B}$ shown below, given that $|\vec{A}|=6N$ and $|\vec{B}|=3N$and the angle between the vectors is as indicated.



20) Determine the angle between two vector forces of equal magnitudes given that their resultant has a magnitude which is half the magnitude of each of the original forces.

21) Consider the velocities $\vec{v}_1 = 2\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{v}_2 = 4\hat{i}+3\hat{j}$. Determine the magnitude of $\vec{v}_3$ such that $\vec{v}_1+\vec{v}_2+\vec{v}_3 = 6\hat{i}+\hat{j}-4\hat{k}$.

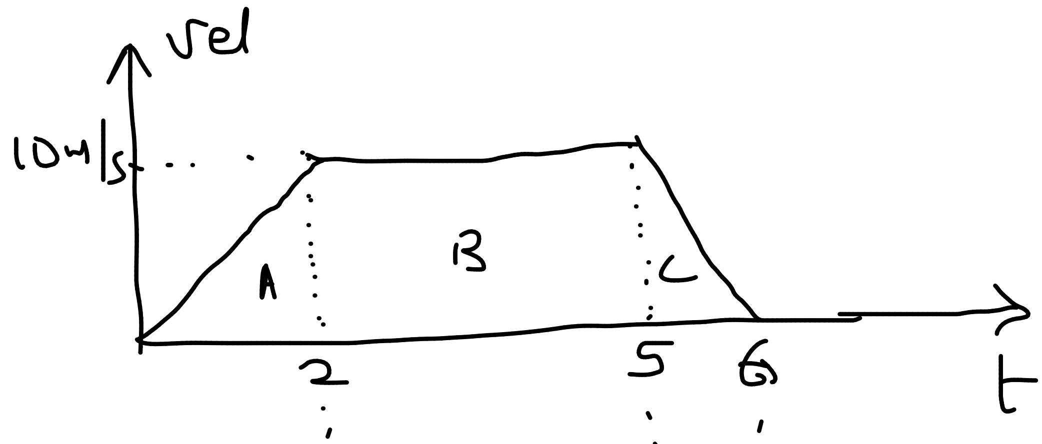
22) A constant force $\vec{F}=3\hat{i}+\hat{j}-2\hat{k}$ moves a point through a displacement $\vec{r}=\hat{i}+2\hat{j}$ Determine (a) the angle that each of the vectors makes with the positive y axis and (b) the work done by the force $\vec{F}$ in moving the point.

HW2 Solutions, Q1 - 15

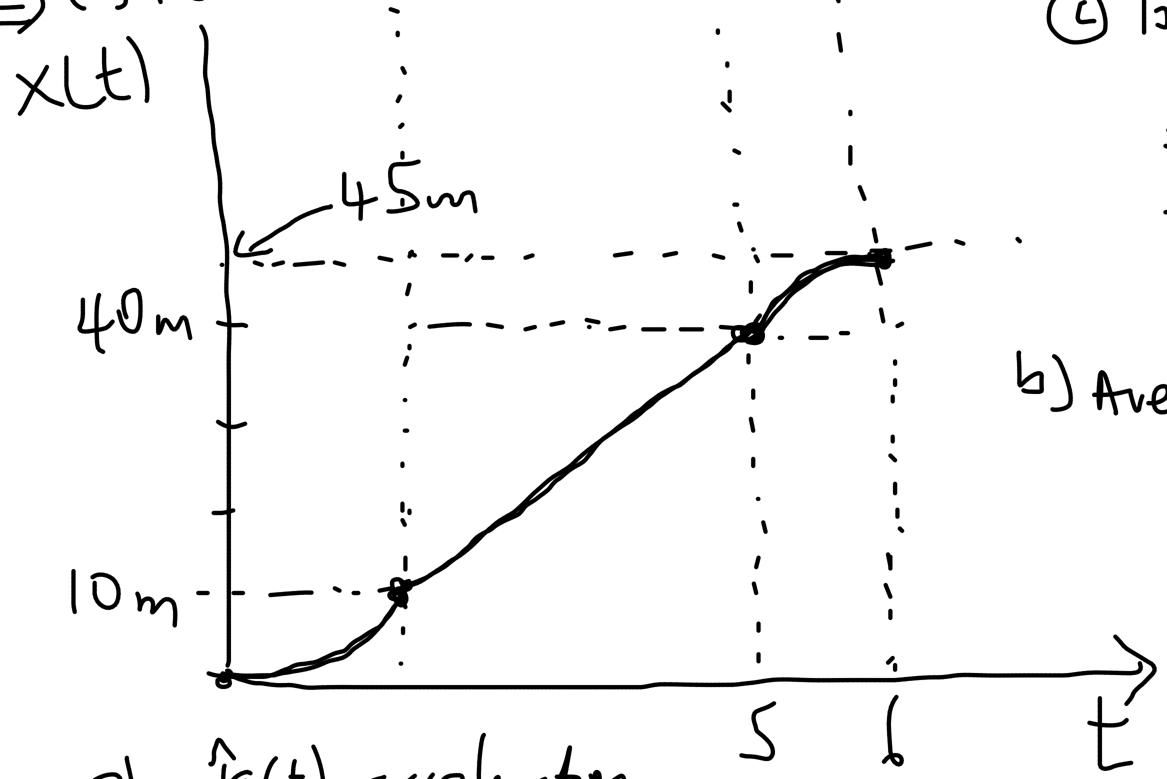
① $v = u + at = 0 + (4\,m/s^2)(12\,s) = \underline{48\,m/s}$

$s = ut + \frac{1}{2}at^2 = 0(12) + \frac{1}{2}(4\,m/s^2)(12\,s)^2 = \underline{\underline{288\,m}}$

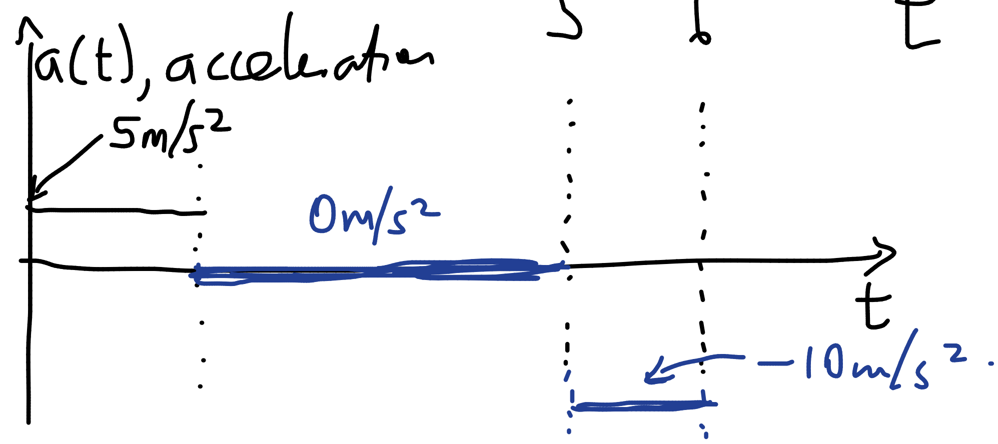
②



⇒ (a) & (e)



c)



ⓒ Total distance travelled = area under v-t graph

$= \text{Area A} + \text{Area B} + \text{Area C}$

$= \frac{1}{2} \times 2 \times 10 + 3 \times 10 + \frac{1}{2} 1 \times 10$

$= 45\,m$

b) Average speed $= \dfrac{\text{total dist.}}{\text{total time}}$

$= \dfrac{45\,m}{6\,s} = 7.5\,m/s$

③ Dropped ⇒ $u = 0$. Use $v = u + at = 0 + (-9.8\,m/s^2)(11\,s) = -107.8\,m/s$. The −ve sign means downwards.

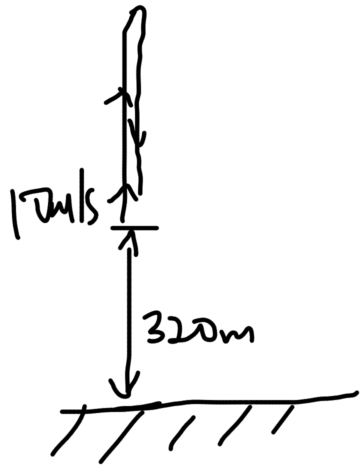
b) $S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(-9.8\,m/s^2)(11\,s)^2 = -592.9\,m$. The −ve sign means downward.

④ It practically took 5.2 seconds for the sound to get to you.

Use $s = ut + \frac{1}{2}\underbrace{at^2}_{=0} = (340\,m/s)(5.2s) = 1768m$

(sound did not accelerate)

⑤ Velocity of stone relative to the ground is $+10m/s$ (but, relative to the bird, it is $0m/s$).



$a = -9.8m/s^2$.

At maximum height, $V_y = 0$. Using $v^2 = u^2 + 2as$,

we get $s = \dfrac{v^2 - u^2}{2a} = \dfrac{0 - (10m/s)^2}{2 \times -9.8m/s} = 5.1m$

$\Rightarrow$ max. height is $320m + 5.1m = 325.1m$

position 4s after it was dropped is: $320m + s(4)$ ← $ut + \frac{1}{2}at^2$

$= 320 + \left[10m/s \times 4s + \frac{1}{2}\times(-9.8m/s^2)(4s)^2\right]$

$= 281.6m$

velocity: Use $v = u + at = 10m/s + (-9.8m/s^2)(4s) = -29.2m/s$

The $-$ve sign means it is moving downwards.

ⓒ When it hits the ground, $s = -320m$. We can use: $s = ut + \frac{1}{2}at^2 \Rightarrow -320 = 10t + \frac{1}{2}(-9.8)t^2$

$\Rightarrow 4.9t^2 - 10t - 320 = 0$

Solve: $t = \dfrac{10 \pm \sqrt{10^2 - 4\times(-320)\times 4.9}}{2 \times 4.9}$.

There are two solutions. One is negative (unphysical), the other is +ve: $t = 9.166s$

t_1 = time taken for stone to fall
t_2 = time taken for sound to reach you

$t_1 + t_2 = 3.3s$

$S = ut + \frac{1}{2}at^2 \Rightarrow S_1 = \underset{\uparrow}{0}t + \frac{1}{2}(-9.8)t_1^2 = -4.9t_1^2$

u=0 since the stone was dropped

Distance traveled by sound $= S_2 = 340\,m/s \times t_2 = 340t_2$

$|S_1| = |S_2| \Rightarrow 4.9t_1^2 = 340\underset{(3.3-t_1)}{t_2} \Rightarrow 4.9t_1^2 = 340(3.3-t_1)$

$\Rightarrow 4.9t_1^2 + 340t_1 - 340\times3.3 = 0$

solve: $t_1 = \dfrac{-340 \pm \sqrt{340^2 - 4\times4.9\times(-340\times3.3)}}{2\times4.9} = \dfrac{-340 \pm 370.933}{9.8}$

$= -72.544s$ or $3.156s$

unphysical correct.

Height of cliff.

$\therefore S_1 = -4.9\times t_1^2 = \underline{\underline{-48.82m}}$. The negative sign means the displacement is downward.

(7) At the maximum height (i.e., top of the flight), $v_y = 0$ (i.e. the stone stops momentarily)

We use: $v^2 = u^2 + 2as$

max. height on earth

On earth: $0 = u^2 + 2(-g)S_{earth} \Rightarrow 2gS_{earth} = u^2$

On moon: $0 = u^2 + 2\left(\frac{-g}{6}\right)S_{moon} \Rightarrow \dfrac{2gS_{moon}}{6} = u^2$

Max. height on moon

$\rightarrow u^2 = 2gS_{earth} = \dfrac{2gS_{moon}}{6}$

$\Rightarrow S_{moon} = 6S_{earth}$
$= 6\times10m$
$= 60m.$

$\Rightarrow$ He can throw the stone up to 60m high

⑧ Vertical | Horizontal

u: $u_y = 150\sin 40°$ | $u_x = 150\cos 40°$

a: $a_y = -g$ | $a_x = 0$

$\boxed{\text{Use } s = ut + \frac{1}{2}at^2}$

s: $y = (150\sin 40°)(10s) + \frac{1}{2}(-9.8m/s^2)(10^2) = 474.184m$

$x = (150\cos 40°)(10s) + \frac{1}{2}(0)10^2 = 1149.1m$

$\Rightarrow$ At 10s, it was 1149.1m along the horizontal from point of release & 474.2m above the ground

V: $V_y = u_y + a_y t$
$= (150\sin 40°) + (-9.8m/s^2)(10s)$
$= -1.58 m/s$. This is negative, meaning, it was moving downward at this time.

Time of flight is: $\frac{2u\sin\theta}{g} = \frac{2 \times 150m/s \times \sin 40°}{9.8m/s^2} = 19.68s$

Range $= (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right) = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{150^2\sin 80°}{9.8} m$
$= 2261.0m$

$\Rightarrow$ it lands 2261.0m from point of release.

9) Let t be the time it takes the stuntman to drop 5m. Then:

$s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow -5m = 0 \times t + \frac{1}{2}(-9.8m/s^2)t^2 \Rightarrow t = \sqrt{\frac{5}{4.9}}\,s = 1.01s$

In this same time, it will move a horizontal distance of:

$s_x = u_x t + \frac{1}{2}a_x t^2 = 4m/s \times 1.01s + \frac{1}{2} \times 0t^2 = 4.04m < 6m$. Thus, If he jumps, he will not land on the second building but fall at a point 4.04m from the first.

If he wants to land on B, his horizontal speed must be at least $\frac{6m}{1.01s} = 5.94 m/s$.

11) Her range was 8.5m

$$R = \frac{u^2 \sin 2\theta}{g}$$ (see class notes)

$$u \simeq \frac{100m}{10s} = 10 m/s$$ (she runs 100m in about 10s)

$$\therefore 8.5m = \frac{(10m/s)^2 \sin 2\theta}{9.8} \Rightarrow \sin 2\theta = 0.833$$

$$\Rightarrow 2\theta = \sin^{-1}(0.833) = 0.9845 rad \equiv 56.4^\circ$$

$$\Rightarrow \theta = \frac{56.4}{2} = \underline{\underline{28.2^\circ}}$$

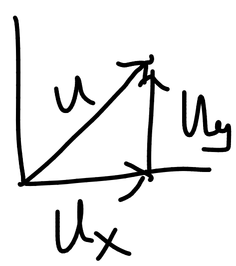
(If she can learn how to take off at 45°, she can increase her range)

10) They all reach the same height ⇒ initial vertical velocities are the same, i.e. $u\sin\theta$ is the same for A, B, C. So, they reach the top at the same time, ⇒ (a) $T_A = T_B = T_C$ where T = total time of flight.

(b) Answered already $u_y^A = u_y^B = u_y^C = u\sin\theta$ since they all have the same max height

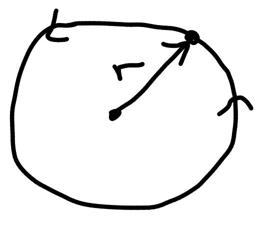
(c) C has the largest reach followed by B then A. ⇒ $u_x^A < u_x^B < u_x^C$

(d) Initial speed is $u = \sqrt{u_x^2 + u_y^2}$



Since u_y is the same for everybody, but u_x are different (see (c) above) then: $u^A < u^B < u^C$.

12)



(a) Distance traveled in one revolution is: $2\pi r$

$$= 2\pi(0.2m) = 1.257m$$

b) Speed $(v) = \frac{2\pi r}{T}$ where T = period = time for one revolution

$T = \frac{60s}{2400}$ (i.e. 2400 revs in 60 seconds $\Rightarrow$ find time for one rev.)

$= 0.025s$

$\therefore v = \frac{2\pi r}{T} = r\left(\frac{2\pi}{T}\right) = 0.2 \times \left(\frac{2\pi}{0.025s}\right) = 0.2m \times 251.327 rad/s$

$\underbrace{\phantom{\left(\frac{2\pi}{0.025s}\right)}}_{w}$

$= \underline{\underline{50.265 m/s}}$

c) $a_{radial} = rw^2 = v^2/r$

$= (50.265)^2/0.2 = 12633.1 m/s^2$, directed radially inward.

d) $T = 0.025s$ [found in (b) above]

period of motion.

13) $(a_r)_{max} = 0.050g$ & $a_r = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_r} \Rightarrow r_{min} = \frac{v^2}{(a_r)_{max}}$

$= \frac{(200 \times 10^3 m/3600s)^2}{(0.050 \times 9.8 m/s^2)} = 6298.8m = 6.3km$

Convert km/h to m/s

b) $a_{radial} = \frac{v^2}{r} \Rightarrow v^2 = a_{radial} r \Rightarrow v = \sqrt{r a_{radial}} \Rightarrow v_{max} = \sqrt{r (a_r)_{max}}$

$= \sqrt{0.5 \times 10^3 m \times (0.05 \times 9.8 m/s^2)}$

$= 15.652 m/s$

$= 56.35 km/h$

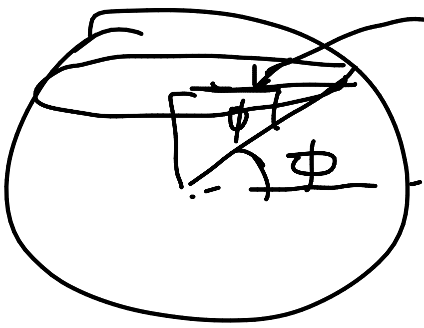
Divide by 1000 & multiply by 3600 to convert to km/h

(14) $R_{earth} \approx 6400\,km = 6.4\times10^{6}\,m/s$

$$w = \frac{2\pi}{T} = \frac{2\pi}{24\times60\times60} = 7.2722\times10^{-5}\,rad/s$$

$$\therefore a_r = r w^2 = 6.4\times10^{6}\times(7.2722\times10^{-5})^2 = 0.0338\,m/s^2$$

(b)



$r = R\cos\phi$

$$\therefore a_r = r w^2 = R\cos\phi\, w^2$$

$$\Rightarrow \cos\phi = \frac{a_r}{R w^2} = \frac{0.8\,cm/s^2}{6400\times10^{3}\times w^2}$$

$$= \frac{0.8\times10^{-2}\,m/s^2}{6400\times10^{3}\times(7.2722\times10^{-5})^2}$$

$$= 0.2364$$

$$\Rightarrow \phi = \cos^{-1}(0.2364) = 1.332\,rad \equiv 76.328^\circ$$

(c) $a_r = R w^2 = R\left(\frac{2\pi}{T}\right)^2 \Rightarrow T = \sqrt{\frac{R(2\pi)^2}{a_r}} = \sqrt{\frac{6.4\times10^{6}\,m}{10\,m/s^2}}\times 2\pi$

$= 5026.55\,s \equiv 1.4$ hours.

(15) Like $v = u + at$ for linear motion, we have:

$$w_f = w_i + \alpha t \quad \text{for uniform circular motion}$$

final w — initial w — constant angular acceleration

$$\therefore 0 = w_i + \alpha(15) \Rightarrow \alpha = -\frac{w_i}{15s} = -\frac{(1800\times2\pi/60s)}{15s} = -12.6\,rad/s^2$$

rpm = revolutions per minute
Divide by 60s to convert to revolutions per second

Also, like $s = ut + \frac{1}{2}at^2$ for linear motion, we have

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$ for uniform circular motion

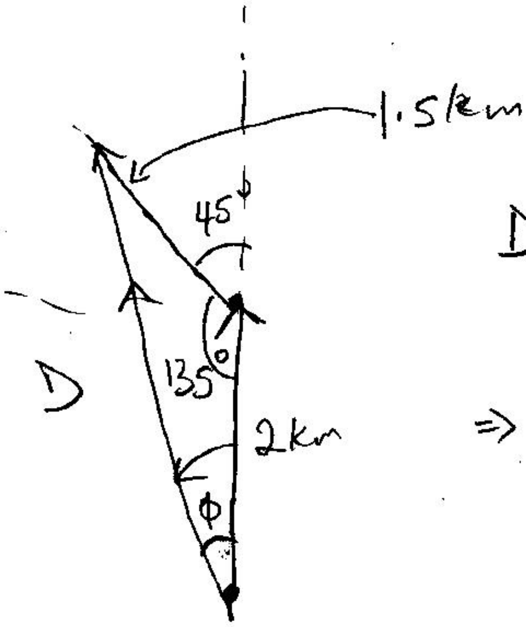
$$= \left(\frac{1800 \times 2\pi}{60}\right)15 + \frac{1}{2}(-12.6)15^2$$

$$= 2827.43 \text{ rad} - 1413.72 \text{ rad}$$

$$= 1413.72 \text{ rad} \equiv 225 \text{ revolutions}$$

Divide by 2π to convert radians to revolutions since one revolution $= 2\pi$ radians

(16)



$D^2 = 2^2 + 1.5^2 - 2 \times 1.5 \times 2 \times \cos 135°$

$= 10.49264 \text{ km}^2$

$\Rightarrow D = \sqrt{10.49264} \text{ km}$

$= 3.239 \text{ km}$

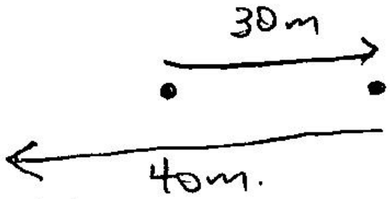
$\frac{D}{\sin 135°} = \frac{1.5\text{km}}{\sin\phi} \Rightarrow \sin\phi = \frac{1.5 \sin 135°}{D} = \frac{1.5 \sin(135°)}{3.239} = 0.3274$

$\therefore \phi = \sin^{-1}(0.3274) = 0.3336 \text{ rad} \equiv 19.114°$

$\Rightarrow \vec{D} = 3.239 \text{ km}$ at $19.114°$ West of North

---

(17) (a)

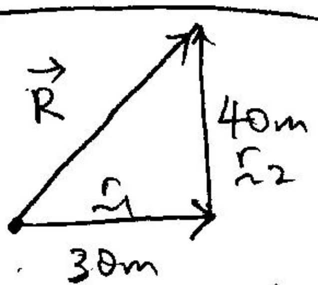


$\vec{r}_1 = 30\text{m}\,\hat{i}$

$\vec{r}_2 = -40\text{m}\,\hat{i}$

$\vec{r}_1 + \vec{r}_2 = -10\text{m}\,\hat{i} \Rightarrow |r_1 + r_2| = 10\text{m}.$

(b)

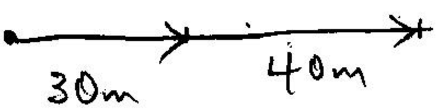


'Walk along $\vec{r}_1$ then along $\vec{r}_2$ as shown

$\vec{r}_1 = 30\text{m}\,\hat{i}$ & $\vec{r}_2 = 40\text{m}\,\hat{j} \Rightarrow \vec{R} = \vec{r}_1 + \vec{r}_2 = (30\hat{i} + 40\hat{j})\text{m}.$

& $|\vec{R}| = \sqrt{30^2 + 40^2}\ \text{m} = 50\text{ m}.$

(c)

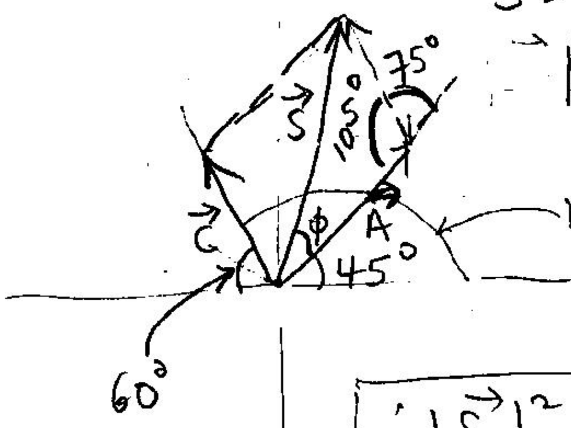


$\vec{r}_1 = 30\text{m}\,\hat{i}$ & $\vec{r}_2 = 40\text{m}\,\hat{i}$

$\vec{R} = \vec{r}_1 + \vec{r}_2 = 70\text{m}\,\hat{i} \Rightarrow |\vec{R}| = 70\text{m}.$

1B)

a)



$\vec{S} = \vec{A} + \vec{C}$

$\rightarrow |\vec{S}|^2 = |\vec{A}|^2 + |\vec{C}|^2 - 2AC\cos 105°$

$\therefore |\vec{S}|^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \cos 105°$
$= 251.76$

$\Rightarrow |\vec{S}| = \sqrt{251.76} = 15.867 N.$

(same magnitude as in Analytical approach below)

Use sine rule:

$$\frac{\sin\phi}{|\vec{C}|} = \frac{\sin 105°}{|\vec{S}|}$$

$$\Rightarrow \sin\phi = \frac{|\vec{C}|\sin 105°}{|\vec{S}|} = \frac{10\sin 105°}{15.867} = 0.6088 \Rightarrow \phi = \arcsin(0.6088)$$
$$= 0.6545 \text{ rad} \equiv 37.5°$$

$\Rightarrow \vec{S}$ makes an angle $45° + \phi = 82.5°$ with the positive x-direction. Again, the same result as the analytical approach below).

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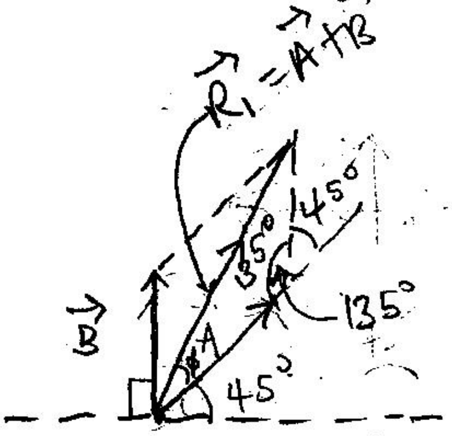
b) $\vec{A} + \vec{B} - \vec{C} = ?$

First, we determine $\vec{A} + \vec{B}$. Let us call it $\vec{R_1}$:



$|\vec{R_1}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2AB\cos 135°$

$= 10^2 + 10^2 - 2 \times 10 \times 10 \cos 135°$

$= 341.42 N^2$

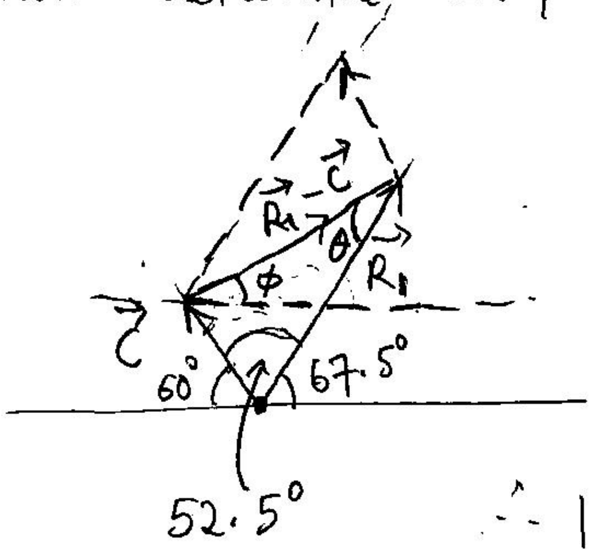
$\therefore |\vec{R_1}| = \sqrt{341.42} N = 18.478 N.$

The angle it makes with the positive x-axis is $45° + \phi$ where ϕ is as shown above.

From sine rule: $\frac{\sin\phi}{|\vec{B}|} = \frac{\sin 135°}{|\vec{R_1}|} \Rightarrow \sin\phi = \frac{10\sin 135°}{18.478} = 0.383$

$\Rightarrow \phi = 0.393 \text{ rad} \equiv 22.5°$ $\therefore \vec{R_1}$ makes angle $45° + 22.5°$
$= 67.5°$ with positive x-axis

Next, we now determine $\vec{R} \equiv \vec{R_1} - \vec{C}$:



$$|\vec{R}|^2 = |\vec{R_1}|^2 + |\vec{C}|^2 - 2 R_1 C \cos 52.5^\circ$$

$$= 18.478^2 + 10^2 - 2 \times 18.478 \times 10 \times \cos 52.5^\circ$$

$$= 216.46$$

$$\therefore |\vec{R}| = \sqrt{216.46}\ N$$

$$= \underline{14.713}\ N.$$

Same result as the analytical approach below.

The angle ϕ shown above can be obtained from the fact that $\phi + \theta = 67.5^\circ$. We will determine θ from the sine rule and, consequently obtain ϕ.

$$\frac{\sin\theta}{|\vec{C}|} = \frac{\sin 52.5^\circ}{|\vec{R}|} \Rightarrow \sin\theta = \frac{10 \sin 52.5}{14.713} = 0.5392$$

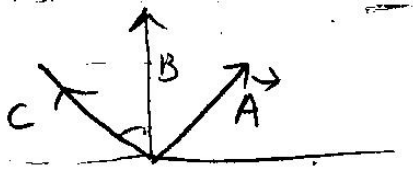
$$\therefore \theta = \sin^{-1}(0.5392) = 0.5695 \text{ rad}$$

$$\equiv 32.6313^\circ.$$

$\therefore \phi = 67.5^\circ - 32.6313^\circ = \underline{\underline{34.869^\circ}}$, same result as the analytical approach below.

$\therefore \vec{A} + \vec{B} - \vec{C}$ has a magnitude of $14.713N$ and makes an angle of 34.869° with the positive x-axis

18) Analytical approach:



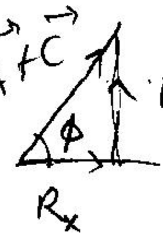
$\vec{A} = 10\cos 45^\circ \hat{i} + 10\sin 45^\circ \hat{j} = 7.07\hat{i} + 7.07\hat{j}$

$\vec{B} = 10\cos 90^\circ \hat{i} + 10\sin 90^\circ \hat{j} = 0\hat{i} + 10\hat{j}$

$\vec{C} = 10\cos 120^\circ \hat{i} + 10\sin 120^\circ \hat{j} = -5\hat{i} + 8.66\hat{j}$

$\therefore \vec{R} = \vec{A} + \vec{C} = 2.07\hat{i} + 15.73\hat{j}$

$|\vec{R}| = \sqrt{2.07^2 + 15.73^2} = 15.866\,N$



& makes an angle of $\cos^{-1}\left(\frac{2.07}{|\vec{R}|}\right)$ ie. $\cos\phi = \frac{R_x}{|\vec{R}|} = 0.1305$

$\Rightarrow \phi = \cos^{-1}(0.1305) = 1.44\,\text{rad} \equiv 82.5^\circ \Rightarrow \vec{R} = 15.866\,N$ at 82.5° to the positive x direction

(b) $\vec{R} = \vec{A} + \vec{B} - \vec{C} = 12.07\hat{i} + 8.41\hat{j}$

$\Rightarrow |\vec{R}|^2 = \sqrt{12.07^2 + 8.41^2} = 14.711\,N$

& makes an angle ϕ such that $\cos\phi = \frac{|R_x|}{|\vec{R}|} = \frac{12.07}{14.711} = 0.8205$

$\Rightarrow \phi = \cos^{-1}(0.8205) = 0.60855\,\text{rad}$

"arccos" $\equiv 34.868^\circ$

$\therefore \vec{A} + \vec{B} - \vec{C}$ has a magnitude of $14.711\,N$ and at angle of 34.868° to the positive x direction

c) $\vec{R} = \vec{A} + \vec{B} + \vec{C} = 2.07\hat{i} + 25.73\hat{j}$

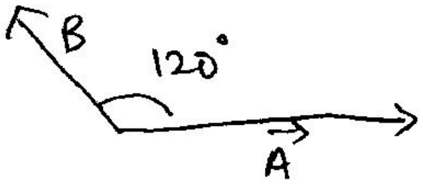
$\therefore |\vec{R}| = \sqrt{2.07^2 + 25.73^2}\,N = 25.813\,N.$

$\vec{R}$ makes an angle ϕ with the positive x-axis.

$\cos\phi = \frac{R_x}{|\vec{R}|} = \frac{2.07}{25.813} = 0.08019 \Rightarrow \phi = \underline{\underline{85.4^\circ}}$

$\therefore \vec{R} = 25.813\,N$ at an angle of 85.4° to the positive x-axis.

19)



$\vec{B} = (|\vec{B}| \cos 120°)\hat{i} + (|\vec{B}| \sin 120°)\hat{j} = (-1.5\hat{i} + 2.598\hat{j})$ N

$|\vec{B}| = 3$ N

$\vec{A} = (|\vec{A}| \cos 0°)\hat{i} + (|\vec{A}| \sin 0°)\hat{j} = (6\hat{i} + 0\hat{j})$ N.

a) $\vec{A} \times \vec{B} = 6\hat{i} \times (-1.5\hat{i} + 2.598\hat{j}) = (0 + 15.588\hat{k})$. N^2

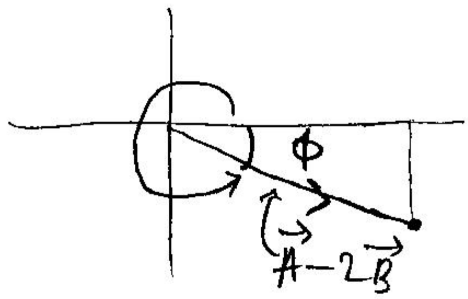
$\hat{i} \times \hat{i} = 0$
$\hat{i} \times \hat{j} = \hat{k}$

b) $\vec{A} \cdot \vec{B} = -1.5 \times 6 + 2.598 \times 0$

$= -9\ N^2$.

c) $\vec{A} - 2\vec{B} = 6\hat{i} - 2(-1.5\hat{i} + 2.598\hat{j}) = \underline{\underline{(9\hat{i} - 5.196\hat{j})}}$ N.

$|\vec{A} - 2\vec{B}| = \sqrt{9^2 + (-5.196)^2} = 10.39$.



$\phi = \cos^{-1}\left(\frac{9}{10.39}\right) = \underline{0.52\text{rad}}$

$\equiv \underline{\underline{30°}}$.

i.e. 10.39° at an angle of $\underbrace{(360 - 30°)}_{=330°}$ to the positive x-axis

Q21

$\vec{v_1} + \vec{v_2} + \vec{v_3} = 6\hat{i} + \hat{j} - 4\vec{k} \Rightarrow \vec{v_3} = 6\hat{i} + \hat{j} - 4\hat{k} - \vec{v_1} - \vec{v_2}$

$= 6\hat{i} + \hat{j} - 4\hat{k} - (2\hat{i} + 3\hat{j} - 2\hat{k}) - (4\hat{i} + 3\hat{j})$

$= 0\hat{i} - 5\hat{j} - 2\hat{k} \Rightarrow |\vec{v_3}| = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29} = \underline{5.385}$.

20) let $\vec{F_1}$ & $\vec{F_2}$ be the two vector forces & θ the angle b/w them.
Their resultant, $\vec{R}$ is $\vec{F_1}+\vec{F_2}$.

From the problem,

$$|\vec{R}| = \frac{1}{2}|\vec{F_1}| = \frac{1}{2}|\vec{F_2}| \quad \text{——— (1)}$$

Clearly, $\vec{R} = \vec{F_1}+\vec{F_2} \Rightarrow |\vec{R}|^2 = |\vec{F_1}|^2 + |\vec{F_2}|^2 + 2\vec{F_1}\cdot\vec{F_2}$

$$\Rightarrow \left(\frac{1}{2}|\vec{F_1}|\right)^2 = F_1^2 + F_1^2 + 2F_1^2\cos\theta. \quad \left(\text{since } F_1 = F_2 \text{ but } \vec{F_1} \neq \vec{F_2}\right)$$

$$\therefore \frac{1}{4}F_1^2 = 2F_1^2 + 2F_1^2\cos\theta$$

Assuming F_1 is not zero, we can divide through by F_1^2 to get

$$\frac{1}{4} = 2 + 2\cos\theta \Rightarrow \frac{1}{8} = 1 + \cos\theta$$

$$\text{or } \cos\theta = \frac{1}{8} - 1 = -0.875$$

$$\therefore \theta = \cos^{-1}(-0.875) = 2.636 \text{ rad}$$

$$\equiv 151.04^\circ$$

22) $\vec{F} = 3\hat{i} + \hat{j} - 2\hat{k} \Rightarrow |\vec{F}| = \sqrt{3^2 + 1^2 + (-2)^2} = 3.74$

$\vec{r} = \hat{i} + 2\hat{j} \Rightarrow |\vec{r}| = \sqrt{1^2 + 2^2} = \sqrt{5} = 2.236$

a) $\underbrace{\vec{F}\cdot\hat{j}}_{=1} = \underbrace{|\vec{F}|}_{3.74}\,\underbrace{|\hat{j}|}_{1}\cos\phi$ where ϕ = angle that $\vec{F}$ makes with the positive y-axis.

$$\Rightarrow \cos\phi = \frac{1}{3.74} \Rightarrow \phi = 1.3 \text{ rad} \equiv 74.50^\circ$$

b) $\underbrace{\vec{r}\cdot\hat{j}}_{=2} = \underbrace{|\vec{r}|}_{2.236}\,\underbrace{|\hat{j}|}_{1}\cos\theta$ where θ = angle that $\vec{r}$ makes with +ve y axis

$$\Rightarrow 2 = 2.236\cos\theta \Rightarrow \cos\theta = \frac{2}{2.236} \Rightarrow \theta = 26.56^\circ.$$

b) Work done $= \vec{F}\cdot\vec{r} = 3\times1 + 1\times2 + (-2\times0) = 5$

HW3 Solutions

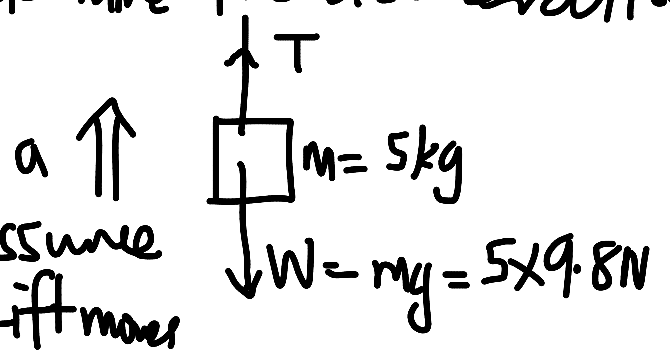
① $F = ma \Rightarrow F = (3kg)(3m/s^2) = 9N$.

for $m = 1kg$, $F = ma \Rightarrow 9N = 1kg \times a \Rightarrow a = \frac{9N}{1kg} = \underline{\underline{1m/s^2}}$

② $W = mg = 80kg \times 9.8m/s^2$ (on earth)
$= 784N$

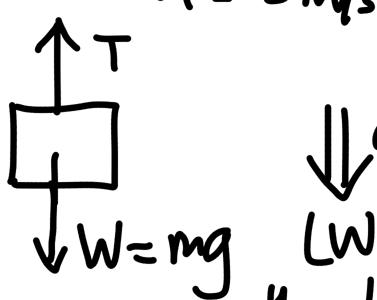
On moon, $g = \frac{9.8m/s^2}{6} \Rightarrow W(\text{on moon}) = 80kg \times \frac{9.8m/s^2}{6}$
$= \underline{\underline{130.67N}}$ on moon.

③ Starts from rest ($\Rightarrow u = 0$) & 3m in 1s. Use $s = ut + \frac{1}{2}at^2$ to determine the acceleration a: $3 = 0 + \frac{1}{2}a \times 1^2 \Rightarrow a = 6m/s^2$



(we assume the Lift moves up)

$\therefore T - mg = ma$
$\Rightarrow T = m(g+a)$
$= 5(9.8+6) = \underline{\underline{79N}}$

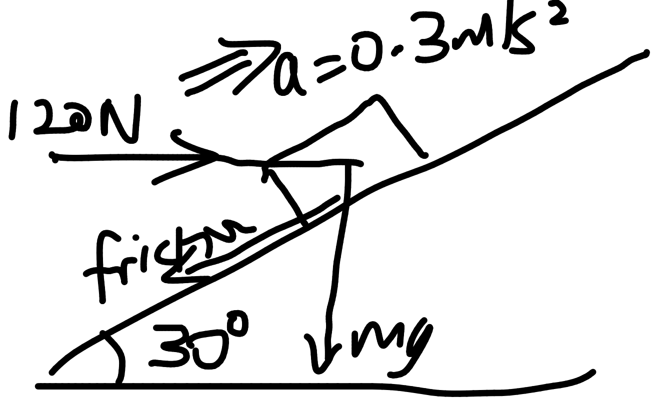


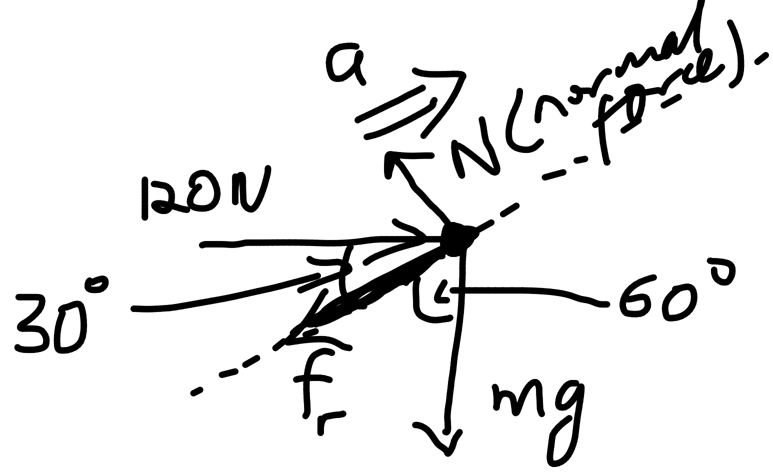
(We assume the lift moves down)

$\therefore mg - T = ma$
$\Rightarrow T = m(g-a)$
$= 5(9.8-6)$
$= \underline{\underline{19N}}$

<u>NB</u>: The problem did not state whether the motion was upward or downward. If upward, the tension in the string will be <u>79N</u>, if downward, it will be <u>19N</u>

④





Along the plane, we have:

$$120\cos 30° - f_r - mg\cos 60° = ma \quad \text{—— ①}$$

Perpendicular to the plane, we have:

$$mg\sin 60° + 120\sin 30° - N = m \times 0 \quad \text{—— ②}$$

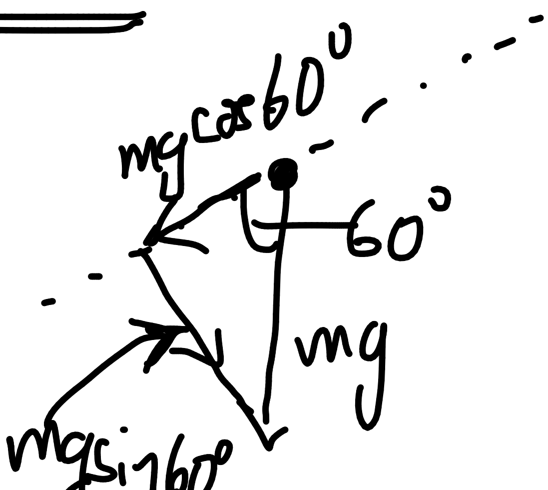
Eq ① gives: $f_r = 120\cos 30° - mg\cos 60° - ma$

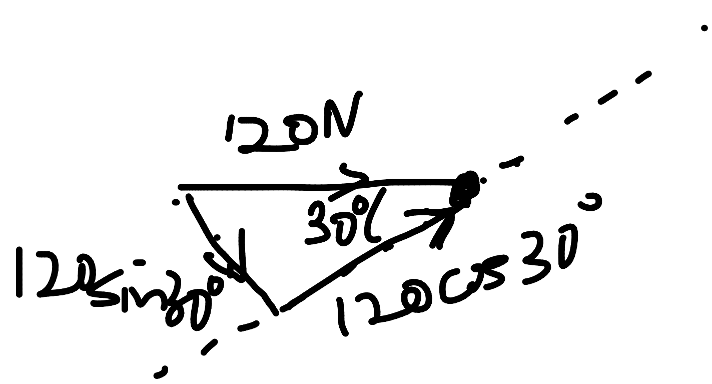
Eq ② gives: $N = mg\sin 60° + 120\sin 30°$

$$\Rightarrow \mu = \frac{f_r}{N} = \frac{120\cos 30° - mg\cos 60° - ma}{mg\sin 60° + 120\sin 30°} = \frac{51.923}{144.870} = \underline{\underline{0.36}}$$

$m = 10\,kg$
$a = 0.3\,m/s^2$

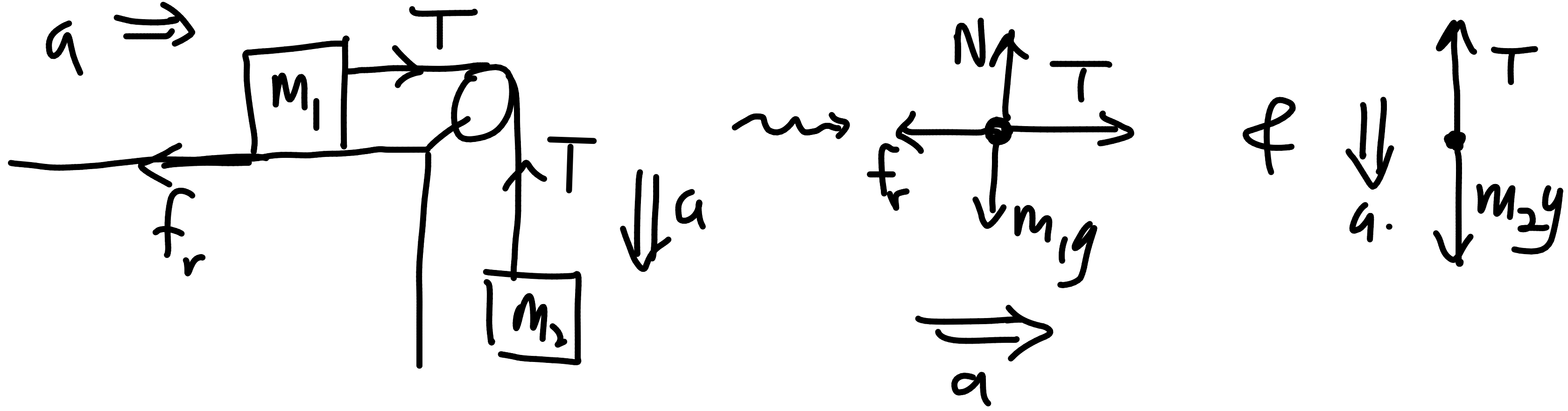
<u>Comment:</u>





"mg" was resolved this way (see Fig. above) | "120N" was resolved this way (see Fig. above)

5



For m_1: Vertical $\Rightarrow N - m_1 g = 0 \Rightarrow N = m_1 g$ $\quad N = m_1 g$

Horizontal $\Rightarrow T - f_r = m_1 a \Rightarrow T - \mu N = m_1 a$

$$\Rightarrow T - \mu m_1 g = m_1 a \quad \text{——— (1)}$$

For m_2: Vertical $\Rightarrow$

$$m_2 g - T = m_2 a \quad \text{——— (2)}$$

Add: $\quad m_2 g - \mu m_1 g = (m_1 + m_2) a$

$$\Rightarrow a = \frac{(m_2 - \mu m_1) g}{m_1 + m_2} = \frac{(5 - 0.25 \times 20) 9.8}{25} = 0$$

$m_1 = 20$ kg

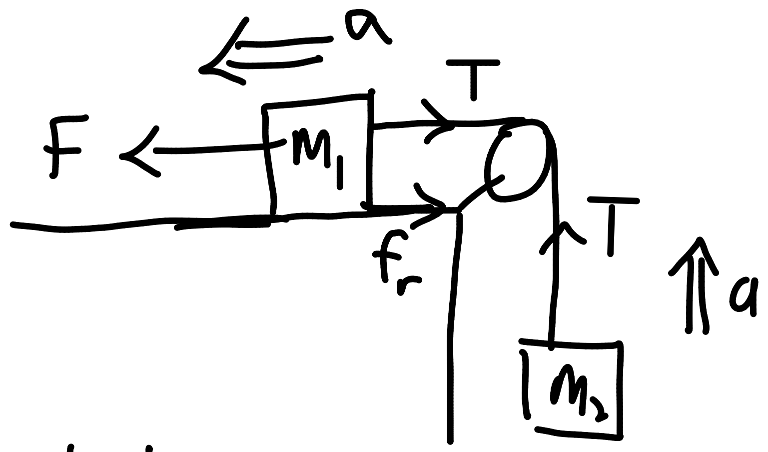
$m_2 = 5$ kg

$\mu = 0.25$ kg

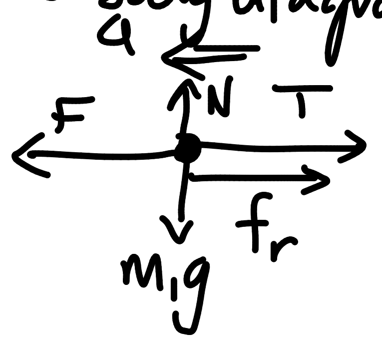
$\Rightarrow a = 0$

Use $s = ut + \frac{1}{2} at^2 = 0 \times 2s + \frac{1}{2} a (2s)^2 = 0$ (since $a = 0$)

$\Rightarrow$ the system does not move (in the first 2 seconds) & will not move at all if released from rest, since $u = 0$ & $a = 0$



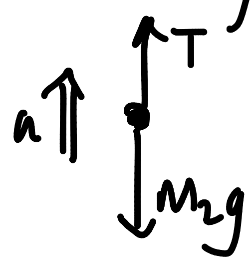
Free-body diagram for m_1:



Vertically: $N - m_1 g = 0 \Rightarrow N = m_1 g$

Horizontally: $F - T - \underbrace{f_r}_{=\mu N} = m_1 a \Rightarrow F = T + \mu m_1 g + m_1 a$ —— ①

Force-body diagram for m_2:



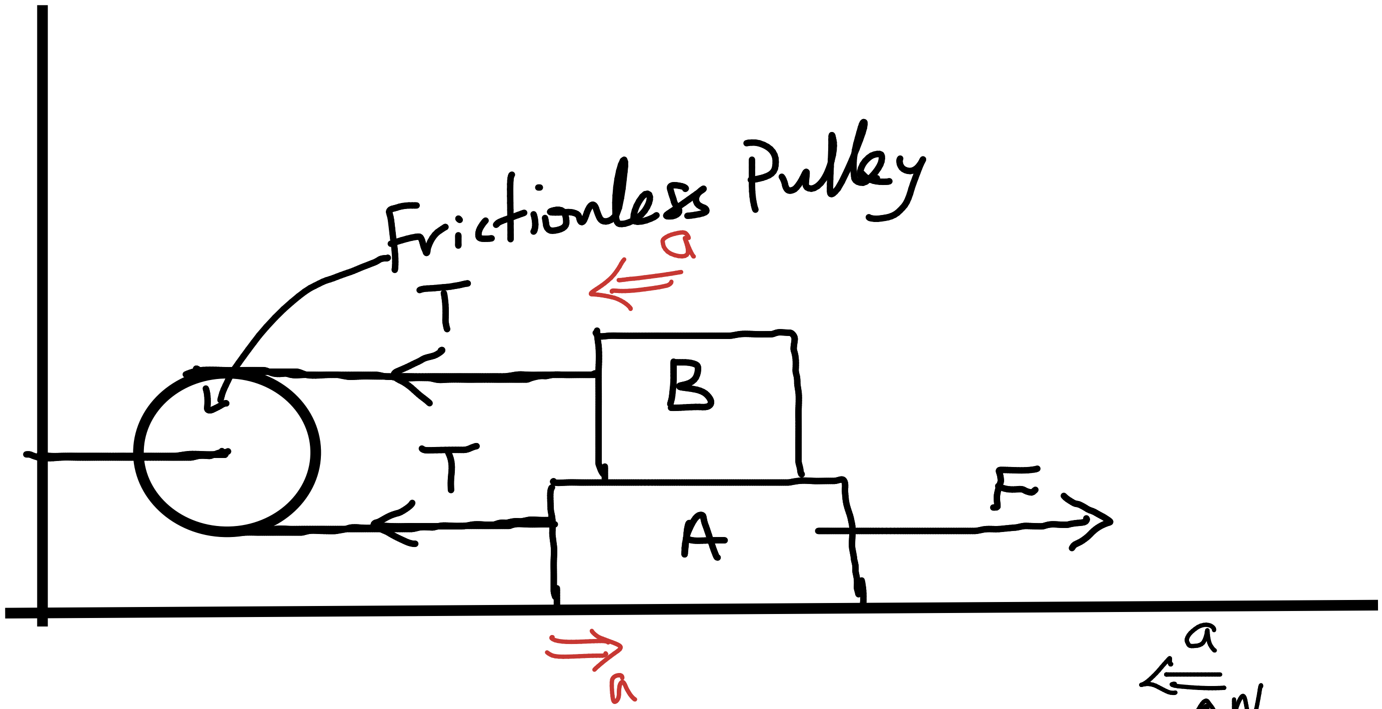
$T - m_2 g = m_2 a \Rightarrow T = m_2(a+g)$ —— ②

Use this T & subtitute in Eq ①:

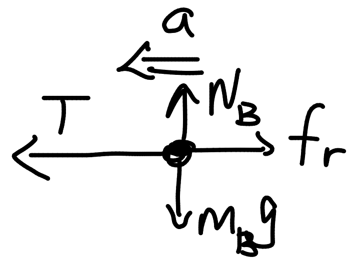
$$F = \underbrace{m_2(a+g)}_{=T} + \mu m_1 g + m_1 a = [5(1+9.8) + 0.25\times 20\times 9.8 + 20\times \cancel{9.8}\,1]\ \text{N}$$

$$= \cancel{299\ \text{N}}$$

$$= 123\ \text{N}$$



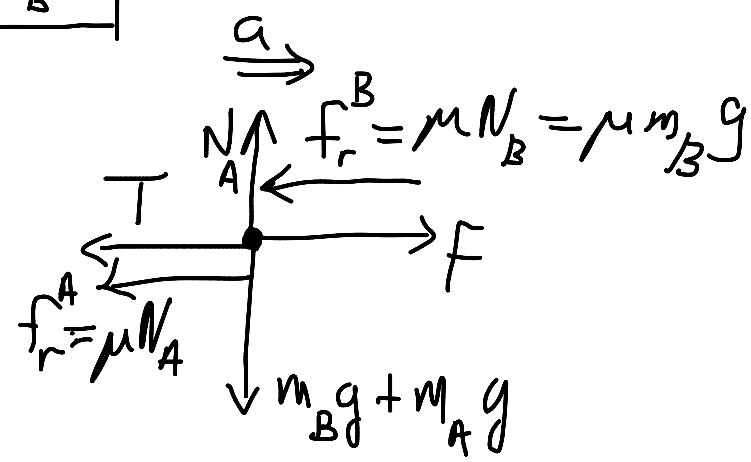
Free-body diagram for B:



$N_B - m_B g = 0 \Rightarrow N_B = m_B g$.

$T - f_r = m_B a \Rightarrow \boxed{T - \mu m_B g = m_B a}$ —— ①

Free-body diagram for A:



Vertically:

$N_A - (m_B g + m_A g) = 0$

$\Rightarrow N_A = (m_A + m_B) g$ —— ②

Horizontally: $F - T - f_r^A - f_r^B = m_A a$

$\Rightarrow F = m_A a + T + f_r^A + f_r^B$

$= m_A a + \underbrace{(m_B a + \mu m_B g)}_{\substack{= T \\ \text{(from Eq 1)}}} + \mu \underbrace{(m_A + m_B) g}_{\substack{= N_A \\ \text{(from Eq 2)}}} + \mu m_B g$

$= \underline{2.908}$ N

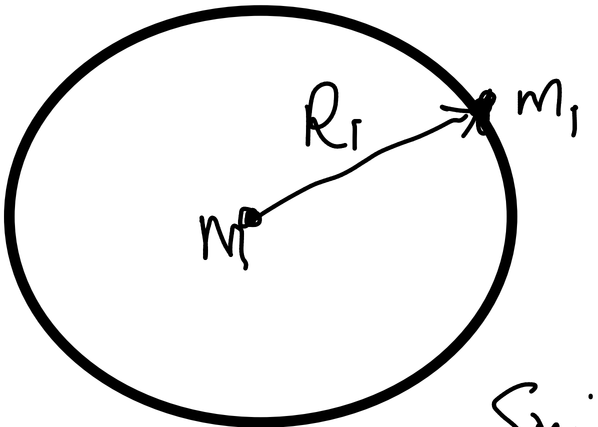
$m_A = 700g = 0.7kg$

$m_B = 200g = 0.2kg$

$\mu = 0.2$

$a = 0.4 m/s^2$

(7) $F_1 = \frac{GMm_1}{R_1^2}$ is the gravitational force & this supplies the centripetal force $m_1\left(\frac{v_1^2}{R_1}\right)$



$$\Rightarrow \frac{GM\cancel{m_1}}{R_1^2} = \frac{\cancel{m_1} v_1^2}{R_1}$$

$$\Rightarrow v_1^2 = \frac{GM}{R_1} \Rightarrow v_1^2 \propto \frac{1}{R_1}$$

Similarly, for the second satellite,

$$v_2^2 = \frac{GM}{R_2} \Rightarrow v_2^2 \propto \frac{1}{R_2}$$

$R_1 = 135\times10^3$ km

$R_2 = 700\times10^3$ km

$R_2 > R_1 \Rightarrow v_2^2 < v_1^2 \Rightarrow$ the satellite at 135,000 km moves faste.

b) $F = \frac{GMm_1}{R_1^2} = m_1 R_1 w_1^2 \Rightarrow w_1^2 = \frac{GM}{R_1^3}$.

Similarly, $w_2^2 = \frac{GM}{R_2^3}$ $\Bigg\} \Rightarrow \frac{w_1^2}{w_2^2} = \frac{R_2^3}{R_1^3}$ — (1)

Since $w = \frac{2\pi}{T}$, Eq (1) gives: $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$

$$\Rightarrow \left(\frac{T_2}{T_1}\right) = \left(\frac{R_2}{R_1}\right)^{3/2} = \left(\frac{700000}{135,000}\right)^{3/2} = \underline{\underline{11.8}}$$

(8) $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M_{earth}}{\frac{4}{3}\pi R_{Earth}^3} = \frac{M_{moon}}{\frac{4}{3}\pi R_{moon}^3} \Rightarrow \frac{M_E}{R_E^3} = \frac{M_m}{R_m^3} \Rightarrow M_E = \frac{M_m R_E^3}{R_m^3}$

For a body of mass m on earth, $F = \frac{GM_E m}{R_E^2} = mg_E \Rightarrow g_E = \frac{GM_E}{R_E^2}$

For a body of mass m on moon, similarly, $g_{moon} = \frac{GM_{moon}}{R_m^2}$

$g_{moon} = \frac{1}{6} g_{earth} \Rightarrow \frac{GM_m}{R_m^2} = \frac{1}{6}\frac{GM_E}{R_E^2} = \frac{1}{6}G$

$g_{moon} = \frac{1}{6} g_{Earth} \Rightarrow \frac{GM_{moon}}{R_m^2} = \frac{1}{6}\frac{GM_{earth}}{R_{earth}^2} = \frac{1}{6}\frac{G}{R_{earth}^2} \quad M_{moon}\frac{R_{earth}^3}{R_{moon}^3}$

$$\therefore \frac{\cancel{GM}_{moon}}{R_m^2} = \frac{1}{6}\frac{\cancel{GM}_{moon} R_{earth}}{R_m^3}$$

$$\Rightarrow R_m = \frac{1}{6} R_{earth} = \frac{6400\,km}{6} = \underline{\underline{1066.7}}\,km$$

9) $E = \underbrace{KE}_{=\frac{1}{2}mv^2} + \underbrace{PE}_{= -\frac{GMm}{R^2}} = \frac{1}{2}mv^2 - \frac{GMm}{R^2}$ for gravitational potential energy

Escape from earth will occur when we have enough energy to escape i.e. when $E \geqslant 0 \Rightarrow \frac{1}{2}mv^2 - \frac{GM_E m}{R^2} \geqslant 0$

$\Rightarrow \frac{1}{2}v^2 - \frac{GM_E}{R^2} \geqslant 0 \Rightarrow v^2 \geqslant \frac{2GM_E}{R^2}$. For escape from the surface of the earth, $R = R_E \Rightarrow v_E^2 \geqslant \frac{2GM_E}{R_E^2}$

$\Rightarrow v_E \geqslant \sqrt{\frac{2GM_E}{R_E^2}} \Rightarrow$ (minimum) escape velocity is: $\sqrt{\frac{2GM_E}{R_E^2}} = \sqrt{2gR_E}$

use $gR_E^2 = GM_E$

Observe that the escape velocity is independent of the mass of the object. Thus, we get the same $\sqrt{2gR_E} = \sqrt{2 \times 9.8\,m/s^2 \times 6400 \times 10^3\,m} = 11200\,m/s = 11.2\,km/s$

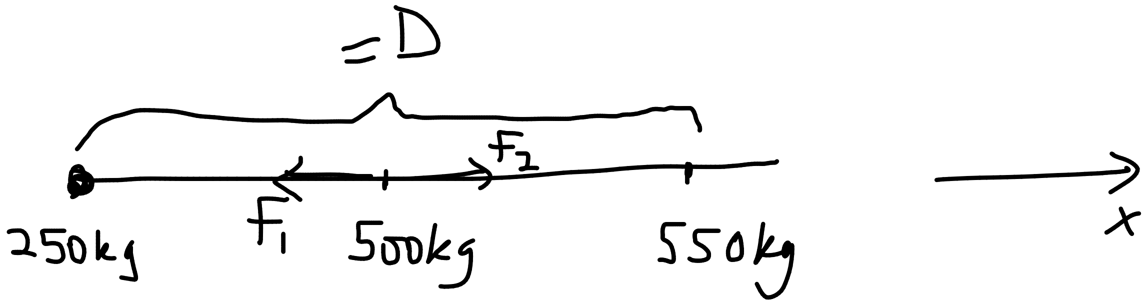
(5) For $m = 200\,kg$, (minimum) energy required $= \frac{1}{2}mv^2 = \frac{1}{2} \times 200\,kg \times (11200\,m/s)^2$

$= 125.44 \times 10^8\,J$

For $m = 800\,kg$, min. energy required $= \frac{1}{2}mv^2 = \frac{1}{2} \times 800 \times (11200\,m/s)^2$

$= 501.76 \times 10^8\,J$

10



$$\vec{F}_1 = -G\frac{250kg \times 500kg}{(D/2)^2}\hat{x} = -\frac{4G}{D^2}(250\times500)\hat{x}$$

$$\vec{F}_2 = \frac{G\, 550kg \times 500kg}{(D/2)^2}\hat{x} = \frac{4G}{D^2}(500\times500)\hat{x}$$

$$\Rightarrow \vec{F}_1 + \vec{F}_2 = \frac{4G}{D^2}(\underbrace{500\times500 - 250\times500}_{125000})\hat{x} = \frac{4G}{D^2}125000\hat{x}$$

$$= \frac{G}{D^2}500000\hat{x} = \frac{6.67\times10^{-11}\times5\times10^{5}}{D^2(m^2)}N = \frac{3.335\times10^{-5}}{D^2[m^2]}N$$

The net force on the 500kg mass is $\frac{3.335\times10^{-5}}{D^2}N$ towards the 550 kg mass.

University of Ibadan

Department of Physics

PHY102 2016/17 Session (2017): HW 4 Due Date: N/A

(Work, Energy, Power, Collisions)

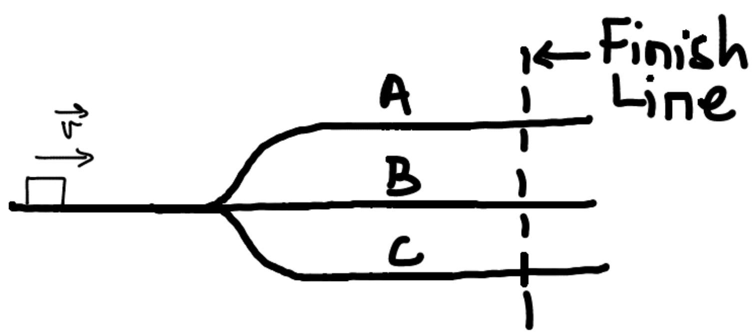
Take the acceleration due to gravity (g) to be 9.8 m/s^2

References: (1) Halliday and Resnick / (2) Farai /(3) etc.

1) A horizontal force F of 10 N pulls a box 12m along the surface of a table. Determine (a) the work done by the force (b) the work done by the force F if the mass of the box is 0.5 kg (c) the work done by the force F if there was a frictional force of 5N opposing the motion (d) the work done by the force if it were directed at an angle of 60° to the horizontal

2) A car moves up a distance of 200m along a plane inclined at an angle of 45° to the horizontal. If its speed is 25 km/h at this distance, determine (a) its kinetic energy (b) its potential energy (c) its total energy

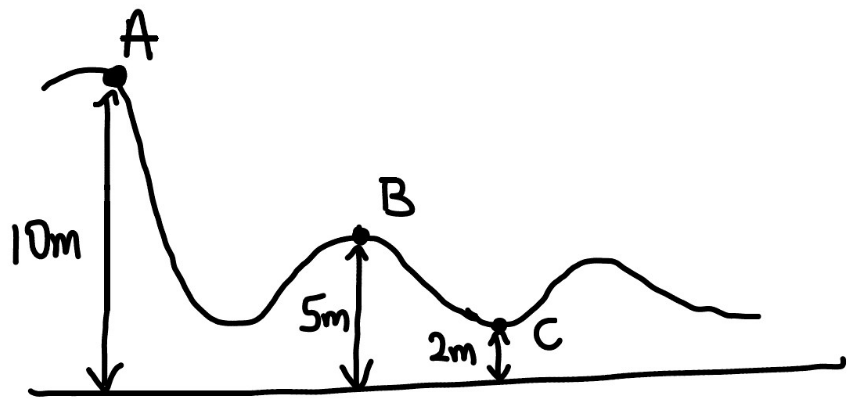
3) H&R: The block shown below can take either of the three frictionless paths A, B, C, each differing in elevation as shown below. Rank the paths in descending order according to (a) speed of the block at the finish line (b) travel time to the finish line.



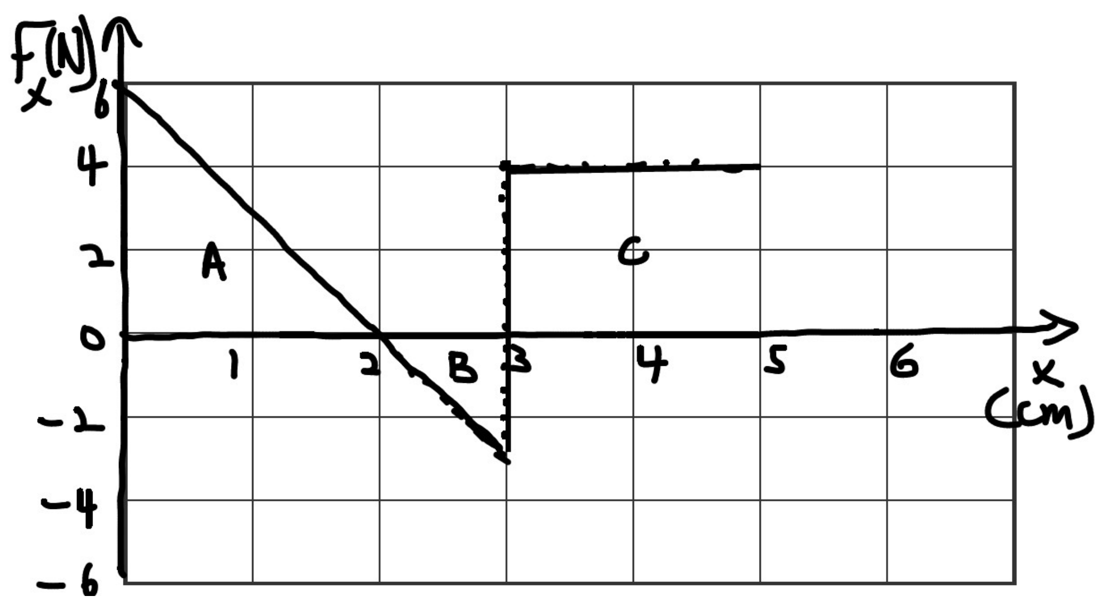
4) A horizontal force F drags a 10kg box a distance 12m along the surface of a table at constant speed. If the coefficient of sliding (kinetic) friction between the table and the box is 0.2, determine the work done by F.

5) An object, acted upon at time t by a force $F(t) = (3t^2 - 2t)$ N, moves at a constant velocity of 3m/s. Determine (a) the power expended at time t (b) the work done in the first five seconds.

6) If the object shown below was released from rest at the point A and the surface is frictionless, determine the velocity of the object at the points B and C as it slides on the track.



7) The horizontal force acting on an object is plotted against displacement as shown below. Determine (a) the work done in each of the segments A (0-2 cm), B (2-3 cm), C (3-5 cm)   (b) the total work done



8) A satellite is released from the surface of the earth with a vertical speed of 10 km/s. Determine its mechanical energies (potential and kinetic) when it is at a height of 2 km above the surface of the earth.

9) Determine the average power required to raise a 120 kg object to a height of 10m in three minutes.

10) H&R: A cannon ball is fired from level ground with a velocity of 18m/s at an angle of 60° to the horizontal. At the top of the trajectory, the cannon ball explodes into two parts of equal mass. One fragment whose speed is zero immediately after the explosion falls vertically. How far horizontally from the point of release does the other fragment land? Neglect air resistance.

11) A neutron of mass 1.67 X 10^{-27} kg traveling with a speed of 8.0 km/s collides with a deuterium nucleus of mass 3.34 X 10^{-27} kg which is at rest. If the collision is elastic, determine (a) the speeds of the particles after collision and (b) the total kinetic energy

12) H&R: A proton (atomic mass 1 amu = 1.67 X 10-27 kg) with a speed of 600 m/s collides elastically with another proton at rest. The projectile proton is scattered 60o from its initial direction. (a) Determine the direction of the velocity of the target proton after the collision and (b) the speeds of the two protons after collision.

13) Two objects, each of mass 2kg traveling at constant speeds of 20m/s and 15m/s, respectively make a head-on collision and coalesce together, moving on in the original direction of the faster object. Determine (a) the speed with which they move off
(b) whether the collision was elastic or inelastic

14) (a) A constant force F = (15,10,1)N moves its point of application through a displacement (2,0,-1) cm. Determine the work done by the force
(b) A force F=(3x,0,0) N moves its point of application along the x-direction from the point x=20 cm to the point x=-20cm. Determine the work done by the force

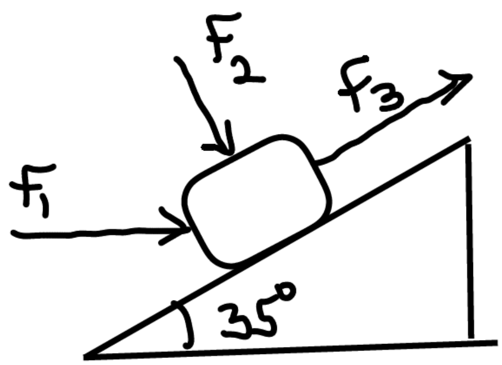
15) A 1.0 kg box is launched horizontally by a compressed spring on a frictionless surface at a speed of 10 m/s. If the spring constant is 1200 N/m, by how much was the spring compressed in order to cause the launch?

16) A 1500 kg car can accelerate uniformly from rest to a speed of 20m/s in a time of 8s. Determine the instantaneous power expended and the average power expended.

17) A 6g bullet moving at a speed of 100m/s strikes a block of wood. Assuming that the bullet decelerates uniformly in the wood and stops within a distance of 5cm, determine (a) the time taken for the bullet to stop (b) the impulse on the wood (c) the average force experienced by the wood

18) You have a 12V battery which happens to have stored 8640 kJ of energy. For how long can the battery be used to power a device whose average power rating is 460 W?

19) A block on an inclined plane is acted upon by three forces of equal magnitudes (20N) as shown below. F_1 is along the horizontal, F_2 is perpendicular to the inclined plane and F_3 is along the inclined plane as shown. If the block slides a distance 50 cm upward along the plane, determine the work done by each of the forces.

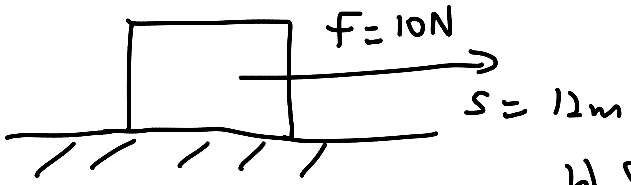


20) An object of mass 20kg was dropped from a height of 400m. Upon reaching the ground, its speed was found to be 80.5 m/s. Determine (a) its potential energy at the top just before it was dropped (b) its kinetic energy just before hitting the ground.
Why are the values in (a) and (b) above different?

21) Work all the questions in Chapter 7 of Prof. Farai's book

HW4 Solns: 1–5 & 10, 16.

#1.



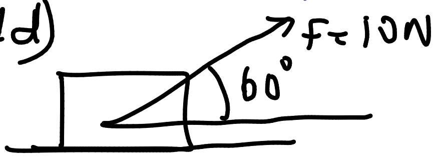
(a) Work done by force is F × (distance moved in direction of force)
$= 10N \times 12m = \underline{\underline{120J}}$

b) Same result irrespective of mass of box: 120J

c) Same result for work done by force F (of 10N): $\underline{120J}$.
This is different from the net work done

[The net work done, which the question is <u>NOT</u> asking for is
Net force • Net displacement $= (10-5)N \times 12m \times \cos 0° = 60J$]
vector   dot   vector

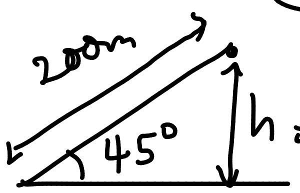
#d)



Work done $= \vec{F} \cdot \vec{D} = FD\cos\theta = 10N \times 12m \times \underbrace{\cos 60°}_{=0.5} = \underline{\underline{60J}}$

#2) $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{25 \times 10^3 m}{60 \times 60 s}\right)^2 = 24.11\,m$ Joules
(m in kg = mass of car)
Convert 25km/h to m/s

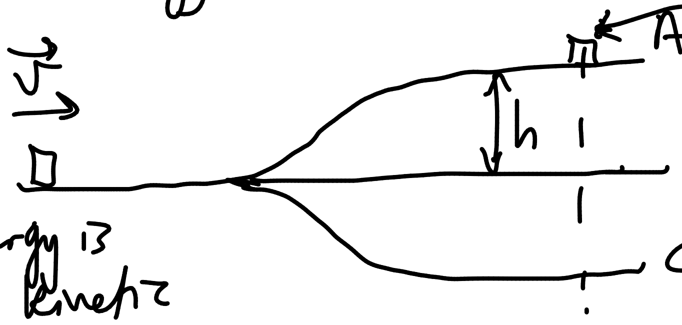
b) PE = mgh



$h = 200\sin 45°$ ⇒ $PE = m \times 9.8m/s^2 \times 200\sin 45°$
$= 1385.93\,m$ Joules

c) Total energy = KE + PE = 24.11m + 1385.93m = 1410.04m Joules
(m in kg = mass of car)

#3



A ← $E_{Total} = KE + PE = KE + mgh$
B ← $E_{Total} = KE + 0 = KE$
C ← $E_{Total} = KE + PE = KE - mgh$

Total Energy is purely kinetic at start.

- The total Energy is constant. In (A), the block rises, hence gains Potential energy but loses KE since total energy is constant. Thus, speed decreases (since KE decreases).
- For B, No change in KE nor PE ⇒ speed is constant
- For C, the block falls, hence, loses Potential Energy (PE) which is gained by the Kinetic Energy (KE) since KE+PE has to remain the same. KE increases ⇒ speed increases

Thus, at the finish line, $v_C > v_B > v_A$

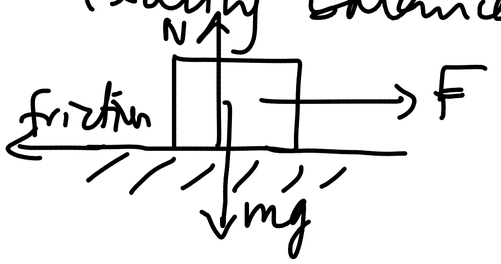
b) What of travel time to finish? B takes a shorter time to the finish line than A since (i) $v_B > v_A$ and (ii) distance travelled by A is larger than distance travelled by B.

$\therefore \; T_B < T_A$.

What of T_C? Clearly, $T_C < T_A$ since $v_C > v_A$ and the distances travelled by A & C are the same. However, it is hard to say whether $T_B > T_A$ or $T_A > T_B$. It depends on the height "h" of the incline.

Conclusion: $\underline{T_B < T_A}$ & $\underline{T_C < T_A}$ but $T_A \gtrless T_B$?

4) Constant speed in same direction ⇒ Balanced forces ⇒ F is exactly balanced by the frictional force.



frictional force $= \mu mg$ (actually, μN but $N - mg = 0$ where $N =$ normal force on block)

& $F - \text{friction} = 0$ ← since no acceleration

$\therefore F = \text{friction} = \mu mg$.

$\therefore$ Work done $= F \times s \cos 0° = Fs = \mu mg \times s = 0.2 \times 10 \times 9.8 \times 12 \text{ J}$
$= 235.2 \text{ J}$.

5) $d\text{Work} = \vec{F} \cdot d\vec{s} \Rightarrow \dfrac{dW}{dt} = \vec{F} \cdot \dfrac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v} = Fv = (3t^2 - 2t)\,3$

velocity; $v = 3 \text{ m/s}$

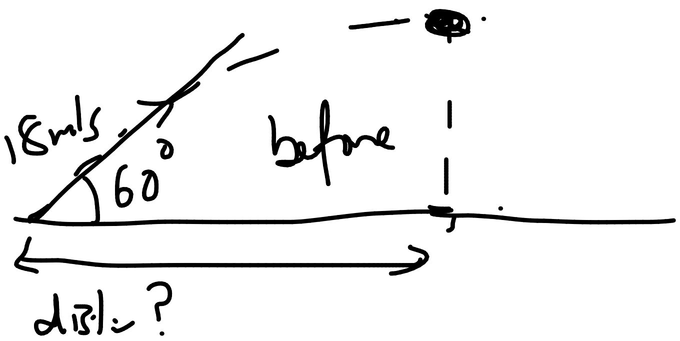
$= (9t^2 - 6t)$ Joules (t is in seconds)

b) $dW = \vec{F} \cdot d\vec{s} = F\,ds = F\,v\,dt = (3t^2 - 2t)\,3\,dt \Rightarrow dW = (9t^2 - 6t)\,dt$

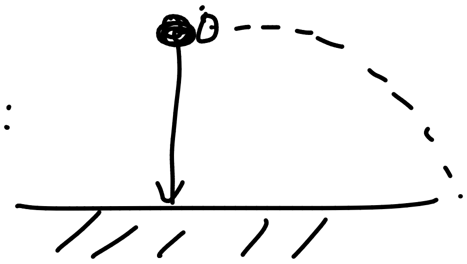
$v = ds/dt \Rightarrow ds = v\,dt$

$\Rightarrow W = \int_0^5 (9t^2 - 6t)\,dt = (3t^3 - 3t^2)\Big|_0^5 = [3(5)^3 - 3(5^2)] - [3(0)^3 - 3(0)^2] = \underline{300 \text{ J}}$

#10:



After:



time taken to top = ?

@ top, $v_y = 0$ ∴ $v = u + at$ gives:

$$v_y = u_y - 9.8t$$

$$\Rightarrow 0 = 18\sin 60° - 9.8t \Rightarrow t = \frac{18\sin 60°}{9.8}$$

∴ Distance moved (along x) $= u\cos 60° \times t$

$$= 18\cos 60° \times \frac{18\sin 60°}{9.8}$$

$$= 14.32\text{m}.$$

After explosion, first mass drops vertically downwards. This means it has no horizontal speed. (It also has no vertical speed. Why?)

horizontal momentum before explosion is:

$$m \times v_x = m\,18\cos 60°$$

Horizontal momentum after collision =

$$\underbrace{\frac{m}{2} \times 0}_{\text{for 1st part}} + \underbrace{\frac{m}{2} v}_{\text{for 2nd part}}$$

v = horizontal momentum after collision

Conservation of horizontal momentum means: $m\,18\cos 60° = \frac{m}{2} 0 + \frac{m}{2} v \Rightarrow v = 2 \times 18\cos 60°$

Next, we need to find how long it will take the object to reach the ground. We can show that it will take $t = \frac{18\sin 60°}{9.8}$ seconds (the same time it took to get up).

∴ horizontal distance covered from point of explosion is $v \times t$

$$= (2 \times 18\cos 60°) \times \frac{18\sin 60°}{9.8} = \underline{28.63\text{m}}.$$

We add this to 14.32m to get: $14.32 + 28.63\text{m} = \underline{42.95\text{m}}$ from point of release.

#16 $dW = F\,ds \Rightarrow P = \frac{dW}{dt} = F\frac{ds}{dt} \overset{\text{since } F = ma}{=} ma\frac{ds}{dt} = ma(at) = \underline{\underline{ma^2t}}$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow \frac{ds}{dt} = u + at = at$ (since $u = 0$ because it started from rest)

∴ Power (@ $t = 8$s) $= ma^2 \times 8$ seconds

We need to determine a: Use $v = u + at$

v = 20m/s, u = 0m/s (from rest), t = 8 seconds

$\Rightarrow a = \frac{v-u}{t} = \frac{20-0}{8}$ m/s^2

$= 2.5 m/s^2$.

* $\therefore$ Power (@ t=8) $= ma^2t = 1500kg \times (2.5 m/s^2)^2 \times 8s = 75,000J$.

= instantaneous power

Work done $= \int P\,dt = \int ma^2t\,dt = \frac{ma^2t^2}{2}$

* Average Power $= \frac{\text{Work done}}{\text{time taken}} = \frac{\frac{1}{2}ma^2t^2}{t} = \frac{1}{2}ma^2t$

$= \frac{1}{2} \times 1500 \times (2.5 m/s^2)^2 \times 8s = \underline{\underline{37,500J}}$

= average power expended

6 Hint: Transformation of mechanical energy

$P.E = K.E$

Since, $P.E = mgh$ and $K.E = \frac{1}{2}mv^2$

Then, $v = \sqrt{2gh}$

Let V_B and V_c be the velocities ~~of~~ at points B and C respectively.

So, $V_B = \sqrt{2gh.}$ $\{g = 9.8 m/s^2; h = 10m\}$

$V_B = \sqrt{(2 \times 9.8 \times 10)}$

$= \underline{14 m/s}$

At B, the ~~total~~ mechanical energy (P.E and K.E)

is: $mgh_B + \frac{1}{2}mv_B^2$

From, the transformation of mechanical energy

$\frac{1}{2}mv_c^2 = mgh_B + \frac{1}{2}mv_B^2$

$\Rightarrow V_c^2 = V_B^2 + 2gh_B$

$V_c = \sqrt{V_B^2 + 2gh_B}$

$V_c = \sqrt{(14)^2 + (2 \times 9.8 \times 5)}$

$V_c = \underline{17.1 m/s}$

7. Hint: Work done = area of the segment ($Nm \equiv$ Joules)

a. Segment A (0 – 2cm)

Work done, W_A = area of segment A (Right-angled Δ)

$= \frac{1}{2} \times 6 \times \left(\frac{2}{100}\right)$

$= \underline{0.06 J}$

Segment B (2-3cm)

Workdone, W_B = area of segment B (Right-angled Δ)

$$= \frac{1}{2} \times -3(N) \times \frac{1}{100}(m)$$

$$= \underline{\underline{-0.015\,J}}$$

Segment C (3-5cm)

Work done, W_c = area of segment C (rectangle)

$$= 4(N) \times \frac{2}{100}(m)$$

$$= \underline{\underline{0.08J}}$$

b. Total work done, W is given by:

$$W = W_A + W_B + W_c$$

$$= 0.06J + (-0.015J) + 0.08J$$

$$= \underline{\underline{0.125}}\ \text{Joules}$$

8. Gravitational P.E, $U = mV$

$$= -\frac{GMm}{r} \quad \left(V = -\frac{GM}{r}\right)$$

<u>where</u>: r – distance from the earth surface

$r = 2km = 2 \times 10^3 m$

M – mass of earth, $= 5.97 \times 10^{24} kg$

$G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

m – mass of object (in this case, a satellite)

So,

$$GP.E = -\frac{6.67 \times 10^{-11} (Nm^2 kg^{-2}) \times 5.97 \times 10^{24} (kg) \times m}{2 \times 10^3 (m)}$$

$$= -1.99 \times 10^{11} m \text{ Joules}$$

For the kinetic energy (k.E), we use:

$k.E = \frac{1}{2} mv^2$

Since $u = 10 km/s = 10^4 m/s$,

then $v^2 = u^2 - 2gh$ $\{h = r = 2km = 2 \times 10^3 m\}$

2km above the earth, $g \neq 9.8 m/s^2$.

For simplicity, $g \approx 9.8 m/s^2$

Then,

$$v = \sqrt{u^2 - 2gh}$$

$$= \sqrt{(10^4)^2 - (2 \times 9.8 \times 2 \times 10^3)}$$

$$= \sqrt{10^8 - 39200}$$

$$v = 9998.04 m/s$$

So, $k.E = \frac{1}{2} \times m \times (9998.04)^2$

$$= 49980400 m \text{ Joules}$$

Hence, the ~~total~~ mechanical energy is given by:

Mechanical energy $= GP.E + k.E$

$$= (-1.99 \times 10^{11} m) + 49980400 m$$

$$\approx -1.99 \times 10^{11} m \text{ Joules}$$

(m - mass of satellite)

(9) $m = 120kg$, $h = 10m$ $t = 3mins = 3 \times 60s = 180s$

The Work done is the Energy ~~expended~~ used to raise 120kg to a height of 10m

$$\text{Power} = \frac{\text{work done}}{\text{time taken}} = \frac{Mgh}{\text{time(s)}} = \frac{120 \times 9.8 \times 10}{180} = 65.33\ J/s$$

(11) $M_N = 1.67 \times 10^{-27} kg$ $V_{Ni} = 8.0 \times 10^3 m/s$

$M_D = 3.34 \times 10^{-27} kg$ $V_{Di} = 0$

For Elastic Collision, Momentum and Kinetic Energy are conserved

$M_N V_{Ni} + M_D V_{Di} = M_N V_{Nf} + M_D V_{Df}$

$\Rightarrow M_N V_{Ni} = M_N V_{Nf} + M_D V_{Df}$ $(V_{Di} = 0)$ — (1)

Similarly $\frac{1}{2} M_N V_{Ni}^2 = \frac{1}{2} M_N V_{Nf}^2 + \frac{1}{2} M_D V_{Df}^2$ — (2)

From (2)

$M_N V_{Ni}^2 = M_N V_{Nf}^2 + M_D V_{Df}^2$

$M_N V_{Ni}^2 - M_N V_{Nf}^2 = M_D V_{Df}^2$

$M_N (V_{Ni}^2 - V_{Nf}^2) = M_D V_{Df}^2$

$M_N [(V_{Ni} - V_{Nf})(V_{Ni} + V_{Nf})] = M_D V_{Df}^2$ ——— (III)

From (1)

$M_N V_{Ni} - M_N V_{Nf} = M_D V_{Df}$

$M_N (V_{Ni} - V_{Nf}) = M_D V_{Df}$

(III) becomes $\cancel{M_D V_{Df}} (V_{Ni} + V_{Nf}) = \cancel{M_D} V_{Df}^{\cancel{2}}$

$V_{Ni} + V_{Nf} = V_{Df}$

$V_{Ni} + V_{Nf} = V_{Df}$

$V_{Ni} = V_{Df} - V_{Nf}$ —— (iv)

Solving (iv) & (1)

$M_N V_{Ni} = M_N V_{Nf} + M_D V_{Df}$

$V_{Ni} = V_{Df} - V_{Nf}$

$V_{Df} = V_{Ni} + V_{Nf}$

$M_N V_{Ni} = M_N V_{Nf} + M_D (V_{Ni} + V_{Nf})$

$M_N V_{Ni} - M_D V_{Ni} = M_N V_{Nf} + M_D V_{Nf}$

$\therefore V_{Nf} = \dfrac{M_N V_{Ni} - M_D V_{Ni}}{M_N + M_D} = \dfrac{V_{Ni}(M_N - M_D)}{M_N + M_D}$

$V_{Nf} = 8\times10^{3} \dfrac{(1.67\times10^{-27} - 3.34\times10^{-27})}{1.67\times10^{-27} + 3.34\times10^{-27}}$

$V_{Nf} = 8\times10^{3} \dfrac{(1.67 - 3.34)}{1.67 + 3.34}$

$V_{Nf} = -2.67\times10^{3}\, m/s$

$V_{Df} = V_{Ni} + V_{Nf} = 8\times10^{3} - 2.67\times10^{3} = 5.3\times10^{3}\, m/s$

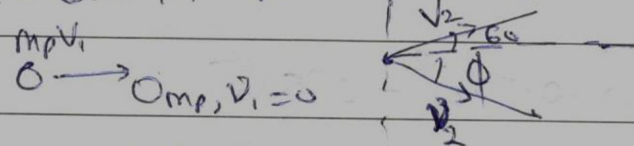
$\therefore$ The velocities (final velocities) are $V_{Nf} = -2.67\times10^{3}\, m/s$ and $V_{Df} = 5.3\times10^{3}\, m/s$.

(b) Total K.E $= \frac{1}{2} M_n V_n^2 = \frac{1}{2}(1.67\times10^{-27})(8\times10^{3})^2 = \underline{\underline{5.34\times10^{-20}\,J}}$.

(12) θ and ϕ be the two angles after collision

$\theta = 60^\circ$ and $\phi = ?$, $m_1 = m_2 = m_p$ = mass of proton

$V_1 = 600\,m/s$, $m_p = 1.67\times10^{-27}\,kg$



Let V = velocity of first proton, v = velocity of second proton (initially at rest)

Horizontal Resolution

$m_pV_1 = m_pV_2\cos60 + m_pv_2\cos\phi$

$\Rightarrow V_1 = V_2\cos60 + v_2\cos\phi$ — (i)

Vertical resolution; $0 = m_pV_2\sin60 - m_pv_2\sin\phi$

$\Rightarrow 0 = V_2\sin60 - v_2\sin\phi$ — (ii)

From conservation of Kinetic Energy $V_1^2 = V_2^2 + v_2^2$ — (iii)

From (1) $600 = V_2(0.5) + v_2\cos\phi$

From (ii) $0 = 0.867V_2 - v_2\sin\phi$

From (iii) $600^2 = V_2^2 + v_2^2$

Taking (1) & (ii) squaring & adding

$$V_2^2 \cos^2\phi = (600 - 0.5V_2)^2$$

$$V_2^2 \sin^2\phi = 0.75 V_2^2$$

$$V_2^2(\cos^2\phi + \sin^2\phi) = (600 - 0.5V_2)^2 + 0.75 V_2^2$$

$$\cos^2\phi + \sin^2\phi = 1$$

$$V_2^2 = 600^2 - 600V_2 + 0.25V_2^2 + 0.75V_2^2$$

$$V_2^2 = 600^2 - 600V_2 + V_2^2$$

From (iii) $600^2 = V_2^2 + V_2^2$

$$\cancel{V_2^2} = V_2^2 + \cancel{V_2^2} - 600V_2 + V_2^2$$

$$0 = 2V_2^2 - 600V_2$$

$$0 = 2V_2(V_2 - 300) \quad \text{Since } V_2 \neq 0$$

$$\Rightarrow V_2 - 300 = 0$$

$$V_2 = 300\,m/s$$

from (4) $0 = V_2 \sin 60 - V_2 \sin\phi$

$$0 = 300(0.867) - V_2 \sin\phi$$

from (iii) $V_2^2 = V_1^2 - V_2^2$

$$V_2^2 = (600)^2 - (300)^2$$

$$V_2 = \sqrt{270000} = 519.6\,m/s$$

$$\sin\phi = \frac{300(0.867)}{519.6} = 0.50056$$

$\phi = \sin^{-1}(0.50056) = 30.037 \simeq \underline{\underline{30^\circ}}$.

The direction of the velocity of the target proton after collision is 30° away from the initial direction of the projectile proton.

(13) $m_1 = 2kg$, $m_2 = 2kg$

$V_{1i} = 20m/s$ $\quad V_{2i} = 15m/s$

$$m_1V_{1i} + m_2V_{2i} = (m_1+m_2)V_f$$

$$2\times20 + 2\times15 = (2+2)V_f$$

$$40 + 30 = 4V_f$$

$$V_f = 70/4 = \underline{17.5 m/s}$$

(b) Inelastic Collision

(14) $F = (15, 10, 1)N$, displacement $x = (2, 0, -1)$ cm

re-writing $r = \frac{1}{100}(2, 0, -1)$ m

work done $W = F\cdot r = (15, 10, 1)\cdot(2, 0, -1)\cdot\frac{1}{100}$

$$= (30 + 0 + (-1))\frac{1}{100}$$

$$W = \underline{\underline{0.29 J}}$$

(b) $F = (3x, 0, 0)N$ moves along x-direction

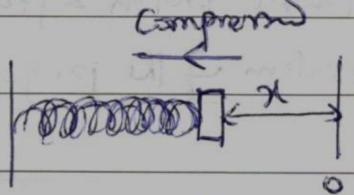
$$W = \int F\,dr \Rightarrow \int_{20}^{-20} F\,dx.$$

$$W = \int_{20}^{-20} 3x\,dx = \frac{3}{2}x^2\Big|_{20}^{-20} = \frac{3}{2}\left[(-20)^2 - (20)^2\right]$$

$$\frac{3}{2}[0] = 0$$

the work done by the force is zero (0)

15



The potential energy stored in the spring due to the compression was used to launch the 1Kg mass

$$\therefore \tfrac{1}{2}Kx^2 = \tfrac{1}{2}mv^2$$

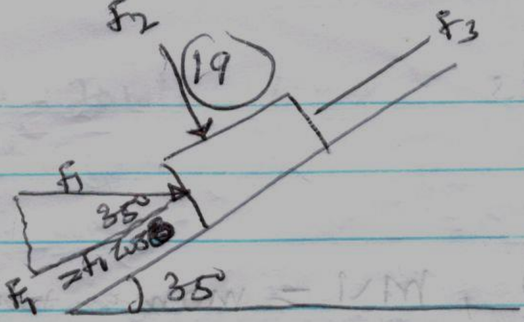
$$Kx^2 = mv^2$$

$$x^2 = \frac{mv^2}{K}$$

$$x = \sqrt{\frac{1 \times 10^2}{12000}} = 0.0913m$$

$\therefore$ The spring was compressed by 0.0913m.

Q
(19)



The component of F_1 along the direction of the displacement

$F_1 = 20 \cos 35 = 20 \times 0.8192 = 16.384N$

Work done by "F_1" $= F_1 \times S$

$W = 16.384 \times 0.5 = 8.192J$

Work done by F_2

$W_2 = F_2 \times 0 = 0J$

Work done by F_3

$W_3 = F_3 \times S = 20 \times 0.5 = 10J$

Q
(17) $u = 100m/s$, $v = 0m/s$, $S = 5cm = 0.05m$

(17) (a) from the third equation of motion

$\cancel{V}^2 = \cancel{U}^2 + 2as$

$0 = 100^2 + 2 \times a \times 0.05$

$0.1a = -100^2$

$a = -\frac{10000}{0.1} = -100000 m/s^2$ - decceleration

from the first equation of motion

$V = u + at$

$0 = 100 - 100000t$

$100000t = 100$

$t = \frac{100}{100000} = 0.001s$

or

Recall that average distance $S = \frac{1}{2} vt$, $S = 0.05m$

$v = 100m/s$

$0.05 = \frac{100t}{2}$

$0.05 = 50t$

$t = \frac{0.05}{50} = 0.001s$

(b)

Impulse $= \Delta(mv)$, mv = momentum

$= m\Delta v = m[v - u]$

Impulse $= \frac{6}{1000} \times 100 = 0.6$ N-s

(c) Impulse $= ft$

$0.6 = f \times 0.001$

$f = \frac{0.6}{0.001} = \underline{\underline{600N}}$

Question (16)

~~Work done = Change~~ [redacted]

Work done = Change in kinetic energy

$W.D = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

v = final velocity u = Initial velocity

Because the car starts from ~~rest~~ rest, $u = 0m/s$

$W.D = \frac{1}{2} \times 1500 \times 20^2 - \frac{1}{2} \times 1500 \times 0^2$ $v = 20m/s$

$W.D = \frac{600000}{2} = 300,000J$

Work = Energy, $E = 300,000$

Recall that

$E = pt$ p – power, t = time, E – energy

Instantaneous ~~Average~~ power $= \frac{E}{t} = \frac{300,000}{8} = 37500W$

[redacted]

power = force × velocity $= fv$

$f = m \times a$

To calculate acceleration, we need to apply first equation of motion

$u = 0$, $v = 20m/s$, $t = 8$

$v = u + at$

$20 = 0 + a \times 8$

$a = \frac{20}{8} = 2.5 m/s^2$

$f = 1500 \times 2.5 = 3750 N$

$p = 3750 \times 20 = 75000 W$

Average power $= 75000 W$

(20) (a) potential energy $= mgh$

$= 20 \times 9.8 \times 400 = 78400 J$

(b) Kinetic energy $(K.E) = \frac{1}{2} mv^2$

$= \frac{1}{2} \times 20 \times 80.5^2$

$= 10 \times 6480.25 = 64802.5 J$

The values different because lost in ~~kinetic~~ ~~potential~~ energy is equal to the gain in kinetic energy. potential energy is calculated when ~~the~~ object does not loss energy that will be gain by kinetic energy.

(18) Energy = power × time

Energy $= 8640 kj = 8640 \times 1000 = 8640000$

power $= 460$

$t = \frac{\text{Energy}}{\text{power}} = \frac{8640000}{460} = 18782.61 (s)$

University of Ibadan
Department of Physics
PHY102 2016/17 Session (2017): HW 5 Due Date: N/A
(RIGID BODIES and SIMPLE HARMONIC MOTION)

Take the acceleration due to gravity (g) to be 9.8 m/s^2

References: (1) Halliday and Resnick / (2) Farai /(3) etc.

1) Determine the moments of inertia for the following:

(a) A flat circular disk about an axis through its center but perpendicular to the plane of the disk.

(b) A sphere about its diameter

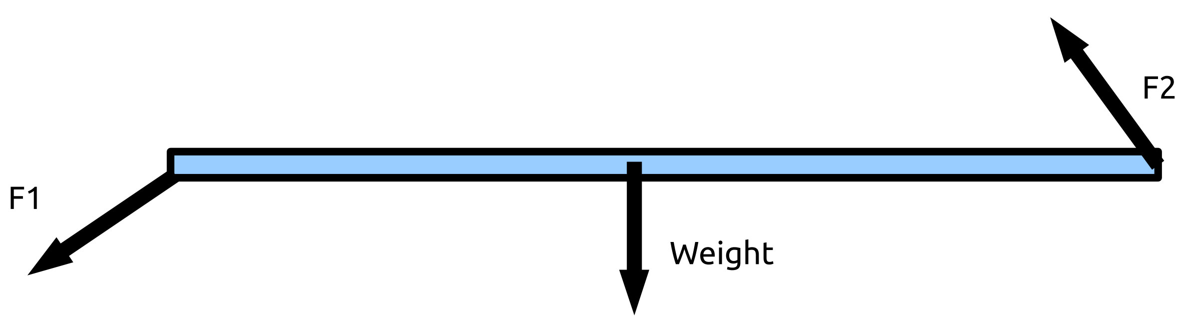
(c) A Slab of length L, breadth B, and height H about an axis through its center parallel its height but perpendicular to its length and breadth.

2) Four points particles are fixed at the corners of a square of length “a” to form a rigid system. If each particle has a mass m, determine the moment of inertia of the rigid system

(a) about an axis through the center of mass of the system and perpendicular to the plane of the masses

(b) about an axis through any of the corners of the square and perpendicular to the plane of the masses

3) Consider the system shown below. Determine its torque (or moments) about the center of the rod. If the rod is uniform with a mass of 0.5 kg and length 0.5m, what is the initial angular acceleration of the rod? F1=60N and makes an angle of 120° with the rod, F2=80N and makes an angle of 40° with the rod as shown below. All forces act in the plane of the paper as shown.



4) A ladder of mass 15kg and length L leans against a smooth (frictionless) wall. If its center of gravity is at a distance 0.7L from the top, determine (a) how much friction there should be between the ground and the ladder for the ladder not to slip (b) the coefficient of static friction

5) A body of mass 0.2kg oscillates at the end of a vertical massless spring of spring constant k = 1000 N/cm
(a) Show whether or not the motion is Simple Harmonic
(b) At what point(s) is its acceleration zero? At what point(s) is its velocity zero?
(c) Can you determine the displacement of the particle at time t?

6) A particle moves such that its position is given by x(t) = B cos(wt +5.0). Show whether or not the motion is Simple Harmonic Motion

7) A circular hoop of mass M and radius R rolls along a plane with a linear speed of 3 m/s. Determine (a) its translational kinetic energy (b) its rotational kinetic energy (c) its total kinetic energy (d) its linear momentum (e) its angular momentum

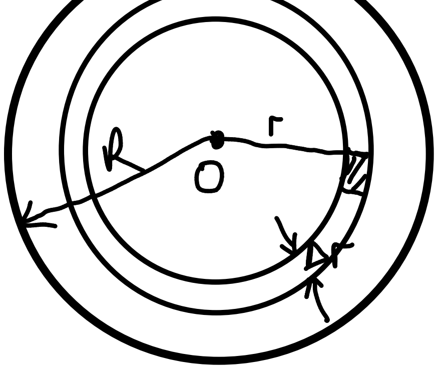
8) An ice skater is spinning with outstretched arms at a rate of 3 radians/s. If she folds her arms so that her moment of inertia is reduced by 20%, a) what is her new spinning rate? b) By what fraction does her kinetic energy change?

9) (H&R) Consider a heavy door of mass 20,000 kg with a moment of inertia of 45000 kg m^2 about an axis through its hinges. The width of the door is 3.2 m. What steady force applied at its outer edge and perpendicular to the plane of the door is needed to move the door from rest through an angle of 60° in 15s? If the force were applied, instead, at the center of the door, what force will be required?

10) (H&R) A diver launched from a board and changed her angular velocity from zero to 5.0 rad/s in 100 milliseconds. If her rotational inertia is 15 kg m^2, determine (a) her angular acceleration, assuming uniform (b) the net torque acting on her during the launch

11) Work all the questions in Chapter 9 of Prof. Farai's book

1a)



$\Delta I = (\delta m) r^2$ = moment of inertia of small part shown shaded

δm ← mass of shaded region

All such small masses in that small ring have the same distance from the origin. If Δr is the width of that small ring as shown, then its area is $2\pi r \Delta r$.

If the disk is uniform, then $\dfrac{M}{\pi R^2} = \dfrac{M_{ring}}{2\pi r \Delta r}$ where M = total mass of disk, R = radius of disk, M_{ring} = Mass of ring shown

$\therefore M_{ring} = \dfrac{M}{\pi R^2} 2\pi r \Delta r.$

Moment of inertia of ring about center $= M_{ring} r^2 = \left(\dfrac{M}{\pi R^2} 2\pi r \Delta r\right) r^2$

So, we sum for many such rings from $r=0$ to $r=R$:

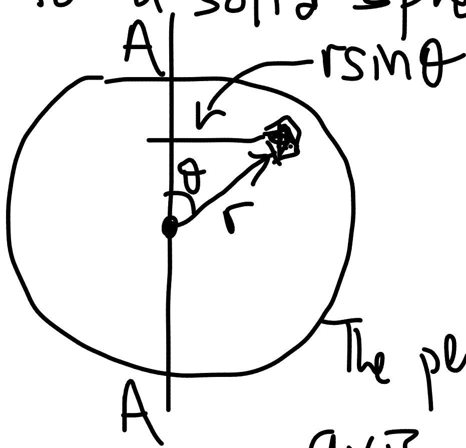
$$I = \underbrace{\int}_{\text{sum}} \left(\frac{M}{\pi R^2} 2\pi r \, dr\right) r^2 = \frac{2M}{R^2} \underbrace{\int_{r=0}^{r=R} r^3 dr}_{=\frac{R^4}{4}} = \frac{2M}{R^2}\frac{R^4}{4}$$

$\Delta r \to dr$.

$$= \frac{MR^2}{2}$$

$\Rightarrow$ Moment of inertia of disk about center, axis perpendicular to plane of disk is $\frac{1}{2}MR^2$.

(b) (ADVANCED – for reference purposes. Don't worry if you cannot derive this. Just know that the final result is not MR^2 but $\frac{2}{5}MR^2$ for a solid sphere.) Consider the solid sphere shown. We take a small piece of mass dm as shown at distance r from the origin & angle θ as shown.



The perpendicular distance of this small mass from the axis AA shown is $r\sin\theta$. Thus, this small mass of mass dm contributes $(dm)(r\sin\theta)^2$ to the moment of inertia $\therefore$ total moment of inertia is the sum (integral) of all such contributions $= \int (dm)(r\sin\theta)^2$.

Now, $dm = \rho\, dV$ where $\rho =$ density of the solid sphere & $dV =$ volume of that small mass. It can be shown that $dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$ where r goes from 0 to R (radius of sphere), θ goes from 0 to π & ϕ goes from 0 to 2π. Then:

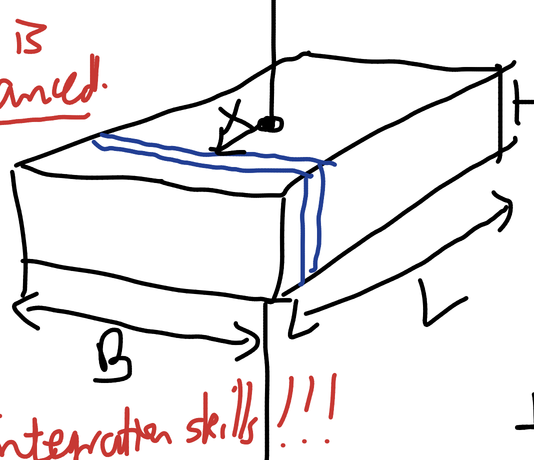
$$I = \int (\rho\, dV)(r\sin\theta)^2 = \int \rho\, r^2 \sin\theta\, dr\, d\theta\, d\phi\, (r\sin\theta)^2$$

$$= \rho \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{R} r^4 \sin^3\theta\, dr\, d\theta\, d\phi$$

$$\overset{\text{The integral gives:}}{=} \rho \frac{R^5}{5} 2\pi \underbrace{\int_0^{\pi} \sin^3\theta\, d\theta}_{=4/3} = \underline{\underline{\rho \frac{R^5}{5} 2\pi\, 4/3}} = I$$

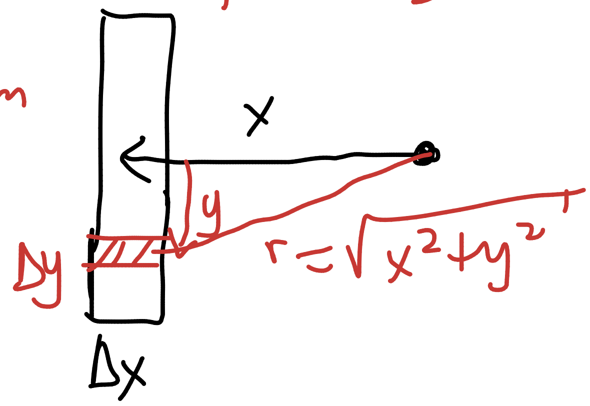
For the uniform solid sphere, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{4/3\,\pi R^3}$. Using this above gives: $I = \left(\frac{M}{4/3\,\pi R^3}\right) \frac{R^5}{5} 2\pi\, 4/3 = \underline{\underline{\frac{2}{5} M R^2}}$

Note: This problem is somewhat advanced. See if you can follow the solution. You need to take a slice of a slice & also, sharpen your integration skills!!!



Take a slice as shown, breadth B, thickness Δx. We let x be the distance from this slice to the axis.

We will determine the moment of inertia of this thin slide about the vertical axis shown. The top view is shown below: We take a slice of this slice (shown in red below). It's distance from the line shown is y and Δy is its width.



Its volume is $(\Delta y \Delta x)H$ so its mass is $\rho \Delta y \Delta x H$ and the distance of this red slice from the origin is $r=\sqrt{x^2+y^2}$ (see figure. Thus, this red slice contributes $(\rho \Delta y \Delta x H)r^2$ to the moment of inertia $\Rightarrow$ the whole black slice contributes (we sum/integrate over many red slices): $\int_{y=-B/2}^{B/2} \rho \, dy \, \Delta x H r^2$

$$= \rho \Delta x H \int_{y=-B/2}^{B/2} (x^2+y^2)\,dy \qquad (\text{recall that } r^2 = x^2+y^2)$$

$$= \rho \Delta x H \left[x^2 y + y^3/3\right]_{y=-B/2}^{B/2} = \rho H (\Delta x)\left[x^2 B + \frac{B^3}{12}\right]$$

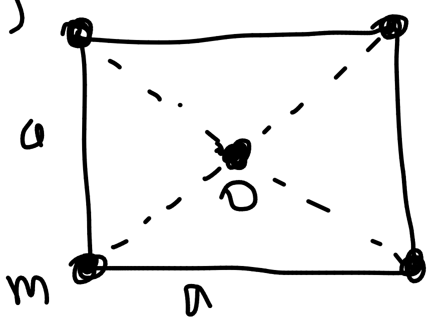
Next, we add up all contributions from all such black slices to get the total moment of inertia: $\int_{x=-L/2}^{L/2} \rho H\left[x^2 B + \frac{B^3}{12}\right] dx$

$$= \rho H \left[\frac{Bx^3}{3} + \frac{B^3}{12}x \right]_{-L/2}^{L/2} = \rho H \left[\frac{BL^3}{12} + \frac{B^3}{12}L \right]$$

Now, $\rho = \frac{M}{\text{volume}} = \frac{M}{LBH}$

$$\Rightarrow I = \frac{MH}{LBH}\left[\frac{BL^3}{12} + \frac{B^3 L}{12} \right] = M\left[\frac{L^2}{12} + \frac{B^2}{12} \right] = \frac{M}{12}\left[L^2 + B^2 \right]$$

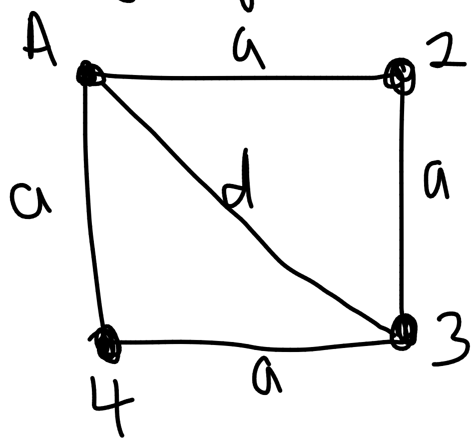
② These are easy: (a)



Let d be the length of the diagonal. Then $d/2$ is the distance from the origin to any of the masses. Contribution from any mass to the moment of inertia about the origin O is $m\left(\frac{d}{2}\right)^2 \Rightarrow$ Contribution from all four masses is: $I = 4m\left(d/2\right)^2 = md^2$.

From Pythagoras, $d^2 = a^2 + a^2 = 2a^2 \Rightarrow I = md^2 = 2ma^2$

(b)



We need to find the moment of Inertia about A: Contribution from mass at A is $m0^2 = 0$

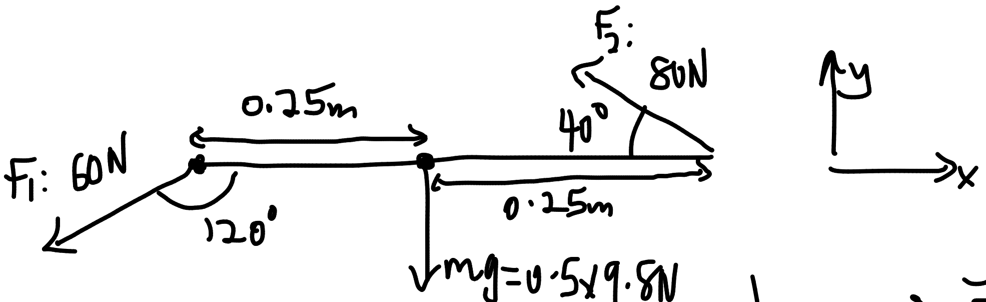
(since its distance from the axis is zero)

Contribution from mass 2 is: ma^2

Contribution from mass 3 is: $md^2 = 2ma^2$

Contribution from mass 4 is: ma^2.

$\therefore$ total contribution is: $0 + ma^2 + 2ma^2 + ma^2 = 4ma^2$



	x	y	$\vec{r}$	$\vec{r} \times \vec{F}$
$\vec{f_1}$:	$-f_1 \cos 60°$	$-f_1 \sin 60°$	$-\hat{x}\, 0.25m$	$\hat{z}\, 0.25 f_1 \sin 60° = 12.99$ Nm
$\vec{f_2}$:	$-f_2 \cos 40°$	$f_2 \sin 40°$	$\hat{x}\, 0.25m$	$\hat{z}\, 0.25 f_2 \sin 40° = 12.86$ Nm
Weight:	0	$mg = 0.5 \times 9.8N$	0m	$= z\, 0 \times mg = 0$

(recall: $\hat{x} \times \hat{x} = 0$
$\hat{x} \times \hat{y} = \hat{z}$)

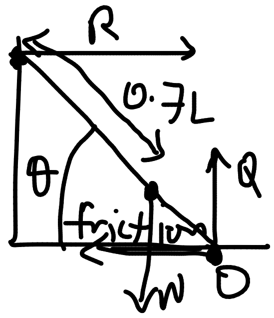
Net torque: $\underline{\underline{\hat{z}\, 25.85 \text{ Nm}}}$

torque ← angular acceleration

$\tau = I\alpha$

Moment of inertia about center is: $\frac{ML^2}{12} = \frac{0.5kg\,(0.5m)^2}{12} = 0.01042\ kgm^2$.

$\therefore \alpha = \tau / I = \frac{25.85\text{ Nm}}{0.01042\ kgm^2} = \underline{\underline{2481.6\ rad\,s^{-2}}}$

4) 

R = reaction force on ladder due to wall. & Q = reaction force on ladder due to floor

forces along x: $R - f_r$ ← friction ——— (1)

forces along y: $Q - W$ ——— (2)

Moments/torque about the point O shown on the floor is: $\underbrace{W(0.3L\cos\theta)}_{\text{clockwise moment}} + \underbrace{(-RL\sin\theta)}_{\text{anticlockwise moments.}}$ ——— (3)

In Equilibrium, Eq (1), (2), & (3) are each zero.

$\therefore R - f_r = 0 \Rightarrow R = f_r$

& $W(0.3L\cos\theta) - RL\sin\theta = 0 \Rightarrow 0.3W\cos\theta - \underset{=f_r}{R}\sin\theta = 0 \Rightarrow f_r = \frac{0.3W\cos\theta}{\sin\theta}$

$f_r = 0.3W\frac{\cos\theta}{\sin\theta}$ is the frictional force required. It depends on the angle θ shown which is not given in the problem.

$= 0.3 \times 15kg \times 9.8 m/s^2 \cot\theta$ $\quad \left(\frac{\cos\theta}{\sin\theta} = \cot\theta = \frac{1}{\tan\theta}\right)$

$= \underline{\underline{44.1\cot\theta}}$ N.

coefficient of static friction

frictional force $= \mu Q \Rightarrow 0.3W\cot\theta = \mu Q$
$= \mu W$ (see Eq 2 @ equilibrium)

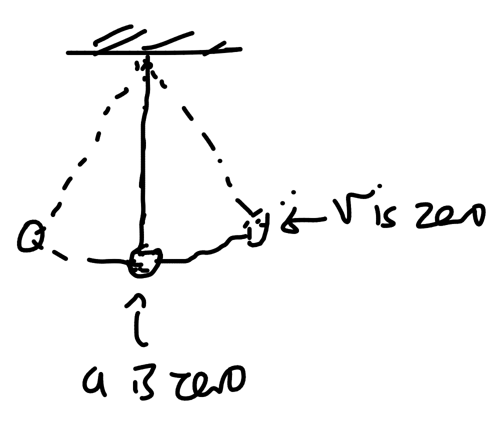
$\Rightarrow \mu = 0.3\cot\theta$

If, for example, $\theta = 45°$, then since $\cos 45° = \sin 45°$, we get: $\cot 45° = 1$
$\Rightarrow f_r = 44.1N$ & $\mu = 0.3$

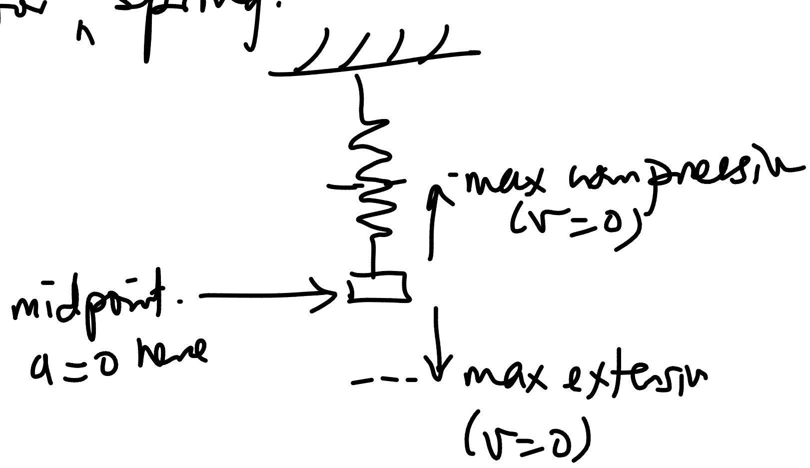
5a) Class Work → see class notes. Yes. It is Simple Harmonic Motion (SHM)

b) For SHM, acceleration (a) $\propto x$ (displacement)
In particular, $a = -w^2 x$. So, a is zero when $x=0$ i.e. when the mass passes through the equilibrium point
(&, obviously, $|a|$ is maximum at the ends, i.e. when x is max.)

For SHM, $v = w\sqrt{A^2 - x^2}$ where A = (maximum) amplitude of oscillation.
Clearly, v is zero when $x = \pm A$ i.e. at the end, when the particle moves to the limits of its displacement, the velocity is zero. For a pendulum, we would have:



for vertical spring:



(c) Can we determine the extension at time t? Ans: NO, unless we are given the maximum amplitude or some more information. The max. amplitude depends, not on the spring but on how much it was pulled or compressed in order to begin/start the oscillations.

6) $x = B\cos(\omega t + 0.5)$

Differentiate with respect to time: $v = \frac{dx}{dt} = -\omega B\sin(\omega t + 0.5)$

Differentiate again: $a = \frac{dv}{dt} = -\omega^2 \underbrace{B\cos(\omega t + 0.5)}_{= x} = -\omega^2 x$

acceleration

$\therefore a = -\omega^2 x$

$\Rightarrow$ (1) $a \propto x$

(2) a is opposite to x (i.e. constant of proportionality is negative)

(1) & (2) $\Rightarrow$ Simple Harmonic Motion (SHM)

7) Hoop, mass $= M$, radius $= R$; (linear) speed $= 3\,m/s$

(a) Translational KE $= \frac{1}{2}Mv^2 = \frac{1}{2}M(3^2) = \frac{9}{2}M$

(b) Rotational KE $= \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 = \frac{1}{2}MR^2(v/R)^2 = \frac{1}{2}Mv^2 = \frac{9}{2}M$

$v = R\omega \Rightarrow \omega = (v/R)$

$I = MR^2$ for hoop about center (see scanned page on website)

Note: In this case, $KE_{trans} = KE_{rot}$. This is not usually the case. This happened in this case because $I = MR^2$. (For the sphere, for example, $I = \frac{2}{5}MR^2$, so, $KE_{trans} \neq KE_{rot}$ for sphere)

(c) Total KE $= KE_{trans} + KE_{rot}$

$= \frac{9}{2}M + \frac{9}{2}M$

$= 9M$

(d) Linear momentum $= Mv = M(3\,m/s) = 3M$

(e) Angular momentum $= I\omega = MR^2\omega = MR^2\left(\frac{v}{R}\right) = MRv = 3MR$

$v = 3\,m/s$

8) $w = 3 rad/s$

$L = Iw$. Assume L is constant. Then,

Angular momentum.

$$L = I_1 w_1 = I_2 w_2.$$

20% less than I_1

$\Rightarrow I_2 = 0.8 I_1$

$$\Rightarrow w_2 = \frac{I_1 w_1}{I_2} = \frac{I_1 w_1}{0.8 I_1} = \frac{w_1}{0.8} = 1.25 w_1 = 1.25 \times 3 rads^{-1}$$

$= \underline{3.75 rad/s}$ is her new spinning rate.

b) Initial $KE = \frac{1}{2} I_1 w_1^2$.

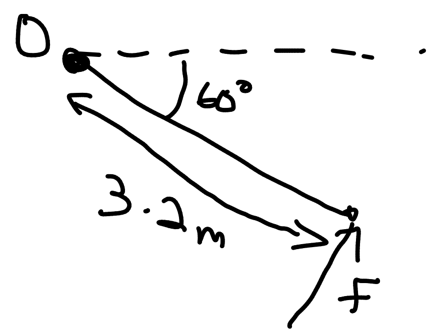
$$\text{final } KE = \frac{1}{2} I_2 w_2^2 = \frac{1}{2}(0.8 I_1)\left(\frac{w_1}{0.8}\right)^2 = \frac{1}{2}\frac{I_1 w_1^2}{0.8} = \frac{\text{Initial } KE}{0.8}$$

since $w_2 = \frac{w_1}{0.8}$ and $I_2 = 0.8 I_1$

$= 1.25 \times \text{Initial } KE$

$\Rightarrow$ KE increases by 25%

9)



60° from rest in 15s.

Just like $s = ut + \frac{1}{2}at^2$,

We have $\theta = w_0 t + \frac{1}{2}\alpha t^2$

$w_0 = 0$ (since from rest)

$$\therefore \frac{2\theta}{t^2} = \alpha = \frac{2\left(\frac{60°}{180°}\pi\right)}{15^2}$$

convert degrees to radians

$= 0.0093 rad/s^2$

Now, $\tau = I\alpha$ & $\tau = Fr\sin\theta = Fr$

since $\vec{r}$ (vector from hinge) is perpendicular to applied force F.

$$\therefore Fr = I\alpha \Rightarrow F = \frac{I\alpha}{r} = \frac{45000 \times 0.0093}{3.2} N = \underline{130.78 N}$$

If the force were applied, instead, half way (1.6m) then:

$$F = \frac{45000 \times 0.0093}{1.6} = 261.56\,N$$, twice as much as before.

10) $I = 15\,kg\,m^2$

$w_0 = 0\,rad/s$; $w_{final} = 5\,rad/s$, $t = 100$ milliseconds $= 100 \times 10^{-3}\,s$

$$\therefore \alpha = \frac{w_{final} - w_0}{\text{time taken}} = \frac{(5-0)\,rad/s}{100 \times 10^{-3}\,s} = \underline{\underline{50.0\,rad/s^2}}$$

$$\tau = I\alpha \Rightarrow \tau = 15\,kgm^2 \times 50\,rad/s^2 = \underline{\underline{750\,Nm}}$$

torque.