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**FEDERAL UNIVERSITY OF TECHNOLOGY OWERRI**  
**SCHOOL OF ENGINEERING AND ENGINEERING TECHNOLOGY**  
**RAIN SEMESTER EXAMINATION 2015/2016 SESSION TIME: 3.00HRS**  
**ENGINEERING MATHEMATICS II. ENG 308**

Instruction: Answer only FOUR questions.

1a. Determine the eigenvalues and eigenvectors for the equation

$$A.X = \lambda X, \quad \text{where } A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix}$$

b. For the set of equations below, use cramer's rule to solve for the value of X

$$X_1 + 3X_2 + 2X_3 = 3$$

$$2X_1 - X_2 - 3X_3 = -8$$

$$5X_1 + 2X_2 + X_3 = 9$$

2a. Calculate the second moment of mass of a hollow shaft whose inner radius is 0.85dm outer radius is 0.125m and density is  $8 \times 10^{-6}$  tonne/cm<sup>3</sup>.

b. A metal door whose dimensions are 4.5dm by 650mm has a mass of 0.0085tonne and is hinged along one 650mm side. Determine the radius of gyration, q, about the 650mm side using the parallel axis theorem.

3a. List any three numerical methods of solving ordinary differential equations and mention one real world application of any one of them. (5marks)

b. Consider the integral  $J = \int_0^1 x^{\frac{1}{2}} \cos x dx$

i. Using any numerical method of your choice, evaluate the integral given that n=10 (10marks)

ii. Determine the error associated with your chosen method in part (i). (Hint: A fairly accurate result may be obtained by using the series method expanding cosx to the fifth term)(10marks)

4a. Convert the second order differential equation to two first order linear differential equations and present it in matrix form;  $y'' + 5y' + 6y = \exp(-3t)$ .

b. Find a solution to the second order differential equation below using the same method in 4a;  $y'' + 6y' + 8y = 0$   $y(0) = 10, y'(0) = 0$

5a. Minimise  $P = 5X + 4Y$

Subject to  $X + 3Y \leq 54$

$$3X + Y \leq 34$$

$$-X + 2Y \geq 12 \quad (X, Y \geq 0) \quad (10marks)$$

b. A company assembling two types of pumps, A and B, is under contract to produce a daily output of at least 35 pumps in all. Assembly and testing times for each type of pump are as follows:

Pump type	Processing time (hours)	
	Assembly	Testing
A	1.0	2.0
B	2.0	1.0

Available staff resources provide a daily maximum of 80 hours for assembly and 55 hours for testing. The profit on the sale of each type A pump is £4.00 and type B pump £5.00. Determine: (i) the daily production schedule for maximum profit.

(ii) the maximum daily profit. (15marks)



2016/2017

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RAIN SEMESTER EXAMINATIONS: COURSE CODE: ENG 308: COURSE TITLE: ENGINEERING MATHEMATICS II; CREDIT LOAD: 3 UNITS

INSTRUCTIONS: ANSWER ALL QUESTIONS; TIME ALLOWED: THREE (3) HRS

1) Solve the equation  $y'' + 2y' - y = 2x^2$ ;  $y(0) = 0$ ,  $y(1) = 1$  for  $h = \frac{1}{4}$  by finite difference method.

2a) List (with the associated formulas) any three numerical methods of evaluating definite integrals.

b) Using the method of *Runge-Kutta*, solve the initial value problem  $y' = -y$  given that  $y(0) = \frac{1}{2}$  for  $x = 0.0(0.1)0.3$ .

Tabulate the results (expressed to 6 places of decimal) and comment on the degree of accuracy of the results.

Hint:  $k_1 = h f(x_n, y_n) = y_n'$ ;  $k_2 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$ ;  $k_3 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$ ;

$k_4 = h f(x_n + h, y_n + k_3)$ ;  $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ .

3a) Find the Fourier integral representation of:  $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & x < 0 \end{cases}$  If this function satisfies the Dirichlet conditions

such that  $f(x)$  is both integrable and absolutely integrable over the interval  $-\infty < x < \infty$ , find the value of the resulting integral when (i)  $x < 0$  (ii)  $x = 0$  and (iii)  $x > 0$ .

b) The following corners:  $(0, 0, 2)$ ;  $(0, 1, 2)$ ;  $(x, 0, 2)$  and  $(x, 1, 2)$  belong to a plane rectangular area function  $P(x)$  where  $x = vt$ ,  $z = ut$ ,  $t$  is the time while  $u$  and  $v$  are constant speeds. Verify the flux transport theorem if  $F = xz k$  and  $k$  is a unit vector in the  $z$ -direction.

4) A company makes three kinds of heat-exchangers, A, B and C.

Process	Time in hours per heat-exchanger		
	A	B	C
Preparation	2	5	4
Assembly	2	3	2
Finishing	5	4	3
Profit in Naira (N) per unit	35	30	24

The producer have 50 men available for Preparation, 40 men for Assembly and 60 men for Finishing, an all staff work a 20 hour week. To remain competitive, at least 300 heat-exchangers in all must be produced each week.

Determine (a) the number of each type needed each week in order to maximize the profit.

(b) the maximum weekly profit.

5) A process is defined by the differential:  $y'''' + 9y''' + 26y'' + 24y' = u(t)$ . Obtain a state space model for the system using the partial fraction/long division model and compute the state transition matrix ( $e^{At}$ ).

$(s+4)(s+2)(s+3)(s+)$   $w^2 = \tan^2 \theta$   $w^2 + 1 = \sec^2 \theta$



FEDERAL UNIVERSITY OF TECHNOLOGY, OWERRI  
 DEPARTMENT OF CHEMICAL ENGINEERING  
 HARMATTAN SEMESTER EXAMINATION  
 ENG 308 (ENGINEERING MATHEMATICS II) 2010/2011 SESSION  
 ATTEMPT ALL QUESTIONS TIME ALLOWED: 3HRS

- 1(a) Solve by direct integration:  $\frac{\partial^2 z}{\partial x \partial y} = xy^2$  (10marks)
- (b) Solve by Lagrange's linear technique:  $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + 2xz = 0$  (10marks)

2(a) Three plants A, B and C produces 100, 120 and 120 tons of a product respectively. The product is to be delivered to Five Warehouses 1, 2, 3, 4 and 5 each of which must get its quota: 40, 50, 70, 90 and 90 respectively. The transportation costs are given in Fig. 1. Obtain the optimal distribution of goods from the plants to the warehouses that minimize the cost of transportation and check if it is unique. (14marks)

(b) Each of the available four lecturers is to be assigned to one of four different courses. Each has indicated the time needed to prepare the lecture materials for the different courses as in Fig. 2. Determine the assignment of lecturers to courses that minimizes the preparation time. (6marks)

3(a) Evaluate  $\int_1^2 \frac{dx}{1+x}$ , by (i) Trapezoidal rule (ii) Simpson's 3/8 rule

(b) Find by Taylor's series method the values of y at x=0.1, 0.2 and 0.3 to five places of decimal from  $\frac{dy}{dx} = x^2 y - 1$ , y(0) = 1

4(a) Given a set of equations in matrix form as shown below, Determine (i) det A = |A|

(ii) Adjoint A<sup>-1</sup> (iii) A<sup>-1</sup> =  $\frac{Adj A}{|A|}$ . Hence find the values of X<sub>1</sub>, X<sub>2</sub> and X<sub>3</sub>. [X = A<sup>-1</sup>.b]

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 1 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 11 \\ 3 \end{bmatrix}$$

that is [A]X = [b] (12marks)

PLANT	WAREHOUSES				
	1	2	3	4	5
A	4	1	2	6	9
B	6	4	3	5	7
C	5	2	6	4	8

FIG. 1

(b) A marketer makes \$10 and \$15 profit respectively per litre of Petrol and Diesel sold, but has two constraints: Petrol costs \$50/litre and Diesel \$100/litre and the marketer has only \$100,000 capital. In addition the total capacity of his reservoirs for Petrol and Diesel is 1200 litres. Using X and Y as litres of Petrol and Diesel respectively, Formulate the Objective function and constraint equations (8marks)

5(a) The Simplex Tableau below was developed by a student, with W, Y and Z as slack variables:

Basis	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	W	Y	Z	b
W	4	3	0	2	1	0	0	8
Y	8	1	0	0	0	1	0	6
Z	0	1	8/3	2	0	0	1	6
P	-4	-2	-1/3	-1	0	0	0	0

LECTURERS	COURSES			
	1	2	3	4
1	26	41	34	37
2	30	38	37	26
3	32	29	32	30
4	29	30	37	34

FIG. 2

Using the Simplex method, find the maximum value of P and the corresponding values of the variables X<sub>1</sub>, X<sub>2</sub>, X<sub>3</sub>, X<sub>4</sub>, W, Y and Z.

(b) Obtain the Integral in Q(3a) above using Monte Carlo simulation of 12 intervals to 4d.p.



INSTRUCTION: ANSWER ANY 5 QUESTIONS

TIME ALLOWED: 3HRS.

1. (a) Utopian system has an energy balance model of the form  $\frac{\partial^2 u}{\partial x^2} + \frac{q}{R} = \frac{1}{\alpha} \frac{\partial u}{\partial t}$   
 (i) Obtain a model herefrom for steady state no heat source. (ii) Solve (i)  
 (b) Solve  $\frac{\partial^2 u}{\partial x \partial y} = 4e^y \cos 2x$  given that at  $y = 0, \frac{\partial u}{\partial x} = \cos x$  and at  $x = \pi, u = y^2$

2. (a) Evaluate  $J = \int_0^1 x^{\frac{1}{3}} e^{-2x} dx$  using rectangular rule given that  $n = 10$ .  
 Determine the error. (Hint: a fairly accurate result may be obtained by series method expanding  $e^{-2x}$  to the seventh term).

- (b) Using the method of Euler solve  $y' = e^{-x} \sqrt{xy}$  for  $x = 0.0(0.1)0.5$  given that  $y(0) = 1$   
 (c) Mention any one area of application for numerical methods.

3. A taxi hire company has one taxi at each of five depots a, b, c, d, and e. A customer requires a taxi in each town, namely A, B, C, D and E. Distances in (km) between depots (origin) and towns (destinations) are given in the following distance matrix.

	a	b	c	d	e
A	140	110	155	170	180
B	115	100	110	140	155
C	120	90	135	150	165
D	30	30	60	60	90
E	35	15	50	60	85

- (a) How should taxis be assigned to customers so as to minimize the distance travelled?  
 (b) Draw a flow chart of the solution sequence.

4. (a) Use Cramer's rule to solve the following system of equations:

$$X_1 + X_2 + X_3 = 2$$

$$2X_1 + X_2 - X_3 = 5$$

$$X_1 + 3X_2 + 2X_3 = 5$$

- (c) Find the eigen values and eigen vectors of the matrix  $[A] = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -3 \\ 1 & -1 & 2 \end{bmatrix}$

5. (a) A firm manufacturing two types of switching module, A & B, is under contract to produce a daily output of at least 35 modules in all. Assembly and testing times for each type of module are as follow

Module type	Processing time (Hours)	
	Assembly	Testing
A	1.0	2.0
B	2.0	1.0

Available staff resources provide a daily maximum of 80 hours for assembling and 55 hours for testing. The profit on the sale of each – A module is \$4.00 and of each B – module \$5.00. Determine

- (i) The daily production schedule for maximum profit. (ii) The maximum daily profit.

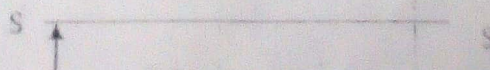
- (b) Minimize  $P = -4x + 3y$

$$\text{Subject to } x + 4y \leq 20$$

$$2x + y \leq 12$$

$$x - y \leq 3$$

6. (a) Calculate the second moment of Area of the shape about the axis "S – S" as shown below with respect to the parallel Axis Theorem.





SIP TYPE

Question one

In a diffusion transport phenomena the following equation is formulated  $\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}(J) + r_1$

where  $r_1$  is first order reaction ( $KC_1$ ) and  $J$  is first Fick's law  $(-D \frac{\partial C}{\partial x})$

- (a) Obtain a 2nd order PDE therefrom.
- (b) Solve the 2nd order PDE using separation of variables technique.

Question Two

(a) Evaluate  $J = \int_0^4 e^{-x} dx$  by means of trapezoidal rule with  $n=20$ . Solve the integral analytically and determine the error

(b) Solve  $y' = x + y$  using Euler's method for  $x = 0.0(0.2) 1.0$  given that  $y(0) = 0$ . Find the exact solution and the error.

Question Three

(a) Using Cramer's rule, solve the following set of linear algebraic equations.

$0.3x_1 + 0.52x_2 + x_3 = -0.01$

$0.5x_1 + x_2 + 1.9x_3 = 0.67$

$0.1x_1 + 0.3x_2 + 0.5x_3 = -0.44$

(b) Determine the eigenvalues and eigenvectors for the equation:

$A \cdot X = \lambda X$ , where  $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{bmatrix}$

Question Four

- (a) Minimize  $P = 4x - 8y + 5z$   
 $2x + 3y + z \leq 70$   
 $x + 2y + 2z \leq 60$   
 Subject to  $3x + 4y + z \leq 84$   
 $x + y + z \geq 33$

(b) A firm manufacturing two types of switching module, A and B, is under contract to produce a daily output of at least 35 modules in all. Assembly and testing times for each type of module are given as follows

Module type	Process time (hours)	
	Assembly	Testing
A	1.0	2.0
B	2.0	1.0

Available staff resources provide a daily maximum of 80 hours for assembly and 55 hours for testing. The profit on the sale of A-module is £4.00 and of each B-module is £5.00. Determine the daily production schedule for maximum profit. (ii) The maximum daily profit.



Question Five

Source	Destination				
	A	B	C	D	
1	6	5	4	3	40
2	4	4	6	7	50
3	7	6	6	5	30

Table 5.0b

Cities

Reservoirs	Cities				
	1	2	3	4	5
1	2	3	4	5	
2	3	2	5	2	
3	4	1	2	3	

Table 5.0a

- (a) We have three reservoirs with daily supplies of 15, 20, 25 million liters of fresh water respectively. On each day we must supply four cities A, B, C, and D whose demands are 8, 10, 12, and 15 respectively. The cost of pumping per million liters is shown in table 5.0a. Use transportation algorithm to determine the cheapest pumping schedule if excess water can be disposed at no cost.
- (b) (i) Determine the optimal solution of the transportation problem in table 5.0b. (ii) Is the solution unique or not? If not, determine the alternative optimal solution.

Question Six

Job	Machines				
	1	2	3	4	5
1	10	11	4	2	8
2	7	11	10	14	12
3	5	6	9	12	14
4	13	15	11	10	7

Table 6.0a

Jobs	Machines		
	1	2	3
1	5	8	4
2	4	3	2
3	8	6	4

Table 6.0b

- (a) A batch of four jobs can be assigned to five different machines. The set-up time for each job in the various machines is given table 6.0a. Find the optimal assignment of jobs to machine which will minimize the total set-up time.
- (b) Three machines are available to execute three jobs. Each job must be done on only one machine. The cost of processing each job on each machine is given in table 6.0b. Determine the minimum cost assignment for each job.



1a. Solve directly the PDE:  $\frac{\partial^2 U}{\partial x \partial y} = a(xy)^n - \sin(x) \cos(y)$  given that at  $y = 0$ ,  $\frac{\partial U}{\partial x} = n$  and at  $x = 0$ ,  $U = \sin(y)$ . (b) Solve the PDE below at steady state:  $\frac{\partial C_A}{\partial t} + u_x \frac{\partial C_A}{\partial x} = D \frac{\partial^2 C_A}{\partial x^2}$

2. Using any numerical method of your choice (a) Evaluate  $J = \int_{-1}^1 2^x e^{-x^4} dx$  given that  $n = 10$ . (b) Solve  $y'' = x - y$  for  $x = 0.0, 0.1, 0.5$  given that  $y(0) = 0$  and  $y'(0) = 1$ . Comment on the result.

3. The University coach is putting together a relay team for a 400-meter relay. Each swimmer must swim 100 meters of breaststroke, backstroke, butterfly or freestyle. The coach believes that each swimmer will attain the times given in the table below. (a) To minimize the team's time for the race, which swimmer should swim which stroke? [15 marks] (b) Determine the worst time available for the team [5 marks].

Swimmer	Time (Seconds)			
	Free	Breast	Fly	Back
Ngozi Okafor	54	54	51	53
Chinedu Onwukwe	51	57	52	52
Chidinma Okorie	50	53	54	56
Ikenna Maduforo	56	54	55	53

4. A Company assembling two brands of cars, A and B, is under contract to produce a daily output of at least 35 cars in all. Assembly and testing time for each type of car are given as:

Car brand	Processing Time (Hours)	
	Assembly	Testing
A	1.0	2.0
B	2.0	1.0

Available staff resources provide a daily maximum of 80 hours for assembly and 55 hours for testing. The profit on the sale of each A-brand car is \$4.00 and of each B-brand car is \$5.00. Determine (a) The Daily production schedule for maximum profit (b) The maximum daily profit.

5a. Using the perpendicular Axis theorem, calculate the second moment area of a circle and the polar second moment if the diameter is  $4\text{cm}^2$ . (b) Proof that the moment of Inertia for a uniform rod of length,  $L$  and thickness,  $t$  rotating about its centre of gravity is given as:  $I_g = \frac{M(L^2 + t^2)}{12}$

6. Two 1000 liter tanks are with salt water. Tank 1 contains 600 liters of water initially containing 30grams of salt dissolved in it and tank 2 contains 1000 liters of water and initially 60 grams of salt dissolved in it. Salt water with concentration of 0.5gram/liter of salt enter tank 1 at rate of 5 liter/hr. Fresh water enter tank 2 at rate of 8 liter/hr. Water flow from tank 2 into tank 1 through a connecting pipe at rate of 10 liter/hr. Through a different connecting pipe 15 liter/hr flows out of tank 1, 12 liter/hr are drained out of the system completely and only 3 liters/hr flow back into tank 2. (a) Set up system of linear DE's that will give the amount of salt in each tank at any given time (b) Solve the system of linear differential equations obtained in (a) above (Assume tank volume as constant).